Optical problems in the electrodynamics of gyrotropic media

O. S. Eritsyan

Erevan State University
Usp. Fiz. Nauk **138**, 645–674 (December 1982)

The theory for the propagation of electromagnetic waves in gyrotropic media with boundaries is reviewed. Media which simultaneously exhibit a natural activity and a magneto-optic activity are discussed. The optical irreversibility effects in such media are analyzed. Magnetic crystals with a natural optical activity are discussed. The meaning of the magnetic permeability at optical frequencies is discussed. The optical properties of magnetically active ferromagnets are examined in the case of coincident roots of the dispersion relation. The propagation of light in a bigyrotropic medium is also discussed. Certain distinctive properties stemming from the existence of two types of gyrotropy, anisotropy, and absorption are discussed. The mechanisms for circular dichroism and the features of the dichroism of naturally gyrotropic media in an external magnetic field are discussed briefly.

PACS numbers: 78.20.Ls, 78.20.Ek

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1. INTRODUCTION

Arago discovered natural optical activity—the first type of gyrotropy—in 1811, and Faraday discovered magnetooptic activity 35 years later. These two types of gyrotropy are superficially similar but very different in physical nature. Natural activity is a manifestation of spatial dispersion; i.e., it occurs because the polarization of a medium at a given point depends on the field not only at that point but also in its vicinity. Magneto-optic activity, in contrast, is a consequence of a frequency dispersion: A magnetic field causes a relative shift of the curves showing the frequency dependence of the refractive index for waves which are (in the simplest case) right-hand and left-hand circularly polarized. The difference between the two types of activity can be seen in the following effect in the simplest case: In isotropic naturally gyrotropic media, the rotation of the polarization plane occurs around the wave vector k, and if the propagation direction is reversed the sense of the rotation also changes (with respect to a fixed coordinate system). In magnetoactive media, in contrast, the rotation is around the magnetic field, and it is identical for propagation parallel and antiparallel to the external magnetic field.

Soon after its discovery, natural activity became a highly useful research tool. Suffice it to say that stereochemistry owes its very origin to this phenomenon: The realization that a molecule must be pictured in three spatial dimensions stems, in particular, from the discovery of enantiomorphism (The existence of a medium in two mirror-symmetry forms). In optics enantiomorphism can be seen in the circumstance that two substances having the same chemical formula can rotate the polarization plane in opposite directions. The theory of natural optical activity and experimental research methods developed continuously. There was much less interest in the Faraday effect, on the other hand, for some time. As Vol'kenshteĭn has pointed out,[1] there may have been a psychological reason for this circumstance, in that the assertions advanced in the 1920's regarding the unique possibilities of the Faraday effect for studying the structure of matter proved incorrect.

Research on the magnetic rotation of the polarization plane grew extensively in connection with research on ferromagnetic media in the microwave frequency range. Other spectral regions, including the visible spectrum, later came under study. Magnetooptics is of considerable interest in connection with research on the electromagnetic properties of plasmas and the ionosphere.[2-5]

Because of the very direct relationship between the optical properties of a medium and its structure, the optical research methods, even the very old ones, have remained effective. On the other hand, the development of ideas regarding the structure of matter which has occurred in a setting of a close mutual encouragement of theory and experiment suggests new experimental problems and stimulates a reexamination of long-familiar phenomena from the standpoint of new ideas. In this regard, gyrotropy is of course a subtle research tool (in the words of I. V. Obreimov, "The natural activity is an

 0038-5670/82/120919-17$01.80

intramolecular interferometer"). We are now seeing a rapid intensification of interest in both spatial dispersion[6-10] and magnetooptic activity.[11-16] An important circumstance which sustains an unflagging interest in these long-familiar phenomena is the possibility of using them to obtain valuable information in a variety of research fields. Along with the major role played by the methods based on the natural optical activity for studying molecular structure (see Refs. 17 and 18, for example), there is the possibility (pointed out by Kizel'[8]) of studying crystal fields by introducing active centers in a crystal and introducing selectively absorbing centers in gyrotropic media (see also Ref. 19). Research in the field of gyrotropy has led to several interesting results, which frequently improve our understanding of not only gyrotropy but the interaction of light with a medium in general. We do not, of course, have room here to list all the corresponding studies. We will simply mention the discovery of several optical effects in crystals.[20-29] e.g., the discovery that there are media which, although they do not rotate a polarization plane, are gyrotropic[20,21]; the discovery of certain distinctive aspects of reflection and refraction at the boundary of a gyrotropic medium[24,25]; the discovery that an external electric field can give rise to gyrotropy[26-28]; the discovery of optical anisotropy of cubic crystals[29]; and the discovery that new waves appear in crystals with a spatial dispersion.[9] Magnetooptic methods have found widespread use in a wide variety of research fields. We might mention their use to study the electron structure of metals,[30] the magnetooptic research on surface layers and the discovery and study of surface magnetism,[31-33] and the prediction of several optical effects in magnetic crystals.[34,35] These examples give some idea of the wide variety of phenomena which can be seen in gyrotropic media.

The macroscopic theory of the optical properties of gyrotropic media, the subject of the present review, encompasses both research on the propagation of electromagnetic waves in such media and the interaction of waves with boundaries. Although we are always dealing with boundary-value problems in using optical methods to study media, these boundary-value problems have not received the attention they deserve (see Refs. 36 and 37, for example). It is true that until recently the relatively poor experimental accuracy (due, for example, to the frequency spread of the light), it was frequently possible to ignore the role played by boundaries. For example, although the rotation of the polarization plane in a magnetically active medium is proportional to the path traversed by the wave (Verdet's law) only in an unbounded medium[38,39] (there are other restrictions as well[40]), this law has been applied to plates; in fact, the law was established in the first place through the use of plates (as would be natural). If the experimental accuracy is good enough, however, we ignore the role of boundaries only at the risk of reaching an incorrect interpretation of experimental results,[41] as was pointed out a long time ago.[39,42] In particular, one way in which boundaries are ignored is to assign the so-called principle of the superposition of gyrotropy and birefringence[36] to plates, while this principle is of limited applicability even in the absence of boundaries[40,43] (that the superposition principle does not operate in the presence of boundaries can be seen by analyzing the expressions for a wave which has traversed an anisotropic magnetically active plate[43]).

To take the boundaries into account in optical problems, particularly in a study of gyrotropy, is of primary importance. It is sufficient to point out that even in the simplest case in which there is no anisotropy in a gyrotropic medium the rotation of the polarization plane and the axial ratio of a wave which has traversed a gyrotropic plate consists of three terms (a surface term, due to the difference between the reflection coefficients for the right-hand and left-hand circularly polarized waves; an interference term due to multiple reflection; and a volume term), two of which stem from the presence of boundaries.[44-46] It is thus not justified to study the polarization of a wave which has traversed a plate without an accurate account of the boundaries, as has been pointed out repeatedly.[39,42-44]

As yet another example we might cite the normal propagation of light through a naturally gyrotropic isotropic plate in a magnetic field directed perpendicular to the plate boundaries. If, as the light propagates from the first boundary to the second, its polarization plane does not rotate (if the natural and magnetooptic rotations cancel out), the transmitted wave nevertheless has a rotated polarization plane—by virtue of the multiple reflection from the boundaries (see Subsection 2b and Ref. 43). We might add to these examples the question of the boundary conditions themselves for naturally gyrotropic media; this question has emerged as an independent object of research in recent years.[24,25]

Several approaches are being taken to study boundary-value problems and the propagation of electromagnetic waves in media. Along with the method which is based on the choice of a definite coordinate system, there is the covariant method,[47] which has been widely used in applications to gyrotropic media.[10] The matrix method of Müller and Jones[52] has been used in several studies[48-51] (see also Ref. 53).

In the solution of boundary-value problems, the particular features of the propagation of electromagnetic waves in these media are automatically taken into account through the use of the dispersion relation and the constitutive equations, which contain in principle all the information about the optical properties of the medium (this assertion may be incorrect in the case in which a naturally gyrotropic medium has a diffuse boundary, in which case it becomes necessary to introduce a parameter to take into account the discontinuity of the tangential component of the magnetic field at the boundary[24,25]). The distinctive features of the propagation, however, may not be explicitly manifested in the boundary-value problem. For this reason, an analysis of the propagation of electromagnetic waves in various media in the absence of boundaries may not only be useful for interpreting experimental results (see Ref. 54 and several papers cited there: Refs. 55–73) but may also be of independent interest.

We might note, for example, the distinctive optical properties of media in situations in which there are coincident roots of the dispersion relation[74,75,77] and a monorefringence arises[1)] (Ref. 76); the possible existence of three waves instead of two[9,78]; the invariance of the dispersion relation (with respect to reversal of the propagation direction) for naturally gyrotropic media in a magnetic field,[79,80] which has been mentioned previously[81-83] (see also Ref. 84); the possible propagation in such media of two waves for which the polarization ellipse is traversed in the same direction[85]; the distinctive features of reflection and refraction at the boundaries of a medium simultaneously exhibiting a gyroelectric and a gyromagnetic activity[86]; the interesting polarization properties of crystals of the planar classes (predicted in Ref. 20), which are stimulating a reexamination of the concept of gyrotropy as the property of necessarily rotating a polarization plane[20,87-89]; and the optical anisotropy of cubic crystals,[29] mentioned earlier, which has expanded our understanding of the anisotropy of optical properties in general.

Systematic theoretical and experimental research has given us a phenomenological theory for the propagation of electromagnetic waves (light) in gyrotropic media and for the interaction of waves with boundaries and has revealed the rich variety of optical properties of gyrotropic media—naturally gyrotropic media and magnetically active media (see Refs. 90–106 and 107–134, respectively, in addition to the papers cited above). Only some of these topics have been covered in reviews and monographs.

Our list of papers cannot, of course, be complete; it simply circumscribes the range of topics covered in the present review.

This review consists of five sections unified by a common theme: the propagation of electromagnetic waves in gyrotropic media in the presence of boundaries. Section 2 deals with media which simultaneously exhibit a natural and a magnetooptic activity. The dispersion relation for such media is not invariant with respect to a reversal of the propagation direction, and for this reason such media exhibit optical irreversibility effects. Magnetic crystals exhibiting a natural optical activity are discussed, along with the meaning of the magnetic permeability at optical frequencies. Section 3 covers the propagation of electromagnetic waves in a magnetically active medium for which the dispersion relation has multiple roots. This situation has been examined previously for absorbing crystals and for gyroanisotropic media. Section 3 also deals with light propagation in a bigyrotropic medium and reflection and refraction at its boundary. Certain distinctive features stemming from the presence of two types of gyrotropy, anisotropy, and absorption are pointed out. For all these media, some boundary-value problems required for an experimental study of the effects are discussed. Section 4 takes up a circular dichroism which is not due to the imaginary parts of the gyrogropy parameters. The distinctive features of the dichroism of naturally gyrotropic media in the presence of an external magnetic field are discussed.

The terms "gyrotropy" and "optical activity" are both applied to the right-left asymmetry in the interaction of light with a medium, usually (but not necessarily) manifested as a rotation of the polarization plane. The two types of gyrotropy which have been recognized for the longest time can be described more specifically by the terms "natural optical activity" and "magnetooptic activity." For greater detail, the terms "gyroelectric magnetooptic activity" and "gyromagnetic magnetooptic activity" are used. For the same purpose, we use the terms "artificial gyrotropy," "intrinsic gyrotropy," and "natural gyrotopy."[10] In connection with the development of the optics of crystals with spatial dispersion,[9] natural optical activity has acquired the meaning of a particular manifestation of a more general phenomenon: spatial dispersion. "Electrogyration" or "electric gyrotropy"[26-28] mean the gyrotropy induced in medium by an external electric field.

