Phase conjugation in stimulated scattering

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Various aspects of optical phase conjugation are discussed: the properties of the conjugate wave, its potential applications, the basic conjugation methods, and a brief history of the question. The theory for phase conjugation in stimulated scattering is set forth in detail. The basic experimental results on this conjugation method are also discussed in detail. Phase conjugation occurs because that configuration of the back-scattered field which has the conjugate wavefront is amplified to the greatest extent (at a doubled gain) in the intense speckle-inhomogeneous conjugate wave in a medium in which stimulated scattering occurs. Because of the large overall amplification in stimulated scattering, all the other, uncorrelated, configurations of the spontaneously scattered nucleating waves are amplified by a factor of 10^7 less and are discriminated against. The intervals of values of the various parameters in which the conjugate configuration (the specklon) exists are discussed theoretically, as is the effect of nonlinear selection and saturation on phase conjugation in stimulated scattering. There is a review of experimental results on the first observation of the effect, on the measurement of the angular structure of the uncorrelated waves and of the extent to which they are discriminated against, on the phase fluctuations of the conjugate wave, on the conjugation of subthreshold and depolarized radiation, and on phase conjugation in stimulated scattering in focused beams and for other scattered-wave amplification mechanisms.

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CONTENTS

1. ELEMENTARY INTRODUCTION

a) Properties of the conjugate wave and its potential applications

The past three or four years have seen an avalanche of studies in a new field in optics: phase conjugation or "wavefront reversal." The reason for the interest in phase conjugation (PC) is the recognition of its extremely attractive potential applications to many important scientific and technological problems. Among these applications are guiding laser beams to small targets for laser controlled fusion, transporting electromagnetic radiation through a turbulent atmosphere or through optical systems afflicted by aberrations without degradation of directionality, and producing highly directional laser beams through the use of (relatively inexpensive) optically inhomogeneous elements (there are many other potential applications).

To give the reader an idea of what a conjugate wave is we will use the following gedanken experiment (which, has actually been performed by many investigators). It is a bright, sunny day in Sochi. A charming young woman is preparing to dive from a high board. On the shore, a tired old man is recording the event with a movie camera. The young woman jumps from the board and flies down; the previously smooth surface of the water becomes covered with waves, water is splashed up, and the young woman disappears into the water. A few days later, the old man looks at the frames on the film and likes them so much he runs the scene over and over again. What does he see when he runs the film backward? The waves created by the dive converge on a central point; all the water which had been splashed out, down to the smallest drop, goes back to the same place; the tanned and shapely body flies back up to the high board; and the surface of the water smoothes out

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again. The movie film has made it possible to run time backwards.

If we were able to record the propagation of a laser beam from the side on movie film, we would see much the same thing. The initial, highly directional beam with a smooth wavefront is scattered ("splashed") in various directions as it passes through an inhomogeneous medium. The beam wavefront becomes distorted and is no longer smooth. If we ran this unusual film backwards, we would see a scene we don't get to view in our everyday lives. As a highly nonuniform beam passes through an inhomogeneous medium, it is not distorted further; on the contrary, the distortions are corrected, and the beam is restored to its initial highly directional form with a smooth wavefront.

Is it possible in reality, as well as on film, to arrange time reversal of a motion? (We hope you saw the quote from Rabelais at the beginning of the text.) For an object which creates waves and splashes water by its fall, this time reversal would be impossible in principle, because of viscosity, turbulence, etc., in other words, because of the important role played by irreversible dissipative processes. We conclude that time reversal of the motion is impossible in this case even without taking up the practical difficulties of specifying exactly reverse directions and magnitudes for the velocities of all the particles involved in the motion.

The situation is different in the optics of transparent media, where the laws governing the propagation of light waves are reversible in the cases of most interest. Accordingly, even in inhomogeneous media there should exist "reversed" or "conjugate" waves which reproduce this picture of a time-reversed propagation of the original wave. There is, however, a second question to be asked here. How do we produce this conjugate wave? As it turns out, for a sufficiently intense and coherent beam there are many different ways to produce a conjugate wave.

