The spectroscopy of light mesons

A. T. Filippov

Joint Institute for Nuclear Research, Dubna
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A review is given of the most recent experimental data on light meson resonances together with a phenomenological analysis incorporating all the basic ideas of QCD. The following topics are discussed in greater detail: (1) the scalar meson problem; (2) the axial meson problem; (3) spin-spin and spin-orbit splitting in meson multiplets; (4) SU_3^f and OZI (Okubo-Zweig-Iizuka) breaking in meson spectra and decays; (5) SU_3^f breaking in Regge trajectories and in radial excitations; (6) quark mixing in isoscalar mesons ($\eta-\eta'$, $\omega-\varphi$, f–f′); (7) experimental determinations of the $\eta-\eta'$ mixing angle, and (8) radiative decays of vector and pseudoscalar mesons.

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CONTENTS

1. INTRODUCTION

This review is devoted to the spectroscopy of light mesons which consist of light quarks (u, d, s) and gluons (g). Although quarks were invented specifically to explain the shape of the spectrum of light hadrons (mesons and baryons), they are now studied mainly in reactions at high energies and momentum transfers ("hard" processes), especially in the scattering of electrons and neutrinos by hadrons, in electron-positron annihilation, and so on. The structure of hadrons is most clearly seen in these reactions and the success of the quark model is particularly striking (the magnitude of R in e^+e^- annihilation, quark and gluon jets, and so on). One of the most outstanding achievements of the quark model is also the prediction and description of the properties of the family of mesons consisting of heavy quarks c and b. All these successes have stimulated the rapid development of quantum chromodynamics (QCD) which was initially invented to explain the paradoxes that arose in the quark spectroscopy of light hadrons. The application of QCD to processes with high-momentum transfer or with participation of heavy quarks has turned out to be fruitful because it is often possible to use standard perturbation theory developed in quantum electrodynamics (QED). Strictly speaking, this "perturbative" QCD (or PQCD) is valid only for elementary events involving the hard interaction between quarks and gluons in which they acquire or give up high momenta. Moreover, the asymptotic freedom of QCD ensures that the effective coupling constant between the quarks and gluons is sufficiently small in the limiting case, and higher-order corrections can be neglected. Although for the currently available energies the effective coupling constant is not as yet very small, and the corrections are large in most cases, this difficulty can, in principle, be obviated by using one of the numerous methods available for the summation of perturbation-theory diagrams that were developed in QED. The more difficult problem is that, in the initial and final states, one observes not quarks and gluons but real hadrons (light mesons and baryons), and PQCD is fundamentally unsuitable for the description of these particles, not only because of the increase in the effective coupling constant at low-momentum transfers but, mainly, because of the presence of nonperturbative effects that do not appear in standard perturbation theory. These effects appear to be connected with the complicated structure of vacuum in QCD, and the problem is to construct a perturbation theory that would take into account vacuum and other nonperturbative effects even in the zero-order approximation. This type of theory, or even a realistic model, has not as yet been developed although the nonperturbative effects have been isolated and estimated more or less reliably in some cases. These studies have shown that light-meson spectroscopy is one of the main sources of information on nonperturbative effects in QCD. This fact alone has determined the importance of experimental studies and theoretical analyses of this problem which is one of the most important in high-energy physics. Apart from this, studies of the spectrum and decays of light mesons enable us to elucidate other effects that are important for the development of a rigorous theory, namely, violation of chiral and SU(3) symmetry, the mixing of quark configurations, spin splitting, and so on. Finally, the discovery of multiquark and, especially, pure gluon states would be of fundamental importance. We emphasize that the observation of pure gluon resonances would be, essentially, an unambiguous confirmation of the validity of the basic QCD ideas and would be the beginning of a new branch of the physics of resonance states. Both

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pure gluon and multiquark states must be sought among light mesons, but the differentiation between them and the "ordinary" mesons which consist of a quark and an antiquark is not at all a simple matter. A clear understanding of regularities revealed by the spectroscopy of ordinary mesons is absolutely essential for the solution of this problem as well. The main aim of this review is to exhibit these regularities at the level of phenomenology based on the quark model and using the fundamental ideas of QCD.

Practically all the meson resonances that have been adequately investigated may be regarded as consisting of a quark q and an antiquark $\bar{q}$ ($q\bar{q}$ state). Some mesons appear to have an admixture of gluon states, but pure gluon states (gluonium) have not as yet been found and their proposed properties will be discussed only briefly below. A more complicated situation obtains in the case of exotic mesons which consist of a larger number of quarks ($\bar{q}\bar{q}q\bar{q}$). Although the existence of these states has not as yet been confirmed with adequate reliability, there are indications that such states, or mixtures of such states with ordinary quark-antiquark states, have already been observed. Unfortunately, all mesons suspected of having exotic properties have quantum numbers (charge Q, isospin I, G-parity, spin J, parity P, and C-parity) that are allowed in the nonrelativistic quark model for the $q\bar{q}$ states as well.

Suspicion can be transformed into certainty only by demonstrating that the suspected mesons cannot be interpreted as quark-antiquark states. The essential foundation for this is a sufficiently complete knowledge of the properties of mesons consisting of a quark and an antiquark. One of the principal aims of this review is to expose these properties at the phenomenological level without introducing complicated or excessively detailed dynamic models. The results obtained with such models will of course be recalled and briefly reviewed below. However, we shall concentrate our attention on simple assumptions and a formalism that will not take us outside the framework of a modern course of general physics (see, for example, Refs. 1 and 2). This selective approach to our material is dictated not only by the limited space available to us but also by the absence of a systematic theory capable of providing a unified explanation of all the basic facts in this branch of physics. Although detailed models have been successful in individual areas, a sufficiently complete and clear picture of the present state of light-meson spectroscopy can at present be outlined only by confining ourselves to the meagre range of color available on the phenomenological palette.

There is not at present a single experimental fact that cannot be fitted into the framework of the quantum structure of hadrons. It is almost equally certain that, in addition to the usual quantum numbers B, Q, I, and J, quarks have a further degree of freedom, namely, their color, so that each quark is a triplet of the color group SU_3^c, and all observed hadrons are "colorless" (i.e., they are singlets belonging to this group). The situation regarding the quark charge is less definite. The most likely conclusion is that quarks have fractional charges ($Q_u = +2/3$, $Q_d = Q_s = -1/3$, $Q_c = +2/3$, $Q_b = -1/3$ in units of the electron charge e), but it is not as yet absolutely certain that colored quarks do not have integral charges. We shall always refer to quarks as objects with the above fractional charges.

Free quarks have not been seen in modern accelerator experiments. Searches for fractionally charged particles among cosmic rays have not as yet led to conclusive results because of the difficulties in interpreting such observations. Most experiments involving searches for fractional charges in different media (measurements of the charges of small particles) have not resulted in their discovery (see, for example, one of the recent papers[3] in which iron particles were used). Experiments with niobium spheres,[4] in which charges of $\pm 1/3$ were observed, form an exception to this. We recall that a similar situation occurred 60–70 years ago when Ehrenhaft, working in parallel with Millikan and others who observed only integral charges, performed analogous experiments with silver particles and reported seeing "subelectronic" charges (see Ref. 5 for a discussion of this; experimental techniques have, of course, advanced appreciably since that time).

It is clear that searches for fractional charges will have to be continued, using different materials. An attempt has also been reported[6] to systematize the chemical properties of the elements that have the highest probability of having quarks "stuck" to them. Lackner and Zweig[6] consider that the negative result of all the experimental searches for quarks in different materials is not an essential limitation on the abundance of quarks in nature.[1)] In any case, if free quarks do exist, their mass would appear to be substantially greater than the mass of the mesons known at present. When we refer to the u, d, s quarks as light, we have in mind their effective mass in the interior of the hadron which, generally speaking, may be different in different hadrons and may also depend on how the quarks are observed. For example, the effective mass of the quark-parton, or the current quark, observed in processes involving high-momentum transfers, may be substantially different from the effective mass of the constituent quark with which we deal in meson spectroscopy in which momentum transfers are usually low. The mass of free quarks, if they exist, is expected to be high and unrelated to the effective mass of a quark in the interior of a hadron.

Henceforth, we shall almost entirely confine our attention to constituent quarks. We shall try to introduce the minimum number of assumptions with regard to their properties in order to leave open the question of the relationship between, say, constituent and current quarks. In certain particular dynamic models, this relationship can be established but at the cost of introducing relatively far-reaching and not fully justified assumptions. The consequence of the desire to avoid this and to use maximally familiar language suitable for the

[1)] See, however, the estimated abundance of quarks reported in Ref. 7. The possibility that this estimate may be too high is discussed in Ref. 8. Experimental searches for quarks are reviewed in Ref. 9. These experiments are discussed in the rapporteur's paper given by Montanet[10] at the Madison Conference in July 1980.

formulation of very different models is, of course, that, in contrast to the current quark considered in the perturbative QCD, the principal object of our analysis—the constituent quark—turns out to be a somewhat nebulous object. Unfortunately, the latter cannot be completely avoided at the present level of our understanding of phenomena involving small momentum transfers in QCD. In actual fact, there is as yet no rigorous theory of the structure of hadrons, and the constituent quark cannot be clearly defined. On the other hand, when this theory becomes available, such a definition is likely to become superfluous.

It is possible to try to compare the constituent quark with an elementary excitation (quasiparticle) in condensed matter. Although this analogy is known to be incomplete and, probably, even incorrect, it does help us to see that the difference between the constituent quark and the current quark is no less than, say, the difference between the electron quasiparticle in the Fermi liquid and the free electron. Such ideas will not be used below. We shall simply gradually introduce the basic phenomenological characteristics of the constituent quark as they become necessary. Of course, the most interesting characteristics are those that are closely related to the confinement of color degrees of freedom and the nontrivial properties of vacuum.

Most theoreticians adhere to the dogma of perpetual confinement of quarks with fractional charges. This dogma can, in principle, be overturned—all that is required is to find free quarks. Its experimental verification is a problem of a fundamentally different type. It is essential to construct the corresponding theory and verify all its basic predictions. Fortunately, such a theory is not essential for the understanding of the spectroscopy of hadrons. It is sufficient, for example, to imagine that free quarks are very heavy and difficult to eject from hadrons. The assumption of quark confinement used in different models can therefore be regarded simply as a good approximation to reality. In this restricted form, the quark confinement hypothesis has undoubtedly been confirmed by hadron spectroscopy, and this in itself is responsible for the many essential differences between the meson and baryon spectroscopy on the one hand and the spectroscopy of molecules, atoms, and nuclei on the other. Although we shall frequently use the analogy with these "old" spectroscopies, such analogies must not be taken literally or too far.

Before we proceed to the main body of this review, let us emphasize that the list of references given at the end is very limited. With the exception of a few excursions into history, which are necessary for an understanding of the modern situation, the author has been forced to abandon references to foundation-laying original papers that are well known and have been examined in review literature and monographs. All the experimental data for which sources are not given are taken from the review in Ref. 11, where references to original papers may be found. The history of the development of compound models of hadrons can be followed in Ref. 12 which also gives much information on numerous calculations in the quark model. A very clear account of the quark model of hadrons, which does not require any preliminary familiarity with the subject, is given in the lectures reprinted in Ref. 13. A brief account of the basic results of the quark model can be found in Ref. 14 together with numerous references to the literature. More recent ideas and results are discussed in the reviewer's paper by the present author at the Tbilisi conference,[15] which also gives a relatively extensive review of the literature. Discussions of experimental data and of the phenomenology of boson resonances can be found in Ref. 10 and Refs. 16–19 and in the proceedings of international conferences on high-energy physics (London, 1974, Tbilisi, 1976, Tokyo, 1978, and Madison, 1980). The proceedings of the London conference[20] give a particularly good discussion of the phenomenological theory of decays of hadronic resonances, based on the transformation from constituent to current quarks (the so-called Melosh transformation—see also Refs. 18 and 19). The proceedings of the Tokyo conference[21] give a good account of the applications of quantum chromodynamics to the spectroscopy of hadronic resonances and a particularly detailed discussion of exotic states and charmed particles. The application of QCD to the spectroscopy of charmonium is discussed in the review paper given in Ref. 22. The basic results of QCD and the history of its development can be gleaned from the reviews given in Refs. 23 and 24. At the end of the present review, we shall touch upon certain new theoretical ideas that give us reason to hope for a deeper understanding of light-meson spectroscopy based on QCD, but we shall begin with a detailed account of experimental data and will analyze them without resorting to excessively detailed dynamic models.

2. MESON RESONANCES AND THEIR MAIN PROPERTIES

The basic data on meson resonances are listed in Table I. Data not given in Ref. 11 or differing from those data are shown in brackets with the appropriate references. They are discussed in the text. The table is confined mainly to sufficiently firmly established data. Whenever the reliability of the data seems in doubt, they are labeled with a question mark and are also discussed in the text. It is important to remember that some of the "sufficiently firmly established data" may, in fact, change appreciably in the future, especially in the case of results obtained in a single experiment with poor statistics. In such cases, we give a reference to the corresponding experiment or print an exclamation mark to draw the attention of the reader. In the latter cases, the corresponding reference can be found in Ref. 11. It is also important to remember that the physics of meson resonances is advancing very rapidly and, at the time of writing, new data are coming to light which may even alter the entire picture. The author hopes to take this into account at the proof stage. Most of the data used in this review were published up to the end of 1980 and at the beginning of 1981. In a few cases, we used the results reported at the Madison conference (July 1980) and discussed in the reviewers' papers by Montanet[10] and Berkelman.[25] In such cases, we quote the designation of the experimental group and (or) the name of the first of the authors to-

TABLE I.

