V. A. Belinskiĭ. *Solitons in the theory of gravitation.* The method of the inverse scattering problem can be applied successfully for the integration of Einstein's gravitational equations in a vacuum (or in a space filled with an ideal fluid with the equation of state $p = \varepsilon$) in the case in which the metric tensor depends on only two variables. Although they are exceptional, such metrics have numerous applications in the theory of gravitation. This class includes the solutions of Schwarzschild and Kerr, the axially symmetric Weyl metrics, the cosmological solutions of Friedmann, the homogeneous cosmological models of types I–VI, and the solutions describing Einstein-Rosen cylindrical waves and Robinson–Bondi plane waves. Such a metric also describes the oscillatory regime of the approach to a cosmological singularity on sections of the so-called "prolonged times" investigated in Ref. 1. In this work, Einstein's equations for the two-variable dependences of the metric tensor under consideration were written in matrix form using coordinates similar to those introduced by Einstein and Rosen for the description of solutions involving cylindrical waves. This form of the equations was found to be most suitable for the application to them of the method of the inverse scattering problem.

The spectral problem (or L–A pair) corresponding to these equations was given in Ref. 2, where a study was made of the case in which the metric depends on the time and on one spacelike coordinate. The case in which both variables are spacelike was considered separately in Ref. 3. The first case involves nonstationary metrices of cosmological and wave type, and the second corresponds to stationary gravitational fields with axial symmetry.

A general n-soliton solution of Einstein's equations was constructed for both the above-mentioned cases, and the simplest one- and two-soliton solutions were studied. In the cosmological and wave soliton solutions, a typical picture of the evolution is as follows. The solution usually represents an exact nonhomogeneous cosmological model and describes the propagation of soliton gravitational waves on some homogeneous cosmological background. These are either plane waves or cylindrical waves. Near the instant of the initial cosmological singularity the soliton perturbation is localized (with respect to one spatial variable), and with the initial expansion it begins to die away. At a certain critical instant, it produces gravitational waves, which then propagate in space away from the position of the initial concentration of the perturbation. Thus, the solutions in question provide exact models describing the generation and propagation in the Universe of soliton gravitational waves of cosmological origin, which appear because of the initial inhomogeneities of the gravitational field. These processes were described in Refs. 4 and 5.

In the case of stationary fields with axial symmetry, the number of solitons in the solution must be even. The first nontrivial step—the construction of a two-soliton solution—leads to the Kerr metric. Thus, the Kerr solution is a dual stationary soliton (on the background of flat space). The n-soliton solution can then be interpreted as a metric describing a stationary configuration of $n/2$ interacting Kerr instantons. The properties of these solutions were studied in Refs. 3 and 6.

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