In terms of monochromatic waves of the type

$$\mathscr{E}_{\mathrm{con}}(\mathbf{R}, t) = |E(\mathbf{R})| \cos(\omega t - kz - \varphi(\mathbf{R})),$$

where z is the central propagation direction, and $\varphi(\mathbf{R})$ is the phase of the field, the conjugate wave corresponds to the replacement $t \to -t$, i.e.,

$$\mathscr{E}_{\mathrm{real}}(\mathbf{R}, t) = |E(\mathbf{R})| \cos(-\omega t - kz - \varphi(\mathbf{R})) \equiv |E(\mathbf{R})| \cos(\omega t + kz + \varphi(\mathbf{R})).$$

In terms of the complex amplitude $E(\mathbf{R})$, introduced by the definition

$$E(\mathbf{R}, t) = \frac{1}{2}[E(\mathbf{R}) e^{-i\omega t} + E^*(\mathbf{R}) e^{i\omega t}], \qquad (1)$$

time reversal is, as we see from (1), equivalent to the replacement of the complex wave amplitude $E(\mathbf{R})$ by its complex conjugate, $E^*(\mathbf{R})$. There is a crucial point here: The temporal reversibility of the system and the reversibility of the paths of all the rays are consequences of the fact that the field $E^*(\mathbf{R})$ is also a solution of the wave equation, even with all possible inhomogeneities being taken into account.

For a plane wave $E(\mathbf{R}) = \exp(i\mathbf{kR} + i\varphi_0)$ with $\varphi_0 = \mathrm{const}$, complex conjugation corresponds to the switch to the

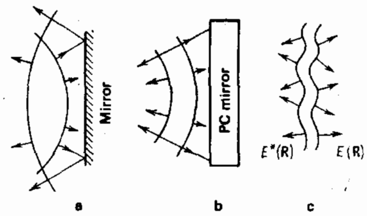

FIG. 1. a—Reflection of a diverging beam by an ordinary mirror. In both the incident and reflected waves, the phase of the central part of the beam is lagging behind; b—reflection of a diverging beam by a mirror which reverses the wavefront: The central part of the reflected beam lags behind the peripheral part in phase, and the beam becomes a converging beam; c—the counterpropagating direct wave $E(\mathbf{R})$ and conjugate wave $E^*(\mathbf{R})$ always have coincident constant-phase surfaces, i.e., wavefront surfaces.

wave $\exp(-i\mathbf{kR} - i\varphi_0)$, which is propagating in precisely the opposite direction. For a wave with a complicated spatial structure, $E(\mathbf{R}) = |A(\mathbf{R})| \exp[i\mathbf{kR} + i\varphi(\mathbf{R})]$, the conjugate wave $E^*(\mathbf{R}) = |A(\mathbf{R})| \exp[i\mathbf{kR} - i\varphi(\mathbf{R})]$ has a reversed wavefront: The surface of constant phase has the same shape, but the wave is propagating in the opposite direction (Fig. 1). This is the reason for the name "optical phase conjugation." The effect is also referred to as "time reversal."

We turn now to a situation in which the incident wave $E(\mathbf{R})$ describes the passage of a highly directional beam through an optically inhomogeneous medium, which scatters the beam from its original direction (Fig. 2). The conjugate wave $E'(\mathbf{R}) = E^*(\mathbf{R})$ then converges back into the medium; propagating back through the same inhomogeneities, it is restored to the original highly directional beam. If the medium is an optical amplifying medium, the conjugate wave can remove, in the form of a highly directional beam, energy which was stored in the amplifying medium, despite the optical inhomogeneities.