Symbol $(I^G J^{PC})[^{2S+1}L]$	M, MeV	Γ_t, MeV	Decays	B, %	Γ, MeV
1. π (1^-0^{-+}) [^{1}S]	137 (3)	7.95 (55) $\times 10^{-6}$ (π^0)	$\pi^0 \to \gamma\gamma$	98.85 (5)	7.86 (54)$\cdot 10^{-6}$
2. η (0^+0^{-+})[^{1}S] [$\eta_u c_\eta - \eta_s s_\eta$]	548.8 (6)	0.85 (12)$\cdot 10^{-3}$ [34] [2.63 (58) $\times 10^{-3}$] [35]	3π $\gamma\gamma$ $\pi^+\pi^-\gamma$ $\pi^0\gamma\gamma$	53.5 (1.3) 38.0 (1.0) 4.89 (13) [$<$ 0.1] [36]	0.455 (65)$\cdot 10^{-3}$ 0.324 (46)$\cdot 10^{-3}$ 0.042 (6)$\cdot 10^{-3}$ [$<1\cdot 10^{-6}$]
3. η' (0^+0^{-+}) [^{1}S] $\eta'_u s_{\eta'} + \eta'_s c_{\eta'}$	957.6 (3)	[0.295 (85)]	$\rho\gamma$ $\omega\gamma$ $\gamma\gamma$	29.8 (1.6) 2.7 (5) 1.9 (2)	[0.88 (26)$\cdot 10^{-3}$] [8.0 (2.8)$\cdot 10^{-3}$] [5.7 (1.5)$\cdot 10^{-3}$]
4. K $(^1/_2 0^{-+})$ [^{1}S] [$u\bar{s}$]	495.7 (2.8)	stable	—	—	—
5. ρ (1^+1^{--}) [^{3}S] [$u\bar{d}$]	776 (3)	158 (5)	$\pi\pi$ $\pi^-\gamma$ $\eta\gamma$ e^+e^-	$\approx$ 100 [0.042 (5)] [3.6 (9)$\cdot 10^{-2}$] [44] 0.0043 (5)	 [67 (7)$\cdot 10^{-3}$] [42] [57 (14)$\cdot 10^{-3}$] 6.8 (8)$\cdot 10^{-3}$
6. ω (0^-1^{--}) [^{3}S] [$\omega_u c_\omega - \omega_s s_\omega$]	782.4 (2)	10.1 (3)	$\pi^+\pi^-\pi^0$ $\pi\gamma$ $\pi^+\pi^-$ $\eta\gamma$ e^+e^- $\pi\mu^+\mu^-$	89.8 (5) 8.8 (5) 1.4 (2) $(3.0^{+2.5}_{-1.8})\cdot 10^{-2}$ [44] 0.0076 (17) [9.6 (2.3) $\times 10^{-3}$] [45]	9.07 (27) 0.889 (62) 0.14 (2) $3.0^{(2.5)}_{(1.8)}\cdot 10^{-3}$ 0.77 (17)$\cdot 10^{-3}$ [0.97 (23)$\cdot 10^{-3}$]
7. φ (0^-1^{--}) [^{3}S] [$\varphi_u s_\varphi + \varphi_s c_\varphi$]	1019.6 (1)	4.1 (2)	K^+K^- $K_L K_S$ $\pi^+\pi^-\pi^0$ ($\rho\pi$) $\eta\gamma$ $\pi^0\gamma$ e^+e^-	48.6 (1.2) 35.2 (1.2) 14.7 (7) 1.5 (2) 0.14 (5)! 0.031 (1)	1.99 (11) 1.44 (10) 0.60 (4) 62 (9)$\cdot 10^{-3}$ 5.7 (2.0)$\cdot 10^{-3}$! 1.27 (7)$\cdot 10^{-3}$
8. K_V $(^1/_2 1^{--})$ [^{3}S] [$u\bar{s}$]	895 (4)	50.3 (8)	$K\pi$ $K^-\gamma$ $K^0\gamma$	$\approx$ 100 [0.12 (3)] 0.15 (7)	 [62 (14)$\cdot 10^{-3}$] [43] 75 (35)$\cdot 10^{-3}$
9. A_2 (1^-2^{++}) [^{3}P] [$u\bar{d}$]	1317 (5)	102 (5)	$\rho\pi$ $\eta\pi$ $K\bar{K}$ $\pi\gamma$ $\eta'\pi$	70.0 (2.2) 14.6 (1.1) 4.8 (5) 0.45 (11)! $<$ 1	71.4 (2.2) 14.9 (1.1) 4.9 (5) 0.46 (11)! $<$ 1
10. f (0^+2^{++}) [^{3}P] [$f_u c_f - f_s s_f$]	1273 (5)	178 (20)	$\pi\pi$ KK $\gamma\gamma$	83.1 (1.9) 2.8 (3) [1.3 (3)$\cdot 10^{-3}$] [47] [2.3 (6)$\cdot 10^{-3}$] [48]	148 (17) 5.0 (8) [2.3 (5)$\cdot 10^{-3}$] [4.1 (1.0)$\cdot 10^{-3}$]
11. f′ (0^+2^{++}) [^{3}P] [$f'_u s_{f'} + f'_s c'_f$]	1516 (12)	67 (10)	$K\bar{K}$ $\pi\pi$	[$\sim$ 70] [0.5—1.0]	
12. K_T $(^1/_2 2^{++})$ [^{3}P] [$u\bar{s}$]	1434 (5)	100 (10)	$\rho\pi$ $\eta\pi$ $K\bar{K}$ $\pi\gamma$ $\eta'\pi$	49.1 (1.6) 27.0 (2.2) 6.6 (1.5) 3.7 (1.6) 2.5 (2.6)!	
13. A_1 (1^-1^{++}) [^{3}P]	[$\sim$1280]?	$\sim$ 300	$\rho\pi$	Principal	
14. D (0^+1^{++}) [^{3}P] [$D_u c_A - D_s s_A$]	1284 (10)	27 (10)	$\eta\pi\pi$ 4π ($\rho\pi\pi$) $K\bar{K}\pi$	49 (6) 41 (13) 10 (2)	

Symbol $(I^G J^{PC})[^{2S+1}L]$	M, MeV	Γ_t, MeV	Decays	B, %	Γ, MeV
15. E (0^+1^{++}) [^{3}P] [$E_u s_A - E_s c_A$]?	1418 (10)	50 (10)	$K_V\bar{K}$, $\delta\pi$	(see text)	
16. Q_1 $(^1/_2 1^+)$ [^{3}P]	$\sim$ 1280	$\sim$ 120	$K\rho$, [$K\omega$]	"	"
17. Q_2 $(^1/_2 1^+)$ [^{3}P]	$\sim$ 1400	$\sim$ 150	$K_V\pi$	"	"
18. B (1^+1^{+-}) [^{1}P]	1231 (10)	129 (10)	$\omega\pi$	the only one	
19. H (0^-1^{+-}) [^{1}P]	1190 (60)	270—370	$\rho\pi$		
20. g (1^+3^{--}) [^{3}D] [$u\bar{d}$]	[1680(20)]	200 (20)	$\pi\pi$ $K\bar{K}$	24.0 (1,3) 1.5 (3)?	
21. ω_g (0^-3^{--}) [^{3}D]	1666 (5)	166 (15)	$\rho\pi$, $B\pi$		
22. K_g $(^1/_2 3^{--})$ [^{3}D]	1785 (6)	126 (20)	$K\pi$	19 (5)!	
23. [ρ_h (1^-4^{++})]?	[$\sim$ 2000]?	[$\sim$ 250]?			
24. h (0^+4^{++}) [^{3}F]	[1980 (20)]	150 (50)	$\pi\pi$ $K\bar{K}$	17 (2)!	
25. K_h $(^1/_2 4^{++})$	[$\sim$ 2060]?	[150—300]?	[$K\pi$]		
26. δ (1^-0^{++})	981 (3)	52 (8)	$\pi\pi$ $K\bar{K}$		
27. S* (0^+0^{++})	980 (10)	40 (10)	$K\bar{K}$ $\pi\pi$		
28. ε (0^+0^{++})	$\sim$ 1300	200—400	$\pi\pi$ $K\bar{K}$	$\sim$ 90 $\sim$ 10	
29. [ε' (0^+0^{++})]?	[1425 (15)] ?	[160 (30)]?	$\pi\pi$, $K\bar{K}$		
30. $\varkappa$ $(^1/_2 0^+)$	$\sim$ 1500	$\sim$ 250	$K\pi$		

Experimental errors are shown in parentheses, for example, $7.95(55)\times 10^{-6} = (7.95 \pm 0.55)\times 10^{-6}$, etc.

The table does not include all the known results. A more complete listing may be found in Ref. 11. A detailed review of data on stable particles or particles decaying as a result of weak or electromagnetic interactions is given in Ref. 26.

In addition to the experimentally determined quantum numbers of mesons, the table also lists (in brackets) the following information: the proposed quark composition, for example, [$u\bar{d}$] and the spectroscopic designation of the quantum numbers connected with the relative motion of quarks in the hadrons; for example, [$^3P_2$] means that the orbital angular momentum L associated with the relative motion and the total spin S of the quarks are both equal to unity. The fact that we are using this nonrelativistic notation does not, of course, mean that the motion of the quarks can be regarded as nonrelativistic. This is simply a convenient way of enumerating the possible states (see Refs. 12–14).

Nine mesons of the form $q\bar{q}$ ($q = u, d, s$) correspond to each $^{2S+1}L_J$ state, where $J = L$ or $J = L \pm S$, namely, the isotriplet ($I = 1$), two charge-conjugate isodoublets ($I = 1/2$) and two isosinglets ($I = 0$). The particle charges are given in this scheme by the Gell-Mann–Nishijima formula $Q = I_3 + (Y/2)$; $C = (-1)^{L+S}$, $P = (-1)^{L+1}$, and $G = C(-1)^I = (-1)^{L+S+I}$. Meson quantum numbers that cannot be represented in this way are referred to as exotic. Moreover, exotic charges, isospins, and hypercharges are exotic quantum numbers of the first kind ($Q \neq 0, \pm 1$; $I \neq 0, \frac{1}{2}, 1$; $Y \neq 0, \pm 1$). For isosinglets and isotriplets, the quantum numbers $J^{PC} = 0^{--}$ and J^{PC} for $J \geq 1$, $P = (-1)^J$, $C = (-1)^{J+1}$ are said to be exotic numbers of the second kind. Of course, there may be states for which all the quantum numbers are equal to the quantum numbers of some $q\bar{q}$ resonance but consist of $q\bar{q}q\bar{q}$ or, say, two gluons gg. Such states are referred to as cryptoexotic. A simple example is: $(s\bar{s})_{0^{++}}(q\bar{q})$, where $s\bar{s}$ is in the $J^{PC} = 0^{++}$ state and $q\bar{q}$ is a nonexotic meson state. The majority of two-gluon and three-gluon S–wave configurations is cryptoexotic. Exotic states of the second kind include the 1^{-+}-two-gluon and three-gluon S-states and the 2^{+-}-three-gluon S-states. The quantum numbers and the hypothetical properties of four-quark mesons are discussed in Refs. 27 and 28 in terms of the Massachusetts bag model. Exotic multiquark states of a somewhat different type, which are strongly coupled to baryon-antibaryon pairs $B\bar{B}$, are discussed in detail in Ref. 29 (see also Ref. 30). The experimental status of meson resonances that are strongly coupled to $B\bar{B}$ is still very uncertain and such resonances will not be discussed here. A review of the proposed properties of exotic (mainly cryptoexotic) gluonic mesons and further literature citations can be found in Refs. 31–33.

Most of the extensively studied mesons are amenable to the $q\bar{q}$ interpretation and some of the "suspected" states are discussed below. It is important to remember that cryptoexotic states can *mix* (in the quantum-mechanical sense) with normal $q\bar{q}$ states in a way similar to the mixing of the states $R_u = (1/\sqrt{2})(u\bar{u} + d\bar{d})$ and $R_s = s\bar{s}$ in isoscalar mesons ($R = \eta, \eta'; \omega, \varphi; f, f'; \ldots$). Only this type of mixing is indicated in square brackets under the particle symbols in the table (we use the abgether with a reference to the reviewer's paper. breviated notation $s_R = \sin\theta_R$, $c_R = \cos\theta_R$, where θ_R is the mixing angle for the strange and nonstrange quarks). In some cases, one could add further states, namely, $(c\bar{c})_R$, $(gg)_R$, and so on. Although we note this possibility, the determination of the corresponding new mixing angles does not seem to be reliable, and such admixtures are not indicated in the table.

A. Pseudoscalar (0^{-+})-mesons

π, K, and η mesons are among the most extensively investigated particles. However, the properties of the η meson have not been adequately studied. The total width of ~850 keV was obtained in a single experiment.[34] In an earlier experiment using the same method (Primakoff effect), the width was found to be greater by a factor of three. It is desirable to determine this quantity by a different method, for example, by using the creation of the η meson in the two-photon process involved in the e^+e^- reaction. For $B(\eta\to\pi^0\gamma\gamma)$, we list a recently published result,[36] whereas the value given in Ref. 11 is $B(\eta\to\pi^0\gamma\gamma) = 3.1\pm1.1\%$. The advantages of the method used in Ref. 36 are so obvious and so considerable that the earlier results can be safely forgotten. The reasons for the discrepancy between the two sets of data are discussed in Ref. 36. The properties of the η' meson were investigated quite recently. References to determinations of the quantum numbers of η' are cited in Ref. 11. The total width was determined in two essentially different experiments[37,38] and the agreement between them is satisfactory. We shall adopt the weighted average of these two results for Γ'_η. Data on the production of η and η' mesons at high energies (see, for example, Ref. 39) are very important for the understanding of the structure of these mesons. The clearest information has been obtained in experiments involving the creation of η and η' in reactions of the form $\pi^-p\to\eta(\eta')n$[40,41]. The cross-section ratio for $t\to 0$ was found to be

$$K_{\eta\eta'} \equiv \frac{\bar{\sigma}(\pi^-p\to\eta'n)}{\bar{\sigma}(\pi^-p\to\eta n)} = 0{,}55\pm0.06^{40} = 0.500\pm0.085\pm0.035^{41}. \quad (1)$$

The IFVÉ-CERN data[40] cover a wide range of energies (p_L = 4–200 GeV/c), whereas data reported in Ref. 41 were obtained for p_L = 8.45 GeV/c. Previous experiments on η and η' production were mostly carried out at lower energies and did not result in a clear picture (see the review in Ref. 39).

B. Vector (1^{--})-mesons

The vector mesons ρ, ω, φ, and K_V (or K*) are the most clearly defined $q\bar{q}$ states. Their properties have been adequately investigated. New data on $\Gamma(\rho^-\to\pi^-\gamma)$[42] and $\Gamma(K^-v\to K^-\gamma)$[43] have recently appeared. The value of $B(\rho\to\pi\gamma)$ given in Ref. 11 is smaller by a factor of two; averaging of the data reported in Refs. 11 and 42 would not be meaningful. Our table lists the results reported in Ref. 42 on the assumption that $\Gamma_{\rho^-} = \Gamma_{\rho^0}$. The quantities $\Gamma(\rho\to\eta\gamma)$ and $\Gamma(\omega\to\eta\gamma)$ have been measured in a single experiment.[44] Of the two solutions given in Ref. 44, we have chosen the one corresponding to constructive ω-ρ interference, which is in better agreement with the quark model. The other solution requires very strong SU_3^f symmetry breaking and does not agree with any of the theoretical models. Data on $B(\omega\to\pi\mu^+\mu^-)$ have recently been obtained at IFVÉ.[45] The latter paper also reports a study of the dependence of the formfactor of the $\omega\pi\gamma$ vertex on the mass $m_{\mu\mu}$ of the virtual photon for $m_{\mu\mu} < 0.65$ GeV, which is important for detailed verifications of the predictions of the vector dominance model (VDM), according to which $\omega\to\pi(\rho)\to\pi(\gamma)\to\pi\mu^+\mu^-$ (virtual particles are enclosed in parentheses). We note that a quantity $B(\varphi\to\pi\gamma)$, important for the theory, has been obtained in a single experiment. An earlier experiment which is not included in the table yielded a result that was lower by a factor of two (see Ref. 11).

Finally, one more word about the width of the extensively investigated $\omega\to\pi\gamma$ decay. The authors of Ref. 46 proposed that the value obtained in Ref. 11 as a result of a fit to all the existing data should be replaced by a somewhat lower value based on an average of all the direct measurements of the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\omega\to\pi^+\pi^-\pi^0)$ (see Ref. 11). It is difficult to agree with this suggestion because one then has to reject the results of nine other experiments which essentially involve the determination of $\Gamma(\omega\to\text{neutr.})/\Gamma(\omega\to\pi^+\pi^-\pi^0)$, and four other experiments involving the determination of $\Gamma(\omega\to\text{neutr})/\Gamma_\omega$ (see Ref. 11). Careful analysis of all the data shows that the statistical analysis performed in Ref. 11 does, in fact, yield the best value for $\Gamma(\omega\to\pi\gamma)$. Together with $\Gamma(\pi^0\to\gamma\gamma)$, this value must be regarded as the most reliably measured radiative width in meson decays. The $K_V^0\to K^0\gamma$, $\rho\to\eta\gamma$, $\omega\to\eta\gamma$, vector-meson decays require further investigation. New, systematic studies of the $\varphi\to\pi\gamma$, $\rho^-\to\pi^-\gamma$, $K_V^-\to K^-\gamma$ decays would be very useful. The $\omega\to\eta\gamma$ decay and the as yet unobserved $\varphi\to\eta'\gamma$ decay are particularly important for the verification of theoretical models (see below).

C. Tensor (2^{++})-mesons

The resonances A_2, f, f', and K_T (or K**) can also be assigned to classical $q\bar{q}$ states whose properties are now very reliably established. Studies of the radiative decays of tensor mesons are only just beginning. Measurements of the decay widths of $f\to\gamma\gamma$, $f'\to\gamma\gamma$, $A_2\to\gamma\gamma$ produced in colliding e^+ and e^- beams are particularly promising. We recall that the $\eta'\to\gamma\gamma$ width has been measured in this type of process.[37] The table lists new results[47,48] obtained by such measurements for $\Gamma(f\to\gamma\gamma)$ (see also Refs. 49 and 50). The creation of f, A_2 (and, possibly, ε as well) in two-photon processes is indicated by the results reported in Ref. 51 and the preliminary data of the MARKII, SPEAR group (see the review paper given by Wiik at the Madison conference[50]). All these preliminary data[2)] will probably be improved in the near future and will then become the subject of theoretical analyses. The foundations of the analysis of radiative decays of mesons with relative quark angular momentum $L = 1(2^{++}, 1^{++}, 0^{++}, 1^{+-})$ are described in Refs. 52 and 53.

[2)] Certain additional assumptions (f-meson helicity and dominance) were introduced in Refs. 47 and 48 to obtain the final results. This will require careful analysis.