Concepts such as optical activity and circular dichroism which are applied to the types of gyrotropy mentioned above are also applied to cholesteric liquid crystals, although in the latter case, in contrast with the media mentioned above and in accordance with a different mechanism for the activity and dichroism, the dielectric tensor can be put in a diagonal form, if only in a local coordinate system.

2. NATURALLY GYROTROPIC MEDIA IN EXTERNAL MAGNETIC FIELDS

a) Dispersion relation and its consequences

We consider the propagation of a light wave

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E} \exp [i(\mathbf{kr} - \omega t)] \tag{2.1}$$

in a naturally gyrotropic medium in an external magnetic field.

In the absence of an external magnetic field, with $\mu_{jl} = \delta_{jl}$ (μ_{jl} is the magnetic permeability, and δ_{jl} is the unit matrix), the electric displacement is related to the electric field by

$$D_j = \varepsilon_{jl} E_l + i\gamma_{jlm} E_l k_m,$$

and the magnetic induction **B** is the same as the magnetic field **H**. When an external magnetic field is applied, a term $i[\mathbf{g}_e \mathbf{E}]$ that takes into account the gyroelectric magnetooptic activity appears on the right side of this equation. If the medium also has a magnetic order, the high-frequency branch of the magnetic-moment precession described by the Landau–Lifshitz equation[135,122,125] leads to nonvanishing off-diagonal components of the magnetic susceptibility, which give rise to a gyromagnetic magnetooptic activity.[65] We therefore replace the relationship $\mathbf{B} = \mathbf{H}$ by $B_j = \mu_{jl} H_l + i[\mathbf{g}_m \mathbf{H}]_j$. Although the difference between μ_{jl} and δ_{jl}, on the one hand, and the differences between the various components μ_{jl}, on the other, are quantities of higher order than g_m, the difference between μ_{jl} and δ_{jl} can give rise to effects of the same order of magnitude as those resulting from the parameter g_m (Ref. 43). We thus find

[1)]See Ref. 47 regarding monorefringence.

the constitutive equations

$$D_j = \varepsilon_{jl}E_l + i\gamma_{jlm}E_l k_m + i\,[\mathbf{g}_e\mathbf{E}]_j, \tag{2.2}$$
$$B_j = \mu_{jl}H_l + i\,[\mathbf{g}_m\,\mathbf{H}]_j. \tag{2.3}$$

Experiment has shown[65] that μ_{jl} and g_m must be retained in the optical frequency range. We will therefore make use of the concept of magnetic permeability in the optical frequency range (see Ref. 136 and Subsection 2c below). The natural optical activity is entirely incorporated in the relationship between D and E; this circumstance simplifies the derivation of the dispersion relation, although the boundary conditions are slightly more complicated in this case than the conditions corresponding to the formulation of the problem in Refs. 138 and 139 (see Refs. 24, 25, and 98).

The properties of media (2.2) and (2.3), which simultaneously exhibit a natural and a magnetooptic activity, can be discussed more simply in the case of an isotropic medium (isotropic in the absence of an external magnetic field) with a scalar magnetic permeability. In an isotropic medium we have $\gamma_{jlm} = -\gamma e_{jlm}$, where e_{jlm} is the totally antisymmetric unit tensor.[137] We can therefore replace (2.2) and (2.3) under the conditions $g_m = 0$, $\mu_{jl} = \mu\delta_{jl}$, $\varepsilon_{jl} = \varepsilon\delta_{jl}$ by

$$\mathbf{D} = \varepsilon\mathbf{E} + i\frac{\gamma}{k}\,[\mathbf{kE}] + i\,[\mathbf{g}_e\mathbf{E}] \tag{2.4}$$
$$\mathbf{B} = \mu\mathbf{H}. \tag{2.5}$$

The second term in (2.4) is not equivalent to the second term in (2.2), since in (2.2), in the case of an isotropic medium, the rotE (rot = curl) (i.e., i[kE]) is multiplied by a quantity which is independent of k, while in (2.4) the corresponding quantity, γ/k, depends on k and has different values for right- and left-hand circular polarizations. This difference causes k to behave in different ways as a function of the gyrotropy parameter, and the difference between the wave vectors in the use of Eqs. (2.2) and (2.4) appears in the terms of order γ^2. Equation (2.4) leads to a simpler dispersion relation than that corresponding to (2.2), so we will use Eq. (2.4), but only in examining questions for which this nonequivalence indicated above does not affect the physical conclusions.

In an external magnetic field the parameters ε, μ, and γ cease to be scalars,[82] and this circumstance must be taken into account in (2.4) and (2.5). The anisotropy of these parameters is a small quantity of second order in their power-series expansion in the external magnetic field, while g_e is a quantity of first order in the magnetic field. Consequently, if γ and g_e are comparable in magnitude, the anisotropy may be ignored in a first approximation in these parameters. (This assertion is valid if we are dealing with propagation in an unbounded medium; it is not valid for determining the amplitudes at boundaries.) As we will see below, it is under the condition $\gamma \sim g_e$ that the distinctive features of these media are expressed most clearly. From (2.4), (2.5), and the field equations we find the following dispersion relation[79,85]:

$$k^{\pm 2} = \frac{\omega^2}{c^2}\,\mu\varepsilon\left[1 \pm \left(\frac{\gamma}{\varepsilon} + \frac{g_e}{\varepsilon}\cos\alpha^{\pm}\right)\right], \tag{2.6}$$

where α^+ and α^- are the angles between the external magnetic field and the propagation directions of the waves with the wave vectors $\mathbf{k}^+$ and $\mathbf{k}^-$.

Let us examine the dispersion relation and its consequences.

1. Reversal of the propagation direction changes the sign of $\cos\alpha^{\pm}$, so that while we have

$$k_{\text{forw}}^{\pm 2} = \frac{\omega^2}{c^2}\,\mu\varepsilon\left[1 \pm \left(\frac{\gamma}{\varepsilon} + \frac{g_e}{\varepsilon}\right)\right] \tag{2.7}$$

for waves which are propagating in the same direction (with, say, $\cos\alpha^{\pm} = 1$), for waves propagating in the opposite direction ($\cos\alpha^{\pm} = -1$) we find

$$k_{\text{back}}^{\pm 2} = \frac{\omega^2}{c^2}\,\mu\varepsilon\left[1 \pm \left(\frac{\gamma}{\varepsilon} - \frac{g_e}{\varepsilon}\right)\right]. \tag{2.8}$$

It follows from (2.7) and (2.8) that all four values of the wave vector are different in magnitude; i.e., the reversibility of the light waves is violated. This is one of the manifestations of the noninvariance of dispersion relation (2.6) with respect to the replacement $\mathbf{k} \to -\mathbf{k}$. The reason for the noninvariance is as follows: When the waves are propagating, say, along the direction of the magnetic field, the natural rotation and the magnetooptic rotation are in the same direction (or the opposite direction), while when the waves are propagating backward the two rotations are in opposite directions (or the same direction), respectively, since the natural rotation reverses direction (if we are always looking along the same direction, e.g., along the direction of the magnetic field), while the magnetic rotation does not reverse. It is this asymmetry which gives rise to the noninvariance of the dispersion relation.

In the case $\gamma = 0$, when the propagation direction is reversed ($\cos\alpha \to -\cos\alpha$) the $\pm$ signs in (2.6) are replaced by $\mp$ signs; this switch does not, of course, change the absolute values of $k^{\pm}$ ($|k^+|$ transforms into $|k^-|$). When $g_e = 0$, on the other hand, $\cos\alpha$ does not appear in (2.6), so that reversal of the propagation direction does not change the absolute values of k^+ and k^-. The noninvariance thus results from the simultaneous presence of two types of gyrotropy: natural and magnetooptic activities.

There is another way to interpret this noninvariance. Time reversal changes the sign of g_e, but the parameter γ remains unchanged.[13] Consequently, changing the sign of the phase velocity (this corresponds to time reversal) changes the absolute values of the velocities. We might note that inversion of the spatial coordinates changes the sign of γ but leaves g_e unchanged.[13] Accordingly, if the waves along a given propagation direction have phase velocities v_1 and v_2 in a right-hand rotating (left-hand rotating) medium in a magnetic field, we find the same velocities in a left-hand-rotating (or, respectively, right-hand-rotating) medium for propagation in the opposite direction or if the medium is placed in a magnetic field in the opposite direction.

2. If the propagation directions and amplitudes of right- and left-hand polarized waves are approximately the same, we may speak in terms of a rotation of the polarization plane of the resultant wave, which is a plane-polarized wave under these conditions. The rotation of the polarization plane over a path length l in

an unbounded medium is (we are replacing $g_e \cos\alpha^+$ and $g_e \cos\alpha^-$ by $g_e \cos\alpha$, thereby introducing an error proportional to g_e^2)

$$\varphi_{\text{forw}} = \left|\frac{k^+_{\text{forw}} - k^-_{\text{forw}}}{2} l\right| = \left|\frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma + g_e\cos\alpha}{2\varepsilon}\, l\right|. \tag{2.9}$$

Reversal of the propagation direction leads to

$$\varphi_{\text{back}} = \left|\frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma - g_e\cos\alpha}{2\varepsilon}\, l\right|. \tag{2.10}$$

The rotation of the polarization plane over a unit path length is different for the forward and backward waves: $\varphi_{\text{forw}} \neq \varphi_{\text{back}}$.

3. When a wave is incident from a gyrotropic medium on a boundary with another medium, the reflected wave splits in two (right- and left-hand polarized waves), which propagate at different angles.[104,111] The reflection angle for one of the two waves is equal to the angle of incidence, but that for the other is not. In a naturally gyrotropic medium in an external magnetic field, this equality of the incidence and reflection angles is violated for both waves, as can be seen by working from the continuity of the tangential components of the wave vector upon reflection and refraction.

To show this, we consider a plate described by Eq. (2.6) and occupying the region $0 \leq z \leq d$. An external magnetic field is directed perpendicular to the plate boundaries. A plane wave with a tangential wave-vector component equal to k_x is incident from the region $z < 0$ on the boundary $z = 0$. In a plate we have four waves: two forward and two backward waves. From the constancy of the tangential components of the wave vectors of all the waves we find the following values for the angles between the wave propagation directions in the plate and the z axis:

$$\sin\alpha^{\pm}_{\text{forw}} = \frac{ck_x}{\omega\sqrt{\varepsilon\mu}}\left[1 \mp \frac{1}{2}\left(\frac{\gamma}{\varepsilon} + \frac{g_e}{\varepsilon}\sqrt{1 - \frac{c^2k_x^2}{\omega^2\varepsilon\mu}}\right)\right], \tag{2.11}$$

$$\sin\alpha^{\pm}_{\text{back}} = \frac{ck_x}{\omega\sqrt{\varepsilon\mu}}\left[1 \mp \frac{1}{2}\left(\frac{\gamma}{\varepsilon} - \frac{g_e}{\varepsilon}\sqrt{1 - \frac{c^2k_x^2}{\omega^2\varepsilon\mu}}\right)\right]. \tag{2.12}$$

The angles $\alpha^{\pm}_{\text{forw}}$ are the refraction angles at the $z = 0$ boundary; they are equal to the angles of incidence of these waves at the $z = d$ boundary. The angles $\alpha^{\pm}_{\text{back}}$ are the reflection angles at the $z = d$ boundary. It follows from (2.11) and (2.12) that none of the reflection angles at the $z = d$ boundary are equal to any of the angles of incidence on this boundary, i.e., that the equality of the angles of incidence and reflection is completely disrupted:

$$\alpha^+_{\text{back}} \neq \alpha^+_{\text{forw}},\quad \alpha^+_{\text{back}} \neq \alpha^-_{\text{forw}},\quad \alpha^-_{\text{back}} \neq \alpha^+_{\text{forw}},\quad \alpha^-_{\text{back}} \neq \alpha^-_{\text{forw}}.$$

With $g_e = 0$ and $\gamma = 0$ we find $\alpha^{\pm}_{\text{forw}} = \alpha^{\pm}_{\text{back}}$ and $\alpha^{\pm}_{\text{forw}} = \alpha^{\mp}_{\text{back}}$, respectively.

