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**G. E. Volovik and V. P. Mineev.** *Textures, vortices and superfluidity of  $^3\text{He}$ .* In 1972 the discovery was made of the new A and B phases of liquid  $^3\text{He}$ , which in subsequent years astonished physicists by their unique properties. Both new phases of  $^3\text{He}$ , like the well-studied  $^4\text{He}$ , are superfluids. The superfluidity of  $^4\text{He}$  is a consequence of the phenomenon of Bose condensation of the macroscopic part of the atoms of the liquid in the ground state. Superfluidity of the normal Fermi liquid  $^3\text{He}$ , like superconductivity of the electron Fermi

liquid in metals, sets in as a consequence of Cooper pairing of the  $^3\text{He}$  atoms. However, there is an important difference between the Cooper pairs in  $^3\text{He}$  and in a superconductor. The total spin of a pair and the relative angular momentum in a superconductor are equal to zero,  $S=L=0$ , i.e., the pair has no magnetic structure and is a spherically symmetric formation. In both phases of  $^3\text{He}$ ,  $S=1$  and  $L=1$ . All pairs are in the same state; as a result, the liquid phases of  $^3\text{He}$  have non-trivial magnetic properties reminiscent of the proper-

ties of antiferromagnetics, and they also have spontaneously broken rotational symmetry, which means that they are liquid crystals. The liquid-crystalline properties manifest themselves particularly clearly in the A phase, whose order parameter corresponds to the state with projection  $L_z = 1$  of the relative orbital angular momentum of the Cooper pairs onto some distinguished axis  $\mathbf{l}$ —the anisotropy axis of the A phase (analogous to the direction of the director in a nematic liquid crystal). The wave function of the relative motion of the atoms of a Cooper pair in the A phase is proportional to the spherical harmonic  $Y_{11}$ , i.e., it has the form  $\psi(\mathbf{r}) = f(r) \sin \theta e^{i\varphi} = f(r) (n_x + in_y)$ , where  $\theta$  is the polar angle, measured from the direction of the axis of quantization  $\mathbf{l}$ ,  $\varphi$  is the azimuthal angle of rotation around the direction  $\mathbf{l}$ , measured from an arbitrary direction  $\Delta$  in the plane perpendicular to  $\mathbf{l}$ , and  $n_x$  and  $n_y$  are the Cartesian components of the vector  $\mathbf{n} = \mathbf{r}/r$  in a coordinate system whose  $z$  axis is directed along  $\mathbf{l}$  and  $x$  and  $y$  axes along  $\Delta'$  and  $\Delta'' = \mathbf{l} \times \Delta'$ . In a weakly nonhomogeneous state, the triplet of orthogonal unit vectors  $\Delta'$ ,  $\Delta''$ ,  $\mathbf{l}$  specifying the orbital part of the order parameter of the A phase varies slowly in space. The phase  $\Phi$  of the order parameter  $\psi = \Delta' + i\Delta''$  is equal to the angle of rotation  $\varphi$  of the vectors  $\Delta'$  and  $\Delta''$  around the direction  $\mathbf{l}$ , and it is this that determines the unique superfluid properties of the A phase. Since three-dimensional rotations do not commute, the ordinary definition of the superfluid velocity  $\mathbf{v}_s = (\hbar/m)\nabla\Phi$  must be replaced in  $^3\text{He-A}$  by  $\mathbf{v}_s = (\hbar/2m)\Delta_i\nabla\Delta_i^*$ , from which it follows at once that the superfluid motion in the A phase cannot be derived from a potential:  $\text{curl } \mathbf{v}_s \neq 0$ . Before the discovery of the A phase, the potential character of the superfluid velocity in  $^4\text{He}$  and in superconductors was considered to be an essential attribute of superfluidity. The abandonment of the potential character and the complexity of the order parameter of the A phase made it necessary to reconsider all superfluid phenomena. Investigation showed that the superfluidity properties of a given superfluid are determined by the topological structure of the region in which the order parameter of this fluid varies. Below, we list some of the results of the topological analysis of the properties of  $^3\text{He-A}$  in comparison with  $^4\text{He}$ .

1. Textures (nonhomogeneous structures). In  $^4\text{He}$ , linear structure defects—quantized vortices—are characterized by a topological invariant  $N$ —the number of quanta of circulation of the superfluid velocity around a vortex, taking arbitrary integral values. Linear defects in the A phase are characterized by a topological invariant  $N$  which takes only the two values  $N = 0$  and  $N = 1$ . All vortices with an even number of quanta of circulation belong to the class  $N = 0$  and can be relaxed continuously into a homogeneous state. All vortices with an odd number of quanta of circulation are topologically essential and belong to the class  $N = 1$ . This same class contains a radial dislocation—a vortex-free state with a linear singularity in the vector field  $\mathbf{l}$ . Within a class, defects can be transformed continuously into one another. The arithmetic of fusion of defects has the following form:  $1 + 0 = 1$ ,  $1 + 1 = 0$ .

The topological analysis was also applied to defects of other types and in other ordered media. As a result, a classification was also given for point defects and singular surfaces, defects without singularities—solitons, defects on the surface of ordered media, and so forth.

2. Persistent fluxes. In  $^4\text{He}$ , there are infinitely many classes of flows in a ring-shaped channel, characterized by an integer invariant  $N$ —the number of quanta of circulation along the channel. A flow with given  $N$  is extremely stable, since  $N$  cannot change continuously, but only through production of defects—vortices. In  $^3\text{He-A}$ , there exist only the two classes of flows with  $N = 0$  and  $N = 1$ . All flows with even  $N$  belong to the class  $N = 0$  and can be relaxed continuously into the rest state. Flows with odd  $N$  belong to the class  $N = 1$  and can be relaxed into the state with lowest energy within this class, i.e., a flow with very small flux. Thus, unlike  $^4\text{He}$ , in  $^3\text{He-A}$  topology does not stabilize the superfluid flux. The topology in the A phase can be changed by applying an external magnetic field; as a result, there exists a unique possibility of controlling the stability of the superfluid flux.

3. Nonstationary Josephson effect. The process of continuous relaxation of the superfluid flux in the A phase is accompanied by oscillations in the vector field  $\mathbf{l}$ . The space-time structure of the field  $\mathbf{l}$  in this process is reminiscent of the instanton which is known from field theory—a particle-like topological formation in the space-time continuum. The instanton in the A phase brings about a continuous process of transition from the “vacuum” with topological “charge”  $N$  into the “vacuum” with topological “charge”  $N - 2$ . If the chemical potentials are different in a channel with the A phase, there is a dissipative flow with a distribution of the field  $\mathbf{l}$  which is periodic in space and time. This flow is an analog of the resistive state in superconductors (the nonstationary Josephson effect) and represents a lattice of instantons in four-dimensional space-time. Periodic oscillations of the field  $\mathbf{l}$  are observed experimentally.

4. Rotation of superfluids. Under rotation, there occurs in  $^4\text{He}$  a lattice of quantized vortices. An analogous lattice of Abrikosov vortices occurs in superconductors of type II subjected to a magnetic field, which plays the same role as the rotation. Under rotation, the A phase exhibits a periodic structure in the field of the vectors  $\mathbf{l}$  and  $\mathbf{v}_s$  without any singularities. In sufficiently strong magnetic fields, this state is replaced by a lattice of singular vortices. An experimental investigation of vortex structures in the A phase is being conducted at the present time in Finland using a unique experimental arrangement involving a rotating cryostat (a rotating minilaboratory).

<sup>1</sup>G. E. Volovik and V. P. Mineev, *Fizika i topologiya* (Physics and Topology), Znanie, Moscow (1980).