Data on strong decays of tensor mesons are taken from Ref. 11, but some of the multiparticle decays have been omitted; it is probable that these also proceed through intermediate two-particle channels which can also be virtual, as in the $\omega \to (\rho)\pi \to 3\pi$ case. For $f' \to \pi\pi$, we reproduce an estimate based on Refs. 54–56. The authors of Ref. 55 analyzed the data in Ref. 54 on the assumption that $B(f' \to K\bar{K}) \sim 70\%$ and found that $B(f' \to \pi\pi) = 0.75 \pm 0.25\%$. The estimate for $B(f' \to K\bar{K})$ follows from exact SU_3^f symmetry and the quark line rule (QLR),[39] which is often referred to as the Okubo-Zweig-Iizuka rule (OZI). The same considerations yield $B(f' \to K_V\bar{K}) \sim 10\%$. This decay, like the $f' \to \eta\eta$ decay which is very dependent on the mixing angle θ_η, has not as yet been observed. The value obtained in Ref. 55 for $B(f' \to \pi\pi)$ is in agreement with the estimate $B(f' \to \pi\pi) = 0.5$–0.9%, reported in a recent paper.[56] A somewhat greater value, namely, $2.7^{+7.1}_{-1.3}\%$, has also been reported.[57] Apart from those mentioned above, the $A_2 \to \eta'\pi$, $K_T \to K\eta$ decays, which are very sensitive to the angles θ_η and θ_η, require further experimental study.

The status of other $q\bar{q}$ states with $L = 1$ will be examined a little later, and we shall begin with states with $L \geq 2$ and small values of J for given $L(3^{--}, 4^{++}, 5^{--})$.

D. Resonances with high spin

The most extensively investigated are the 3^{--} states (g-meson family). Only one isoscalar state φ_g is still required to fill this SU_3^f multiplet. The mass of the g-meson listed in the table is the average over all the experiments and is close to the weighted average given in Ref. 11. The uncertainty indicated in the table is not the statistical uncertainty and is intended simply to be an indication of the spread of the data (within three standard errors). The g-meson decays do not include data on the main channel $g \to 4\pi(\sim 72\%)$ which probably consists of the two-particle channels $\rho\rho, A_2\pi, \omega\pi$ (see Ref. 11). The value $B(g \to K\bar{K}) = 1.3 \pm 0.3\%$ obtained in Ref. 56 is in good agreement with the results listed in the table (see Ref. 11). We note that the result quoted in Ref. 57, namely, $\Gamma(g \to K\bar{K})/\Gamma(g \to \pi\pi) = 19 \pm 4\%$, is very different from that quoted in Ref. 11 ($6.3 \pm 1.3\%$; see Table I). In contrast to $B(g \to 2\pi)$, the quantity $B(g \to K\bar{K})$ is difficult to regard as firmly established, and this is indicated by the question mark in the table. The partial width of K_g has not as yet been determined. There are data on the existence of the $K_g \to K\rho, K_g \to K_V\pi, K_g \to K\pi\pi$ decays,[11] where the last decay may proceed through the $K\rho, K_V\pi, K_T\pi$ channels. The main two-particle decays of ω_g are probably $\rho\pi$ and $B\pi$. It is not as yet possible to estimate their partial widths.

The 4^{++} multiplet (h-meson family) has recently begun to be filled. The existence of the h meson is firmly established and its mass is known with a reasonable relative precision.[3)] However, the absolute mass is not well known. The weighted average of the masses obtained in different experiments[11] is shown in our table. The decay channels have not as yet been adequately investigated. Preliminary data suggesting the existence of the 1^-4^{++}-resonance in the $K_SK^\pm$ system were reported at the Madison conference (W. Cleland *et al.*; see Ref. 10). This resonance is designated as ρ_h and is included in the table although its existence cannot as yet be regarded as established. The data on the K_h-resonance which has been seen in $K_S\pi^\pm$ [58] and $K^-\pi^+$ [59] are more reliable. The masses and widths of the K_h obtained in these experiments are somewhat different, and we have listed the simple average of the masses and a rough estimate of Γ_{K_h}.

Some evidence has recently appeared for the existence of resonances with spins of 5 and 6. The K^+K^- system exhibits a peak, and the analysis of the angular distribution of the decay products suggests the presence of a resonance with $M = 2307(6)$, $\Gamma = 245(20)$, $J^{PC} = 5^{--}$; it is probable that $I^G = 1^+$ (see Ref. 60). Preliminary data on the $K_S^0K^\pm$-resonance (1^-6^+, $M \sim 2515$, and $\Gamma \sim 450$) were reported at the Madison conference (W. Cleland *et al.*; see Ref. 10).

E. Axial mesons

The status of axial mesons with $J^P = 1^+$ is less clear than that of the tensor mesons. The most fully investigated are the D and B mesons. The Q_1 and Q_2 resonances are firmly established but require further detailed investigation. The H meson has recently been seen in the $\rho\pi$ channel.[61] The A_1 and E mesons have given rise to some interesting problems with which we shall begin our discussion.

The existence of A_1 can now be regarded as firmly established, but measurements of its mass have resulted in values that are grouped near 1100 and 1300 MeV. Our review of early data and their analysis can be found in Ref. 11. Recent data[61–63] have been obtained with much better statistics (better by an order of magnitude) but discrepancies remain. The A_1 resonance was sought in both the recent and earlier experiments on reactions of two types, namely, in diffraction production of $\rho\pi$ in $\pi^-p \to \pi^+\pi^-\pi^0n$ (see, for example, Refs. 61 and 62) and in the baryon-exchange reaction $K^-p \to \Sigma^-\pi^+\pi^+\pi^-$ (see, for example, Ref. 63). The basic difficulty in analyzing the data of the former experiments is the presence of a high nonresonant background (this is the Deck effect). It would appear that observations reveal not the pure resonance but the resonance plus background. The method proposed in Ref. 64 is currently used to subtract the background. An analogous approach has been used to reexamine some of the older data (see Ref. 11). The application of this analysis to the main experiment belonging to the first of the above two types indicates that the A_1 mass lies in the range 1250–1300 MeV. For example, the experiment reported in Ref. 62, which has the highest statistics, has yielded $M_{A_1} = 1280 \pm 30$ MeV. A lower value lying in the range 1050–1100 MeV has been obtained for reactions belonging to the second type. For example, the value reported in Ref. 63 is $M_{A_1} = 1040 \pm 13$ MeV. We note, however, that there are specific problems when the baryon-exchange reactions are analyzed (see, for ex-

[3)] We note that much higher precision is necessary for estimates of fine effects, such as spin-splitting of the mixing angles and so on, because the absolute uncertainty in m_h^2 for a particle with a large mass is very large.

ample, Refs. 65 and 10) which have not as yet been adequately investigated. The A_1 meson appears to have been observed in $\tau^{\pm}\to A_1^{\pm}\nu_\tau$ decays (see Ref. 11), but it is still difficult to establish its mass reliably from such data (see the discussion given in Ref. 66). A rough estimate is 1100–1200 MeV. One still cannot exclude the possibility that two different 1^{++} particles are being observed. If this is so, one of them is probably exotic (see Ref. 27) and the observed states can be mixtures of $q\bar{q}$ and $q\bar{q}q\bar{q}$. A more detailed review of experimental data and further references to the literature will be found in Ref. 11 (see also Refs. 63 and 65).

Somewhat more subtle problems have arisen in connection with the E meson whose quantum numbers, mass, and width are now firmly established. The mass given in our table is slightly different from that in Ref. 11 since we have taken into account the most recent results.[67] For $\Gamma(E\to K_V\bar{K}+\text{c.c.})/\Gamma(E\to K\bar{K}\pi)$, we can take the weighted average of two recent results,[67,68] which are in good agreement with one another, and the final figure turns out to be $80\pm 8\%$. Approximately 20% of the $E\to K\bar{K}\pi$ decays appear to be $E\to\delta\pi\to K\bar{K}\pi$ (see Ref. 67). These data are not included in the table because they cannot be regarded as reliable enough. A particle with a mass of ~1440 MeV and width ~50 MeV has recently been observed[69] in $J/\psi\to\gamma X$, $X\to\eta\pi\pi$, and $X\to K\bar{K}\pi$ decays.[4)] It follows from these data that $\Gamma(X\to\eta\pi\pi)/\Gamma(X\to K\bar{K}\pi)\sim\frac{2}{3}$, i.e., $B(X\to\delta\pi)$ is greater than $B(E\to\delta\pi)$, and it is difficult to say anything more precise at present. An important point is that $J/\psi\to\gamma D$ has not been observed even though $B(J/\psi\to\gamma X)\sim B(J/\psi\to\gamma\eta')$. All this gives rise to the suspicion that X and E are not the same particles. We recall that the $K\bar{K}\pi$ enhancement with $M_{K\bar{K}\pi}\sim 1425$ MeV and width ~80 MeV has also been observed in $p\bar{p}$ annihilation at rest, which was not accompanied by the D-meson effect[70] and was strongly coupled to the $\delta\pi$ channel; the D and E mesons are readily seen in $p\bar{p}$ annihilation and in $\pi^- p$ processes at high energies (see, for example, Ref. 67). It is possible that the J/ψ decays and the $p\bar{p}$ annihilations at rest reveal not the E meson but another particle with $J^{PC}=1^{++}$ or 0^{-+}. If this is so, the state can be a radial 0^{-+} excitation, an exotic $q\bar{q}q\bar{q}$ meson (we recall that $M_{K_V}+M_K\sim 1400$ MeV), or even gluonium[71-73] (see also Refs. 10, 25, and 49). Before we can say anything more definite, we shall, of course, require more detailed studies of the E-meson decays. As in the case of the A_1 meson, it would be particularly useful to have information on radiative decays because quite reliable predictions can be made for the $q\bar{q}$ states (see, for example, Refs. 52 and 53).

In contrast to the A_1 and E, the D meson has been investigated quite well and we may suppose that this is mainly a $q\bar{q}$ state. The D meson is strongly coupled to the $\delta\pi$ channel, and roughly three-quarters of the $D\to\eta\pi\pi$ decays proceed through the intermediate $D\to\delta\pi$ decay.[11] It is possible that the $D\to K\bar{K}\pi$ decays are also coupled to the $D\to\delta\pi\to K\bar{K}\pi$ decay.

The strange meson Q_A, which is present with A_1 in the same SU_3^f multiplet, has been observed in a mixture with Q_B, which belongs to the SU_3^f multiplet of the $B(1^{+-})$ meson. Experimentally, Q_1 has been observed with the $K\rho$ main decay channel and Q_2 with the $K_V\pi$ main channel. The following simple mixing formalism has been used to analyze the Q-meson data[74]:

$$Q_1=\cos\theta_Q\cdot Q_A+\sin\theta_Q\cdot Q_B,\qquad Q_2=-\sin\theta_Q\cdot Q_A+\cos\theta_Q\cdot Q_B. \tag{2}$$

If the masses and widths of the Q_1 and Q_2 are known with sufficient precision, one can determine the mixing angle θ_Q and, subject to certain additional assumptions, the masses of Q_A and Q_B.[74] Currently available data have not as yet enabled us to find θ_Q with sufficient precision. As a rough guide, we note that θ_Q is most likely to lie in the range 35–40°[75,76] (however, see Ref. 77, where a lower value is reported).

It is important to emphasize that the determination of the data on the Q_A and Q_B mesons from data on Q_1 and Q_2 is, in fact, more complicated than might appear at first sight. To obtain sufficiently reliable data on the mass and width of these resonances, one must begin with a detailed study of the mechanisms responsible for their creation. For example, both resonances are usually observed in diffraction creation. In nondiffraction processes, for example, $\pi^- p\to K\pi\pi\Lambda$ [78], one observes either Q_1 or Q_2. A more detailed discussion can be found in Ref. 79.

$Q_1\to K\omega$ has recently been seen[75] in the $K^{\pm}p$ reaction, and the ratio of the coupling constants was found to be $g^2_{Q_1K\omega}/g^2_{Q_1K\rho}=0.21-0.04$ which is in agreement with SU_3^f symmetry predictions for $q\bar{q}$ states. The $Q_1\to K\omega$ decay has not as yet been observed in nondiffraction processes.[78]

The axial isovector meson B with negative charge parity is firmly established. Its probable isoscalar partner H has recently been detected in the $\rho\pi$ system.[61] The heavier isoscalar meson H′ has not as yet been seen.

F. Scalar resonances

The properties of scalar resonances are, at present, the darkest corner of light-meson spectroscopy. Although the $\delta, S^*, \varkappa, \varepsilon$ resonances can be regarded as well established, their interpretation in terms of the quark model is not clear. At first sight, the natural assumption is that they form a nonet of $q\bar{q}$ states, but this is not consistent with their masses. The ε mass is too high as compared with that of δ and S^*, but even for $M_\varepsilon=1300$, the mass of the $\varkappa$ should be about 1150, whereas for $m_{K\pi}<1400$ MeV, there is no indication that a strange scalar $K\pi$ resonance does, in fact, exist (see Refs. 10 and 11). On the other hand, there is recent serious indication of the existence of the ε' resonance[80,81] (see also Ref. 10). One could try to fit the ε, ε', and $\varkappa$ resonances into a single $q\bar{q}$, $^3P_0$ multiplet if $M_\varkappa\lesssim 1400$ MeV, but then one would have to decide the fate of the S^* and δ. These resonances lie near the threshold for the creation of a pair of K and $\bar{K}$ mesons, and there is therefore a suspicion that they may be states of a "molecular" type,[82] i.e., $(u\bar{s})(u\bar{s})$, and so on. Such highly simplified ideas are, of course, difficult to

[4)] See also Refs. 10, 25, and 49.

use in the development of a quantitative theory of the very complicated range of phenomena involving these resonances (coupled channels $\pi\pi$, $K\bar{K}$, $\pi\eta$, $\eta\eta$). The approach developed in Refs. 83 and 84 is more promising and takes into account the coupling between the different channels, the finite width of the resonances, and threshold effects. It turns out that exotic $\bar{q}\bar{q}qq$ states, which have a large width and mass close to $2M_K$, can simulate the observed narrow resonances S* and δ.[83,84] The occurrence of this effect is connected precisely with the large width of the original state. The final elucidation of the nature of the scalar resonances will have to await the availability of more accurate data on the phase shifts for $\pi\pi$, $K\bar{K}$ scattering and an analysis of such data with allowance for the above effect (see Ref. 83 for further details).

Another approach to the analysis of the scalar resonances is based on the use of the P-matrix instead of the S-matrix.[28] In the single-channel, nonrelativistic theory, the P-matrix reduces simply to

$$(P = k \operatorname{ctg} (kb + \delta (k)),$$

where k is the momentum in the center of mass system and b is the distance at which the wave function vanishes. In the general multichannel case, the P-matrix was first used by Wigner and Eisenbud[85] (see also Ref. 86) to analyze nuclear reactions. It was shown in Ref. 28 that the P-matrix contained the most complete information on the $q\bar{q}$ and $q^2\bar{q}^2$ states. In particular, even the $q^2\bar{q}^2$ states that do not appear at present as resonances can be observed as the poles of the P-matrix; they are referred to as "primitives" in Ref. 28. Some of the primitives correspond to real resonances, for example, all the $q\bar{q}$ states (ρ mesons) and also the S* meson, and, probably, the δ meson, regarded as $q^2\bar{q}^2$ states. Among the scalar primitives corresponding to $q^2\bar{q}^2$, Jaffe and Low[28] found not only the cryptoexotic states ($I = 0$: $M = 690$ and $M = 980$; $I = 1/2$: $M = 960$, where the second isoscalar state corresponds to S*), but also primitives with $I = 3/2(1190)$ and $I = 1(1040)$, i.e., properly exotic states of the first kind. These primitives have masses close to the predictions for the $q^2\bar{q}^2$ states.[27] It may be considered that the isoscalar primitive with the mass of 980 MeV appears as the S* resonance. To find the isovector primitive connected with the δ resonance by this procedure, we must have data on phase shifts in the $\pi\eta$ system.

