

Fundamental properties of the neutron: Fifty years of research

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This review, occasioned by the fiftieth anniversary of the discovery of the neutron, considers the most important papers on fundamental studies of the properties of the neutron. Some of the researches are briefly expounded, and the present values of the fundamental quantities that characterize the neutron as an elementary particle are given.

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1. INTRODUCTION

The discovery of the neutron fifty years ago was probably one of the most significant scientific events of the present century. It was the stimulus for many surprising new discoveries and led to the development of such fundamental branches of science as nuclear physics and elementary-particle physics. Neutron physics is now an independent branch of physics that began with the discovery of the neutron. In trying to formulate in general terms the remarkably wide range of interests in current neutron physics, we can name the following fields of research: a) The study of nuclear structure as manifested in the interactions of neutrons with nuclei and with isolated nucleons; b) the study of biological and other structures by means of neutrons; c) the study of the dynamics of condensed media, magnetic properties of matter, and phase transitions; d) nuclear reactor physics and nuclear technology.

Throughout the entire fifty years since the neutron's discovery there has also been continued study of the neutron itself as an elementary particle. At different times this research has been of varying degrees of importance in the total stream of neutron research, but it never ceased altogether. It did not stop even when the

main efforts of the neutron physicists were mainly directed toward the solution of the uranium problem in their countries. The study of the neutron is, of course, still going on now, and is producing new fundamental results, although the number of physicists that devote themselves to this activity is relatively small.

The aim of this article is a rather brief survey of the results that have been obtained in research on the neutron. It cannot lay claim to completeness, owing to the breadth and variety of the problems considered. Without doubt many of the questions considered here could be the objects of separate reviews, and even books, and such articles and books indeed exist. Some newly written review articles of this sort are being published simultaneously with this paper. The celebrative purpose of the present paper has required an exposition on a historical plan, to the detriment of a more detailed analysis of the status of the problems. Space limitations have necessitated giving up the original intention to tell about all the most remarkable experiments with neutrons that have been made over fifty years. The selection of material has been based on the somewhat artificial principle of dealing only with those experiments in which the neutron was more the object of the research than the means for it. It was

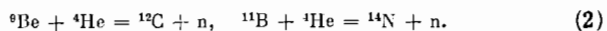
thus necessary to omit experiments that have led to such fundamental discoveries as that of artificial radioactivity, that of fission, and the comparatively recent discovery of parity nonconservation in nuclear interactions.

2. THE MASS OF THE NEUTRON

Measurements of the mass of the neutron, initially rather approximate, were the first and most reliable evidence that induced Chadwick to announce the existence of a new elementary particle, the neutron. We recall that Chadwick's work was immediately preceded by the work of Curie and Joliot,¹ which showed beyond doubt that the radiation emitted from boron and beryllium under the action of α rays, which had been found earlier by Bothe and Becker,² was capable of knocking out protons from hydrogen-containing substances. When they assumed that this radiation was very hard γ rays, and that the process of knocking out protons was like the Compton effect on electrons, Curie and Joliot had to suppose that the energy of these γ rays was very large—about 50 and 35 MeV for Be and B, respectively. Chadwick repeated these experiments, and also observed knocking out of other nuclei, in particular those of helium, beryllium, and carbon. Measuring the energies of the recoil nuclei of various elements in terms of their ranges in air, he concluded that the hypothesis that the primary radiation is γ rays is in contradiction with the laws of conservation of energy and momentum, and suggested that the radiation consists of neutral particles with mass 1, i.e., of neutrons.³ The results of very detailed measurements of the energies of recoil protons and nitrogen nuclei were published three months later.⁴ Starting from the resulting values of the speeds of recoil particles of two different masses and using the relations

$$u_p = \frac{2M}{M+1} V, \quad u_n = \frac{2M}{M+14} V, \quad \frac{M+14}{M+1} = \frac{u_p}{u_n}, \quad (1)$$

where M and V are the mass and velocity of the neutron, and u_p and u_n are the maximum velocities of recoil protons and nitrogen nuclei from head-on collisions, Chadwick found that $M = 1.15$ mass units, to an accuracy of 10%. In the same paper he proposed a different method for measuring the mass of the neutron, based on the balance of masses and energies of the objects involved in reactions in which neutrons are produced. All subsequent work on determining the neutron mass, up to the present time, has been based on this principle. Chadwick assumed that when beryllium and boron are irradiated with α particles the following reactions occur:

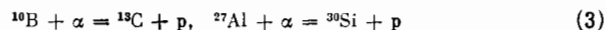


At that time there was no information on the mass of ${}^9\text{Be}$, and the reaction with boron was used to obtain the neutron mass. One has the following equation:

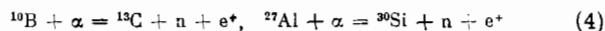
$$\text{mass } {}^{11}\text{B} + \text{mass } {}^4\text{He} + \text{KE } {}^4\text{He} = \text{mass } {}^{14}\text{N} + \text{KE } {}^{14}\text{N} + \text{mass } n + \text{KE } n$$

(where KE means "kinetic energy of"). Using the values of the atomic masses from mass-spectrometer data, the values of the particles' energy from their

ranges, and the value of the neutron's energy from the maximum range of the recoil protons, Chadwick obtained for the neutron mass the value $M = 1.0067$ mass units. Allowing for inaccuracies, he gave 1.005–1.008 as the most probable range of mass values. This result fully agreed with Chadwick's belief that the neutron is a structure composed of a tightly bound proton and electron.¹⁾ Comparing his value of the neutron mass with the sum 1.0078 of the proton and electron masses, Chadwick obtained the quite reasonable value 1–2 MeV for the binding energy of such a structure. However, in his article "The existence of a neutron" Chadwick wrote a significant statement: "It is, of course, possible to suppose that the neutron may be an elementary particle. This has little to recommend it at present, except the possibility of explaining the statistics of nuclei such as ${}^{14}\text{N}$." Very soon afterward, however, serious doubts were raised as to the accuracy of Chadwick's value of the neutron mass. Right after Chadwick's note appeared in *Nature*, the hypothesis of the proton-neutron model of nuclei was suggested, and it rapidly received confirmation. It was stated almost simultaneously, and independently, by Ivanenko⁷ (paper received 21-4-1932) and by Chadwick⁴ (10-5-1932), and somewhat later by Heisenberg⁸ (7-7-1932).²⁾ According to this new concept the ${}^9\text{Be}$ nucleus must be regarded as consisting of two α particles and a neutron. If Chadwick's value is used for the neutron mass, the experimental mass of ${}^9\text{Be}$, determined in 1933, is larger than the sum of the masses of the α particles and the neutron, in contradiction with the existence of ${}^9\text{Be}$. This was pointed out in the same year, 1933, by Curie and Joliot.^{9,10} Studying the interaction of α particles with boron and aluminum, they assumed that along with the reactions



the reactions



also occurred. We note that up to that time there had been no known case in which a reaction went by two channels, nor any involving positron emission, so that this was a rather bold assumption. Using the mass balance from reactions (3) and (4), Curie and Joliot obtained the value 1.011–1.012 for the neutron mass, which is considerably greater than the mass of the hydrogen atom. In 1934 Curie and Joliot were evidently the first to make the assumption that the emission of the neutron and the positive electron occurs not in a single process, as in Eq. (4), but in two stages; the (α, n) process on the ${}^{10}\text{B}$, ${}^{27}\text{Al}$, or ${}^{24}\text{Mg}$ occurs first, and is followed by the positron decay, with a neutrino

¹⁾The hypothesis of the neutron existing as a bound state of a proton and an electron had been advanced in 1920 by Rutherford.⁵ Chadwick was greatly influenced by this idea and was looking for a neutron long before the work of Bothe and Becker and of Curie and Joliot.⁶

²⁾Heisenberg regarded the proton and the neutron as a single particle which could exist in nuclei in two quantum states. In this paper Heisenberg already refers to the work of D. D. Ivanenko.⁷

emitted along with the positron. This did not cause any essential change in the earlier conclusions about the mass of the neutron, based on measurements of the masses of the final products. As for the energy carried off by the neutrino, it was pointed out that in setting up the energy balance one must take the maximum energy of the positron, corresponding to a very small kinetic energy of the neutrino, which was assumed to have mass zero. This truly remarkable work gave an improved value of the neutron mass, $M = 1.010 \pm 0.005$. The conclusion of Curie and Joliot about the neutron's having more mass than proton plus electron was confirmed very soon. In 1934 Chadwick and Goldhaber discovered the photodisintegration of the deuteron.¹² The energy and mass balance of this newly discovered process gave a much more precise value of the mass of the neutron: $M = 1.0080 \pm 0.0005$. Continuing this research, Chadwick and Goldhaber in 1935 published the mass value 1.0084, but pointed out that this value might be increased up to 1.0090 if a correction to the masses of the light elements discovered at about that time turned out to be really necessary.¹³ This correction, coming from the mass ratio He/O, was found to be genuine. The paper¹³ of Chadwick and Goldhaber was also remarkable in containing what was evidently the first suggestion that the neutron might be radioactive. The process of the decay of the free neutron was observed only after the passage of thirty years (see Chapter 5 of this paper). Other research in 1935 agreed with the results of Ref. 13. Interrupting at this point the historical order of the exposition, we note that the reader can find a survey of the early work on measurement of the neutron mass in a book by Stranathan¹⁴ and a review article.¹⁵

Present methods for determining the mass of the neutron are based on measurements of the threshold of the reaction ${}^3\text{H}(p, n){}^3\text{He}$, and also on combining mass-spectrometer measurements of the mass doublet $\text{H}_2 - {}^2\text{D}$ with γ -spectrometer measurements of the binding energy of the deuteron. The most detailed accounts of the present state of the problem can be found in Refs. 16 and 17. The values of the neutron mass and the mass difference of the neutron and proton recommended in 1980 by an elementary particle group¹⁸ are based on the results of Ref. 17 and are as follows:

$$\begin{aligned} m_n &= 939.5527 \pm 0.0052 \text{ MeV}, \\ m_n - m_p &= 1.293429 \pm 0.000036 \text{ MeV}. \end{aligned}$$

For further reference we give some data from Ref. 17:

$$\begin{aligned} m_e &= 0.5110034 \pm 0.0000014 \text{ MeV}, \\ 1 \text{ amu} &= 931.5016 \pm 0.0026 \text{ MeV}. \end{aligned}$$

