

K. B. Efetov. *Field-theory description of Anderson localization and the conductivity of two-dimensional disordered metals.* The properties of disordered metals are usually described with the aid of perturbation theory, which is valid at impurity concentrations so low that $\tau\epsilon_0 \gg 1$, where τ is the free path time and ϵ_0 is the Fermi energy. There is a well-developed diagram technique in this limit.¹ The diagrams are constructed from electron and impurity lines. It has become clear in recent years that summation of the standard diagrams is extremely complex for a broad range of problems (conductivity of two-dimensional metals, thick wires, calculation of correlations between levels in a limited volume, etc.) owing to the divergences of the type²

$$\int \frac{d\mathbf{K}}{DK^2 + i\omega} \quad (1)$$

that appear at low frequencies ω .

These divergences result from the existence of diffusion modes that appear formally on summation of a certain infinite sequence of diagrams.

It has been proposed that the above problems be solved by a supersymmetry method that makes it possible to escape the bounds of ordinary perturbation theory. In this method, integration can be carried out at once over the electron lines and the problem can be reduced to investigation of a Lagrangian that describes only the diffusion modes. The basis for all the proposed calculations is description of the interaction of the electrons with a random potential with the aid of the field-theory Lagrangian

$$L = \int \left[-i\bar{\psi} H_0 \psi + \frac{1}{2\pi\nu\tau} (\bar{\psi}\psi)^2 + \frac{i(\omega - i\delta)}{2} \bar{\psi} \Lambda \psi \right] dr. \quad (2)$$

In (2), ψ is an eight-component vector that has boson and fermion components, ν is the density of states, H_0 is the kinetic energy, and

$$\Lambda = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

where the I are unit 4×4 matrices. The Lagrangian L (2) yields the usual perturbation-theory series on expansion in the interaction.¹ The use of both boson and fermion components produces the necessary reduction of the corrections applied to the impurity lines (the boson and fermion loops are of different signs). The Lagrangian L (2) is similar to the Lagrangians that arise, for example, in superconductivity theory. Analysis of (2) indicates that, as in superconductivity, symmetry is spontaneously violated in (2) with the appearance of the averages³

$$Q_{\alpha\beta} = \langle \psi_\alpha \bar{\psi}_\beta \rangle. \quad (3)$$

The matrix Q in (3) contains both boson and fermion elements. Such matrices have come to be known as supermatrices. The definition and properties can be found in the review in Ref. 4.

The ground state of the Lagrangian (2) at $\omega=0$ is strongly degenerate. Any supermatrix Q of the form

$$Q = \bar{u}_\lambda u, \quad \bar{u}u = 1 \quad (4)$$

satisfies the self-consistency condition (3). Degeneracy of (4) results in the appearance of Goldstone (at $\omega=0$) excitations. These excitations are described by the effective Lagrangian

$$L_{\text{eff}} = \frac{\pi\nu}{8} \int \text{SSP} [D(\nabla Q)^2 - 2i(\omega - i\delta)\Lambda Q] dr. \quad (5)$$

See Ref. 4 for definition of the supertrace SSP. The effective Lagrangian L_{eff} is a generalized nonlinear supersymmetric σ model. The symmetry of the matrix Q may change under the action of magnetic and spin-orbit interactions.

The Lagrangian (5) has now been investigated in two problems:

1. *Conductivity in two-dimensional metals.* In this problem, the gradient in (5) is two-dimensional. The renormalization-group equations that relate the effective diffusion coefficient D_{eff} to frequency were written and solved. It was shown that as the frequency decreases, D_{eff} also decreases for scattering by ordinary and magnetic impurities. If there are no magnetic interactions, but spin-orbit interactions are present, D_{eff} increases.

2. *Correlations between levels in a metal with limited volume.*⁵ This problem corresponds to the zero-dimensional limit in (5). The gradient term can be neglected. Simple integrals are evaluated instead of the functional integrals with the energy (5). It has been shown that the level correlation function depends on the presence or absence of magnetic and spin-orbit interactions and agrees in three possible cases with the results obtained by Dyson⁶ in the phenomenological theory of levels in complex systems for orthogonal, unitary, and symplectic ensembles. The calculation made in Ref. 5 represents the first direct verification of Dyson's statistical hypotheses.

¹A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Metody kvantovoi teorii polya v statisticheskoi fizike* (Methods of Quantum Field Theory in Statistical Physics). Fizmatgiz, Moscow, 1963.

²L. P. Gor'kov, A. I. Larkin, and D. E. Khmel'nitskii, *Pisma Zh. Teor. Fiz.* **30**, 248 (1979) [*JETP Lett.* **30**, 228 (1979)].

³K. B. Efetov, *Zh. Eksp. Teor. Fiz.* **82**, 872 (1982) [*Sov. Phys. JETP* **55**, 514 (1982)].

⁴F. A. Berezin, *Yad. Fiz.* **29**, 1670 (1979) [*Sov. J. Nucl. Phys.* **29**, 857 (1979)].

⁵K. B. Efetov, *Zh. Eksp. Teor. Fiz.* (1982) [*sic*].

⁶F. J. Dyson, *Statistical Theory of the Energy Levels of Complex systems*: *J. of Math. Phys. (N.Y.)* **3**, (I) 140-156, (II) 157-165, (III) 166-175 (Jan.-Feb. 1962) [*Russ. translation, IL, Moscow* (1963)].

N. F. Shul'ga. *Development of electrodynamic processes in space and time at high energy.* Several new ef-

fects that may unfold in the interaction of high-energy particles with matter have recently been predicted.