Another important application of phase conjugation, and one which is being developed rapidly at the moment, is based on the concept of PC self-guiding of radiation to a target. Let us assume that we are required to deliver the beam from a laser system to a small target. The conventional aiming methods impose extremely

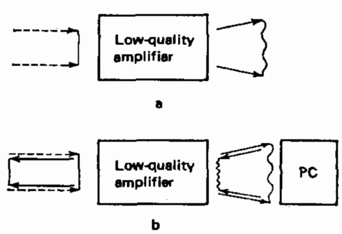

FIG. 2. Use of phase conjugation to cancel phase distortions in inhomogeneous optical amplifiers. a—In single-pass amplification the divergence of a highly directional beam increases, so that the brightness of this beam usually decreases; b—In double-pass amplification with phase conjugation between the two passes, the beam acquires energy from the amplifier without suffering a distortion of its directionality.

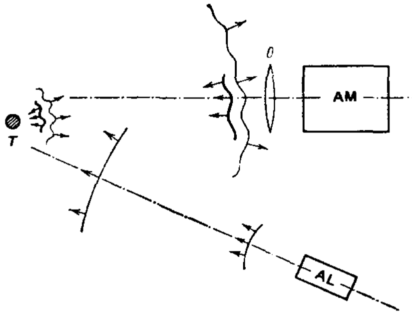

FIG. 3. Arrangement for phase-conjugation self/guidance of a laser system onto a target, T.

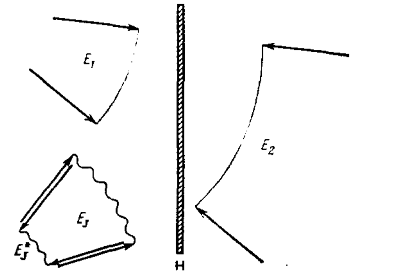

FIG. 4. Arrangement for holographic phase conjugation. A hologram (H) is recorded by a signal wave E_3 and a reference wave E_1. When the reference wave E_2 is applied in the opposite direction to the developed hologram, the result is the wave E_3^*, the conjugate of the signal wave.

stringent requirements on the alignment, aberrations, and optical inhomogeneities of all elements of the system. In high-power laser systems, these requirements are essentially impossible to meet by conventional methods.

In the PC self-guiding situation (Fig. 3) we are to illuminate a target T with the beam, not highly directional, from an auxiliary laser (AL). The secondary waves scattered by the target are caught in part by the objective O of the laser system, passed through the amplifying medium (AM) (and amplified in the process), and sent into a device which performs the phase conjugation. On its return path, the conjugate wave picks up some energy which has been stored in the amplifying medium and strikes the target. It is important to note that the phase inhomogeneities of the amplifying medium, the aberrations of the optical elements, the alignment errors, etc., are automatically cancelled out in the conjugate wave. All the energy is delivered to the target, with a diffraction-level focusing and aiming, regardless of the spatial position of the target and even if there are optical inhomogeneities between the target and the laser.

Yet another possible application of PC is to use laser beams to burn complicated profiles on the surface of a microcircuit during its fabrication: photolithography. In the conventional approach, extremely stringent requirements (in terms of alignment, aberration correction, etc.) are placed on the optical elements through which the intense beam passes. Furthermore, with any type of shaping of the beam by means of an amplitude transparency, most of the energy of an intense beam will be absorbed, and the efficiency of the system will be low. The idea underlying the use of PC in this problem is that all the losses of this type are losses from the low-intensity beam from an auxiliary laser which illuminates the surface to be processed through the appropriate transparency. The secondary waves from the surface are captured by the objective of the high-power laser, amplified, and—after PC—returned to the appropriate points on the surface without any loss of energy. The PC automatically cancels out the inhomogeneities of the laser and the optical errors, as in the case of PC self-guiding.