Comparison of the analysis carried out in Refs. 83 and 84 with the results of the P-matrix approach[28] leads to the conclusion that the S* meson is probably cryptoexotic. It is possible that the δ-resonance also corresponds to a cryptoexotic state. Final elucidation of the nature of these resonances will have to await more complete and accurate data on phase shifts in their main decay channels and a complete analysis in both the P-matrix formalism[28] and the more traditional dispersion approach but including corrections for finite width and threshold effects.[83] It will be particularly important to find and investigate radiative decays of the scalar mesons, which should reveal most directly their quark structure. Decays into two photons are probably most likely to be seen in e^+e^- reactions (see Refs. 47–51). It is, of course, essential to continue the searches for $q\bar{q}$ scalar states. The ε, κ resonances and, possibly, the ε′ resonance, remain likely candidates for these states, but their properties have not as yet been adequately investigated and there is as yet no candidate for the isovector $q\bar{q}$ resonance.

G. Radial excitations

In the nonrelativistic theory, radial excitations correspond to states whose radial wavefunctions have zeros and for which the number N of a radial excitation is equal to the number of such zeros; for the ground state, $N = 0$. Radial excitations of the $q\bar{q}$ states are well established for the heavy quarks c and b—these are, for example, ψ′, Υ′, and so on (see Refs. 11 and 25). The situation is less clear for light quarks. The most likely candidates for the radial excitation of the ρ mesons is the 1^{--}-isovector resonance ρ′(1600): $M_\rho \sim 1600$ MeV, $\Gamma_\rho \sim 300$, decay into $2\pi(\sim 15\%)$ and $4\pi(\sim 85\%)$. The most recent data confirm the existence of the ρ′(1600) and the above parameter values. For example, the photoproduction of ρ′(1600) and the decay $\rho' \to \rho^0\pi^\pm\pi^\mp \to \pi^+\pi^-\pi^\pm\pi^\mp$ were investigated in Ref. 87. Analysis of the data showed[87] that $M_\rho = 1520(30)$, $\Gamma_\rho = 400(50)$. On the other hand, if it is supposed that $\rho' \to \rho^0\pi^\pm\pi^\mp$ proceeds through the two-particle decay $\rho' \to A_1^\pm\pi^\mp \to \rho^0\pi^\pm\pi^\mp$, then this experiment yields the following values for the parameters of the A_1 meson: $M_{A_1} \sim 1300$, $\Gamma_{A_1} \sim 300$.

The fate of the ρ′(1250) meson which has been discussed for some considerable time remains unclear. Although this meson does not appear to be necessary[88] for the description of data on the creation of 2π- and 4π-systems in e^+e^- reactions in the range $\sqrt{s} = 1$–1.5 GeV, it is difficult to exclude it altogether for sufficiently small $\Gamma(\rho' \to e^+e^-)$ (see Refs. 10 and 88). There have been some new indications that it might, nevertheless, exist (see Refs. 89 and 10) but, if this is so, the leptonic width of the ρ′(1250) is very small. According to Refs. 90 and 91, $\Gamma(\rho' \to e^+e^-) \sim 0.5$ keV. Its interpretation as a radial excitation of the ρ meson must then be regarded as relatively unlikely.

The situation relating to the radial excitations of the ω and φ mesons is less clear. There has been some new evidence for the presence of a resonance (or resonances) with a mass of the order of 1650 MeV in the $\omega\pi^+\pi^-$ and $K\bar{K}$ channels (J.C. Bizot *et al.*; see Refs. 10 and 92–94; references to earlier work can be found in Ref. 11). Data are accumulating on the existence of an isoscalar vector meson in the range 1800–1850 MeV, strongly coupled to $K\bar{K}$,[10,11,95] and there are also indications that a vector $K^-\pi^+$-resonance with a mass of ~1650 MeV exists (D. Aston *et al.*; see Ref. 10). If these resonances are confirmed, then, together with the ρ′(1600), they will probably complete the SU_3^f multiplet of radially excited vector mesons ρ′, ω′, φ′, K_V'. It is difficult to make any final conclusions as yet.

Data on radial excitations of pseudoscalar mesons are even less clear. The isoscalar $\eta\pi\pi$-resonance with $J^{PC} = 0^{-+}$, $M \sim 1275$ MeV, $\Gamma \sim 70$ MeV has been seen in one experiment. This is a probable candidate for the radially excited state of the η-meson. Quite recently, IFVE obtained evidence for the existence of the π' resonance in the 3π system with the following quantum numbers of the π meson: $M_{\pi'} \sim 1240$, $\Gamma_{\pi'} \sim 300$ MeV.[97] Data on the creation of three π mesons in diffraction[62] and charge-exchange reactions (unpublished data from the Canadian-American collaboration CEX) were analyzed in Ref. 98. It is reported that two resonances are necessary for the description of these data, namely,

$$A_1,\ M_{A_1} = 1230(30),\ \Gamma_{A_1} = 350(60),$$

$$\pi',\ M_{\pi'} = 1273(50),\ \Gamma_{\pi'} = 580(100).$$

Because of the special nature of the pseudoscalar mesons (small mass of the π, large mass of the η, strong mixing of strange and nonstrange quarks in the η- and η'-mesons), it would be very interesting to have further studies of their possible radial excitations $\eta_{R_1} \equiv \eta(1275)$ and $\pi'(1240)$.

H. Additional remarks

At this point, we have to end our somewhat cursory and incomplete account of the main experimental data on light mesons. We have not discussed data on the $B\bar{B}$ resonances whose experimental status and theoretical interpretation are far from clear (see Refs. 10, 29, and 30). We have also left out some results which, while interesting in themselves, are not very important for the description of the overall picture of light-meson spectroscopy.

Thus, with a few exceptions, we shall not use data on the creation of resonances in inclusive and exclusive reactions, we shall not report on the parameters of Regge trajectories, deduced from data on interactions at high energies, and shall not examine electromagnetic and weak meson formfactors. Of the data omitted from the table and the text of this section, we will now mention only those facts that will be useful later.

The resonance nature and the quantum numbers are now fairly well established for the state $A_3(1660)$: $I^G J^{PC} = 1^-2^{-+}$, $M = 1660(10)$, $\Gamma = 200(50)$, $B(A_3 \to f\pi) \sim 60\%$, $B(A_3 \to \rho\pi) \sim 30\%$ (see Ref. 11 and the new data in Ref. 97).

This is almost certainly the $^1D_2$ state of $q\bar{q}$, split from the g-meson by the spin-spin interaction.

The table does not include recent IFVE data[99] on rare decays of the η and η' mesons, namely, $\eta \to \mu^+\mu^-$, $\eta \to \mu^+\mu^-\gamma$, $\eta' \to \mu^+\mu^-\gamma$. The partial widths and the dependence of the formfactors of these decays on $m_{\mu^+\mu^-}$ are in good agreement with the vector dominance model (VDM).

3. MAIN REGULARITIES IN MESON SPECTROSCOPY

The clearest feature of the spectrum of meson states described above is the presence of relatively narrow resonances with quantum numbers that can be predicted by the simple, nonrelativistic model of excitations in the $q\bar{q}$ system. The $^1S_0$-, $^3S_1$-, and $^3P_2$-$^3P_1$-multiplets are complete. The $^1P_1$ and $^3D_3$ are almost complete, and $^3F_4$ is being completed. The $q\bar{q}$ system definitely has radial excitations. The cryptoexotic states, $q^2\bar{q}^2$, probably exist, but are seen as ordinary resonances under special conditions. Exotic values of I^G and J^{PC} are not observed in the spectrum of the resonances. Modern data do not exclude the existence of pure gluon resonances, but their creation in ordinary hadron reactions appears to be suppressed. It is interesting that the main decay channels of $q\bar{q}$ resonances are two-particle channels. Practically all the known multiparticle decays proceed through intermediate two-particle channels. Occasionally, as in the case of the $\omega \to 3\pi$ decay, the intermediate two-particle state can be virtual.

The structure of the SU_3^f multiplet $1^{--}, 2^{++}, 1^{++}, 1^{+-}, 3^{--}, 4^{++}$ seems to be very simple. The isovector and the light isoscalar states have similar masses $M_1 \sim M_0$, and the mass of the state with isospin 1/2 and of the heavy isoscalar is such that $M_1 < M_{1/2} < M_0'$ and

$$M_1 + M_0' \sim M_0 + M_0' \sim 2M_{1/2}.$$

Simple interpretation of these relations yields

$$M_1 \sim M_0 \sim 2u_R,\quad M_{1/2} \sim (u_R + s_R),\quad M_0' \sim 2s_R,$$

where u_R and s_R are the effective masses of the corresponding quarks and it is assumed that $u_R \neq d_R$. The pseudoscalar nonet appears to be constructed in a different way: $M_\pi < M_\eta \sim M_K < M_{\eta'}$, where the masses M_π and M_η differ by almost an order of magnitude. This means that the quark line rule (QLR)[39] is broken quite strongly in the pseudoscalar multiplet, whereas its breaking in the remaining known multiplets is very slight. According to this rule, $K\bar{K}$ and $K_V\bar{K}$ are the main channels in the decays of φ, f′, and E.

Spin-spin splitting is very large for the S state. A rough estimate can be obtained by taking the difference between the mean squares of the masses of the $^3S_1$ and $^1S_0$ nonet: ~0.49 GeV^2. A more accurate estimate will be obtained later, after we examine in detail the structure of the pseudoscalar nonet. This splitting is smaller for the P-multiplet. If we estimate the average square of the mass of the $^3P_{2,1,0}$ states from the A_2, f, A_1, and D, and subtract the square of the mass of the B-meson, we obtain ~0.14 GeV^2. For the D-multiplet (splitting of g and A_3), this quantity is ~0.07–0.013 GeV^2. From the standpoint of nonrelativistic ideas, this splitting is most likely to continue to fall with increasing L.

Spin-orbit splitting is probably present but not very strong. We can estimate it by examining the structure of the $^3P_2$- and $^3P_1$-nonets in greater detail. It is possible that tensor forces must also be included in the nonrelativistic description of these interactions, but it is hardly possible to show this at the present level of our knowledge of the P states.

It may be that the most striking feature of the spectrum of observed resonances is the linear rise of the Regge trajectories. Crudely speaking, the Regge trajectory is the relationship between M^2 and L. It is readily verified that the squares of the masses of the resonances ρ, ω, A_2, f, g, ω_g, h, ρ_h and of the recently found candidates for the isovector resonances with $J^P = 5^-$ and 6^+ fit quite well a single straight line of the form

$$M^2(L) = \mu_0^2 + \mu_1^2 L,$$

where $\mu_0^2(u\bar{u}) \sim 0.548$ GeV^2, $\mu_1^2(u\bar{u}) \sim 1.153$ GeV^2. For the K-resonances K_V, K_T, and K_g, these parameters are $\mu_0^2(u\bar{s}) \sim 0.829$, $\mu_1^2(u\bar{s}) \sim 1.193$. For φ and f′, $\mu_0^2(s\bar{s}) \sim 1.040$, $\mu_1^2(s\bar{s}) \sim 1.259$. Although the parameters μ_0^2 (crossings) for these trajectories are quite different, the slopes are very close to one another. To within experimental error in $\mu_1^2(\sim 10\%)$, we may suppose that the slopes are equal but, as will be shown below, the simple relationship between μ_1^2 and the quark composition of the particles lying on the trajectory is not accidental.

Similarly to the masses of the particle themselves, the slope parameters satisfy the following inequalities:

$$\begin{gathered} \mu_1^2(u\bar{u}) < \mu_1^2(u\bar{s}) < \mu_1^2(s\bar{s}'), \\ \mu_1^2(u\bar{u}) + \mu_1^2(s\bar{s}) \approx 2\mu_1^2(u\bar{s}). \end{gathered} \tag{3}$$

We note that the parameter $\mu_1^2(u\bar{u})$ represents the slopes of both the isovector and the isoscalar (ω, f, h) trajectories which, in the first approximation, can be regarded as degenerate. Similar relationships are satisfied by $\mu_0^2(q\bar{q}')$ but SU_3^f-symmetry breaking is very large in μ_0^2. If we use the masses of the χ/P_c and J/ψ particles to find the slope $\mu_1^2(c\bar{c})$ for the charmed particles (see Ref. 11), we see that $\mu_1^2(c\bar{c}) \sim 2.8$. Although this slope is much greater than the slope of the usual trajectories, the following relationship is satisfied:

$$\mu_1^2(c\bar{c}) - \mu_1^2(s\bar{s}) \ll \mu_0^2(c\bar{c}) - \mu_0^2(s\bar{s}) \sim M_\psi^2 - M_\varphi^2.$$

The linearity of the Regge trajectories is also well defined in the baryon spectrum. By analyzing the sequence of resonances $\Delta(J^P = 3/2^+, 7/2^+, 11/2^+)$, $N(1/2^+, 5/2^+, 9/2^+)$, $N'(3/2^-, 7/2^-, 11/2^-)$, $\Lambda(1/2^+, 5/2^+, 9/2^+)$, $\Sigma(1/2^+, 5/2^+)$, $\Sigma'(3/2^+, 7/2^+)$ (see Ref. 11), we obtain the following average slopes for the Δ, N and Λ, Σ trajectories, respectively:

$$\mu_1^2(uuu) = 1.02\,(6), \quad \mu_1^2(uus) = 1.10\,(3).$$

The linear rise of the Regge trajectories is one of the strongest arguments in favor of the confinement of quarks. In the nonrelativistic theory, linearly rising trajectories are obtained only for the oscillator potential. However, the nonrelativistic model says nothing about the spin dependence of the potential. The presence of a large spin-splitting, comparable with the splitting associated with the excitation of the orbital angular momentum L, will probably require an essentially relativistic description.

Let us now formulate a more accurate theory of the mass formulas, taking into account the effects mentioned above (see Refs. 15, 100, 101, and 102 and the more complete account given in Ref. 103). We shall assume that the wavefunction of the quarks q_i and $\bar{q}_j$ in the state R satisfies the following set of equations:

$$[\hat{\mathscr{K}}^2_{ijR} - k^2(M_R^2, m_i^2, m_j^2)]\,\Psi_{ijR} = \sum_{kl} \mathscr{M}^2_{ij,\,kl}(R)\,\Psi_{klR}. \tag{4}$$

The wavefunction Ψ depends on the relative coordinates of the q_i and $\bar{q}_j$ and on $R = (J, L, S, N)$. In the ensuing account, we shall often replace the set of quantum numbers with the symbols for the corresponding multiplets R = P, V, T, A, S, P′, V′, ... (P—pseudoscalar, T—tensor, V′—radially excited vector, and so on). In the phenomenological approach, the operator $\hat{\mathscr{K}}^2_{ijR}$ is not specified in an explicit form, but certain assumptions are introduced about the dependence of its eigenvalues $\mathscr{K}^2_{ijR}$ on i, j, and R. The quantity $k^2(M_R^2, m_i^2, m_j^2)$, i.e., the square of the relative momentum of the quarks q_i and $\bar{q}_j$ in the center-of-mass system, is given by the following expression:

$$k^2(M_R^2,\ m_i^2,\ m_j^2) = \frac{1}{4} M_R^2 - \frac{1}{2}(m_i^2 + m_j^2) + \frac{(m_i^2 - m_j^2)^2}{4M_R^2}; \tag{5}$$

where m_i is the mass of the i-th quark and M_R is the mass of the bound quark state, i.e., the mass of the meson, which, in principle, is determined by the solution of the operator equation (4). The right-hand side describes the mixing of the quark configurations, for example, $u\bar{u} \to s\bar{s}$, i.e., the breaking of the QLR. We shall leave the right-hand side until we come to the next section and, for the moment, we shall assume that $\mathscr{M}^2_{ij,kl}(R) = 0$, which gives a good approximation to the description of the vector mesons (R = V) and an adequate approximation for R = T.