After the appearance of Ref. 17 two more papers^{19,20} appeared which were not included in Ref. 18. The following are the results of these papers for the mass difference of the neutron and the hydrogen atom:

$$\begin{aligned} m_n - m_{\text{H}} &= 782.340 \pm 0.040 \text{ keV}^{19}, \\ m_n - m_{\text{H}} &= 782.332 \pm 0.017 \text{ keV}^{20}. \end{aligned}$$

3. THE MAGNETIC MOMENT OF THE NEUTRON

a) The discovery of the magnetic moment

At the beginning of the 1930s, when the neutron was discovered, it obviously was hard to expect that a neu-

tral particle could have a magnetic moment. Surprise at this was expressed even a decade later (cf. e.g., Ref. 14). It is then surprising that discussion in the literature of the hypothesis that the neutron has a magnetic moment started in 1934. The technique for measuring the magnetic moments of molecules developed by Stern and his colleagues, and based on deflection of a molecular beam in a nonuniform magnetic field (the Stern-Gerlach experiment), made it possible to measure the magnetic moments of the proton and of the deuteron.²¹⁻²³ A large difference between these quantities was found. And then in a paper²³ by Estermann and Stern reported at a meeting of the American Physical Society in April 1934 the hypothesis was suggested, apparently for the first time, that the magnetic moment of the deuteron should be equal to the sum of the magnetic moments of the proton and the neutron. These writers estimated μ_n to be 1.5-2 times the nuclear magneton (NM). Almost simultaneously, and without doubt independently, there appeared a paper in the *Doklady* of the Soviet Academy of Sciences by Tamm and Al'tshuller.²⁴ Analyzing the data on the magnetic moments of nuclei obtained from spectroscopic work on hyperfine structure, Tamm and Al'tshuller also came to the conclusion that a magnetic moment of the neutron exists, with a value of about 0.5 NM. It was not known, however, how to approach the search for a neutron magnetic moment experimentally, since it was impossible to do an experiment of the Stern-Gerlach type owing to the low intensities of the neutron sources that existed then. This was the situation until in 1936 a paper by Bloch appeared,²⁵ in which he showed that the presence of a magnetic moment of the neutron must lead to the appearance of a specific magnetic scattering, and in particular to a spin-dependent component in the cross sections for interaction of neutrons with ferromagnetic substances. Quantitative calculations were made, and it was proved directly that this new effect could be applied to the search for the neutron's magnetic moment. In 1937 Laslett tried to observe a difference in the scattering of neutrons by iron above and below the Curie point, but found no effect.²⁶ At the same time (in the same number of the *Physical Review*) Dunning, Powers, and Beyer also reported a search for magnetic scattering.²⁷ In their experiment neutrons were sent through a piece of magnetized iron, and were then scattered by another piece of magnetized iron. It was found that the intensity of the scattered neutrons was practically independent of the direction of magnetization of the scatterer³⁾ but that there was an undoubted change of the counting rate when the scatterer was demagnetized. Thus it was shown that the neutron has magnetic properties. The presence of magnetic effects was confirmed in subsequent experiments,^{28,29} but the magnitude and sign of the magnetic moment still could not be measured. Soon Rabi suggested a new way to change the sign of the polarization of neutrons, the

³⁾ Reversal of the specimen's magnetization may fail to lead to a change of the sign of the neutron's polarization, if certain definite conditions are not satisfied. These requirements of so-called nonadiabaticity apply to the rate of rotation of the field as measured in the coordinate system of the neutron.

radio-frequency resonance method,^{30,31} and Schwinger³² gave the theory of depolarization of neutrons by passage through a demagnetized ferromagnetic substance. Experimenters now had available everything needed for work with polarized neutrons: Polarizer and analyzer, magnetized ferromagnetic substances (Bloch); spin-flipper, a field rotating or oscillating at the precession frequency (Rabi); depolarizer, a demagnetized ferromagnetic substance (Schwinger). At practically the same time Frisch and Halban reported that they had succeeded in depolarizing a neutron beam by placing in its path a thin solenoid in which the magnetic field was perpendicular to the field direction in the polarizer and analyzer magnets. The sufficiently rapid (nonadiabatic) change of direction of the field near the winding of the solenoid brought about precession of the neutrons inside the solenoid around the new direction of the field inside the solenoid, and they entered the region of action of the main analyzer field when the polarization vector made an angle with the field; i.e., they were depolarized.³³ A month later Frisch reported that he had succeeded in determining the direction of the precession of the neutrons in the solenoid, and thus finding the sign of the magnetic moment, which, as expected, was negative (Ref. 34).⁴⁾ Finally, still in 1937, a paper appeared by Powers and his colleagues, in which the sign was firmly established and an experimental estimate of the magnitude of the magnetic moment was obtained.³⁵ The results of this work were: Most probable value $\mu_n = -2$ NM, and range of possible values $-3 < \mu_n < 1$ NM. For neutron physics 1937 was truly the year of the magnetic moment.

b) Measurement of the value of the magnetic moment

The first accurate measurement of the magnetic moment of the neutron was made in 1939 by Alvarez and Bloch.³⁶ This work was unquestionably a landmark in the history of neutron research. The high experimental technique, the use of an accelerator as a neutron source, the verification and control tests made by the authors, the use of time-modulation of the measured effect to decrease the influence of instabilities—all these things make the work of Alvarez and Bloch quite up-to-date, even by present standards. The arrangement of the experiment is shown in Fig. 1. The neutrons were produced by bombarding a beryllium target with deuterons accelerated in a cyclotron. The neutron beam from the moderator passed through two blocks of strongly magnetized iron, which served as polarizer and analyzer (the Bloch method³⁵). Between these blocks was placed a magnet which produced a high stable field H_0 . In the gap of this magnet was located a coil with a field H_1 oscillating at frequency ω_1 , perpendicular to the field H_0 . This served as the Rabi resonance spin flipper. When the resonance condition

$$\hbar\omega_1 = \hbar\omega_{\text{Larm}} = 2\mu_n \mu_{\text{nuc}} H_0 \quad (5)$$

is satisfied transitions are probable between the quan-

⁴⁾Subsequently Alvarez and Bloch³⁶ cast doubt on the cogency of the argument in Ref. 24, and suggested that the result was accidental.

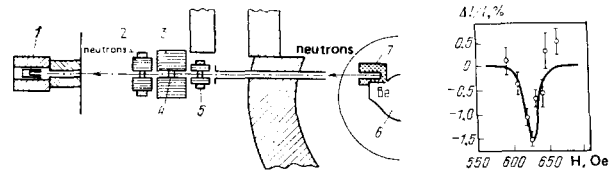


FIG. 1. Diagram of the Alvarez-Bloch experiment to measure the magnetic moment of the neutron; 1—detector, 2—analyzer magnet, 3—magnet producing constant magnetic field, 4—region occupied by oscillating field, 5—polarizer magnet, 6—cyclotron chamber, 7—moderator.

tum levels corresponding to the two different values of the component of the magnetic moment along the direction of the field. The energy of the transition is

$$\Delta E = 2\mu_n \frac{e\hbar}{2m_p c} H_0 = 2\mu_n \mu_{\text{nuc}} H_0, \quad (6)$$

where $\mu_{\text{nuc}} = e\hbar/2m_p c$. The occurrence of resonance was determined from the change of the counting rate of neutrons passing through the analyzer (see Fig. 1). From the resonance values of ω_1 and H_0 one can determine the value of μ_n :

$$\mu_n = \frac{\hbar\omega_1}{2\mu_{\text{nuc}} H_0}. \quad (7)$$

To avoid absolute measurement of the frequency and field strength, the authors made relative measurements of these quantities, comparing them with the frequency and field strength in the cyclotron when the conditions for acceleration of protons were satisfied. As is well known, one then has

$$\omega_p = \frac{eH_p}{m_p c}, \quad (8)$$

where ω_p is the cyclotron frequency for protons. Setting $m_n = m_p$ we easily get

$$\mu_n = \frac{\omega_1}{\omega_p} \frac{H_p}{H_0}. \quad (9)$$

This relation was used to determine the value of the neutron's magnetic moment. Using the result of Powers' paper³⁵ on the sign of the moment, Alvarez and Bloch obtained $\mu_n = -1.935 \pm 0.020$ NM.

This result was naturally analyzed in relation to the accurate values which Rabi had found somewhat earlier for the proton and deuteron magnetic moments³⁷: $\mu_p = 2.875 \pm 0.020$, $\mu_d = 0.855 \pm 0.006$. The similarity of the values of $(\mu_p + \mu_n)$ and μ_d indicated that the proton and neutron magnetic moments are additive in the deuteron, so that the deuteron is in a 1S state. At this time, however, Rabi and his group had already discovered the quadrupole moment of the deuteron, which was incompatible with a pure 1S state for the deuteron. Therefore the question of improving the value of the neutron's magnetic moment remained as urgent as ever.

More accurate work appeared in the postwar period.^{39,40} In experiments of Bloch, Nicodemus, and Staub the magnetic moment was also measured in units of the magnetic moment of the proton. The principle of the experiment was not very different from that of the Alvarez-Bloch apparatus. Using the same magnetic

field, a resonance experiment was done with neutrons, and then the method of nuclear induction was used to measure the Larmor frequency of the protons in water. The resonance frequencies were compared. The following value was found for the ratio of the magnetic moments: $|\mu_n|/|\mu_p| = 0.685\,001 \pm 0.000\,030$. Using the value of μ_p from Ref. 41, the authors got $|\mu_n| = 1.931\,07 \pm 0.000\,06$. This experiment and the results of new measurements of the magnetic moments of the proton⁴¹ and the deuteron⁴² firmly established the fact that the magnetic moments are not additive in the deuteron.