It also follows from (2.11) and (2.12) that with a suitable choice of the signs of the parameters γ and g_e we can satisfy the condition $\sin\alpha^{\pm}_{\text{back}} > \sin\alpha^{+}_{\text{forw}}$ or $\sin\alpha^{\pm}_{\text{back}} > \sin\alpha^{-}_{\text{forw}}$; i.e., we can arrange a situation such that both reflection angles are larger than the incidence angle α^{+}_{forw} or α^{-}_{forw}. If only the wave corresponding to α^{+}_{forw} or α^{-}_{forw} is incident on the boundary, then if these angles are large enough the reflection angles reach 90°. With a further increase in the angle of incidence, the reflection angles do not correspond to real values. Let us assume $\gamma > 0$ and $g_e > 0$, for example. If a wave with

$$\sin\alpha^{+}_{\text{forw}} = \frac{ck_x}{\omega\sqrt{\varepsilon\mu}}\left[1 - \frac{1}{2}\left(\frac{\gamma}{\varepsilon} - \frac{g_e}{\varepsilon}\sqrt{1 - \frac{c^2k_x^2}{\omega^2\varepsilon\mu}}\right)\right],$$

is incident on the $z = d$ boundary, we then have

$$\sin\alpha^{\mp}_{\text{back}} > \sin\alpha^{+}_{\text{forw}},$$

i.e., both reflected waves propagate closer to the boundary than does the incident wave. If $\sin\alpha^{+}_{\text{forw}}$ is sufficiently close to 1, we find from $\sin\alpha^{\pm}_{\text{back}} > \alpha^{+}_{\text{forw}}$, the result $\sin\alpha^{\pm}_{\text{back}} > 1$. In other words, the reflected angles do not correspond to real values; the reflected waves are damped (or grow) with distance from the boundary. To analyze this situation we need to resort to higher-order approximations to derive the dispersion relation. If $\sin\alpha^{+}_{\text{forw}}$ is sufficiently close to 1, then $\cos\alpha^{+}_{\text{forw}}$ is a small quantity, and the term $(g_e/\varepsilon)\cos\alpha^{+}_{\text{forw}}$ in the dispersion relation is a small quantity of higher order than g_e/ε. In this case we must take into account, along with this term, other quantities of higher order.

Before we go on to the higher-order approximation, let us examine the geometric reason for the absence of real reflection angles. We assume an isotropic, naturally gyrotropic medium. The wave-vector surfaces in the absence of an external magnetic field are spheres of radii k^+ and k^-, where k^+ and k^- are the numerical values of the wave vectors for right- and left-hand circularly polarized waves. We consider the reflection of a wave incident from this medium on a boundary with another medium. We place the xz plane in the propagation plane of the wave. Figure 1a shows the intersection of the wave-vector surfaces with the propagation plane. The wave incident on the boundary at a glancing angle ϑ excites reflected waves which propagate at angles ϑ' and ϑ''.

We now apply an external magnetic field along the z axis. In this case the wave-vector surfaces are displaced in opposite directions parallel to the magnetic field (Fig. 1b). If a wave with a wave vector $\mathbf{k}^+$ is incident on the boundary at an angle such that the tip of this vector lies on the arc $1m2$ (Fig. 1b), the perpendicular drawn from the tip of this vector to the x axis does not intersect the wave-vector surfaces anywhere in regions $1n3$ and $4p5$ (the region of reflection angles). This result means that the z components of the wave vectors of the reflected waves are imaginary; i.e., the reflection angles do not correspond to real values, as was stated above.

4. To go to a higher-order approximation, we work from (2.2) and (2.3). If the medium is isotropic ($\varepsilon_{xx} = \varepsilon_{yy}$

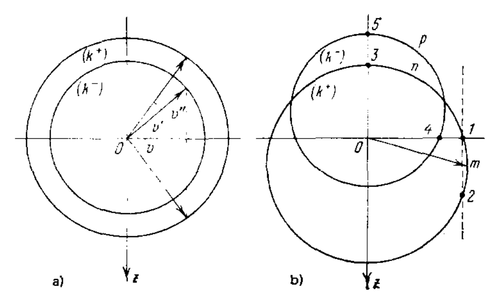

FIG. 1.

$= \varepsilon_{zz}$, $\mu_{xx} = \mu_{yy} = \mu_{zz}, \gamma_{xyz} = \gamma_{yzx} = \gamma_{zxy}$) in the absence of a magnetic field, the imposition of a magnetic field leads to[82] (the external magnetic field is directed along the z axis)

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_{yy} = \varepsilon \neq \varepsilon_{zz} = \varepsilon_3, \\ \mu_{xx} &= \mu_{yy} = \mu \neq \mu_{zz} = \mu_3. \end{aligned} \tag{2.13}$$

These conditions hold when the isotropic medium is converted in a magnetic field to a medium with the symmetry of a uniaxial crystal (generally speaking, in the case of magnetic media there may also be a conversion to a medium with the symmetry of a biaxial crystal[72]). The differences $\varepsilon_3 - \varepsilon$ and $\mu_3 - \mu$ cause a linear birefringence in the magnetic field and are quantities of second order in their expansion in powers of the magnetic field. The anisotropy introduced in γ_{ijl} is also a quantity of second order.[82]

We turn now to the dispersion relation, working from the constitutive equations (2.2) and (2.3) and assuming that the medium is isotropic before the magnetic field is applied. If the medium has only the gyroelectric part of the magnetooptic activity $(g_e \neq 0, g_m = 0)$, we find the equation

$$\begin{aligned} \frac{\mu}{\mu_3} x^4 + \Big\{\Big[\Big(1+\frac{\varepsilon_3\mu}{\varepsilon\mu_3}\Big) + \frac{\mu}{\varepsilon}(\gamma_1-\gamma_2)^2\frac{\omega^2}{c^2}\Big]\beta^2 \\ + 2\sqrt{\frac{\mu}{\varepsilon}}\frac{g_e}{\varepsilon}\frac{\omega}{c}(\gamma_1-\gamma_2)\beta - \Big(1+\frac{\varepsilon_3\mu}{\varepsilon\mu_3}\Big) - \frac{\varepsilon}{\mu}\Big(\frac{\omega}{c}\gamma_2\Big)^2 + \frac{g_e^2}{\varepsilon^2}\Big\} \quad x^2 \\ + \frac{\varepsilon_3}{\varepsilon}\Big[(\beta^2-1)^2 - \Big(\sqrt{\frac{\mu}{\varepsilon}}\frac{\omega}{c}\gamma_1\beta + \frac{g_e}{\varepsilon}\Big)^2\Big] = 0, \\ x^2 = k_x^2\Big(\frac{\omega^2}{c^2}\varepsilon\mu\Big)^{-1}, \ \beta^2 = k_z^2\Big(\frac{\omega^2}{c^2}\varepsilon\mu\Big)^{-1}, \quad k_y = 0, \quad \gamma_1 = \gamma_{xyz}, \quad \gamma_2 = \gamma_{yzx}. \end{aligned} \tag{2.14}$$

The noninvariance of the dispersion relation, expressed by the presence of odd powers of β in (2.14), is caused by the two terms

$$\Big(\sqrt{\frac{\mu}{\varepsilon}}\frac{\omega}{c}\gamma_1\beta + \frac{g_e}{\varepsilon}\Big)^2 \text{ and } \sqrt{\frac{\mu}{\varepsilon}}\frac{g_e}{\varepsilon}\frac{\omega}{c}(\gamma_1-\gamma_2)\beta.$$

The second of these terms results from the anisotropy of the medium, since it vanishes if $\gamma_1 = \gamma_2$. In the absence of a magnetooptic activity ($g_e = 0$) or of a natural activity ($\gamma_1 = \gamma_2 = 0$), Eq. (2.14) contains no odd powers of β, i.e., is invariant with respect to reversal of the propagation direction.

The possibility mentioned above, that both reflected waves will graze along the boundary, corresponds to the possible existence of values of k_x for which the real values of k_z are exclusively positive (values $k_z > 0$ correspond to waves incident on the $z = d$ boundary of the plate form the $z = 0$ boundary). To determine the possibility of arranging such a situation, we examine Eq. (2.14). Dispersion relation (2.14) is biquadratic in x (i.e., in k_x), so that we can express x in terms of β (i.e., k_x in terms of k_z):

$$\begin{aligned} x^{\pm 2} = \frac{\mu_3}{2\mu}\Big[\Big(1+\frac{\varepsilon_3\mu}{\varepsilon\mu_3}\Big) - 2\beta^2 + \frac{\varepsilon}{\mu}\Big(\frac{\omega}{c}\gamma_2\Big)^2 - \Big(\frac{g_e}{\varepsilon}\Big)^2 \\ \pm \sqrt{4\frac{\varepsilon}{\mu}\Big(\frac{\omega}{c}\gamma_2\Big)^2 + 8\frac{\varepsilon_3\mu}{\varepsilon\mu_3}\sqrt{\frac{\mu}{\varepsilon}}\Big(\frac{\omega}{c}\gamma_1\Big)\beta\frac{g_e}{\varepsilon} + O(4)}\Big], \end{aligned} \tag{2.15}$$

where $O(4)$ means small terms of fourth order in the magnetic field. To single out these terms we have assumed that g_e/ε and $\omega/c\gamma_{1,2}$ are small quantities of the same order. Differentiation shows that x^{+2} and x^{-2} go through maxima. The maximum values of $x^{\pm 2}$ are

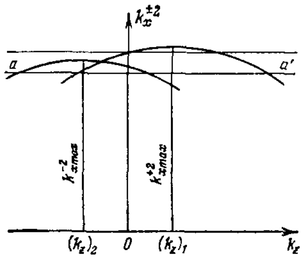

FIG. 2.

reached at

$$\beta = \pm\frac{1}{\sqrt{2}}\sqrt{\frac{\mu}{\varepsilon}}\frac{g_e}{\varepsilon}$$

and are

$$x^{\pm 2}_{\max} = 1 \pm \sqrt{8\frac{\varepsilon}{\mu}\frac{\omega}{c}\gamma_2}.$$

We thus see the following picture: The k_z dependence of k_x^2 has two branches, in accordance with the two signs in front of the radical in (2.15). One branch corresponds to a wave with a right-hand elliptic (or, in a particular case, circular) polarization, while the second corresponds to a wave with a left-hand elliptic polarization, as usual. The points at which the maxima occur, however, have a relative displacement along both the abscissa and the ordinate. The maxima of $k_x^{\pm 2}$ are reached at

$$(k_z)_{1,2} = \pm\frac{\omega}{c}\frac{\mu g_e}{\sqrt{2}\varepsilon}$$

and are

$$k^{\pm 2}_{x\,\max} = \frac{\omega^2}{c^2}\varepsilon\mu\Big(1 \pm \sqrt{8\frac{\varepsilon}{\mu}\frac{\omega}{c}\gamma_2}\Big). \tag{2.16}$$

Figure 2 is a rough sketch of the functions $k_x^{\pm 2} = f^{\pm}(k_z)$ (for $g_e > 0, \gamma_2 > 0$).