In this review we will discuss the present theoretical understanding of the physical mechanisms underlying the basic PC methods, and we will review the extensive experimental information which has been obtained. Research on the use of PC methods to solve the practical problems mentioned earlier is still in a beginning stage and thus lies outside the scope of this review.

b) Basic phase-conjugation methods

The holographic methods for phase conjugation are the simplest to understand. We assume that an inhomogeneous wave $E_3(\mathbf{r})$, which is to be conjugated, and a reference wave $E_1(\mathbf{r})$, with an amplitude constant over the cross section, $|E_1(\mathbf{r})|^2=\text{const}$, are incident on a holographic, i.e., photosensitive, medium H (Fig. 4). If the waves $E_1(\mathbf{r})$ and $E_3(\mathbf{r})$ are coherent, they record interference perturbations of the following type in the hologram:

$$\delta\varepsilon(\mathbf{r}) = \text{const}\cdot[E_1^*(\mathbf{r})E_3(\mathbf{r}) + E_1(\mathbf{r})E_3^*(\mathbf{r})]. \tag{2}$$

If, in the readout step, this hologram is illuminated from the opposite side by a wave $E_2(\mathbf{r})$ which is precisely the opposite of the reference wave used in the recording, $E_2(\mathbf{r}) = E_1^*(\mathbf{r})$, the hologram will reconstruct the conjugate wave by virtue of the second term in (2):

$$E_4(\mathbf{r}) = \text{const}\cdot E_1E_2E_3^*(\mathbf{r}) = \text{const}\cdot|E_1|^2E_3^*(\mathbf{r}). \tag{3}$$

The arrangement in Fig. 4 might be modified slightly to have the hologram record the interference of waves E_2 and E_3 and then be read out by the wave E_1; this modification works well for thick-layer reflection holograms.

We are primarily interested in dynamic, i.e., real-time, recording and readout of holograms. The medium for dynamic holography might be a layer of a dye which darkens or becomes more transparent, a photochromic glass, or, in general, any medium in which the complex dielectric permittivity is changed by the light directly during the exposure time. Among these media are some exhibiting a cubic optical nonlinearity.

In nonlinear-optics terms this case can be described as follows. In a nonlinear medium with a cubic response of the polarization P to the field, $\mathbf{E}$, i.e., with $P_i^{NL} = \chi_{iklm}^{(3)}E_kE_lE_m^*$, two counterpropagating plane reference waves $(\mathbf{E}_1e^{ikz} + \mathbf{E}_2e^{-ikz})e^{-i\omega t}$ and a signal wave $\mathbf{E}_3(\mathbf{R})e^{-i\omega t}$ induce in the polarization a term

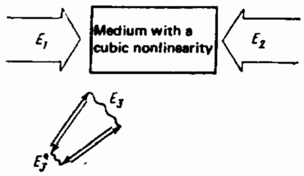

FIG. 5. Arrangement for four-wave phase conjugation.

$$P_i = \chi^{(3)}_{iklm} E_{1_k} E_{2_l} E^*_{3_m}(\mathbf{R})\, e^{-i\omega t}. \tag{4a}$$

This term emits a conjugate wave in a synchronized manner over the entire volume. The customary approach is to use counterpropagating plane reference waves E_1 and E_2 in extended media (Fig. 5). In this approach, two processes occur at the same time: the recording of the hologram $E_1E_3^*$, with readout by wave E_2, and the recording of hologram $E_2E_3^*$, with readout by E_1.

There are four waves in the interaction volume: the three incident waves $E_1(\mathbf{r})$, $E_2(\mathbf{r})$, and $E_3(\mathbf{r})$ and the conjugate wave $E_4(\mathbf{r})$. This PC method is thus also referred to as "four-wave mixing."

If a hologram is recorded by a plane reference wave E_1 and by the signal $E_3(\mathbf{r})$, as a modulation of the reflection coefficient of a plane mirror (Fig. 6), and if the reference wave E_1 is incident strictly along the normal to the mirror, then an auxiliary reference wave E_2 is not required. In this method, "phase conjugation by a reflecting surface," the readout of the dynamic surface reflection hologram is performed by the recording wave E_1 itself.