All further measurements of the magnitude of the neutron have been made with the radiofrequency resonance method, which was perfected by Ramsey.^{43,44} The determination of the field is made, as in Ref. 40, in terms of the resonance frequency for protons. Bibliography for the later work can be found in books by Gurevich and Tarasov⁴⁵ and by Aleksandrov.⁴⁶

The currently accepted value of the magnetic moment, as of 1980,¹⁸ was found in Ref. 47 and is

$$\mu_n = -1.91304184 \pm 0.00000088.$$

c) The nature of the magnetic moment

Leaving aside discussion of the theoretical research connected with the question of the values of the anomalous magnetic moments of the neutron and the proton, we shall consider another problem, which is decided experimentally and relates to the nature of the magnetic moment.⁵⁾ The point is that in the paper by Bloch cited earlier²⁵ (see also Ref. 48) and in Schwinger's paper³² different models of the magnetic moment of the neutron were used. Bloch regarded the neutron as a true magnetic dipole, whereas in Schwinger's model the neutron is thought of as an object with a current distribution, which causes the appearance of the magnetic moment. In many cases these models lead to different consequences, which are testable experimentally, since the true dipole and the magnetic moment of a current are acted on in a medium by different effective fields. In the case of the dipole the interaction energy is $-\mu_n B = -\mu_n(H + 4\pi M)$, where M is the magnetic moment per unit volume of the medium. Therefore a measurement of the energy of interaction of the neutron with a ferromagnetic substance gives an unambiguous answer as to the correctness of one model or the other. Since for ferromagnetic substances the values of H and B differ by some orders of magnitude, the magnetic interaction energies are very different for the two models. Besides this, the quantities H and B behave differently near the boundaries of the substance. The tangential field component $H_{||}$ is continuous across a boundary, while this component $B_{||}$ of the induction has a discontinuity. A particular result of this fact is that when neutrons are reflected from a magnetized ferromagnetic mirror two critical angles of incidence should be observable⁴⁵

$$\varphi_{cr}^{\pm} = \lambda \sqrt{\frac{N b_{coh}}{\pi} \pm \frac{m}{2\pi^2 \hbar^2} \mu B}, \quad (10)$$

⁵⁾ This question of the nature of the magnetic moment of the neutron has been examined in detail in a book by I. I. Gurevich and L. V. Tarasov (Ref. 45).

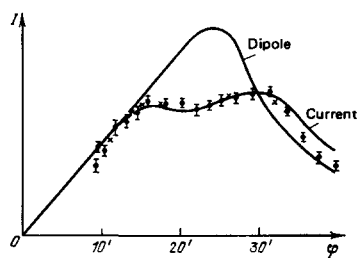


FIG. 2. Data of Hughes-Burgy experiment on the specular reflection of neutrons from magnetized iron. The curves show the predictions of the theory for two models of the magnetic moment of the neutron, the current model and the dipole model.

where λ is the wavelength of the neutron, N is the concentration of nuclei in the material of the mirror, and b_{coh} is the coherence range among the nuclei in this material. The first term in Eq. (10) is due to coherent scattering from the nuclei in the mirror material and the second is of magnetic origin. In the absence of magnetic reflection, at angles of incidence less than the value φ_{mnc} given by the first term in Eq. (10) all the neutrons are reflected from the mirror. When the second term in μB is included, total reflection occurs at the smaller of the two angles, $\varphi < \varphi_{cr}^-$, and for $\varphi_{cr}^- < \varphi < \varphi_{cr}^+$ the neutrons of one polarization are reflected.⁶⁾ In the case of the dipole model the quantity μB must be replaced by the much smaller quantity μH , and there will be no sharp change of potential at the surface of the material. This gives a different dependence of the reflection coefficient on the angle.

The first experiment on reflection of neutrons from a magnetized ferromagnetic mirror was made by Hughes and Burgy⁴⁹ in 1951 (Fig. 2). It fully confirmed the validity of the current model. Subsequently the fact that the magnetic energy of neutrons is due to the induction field B was applied in many experimental arrangements and installations.

4. THE SPIN OF THE NEUTRON

The conviction that the spin of the neutron is $\frac{1}{2}\hbar$, so that this particle obeys Fermi-Dirac statistics, appeared immediately after the discovery of the neutron. This idea was an important basis for the proton-neutron model of the nucleus. In the Bakerian lecture in 1933 Chadwick, on the basis of the spins and statistics of light nuclei, expresses the firm belief that the neutron is an elementary particle with spin $\frac{1}{2}$ (Ref. 50; cf. our earlier quotation from Chadwick's 1932 paper⁴). The measurement of the spin and magnetic moment of the neutron confirmed this idea. There are now innumerable experimental facts that can be reconciled only when the spin of the neutron is taken to be $\frac{1}{2}$. The most convincing data, apart from those on the deuteron, are obtained from the measurements of the scattering of neutrons by protons and by ortho and para hydrogen.

⁶⁾ That there are two critical angles for the reflection of neutrons from a ferromagnetic substance was first pointed out by A. I. Akhiezer and I. Ya. Pomeranchuk.¹⁴⁸ They also suggested using this phenomenon to polarize neutrons.

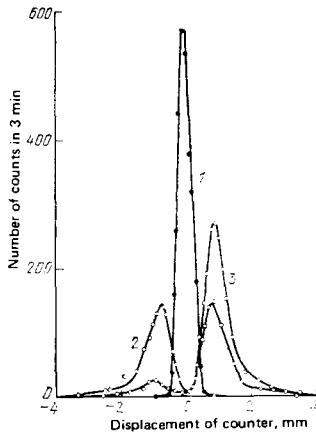


FIG. 3. Experiment of the Stern-Gerlach type on the neutron; splitting of a beam of neutrons in a nonuniform magnetic field (from Ref. 52). 1—Beam without the magnetic field; 2—splitting of an unpolarized beam; 3—splitting of a polarized beam.

There are also experiments that show directly that the component of the magnetic moment of the neutron along a physically defined axis can take two and only two values; this is a direct confirmation that the spin is one-half. We list these experiments.

a) The Hughes-Burgy experiment on reflection of neutrons from a magnetized mirror, which showed that there are two values of the critical angle.⁴⁹

b) Direct experiments of the Stern-Gerlach type on free neutrons, in which a neutron beam is split into spin components in a nonuniform magnetic field. This was first done in 1954 in Oak Ridge.⁵¹ Later it was proposed to use the Stern-Gerlach effect to measure the degree of polarization of a neutron beam,⁵² and this is now established practice in work with polarized neutrons (Fig. 3).

c) The demonstration by Shull of the phenomenon of double refraction of a neutron wave in a prism of magnetic material⁵³ (Fig. 4). This effect is due to the different indices of refraction for neutrons with two different directions of the projection of the magnetic moment. Like the experiments on the reflection from a magnetized mirror, this is based on the difference in the magnetic energies for the two values of the spin component.

d) Experiments on the deceleration or acceleration of neutrons in a magnetic field. The idea of this experiment was proposed by Drabkin and Zhitnikov in 1960.⁵⁴ The essential point is that upon entering a

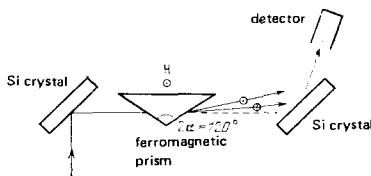


FIG. 4. Diagram of Shull's experiment for observing double refraction of neutrons in a ferromagnetic prism.

magnetic field a monochromatic neutron wave is split as to energy into two components with the energy values $E_0 \pm \mu B$. If within the region in which the field exists one produces a spin reversal by radio-frequency methods, then the kinetic energy of the neutrons is unchanged, but the potential energy of the interaction with the field is changed. The result is that one component of the wave is slowed down not only on entering the field, but also on leaving the field after the spin reversal. The other component is accelerated twice. Accordingly the energy difference is $4\mu B$. The same suggestion was made in Ref. 55. Recently the speeding up and slowing down of neutrons in such a system was observed experimentally.⁵⁶

5. THE DECAY OF THE FREE NEUTRON

a) The discovery of the radioactivity of the neutron. The half-life

The question of the instability of the neutron arose in the middle of the 1930s, when the excess of its mass over the sum of the masses of the proton and the electron became an established fact. As we have already stated, Chadwick and Goldhaber were evidently the first to raise this question. The present writer does not know whether experimental work on this was attempted before the end of the 1940s, but it is quite clear that the radioactivity of the neutron could be observed only after the appearance of atomic reactors as powerful sources of neutrons. Experimental attempts to observe neutron decay were extraordinarily difficult. The decay products that can be observed have low energies (protons up to 705 eV, electrons up to 780 keV), and it is very hard to distinguish effects from the decay from the background of γ rays and electrons accompanying the neutron beam. The relatively long half-life of the neutron and the low density of the source, a beam of neutrons, complicated the problem greatly.

The first short report on observation of neutron decay was made in 1948 by Snell and Miller.⁵⁷ These authors could estimate the half-life as 15 to 30 minutes. By 1950 three groups had observed neutron decay: Snell and his group in Oak Ridge,⁵⁸ Robson with the Chalk River reactor in Canada,^{59,60} and Spivak's group in the U.S.S.R.⁷¹ Snell observed coincidences of electrons with previously accelerated protons, while in the work of Robson and of Spivak only the decay protons were registered. All the authors gave rather crude estimates of the half-life leading to an average value of $T_{1/2} = 8-15$ min. Accordingly, the fact that the neutron is unstable had been reliably established. Without doubt, however, improvement in the half-life was extremely important. As a rule, an experiment for measuring a half-life divides into two independent tasks, exact measurement of the density of the neutron beam and measurement of the absolute activity of the beam. It is usually very difficult to determine the effective volume of the beam, i.e., the region from which the

⁷¹Work done by P. E. Spivak in 1950 was published, along with some later work of his, some years afterward at the Geneva Conference on the Peaceful Uses of Atomic Energy.⁶¹

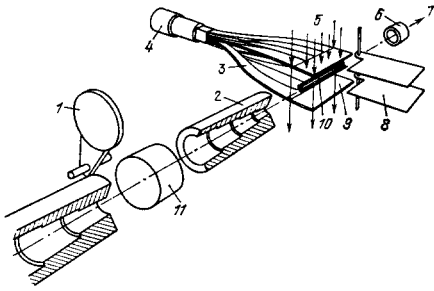


FIG. 5. Diagram of experiment performed by Christensen *et al.*⁶⁴ to measure the lifetime of the neutron. 1—Shutter for beam; 2—collimator; 3—light guides; 4—photomultiplier; 5—magnetic field lines; 6—monitor; 7—direction of neutron beam; 8—shutters for electrons; 9—scintillator; 10—effective volume for registering decay electrons; 11—bismuth filter.