Let us assume that a plane wave with a tangential wave-vector component $k_{x\,\mathrm{inc}}$ is incident from the region $z < 0$ on a plate described by (2.14) which occupies the region $0 \leq z \leq d$. To determine the propagation directions of the waves in the plate, we must draw a line parallel to the k_z axis which cuts out from the k_x axis a segment equal to $k_{x\,\mathrm{inc}}$. The abscissas of the intersections of this line with the curves give us the z components of the wave vectors of the wave propagating in the plate. If $k^2_{x\,\mathrm{inc}} < k^{-2}_{x\,\max}$, we have four intersection points (the line aa') in accordance with the four waves in the plate. If, however,

$$k^{-2}_{x\,\max} < k^2_{x\,\mathrm{inc}} < k^{+2}_{x\,\max}, \tag{2.16'}$$

then we have only two intersection points, both corresponding to the same branch. If $k^2_{x\,\mathrm{inc}}$ and $k^{+2}_{x\,\max}$ are sufficiently close together, positive values of k_z correspond to both intersections. This result means that only forward waves are excited in the plate, and the z components of the wave vectors of the backward waves are complex. Since the complex solutions are complex conjugates, the imaginary part of the z component of one of the "reflected" waves is positive, while that for the other wave is negative. The amplitude of one of these waves will thus increase, while that of the other will decrease along z. In connection with the presence of a wave of increasing amplitude we should point out that in solving the boundary-value problem in this situation we do not run into any infinite amplitudes, since the determinant of the system of equations determining the amplitudes at the boundaries also contains an increasing term, which suppresses the unbounded growth of the waves inside the plate.

These properties of the medium result from the simultaneous existence of natural and magnetooptic activities. Because of the natural activity, the maxima of the branches of the dispersion relation are displaced vertically from each other, while the magnetooptic activity causes a horizontal displacement. With such an arrangement of branches there may also be an intersection of a line running parallel to the k_z axis with the curves, at which only two intersection points would be obtained (instead of four), and the two would have the same abscissa sign.

We also note that the reflection and refraction geometry at the boundary of a naturally gyrotropic medium in a magnetic field was studied in Ref. 85 by means of Sturm function series. Small terms of second order were retained in the dielectric tensor, while the anisotropy of the magnetic permeability and of the natural activity were ignored. The results derived in Ref. 85 lead to the same picture of the reflection of the boundary as we have seen here. Furthermore, when only forward waves with real wave numbers are excited in a plate (and the wave vectors of the backward waves are complex) the two forward waves will trace out their polarization ellipses in the same direction, since they correspond to the same branch of the dispersion relation.

b) Boundary-value problem for a plate

As we saw in the preceding section, the existence of two types of activity (natural and magnetooptic) causes an irreversibility of the light waves, disrupts the law of sines, changes the absolute value of the rotation of the polarization plane upon a reversal of the propagation direction, and leads to the distinctive properties discussed in Subsection 1a4. Here we will examine the propagation of light through a plate in order to determine how noninvariance of the dispersion relation affects the optical properties of the plate. For simplicity, we assume normal transmission. We will thus not take up the particular properties of the medium which are related to the propagation geometry of the incident and reflected waves in the plate. Those aspects of the problem which are related to the change in the absolute value of the rotation of the polarization plane upon a reversal of the propagation direction, however, will be seen most clearly.

We assume that a plane monochromatic wave

$$\mathbf{E}(z, t) = \mathbf{E} \exp i \left(\frac{\omega}{c} z - \omega t \right) \tag{2.17}$$

is incident from the region $z<0$ on the $z=0$ boundary of a plate which occupies the region $0 \leq z \leq d$ and which is described by dispersion relation (2.6). There is a vacuum at $z<0$ and $z>d$. Inside the plate we have four waves: two forward waves, corresponding to $\cos\alpha^{\pm}=1$ in (2.6) and two backward waves, with $\cos\alpha^{\pm}=-1$. According to (2.9) and (2.10), for forward waves propagating from the $z=0$ boundary toward the $z=d$ boundary the rotation of the polarization plane over a unit length of the beam is proportional to the sum $\gamma+g_e$, while for backward waves it is proportional to the difference $\gamma - g_e$. We consider the case in which a wave propagating from the $z=0$ boundary toward the $z=d$ boundary undergoes no polarization-plane rotation, i.e.,

$$\gamma + g_e = 0.$$

Using the relations derived in Ref. 43, we then have the following expressions for the components E_{4x} and E_{4y} of the field of the transmitted wave (in the amplitudes we are ignoring terms proportional to the gyration parameters[2)] when we have $E_x=0$, $E_z=0$, and $E_y \neq 0$ in the incident wave):

$$\begin{aligned} E_{4x} = & -4\Delta_0^{-1} \sqrt{\frac{\varepsilon}{\mu}} E_y \Big\{ \Big[\Big(1 - \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \cos \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \\ & + i \Big(1 - \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \sin \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \Big] \sin \Big(\frac{g_e - \gamma}{2\varepsilon} \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \Big) \Big\} e^{i(\omega/c)d}, \\ E_{4y} = & -4\Delta_0^{-1} E_y \sqrt{\frac{\varepsilon}{\mu}} E_y \Big\{ \Big[\Big(1 + \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \\ & - \Big(1 - \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \cos \Big(\frac{g_e - \gamma}{2\varepsilon} \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \Big) \Big] \cos \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d - i \Big[\Big(1 + \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \\ & + \Big(1 - \sqrt{\frac{\varepsilon}{\mu}}\Big)^2 \cos \Big(\frac{g_e - \gamma}{2\varepsilon} \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \Big) \Big] \sin \frac{\omega}{c} \sqrt{\varepsilon\mu}\, d \Big] \Big\} e^{-i(\omega/c)d}, \end{aligned} \tag{2.18}$$

where Δ_0 is the determinant of the system of equations representing the boundary conditions (the continuity conditions on the tangential components of the electric and magnetic fields).

It follows from (2.18) that the polarization plane of the transmitted wave is rotated with respect to that of the incident wave ($E_{4x} \neq 0$) although the wave propagating in the plate from the $z=0$ boundary to the $z=d$ boundary does not undergo a polarization-plane rotation. The rotation of the polarization in the transmitted wave in this case results from multiple reflection. The multiple reflection rotates the polarization plane in the transmitted wave because of a polarization-plane rotation in the backward wave in the plate (under the condition $\gamma+g_e=0$ the expression $\gamma - g_e$, to which the rotation in the backward wave is proportional, is nonzero). If $\sqrt{\varepsilon/\mu}=1$ (i.e., if there are no reflections from the plate boundaries) we in fact find $E_{4x}=0$. Consequently, noninvariance of the dispersion relation gives rise to a polarization-plane rotation in the transmitted wave in the absence of a rotation in the wave propagating from the first boundary of the plate to the second.

This example shows how wrong we can be if we try to discuss the wave transmitted through the plate by working from propagation in an unbounded medium. In Ref. 83, for example, it is concluded that the natural rotation of the polarization plane can be cancelled by a magnetic rotation. It follows from (2.18) that under the conditions $\sqrt{\varepsilon/\mu}=1$, i.e., when there are reflections

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[2)]The discontinuity of the tangential component of the magnetic field[24,25] was not taken into account in Ref. 43, so that to incorporate terms proportional to the gyration parameters in the amplitudes would be to go beyond the attainable accuracy.

from the boundaries, it is not possible to arrange cancellation of the natural and magnetic rotations.

c) Magnetic crystals exhibiting a natural optical activity

Natural optical activity is seen in the visible wavelength region when the change in the wave phase over characteristic atomic distances cannot be ignored. In this region the magnetic susceptibility cannot be assumed equal to zero.[137] It may thus appear at first glance that a joint analysis of the magnetic properties and the natural activity is meaningless. As we mentioned above, however, it has been shown that in the optical region the magnetic susceptibility should be assumed nonzero, at least for ferromagnetic media.[65] In particular, it has been shown that the Landau–Lifshitz equation remains applicable up to the optical frequency range[65] (see also Ref. 64). Since this conclusion contradicts the assertion that the magnetic susceptibility must be assumed equal to zero at high frequencies (beginning in the far-IR region) (§60 in Ref. 137), we should point out the following[136]: a) The most favorable conditions for manifestation of the magnetic properties of a material arise in magnetooptics. The Faraday effect in an external magnetic field makes it possible to see evidence of some rather small off-diagonal components of the magnetic susceptibility, which are caused by rotation of the polarization plane. For example, for off-diagonal components of the order of 10^{-6} the rotation of the polarization plane over 1 cm at the wavelength $\lambda \sim 6\cdot 10^{-5}$ cm in a dielectric ($\varepsilon \sim 5$) is of the order of 0.1 rad. b) The case for retaining the magnetic susceptibility at high frequencies must include, in addition to numerical estimates, proof that the very meaning of magnetic susceptibility is retained at these frequencies. The proof that the meaning of magnetic susceptibility is untenable is based on the requirement that two values of the magnetic moment of an object, expressed respectively in terms of currents and in terms of the magnetic-moment density, must be equal (§60 in Ref. 137). This requirement is valid, however, if there are no convection currents. In the proof in §27 in Ref. 137 that these two values of the total magnetic moment of an object are equal for the case of static magnetic fields it was assumed that there are no convection currents capable of transporting charge across the total cross section of the object. Consequently, when there are direct convection currents (zero-frequency currents) (§29 in Ref. 137) these two values of the total magnetic moment cannot be the same, although the magnetic susceptibility retains its meaning in a static field. These two values of the total magnetic moment of an object may thus disagree, regardless of whether the frequencies are high or low, although in §60 this discrepancy and the related lack of meaning of the magnetic susceptibility are attributed to high frequencies (at which the polarization currents become significant). These arguments and the applicability of the Landau–Lifshitz equation at high frequencies furnish a basis for use of the magnetic susceptibility if only for ferromagnetic media, provided only that the macroscopic approach is applicable.

On the basis of all these arguments, we feel completely justified in examining media which simultaneously exhibit a natural optical activity and a nonzero magnetic susceptibility. Such media are described by constitutive equations (2.2) and (2.3). We used these equations above under the assumption $g_m = 0$, however, and under the assumption of a small anisotropy $\mu_{xx} - \mu_{zz}$. We turn now to a medium described by Eqs. (2.2) and (2.3); everywhere below we assume $g_m \neq 0$, which means that the magnetooptic activity has a gyromagnetic component.

1. To simplify the problem while retaining the noninvariance of the dispersion relation—a characteristic property of the medium—we assume that the medium is uniaxial.