The phenomenological relativistic equation (4) can be made more specific by introducing a particular dynamic model. For example, if the interaction between the quarks is described by relativistic quasipotential equations, then $\hat{\mathscr{K}}^2$ appears as a differential or integro-differential operator (see, for example, Refs. 15 and 104 where further references can be found). In the simplest cases, the Bethe-Salpeter equation will also reduce to a differential equation for the quark wave function (see, for example, Refs. 15 and 105). It is sometimes useful to use the simplest form of the operator $\hat{\mathscr{K}}^2$:

$$\hat{\mathscr{K}}^2_{ijR} = -\frac{d^2}{dr^2} + \frac{L(L+1)}{r^2} + V_{ijR}(r). \tag{6}$$

When we refer to a "potential," this will mean a function $V_{ijR}(r)$ determined by this particular form of $\hat{\mathscr{K}}^2$. We emphasize, however, once again that the phenomenological description that we are giving requires not only a potential but a particular form of (4). SU_3^f symmetry breaking is already included in the main equation given by (4), since the masses of the quarks m_i are not assumed to be equal. Moreover, as indicated above, symmetry breaking in the slopes of the Regge trajectories is also possible, and this must be reflected in the assumptions made about the dependence of $\mathscr{K}^2_{ijR}$ on i, j, and L. We therefore assume that

$$\mathscr{K}^2_{ijR} = k_R^2 + \frac{1}{2}(\mu_i^2 + \mu_j^2)\,L_R, \tag{7}$$

where L_R is the orbital angular momentum in the relative motion of the quarks in the multiplet R. Isotopic invariance demands that $\mu_u = \mu_d$; the dependence of k_R^2 on R will be discussed a little later. The assumption expressed by (7) was first put forward and justified phenomenologically in Refs. 15, 100–103. An analogous assumption relating to the reciprocal quantities (the usual Regge trajectories) was introduced in Ref. 106. Multiplicative symmetry breaking on the Regge trajectories [$\mu_i \mu_j$ instead of $(\mu_i^2 + \mu_j^2)/2$ in (7)] was introduced in Refs. 107 and 108 where an attempt was made to justify this hypothesis. It is shown in a recent paper[109] that (7) follows from the condition of s-channel factorization of planar diagrams of the dual topological expansion (see the review paper given in Ref. 110 for further details of this expansion). As noted above, the difference between μ_s^2 and μ_u^2 is difficult to estimate directly from the slopes of the Regge trajectories—the errors are too large. However, an estimate readily follows from (4) if we neglect mixing (which will be taken into account later). The mass formulas then follow from the condition

$$\mathcal{K}_{ijR}^2 = k^2 (M_R^2,\ m_i^2,\ m_j^2). \tag{8}$$

Let

$$m_{iR}^2 = m_i^2 + k_R^2 + \mu_i^2 L_R,\quad m_{iR}^2 - m_{jR}^2 = m_i^2 - m_j^2 + (\mu_i^2 - \mu_j^2) L_R \equiv \Delta_{ijR}^2, \tag{9}$$

so that

$$\Delta_{ijR}^2 = \Delta_{ij}^2 + (\mu_i^2 - \mu_j^2) L_R,\quad \Delta_{ij}^2 \equiv m_i^2 - m_j^2. \tag{10}$$

It follows from (5), (7), (8), and (9) that

$$M_{ijR} = m_{iR} + m_{jR}, \tag{11}$$

and this leads to linear mass formulas (the masses are indicated by the particle symbols, i.e., $M_\rho \equiv \rho$, and so on):

$$\rho = \omega,\quad K_V = \frac{1}{2} (\varphi + \rho), \tag{12}$$

$$A_2 = f,\quad K_T = \frac{1}{2} (f' + A_2). \tag{13}$$

The masses of the vector mesons satisfy (12) with good precision, but (13) is less satisfactory.

We emphasize that all the main solutions involve the squares of the masses, as required by the linearity of the Regge trajectories, but all the mass formulas turn out to be linear, nevertheless. This is connected with the relativistic relationship (5) between k^2 and the quark masses and with the absence of mixing. When mixing is taken into account, the mass formulas turn out to be intermediate between linear and quadratic.

It is assumed in (9) and (10) that the entire SU_3^f breaking in $\mathcal{K}^2$ is described by the slope parameters $\mu_u = \mu_d$ and μ_s. Symmetry breaking in spin-splitting must also be taken into account when we use the analogous relationships for SU_4^f or SU_5^f. This SU_3^f breaking can be neglected for light mesons.

Using (11) and the $\Delta_{suV}^2 \equiv \Delta_{su}^2$ (from now on, $\Delta_{su}^2 \equiv \Delta^2$) and Δ_{suT}^2 and hence obtain $\Delta^2 \sim 0.107$ GeV^2, $(\mu_s^2 - \mu_u^2) \sim 0.06$ GeV^2, i.e., the SU_3^f symmetry breaking in the slopes of the Regge trajectories, is ~20–30%. We note that L_R cannot be replaced with J_R in (7) because the observed splitting between the 2^{++} and 1^{++} (or 1^{+-}) states is small.

Let us now consider the dependence of k_R^2 on R. The simplest to explain is the dependence of k_R^2 on N. In any model with a given potential, the dependences on L and N are coupled. For example, if the meson is described by (6) with the potential $V(r) \sim r^\gamma$, then $\mathcal{K}_R^2$ depends on the combination $L_{eff} = \sqrt{2+\gamma}(N + \frac{1}{2}) + (L + \frac{1}{2})$. This can readily be verified with the aid of the quasiclassical approximation (see, for example, Ref. 111). However, when $L \gg 1$, the Regge trajectory corresponding to the potential of the form $\sim r^\gamma$ behaves as $L^{2\gamma/(2+\gamma)}$, so that γ must be close to 2 (oscillator potential). To ensure the linearity of the Regge trajectories, it is sufficient to suppose that $V \sim r^2$ for large r. For small r, the potential can be, for example, proportional to $1/r$, as follows from QCD. In view of the foregoing, we might try to replace L_R in (7) with $L_R + \beta N_R$, where β may differ from 2. The data on ρ', π', η_{R_1} discussed above do not as yet enable us to establish with complete clarity the structure of the spectrum of radial excitations. All we can say is that $\rho'(1600)$ gives $\beta \sim 2$, whereas the description of the π' and η_{R_1} requires a smaller value ($\lesssim 1.5$). It is possible, however, that, for small values of L, the Regge trajectories cannot be regarded as linear, and the description of the spectrum of radial excitations will require more complex assumptions. At any rate, nonlinear effects of this type are essential for the description of the excited states of charmed particles and the Υ-family. If the combination of Coulomb and oscillator potentials is replaced with the effective potential $V \sim r^\gamma$ mentioned above, we find that $k_R^2 \sim L_{eff}^{2\gamma/(2+\gamma)}$. It is readily verified that γ must be relatively small (<1/4) and the nonlinear effect is quite large. Although this potential is occasionally used for the description of the excited states of the ψ and Υ-particles,[112,113] (see the discussion and further references in Refs. 21 and 25), it is completely unsuitable for light mesons. The potential connected with one-gluon exchange[114] seems to be more justified. This potential will also give the dependence of the eigenvalues on R, which can readily be obtained by using existing results of positronium theory (see, for example, Ref. 115). Unfortunately, this simple approach will not reproduce the observed splitting in R not only for the light but for the charmed mesons as well. In view of this, more successful descriptions of the spectra of the $c\bar{c}$ and $b\bar{b}$ states have been achieved with more complicated potentials containing not only vector but also scalar or pseudoscalar parts (see Refs. 116 and 117, and the review paper by J.D. Jackson *et al.*; see Ref. 21). The parameters that have to be introduced in this approach cannot be determined from the observed light-meson spectrum because the structure of the $L = 1$ multiplets is not sufficiently well understood.

The dependence on R in k_R^2 could be introduced in the phenomenological scheme as follows: (1) isolate the term $\mu_F^2(s_i, s_j)$ which describes the splitting of the V- and P-multiplets, (2) isolate the term $\mu_F'^2(s_i, s_j)(1 - \delta_{L0}) + (\mu_L^2/4)(L, S)$ which describes the spin and the spin-orbit splitting in multiplets with $L \neq 0$, and (3) take into account the tensor interaction: $\frac{1}{4}\mu_T^2 X(J)$, where $X(J) = (J + 2)/(2J + 1)$ for $J = L - 1$, $X(J) = -1$ for $J = L$, $X(J) = (J + 1)/(2J + 1)$ for $J = L + 1$, L fixed. The quantity μ_L^2 will be found below and we shall also obtain some estimates for $\mu_F'^2$ and μ_L^2 for multiplets with $L \geqslant 1$. We note that these parameters must be regarded as different for different L and, moreover, they may depend on the quark composition of the mesons but, for light mesons, this dependence can be neglected. In view of all the assumptions introduced above, the mass of states with $I = 1$ and $I = 1/2$ can be described by

$$M_R^2 = m_0^2 + 2(m_i^2 + m_j^2) - \frac{(m_i^2 - m_j^2)^2}{M_R^2} + 2(\mu_i^2 + \mu_j^2)(L + \beta N)$$
$$+ 4[\mu_F^2 \delta_{L0} + \mu_F'^2 (1 - \delta_{L0})](s_i,\ s_j) + \mu_L^2 (L,\ S) + \mu_T^2 X(J). \tag{14}$$

Corrections that are not included in (14), for example, the correction for the nonlinearity of the Regge trajectories, are contained implicitly in m_0^2. The quadratic mass formulas[12-14] that are frequently employed can be obtained from (14) by neglecting the third term on the right-hand side. If, in addition, we replace the squares of all the quantities with the dimensions of mass by linear terms, we obtain the linear mass formulas.

4. MIXING OF QUARK CONFIGURATIONS. DECAYS

Quantum chromodynamics leads to a simple expression for the mixing matrix, namely,

$$\mathcal{M}^2_{ij,\,kl}(\mathrm{R}) = -\varepsilon_{\mathrm{R}}^2\delta_{ij}\delta_{kl}. \qquad (15)$$

This is obvious in perturbation theory (see Refs. 114, 118, and 119) but perturbation theory does not yield even the correct sign for the quantity $\varepsilon_{\mathrm{V}}^2$ not to mention $\varepsilon_{\mathrm{P}}^4$.[120] However, the same mixing matrix results from nonperturbative effects that are taken into account with the aid of the effective Lagrangian[121-123] (see also the lectures reported in Ref. 124; nonperturbative effects have also been investigated by other methods[125,126,33]). The mixing matrix (15) was first put forward by Schwinger,[127] who used it to derive the well-known expression for the mass of vector mesons

$$(\omega^2-\rho^2)(\varphi^2-\rho^2) = 2\,[\varphi^2-(2\mathrm{K}_{\mathrm{V}}^2-\rho^2)]\,[(2\mathrm{K}_{\mathrm{V}}^2-\rho^2)-\omega^2].$$

The linear variant of the Schwinger formula works somewhat better than the quadratic formula, but either gives reasonable results: the prediction for the mass of the ρ meson based on the masses of the ω, φ, and K is $\rho = 0.774$ GeV. However, the Schwinger formula is not suitable for pseudoscalar mesons: the predicted η' masses based on M_{π_1}K and η are: $\eta' = 1.610$, $\eta' = 2.340$. The conclusion reported in Refs. 118, 119, 123, and 124 that the mixing matrix (15) can be used to describe the masses of the pseudoscalar octet is therefore incorrect. The mass formulas used in these references are the same as the Schwinger mass formulas and the mass of the η' should be very large. It was suggested in Ref. 114, where linear formulas were used so that the discrepancy was not noticeable, that the parameter $\varepsilon_{\mathrm{R}}^2$ is very dependent on the meson mass, $\varepsilon_{\mathrm{R}}^2 = \varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}^2)$. This dependence appears very naturally in QCD and can, in principle, be connected with the dependence of the effective strong-interaction coupling constant α_s on M_{R}^2. It turns out, however, that this dependence must be exceedingly strong: $\varepsilon_{\mathrm{P}}^2(\eta^2)/\varepsilon_{\mathrm{P}}^2(\eta'^2) \sim 8$.[114] For quadratic formulas, this dependence can be somewhat weaker, $\varepsilon_{\mathrm{P}}^2(\eta^2)/\varepsilon_{\mathrm{P}}^2(\eta'^2) \sim 4$, but, even in this case, it is difficult to reconcile it with the practical absence of a similar dependence for the ω and φ mesons so that one can hardly hope for an explanation of such a strong effect in QCD.

The idea that $\varepsilon_{\mathrm{R}}^2$ depends on the mass was used in Refs. 101–103 to formulate a new mixing model in which the SU_3^f singlet matrix (15) was replaced with the SU_3^f singlet matrix

$$\mathcal{M}^2_{ij,\,kl}(\mathrm{R}) = -\varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}^2)\left(\delta_{ij}\delta_{kl}-\frac{1}{3}\,\delta_{ik}\delta_{jl}\right). \qquad (16)$$

If $\varepsilon_{\mathrm{R}}^2$ is independent of M_{R}^2, the difference between (16) and (15) at the phenomenological level is purely formal: *the additional term $\varepsilon_{\mathrm{R}}^2\delta_{ik}\delta_{jl}/3$ can be included in m_0^2* [see (14)] and all the relationships between the masses remain. If $\varepsilon_{\mathrm{R}}^2$ depends on M_{R}^2, it follows from (4) and (14) that the masses of particles with isospins 1 and 1/2 are given by

$$M^2_{\mathrm{R},\,1} = m_{\mathrm{R}}^2 - 2\Delta_{\mathrm{R}}^2 - \frac{4}{3}\,\varepsilon_{\mathrm{R}}^2(M^2_{\mathrm{R},\,1}), \qquad \Delta_{\mathrm{R}}^2 = \Delta^2_{su\mathrm{R}}; \qquad (17)$$

$$M^2_{\mathrm{R},\,1/2} = m_{\mathrm{R}}^2 - \frac{\Delta^4}{M^2_{\mathrm{R},\,1/2}} - \frac{4}{3}\,\varepsilon_{\mathrm{R}}^2(M^2_{\mathrm{R},\,1/2}), \qquad \Delta^2 = \Delta^2_{su} = s^2 - u^2; \qquad (18)$$

whereas the masses M_{R}, M'_{R} of the isosinglet particles ($\varepsilon_{\mathrm{R}}^2 = \varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}^2)$, $\varepsilon_{\mathrm{R}}'^2 = \varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}'^2)$) are given by

$$M_{\mathrm{R}}'^2 = m_{\mathrm{R}}^2 + (14/3)\,\varepsilon_{\mathrm{R}}'^2 + 2\sqrt{(\Delta_{\mathrm{R}}^2-\varepsilon_{\mathrm{R}}'^2)^2+8\varepsilon_{\mathrm{R}}'^4}, \qquad (19)$$

$$M_{\mathrm{R}}^2 = m_{\mathrm{R}}^2 + (14/3)\,\varepsilon_{\mathrm{R}}^2 - 2\sqrt{(\Delta_{\mathrm{R}}^2-\varepsilon_{\mathrm{R}}^2)^2+8\varepsilon_{\mathrm{R}}^4}. \qquad (20)$$

The mixing angle for the quark functions R_s and R_u in a light isosinglet particle is given by

$$\mathrm{tg}\,\theta_{M'_{\mathrm{R}}} = 2\sqrt{2}\,\varepsilon_{\mathrm{R}}^2\,[\Delta_{\mathrm{R}}^2 - \varepsilon_{\mathrm{R}}^2 + \sqrt{(\Delta_{\mathrm{R}}^2-\varepsilon_{\mathrm{R}}^2)^2+8\varepsilon_{\mathrm{R}}^4}]^{-1}, \qquad (21)$$

where the expression for $\theta_{M_{\mathrm{R}}}$ is given by (21) with $\varepsilon_{\mathrm{R}}^2 = \varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}^2)$ replaced with $\varepsilon_{\mathrm{R}}'^2 \equiv \varepsilon_{\mathrm{R}}^2(M_{\mathrm{R}}'^2)$. *If ε_{R} is independent* of M_{R}, then, by neglecting the second term on the right-hand side of (18) and eliminating the parameters Δ^2, m_{R}^2, and $\varepsilon_{\mathrm{R}}^2$ from (17)–(20), we obtain the Schwinger formula mentioned above. When the term $\Delta^4/M_{\mathrm{R}}^2$ is taken into account, the corresponding formula is (see Ref. 103)

$$(\omega^2-\rho^2)(\varphi^2-\rho^2) = 2\,[\varphi^2-(2\mathrm{K}_{\mathrm{V}}-\rho)^2]\,[(2\mathrm{K}_{\mathrm{V}}-\rho)^2-\omega^2].$$

When this expression is used to calculate the mass of one of the vector mesons from the other three, the result is practically the same as the prediction of the Schwinger formula. Hence, it follows that $\varepsilon_{\mathrm{V}}^2(M^2) = \varepsilon_{\mathrm{V}}^2 \approx \mathrm{const}$ and, since the masses $\rho = M_{\mathrm{V},1}$ and $\omega = M_{\mathrm{V}}$ are very close, the quantity $\varepsilon_{\mathrm{V}}^2$ must be small. The masses of the ω, $\mathrm{K_V}$, and φ can readily be used to find $\Delta_{\mathrm{V}}^2 = \Delta^2 = s^2 - u^2$, $\varepsilon_{\mathrm{V}}^2$, and m_{V}^2:

$$\Delta^2 = 0.1085, \quad m_{\mathrm{V}}^2 = 0.8161, \quad \varepsilon_{\mathrm{V}}^2 = 0.0017. \qquad (22)$$

We note that Δ^2 and m_{V}^2 are determined with good precision (~0.6%), whereas the precision of $\varepsilon_{\mathrm{V}}^2$ is low (~50%), where the uncertainties are largely connected with the experimental uncertainty in the mass of the $\mathrm{K_V}$. The mass of the ρ meson is predicted to be $\rho_{\mathrm{pred}} = 0.774$. The angle $\theta_{\omega} = \theta_{\varphi}$ can be found from (21) and the result is $\theta_{\omega} = \theta_{\varphi} = 0.6$–$1.8°$, which is appreciably lower than the value obtained from the quadratic mass formulas (~5°) and is close to the angle associated with the linear formulas (~2 °C) (see Ref. 11). The parameter Δ^2 is equal to $\mathrm{K_V}(\mathrm{K_V}-\rho_{\mathrm{pred}})$, which follows from (9) and (11). Since $\varepsilon_{\mathrm{V}}^2$ is small, Eqs. (11) and (12) are well satisfied, and to a good precision $\Delta^2 \approx (\varphi^2-\rho^2)/4$.