In crystals lacking an inversion center we know that there is a quadratic term in the dependence of the nonlinear polarization $P(\mathbf{r})$ on the field $E(\mathbf{r})$ at the same point: $P_i = \chi^{(2)}_{ikl}E_kE_l$. If there are fields at the fundamental frequency and at twice this frequency, $E^\omega(r)$ and $E^{2\omega}(r)$, in the medium, the quadratic polarization at the frequency ω is

$$P_i^\omega(\mathbf{r}) = \chi^{(2)}_{ikl} E_k^{2\omega}(\mathbf{r})\,(E_l^\omega(\mathbf{r}))^*. \tag{4b}$$

If the waves E^ω and $E^{2\omega}$ are propagating in the same direction, and if the wave $E^{2\omega}$ is a plane wave with a constant amplitude, then the polarization P^ω in (4) effectively excites a conjugate wave which propagates in the same direction. Upon reflection from a mirror in direct contact with the medium and oriented precisely normal to the direction of the reference wave $E^{2\omega}$, this wave becomes the phase-conjugated wave of interest (Fig. 7).

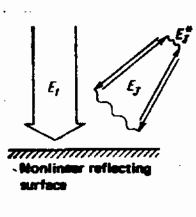

FIG. 6. Arrangement for phase conjugation by means of a reflecting surface.

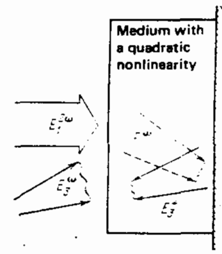

FIG. 7. Arrangement for three-wave phase conjugation.

In practice, all these methods require plane or spherical reference waves of high (diffraction-level) quality and of high intensity. In contrast, the method of phase conjugation in stimulated backscattering (PCSS), the subject of Sections 2 and 3 of this review, does not have this limitation. In PCSS there is a self-reversal of the wavefront. The most common PCSS arrangement is shown in Fig. 8. The radiation which is to be conjugated passes through an inhomogeneous phase plate Φ, which introduces some fine-scale intereference inhomogeneities (i.e., a speckle structure) in the beam. The beam is then sent into a cell holding a transparent liquid, a compressed gas, or a solid. If the original beam is sufficiently intense (above a threshold), stimulated backscattering is excited from the spontaneous-scattering nucleation centers in the medium. As it turns out, the stimulated-scattering amplification of the back-scattered waves in the field of the speckle-inhomogeneous exciting beam singles out that backward-propagating configuration which has the precisely reversed wavefront. We wish to emphasize that this spatial inhomogeneity of the beam exciting the stimulated scattering is of fundamental importance to PCSS. For example, if the stimulated scattering is excited by an unfocused plane wave with a broad cross section, the scattered waves will fill a broad solid angle, set by the experimental geometry, and phase conjugation will not occur. By way of comparison we might note that the holographic methods for phase conjugation are equally good for conjugating a plane signal wave. The physical mechanism underlying phase conjugation in stimulated scattering is decidedly different in nature from that for holographic phase conjugation and is described in Subsection 2a.

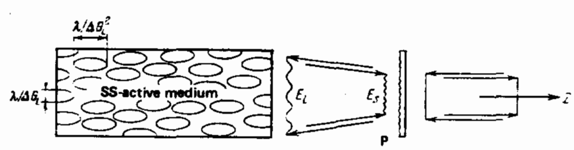

FIG. 8. Arrangement for phase conjugation accompanying stimulated scattering (SS). The beam E_L, to be conjugated, is distorted by the phase plate P and sent into the stimulated-scattering (SS) medium. The distorted beam, with a divergence $\Delta\theta_L$, has a speckel-inhomogeneous structure with longitudinal and transverse inhomogeneity scale dimensions $\lambda/\Delta\theta_L^2$ and $\lambda/\Delta\theta_L$, respectively. Under certain conditions, the back-scattered wave E_S is the conjugate of the incident wave: $E_S \propto E_L^*$.