Assuming that the optic axis (the z axis) is along the external magnetic field, we can write

$$\left.\begin{array}{l}\mu_{xx}=\mu_{yy}=\mu\neq\mu_{zz}=\mu_3,\\ \varepsilon_{xx}=\varepsilon_{yy}=\varepsilon\neq\varepsilon_{zz}=\varepsilon_3,\\ \gamma_{xyz}=-\gamma_{yxz}=-\gamma_1,\\ \gamma_{zyx}=-\gamma_{yzx}=-\gamma_2,\\ \gamma_{xzy}=-\gamma_{zxy}=\gamma_2,\\ (g_e)_x=(g_e)_y=0,\quad (g_e)_z=g_e,\\ (g_m)_x=(g_m)_y=0,\quad (g_m)_z=g_m.\end{array}\right\} \tag{2.19}$$

Using (2.2), (2.3), (2.19), and the field equations, we find the dispersion relation

$$a_4k_z^4+a_3k_z^3+a_2k_z^2+a_1k_z+a_0=0, \tag{2.20}$$

where k_z is the z component of the wave vector of wave (2.1), which is propagating in the medium.

The exact expressions for a_i $(i=1,2,3,4,0)$ are quite lengthy. If we ignore in them terms proportional to the parameters $g_e/\varepsilon, g_m/\mu$, and $(\omega/c)\gamma_{ijl}\sqrt{\mu/\varepsilon}$ raised to the third or higher orders, we find some comparatively simple expressions:

$$a_4=\frac{\omega^2}{c^2}\varepsilon_3\mu\left(1-\frac{g_m^2}{\mu^2}\right), \tag{2.21a}$$

$$a_3=\frac{\omega^2}{c^2}g_m\gamma_1\left(k_x^2-2\frac{\omega^2}{c^2}\varepsilon_3\mu\right)+\frac{\omega^2}{c^2}g_m\gamma_2k_x^2, \tag{2.21b}$$

$$\begin{aligned}a_2={}&-2\frac{\omega^4}{c^4}\varepsilon\varepsilon_3\mu^2+k_x^2\left(\frac{\omega^2}{c^2}\varepsilon\mu+\frac{\omega^2}{c^2}\varepsilon_3\frac{\mu^2}{\mu^3}\right)+\frac{g_m^2}{\mu^2}\Big[3\frac{\omega^4}{c^4}\varepsilon\varepsilon_3\mu^2\\&-\left(k_x^2-\frac{\omega^2}{c^2}\varepsilon_3\mu\right)\left(\frac{\omega^2}{c^2}\varepsilon\mu-k_x^2\frac{\mu}{\mu_3}\right)-\frac{\omega^2}{c^2}\varepsilon\mu k_x^2-k_x^4\frac{\mu}{\mu_3}-k_x^2\frac{\omega^2}{c^2}\varepsilon_3\mu\frac{\mu}{\mu_3}\Big]\\&-2\frac{\omega^4}{c^4}g_eg_m\varepsilon_3\mu+\frac{\omega^4}{c^4}\mu^2\gamma_3^2k_x^2+\frac{\omega^4}{c^4}\mu^2\gamma_1^2\left(k_x^2-\frac{\omega^2}{c^2}\varepsilon_3\mu\right)-2\frac{\omega^4}{c^4}\mu^2\gamma_1\gamma_2k_x^2,\end{aligned} \tag{2.21c}$$

$$a_1=-2\frac{\omega^4}{c^4}\varepsilon\mu g_m\gamma_2k_x^2+2\frac{\omega^4}{c^4}\mu^2g_e\gamma_1\left(k_x^2-\frac{\omega^2}{c^2}\varepsilon_3\mu\right)-2\frac{\omega^4}{c^4}\mu^2g_e\gamma_2k_x^2, \tag{2.21d}$$

$$\begin{aligned}a_0={}&\frac{\omega^2}{c^2}\varepsilon\mu\left(\frac{\omega^2}{c^2}\varepsilon\mu-k_x^2\frac{\mu}{\mu_3}\right)\left(\frac{\omega^2}{c^2}\varepsilon_3\mu-k_x^2\right)\\&+\frac{g_m^2}{\mu^2}\Big[\left(\frac{\omega^2}{c^2}\varepsilon\mu-k_x^2\frac{\mu}{\mu_3}\right)\frac{\omega^2}{c^2}\varepsilon\mu\left(k_x^2-2\frac{\omega^2}{c^2}\varepsilon_3\mu\right)\\&+\frac{\omega^2}{c^2}\varepsilon\mu\left(k_x^2-\frac{\omega^2}{c^2}\varepsilon_3\mu\right)\left(\frac{\omega^2}{c^2}\varepsilon\mu-k_x^2\frac{\mu}{\mu_3}\right)\Big]-\frac{\omega^6}{c^6}\varepsilon^3\mu\gamma_2^2k_x^2\\&+\frac{\omega^4}{c^4}\mu^2g_e^2\left(k_x^2-\frac{\omega^2}{c^2}\varepsilon_3\mu\right).\end{aligned} \tag{2.21e}$$

The quantity k_x in (2.21a)–(2.21e) is the x component of the wave vector of wave (2.1), and k_y is assumed to be zero. It follows from (2.20) and (2.21) that when there is a gyromagnetic rotation ($g_m \neq 0$) a new term with an odd power of k_z ($a_3 \neq 0$) appears along with the term containing the first power of k_z ($a_1 \neq 0$). The appearance of this new term should change the shape of the index ellipsoid and of the gyration surface.

2. *Boundary-value problem.* We consider now the normal propagation of light through a plane-parallel plate described by constitutive equations (2.2) and (2.3) under simplifying conditions (2.19). The plate fills the region $0 \leq z \leq d$ and is bordered by vacuum on each side. The plane wave

$$\mathbf{E}(z, t) = \mathbf{E}\exp\left[i\left(\frac{\omega}{c}z - \omega t\right)\right], \quad E_x = 0, \quad E_y \neq 0 \tag{2.22}$$

is incident from the $z < 0$ half-space.

The boundary conditions depend strongly on the form of the constitutive equations. With the constitutive equations as in (2.2) and (2.3), where the entire contribution of the natural activity is taken into account in the expression for the electric displacement D (as was mentioned earlier, the constitutive equations of a naturally active medium can be formulated in another way also; see Ref. 10, where the various possible formulations are discussed), the tangential components of the magnetic field are discontinuous.[24,25,98] The reason is that the constitutive equation relating D and E contains a gradient parameter which is a characteristic of the medium and which is naturally discontinuous at the sharp boundary as we go from one medium to another. In the simplest case of an isotropic medium, for example, D and E are related by[24,25,98]

$$\mathbf{D} = \varepsilon\mathbf{E} + \delta_1 \operatorname{rot}\mathbf{E} + \operatorname{rot}(\delta_2\mathbf{E}) = \varepsilon\mathbf{E} + (\delta_1 + \delta_2)\operatorname{rot}\mathbf{E} + [\operatorname{grad}\delta_2\mathbf{E}]. \tag{2.23}$$

In each of the homogeneous media adjacent to each other we have grad $\delta_2 = 0$, but if the parameters δ_2 for these media are different, then grad δ_2 is discontinuous at the tangential components of the magnetic field. This field is related to D by rot $\mathbf{H} = (1/c)\partial\mathbf{D}/\partial t$. [The case of an inhomogeneous medium was taken up in Ref. 98, and the meaning of the parameter δ_2 was determined (the tensor β_{ijl} in Ref. 98)].

Working from the boundary conditions, and using the discontinuity of the tangential component of the magnetic field,[98] we find the following expressions for the field amplitudes of the transmitted wave:

$$E_{4y} = (A^+ + A^-)E_y, \tag{2.24}$$

$$E_{4x} = -i(A^+ - A^-)E_y, \tag{2.25}$$

where

$$\left.\begin{aligned}
&A^{\pm} = (\Delta^{\pm})^{-1}(\alpha_2^{\pm} - \alpha_3^{\pm})\exp[i(k_{2z}^{\pm} + k_{3z}^{\pm})d],\\
&\Delta^{\pm} = (1 + \alpha_2^{\pm})(1 - \alpha_3^{\pm})\exp(ik_{3z}^{\pm}d) - (1 - \alpha_2^{\pm})(1 + \alpha_3^{\pm})\exp(ik_{2z}^{\pm}d),\\
&\alpha_{2,3}^{\pm} = \frac{\omega}{ck_{2,3z}^{\pm}}[\varepsilon \pm (\gamma_1 k_{2,3z}^{\pm} + g_e)] \mp \frac{\omega}{2c}\gamma_1.
\end{aligned}\right\} \tag{2.26}$$

Within small terms of first order in the activity parameters, the quantities $k_{2,3z}^{\pm}$ are

$$k_{2z}^{\pm} = \frac{\omega}{c}\sqrt{\varepsilon\mu}\left\{1 \pm \frac{1}{2}\left[\left(\frac{g_e}{\varepsilon} + \frac{g_m}{\mu}\right) + \frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1}{\varepsilon}\right]\right\} \tag{2.27a}$$

for forward waves and

$$k_{3z}^{\pm} = -\frac{\omega}{c}\sqrt{\varepsilon\mu}\left\{1 \pm \frac{1}{2}\left[\left(\frac{g_e}{\varepsilon} + \frac{g_m}{\mu}\right) - \frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1}{\varepsilon}\right]\right\} \tag{2.27b}$$

for backward waves.

It follows from (2.27) that the reversibility of the light waves is disrupted, since neither k_{3z}^{+} nor k_{3z}^{-} is equal in absolute value to k_{2z}^{+} or k_{2z}^{-}. In contrast with the medium discussed in the preceding subsection, we are interested here in the sum $(g_e/\varepsilon) + (g_m/\mu)$ instead of g_e/ε. This statement means that the gyroelectric and gyromagnetic rotations are indistinguishable from the standpoint of wave propagation. A difference is revealed, however, upon wave reflection (see Refs. 39, 43, and 86 and Subsection 3b below).

The rotation of the polarization plane over a unit path length during wave propagation in the forward and backward directions is, respectively,

$$\varphi_{1,2} = \left(\frac{g_e}{\varepsilon} + \frac{g_m}{\mu} \pm \frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1}{\varepsilon}\right)\frac{\omega}{c}\sqrt{\varepsilon\mu}. \tag{2.28}$$

Since $|\varphi_1| \neq |\varphi_2|$, the rotations during forward and backward propagation are not equal in absolute value.

The absence of a rotation of the polarization plane for the forward wave corresponds to the relation

$$\frac{g_e}{\varepsilon} + \frac{g_m}{\mu} + \frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1}{\varepsilon} = 0. \tag{2.29}$$

From (2.25) and (2.26) we have $E_{4x} \neq 0$; i.e., as in the case discussed in Subsection 2b, multiple reflection causes a rotation of the polarization plane of the transmitted wave. In the absence of reflections from the plate boundary ($\sqrt{\varepsilon/\mu} = 1$) we find

$$A^+ = A^-,$$

and, by virtue of (2.25), $E_{4x} = 0$.