Since the spread of the masses in the tensor multiplet is not very large, we may suppose that $\varepsilon_{\mathrm{T}}^2(M^2) = \varepsilon_{\mathrm{T}}^2 = \mathrm{const}$. The masses of the f, f′, and $\mathrm{A_2}$ then yield

$$\Delta_{\mathrm{T}}^2 = 0.1521, \quad m_{\mathrm{T}}^2 = 2.038, \quad \varepsilon_{\mathrm{T}}^2 = -0.0132. \qquad (23)$$

The mass of the $\mathrm{K_T}$ is predicted to be $\mathrm{K_T} = 1.426$. This result is somewhat different from that reported in Ref. 11 but, in Table I, it is in good agreement with the weighted average taken over all the experiments analyzed in Ref. 11. Once we know $\Delta_{\mathrm{T}}^2 = \Delta^2 + (\mu_s^2 - \mu_u^2)$ and Δ^2, we can show that

$$\mu_s^2 - \mu_u^2 = 0.0436, \qquad (24)$$

which is appreciably greater than the very rough estimate obtained in Sec. 3 ($\mu_s^2 - \mu_u^2 = [\mu_1^2(s\bar{s}) - \mu_1^2(u\bar{u})]/4 \sim 0.027$). The value given by (24) is, however, in good agreement with the differences between the slopes of the baryon trajectories: $\mu_s^2 - \mu_u^2 = (1/2)[\mu_1^2(suu) - \mu_1^2(uuu)] \sim 0.04$. The spread among the rough estimates is very considerable, whereas the uncertainty in (24) is small. It is clear that (24) is, at present, the most reliable estimate of SU_3^f symmetry breaking in the slopes of the Regge trajectories. Equations (23) and (21) yield $\theta_f = \theta_f \approx -6.4°$ for the mixing angle, which is in agreement with the value obtained for the linear mass formulas.

Since $\Delta_A^2 = \Delta_T^2$, the above results can be applied to axial multiplets. If E and D are the isoscalar states of this multiplet, then (19) and (20) show that

$$E^2 - D^2 = 4\sqrt{(\Delta_T^2 - \varepsilon_A^2)^2 + 8\varepsilon_A^4} \geqslant \frac{8\sqrt{2}}{3}\Delta_T^2, \tag{25}$$

where the equality sign is achieved for $\varepsilon_A^2 = \frac{1}{9}\Delta_T^2$. Therefore, $E \geqslant 1.490$ (8). If $\varepsilon_A^2 = 0$, then $E = 1.502$ (8). In the latter case, $A_1 = D$, whereas for $\Delta_A^2 = (1/9)\Delta_T^2 \simeq 0.017$, we have $A_1 = 1.236$. Correspondingly, $Q_A = 1.395$ and $Q_A = 1.351$. In the latter case, the A_1 is almost degenerate with the B meson, and this means that $Q_A \approx Q_B$. The mixing angle is then $\theta_Q = \pm 45°$, which is not inconsistent with experimental data, but we still cannot exclude the possibility that $\varepsilon_A^2 \simeq 0$, i.e., $A_1 \approx D$. All we can do is to say, with confidence, that, for the pure $q\bar{q}$ state, we have excluded the possibility that $A_1 \lesssim 1.16$ and that it is very probable that the E meson with the mass of about 1.43 is not a pure $q\bar{q}$ state. The conclusion about the mass of the A_1 is based on the fact that (20) and (17) lead to $A_1 \gtrsim 1.16$ for $\varepsilon_A^2 \lesssim (1/2)\Delta_T^2$. The value of $(1/2)\Delta_T^2$ is then unacceptably large—six times greater than $|\varepsilon_T^2|$—and, as we shall show shortly, it is even greater than ε_P^2 for η and η' mesons.

The difference $\mu_s^2 - \mu_u^2$ can be found from the masses g and K_g in a similar way. Neglecting ε_g^2 mixing, and using (17) and (18), we obtain $\Delta_g^2 \equiv \Delta^2 + 2(\mu_s^2 - \mu_u^2)$, and hence $\mu_s^2 - \mu_u^2 \approx 0.038$. This is in good agreement with (24). The masses of mesons with higher spins are not accurate enough to compare them with the mass formulas. Having determined the slope of the Regge trajectories, and neglecting mixing, we can reverse the process and predict the masses of particles with high spins. We shall not pause to do this here and proceed to the analysis of the pseudoscalar multiplet.

To begin with, we assume that the dependence of ε^2 on the mass of the K mesons can be neglected, i.e., $\varepsilon_K^2 \approx \varepsilon_\eta^2 \approx \varepsilon_{\eta}^2 = \varepsilon_P^2$, where $\varepsilon_K^2 \equiv \varepsilon_P^2(K^2)$, and so on. Having determined the two unknown parameters m_P^2 and ε_P^2, we can use $(\eta^2 + \eta'^2)$ and K^2 to show that $m_P^2 = 0.3637$, $\varepsilon_P^2 = 0.0526$ which gives $\eta' \approx 963$, $\eta \approx 0.540$, and $\theta_\eta = \theta_\eta = 34.7°$ (see Refs. 15 and 101). The masses of the η and η' are sufficiently close to the observed values and the mixing angle is in good agreement with the results obtained from data on η and η' production [see (1)]. In fact, for small departures from the QLR in the matrix elements for η and η' production, the ratio $K_{\eta\eta}$ in (1) is related to θ_η and θ_η by[39]

$$K_{\eta\eta'} = \frac{\sin^2\theta_{\eta'}}{\cos^2\theta_\eta}\,\frac{1-\varepsilon'^2}{1-\varepsilon^2}, \tag{26}$$

where ε and ε' determine the impurity of the η and η' states differing from qq':

$$\eta = \sqrt{1-\varepsilon^2}\,(\eta_u c_\eta - \eta_s s_\eta) + \varepsilon\xi, \tag{27}$$

$$\eta' = \sqrt{1-\varepsilon'^2}\,(\eta_u s_{\eta'} + \eta_s c_{\eta'}) + \varepsilon'\xi'. \tag{28}$$

We recall that $\eta_u = \eta'_u = (u\bar{u} + d\bar{d})/\sqrt{2}$, $\eta_s = \eta'_s = s\bar{s}$, $s_\eta = \sin\theta_\eta$, and so on; ξ and ξ' represent all the remaining states, for example, $2g$; the states ξ, ξ' are orthogonal to η_u and η_s. The fact that the masses of the η and η' are satisfactorily described even for constant ε_P^2 leads us to the conclusion that θ_η is not very different from θ_η, so that ε and ε' are sufficiently small quantities. Substituting $\varepsilon = \varepsilon' = 0$ and $\theta_\eta = \theta_{\eta'} = 34.7°$ in (26), we obtain $K_{\eta\eta'} = 0.48$, which is in reasonable agreement with (1). This value of $K_{\eta\eta'}$ does not, however, agree with the usually adopted angles $\theta_\eta = \theta_{\eta'} \simeq 45°$ (see, for example, Refs. 11 and 14).

We note that the singlet-octet mixing angles $\theta_P(\eta)$, $\theta_P(\eta')$ are more frequently used in the literature. Equations (27) and (28) can be rewritten in the form

$$\eta' = \sqrt{1-\varepsilon'^2}\,(\eta_8 \sin\theta_P(\eta') + \eta_0 \cos\theta_P(\eta')) + \varepsilon'\xi', \tag{29}$$

$$\eta = \sqrt{1-\varepsilon^2}\,(\eta_8 \cos\theta_P(\eta) - \eta_0 \sin\theta_P(\eta)) + \varepsilon\xi, \tag{30}$$

where

$$\eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}), \tag{31}$$

$$\eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \tag{32}$$

$$\theta_P(\eta) = \theta_\eta + \theta_0 - 90°, \quad \theta_P(\eta') = \theta_{\eta'} + \theta_0 - 90°, \quad \operatorname{tg}\theta_0 = \frac{1}{\sqrt{2}}. \tag{33}$$

The result $\theta_\eta = \theta_{\eta'} \approx 45°$ corresponds to $\theta_P(\eta) = \theta_P(\eta') \approx -10°$, whereas (21) leads to $\theta_P(\eta) = \theta_P(\eta') \simeq -20°$.

The mixing described by (16) and (19)–(21) differs from the Schwinger description by one interesting property, namely, there is a finite value of ε_P^2 for which the mass of the η meson is a maximum and, at the same time, the η' meson contains the maximum possible $s\bar{s}$ impurity. If ε_P^2 varies from $-\infty$ to $+\infty$, then $\tan\theta_\eta$ increases monotonically from $-1/\sqrt{2}$ to $\sqrt{2}$ and θ_P increases monotonically from $-90°$ to $0°$. When $\varepsilon_P^2 = -\infty$, we have $\eta' = -\eta_8$, $\eta = \eta_0$ and, when $\varepsilon_P^2 = +\infty$, the result is $\eta' = \eta_0$, $\eta = \eta_8$, so that the admixture of strange quarks in the η' is a maximum for $\varepsilon_P^2 = -\infty$. However, this impurity is also a maximum for $\varepsilon_P^2 = \Delta^2/2$, for which $\operatorname{tg}\theta_\eta = +1/\sqrt{2}$. In that case, η and η' are, respectively, equal to η_0 and η_8 but with a different sign in front of $s\bar{s}$,[5)] and the mass of the η meson determined from (20) assumes the maximum value

$$\eta^2 = m_P^2 - \frac{2}{3}\Delta^2; \tag{34}$$

for which $\eta'^2 = m_P^2 + 16/3\,\Delta^2$, $K^2 = \eta^2 - \Delta^4/K^2$. The octet-singlet mixing angle is $\theta_P \simeq -19.5°$ and $K_{\eta\eta} = 1/2$, which is in good agreement with (1). If this description gives

[5)] Since the wavefunctions of η_0 and η_8 are proportional to the matrix elements of the operators $\bar{q}\lambda_0\gamma_5 q$ and $\bar{q}\lambda_8\gamma_5 q$, we can obtain η and η' from η_0 and η_8 by means of the γ_5 transformation of the u and d quarks: $q \to \mathscr{P}\gamma_5 q$, $\bar{q} \to -\bar{q}\gamma_5\mathscr{P}$, where $\mathscr{P} = (\lambda_8 + \sqrt{2}\lambda_0)/\sqrt{3}$ is the projection operator for the u and d quarks. Since the masses of the current quarks u and d are very small,[128] the corresponding Lagrangian is almost invariant.

the correct mass of the π meson, then it can be regarded as fully satisfactory. However, the value of M_π turns out to be greater than the observed result by a factor of two [see (17)] and it is essential to take into account the dependence of ε_P^2 on M^2. A reasonable description of the mixing and of the K, η, η' masses that differs from the results reported in Refs. 114, 119, 124, and 127 has been achieved because the mass of the π mesons has not been used in the determination of the parameters. If we adopt the usual quadratic mass formulas [i.e., omit Δ^4/K^2 from (18)], then, as already noted, we obtain the Schwinger formula which predicts that $\eta' \sim 1.6$. This was not noted in the references cited above because the procedure adopted there was to determine ε_P^2 from the sum of the squares of the masses $\eta^2 + \eta'^2$. All this finally gives the impression that the mass formulas give a reasonable description of the pseudoscalar multiplet. The reason for this curious phenomenon is that the mass of the η is close to the maximum value, and a small change in the mass of the η produces a very considerable change in the mass of the η'.

Before we proceed to the investigation of the dependence of ε_P^2 on M^2, we note that there is another possibility whereby the description of the pseudoscalar multiplet can be improved, namely, one could try and introduce SU_3^f symmetry breaking into the mixing matrix (15).[129] Instead of the single parameter ε_P^2, one must then introduce the three parameters $\varepsilon_P^2(u\bar{u})$, $\varepsilon_P^2(u\bar{s})$, $\varepsilon_P^2(s\bar{s})$ and, to determine all the unknown quantities, one must specify not only the π, K, η, η' masses, but also the mixing angle θ_P. Using the mixing model based on the effective Lagrangian in QCD,[121-124] it is possible to reduce the number of unknown parameters down to two[130] (see also Ref. 131). However, the symmetry breaking in ε_P^2 turns out to be very large (>50%) and the source of this is not clear.

The source of the dependence of $\varepsilon_P^2(M^2)$ on M^2, on the other hand, seems almost obvious, namely, the dependence of the strong interaction constant $\alpha_S(M^2)$ on M^2. Since ε_P^2 is determined by nonperturbative effects (see Refs. 33, 121–126, 130, and 131), it was reasonable to suppose that

$$\varepsilon_P^2(M^2) = M_0^2 \exp\left(-\frac{\bar{\lambda}}{\alpha_s(M^2)}\right), \tag{35}$$

where $\bar{\lambda}, M_0$ are unknown parameters. The function $\alpha_S(M^2)$ is known only for $M^2 \to \infty$ [23,24]:

$$\alpha_s^{-1}(M^2) \sim c_f^{-1} \ln \frac{M^2}{\Lambda^2}, \quad c_f = \frac{12\pi}{33-2n_f}.$$

This expression cannot, of course, be used for $M^2 \lesssim \Lambda^2$, but one could try to extrapolate it to small M, so that α_S becomes infinite only for $M^2 = 0$:

$$\alpha_s^{-1}(M^2) = c_f^{-1} \ln\left(\frac{M^2}{\Lambda^2} + 1\right). \tag{36}$$

A similar extrapolation was proposed in a slightly different form in Ref. 132 and 103 and was subsequently used to extrapolate the QCD potential for heavy quarks from the region of small distances, where it is known, to the region of large distances.[133-135] In the latter references, Λ was assumed to be 400–500 MeV, whereas the value adopted in Refs. 103 and 132 was $\Lambda \sim 100$ MeV. It will be shown below that this value of Λ is actually necessary for the description of pseudoscalar mesons. A small value of Λ is obtained from data on e^+e^- annihilation in the region of ψ and Υ (see Refs. 136 and 137), whereas the large value is obtained from data on deep inelastic processes. Determinations of Λ from e^+e^- annihilation are theoretically more justified (see Refs. 137, 138, 33, and 125).