decay products are registered. The various investigators solved this problem in different ways. Robson and Spivak's group attained about ten-percent accuracy with results that agree: 12.8 ± 2.5 min⁶² and 12.0 ± 1.5 min.⁶¹ A similar result was found by D'Angelo⁶³ in 1959. Unlike all the other work, D'Angelo's measurement was made with a diffusion chamber located directly on the beam. The result was $T_{1/2} = 12.7 \pm 1.9$ min. More accurate results obtained up to the end of 1980 are as follows: $T_{1/2} = 10.61 \pm 0.16$ min by Christensen's group,⁶⁴ $T_{1/2} = 10.13 \pm 0.09$ min by Spivak's group,⁶⁵ and $T_{1/2} = 10.82 \pm 0.21$ min by Byrn *et al.*⁶⁶ The method used in these papers is quite different (Fig. 5). In Christensen's work the decay electrons were registered with a scintillation 4π detector. There is no great actual advantage in this, because of the large background of the detector, of the order of the effect itself. The decay electrons were transported from the place of the decay in the beam to the detector along the lines of force of an applied uniform magnetic field. The effective volume of the beam was determined in first approximation simply by the dimensions of the detector. In Spivak's apparatus the decay protons were registered. The protons move with their initial velocities from the points where they are produced to a diaphragm which defines a strictly specified solid angle, after which an accelerating and focussing system assures that all the protons from this solid angle are registered, with a very high ratio of observed effect to background. The effective volume of the beam is determined from simple geometrical considerations. In Ref. 66 use was made of a trap for the decay protons. The protons follow magnetic lines of force while executing an oscillatory motion between two electrostatic barriers, which are the "plugs" of the trap. On application of a voltage pulse one of the "plugs" opens and there is a discharge of the accumulated protons to the detector. This periodic pulsed discharge greatly improves the background situation. The efficiency of the collection of protons is stated by the authors as ~75%, so that the problem of calibrating the registration efficiency and the effective beam volume would seem to have been rather complicated. It is unpleasant that the results obtained in various statistical runs of this work show poor agreement among themselves.

As can be seen from the stated results, the desired agreement between the results of the various researches has not been achieved. If we nevertheless average these data, we get as the world average value $T_{1/2} = 10.32 \pm 0.07$ min, $\tau_n = 893.0 \pm 6.3$ sec, where τ_n is the decay constant, $\tau_n = T_{1/2} \cdot 60 / \ln 2$. This value differs from that recommended by the elementary-particle group, $\tau_n = 917 \pm 14$ sec.¹⁸ This last figure is mainly based on the results of Christensen's group,⁶⁴ since Ref. 66 did not appear in time to be included in the collection of data, and the results of Spivak's group owing to a misprint in the Americal version (JETP Letters) of *Pis'ma v ZhETF*, are used in Ref. 18 with an error overstated by an order of magnitude.

b) The detailed study of the β decay of the neutron

The study of the β decay of the neutron played an important role in the formation of views about β decay in general. The reader will find a full account of the development of these views in a book by Wu and Moszkowski.⁶⁷ The foundations of β decay theory were laid by Fermi,⁶⁸ who postulated that an electron and a neutrino are produced at the moment of decay, in analogy with the emission of γ -ray quanta. Beta decay was introduced as an interaction among four fermions. The transition probability is proportional to the squares of the absolute values of the wave functions of the four particles involved in the interaction. Later it was shown that if one includes all relativistically invariant combinations of these wave functions the Hamiltonian for decay can contain five types of operators, called the scalar, S, vector, V, axial-vector, A, tensor, T, and pseudoscalar, P terms on account of their transformation properties. It is known that in the case of the neutron the P type is unimportant. The question as to which of the possible decay types are realized in nature and what relative roles they play was for a long time, and in some ways still is, one of the important problems of β decay. At the end of the 1950s it became clear from analysis of the shape of the electron spectra from nuclear β decays that only two types of interaction are dominant: S and T, or V and A, and preference was given to the V-A type. After the discovery of parity nonconservation⁶⁹ it became clear that the choice could be made by measuring the helicity of the neutrino, which was done by Goldhaber, Grodzins, and Sunyar,⁷⁰ or else by measuring the angular correlations between the decay products or between the directions of polarization and the momentum of the decay particles. The purest results can be obtained by studying the decay of polarized neutrons. Such an experiment was first made in 1960 by Burgy *et al.*

As is well known, the decay probability per unit time can be written in the form

$$W(E, p_e, p_{\bar{\nu}}) = F(E) \left[1 + a \frac{v}{c} (\mathbf{p}_e \mathbf{p}_{\bar{\nu}}) + A \frac{v}{c} (\sigma \mathbf{p}_e) + B (\sigma \mathbf{p}_{\bar{\nu}}) + D \frac{v}{c} \sigma (\mathbf{p}_e \mathbf{p}_{\bar{\nu}}) \right], \quad (11)$$

where $\mathbf{p}_e, \mathbf{p}_{\bar{\nu}}$ are the unit vectors of the directions of the momenta of the electron and antineutrino, σ is the unit vector of the direction of polarization of the neutron beam, F is the form factor of the electron spectrum, v is the speed of the electron, and $a, A, B,$ and D are

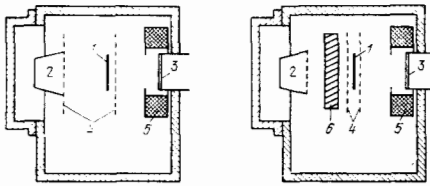


FIG. 6. Diagram of experiment of Burgy *et al.*⁷¹ to study the beta decay of polarized neutrons. On left, for measuring correlation of spin electron momentum; on right, for measuring correlation between spin and antineutrino momentum; 1—section of neutron beam; 2—proton detector; 3—electron detector; 4—grids; 5—coil; 6—diaphragm with slits for protons.

the constants of the various angular correlations, which are functions of the fundamental coupling constants appearing in the Hamiltonian for β decay. Measuring these correlation coefficients was the aim of Burgy's experiment. To give an idea of the difficulty of the experiment, we recall that several years before it was accomplished the mere observation of the fact of neutron decay was a matter of great difficulty. Here one had to work, not with the direct beam from a reactor, but with a polarized beam, which greatly reduced the intensity. The success of the work was largely due to a method, developed earlier in the same laboratory (Argonne), for polarizing neutrons by reflecting them from a magnetized ferromagnetic mirror. The diagram of the experiment of Burgy *et al.*, shown in Fig. 6, has been given in most books devoted to β decay,⁶⁷ and also in those on the properties of the neutron.^{45,46} It was with this apparatus that the correlation between the spin of the neutron and the direction in which the electron is emitted was detected and measured. It was found that the electrons are mainly emitted in directions opposite to the spin. The correlation coefficient is $A = -0.11 \pm 0.02$. Since it was not possible to register the antineutrinos directly, the predominant direction of the recoil protons was observed in order to get the correlation between spin and antineutrino directions. For this purpose a special collimator with slits directed downward was placed in front of the proton detector. The protons could pass through it only in cases when the antineutrino was emitted in the upper half-plane, i.e., along or opposite to the direction of the polarization. The corresponding correlation coefficient was found to be $B = +0.88 \pm 0.15$. In the case of the exact $V-A$ interaction type (with the constants g_V and g_A equal in magnitude and opposite in sign, $\lambda = g_A/g_V = -1$) the expected values of A and B were respectively -0.1 and $+1$. For all other combinations of types the values of A and B were very different from those observed. Accordingly, it could be concluded that A and V are the basic types. Assuming that the other types are absent, the authors found for the ratio of the coupling constants $\lambda = g_A/g_V = -1.25 \pm 0.05$.

Later the measurements of the correlation coefficients were repeated with greater accuracy both at the Argonne Laboratory and by Erokolimskii's group in Moscow, with decidedly improved methods. The most precise values of the spin-to-electron correlation are now $A = -0.112 \pm 0.006$ (Ref. 72) and $A = -0.114 \pm 0.005$

(Ref. 73). The present value for this quantity, obtained by averaging these two, is $A = -0.1132 \pm 0.0032$. From this it is easy to get the value of the ratio of constants, $\lambda = -1.259 \pm 0.009$. The most precise value of the anti-neutrino-spin correlation was obtained in the work of a group at the Kurchatov Institute of Atomic Energy in Moscow: $B = 0.995 \pm 0.035$ (Ref. 74). The world-average result, including also a result of the Argonne group,⁷⁵ is $B = 0.995 \pm 0.028$.

The correlation coefficient between the momenta of the antineutrino and the electron is also rather sensitive to the value of g_A/g_V . This quantity is measured with beams of unpolarized neutrons. The main contribution to the determination of the electron-antineutrino correlation coefficient has been made by a group at the Institute of Theoretical and Experimental Physics in Moscow and Dobrozemsky's group in Austria. The results of the Moscow group is⁷⁶ $a = -0.091 \pm 0.039$. The quantity a is measured from the shape of the spectrum of recoil protons which is sensitive to this correlation, since the momentum of the recoil proton is equal to the sum of those of the light particles (to good accuracy the neutron can be considered at rest). However, this sensitivity is very small, and very high-level spectrometric technique is necessary to obtain sufficient accuracy for this quantity. The Dobrozemsky group undertook a really precise study of the shape of the proton spectrum and obtained a very precise result for the correlation coefficient⁷⁷ $a = -0.1017 \pm 0.0055$. From this one gets $\lambda = g_A/g_V = -1.259 \pm 0.017$, in excellent agreement with the result obtained from the measurement of the spin-electron correlation. Therefore the value of λ as found from neutron decay research only, is now $\lambda = -1.259 \pm 0.008$.