We will complete this subsection with the approximate dependence of k_z on k_x (accurate to the second power of the activities parameters). Although the exact functional dependence can be found from (2.20) and (2.21) as the solution of the complete fourth-degree equation, the corresponding solutions are not very informative because the expressions for the coefficients a_i are very complicated. The function $k_z = F(k_x)$ must be found, for example, in solving the boundary-value problem in the case of oblique incidence. Assuming that the anisotropy parameters $(\varepsilon - \varepsilon_3)/\varepsilon_3$ and $(\mu - \mu_3)/\mu_3$ are no greater than the squares of the activity parameters $(\gamma_{jlm}\varepsilon^{-1}\cdot(\omega/c)\sqrt{\varepsilon\mu}, g_e\varepsilon^{-1}, g_m\mu^{-1})$ in order of magnitude, and writing k_z as

$$k_z = k_{0z}(1 + s), \tag{2.30}$$

we find from (2.20) and (2.21)

$$s = \pm\frac{1}{2}\sqrt{f(k_{0z}, k_x)\left(\frac{\omega^2}{c^2}\varepsilon\mu k_{0z}^4\right)^{-1}}, \tag{2.31}$$

where

$$\begin{aligned}
f(k_{0z}, k_x) = {}& \frac{\omega^2}{c^2}g_m\gamma_1\left(\frac{\omega^2}{c^2}\varepsilon\mu + k_{0z}^2\right)k_{0z}^3\\
&+ \frac{\omega^2}{c^2}g_m\gamma_2 k_x^2\left(2\frac{\omega^2}{c^2}\varepsilon\mu - k_{0z}^2\right)k_{0z} + \frac{g_m^2}{\mu^2}\frac{\omega^4}{c^4}\varepsilon^2\mu^2 k_{0z}^2\\
&+ 2\frac{\omega^4}{c^4}\varepsilon\mu g_e g_m k_{0z}^2 + \frac{\omega^4}{c^4}\mu^2(\gamma_2^2 k_x^4 + \gamma_1^2 k_{0z}^4) + 2\frac{\omega^4}{c^4}\mu^2\gamma_1\gamma_2 k_x^2 k_{0z}^2\\
&+ 2\frac{\omega^4}{c^4}\mu^2 g_e\gamma_1 k_{0z}^3 + 2\frac{\omega^4}{c^4}\mu^2 g_e\gamma_2 k_x^2 k_{0z} + \frac{\omega^4}{c^4}\mu^2 g_e^2 k_{0z}^2.
\end{aligned} \tag{2.32}$$

For the forward and backward waves, respectively, k_{0z} is

$$k_{0z} = \pm\sqrt{\frac{\omega^2}{c^2}\varepsilon\mu - k_x^2}. \tag{2.33}$$

If $\gamma_{jlm} = 0$, the medium becomes bigyrotropic, and for s we find

$$s = \pm\frac{1}{2}\frac{k_0}{k_{0z}}\left(\frac{g_e}{\varepsilon} + \frac{g_m}{\mu}\right), \quad k_0 = \frac{\omega}{c}\sqrt{\varepsilon\mu}. \tag{2.34}$$

These relations hold if s is of the order of the activity parameters. This condition is violated for propagation directions making angles with the z axis near $\pi/2$, for which k_{0z} is small.

It follows from (2.30)–(2.34) that the wave-vector surface and the gyration surface are symmetric with respect to the plane perpendicular to the external magnetic field. These surfaces can be seen to have different shapes, depending on whether the medium has a gyromagnetic activity in addition to its gyroelectric activity.

3. MAGNETOACTIVE MEDIA

a) Propagation of an electromagnetic wave in a magnetized ferromagnet near a point of coincident roots of the dispersion relation

1. Generally speaking, the field equations for anisotropic and gyrotropic media yield two values for the wave vector of waves propagating in the same direction. These waves differ in polarization, and each has a single independent field component (electric or magnetic), in terms of which the other components can be expressed. In determining the field amplitudes of the reflected and refracted waves we deal with a system of equations (boundary conditions) equal in number to the number of independent components. If, for example, a wave is incident from vacuum on a gyrotropic half-space, we have four field-continuity conditions and four unknown components for a given amplitude of the incident wave: two components (of, say, the electric field) in the reflected wave and one component in each of the two refracted waves. All the other components of the electric field can be expressed in terms of these by means of the constitutive equations and the equation $\operatorname{div}\mathbf{D}=0$.

The magnetic field components are expressed in terms of the electric field components by means of one of the first two Maxwell equations.

Under certain conditions, however, this ordinary situation may be disrupted. For example, if roots of the dispersion relation coincide, the number of unknown components may not be the same as the number of boundary conditions. However, we must distinguish from the outset between two cases of coincident roots. In the first, one of the relationships between components disappears because of the coincidence. It turns out in this case that two waves with independent polarizations correspond to the two coincident roots. This case (degeneracy[9]) does not pose any difficulties in the boundary-value problem or any singularities in the wave propagation. In the second case, the polarizations of the waves are also equal when the roots are coincident. In this case the roots are called "essentially multiple."[9]

The propagation directions for which the roots of the dispersion relation are essentially multiple are generally called "singular." There may be isolated singular directions[74,140,141] or cones of such directions.[142]

Situations of the second type are interesting not only because they complicate the formulation of the boundary-value problem but also because the media exhibit distinctive optical properties under such conditions. The case of coincident roots of the dispersion relation in absorbing crystals was examined by Khapalyuk.[74] He concluded that there are, in addition to ordinary normal waves, waves of a new type, of the form[9,7]

$$\mathbf{E}(r,\ t)=\mathbf{A}\cdot(\mathbf{kr})\exp[i(\mathbf{kr}-\omega t)]. \tag{3.1}$$

Such waves may arise when absorption is taken into account, and this circumstance prevents an unbounded growth of the field upon an unbounded increase in (kr): As $(\mathbf{kr})\to\infty$, the exponential function tends toward zero more rapidly than the product (kr) tends toward infinity, and the field amplitude decreases.

Kaganov and Yankelevich[75] examined the case of coincident roots of the dispersion relation in a nonabsorbing gyroanisotropic medium. In such media, without absorption, waves of the type in (3.1) cannot arise.[9] It turns out that when the roots of the dispersion relation coincide the picture of the wave field is completely different: The waves propagate undamped in the medium, and the energy flux is zero. The reason why there is no energy flux is that the electric and magnetic field vectors are parallel to each other.

We turn now to the situation which arises in the case of essentially multiple roots in a magnetized ferromagnet with a scalar dielectric permittivity.[77]

2. We assume that a plane wave (2.1) is incident from the region $z<0$ on a ferromagnetic medium occupying the $z\geqslant 0$ half-space in an external magnetizing field H_0. We denote by ε_2 the dielectric permittivity of the ferromagnetic medium, and we write the magnetic permeability as[122,125]

$$\left.\begin{array}{l}\mu_{xx}=\mu_{yy}=1+a(1-q^2)^{-1}=\mu_2,\\ \mu_{xy}=-\mu_{yx}=-ig=iaq(1-q^2)^{-1},\\ \mu_{zz}=\mu_3=1,\quad q=\dfrac{\omega}{\omega_H},\quad \omega_H=\gamma_0 H_0,\end{array}\right\} \tag{3.2}$$

$$a=4\pi\frac{M_0}{H_0},\quad \mu_{xz}=\mu_{yz}=\mu_{zx}=\mu_{zy}=0,$$

where ω_H is the ferromagnetic resonance frequency, ω is the frequency of the wave propagating in the medium, and M_0 is the magnetic moment per unit volume. Equations (3.2) hold if

$$|1-\omega^2\omega_H^{-2}|\gg\omega_r\omega_H^{-2}\omega,\quad \omega_r^2\omega_H^{-2}\ll 1, \tag{3.3}$$

where ω_r is the relaxation frequency.[122]

We denote by k_x the tangential component of the wave vector of the incident wave, and we assume $k_y=0$. We then find the following expressions for the z components of the wave vectors of the refracted waves:

$$k_{2z}^{\pm 2}=\omega^2c^{-2}\varepsilon_2\mu_2-(\mu_2+\mu_3)(2\mu_3)^{-1}k_x^2\pm\sqrt{\eta_{2z}}, \tag{3.4}$$

$$\eta_{2z}=(\mu_2-\mu_3)^2(2\mu_3)^{-2}k_x^4-\omega^2c^{-2}\varepsilon_2 g^2\mu_3^{-1}k_x^2+\omega^4c^{-4}\varepsilon_2^2 g^2. \tag{3.5}$$

Dispersion relation (3.4) has multipole roots if

$$\eta_{2z}=0. \tag{3.6}$$

Condition (3.6) is satisfied along with the condition of real multiple roots (in the absence of intrinsic absorption) on the following curve:

$$k_x^2\left(\frac{\omega^2}{c^2}\varepsilon_2\right)^{-1}=2\left[\frac{\omega^2}{\omega_H^2}+\sqrt{\frac{\omega^4}{\omega_H^4}-\frac{\omega^2}{\omega_H^2}}\right], \tag{3.7}$$

if the frequency interval is determined by

$$1<\frac{\omega^2}{\omega_H^2}<1-a^2(1+2a)^{-1}. \tag{3.8}$$

The wave is elliptically polarized.[77] We might note that by varying k_x we could change ω in (3.7); i.e., the situation in which multiple roots appear may occur not at a single frequency but over an entire frequency interval. There is the possibility of modulating the wave polarization and the energy transport across the boundary in this interval (see the following subsection).

3. Differentiating (3.4), we see that under condition (3.7) the z components of the group velocities vanish:

$$u_{2z}^{\pm}=\frac{\partial\omega}{\partial k_{2z}^{\pm}}=0. \tag{3.9}$$

It is interesting to examine the way in which the energy flux in the z direction vanishes. In a gyrotropic medium the Poynting vector frequently rotates over space at the wave frequency ω, and we are interested in its average value $\overline{\mathbf{S}}$ rather than its instantaneous value. Constructing the Poynting vector, we find the following condition on curve (3.7):

$$\overline{S}_{2z}=0 \tag{3.10}$$

in accordance with (3.9). The vanishing of the z component of the group velocities in this case results from the phase difference (of $\pi/2$) between the components of the electric and magnetic fields, in contrast with the case of a gyroanisotropic medium.[75] This phase difference causes the z component of the vector product [EH] to vanish when averaged over time.[77]

We assume that k_x is fixed. If $\omega\omega_H^{-1}$ satisfies (3.7), we then have $u_{2z}^{\pm}=0$. We denote this value of $\omega\omega_H^{-1}$ by q_0. If we now vary ω or ω_H, we move away from (3.7). We assume that the value of $\omega\omega_H^{-1}$ differs from q_0 by $\Delta q=\omega\omega_H^{-1}-q_0$. A displacement of q from q_0 changes the z components of the group velocities. At small values of the ratio $\Delta q/q_0$ we find the following expressions for $u_{2z}^{\pm}$:

$$u_{2z}^{\pm}=\mp c^2k_{2z}(1-q_0^2)\left[\omega_H\varepsilon_2 a\sqrt{2\left(q_0^2+\sqrt{q_0^4-q_0^2}\right)-1}\right]^{-1}\sqrt{\frac{2\Delta q}{q_0}}, \tag{3.11}$$

where $k_{2z}=k_{2z}^{+}=k_{2z}^{-}$ on curve (3.7). If $\Delta q<0$, then the z components of the group velocities of the two waves propagating in the same direction have different signs near the frequencies at which the roots of the dispersion relation coincide: The energy of one wave is propagating forward, and that of the other is propagating backward [the direction of these waves is determined by whether k_{2z} is positive or negative in (3.11)]. On curve (3.7) itself, there is no energy flux along the direction of the z axis, although the waves propagate undamped. If $\Delta q>0$, then the amplitude of one of the waves increases exponentially, and the amplitude of the other decays. The components $u_{2z}^{\pm}$ are imaginary when $\Delta q>0$. Clearly, we can move away from curve (3.7) not only by changing ω or ω_H but also by changing k_x.

