B. I. Ivlev and N. B. Kopnin. *Theory of resistive states in narrow superconducting channels.* Consider a narrow superconducting channel with transverse dimensions smaller than the depth of penetration of the magnetic field and the coherence length $\xi(T)$. The Ginzburg-Landau theory predicts that when the current density j through this channel exceeds a value j_c, which is called the critical Ginzburg-Landau current, superconductivity failure occurs in the channel and the channel transfers to the normal state. However, experiment indicates that there is a range of currents above j_c where superconductivity does not vanish totally but continues to exist in spite of the fact that the specimen offers nonzero resistance to direct current.[1,2] This state of the superconductor is called the resistive state (RS). An electric field that accelerates electrically charged Cooper pairs exists in a superconductor in the RS. The increase in the velocity $v_s=(\hbar/2m)\nabla\chi$ of the Cooper pairs, where χ is the phase of the order parameter, is related to the increase in the difference $\chi_2-\chi_1$ of the phases at the ends of the superconducting channel. Since superconductivity does not fail in the RS, this implies the existence of a mechanism that offsets the increase of the phase difference under the action of the electric field. Phase slip is such a mechanism. It consists essentially of the following. The absolute value of the order parameter Δ begins to decrease at a certain place in the specimen, and at a certain time it vanishes. At this time, the difference between the phases to the right and left of this point changes stepwise by 2π, after which superconductivity is restored at this point and the order parameter begins to increase. The process is repeated after a certain time. The point at which the order parameter vanishes as it oscillates and its phase experiences the 2π jump is called the phase-slip center[3] (PSC). If the specimen is long enough, it may contain several PSCs. In the limit of an infinitely long specimen, PSCs occur periodically along its length with a certain density that depends on the current.

The phase-reversal process desscribed above is conveniently analyzed in two-dimensional space-time $\{x, ct\}$, where x is the coordinate along the specimen. We introduce the two-dimensional vectors

$$\boldsymbol{\rho} = \{x; ct\}, \quad \mathbf{q} = \{Q_x; -\Phi\}, \quad \mathbf{a} = \{A_x; -\varphi\},$$

where

$$Q_x = A_x - \frac{\hbar c}{2e}\frac{\partial\chi}{\partial x} \text{ and } \Phi = \varphi + \frac{\hbar}{2e}\frac{\partial\chi}{\partial t}$$

are gradient-invariant vector and scalar potentials (A and φ are the ordinary electromagnetic potentials). The vectors $\mathbf{q}$ and $\mathbf{a}$ are related by $\mathbf{q} = \mathbf{a} - (\hbar c/2e)\,\partial\chi/\partial\boldsymbol{\rho}$. The phase-slip time is represented by a point $\{x; ct\}$ in space-time. Let us surround this point with a closed contour l and consider the integral

$$\oint_l \mathbf{q}\,d\boldsymbol{\rho} = \oint_l \mathbf{a}\,d\boldsymbol{\rho} - \frac{\hbar c}{2e}\oint_l \frac{\partial\chi}{\partial\boldsymbol{\rho}}\,d\boldsymbol{\rho}. \tag{1}$$

The phase-reversal process is equivalent to the condition that the order-parameter phase change by $2\pi n$, where n is an integer, as we move around this point. Using the definition of the electric field

$$E = -\frac{1}{c}\frac{\partial A}{\partial t} - \frac{\partial\varphi}{\partial x} = (\mathrm{rot}\ \mathbf{a})_3$$

we obtain a "quantization rule"[4] from (1):

$$\int_S E\,dx\,c\,dt = \varphi_0 n.$$

where $\varphi_0 = \pi\hbar c/e$ is the quantum of the flux, and the integration extends over the area occupied by a single PSC in the space $\{x, ct\}$. This expression is a generalized Josephson relation.

It is necessary to resort to the dynamic equations of superconductivity to determine the structure of the PSC and to calculate the volt-ampere characteristic (VAC) of the specimen. Another characteristic length appears here: the penetration depth l_E of the electric field,[5,6] which is the relaxation length of the difference between the chemical potential $\mu_e = -e\varphi$ of the normal excitations and the chemical potential of the Cooper pairs $\mu_r = (\hbar/2)\,\partial\chi/\partial t$: $\Phi = (\mu_r - \mu_e)/e$. The penetration depth l_E was calculated in Refs. 5 and 6 for temperatures near T_c. It is considerably greater than $\xi(T)$.

The distance L between successive PSCs in the RS is of the order of magnitude of the electric-field penetration depth: $L \sim l_E$. Analysis of the dynamic equations of superconductivity indicates[7] that three segments can be distinguished on the interval $-L/2 \leq x \leq L/2$ (the point $x = 0$ corresponds to the PSC):

1. $|x| \leq x_1 \sim \xi^2/l_E$—a range of oscillations of the order parameter Δ. It is significantly suppressed in this range as compared to the equilibrium value Δ_0.

2. $\xi^2/l_E \leq |x| \leq x_2 \sim \sqrt{\xi l_E}$. The superconducting current j_s oscillates in this range. The order parameter does not depend on time and takes the form $\Delta(x) = \Delta_0 \tanh(x/\sqrt{2}\xi)$. The lengths of the dynamic regions x_1 and x_2 are considerably smaller than L: $x_1, x_2 \ll L$.

3. $\sqrt{\xi l_E} \leq |x| \leq L/2 \sim l_E$. None of the quantities depends on time in this range. Here a voltage forms at the PSC. Because of the narrowness of the dynamic ranges x_1 and x_2, the main contributions to the potential drop on the PSC comes from the static region, in which the simple equations

$$j = \sigma E + j_s, \quad E = -\frac{\partial\Phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2\Phi}{\partial x^2} = \frac{1}{l_E^2(\Delta)}\Phi, \tag{3}$$

hold and the superconducting current j_s is related to Δ by the Ginzburg-Landau equation: $j_s = j_s(\Delta)$. Equations (2) and (3) are easily integrated:

$$\Phi^2(j_s) = \frac{2}{\sigma^2}\int_{j_s}^{j_c} l_E^2(\Delta)(j - j_s)\,dj_s, \quad L = \frac{2}{\sigma}\int_0^{j_c} l_E^2(\Delta)\frac{dj_s}{\Phi(j_s)}, \tag{4}$$

from which we determine the average electric field in the specimen[7,8]

$$E = \frac{2\Phi}{L}; \quad j_s = 0. \tag{5}$$

The VAC (4), (5) was obtained for a long channel with a large number of PSCs. At large currents $j \gg j_c$, it runs parallel to Ohm's law, $j = \sigma E + j_{exc}$ with an excess current $j_{exc} = 0.68 j_c$. The initial VAC segment is described by the relation

$$j - j_c = j_c \exp\left(-0.91\frac{j_c}{\sigma E}\right).$$

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