The β -decay coupling constants can of course be determined from the β decay of nuclei. In this case the interpretation of the results is often made difficult by uncertainties in the values of nuclear matrix elements. In some cases this difficulty can be resolved, in particular in superallowed transitions or in transitions between analogous states. From the half-value period of superallowed $O^+ - O^+$ transitions one can determine the fundamental constant of β decay [$G = 1.41222 \pm 0.00046 \times 10^{-49}$ erg cm³ (Ref. 78)]. These transitions involve only the Fermi interaction, the vector V , or, if it is present, the scalar S . In the case when the so-called Gamow-Teller interaction is present (axial-vector and tensor types), for the reasons we have indicated one cannot get exact information from nuclear decay. In this sense the neutron is a completely unique object. All the interactions take part in its decay, and the magnitudes of the Fermi and Gamow-Teller matrix elements are determined by purely statistical considerations: $|M_F|^2 = 1$, $|M_{GT}|^2 = 3$. Besides this, since the products of the decay have the smallest possible charges, the usually complicated problem of electromagnetic corrections is much less difficult. From the data on the quantities $FT^{(0-0)}$ for $O^+ - O^+$ transitions and the quantities $FT^{(n)}$ for the neutron we can determine the ratio g_{GT}/g_F (here F is a normalizing factor owing to the different phase volumes and giving the dependence of the decay probability on the energy of the transition,

and $T = \tau \ln 2$ is the half-life). Using the value of τ_n given before as the world average and the value of $FT^{(0-0)}$ from Ref. 79, we obtain for the quantity λ' the value

$$\lambda' = \sqrt{\frac{|g_A|^2 + |g_T|^2}{|g_V|^2 + |g_S|^2}} = 1.266 \pm 0.005,$$

which is in good agreement with the value λ obtained from the study of correlations in neutron decays. We point out that in the calculation of the quantity λ from data on correlation coefficients it was assumed that the S and T types were absent, whereas the value obtained from the ratios of FT gives the ratio of the Gamow-Teller and Fermi constants rigorously. The good agreement between the two quantities is evidence of the predominance of the V and A interactions.

Accordingly, the study of the half-life and the correlation coefficients in neutron decay and the study of nuclear β decay provides a test of the validity of the predictions of the theory of β decay. There are several papers on this question (cf. e.g., Refs. 79-81). However, simply from the data on correlations in neutron decay one can get the results, without bringing in other data. For example, in Ref. 82 it is shown that in the case of the $V-A$ type taken as valid there must be a simple relation between the correlation coefficients:

$$\begin{aligned} 1 + A &= B + a, \\ A(1 + A) &= aB. \end{aligned} \quad (12)$$

Substitution of the values we have given shows that these relations are well satisfied. We must not, however, conclude from this that the S and T interaction types are certainly absent. A more detailed analysis of this problem can be found in Refs. 79-81. A rather complete survey of the history and current state of neutron decay research can be found in a review by Eroziolimskii.⁸³

c) Search for violation of T parity in neutron decay

It can be seen that the last term in the expression (11) for neutron decay changes sign when the sign of the time is changed. This means that the observation of a correlation between the neutron spin and the plane in which the light particles are emitted would indicate a violation of T parity in the weak interactions. Formally this would lead to a complex value of the coupling constant or to an additional phase constant in the ratio of the constants. In particular, the ratio of the axial-vector and vector constants is usually written in the form $\lambda = g_A/g_V = |\lambda| e^{i\varphi}$. If time parity is strictly conserved, then $\varphi = \pi$. The first attempt to detect a T -odd correlation was undertaken in the previously cited paper by the Argonne group.⁷¹ No effect was found, and the value obtained for D was $D = 0.04 \pm 0.05$. Interest in this experiment increased greatly after the discovery of violation of T invariance in the decay of neutral K mesons.⁸⁴ An important improvement in the accuracy of the coefficient of the T -odd correlation was achieved by the group at the Kurchatov Institute of Atomic Energy.⁸⁵ This experiment involved the development of an original approach to the problem of deter-

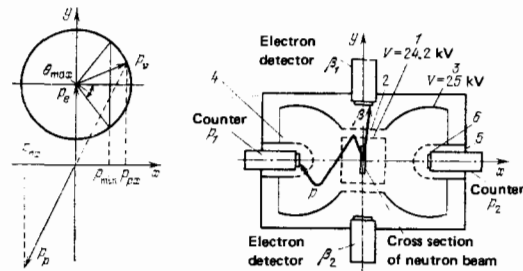


FIG. 7. Diagram of apparatus of Eroziolimskii *et al.* for measuring three-vector correlation⁸⁵ and the spin-antineutrino correlation⁷⁴ in the decay of polarized neutrons. 1—Screen-mesh cylinder forming field-free region; 2—cylindrical grid with potential reflecting protons; 3—outer spherical focussing electrode; 4—inner grid electrode; 5—chamber; 6—scintillator proton detector. At left: momentum diagram of products of neutron decay. With registration of electrons in a narrow solid angle, time-of-flight measurement of one component of the proton momentum defines a cone of directions of the neutrino emergence.

mining the direction of emission of the antineutrino. Whereas in the Argonne experiment cases of decay with a definite direction of the recoil proton were registered, here there was carried out an analysis of one component of the momentum of the proton with respect to the time of flight, with a fixed direction of emission of the electron. This made it possible directly to distinguish the angle of emission of the antineutrino by using the law of conservation of momentum (for details see Ref. 83). The use of two pairs of proton and electron detectors made the arrangement remarkably symmetrical and reduced sharply the possibilities for methodological errors. The accuracy of the experiment was limited by statistics and reached ± 0.01 , but no effects was found. Further progress was possible by work with more intense beams of polarized neutrons. At the medium-flux reactor of the Institute a new vertical channel of polarized neutrons was constructed; this made it possible to lower further the experimental limit on the possible existence of such an effect. The result obtained was $D = (-2.7 \pm 3.3) \cdot 10^{-3}$ (Ref. 86). An analogous experiment was done by Steinberg *et al.* at the high-flux reactor in Grenoble, with the result $D = (-1.1 \pm 1.7) \cdot 10^{-3}$ (Ref. 87). The world average value of the coefficient, taking into account two other early papers, is now¹⁸: $D = -0.0009 \pm 0.0013$, and the phase of the ratio of the constants is $\varphi = 180.1^\circ \pm 0.17^\circ$.

6. THE SEARCH FOR AN ELECTRIC DIPOLE MOMENT OF THE NEUTRON

Along with the problem of violation of T -invariance there is also the related problem of the existence of an electric dipole moment (EDM) of the neutron. Long before any such relations was recognized, however, it was understood that the existence of an electric dipole moment of an elementary particle contradicts the law of conservation of spatial parity. The EDM of a particle is directed along its spin, which is the only distinguished direction for an elementary particle. But the axial vector of the spin and the vector of the EDM

have different transformation properties under the operation of P inversion, so that there is a contradiction.

The history associated with the search for an EDM evidently began in 1950, when Purcell and Ramsey stated the opinion that the theoretical arguments against the existence of an EDM of an elementary particle are based on assumptions that have not been sufficiently well tested experimentally.⁸⁸ In 1951 Smith, Purcell and Ramsey made the first experiment to look for an EDM of the neutron. It is interesting that the negative result which these authors found did not seem to them at the time so important as to require immediate publication. The description of the experiment and its result were contained only in Smith's doctoral dissertation (cf., e.g., Refs. 44, 89, 90). This work was published only after six years, when the possibility of violation of spatial parity conservation had become an established fact. The upper limit found in the experiment for the EDM was $|d| < 5 \cdot 10^{-20} e \cdot \text{cm}$. At about this time Landau showed that an EDM of an elementary particle can exist only in the case when there is violation of parity conservation not only for space but also for time.⁹¹ After the discovery of T -parity nonconservation in the decay of K mesons the search for an EDM became an urgent matter, since its discovery would be a proof of the universality of T parity nonconservation.

Almost all experiments on the search for an EDM of the neutron have been based on the use of a magnetic resonance method,⁸⁾ based on the following general principles. A beam of polarized neutrons is sent into a region with a uniform magnetic field H_0 , in such a way that the direction of the polarization vector of the neutrons is perpendicular to the magnetic field. At the same time a uniform electric field E is applied parallel to the magnetic field. If there is no EDM the neutrons precess relative to the direction of the magnetic field with the Larmor frequency. If the neutron has an electric dipole moment d , the expression for the energy of the neutron in the field contains an additional term proportional to $d \times E$, and the precession frequency is changed; this can be observed through a change of the phase of the precession at a certain point on emergence from the field. The preparation of a beam with the magnetic moment directed perpendicular to the field is accomplished by sending the beam through a radiofrequency coil with its axis perpendicular to the direction of the field H_0 , which produces an oscillating field H_1 at the frequency of the Larmor precession. There is a similar coil at the exit from the field, so that the total rotation of the polarization vector as the neutron passes along the apparatus depends on the phase of the oscillations in the coils and on the phase angle of the precession over the path between the coils. In the experiment one measures the change of this phase angle when the relative direction of the magnetic and electric fields is reversed.

⁸⁾The only work in which the crystal-diffraction method is used to measure the EDM is Ref. 92. It is also expounded in detail in Ref. 46.

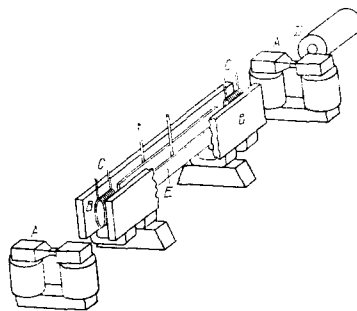


FIG. 8. Diagram of experiment of Smith, Purcell, and Ramsey to measure electric dipole moment of the neutron. A—Polarizer and analyzer magnets, B—magnet with uniform field, C—coil to produce high-frequency field, D—neutron counter, E—plates between which electric field is produced.

This was the procedure used by Smith, Purcell and Ramsey (Fig. 8). Subsequently many measurements were made in a similar way (for a survey of these papers see Refs. 46, 90, 93). The most accurate result obtained with this method is: $|d| < 3 \cdot 10^{-24} e \cdot \text{cm}$ at the 90% confidence level.⁹³ Thus during twenty years since the publication of the first paper the upper limit on the possible value of the EDM of the neutron has been lowered by four orders of magnitude. However, the possibilities of the method were practically exhausted, and there are at least two serious obstacles to further progress. It is obvious that the phase angle to be measured is proportional to the duration of the precession, i.e., the time the neutron spends in the apparatus. Therefore the sensitivity of the system to the looked-for effect can be increased by lengthening the region of uniform field and lowering the speed of the neutrons. The research of Ref. 93 used cold neutrons with speeds 200 m/s from the refrigerated moderator at the high-flux reactor in Grenoble. The use of still slower neutrons involves losses of statistical accuracy owing to the low currents of cold neutrons, and lengthening the apparatus encounters considerable technical difficulties. The second thing that hinders the increase of accuracy with the method described is that when the magnetic moment moves in an electric field there is magnetic action on the neutron with the effective interaction

$$H_E = \mu_n \frac{|VE|}{c}. \quad (13)$$

This leads to an observed spurious effect

$$d = \mu_n \frac{r}{c} \sin \beta, \quad (14)$$

where β is the angle between the directions of the magnetic and electric fields. This effect was the main limiting factor on the accuracy of the last experiments in Grenoble.