Before we determine the parameters $\bar{\lambda}$, M_0, and Λ from the masses of the pseudoscalar mesons, we note that (17), which gives the mass of the π meson, has the following remarkable property.[103] If $-\varepsilon_P^2(M^2)$ – is a convex increasing function of M^2 [this property is satisfied in (35)], then (17) has no solutions for a sufficiently large value of M_0^2. As M_0^2 is reduced, a point is reached at which a solution which determines the minimum possible value of M_π is found to appear. (Further reduction of M_0 leads to two solutions but the solution with the lower mass is unstable.) The condition for the minimum mass of the π meson has the simple form[103]

$$\frac{d\varepsilon_P^2(M_\pi^2)}{dM_\pi^2} = -\frac{3}{4}. \tag{37}$$

When ε_P^2 is given by (35) and (36), the result is the simple equation

$$\Lambda^2 + M_\pi^2 = \frac{4}{3}\lambda\varepsilon_P^2(M_\pi^2), \quad \lambda \equiv \frac{\bar{\lambda}}{c_f}. \tag{38}$$

Knowing π, η, and η' and using (17), (34), (20), and (38) together with the above value of Δ^2, we find that

$$\Lambda = 0.0977, \quad M_0 = 0.3594, \quad \lambda = 0.2055, \quad m_P^2 = 0.3735. \tag{39}$$

The following prediction results therefrom:

$$K_{\eta\eta'} = 0.486, \quad \theta_\eta = 37.9^\circ, \quad \theta_{\eta'} = 33.9^\circ. \tag{40}$$

The resulting value $\Lambda \simeq 98$ MeV is also in good agreement with experiment.[136]

Thus, the structure of the pseudoscalar multiplet can be understood at the semiphenomenological level. QCD provides a partial explanation of the phenomenological parameters that have been introduced but, unfortunately, further parameters are inserted at the same time and these cannot be calculated on a theoretical basis. Nevertheless, it is now clear that the complex structure of the pseudoscalar multiplet reflects the complex structure of vacuum in QCD and is closely related to the confinement of the color of, in particular, quarks. The mixing parameter ε_P^2 is directly related to vacuum fluctuations that are not described by perturbation theory (see Refs. 33, 121–126, 130, and 131 for further details). It is probable that analogous fluctuations govern the SU_3^f symmetry breaking as well; this is indicated in particular by the expression $\varepsilon_P^2(\eta^2) \sim \frac{1}{2}\Delta^2$ found above [and, incidentally, by $\varepsilon_P^2(M_\pi^2) \sim \Delta^2$ and $M_0^2 = \varepsilon_P^2(0) - \varepsilon_P^2(M_\pi^2)$, as well]. We note that the parameter μ_F^2 which determines the spin-spin splitting is almost exactly equal to Δ^2: $4\mu_F^2 \approx m_V^2 - m_P^2 = 0.445$. If we substitute $m_0^2 = 0$ in (14), we can determine the masses of u and s: $u \simeq 350$ MeV, $s \simeq 480$ MeV, which are very close to the masses of the constituent quarks used in baryon spectroscopy. Moreover, $u^2 \simeq 0.12 \sim M_0^2 \sim \Delta^2$. One thus gains the impression that, in addition to the main dimensional parameter $\Lambda \sim 0.1$ GeV, we need a further dimensional parameter, say, $M_0 \sim 350$ MeV, which determines the masses of the quarks, SU_3^f symmetry breaking, mixing, and so on. This parameter is determined by vacuum fluctuations and cannot be calculated from perturbation theory. In rigorous theory, the two parameters are probably related.

Having separated the mixing and spin-splitting effects in pseudoscalar and vector mesons, we can estimate the spin-orbit and spin splitting in the $L=1$ multiplet, and find the slope of the Regge trajectory $4\mu_u^2$. If, as discussed above, $\varepsilon_A^2 = \frac{1}{9}\Delta_T^2 \sim |\varepsilon_T^2|$, $A_1 \sim B$ and $\varepsilon_B^2 = -0.012 \approx \varepsilon_T^2$, then $\mu_L^2 \sim 0.083$, $\mu_F'^2 \sim 0.033$, and $4\mu_u^2 \sim 1.11$. These estimates of μ_L and μ_F' cannot, of course, be regarded as final, but the slope of the Regge trajectory is determined with sufficient accuracy (to within better than about 10%). This slope is in agreement with the average value of about 1.15, found above from the high-spin particle masses. It is curious that $\varepsilon_B^2 \approx \varepsilon_T^2 < 0$ whereas $\varepsilon_A^2 > 0$ and has the same order of magnitude.

We recall that certain assumptions that cannot be justified (say, $m_0 = 0$) have to be introduced to determine the absolute quark masses. It is possible, however, to find the minimum mass of the s quark: $s \geq s_{min} = \Delta = 0.330$. When s is close to s_{min}, the masses of the u and d quarks should be small and the chiral limit is reached.[139] The masses cannot, however, be compared directly with the current masses[128] $s^* = 150$ MeV, $d^* = 7.5$ MeV, $u^* = 4.2$ MeV because they differ by the renormalization factor Z. The estimate for this factor is $Z = s^*/s_{min} \approx 0.46$. The constant Z can be calculated only in specific models. For example, in the Massachusetts bag model, it can be shown that[140] $Z \sim 0.5$, which is in good agreement with our estimate.

The above phenomenology can be applied to the analysis of the spectrum of charmed particles. If $(\mu_F^2(c\bar{c}) \sim \mu_F^2(c\bar{u}) \sim \mu_F^2(u\bar{u}))$, the masses of J/ψ and D_V can be used to show[11] that $\Delta_{cu}^2 = c^2 - u^2 \approx 2.187$ and $m^2 \equiv \bar{m}_0^2 = 2u^2 + 2c^2 \approx 5.106$. (We are not assuming that $\bar{m}_0^2 = m_0^2$.) This leads to the following predictions: $D \approx 1.83$, $\eta_c \approx 3.02$, $F \approx 1.97$, $F_V \approx 2.1$. The agreement with experimental values[11] must be admitted to be sufficiently good, especially if we recall the exceptional simplicity of the model. We draw attention to the fact that η_c is relatively large. Its value cannot be reduced even by taking into account symmetry breaking in the spin interaction. This phenomenological result was obtained in Ref. 15 and was then discussed in detail in Refs. 100–102, where it was shown that the mass of the pseudoscalar state $c\bar{c}$ cannot be made less than 3 GeV. In the simplest potential models, the $J/\psi - \eta_c$ splitting was also predicted to be small[114] but, when the number of parameters was later increased, the mass of η_c was reduced to about 2.8 GeV, as seemed necessary in the light of experimental data. Phenomenology does not allow us to do this provided J/ψ, D_V, D, and η_c are pure quark-quark states.[100-102] Inclusion of $\psi - \psi'$, $\eta_c - \eta_c'$ GeV mixing can slightly modify the predictions but, whatever happens, we are still left with the fact that $\eta_c \gtrsim 3$ GeV. This result was subsequently obtained with the aid of the dispersion sum rules in QCD[141]; the most reliable mass estimate is $2.98 \leq \eta_c \leq 3.02$.[142]

Having taken the average of the squares of the masses of the χ/P_c particles[11] in order to exclude the contributions of spin-orbit and tensor terms, one can then show that the slope of the Regge trajectory is $4\mu_c^2 \approx 2.89$. The mass of the ψ' can be used[11] to estimate the parameter β in (14) for the $c\bar{c}$ state, and the result is $\beta(c\bar{c}) \sim 1.37$.[6)] As noted above, the value corresponding to the radial excitation of $\rho'(1.6)$ is $\beta(u\bar{u}) \sim 1.80$. This estimate is not very accurate because the mass of the ρ' is uncertain and so is the spin interaction between radial excitations, but the inequality $\beta(c\bar{c}) < \beta(u\bar{u})$ seems reasonable if we take into account the above effect associated with the nonlinearity of the trajectories. We note also that the slope $4\mu_c^2$ determined from the mass of the D state of $\psi'(3.77)$ is appreciably smaller than the value found from the P state.

This brief digression into an area that is not covered by the theme of the present review was made in order to estimate the reliability of the phenomenology. We have satisfied ourselves that, even when symmetry breaking is considerable, the phenomenological mass formulas yield reliable predictions and reasonable estimates for the quark masses.

In conclusion, let us briefly consider light-meson decays. Existing data will, in principle, allow us to estimate QLR violations (in particular, the mixing angles) and the SU_3^f symmetry breaking. SU_3^f breaking in $V \to e^+e^-$ is the most noticeable, which is not surprising because these processes are connected with the annihilation of quarks whose masses are very different. To achieve a clearer picture of the overall situation, it is useful to consider the decays of not only $V = \rho, \omega, \varphi$ but also $V = \psi, \Upsilon$. In the nonrelativistic quark model, $\Gamma_V^e \equiv \Gamma(V \to e^+e^-) \sim Q_V^2 |\Psi_V(0)|^2/m_V^2$ [143,144] (see also Refs. 13 and 14), where Q_V is the effective charge of the quarks in the V meson: $Q_\rho^2 = 1/2$, $Q_\omega^2 = 1/18$, $Q_\varphi^2 = Q_\Upsilon^2 = 1/9$, $Q_\psi^2 = 4/9$.[7)] A similar expression for $\Gamma(V \to e^+e^-)$ is obtained in relativistic theory as well but, instead of $|\Psi_V(0)|^2$, the relativistic result contains an integral of $|\Psi_V(r)|^2$ over a region whose size is of the order of the Compton wavelength of the corresponding quark, and the nonrelativistic expression is obtained when the quark mass is large enough.[145-147] Experiment indicates that the ratio Γ_V^e/Q_V^2 is approximately constant for $V = \rho, \omega, \varphi, \psi, \Upsilon$, and is equal to 11–13 keV. This means that $SU_{n_f}^f$ symmetry is strongly violated in the wavefunctions Ψ_V. One could also try to relate the symmetry breaking with the quark mass.[148-150] We note that it can be related to symmetry breaking in the slopes of the Regge trajectories by taking the oscillator potential which correctly describes the empirical slopes. Similar SU_3^f symmetry breaking is observed in $P \to l\bar{\nu}_l$ decays, for example, $\pi \to \mu\bar{\nu}_\mu$, $K \to \mu\bar{\nu}_\mu$, and so on.[143-150]

The effects of SU_3^f breaking in strong decays of mesons are much more difficult to exhibit. Firstly, the measured widths are not sufficiently accurate and, secondly, it is not entirely clear how kinematic factors, which, of course, contain the frequently very considerable SU_3^f breaking, are to be taken into account. In the

6) These values of μ_c and β can be used to predict the mass of the radial excitation $\eta_c' \approx 3.59$ GeV.

7) The small mixing angle $\theta_\omega = \theta_\varphi$ can be neglected in this context.

most common approach to the two-particle decays, it is taken into account with the aid of the kinematic factor $\Gamma = |\mathcal{M}_{fi}|^2 p_f^{2J+1}/M^2$ where $\mathcal{M}_{fi}$ is the matrix element for the decay, p_f is the momentum of the final decay product in the center-of-mass system, and M is the mass of the decaying particle.[16,39] Other kinematic factors[18,19] may turn out to be natural from the standpoint of the quark model. It is also necessary to take into account corrections for the finite width of the resonances and the coupling between different decay channels. If we ignore all these obscurities and corrections, and simply evaluate the ratio of the matrix elements using SU_3^f[151] and the known mixing angles for the P, V, and T nonets, we can verify that the SU_3^f predictions are satisfied with reasonable precision. A sufficient volume of reliable data is available for $V \to PP$, $T \to VP$, $T \to PP$, and $3^- \to PP$ decays; the SU_3^f ratios of the corresponding widths are given in Okubo's paper[39] with allowance for the mixing angles. Comparison with experimental data shows that SU_3^f symmetry can be regarded as unbroken in these decays to within experimental error. Actually, the theoretical and experimental ratios $\Gamma(f \to K\bar{K})/\Gamma(f \to \pi\pi)$, $\Gamma(g \to K\bar{K})/\Gamma(g \to \pi\pi)$, $\Gamma(\rho \to \pi\pi)/\Gamma(K_V \to K\pi)$, $\Gamma(A_2 \to K\bar{K})/\Gamma(K_T \to K\pi)$ differ by two to three standard errors. However, the g, A_2, $f \to K\bar{K}$ decay widths cannot be regarded as finally established and, moreover, corrections for the finite widths in f, g, $\rho \to \pi\pi$ decays can be quite large. The discrepancy can be connected with the kinematic factor which is particularly important in the first two of the above ratios. A model based on the Melosh transformation can be used to obtain further ratios of the widths of two-particle decays, which do not contradict existing data with roughly the same level of precision with which the SU_3^f symmetry ratios were obtained (see Refs. 18 and 19).

Unfortunately, it has not been possible to deduce the mixing angles from these data. All that can be done is to verify that the angles determined from the mass formulas are not in conflict with the decay data. More accurate information on the mixing angle $\theta_\omega = \theta_\varphi$ can be obtained by comparing the φ, $\omega \to \pi^+\pi^-\pi^0$ decay widths. Assuming that both decays proceed through intermediate two-particle $\rho\pi$ states, we find that $\theta_\omega = 2.4 \pm 0.3°$ (see Ref. 152).

Let us now consider the radiative decays $V \to P\gamma$, $P \to V\gamma$, $P \to \gamma\gamma$. Extensive literature is available on the analysis of these decays (see, for example, Refs. 12, 14, 19, 39, and 151–156). In the quark model, the widths of these decays are expressed in terms of the magnetic moments of the quarks, which are determined from the magnetic moments of the baryons (see Refs. 12, 14, 147, 154, and 156). However, this involves the utilization of highly simplified hypotheses on the structure of mesons and baryons, which are difficult to justify. Moreover, existing data on the magnetic moments of baryons[11] appear[8)] to be in conflict with these very simple ideas on baryon structure. If we calculate the magnetic moments of the u, d, s from the magnetic moments μ_p, μ_n, and μ_Λ, the predicted values for μ_{Ξ^-} and μ_{Σ^-} differ from the experimental results by a factor of 1.5 (the discrepancy exceeds four standard errors in the case of the μ_Ξ). Finally, SU_3^f symmetry breaking can readily be taken into account in this primitive model, but QLR breaking cannot be introduced into the matrix elements (with the exception of the breaking described by the mixing angles). It has been shown[157] that this QLR breaking cannot be neglected. It is not surprising that the agreement between the model and experiment is not too good, especially for the $\omega \to \pi\gamma$, $\rho \to \pi\gamma$ decays which depend neither on $\eta - \eta'$ mixing nor on SU_3^f breaking in magnetic moments (see Ref. 154). It can, of course, be assumed that the quark magnetic moments in mesons are different from those in baryons, but the model then becomes less attractive and essentially reduces to the phenomenological description of $V \to P\gamma$, $P \to V\gamma$ decays with the simplest SU_3^f symmetry breaking.

We therefore turn to the phenomenological analysis of radiative decays, taking into account QLR breaking and certain special SU_3^f symmetry breaking. If SU_3^f is retained, the widths $\Gamma(V \to P\gamma)$ and $\Gamma(P \to V\gamma)$ can be expressed in terms of the matrix elements of the octet vector current J_i^μ $(i = 1, \ldots, 8)$:

$$\langle V_i \mid J_j \mid P_k \rangle = g d_{ijk}, \quad \langle V_0 \mid J_j \mid P_i \rangle = (g + \varepsilon) d_{0ij}, \quad \langle V_i \mid J_j \mid P_0 \rangle = (g + \delta) d_{0ij}, \tag{41}$$

where $d_{0ij} = \sqrt{2}/3\delta_{ij}$, V_i, P_i are the octet states, and V_0, P_0 are the singlet states. The dependence on polarization and momenta, and the normalization factors, are omitted. The exact QLR corresponds to $\varepsilon = \delta = 0$, so that ε and δ describe the QLR breaking that was not taken into account in $\omega - \varphi$ and $\eta - \eta'$ mixing. In standard notation,[158] $g = g_{\rho\pi\gamma} = -\frac{1}{2} g_{K_V^0 K^0 \gamma} = g_{K_V^- K^- \gamma}$, $g_{\omega\pi\gamma} \approx 3g + 2\varepsilon$, $g_{\varphi\pi\gamma} \approx 3g\,\mathrm{tg}\,\theta_\omega + \sqrt{2}\varepsilon$, and so on (the expressions for $g_{\omega\pi\gamma}$ and $g_{\varphi\pi\gamma}$ take into account the fact that θ_φ and ε are small).

The following SU_3^f symmetry breaking mechanism, which changes the ratio of the constants $g_{\omega\pi\gamma}$ and $g_{\rho\pi\gamma}$, is noted in Ref. 15. Consider the transitions $V \to (V'P'^*) \to (V'P'^*)\gamma \to P\gamma$, $P \to (V'P'^*) \to (V'P'^*)\gamma \to V\gamma$, where the parentheses indicate virtual particles and P'^* is π^* or K^*. Since the masses π and K are sufficiently small, these transitions can introduce appreciable corrections. These corrections are very large (about 100%) when $M_\pi = M_K = 0$, but they fall rapidly as the mass of P′ increases.