4. The fields of the waves propagating in the medium under discussion depend on the coordinates and the time in accordance with $\mathbf{E}_0^{\pm}\exp[i(k_x x+k_{2z}^{\pm}z-\omega t)]$. These waves coalesce to form a single wave on curve (3.7). To determine how the field depends on the coordinates in the case of coincident roots of the dispersion relation we need to know the amplitudes E_0^{+} and E_0^{-}. To find them, we briefly examine the boundary-value problem.[77]

In determining the field amplitudes we ordinarily have two waves in a semi-infinite refracting medium. Of the four possible waves we select those which are damped and do not grow with distance from the boundary (damping and growth always occur, regardless of other factors, because of the unavoidable presence of absorption; in the case at hand, the absorption results from the imaginary part of the dielectric permittivity). We thus have two independent amplitude components in a refracting medium. If the roots of the dispersion relation are essentially multiple, on the other hand, the two waves coalesce to form a single wave, and instead of two unknown amplitudes we have one. To determine the amplitudes when the roots of the dispersion relation coincide, i.e., when $\Delta q=0$, we find expressions for these amplitudes under the condition $\Delta q\neq 0$ (in which case the number of unknown amplitude components is equal to the number of conditions at the boundary), and we let Δq tend toward zero in these expressions. We then find an expression of the following type for the field of the refracted wave:

$$\mathbf{E}_{\mathrm{refr}}(\mathbf{r},t)=(\mathbf{A}z+\mathbf{B})\exp[i(k_x x+k_{2z}z-\omega t)]. \tag{3.12}$$

The sum of the two refracted normal waves of the type $\mathbf{E}_0^{\pm}\exp[i(k_x x+k_{2z}^{\pm}z-\omega t)]$ thus transforms to an expression like (3.12) as we tend toward curve (3.7), on which the condition $k_{2z}^{+}=k_{2z}^{-}$ holds. Since $\mathrm{Im}\,k_{2z}>0$, the field remains finite despite the term $\mathbf{A}z$; the reason is that the function $\exp(\mathrm{Im}k_{2z}z)$ tends toward zero more rapidly than $|\mathbf{A}z|$ tends toward infinity.

b) Bigyrotropic media

Such media are described by constitutive equations (2.2) and (2.3) with $\gamma_{jlm}=0$. The variety of optical properties of bigyrotropic media have been revealed in a series of studies.[39,75,76,86,117,121,123,129] For example, the ellipticity of a wave transmitted through a bigyrotropic plate is caused not only by the different absorption levels for right- and left-hand polarized waves but also by the difference between the impedances for these waves.[39] Several distinctive features have been found for propagation in bigyrotropic media; in particular, there is the possibility of monorefringence,[76,47] and there is the possibility that the electric and magnetic fields may be parallel in a standing wave in a semi-infinite gyroanisotropic medium.[75] It has been shown that it is possible to distinguish between the gyroelectric and gyromagnetic effects[86,117] (which are caused by the parameters g_e and g_m, respectively). We now consider the amplitude relations for a bigyrotropic medium with absorption and with a (usually neglected) slight anisotropy, and we will briefly discuss certain polarization properties of bigyrotropic media which result from the presence of the two gyrotropy parameters g_e and g_m.

We assume that a bigyrotropic medium described by the dielectric permittivity and magnetic permeability tensors

$$\varepsilon_{2ik}=\begin{pmatrix}\varepsilon_2 & -ig_e & 0\\ ig_e & \varepsilon_2 & 0\\ 0 & 0 & \varepsilon_3\end{pmatrix},\quad \mu_{2ik}=\begin{pmatrix}\mu_2 & -ig_m & 0\\ ig_m & \mu_2 & 0\\ 0 & 0 & \mu_3\end{pmatrix}, \tag{3.13}$$

occupies the region $z\geqslant 0$. A wave

$$\mathbf{E}(\mathbf{r}, t) \quad \mathbf{E}\exp[i(\mathbf{k}\mathbf{r} - \omega t)] \tag{3.14}$$

is incident from vacuum on the $z=0$ boundary. We seek a refracted field in the form $\mathbf{E}_2(r, t) = \mathbf{E}_2\exp[i(\mathbf{k}_2\mathbf{r} - \omega t)]$. We then find the dispersion relation

$$k_2^{\pm 2} = \frac{\omega^2}{c^2}\varepsilon_2\mu_2 - \frac{1}{2}\left(\frac{\mu_2}{\mu_3} + \frac{\varepsilon_2}{\varepsilon_3} - 2\right)k_x^2 \pm \sqrt{\eta},$$

$$\eta = \frac{1}{4}\left(\frac{\mu_2}{\mu_3} - \frac{\varepsilon_2}{\varepsilon_3}\right)^2 k_x^4 - \frac{\omega^2}{c^2}\left[\left(\frac{\mu_2}{\mu_3} + \frac{\varepsilon_2}{\varepsilon_3}\right)g_e g_m + \frac{\varepsilon_2}{\varepsilon_3}g_m^2 + \frac{\mu_2}{\mu_3}g_e^2\right]k_x^2 + \frac{\omega^4}{c^4}(\varepsilon_2 g_m + \mu_2 g_e)^2. \tag{3.15}$$

In a discussion of the frequency interval with

$$\left|\frac{\mu_2 - \mu_3}{\mu_2}\right| \sim \left|\frac{g_m}{\mu_2}\right|^2, \quad \left|\frac{g_e}{\varepsilon_2}\right| \sim \left|\frac{g_m}{\mu_2}\right|, \quad \left|\frac{\varepsilon_2 - \varepsilon_3}{\varepsilon_2}\right| \sim \left|\frac{g_e}{\varepsilon_2}\right|^2, \tag{3.16}$$

and with terms of the order of the gyration parameters retained, the customary approach has been to ignore the anisotropy.[123] This simplification is justified in a study of light propagation in an unbounded medium. As *mentioned above, however, in the presence of boundaries* it turns out that an anisotropy gives rise to terms in the amplitudes which are of the same order as those caused by the gyrotropy.[43] Consequently, anisotropy has been retained in the derivation of the dispersion relation (3.15), so that these terms can subsequently be taken into account.

For the components E_{1x} and E_{1y} of the electric field amplitude of the reflected wave we find[43]

$$E_{1x} = \frac{k_{2z} - \varepsilon_2 k_z}{k_{2z} + \varepsilon_2 k_z}E_x + \frac{2iE_y}{\Delta_0}\frac{\varepsilon_3 k_z k_{2z}}{k_x^2}\left[\left(\frac{g_m}{\mu_2} - \frac{g_e}{\varepsilon_2}\right)\frac{k_2}{k_{2z}} + \frac{2(\mu_3 - \mu_2)k_x^2}{\mu_3[(g_e/\varepsilon_2) + (g_m/\mu_2)]k_{2z}k_2}\right],$$

$$E_{1y} = \frac{\mu_2 k_z - k_{2z}}{\mu_2 k_z + k_{2z}}E_y - \frac{2iE_x}{\Delta_0}\frac{\varepsilon_2 k^3 k_2}{k_x^2 k_z}\left(\frac{g_m}{\mu_2} - \frac{g_e}{\varepsilon_2}\right), \tag{3.17}$$

where

$$k_2 = \frac{\omega}{c}\sqrt{\varepsilon_2\mu_2}, \quad k_{2z} = \sqrt{k_2^2 - k_x^2},$$

$$\Delta_0 = -2\frac{k_{2z}k_2}{k_x^2}\left(1 + \frac{k_{2z}}{\mu_2 k_z}\right)\left(1 + \frac{\varepsilon_2 k_z}{k_{2z}}\right).$$

The second term in brackets in the expression for E_{1x} results from the anisotropy. According to (3.16), it is a quantity of the same order of magnitude as the gyration parameters $g_e\varepsilon_2^{-1}$ and $g_m\mu_2^{-1}$. Consequently, in determining the field amplitudes within terms of the order of the gyration parameters, we must also take into account the small parameters of higher order which are caused by the anisotropy.

The geometry of this ellipticity which results from the anisotropy can be understood from the following arguments. We assume that a wave with an electric field parallel to the y axis is incident normally on the $z=0$ boundary of a gyrotropic medium. In the absence of anisotropy, the refracted wave is elliptic because the amplitudes of the waves with right- and left-hand circular polarizations are not equal: An x component of the field appears with an amplitude numerically equal to the segment AB (Fig. 3a). This anisotropy "deforms" the polarization circles into ellipses, compressing one circle in one direction and the other in a perpendicular direction (Fig. 3b). We thus have a further difference between the parallel semiaxes of the two polarization ellipses: The amplitude of the x component of the field is now numerically equal to the segment A_1B_1. By virtue of the boundary conditions, an additional ellipticity also arises in the reflected wave.

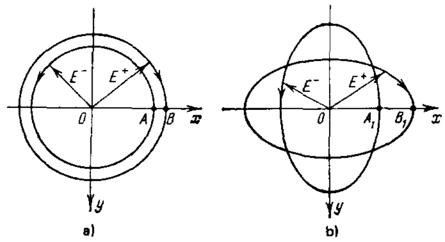

FIG. 3.

It can be seen that there are two ways in which the ellipticity can be cancelled: because of a difference between the radii of the polarization circles and because of the "deformation" of these circles. Ellipticity is thus a necessary property of a wave reflected from a magnetoactive medium on which a plane-polarized wave is incident.

There is also the possibility that there will be no ellipticity in the case of a medium which is not anisotropic. To pursue this possibility we consider the case $k_x = 0$. There is no ellipticity in the reflected wave if $(g_e/\varepsilon_2) - (g_m/\mu_2) = 0$. The polarization plane rotates in the medium $[(g_e/\varepsilon_2) + (g_m\mu_2) \neq 0]$. If, on the other hand, we have $(g_e/\varepsilon_2) + (g_m/\mu_2) = 0$, then the medium does not rotate the polarization plane, although it clearly has two types of gyrotropy and must therefore be assumed gyrotropic. The gyrotropy is seen in this situation in the reflected wave: in its ellipticity. Here we see yet another case which shows that the concept of a gyrotropic medium as one which rotates the polarization plane is of limited applicability, as was pointed out in Refs. 7, 10, and 20. This limitation is incorporated in the definition of gyrotropy given in Ref. 10.

We now consider the role played by absorption in the polarization of the reflected wave. We assume that a plane wave with electric vector in the plane of incidence is incident on a medium. If we take into account the absorption caused by the imaginary parts $\varepsilon''_{2,3}$ and $\mu''_{2,3}$ of the components $\varepsilon_{2,3} = \varepsilon'_{2,3} + i\varepsilon''_{2,3}$, $\mu_{2,3} = \mu'_{2,3} + i\mu''_{2,3}$, we find an additional phase difference in the reflected wave.

Ignoring the terms proportional to $\varepsilon''_{2,3}$ and $\mu''_{2,3}$ raised to the second and higher powers, and also ignoring the products of $\varepsilon''_{2,3}$ and $\mu''_{2,3}$ with g_e or g_m, we find the following expression for the additional phase difference Φ:

$$\Phi = \Phi_1 - \Phi_2, \tag{3.18a}$$

where $\Phi_{1,2}$ are the arguments of the complex numbers $\rho_{1,2}$:

$$\rho_{1,2} = \sqrt{\varepsilon'_2\mu'_2 - \sin^2\vartheta} \mp \varepsilon'_2\cos\vartheta + i\left(\frac{\varepsilon'_2\mu''_2 + \varepsilon''_2\mu'_2}{2\sqrt{\varepsilon'_2\mu'_2 - \sin^2\vartheta}} \mp \varepsilon''_2\cos\vartheta\right) \tag{3.18b}$$

(the subscript "1" corresponds to the upper sign), and ϑ is the angle of incidence.