A way out of this situation, providing solutions for both difficulties, was found in 1968 by Shapiro.⁹⁴ He proposed to use an idea of Zel'dovich, who stated ten years earlier that it would be possible to store very slow so-called ultra-cold neutrons (UCN) in a closed cavity.⁹⁵ (On the discovery of UCN see Sec. 8.) Using such a container with UCN instead of a neutron beam one can increase the time the neutrons spend in the

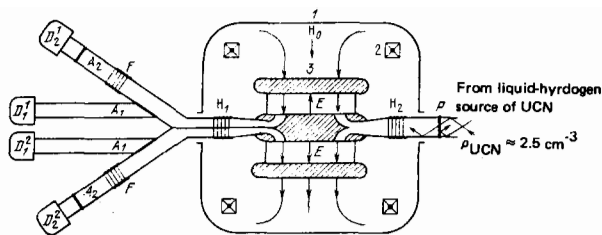


FIG. 9. Diagram of experiment of group at Leningrad Institute of Nuclear Physics to look for dipole moment of the neutron^{96,97} 1—magnetic screen, 2—coils producing constant magnetic field, 3—chamber for storing UCN. P —polarizer, A_1, A_2 —analyzers, $D_1^1, D_1^2, D_2^1, D_2^2$ —detectors registering neutrons of different polarizations from the two halves of the storage chamber, H_0 —constant magnetic field, H_1 —oscillating magnetic field, E —electric field.

fields by several orders of magnitude and thus raise the sensitivity of the experiment. At the same time the spurious effect is sharply reduced, being proportional to v/c , and in some cases vanishes. The increase in sensitivity should considerably outweigh the appreciable loss in the statistical properties of the experiment.

A measurement of the EDM of the neutron by means of UCN was made by Lobashev's group in Leningrad^{96,87} (Fig. 9). A double chamber was used for storing the UCN; it was located in a uniform magnetic field, but the electric fields in the two parts were opposite in direction. The use of such a double chamber and four detectors, which made it possible to register simultaneously the neutrons having opposite polarizations for each of the two halves independently, with regular reversals of the directions of the electric fields, makes the arrangement highly symmetrical and insensitive to every sort of random disturbance. The time spent by the neutrons in the apparatus was 5 s instead of the value 10^{-2} s in Ref. 93, and the magnetic resonance line was correspondingly narrower. All this made possible a considerable advance in the search for the EDM. The last result obtained with this method is $d = (2.3 \pm 2.3) \cdot 10^{-25} e \cdot \text{cm}$ or $|d| < 6 \cdot 10^{-25} e \cdot \text{cm}$ at the 90% confidence level. It must be pointed out that this increase of accuracy by a factor five was obtained with a reactor with neutron flux an order of magnitude smaller than that of the Grenoble reactor used for the work of Ref. 93. This means that a further advance in accuracy is to be expected in the not too distant future. Such progress is of undoubted interest, even if no EDM is thus found. The point is that there are now many papers which predict values for the EDM on the basis of various theoretical models. (For reviews of these papers cf., e.g., Refs. 46, 93, 97.) Some of them give values of the EDM that cannot be tested in the very near future, and some have been disproved, being inconsistent with the present experimental upper limit. However, many models give values of the EDM of the order of $10^{-25} e \times \text{cm}$, and thus their acceptability will be tested fairly soon.

7. THE PROBLEM OF THE NEUTRALITY OF THE NEUTRON

While the question concerning the dipole moment of the neutron is that of its value, the existence of an EDM being scarcely doubted, there is no theoretical basis for the supposition that the neutron has any electric charge. It can be stated that there is no theoretical need so far for a charge of the neutron. But neither are there any very strong theoretical arguments against its existence. Therefore experiments to look for a charge are by no means senseless. From the form of the neutron decay reaction



we can obtain in the most general form the possible relations for the charges Q_i involved in the reaction and leading to $Q_n \neq 0$:

$$\left. \begin{array}{l} \text{a) } |Q_p| \neq |Q_e|, \quad Q_{\tilde{\nu}} = 0, \\ \text{b) } |Q_p| = |Q_e|, \quad Q_{\tilde{\nu}} \neq 0, \\ \text{c) } |Q_p| = |Q_e|, \quad Q_{\tilde{\nu}} = 0 \end{array} \right\} \quad (16)$$

(in the last case the law of charge conservation is not satisfied).

Each of these cases is in principle testable by experiment. The best test of the relation a) is made in Ref. 98. Equality of the proton and electron charges has been established to an accuracy of 10^{-21} . The validity of charge conservation is tested by searching for processes that are forbidden only by this law.⁹⁹⁻¹⁰¹ If we put the results of these papers in the form of a ratio of the probabilities of neutron decay by two channels

$$R = \frac{\Gamma(n \rightarrow p + \nu_e + \tilde{\nu}_e + \text{neutral particles})}{\Gamma(n \rightarrow p + e^- + \tilde{\nu}_e)} \quad (17)$$

the best estimate of R , obtained in the last of these papers¹⁰¹ is $R \leq 9 \cdot 10^{-24}$. Of course, there have also been, and still are, direct searches for a charge of the neutron.

The first experiment of this kind was made in 1956 by Shapiro and Éstulin.¹⁰² It was based on measurement of any possible deflection of a well collimated beam of neutrons passing between the plates of a condenser. The deflection is given by

$$x = \frac{Q_n E l^2}{2m v^2} \quad (18)$$

where E is the intensity of the electric field, l is the length of the condenser, and v is the speed of the neutron. The upper limit for the charge found in this work is $Q_n \leq 6 \cdot 10^{-12} e$. A similar experiment was done in 1960 by Zorn, Chamberlain, and Highes,¹⁰³ with the result $Q_n < 1.3 \cdot 10^{-13} e$. Shull¹⁰⁴ looked not for a linear displacement of the beam, but for a small change of angle coming from the bending of the neutron's path, if it has a charge, when it passes through the condenser. This angle is given by

$$\alpha = \frac{Q_n E l}{m v^2} \quad (19)$$

A beam with a very small angular divergence was formed by Bragg reflection from a silicon crystal of very high quality. A second Bragg reflection was used to

analyze the deflection angle, and the condenser was placed between the crystals. Shull's result is $Q_n = (-1.9 \pm 3.7) \cdot 10^{-18}e$, the best up to the present time. Evidently, however, we can expect that very soon there will be considerable improvement in the sensitivity of experiments to search for a charge. A new optical instrument for detecting a neutron charge has been built in Grenoble.¹⁰⁵ The authors state that they count on greatly reducing the estimate of the possible value of the charge. There are a number of other proposals which, if carried out, could lower the estimate for Q_n to $(10^{-22} - 10^{-23})e$.

8. THE DISCOVERY OF ULTRACOLD NEUTRONS

In telling about remarkable experiments in the field of neutron physics, we must not fail to tell about the discovery of ultracold neutrons (UCN). We have already mentioned UCN in connection with experiments in the search for an EDM of the neutron. According to the latest standard terminology, neutrons are called ultracold if they are capable of being totally reflected from a vacuum-medium interface at all angles of incidence. This feature of the reflection of UCN was first pointed out by Zel'dovich in 1959.⁹⁵ Here Zel'dovich called attention to the fact that neutrons with energy less than some limiting value E_{lim} , which is characteristic of a given substance, will be reflected from a boundary of that substance with an extremely small absorption in the wall. Owing to this it was possible to accumulate ultracold neutrons in a closed container. The value of the limiting energy is given by

$$E_{lim} = \frac{h^2}{2\pi m} Nb, \quad (19')$$

where m is the mass of the neutron, N is the density of the nuclei in the substance, and b is the coherent scattering length of the substance. For most nuclei b is positive and of the order of a few fermis. Substituting suitable values in Eq. (19), we find that E_{lim} amounts to a small multiple of 10^{-7} eV. The speed of neutrons having this energy is about 5 m/s. It seemed extremely difficult to test Zel'dovich's idea, because the fraction of neutrons in the Maxwell distribution from an ordinary reactor that have such energies is very small. It could be expected that the neutron gas stored in a trap would have a density of 1-2 neutrons per cm^3 at a temperature $T \sim 10^{-3}$ K. It is important here that at the beginning of the 1960s not enough attention had been given to a rather important physical problem, whose solution could have stimulated the beginning of these difficult studies. Soon after Zel'dovich's paper appeared, Vladimirskii showed that very slow neutrons can also, in principle, be kept in magnetic traps.¹⁰⁶ The presence of the neutron magnetic moment leads to an additional term in the neutron energy, $E = -\mu B$. The value of μ in the appropriate units is $\mu = 6.03 \cdot 10^{-12}$ eV/G. For ordinary fields in laboratory work, $B \approx 10-20$ kG, the magnetic energy is $E_\mu = (0.6-1.2) \cdot 10^{-7}$ eV, of the order of magnitude of E_{lim} .