Estimates of the simplest diagrams describing these transitions (VV′P and V′P′P vertices are estimated with the aid of SU_3^f from $g_{\rho\pi\pi}$ and $g_{\omega\rho\pi}$ or $g_{K_V K \pi}$) show that this mechanism can increase the constant $g_{\omega\pi\gamma}$ by 15–20%.[9)]

It is possible to introduce the single parameter λ so that the most important effects are taken into account by shifting the constant $G \equiv 3g$ in the $\omega\pi$ transition by 8λ,

[8)] The magnetic moment of the Ξ^- has recently been determined with good precision (see Ref. 10).

[9)] Divergent integrals were truncated in the π-meson momentum as follows: $|P\pi| \leq \Lambda \sim M_\rho$. The justification for this cutoff can be found in Ref. 159, where the sum of the ladder diagrams is evaluated in another model which is equivalent to that discussed here. It is shown there that the sum does not contain any divergences and that it can be estimated by evaluating the simplest diagram subject to this cutoff.

in the $K_V K^-$ transition by 3λ, in the $\eta'\omega$ transition by 8λ, and so on. Theoretical estimates of λ yield $\lambda \sim 0.04$–$0.05\ \mathrm{GeV}^{-1}$. The parameters $G = 3g$, ε, and δ were determined in[157] from all the existing data for given $\omega - \varphi$ and $\eta - \eta'$ mixing angles [see (22) and (40)]. The λ-correction was included only in the $\omega\pi\gamma$, and the $P \to \gamma\gamma$ decay was evaluated with the aid of the vector dominance model. All decays other than $\eta \to \gamma\gamma$ could be reasonably described, and the prediction $\Gamma(\eta \to \gamma\gamma) \approx 720$ eV is close to the average obtained in the two experiments reported in Refs. 34 and 35 and is greater by about a factor of 2 than the value adopted in Ref. 11. The latter is in good agreement with the standard angle $\theta_P = -10^\circ$ if there is weak SU_3^f breaking in $P \to \gamma\gamma$ decays. This point was also noted in Refs. 160 and 161, where, however, the $V \to P\gamma$ and $P \to V\gamma$ decays were not investigated. We note that VDM, current algebra, and chiral model[161] give equivalent results[157] for $P \to \gamma\gamma$ decays. Bearing in mind the good agreement between the $V \to P\gamma$, $P \to V\gamma$, $\pi^0 \to \gamma\gamma$, $\eta' \to \gamma\gamma$ data and (1) and (40), we may conclude that a new measurement of $\Gamma(\eta \to \gamma\gamma)$ is essential, possibly by a new method (for example, by using the two-photon process $\gamma\gamma \to \eta$ in e^+e^- collisions). Since the data used in Ref. 157 were improved in the course of the last year ($K_V^- \to K^-\gamma$, $\rho^- \to \pi^-\gamma$, $\eta^- \to \omega\gamma$), they were reanalyzed by the present author together with O. Rasizade. The values of the angles θ_η, θ_η were not fixed, all the λ corrections were taken into account (λ was not fixed), the $V \to e^+e^-$ widths were used in the analysis (γ_V was not fixed), the width $\Gamma(\eta \to \gamma\gamma)$ was not included in the "fit," and only the relative width $B(\eta \to \pi^+\pi^-\gamma)$ calculated in accordance with VDM was employed; $\Gamma(\pi \to \gamma\gamma)$ and $\Gamma(\eta' \to \gamma\gamma)$ were also calculated by VDM. Altogether, and including (1) [we substituted $\varepsilon = \varepsilon' = 0$ in (26)–(28)], we used 18 experimental values. The independent variable was the ratio $\Gamma(\eta' \to \rho\gamma)/\Gamma(\eta' \to \omega\gamma)$, which is known to a better precision than the widths themselves. The best values of the 9 free parameters (for $\theta_\omega = \theta_\varphi = 1.5^\circ$) are as follows:

$$\left.\begin{array}{c} G = 1.983, \quad \delta = 0.228, \quad \varepsilon = 0.054, \quad \lambda = 0.059\ (\mathrm{GeV}^{-1}), \\ \theta_\eta = 34.0^\circ, \quad \theta_{\eta'} = 37.5^\circ, \\ \frac{\gamma_\rho^2}{4\pi} = 0.509, \quad \frac{\gamma_\omega^2}{4\pi} = 5.25, \quad \frac{\gamma_\varphi^2}{4\pi} = 3.57. \end{array}\right\} \tag{42}$$

For this fit, $\chi^2/9 = 1.66$ and the maximum contribution to χ^2 was due to the $K_V^0 \to K^0\gamma$ and $\omega \to \eta\gamma$ decays, whose widths were obtained in a single experiment with poor statistics. When these are excluded, we obtain $\chi^2/7 = 0.893$ for the same fit. The predicted $\rho^- \to \pi^-\gamma$ width is smaller than the experimental value by ~1.5 standard errors, and the corresponding figure for $K_V^- \to K^-\gamma$ decays is 1.3. The widths of these decays were determined in a single although very difficult experiment using the Primakoff effect.[10)] The remaining predictions differ from the experimental values by less than one standard error, so that, on the whole, the fit is very good. The angles θ_η, θ_η are in good agreement with (40); and the value of λ is close to the theoretical estimate 0.04–0.05. The maximum violation of QLR ($2\delta/G \sim 0.23$) is comparable with the maximum SU_3^f breaking ($8\lambda/G \sim 0.24$). It would be interesting to elucidate the origin of the δ correction in QCD language. We recall that this correction describes QLR violation in the transition of the P singlet into the V octet, and its relatively large value may well be connected with the same mechanisms that create the considerable mixing in the P multiplet. The following numerical relationships are interesting from this point of view:

$$\left(\frac{2\delta}{G}\right)^2 \approx \frac{\varepsilon_{\eta'}^2}{\bar{M}^2}, \quad \left(\frac{2\varepsilon}{G}\right)^2 \sim \frac{\varepsilon_\varphi^2}{\bar{M}^2},$$

where $\bar{M}^2 = m_0^2 + 2(u^2 + s^2) \approx 0.71$, and $\bar{M}$ is of the order of the mean mass of the vector multiplet.

The other decays of pseudoscalar mesons, $\eta \to 3\pi$, $\eta' \to \eta\pi\pi$, $\eta \to \pi^0\gamma\gamma$, are also very interesting. Useful information on the structure and interaction of P mesons can be extracted from data on weak and electromagnetic-weak decays of π and K. The most complete results on these decays have been obtained in chiral quark models (see, for example, Refs. 139 and 162; the connection with QCD is discussed in Refs. 121–124, 130, and 131). Unfortunately, phenomenology alone does not allow us to say practically anything about complex processes such as $\eta \to 3\pi$, $\eta' \to \eta\pi\pi$. However, one can readily estimate $\Gamma(\eta \to \pi^0\gamma\gamma)$ by using VDM and the known constants $g_{V\eta\gamma}$ and $g_{V\pi\gamma}$: $\eta \to (V)\gamma \to \pi^0\gamma\gamma$, $V = \rho, \omega, \varphi$. This yields $\Gamma(\eta \to \pi^0\gamma\gamma) \sim 0.5$ eV, which is roughly equal to the upper limit obtained at IFVE[36] (see Table I). It is stated in Ref. 164 (see also Ref. 162) that the chiral model predicts a much smaller width, namely, ~0.01 eV. If this estimate is correct, there must be some unknown mechanism that reduces the contribution of the $\eta \to (V)\gamma \to \pi^0\gamma\gamma$ transition.

Data on radiative decays of other mesons will probably be just as interesting for the theory as those examined above. For example, measurements of the width $\Gamma(f \to \gamma\gamma)$[47,48] (see Table I) can be used to discriminate against models in which the predicted width is $\lesssim 1$ keV[165] or $\gtrsim 7$ keV (see, for example, Refs. 166 and 167). Simple quark-model estimates yield the best agreement (for example, Refs. 160 and 168).

New data have recently appeared which substantially modify some of the details of the overall picture in light-meson spectroscopy. We shall reproduce those that were reported at the International Conference on Lepton and Photon Interactions at High Energies (Bonn, August 1981) and are described in the review papers in Refs. 182–184.

The Crystal Ball group[182] has established that the particle X(1440), discussed above, which was previously seen[69] in $J/\psi \to \gamma X$ decays, is a pseudoscalar $\delta\pi$ resonance $J/\psi \to \gamma\pi^\mp\delta^\pm \to \gamma\pi^\mp K_0 K^\pm$ and cannot, therefore, be identified with the axial meson E(1420). They have examined the possible interpretation of this particle, given the designation $\iota(1440)$ in Ref. 182, as a pseudoscalar gluonium, but it is very probable that this is the radial excitation $s\bar{s}(0^{-+})$. We recall that there are candidates for other pseudoscalar radial excitations, namely, $\pi'(1200)$, $\eta_{R_1}(1275)$. The same group has found the new resonance $\theta(1640)$ in the decay $J/\psi \to \gamma\theta \to \gamma\eta\eta$; $M_\theta = 1640(50)$, $\Gamma_\theta = 220^{+100}_{-70}$; the most probable inter-

[10)]We note that our fit gives $\Gamma(\rho^- \to \pi^-\gamma)/\Gamma(K_V^- \to K^-\gamma) = 1.28$, whereas the experimental value for this ratio is 1.08 ± 0.27. The agreement is appreciably better.

pretation is that $J^{PC} = 2^{++}$.[182] One cannot exclude the possibility that this particle is gluonium. To verify this suggestion, it is necessary to show that θ is an SU_3^f singlet. This means that one will have to find the other decay channels and verify the SU_3^f selection rules.[185] The other possibility is that θ is the exotic state $q\bar{q}q\bar{q}$. Such states (mass ~1650) were predicted previously,[27] and attempts are being made at present to explain known anomalies in the $\gamma\gamma \to \rho^0\rho^0$ reaction near the threshold[183] in terms of the creation of these exotic mesons.[186] The difficulty in this interpretation is the relatively low value of Γ_θ. On the other hand, if θ is gluonium, then one would expect a relatively small width $\Gamma(\theta \to \gamma\gamma)$, and the creation of θ in two-photon processes should not occur at a high rate.

The Crystal Ball detector has also been used to obtain clear evidence for the existence of η_c and η_c' in $\psi' \to \gamma\eta_c, \gamma\eta_c'$ decays. The masses $M_{\eta_c} = 2984(4)$ and $M_{\eta_c'} = 3592(5)$ are in good agreement with the above predictions, and the relation $M_{\psi'}^2 - M_\psi^2 = M_{\eta_c'}^2 - M_{\eta_c}^2$, which follows from the above mass formulas, is particularly well satisfied. The widths $\Gamma_{\eta_c} = 12.4(4.1)$ MeV (95% confidence level) are also consistent with the theoretical estimates.[22,142]

New data on f^0, A_2^0, and η' decays have been obtained from two-photon processes produced in e^+e^- beams[183]: $\Gamma(f^0 \to \gamma\gamma) \approx 4$ keV (4 experiments); $\Gamma(A_2^0 \to \gamma\gamma) \approx 1$ keV (2 experiments); $\Gamma(\eta' \to \gamma\gamma) = 5.9 \pm 1.6 \pm 1.2$ keV; $\Gamma(\eta' \to \gamma\gamma) = 7.5 \pm 0.7$ keV. The results for f^0 and η' are in agreement with those listed in our Table I.

Many new data have been reported[184] on $\rho'(1600)$. In particular, it was established that, in the $\rho' \to \rho\pi\pi$ decay, the main channel cannot be $\rho' \to \rho\varepsilon \to \rho\pi\pi$ and, in fact, the main channel is probably $\rho' \to \pi A_1 \to \pi\pi\rho$. There is some evidence for the $\rho' \to \eta\pi^+\pi^-$ decay, and a rough estimate has been obtained for the leptonic width of ρ': $(\rho' \to e^+e^-) \sim 5$ keV.[184] The status of the resonance $\rho'(1250)$ is still unclear. There are as yet no data for which this resonance is essential, but it cannot be excluded absolutely. We note that a theoretical model has recently been developed in detail in which $\rho'(1250)$ is the first radial excitation and $\rho'(1600)$ is the second.[187] To elucidate the situation, it is very desirable to find the K_V', ω', and φ' radial excitations. New indications have recently appeared for the existence of a vector resonance with a mass ~1650 in the $K_S^0 K^\pm \pi^\mp$ and K^+K^- systems (see Ref. 184). This may be the radial excitation φ'.

The authors of Ref. 36 have completed their analysis of the data on $\eta \to \pi^0\gamma\gamma$ decays and found that $B(\eta \to \pi^0\gamma\gamma) = (0.095 \pm 0.025)\%$.[190] We note that the results mentioned in the text of the calculations, which gave the low result $\Gamma(\eta \to \pi^0\gamma\gamma) \sim 0.01$ eV in the chiral model,[162] have been reexamined in Ref. 188, in which the chiral model was modified in the spirit of the hybrid quark model.[189] The width $\Gamma(\eta \to \pi^0\gamma\gamma)$ was estimated in Ref. 188 from $\eta \to \gamma(\rho, \omega) \to \gamma\gamma\pi^0$ vector dominance, but the constants $g_{VP\gamma}$ were calculated with the aid of quark loops, and the estimated $\Gamma(\eta \to \pi^0\gamma\gamma)$ was close to the vector dominance width[163] (see the text).

5. CONCLUSIONS

The data presented above and their phenomenological analysis undoubtedly show that the quark model is capable of explaining the main regularities in meson spectroscopy. Most probably, a quantitative theory can be constructed only on the basis of QCD. The essential step in this is the justification of the quark confinement hypothesis if only with the aid of numerical calculations, and one must learn how to evaluate the fundamental phenomenological parameters such as the slope of the Regge trajectories, the quark masses, the mixing parameters, the spin and spin-orbit splitting, and so on, without resorting to perturbation theory. Judging by what is known today (see Refs. 33, 103, and 121–131), one is reasonably safe in suggesting that this aim will be achieved in the near future.

The idea involving the expansion in terms of the number of colors (the $1/N_c$ expansion; see, for example, Refs. 169–172) brings us closer to the solution of these problems than other approaches. This expansion enables us to understand the main qualitative features of light-meson spectroscopy, namely, the small width $\Gamma_R \sim m_R/N_c$ and the very existence of the $q\bar{q}$ resonances, the suppression of multiquark and gluon components in them, the QLR rule, and the suppression of multiparticle decays of the resonances.[172] The color confinement hypothesis fits naturally into the $1/N_c$ expansion, and the principal approximation contains all the main ingredients of the Regge and dual phenomenologies.

Substantial advances in the quantitative understanding of light-meson spectroscopy have been achieved by elucidating the role of nonperturbative vacuum fluctuations whose classical examples are the instantons[173,174] and the Gribov ambiguities.[175] These ideas have led to a justification and improvement, at least in general outline,[176-178] of the QCD-bag model[179,180] and a qualitative explanation of the origin of strong mixing in the pseudoscalar multiplet.[33,125,126] The unification of these ideas with the $1/N_c$ expansion[121-124,130,131] has been particularly fruitful and has brought us very close to the solution of the well-known U_1 problem[128] and to a deeper understanding of the role of the pseudoscalar multiplet in QCD.

It is less clear how one can tackle problems connected with SU_3^f symmetry breaking, namely, the origin of the quark masses and their splitting, the origin of symmetry breaking in the slopes of Regge trajectories and in particle decays, and so on. There is a fairly generally held view that it will be sufficient to solve the QCD problems in a world without quarks. It is possible, however, that quarks with zero bare mass play an appreciable role in the color confinement mechanism (see Gribov's paper[181] on the screening of massless charges in gauge theories). At any rate, a detailed quantitative theory of light mesons can hardly be constructed without a clear understanding of the mechanism responsible for SU_3^f symmetry breaking. This must also be said about dynamic effects associated with the spin of the quarks. The peculiar structure of the spin and spin-orbit splitting in mesons is probably an indication of a deeper interrelation between these effects and the basic properties of QCD. The nature of this connection is still not clear. It is probable that a relativistic theory of bound states that is more rigorous and takes into account both gauge invariance and color confinement will turn out to be useful in this context.

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