When absorption is taken into account, the phase difference between the components of the electric field in the reflected wave (one of these components lies in the propagation plane, and the other is perpendicular to this plane) ceases to be a constant (equal to $\pi/2$) and becomes dependent on the angle of incidence. If the electric vector of the incident wave lies in the propagation plane or is perpendicular to it, then the major semi-axis of the polarization ellipse of the reflected wave correspondingly departs from this plane or slopes toward it.

Expressions (3.18) hold if

$$\left|\frac{\varepsilon_2'\mu_2''+\varepsilon_2''\mu_2'}{\varepsilon_2'\mu_2'-\sin^2\vartheta}\right|\ll 1.$$

We will complete this subsection by examining yet another property of magnetoactive media. If the medium is isotropic in the x, y plane, and if we have $E_y=0$ in the incident wave, then the ratio E_{1y}/iE_{1x} in the reflected wave is (with $k_x=0$)

$$\alpha_1=\frac{E_{1y}}{iE_{1x}}=\frac{2Z_0}{1-Z_0^2}\left(\frac{g_e}{\varepsilon_2}-\frac{g_m}{\mu_2}\right),\quad Z_0=\sqrt{\frac{\varepsilon_2}{\mu_2}}. \tag{3.19}$$

The rotation of the polarization plane over a unit path length in an unbounded medium, which we can use to estimate the rotation during propagation of a wave through a plate, is proportional to

$$\alpha_2=\frac{\omega}{c}\sqrt{\varepsilon_2\mu_2}\left(\frac{g_e}{\varepsilon_2}+\frac{g_m}{\mu_2}\right). \tag{3.20}$$

We assume for definiteness $Z_0>1$. Then it follows from the last relations that if α_1 and α_2 have different signs then the medium is gyroelectric (or that the gyroelectric effect is stronger than the gyromagnetic effect), while if the signs of α_1 and α_2 are the same then the medium is gyromagnetic (or the gyromagnetic effect is stronger than the gyroelectric effect). This assertion remains valid in the case of oblique incidence, although in this case α_1 and α_2 are not the same as in (3.19) and (3.20). We can thus draw conclusions about the nature of the rotation simply by comparing the signs of α_1 and α_2.

4. CIRCULAR DICHROISM, DICHROISM MECHANISMS, AND IRREVERSIBILITY

1. The absorption related to the gyrotropy parameter is known to cause circular dichroism: different absorption levels for waves with right- and left-hand circular (or elliptic) polarizations.[17,18,138] This is the simplest type of dichroism, and the one known for a long time, but it is not the only type.

In an anisotropic medium, both the rotation of the polarization plane and the different absorption levels for right- and left-hand elliptic polarizations depend on both the imaginary parts of the gyrotropy parameters and the imaginary parts of the dielectric permittivity tensor.[143,144] For example, if light is propagating along one of the primary directions in an anisotropic crystal, a direction along which an external magnetic field is applied, then the wave vectors are

$$k^{\pm 2}=\frac{\omega^2}{c^2}\frac{\varepsilon_1+\varepsilon_2}{2}\pm\sqrt{\left(\frac{\omega^2}{c^2}\frac{\varepsilon_1-\varepsilon_2}{2}\right)^2+\frac{\omega^4}{c^4}g^2}, \tag{4.1}$$

where g is the projection of the gyration vector onto the propagation direction, and ε_1 and ε_2 are the principal values of the tensor ε_{ij} in the plane perpendicular to this direction. It follows from (4.1) that when ε_1 and ε_2 have imaginary parts the wave vectors k^+ and k^- will have different values even if the parameter g is real. The rotation and the circular dichroism depend on the linear dichroism because the circular polarization of the waves becomes elliptic in an anisotropic medium.[143] The effect of linear dichroism on circular dichroism should apparently be regarded as yet another dichroism mechanism.

2. There is a circular dichroism different from those discussed above.[145] It operates even if there is neither an anisotropy nor an imaginary part of the gyrotropy parameter. To explain this mechanism we consider the propagation of light along an external magnetic field in a medium which is isotropic in the absence of the magnetic field and described in the presence of the field by

$$\mathbf{D}=\varepsilon\mathbf{E}+i[\mathbf{g}_e\mathbf{E}],\quad \mathbf{B}=\mu\mathbf{H}. \tag{4.2}$$

For the wave vectors of the waves with right- and left-hand circular polarizations we find

$$k^{\pm}=\frac{\omega}{c}\sqrt{\mu(\varepsilon\pm g_e)}. \tag{4.3}$$

If ε has an imaginary part ($\varepsilon=\varepsilon'+i\varepsilon''$), the imaginary parts of k^+ and k^- will not be the same. Specifically, they will differ by an amount proportional to the product $\varepsilon'' g_e$. Although the imaginary parts of the expressions in the radicals,

$$\varepsilon'+i\varepsilon''+g_e \text{ and } \varepsilon'+i\varepsilon''-g_e$$

are equal, the values of the corresponding roots are not, because of the difference between the real parts of the expressions in the radicals.

This type of dichroism occurs because the condition $g_e\neq 0$ makes the optical path lengths traced out by the right- and left-hand circularly polarized waves different over the same geometric path lengths (because of the difference between the wavelengths); as a result, the waves experience different absorption levels at a given value of the damping parameter ε''. With a real gyration parameter, the circular dichroism may thus occur even in the absence of anisotropy in a magnetized medium (in contrast with isotropic naturally gyrotropic media[10]).

To observe this type of dichroism might require large imaginary parts of the tensor ε_{ij} (or μ_{ij}; see Ref. 145, where this type of dichroism is discussed for the case of a bigyrotropic medium) and large values of the gyrotropy parameters. Large values of the gyrotropy parameter were measured in Ref. 146, for example. The large values of the parameter g_e in magnetooptic experiments in megagausss fields[147] should also give rise to this type of dichroism in the presence of an absorption due to the imaginary part of the dielectric permittivity.

3. In naturally gyrotropic media in an external magnetic field, with waves propagating in opposite directions (not perpendicular to the magnetic field), the magnitude of the circular dichroism is not the same. From (2.27a) and (2.27b), with real ε and μ, for example, we find

$$\begin{aligned}|k_{2z}^{+''}-k_{2z}^{-''}|&=\left|\frac{\omega}{c}\sqrt{\varepsilon\mu}\left(\frac{g_e''}{\varepsilon}+\frac{g_m''}{\mu}+\frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1''}{\varepsilon}\right)\right|,\\ |k_{3z}^{+''}-k_{3z}^{-''}|&=\left|\frac{\omega}{c}\sqrt{\varepsilon\mu}\left(\frac{g_e''}{\varepsilon}+\frac{g_m''}{\mu}-\frac{\omega}{c}\sqrt{\varepsilon\mu}\,\frac{\gamma_1''}{\varepsilon}\right)\right|\end{aligned} \tag{4.4}$$

(the double prime means the imaginary part of the corresponding quantity). The inequality $|k_{2z}^{+''} - k_{2z}^{-''}| \neq |k_{3z}^{+''} - k_{3z}^{-''}|$ is responsible for a difference in the absorption levels for plane-polarized and unpolarized light propagating through a plate in opposite directions.[145] We thus see an irreversibility of the absorption with respect to a reversal of the propagation direction.

4. Yet another type of dichroism appears in a bigyrotropic medium. Let us consider the propagation of light along the magnetic field in the medium described by (3.13). For the wave vectors we find

$$k^{\pm} = \frac{\omega}{c}\sqrt{(\varepsilon \pm g_e)(\mu \pm g_m)}. \tag{4.5}$$

If $\varepsilon = \varepsilon' + i\varepsilon''$, $\mu = \mu' + i\mu''$, and g_e and g_m are real, then $k^{\pm} = \omega/c\{(\varepsilon'\mu' + g_e g_m - \varepsilon''\mu'') \pm (\varepsilon' g_m + \mu' g_e) \pm i[(\varepsilon'' g_m + \mu'' g_e) \pm (\varepsilon'\mu'' + \varepsilon''\mu')]\}^{1/2}$.

The quantity $i(\varepsilon'' g_m + \mu'' g_e)$ appears with different signs in the expressions for k^+ and k^- and thus causes different absorption levels for waves with right- and left-hand circular polarizations. Each of the terms $\varepsilon'' g_e$ and $\mu'' g_m$, we might note, is the product of two parameters, one appearing in the equation relating **D** and **E** and the other appearing in the equation relating **B** and **H**. We should emphasize that the possible appearance of terms with a structure of this sort was pointed out by Bokut' and Fedorov[90] for naturally gyrotropic media with constitutive equations formulated in such a manner that the gyrotropy parameter appears in both the equation relating **D** and **E** and that relating **B** and **H**.

5. CONCLUSION

We have touched on some of the many problems in the optics of gyrotropic media. The trends seen today in research fields bearing more or less directly on the optics of gyrotropic media suggest certain specific questions as worthwhile to pursue. The first is the optics of boundary layers.[25] Generally speaking, an interface presents new conditions for studying the structure of matter (say the intermolecular interaction), the reason being simply the "cutoff" of the medium at its boundary (which, for example, changes the crystal wave functions because of the disruption of the periodicity). Magnetooptic methods constitute a subtle instrument for studying boundary layers.[31,32] A detailed derivation of a theory for the optics of boundary layers incorporating gyrotropy and the anisotropy of the dielectric function would clearly be worthwhile. The number of papers on this topic is small, and the overwhelming majority have so far addressed nongyrotropic media.[148,149]

A second group of questions involves a detailed derivation of a phenomenological theory for reflection and refraction at the interfaces of half-spaces and plates incoporating the optical properties of unbounded gyrotropic media with which we are familiar at this point (the appearance of new roots of the dispersion relation, the coincidence of roots of the dispersion relation, etc.). Research in this field is required not only for studying these effects and identifying them experimentally but also for taking boundaries into account properly. Such research may also reveal some new effects due to boundaries. The derivation of a detailed phenomenological theory incorporating the observed effects would also eliminate some difficulties which arise in experimental studies of these effects, e.g., the difficulties resulting from the appearance of anisotropy in research on the gyrotropy caused by mechanical stress.

There is a perceptible need for effective new (noncomputer) methods for solving boundary-value problems and for refining the existing methods for problems involving many boundaries.

Several interesting new optical effects have recently been predicted,[150,152] and gyrotropy effects occupy an important position among them. The derivation of a theory for light propagation through bounded media incorporating these effects would accelerate their experimental discovery, a matter of much interest.

I am deeply indebted to B. M. Bolotovskiĭ for discussions and advice and to V. M. Agronovich and V. L. Ginzburg for interest in this study and for several useful comments. Discussions with M. I. Kaganov regarding multiple roots in ferromagnets and the dispersion of the magnetic permeability and the advice offered by G. S. Krinchik on magnetooptics and (in particular) the magnetic susceptibility at optical frequencies were extremely useful, and again I am deeply indebted.

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Translated by Dave Parsons.