As was mentioned earlier, in 1968 Shapiro turned to the possibility of storing neutrons in connection with the problem of increasing the sensitivity of experimental

searches for the EDM of the neutron.⁹⁴ Proceeding to experiment, Shapiro and his group had already shown by the end of 1968 that UCN can be brought out of a reactor with strongly curved neutron guides which did not transmit faster neutrons, and that the neutrons remained in the apparatus for appreciable times.¹⁰⁷ The results of more detailed studies, including direct experiments on storing neutrons in traps and measuring spectra of UCN were published at the end of 1970.¹⁰⁸ The storage time for neutrons achieved in this experiment was about 30 s, and subsequently was considerably increased. Simultaneously with the Shapiro group, Steyerl in Munich began to work with very cold neutrons. Taking as his goal the measurement of cross sections for interactions of neutrons with matter at very low speeds, Steyerl, at the suggestion of Maier-Leibnitz, set up a spectrometer for very slow neutrons and began to work with neutrons in an energy range extending down to the region of UCN.¹⁰⁹

These efforts were the foundation of research with UCN. Today UCN physics is an independent and rapidly developing branch of neutron physics. A survey of early work on ultracold neutrons has been made by Shapiro.¹¹³ There are rather detailed reviews by Steyerl¹¹¹ and by Golub and Pendlebury.¹¹²

9. WAVE PROPERTIES OF THE NEUTRON

a) Neutron diffraction

The first person to express the conviction that the motion of a neutron must be determined by wave mechanics, and that consequently neutrons, like x-rays, must be diffracted by crystals was evidently Elsasser.¹¹³ His paper appeared in 1936. Almost immediately there followed a paper by Halban and Preiswerk.¹¹⁴ They observed a change of the angular distribution of neutrons from a scatterer, an iron cylinder, depending on the temperature of the scatterer, in qualitative agreement with Elsasser's predictions. However, the most convincing experiment confirming the existence of the phenomenon of neutron diffraction was made by Mitchell and Powers in 1936.¹¹⁵ Using neutrons from a $Be(\alpha)$ source placed in paraffin, they registered the neutrons scattered from a system of 16 crystals of MgO placed at the angle of Bragg reflection for the maximum of the neutron spectrum from the source. The reflected intensity decreased when the angle was changed.

In subsequent years work on the detection and study of neutron diffraction continued. In particular, it was found that the cross sections for scattering from polycrystalline specimens and from single crystals were different, and that the cross sections of individual atom atoms were not additive in chemical compounds formed from them; this indicated that coherent effects are important in the interaction of neutrons with matter.¹¹⁶⁻¹¹⁹ But the most beautiful and impressive experiments in the new field of neutron optics were made after the appearance of atomic reactors, and the largest contribution to the development of this science was made by Fermi and his school. In 1944 Fermi and his group measured the spectrum of the neutrons from the graphite thermal column of the Argonne reactor and

found that it was very strongly enriched with slow neutrons. Special experiments on the passage of thermal neutrons through a graphite filter showed that such a filter transmits only quite cold neutrons. The effective temperature of the transmitted neutrons was only 18°K, with the filter at room temperature. It was quite clear that polycrystalline graphite effectively scatters thermal neutrons owing to the Bragg reflection. Neutrons with wavelengths more than twice the distance between crystal planes are not scattered and pass through the filter, as follows at once from the Bragg formula

$$n\lambda = 2d \sin \theta. \quad (20)$$

This explanation was well confirmed quantitatively, since the value of $2d$ for graphite is 6.69 Å, and the filtered neutrons had effective wavelength 7.15 Å.¹²⁰ After the appearance of reactors there were powerful and well collimated beams of neutrons, so that a clean experiment on neutron diffraction could be made. A beam with angular divergence about 10' was used. Zinn¹²¹ observed a peak of the Bragg reflection of the expected width from a CaCO₃ crystal. This technique soon made possible the construction of a crystal monochromator. Using for monochromatization reflection from the (100) planes of CaF₂, Fermi and Marshall then observed Bragg reflection from another crystal placed in the monochromatic beam. Measuring the intensities of various orders of reflection from various specimens, they were able to determine the phase of the reflected wave for several substances. The phase of the scattering by a nucleus is related to the sign of the coherent scattering length for the given nucleus. For most of the specimens studied the phase corresponded to $b > 0$, but for the first time a substance was found, namely Mn, which scattered neutrons with the opposite phase. This was probably the first crystallographic research.¹²²

After this work with crystal spectrometry developed rapidly.^{123,124} Soon spectrometers, or rather refractometers with two crystals, one of which is a monochromator, became indispensable accessories of a research reactor.

Neutron diffraction has now become a powerful means for research on properties of matter. Concerning neutron diffraction see, for example, Refs. 45, 125–127.

b) Index of refraction of neutron waves

The experimental discovery of a major manifestation of wave properties of neutrons suggested that the wave nature of neutrons could be shown in other ways than by diffraction. Fermi suggested that various substances might have different indices of refraction for neutron waves. Physically, an index of refraction different from unity arises because of the coherent scattering of neutrons by the atoms of a substance:

$$n^2 = 1 - \lambda^2 \frac{Nb}{\pi}, \quad (21)$$

where N is the density of the scatterers, λ is the wavelength of the neutron, and b is the coherent scattering length. For $n - 1 \ll 1$

$$n - 1 = -\lambda^2 \frac{Nb}{2\pi}. \quad (22)$$

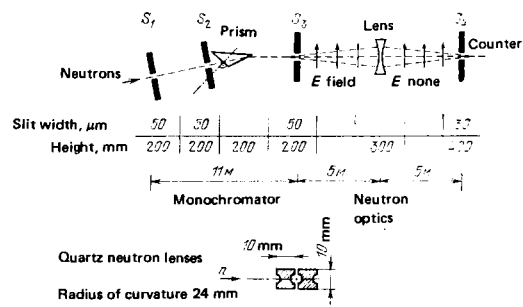


FIG. 10. Optical apparatus for searching for charge of neutron.

The sign of the quantity $n - 1$ depends on the sign of b , or on the phase of the scattered wave. In papers on this problem Fermi showed that whereas for x-rays the scattering phase is negative, for neutrons it can have either sign. If $b > 0$ there can be total reflection from the surface of the substance. Fermi and Zinn undertook experiments looking for reflections from various substances and discovered the phenomenon of total reflection, thus proving the existence of the index of refraction.^{128,129} This was in 1944. Later, as already stated, the fact that the scattering phase could have either sign was experimentally confirmed,¹²² and consequently the index of refraction can actually be either larger or smaller than unity. The scattering lengths were subsequently measured for practically all nuclei, and it turned out that the great majority have refraction indices smaller than unity. The existence of refraction of neutron waves makes it possible in principle to use lenses and prisms for neutrons. In the remarkable Lectures on Neutron Physics given by Fermi in 1945 we find: "But the index of refraction for neutrons is very close to one. This means that a converging lens for neutrons would have to bulge very much along the axis to do any good if it were made of substances where n is slightly greater than one. For substances in which $n < 1$ a converging lens would look like the diverging lenses of optics. These lenses are possible in principle but because $|n - 1|$ is so small they are not at all practical." (Ref. 130.)

Fermi's pessimism is quite understandable, inasmuch as he had in mind thermal neutrons. But owing to the specific form of the dispersion law, the situation changes very sharply when we go over to very slow neutrons. [It can be seen from Eq. (21) that $n^2 - 1 \propto \lambda^2$.] For cold neutrons both prisms and lenses were later used. The most recent and quite beautiful use of neutron-optical elements is the optical apparatus mentioned earlier for looking for a neutron charge.¹⁰⁵ In this arrangement both a prismatic monochromator, quite analogous to the simple monochromator generally used in optics, and focussing double concave lenses are used.

The dispersion law (21) also shows that total reflection can occur for ultracold neutrons. In fact, the expression for the limiting wavelength is found from the condition $n^2 = 0$:

$$\lambda^2 = \frac{\pi}{Nb}. \quad (23)$$

From this we easily get the values of the limiting speed and the limiting energy, in agreement with Eq. (19). In fact, if the interaction energy between a neutron and the medium or the field is E , we have the simple relation

$$n^2 = 1 - \lambda^2 \frac{2m}{\hbar^2} E. \quad (24)$$

In particular, when magnetic fields are involved, there is a term $\pm \mu B$ in the expression for E , and the index of refraction has different values for the two directions of the polarization. This leads to the phenomenon of dichroism, in this case magnetic dichroism.⁹⁾ We have already mentioned experiments that demonstrate this phenomenon, such as the presence of two critical angles in reflection from a magnetic mirror in the experiment of Hughes and Burgy⁴⁹ or Shull's demonstration of double refraction in a magnetized lens.⁵³ The phenomenon of dichroism must appear in all cases when the quantity b depends on the direction of the polarization. In particular, the scattering length depends on the sign of the mutual orientation of the spins of the neutron and of the scattering nucleus. Therefore dichroism, manifesting itself in the phenomenon of rotation of the plane of polarization, must occur whenever polarized neutrons are sent through a polarized target. This effect, predicted in 1964 by Baryshevskii and Podgoretskii,¹³¹ was observed experimentally by the Abragam group in France.¹³² A connection between the direction of neutron polarization and the amplitude of coherent scattering can also appear owing to effects of nonconservation of parity. In this case a rotation of the plane of polarization will be observed when neutrons pass through an unpolarized target. Predicted by Michel and by Stodolsky,¹³³ this phenomenon was quite recently observed experimentally.¹³⁴

Thus in the case of neutrons, the phenomenon of refraction, which is due to interference between incident and scattered waves and is well known in optics, takes on a number of new and remarkably beautiful features.

c) Neutron experiments and free-particle quantum mechanics

Neutrons, with low velocities, relatively large mass, and consequently large wavelength, and also being without charge, provide very convenient objects for experiments used to demonstrate or investigate quite a number of quantum-mechanical effects.

As of today, we already have a rather large number of such experiments, and both reports of new results and suggestions for new experiments are constantly appearing. Therefore any sort of detailed survey of experiments of this kind would have to be the subject of a separate article. We here list only some of the most interesting results and give a very brief description of individual experiments, and our list of literature on this matter lays no claim to completeness.

⁹⁾The phenomenon of dichroism when neutrons are sent through a magnetized ferromagnetic substance had been predicted as early as 1941.¹⁴⁹

1) *The potential barrier for a free particle.* As has already been pointed out the coherent interaction of a neutron with the nuclei of a substance leads to the appearance of a discontinuity of the potential at the boundary of the substance; the magnitude of the potential change is proportional to the coherent scattering length for the given substance. Therefore the behavior of the reflection and transmission coefficients for a neutron at the surface of a substance is given by the known quantum-mechanical relations for the reflection of a particle from a potential barrier. The formulas have been tested experimentally with satisfactory accuracy by Steyerl for ultracold neutrons with energies close to the height of the barrier.¹³⁵ In the case of a thin film of the substance, with thickness comparable with the neutron's wavelength, the expressions for the transmission and reflection of the neutrons include terms corresponding to the interference of waves reflected from the two boundaries of the film. Besides this, the tunnel effect becomes important. When there are two potential barriers, energy levels appear between them; the positions and widths of these levels depend, as is well known, on the degree of transparency of the barriers and the distance between them. The presence of such quasistationary levels has also been observed experimentally through resonances in the transmission of neutrons through a three-layer system made from substances having different values of the potential.¹³⁶ For three barriers spaced at equal intervals there is a splitting of the levels, and in the limit of an infinite periodic structure this goes over into a continuous energy band. This sort of splitting of the levels has recently been demonstrated in a direct experiment.

