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A joint scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the USSR Academy of Sciences was held on 30 September and 1 October 1981 at the P. N. Lebedev Physics Institute, Moscow. The following papers were delivered:

30 September

1. *I. M. Khalatnikov,* The scientific activity of the L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences.

2. *P. B. Vigman,* Exact solution of the Kondo problem.

3. *S. A. Brazovskiĭ,* The theory of conducting polymers.

4. *A. I. Larkin and D. E. Khmel'nitskiĭ,* Anderson localization and anomalous magnetoresistance at low temperatures.

1 October

5. *A. M. Polyakov,* Phase transitions and the Universe.

6. *G. E. Volovik and V. P. Mineev,* Textures, vortices and superfluidity of ^{3}He.

7. *V. A. Belinskiĭ,* Solitons in the theory of gravitation. Summaries of six of the papers are published below.

P. B. Vigman. *Exact solution of the Kondo problem.* It is well known that ions of transition elements and certain rare-earth elements having a well-defined magnetic moment become nonmagnetic when placed in simple metals. The transition from the magnetic to the nonmagnetic state is related to the well-known anomalies in the thermal and kinetic properties of dilute alloys.

The properties of dilute magnetic alloys are determined mainly by the antiferromagnetic exchange interaction between the conduction electrons and the magnetic moment of the impurity. It is customary to study this interaction using the example of the so-called *s-d* exchange model:

$$\mathcal{H}_{s\text{-}d}=\sum_{k,\sigma=\uparrow\downarrow}\epsilon_k c^{+}_{k\sigma}c_{k\sigma}+J\sum_{k,k'}c^{+}_{k\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}c_{k'\sigma'}\mathbf{S}, \tag{1}$$

in which the impurity is regarded as a localized moment **S**.

The difficulties which occur in the study of this Hamiltonian at low temperature are well known. They consist of the fact that the amplitude of the interaction grows with decreasing temperature[1] and, when calculated in the leading logarithmic approximation, has a pole at the Kondo temperature $T_K=\epsilon_F\exp(-1/J)$ (see Ref. 2). Therefore the problem of the low-temperature properties of magnetic alloys—the "Kondo problem" proper—has remained open.

Two years ago, it was shown that the traditional *s-d* model is completely integrable, and the Hamiltonian (1) was diagonalized exactly.[3–6] The method of diagonalization goes back to H. A. Bethe, who in 1931 constructed a solution of the one-dimensional chain of spins. The application of Bethe's analysis to problems of magnetic alloys has made it possible not only to solve the problem of the ground state of an impurity in a metal, but also to calculate the dependence of the thermodynamic functions on the temperature and on the magnetic field.[5–8]

The Hamiltonian (1) is in essence one-dimensional, since the potential of the impurity is regarded as pointlike, so that only the *s* wave interacts with the impurity.[1)]

A system of N particles with spins $\sigma_1,\ldots,\sigma_N$ and an impurity of spin s is described by a wave function having the form of the *Bethe ansatz*: in the region $X_Q=\{x_{q_0}<x_{q_1}<\ldots<x_{q_N}\}$, where $Q=\{q_0,\ldots,q_N\}$ is a permutation of the integers $\{0,1,\ldots,N\}$,

$$\Psi_{\sigma_1,\ldots,\sigma_N,s}(x_1,\ldots,x_N)=\sum_P A_{\sigma_{q_1},\ldots,\sigma_{q_N},s}(Q'P)(-1)^P\exp\Big(i\sum_{j=1}^{N}k_{P_j}x_j\Big); \tag{2}$$

here Q' is identical to Q, from which $q_i=0$ is eliminated, $P=\{p_1,\ldots,p_N\}$ is a permutation of the integers $\{1,\ldots,N\}$, and $x_0=0$. The content of Bethe's hypothesis is that in each of the regions x_Q in which the particles are ordered the wave function is a superposition of a finite number of waves. The validity of this hypothesis

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[1)] Of course, this is true only in the linear approximation in the concentration of the impurities.

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involves the existence of an infinite series of conservation laws and leads to a factorized scattering theory.[9]

A knowledge of the wave function makes it possible to find the entire spectrum of the Hamiltonian (1) and to determine the equilibrium properties at finite temperature. They are given by the solutions of the system of nonlinear integral equations[7,8]

$$\varepsilon_n(x) = \int_{-\infty}^{+\infty} \frac{1}{2\mathrm{ch}\,[(\pi/2)(x-x')]} \ln(1+e^{\varepsilon_{n+1}(x)})(1+e^{\varepsilon_{n-1}(x)}) - \delta_{n1} e^{-\pi x},$$

$$\lim_{n\to\infty} \frac{\varepsilon_n(x)}{n} = \frac{H}{T}, \quad \varepsilon_0 = -\infty; \tag{3}$$

the impurity part of the free energy has the form

$$F(T, H) = -T \int_{-\infty}^{+\infty} \frac{\ln(1+e^{\varepsilon_{2S}(x)})}{2\mathrm{ch}\,(\pi x/2)}\, dx. \tag{4}$$

These equations were studied in Ref. 8, where the asymptotic behavior of the solutions for $T \gg T_K$ and $T \ll T_K$ were given. At $T=0$, the equations (3) become linear and can be solved explicitly.[5,6] Here we give the result for the impurity part of the magnetic moment at $T=0$ as a function of the magnetic field H:

$$M(H) = S + \frac{i}{4\pi^{3/2}} \int_{-\infty}^{+\infty} \Gamma\left(\frac{1}{2}+i\omega\right)\left(\frac{-i\omega+0}{e}\right)^{i\omega 2S} \times \left(\frac{i\omega+0}{e}\right)^{-i\omega(2S-1)} e^{-i2\omega \ln(H/T_K)} \frac{d\omega}{\omega - i0}. \tag{5}$$

When $H \gg T_K$, this quantity has an asymptotic expansion in inverse powers of $\ln(H/T_K)$, which is known from perturbation theory. When $H \ll T_K$, the properties of $M(H)$ and all other physical quantities for $S \neq \frac{1}{2}$ differ from those of the case $S=\frac{1}{2}$. In the first case, $M(H)$ is the same as for $H \gg T_K$, and we have an expansion in inverse powers of $\ln(H/T_K)$, whose coefficients agree under the substitution $S \to S - \frac{1}{2}$. In particular, $M(H\to 0) = S - \frac{1}{2}$, i.e., the electrons do not completely compensate the impurity magnetism, and the ground state is $2S$-fold degenerate. But if $S=\frac{1}{2}$, then $M(H)$ can be expanded in a series in integral powers of H/T_K. For $H \to 0$, we have $M(H) \sim H/T_K$ and this quantity vanishes as the field tends to zero. The ground state of the impurity is a singlet.

In addition to the exchange Hamiltonian (1), the Anderson model describing the formation of a localized moment is also integrable[10]; the s-d exchange model ignores the orbital degeneracy of the unfilled impurity shell and is therefore applicable only to those alloys in which the crystal field is greater than T_K, and only for $S=\frac{1}{2}$ {for example, (LaCe)B_6 and (LaCe)Al_2]. More realistic orbitally degenerate exchange Hamiltonians are also integrable.[11]

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