2) *A neutron interferometer.* The first neutron interferometer was constructed by Maier-Leibnitz and Schieringer in 1961. In this device two coherent beams were formed by refracting the original beam with a biprism. The diffraction pattern was observed by shifting the detector in a direction transverse to the axis of the original beam into the region where the beams overlapped. The zeroth and first orders of diffraction were observed.¹³⁸ A much more convenient type of interferometer, which serves as the basis for making a number of remarkable experiments, is based on diffraction by a perfect single crystal of silicon. The device consists of three plane plates of silicon, processed with minute precision, constructed as an E -shaped single crystal with portions removed to leave the three joined and spaced plates. The splitting of the wave occurs at the first plate, after which the transmitted and the diffracted waves undergo Bragg reflection from the second plate and come together at the third, which is the analyzer. The amount of space separation between the two waves attains values of the order of 1 cm (Fig. 11). For details on interferometers of this type see Refs. 126, 127.

3) *A demonstration of the 4π symmetry of the neutron wave function.* The intensity of the resulting beam of neutrons after passage through the interferometer is determined by the coherent superposition of the two waves which have followed paths I and II:

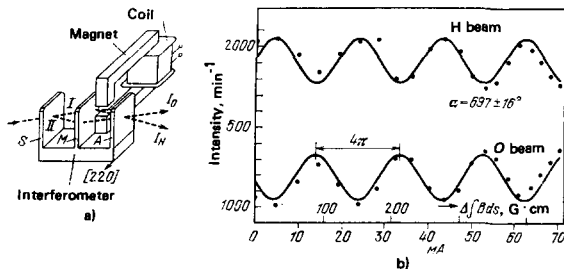


FIG. 11. Diagram of experiment of Rauch *et al.*, to demonstrate the 4π symmetry of the neutron wave function by means of a neutron interferometer.

$$I = |\psi_I + \psi_{II}|^2. \quad (25)$$

The phase of the wave function in each of the paths can be changed by introducing a region with a nuclear, gravitational, or magnetic potential. If in one of the arms of the interferometer one produces some local region with a uniform magnetic field B , a precession of the spin will occur in this field, and the angle through which it turns is given by the following expression:

$$\alpha = \pm \omega_L \tau = \pm \frac{\gamma B D m \lambda}{h} \quad (26)$$

($\omega_L = \gamma B$ is the Larmor frequency; D is the length of the region with the field through which the neutron passes in the time $\tau = D/v$; m is the mass; and γ is the gyromagnetic ratio). Then the spinor wave function of the neutron, altered by the action of the magnetic field inserted into the path I is

$$\begin{pmatrix} +\psi^I(x) \\ -\psi^I(x) \end{pmatrix} = \begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix} \begin{pmatrix} +\psi^I(0) \\ -\psi^I(0) \end{pmatrix}. \quad (27)$$

From this we get the expression for the way the counting rate beyond the interferometer depends on the angle of precession,

$$I_0(\alpha) = \frac{I_0(0)}{2} \left[1 + \cos\left(\frac{\alpha}{2}\right) \right]. \quad (28)$$

As we see, the period of this intensity function is 4π . This 4π symmetry of the wave function of the neutron was verified in an experiment with a neutron interferometer. The most precise experiment gave as the value of the period $T = 716^\circ \pm 3.8^\circ$ (Fig. 11).¹³⁹

4) *The gravitational phase shift, the effect of the Earth's rotation, and the principle of equivalence in quantum mechanics.* If the arms of the interferometer are at different heights, i.e., the motion occurs at different gravitational potentials, there is a phase shift associated with gravitation. In the general case the phase difference for the wave after going through the interferometer is

$$\Delta\varphi = \frac{1}{h} \int_I \mathbf{p} \, d\mathbf{r} - \frac{1}{h} \int_{II} \mathbf{p} \, d\mathbf{r} = \frac{1}{h} \oint \mathbf{p} \, d\mathbf{r}. \quad (29)$$

Since the experiment is performed on the surface of the rotating Earth, it is necessary to take this rotation into account. As is well known, the classical Hamiltonian for a particle in a rotating coordinate system is¹⁴⁰

$$H = \frac{p^2}{2m_I} - G \frac{M m_g}{r} - \omega L, \quad (30)$$

where $L = [\mathbf{r} \times \mathbf{p}]$, ω is the frequency of rotation of the system, and m_I and m_g are the inertial and gravitational masses. Calculating p and r from the classical Hamiltonian equations and using Eq. (30), the quasi-classical approximation we get for the phase shift

$$\Delta\varphi = \frac{m_I}{h} \oint \dot{\mathbf{r}} \, d\mathbf{r} - \frac{m_I}{h} \oint (\omega \dot{\mathbf{r}}) \, d\mathbf{r}, \quad (31)$$

where the first term takes into account a possible change of the phase along the path of integration, including that owing to the action of the force of gravity, and the second owes its origin to the rotation of the coordinate system. The existence of the second effect was first demonstrated for light in 1913 by Sagnac¹⁴¹ in an experiment in which an optical interferometer was placed, along with the source, on a rotating platform. Thereafter this effect was observed by Michelson for the case of the Earth's rotation.¹⁴² The phase shift caused by gravitation is¹⁴³

$$\Delta\varphi_{\text{grav}} = -2\pi m_I m_g (gh^2) \lambda A' \sin \beta, \quad (31')$$

where λ is the wavelength, A' is a quantity of the dimensions of an area and order of magnitude close to that of the area occupied by the interferometer, and β is the angle between the plane of the path and the horizontal. When β is changed, i.e., the apparatus is rotated around a horizontal axis, the gravitational phase shift changes and can be measured from the periodic modulation of the intensity. The phase shift associated with the Sagnac effect is

$$\Delta\varphi_{\text{Sgn}} = \frac{4\pi m_I}{h} \omega A, \quad (32)$$

where A is the area of the interferometer and ω is the frequency of the Earth's rotation, $\omega = 7.29 \cdot 10^{-5} \text{ s}^{-1}$. Both effects were observed and measured rather accurately. From the gravitational shift one can determine the factor $(m_I m_g)^{1/2}$ and compare it with the value of the neutron mass, $1.6747 \cdot 10^{-24} \text{ g}$. This comparison is a test of the principle of equivalence for microscopic objects. The most precise results for measurements of both the gravitational phase shift and the Sagnac effect were obtained in Ref. 143. The value found for $(m_I m_g)^{1/2}$ was $(1.675 \pm 0.003) \cdot 10^{-24} \text{ g}$. For the Sagnac effect the theoretical value is $q^{\text{theor}} = 1.604 \text{ rad}$, and that found from the experiment was $q = 1.689 \pm 0.003$ where q is the period of the intensity oscillation when the apparatus is turned about a vertical axis.

5) *A test of the linearity of quantum mechanics.* The question of the degree to which the equations of quantum mechanics are linear is not new. One of the reasons for the interest in the question of the linearity of the Schrödinger equation is the fact that the introduction of a nonlinear term $F(|\psi|^2)$ in the Hamiltonian leads in some cases to solutions with a nonspreading wave packet. There are grounds for the assumption that this nonlinear term must be logarithmic in form.¹⁴⁴ Then

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}, t) \right] \psi - b \ln(a^3 |\psi|^2) \psi, \quad (32')$$

where a is a quantity which does not have to be a universal constant, and b is a fundamental constant with the dimensions of energy. Estimates of the quantity b from measurements of the Lamb shift in the hydrogen

spectrum gave $b < 4 \cdot 10^{-10}$ eV.

Recently it has been shown that the presence of a nonlinearity in the Schrödinger equation leads to an additional phase shift in an interferometer in the case that there is an attenuator, i.e., a semitransparent absorber, in one of the arms. This phase shift is¹⁴⁵

$$\Delta\varphi = \frac{d}{\hbar} \sqrt{\frac{m}{2E}} [F(|\psi|^2) - F(\alpha)|\psi|^2], \quad (33)$$

where d is the path length after the attenuator with absorption factor α^2 , and m and E are the mass and energy of the neutron. An interferometer experiment of this kind has been made. The estimates obtained for b is $b < 3.4 \cdot 10^{-13}$ eV.¹⁴⁶ A still more precise estimate has been obtained from an experiment on the exact measurement of the intensity distribution of neutrons in Fresnel diffraction at the edge of a screen. The presence of a nonlinearity would lead to a deviation of the intensity distribution in a direction transverse to the beam from the theoretical form. The estimate obtained in this experiment was $b < 3.3 \cdot 10^{-15}$ eV.¹⁴⁷

10. CONCLUSION

In concluding this survey, we would like to make some remarks of a general nature. It can be seen that many of the problems relating to various aspects of the study of the neutron, which have arisen in the course of the last fifty years, are still of current interest. For example, work on increasing the accuracy of the value of the neutron mass continues. The existence of the neutron's magnetic moment was firmly established in 1937, but work on measuring it precisely still goes on. The study of the β decay of the neutron is still exciting, though the fact that it is unstable was established in the 1950s. The electromagnetic structure of the neutron is still being investigated, and the searches for an electric charge and for an electric dipole moment continue. The neutron has been found to be an ideal object for experiments in which the wave properties of material particles are clearly and impressively manifested. Neutron interferometry has been originated and is being rapidly developed.

Why has the neutron been so popular? Why have physicists not lost their zest for studying its properties after fifty years? A brief answer to these questions might be as follows. The neutron is elementary enough to be a solid representative of a broad family of elementary particles, but at the same time it has a rather complicated structure, and takes part in all the known main types of interactions. It is neutral, but has non-zero spin and magnetic moment. It is stable enough to be experimented with easily, even when its speed is very small, and at the same time has a not-too-long half-life, so that its transformations can be studied relatively easily. The neutron has a relatively large mass. It is the cheapest of the massive neutral elementary particles.

These facts give us reason to expect that also in future years neutron physics will remain a fundamental branch of science, continuing its development along with and in close association with the younger and newer developments in research.

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