c) Brief historical review

The existence of complex-conjugate fields upon reconstruction of holograms was mentioned back in 1949 by Dennis Gabor in his pioneering papers proposing the method of holography.[1] In those papers the complex image propagated in the same direction as the fundamental wave (this situation might be compared today with a forward-propagating conjugate field in three-wave mixing). Gabor regarded this conjugate field as noise. In 1950, W. L. Bragg (of equal distinction) pointed out,[2] with reference to Gabor's work, that if a wave which was the conjugate of the reference wave used in the recording was used for readout of a thin hologram two fields would be produced, one precisely the phase-conjugate field (in today's terms). In his 1951 patent application[3] Gabor describes the operating principles of an apparatus in which thin (in today's terms) holograms are used to arrange the selective reconstruction of only that of the two images which corresponds to the conjugate wave.

In a paper laying the groundwork for thin-layer reflection holograms, the Soviet researcer Denisyuk[4] noted that if such a hologram is illuminated with a reconstructing wave which is propagating opposite the wave used for the recording the result would be the reconstruction of only a single image: the conjugate image (i.e., the wavefront-reversed field).

Back in the US, Kogelnik at Bell Laboratories and Leith and Upatnieks,[5,6] pioneers in laser holography, demonstrated the possibility of cancelling optical phase inhomogeneities through the use of the conjugate fields produced during reconstruction from static holograms.[1)]

The idea of self-guiding and cancellation of atmospheric inhomogeneities in real time was suggested by Cathey[8] as well as Kogelnik.[7a]

Independent work in the rf range, we might note, led to the idea of shaping conjugate fields through active control by the elements of a phased antenna array. An extension of the concept of active wavefront control by means of computers to real-time optical problems took the form of the development of the entirely new field of active or adaptive optics.[9] Unfortunately, neither the speed nor the number of resolvable elements is particularly impressive in the adaptive systems which have been developed to date.

Meanwhile, working with dye solutions, Gerrietsen *et al.*[10] experimentally demonstrated (without reference to phase conjugation) that it was possible to record and read dynamic holograms.

[1)] In a patent application[7a], also made in 1965, Kogel'nik raised the possibility of self-guidance to a target by means of phase conjugation with cancellation of time-varying inhomogeneities and through the use of a dynamic holographic medium (a reversibly darkening dye). Sincerbox[7b] suggested a holographic arrangement, now used quite frequently, in which a counterpropagating plane reference wave is produced through the exactly backward reflection of a first plane reference wave by a mirror.

Experiments in an arrangement involving reflection of a first reference wave from a plane mirror in real time were first carried out in 1970. A scientific team from Minsk (Stepanov, Ivakin, and Rubanov[11]) used a dye solution to record holograms; Woerdman (who submitted his paper[12] a few days later) used crystalline silicon for the same purpose. For phase conjugation to be achieved in this arrangement, the reference waves must of course be very accurate conjugates of each other.

Various stimulated-scattering mechaisms were discovered in 1962–1965: stimulated Raman scattering[13] (or "stimulated combinational scattering"), stimulated Brillouin scattering[14] (or "stimulated Mandel'shtam-Brillouin scattering"), and stimulated Rayleigh-line-wing scattering,[15] among others. It was mentioned in several papers that the scattered light is highly directional, often comparable in this regard to the exciting beam (see the papers by Brewer,[16] Rank *et al.*,[17] Bespalov and Kubarev,[18] and Kurdryavtseva, Sokolovskaya, and Sushchinskiĭ[19]).

In some work carried out at the Lebedev Physics Institute in Moscow in 1971, Zel'dovich, Popovichev, Ragul'skiĭ, and Faĭzullov discovered wavefront *self-reversal* accompanying stimulated Brillouin scattering.[20] They clarified the discrimination mechanism for the self-reversal. The phase conjugation in stimulated Brillouin scattering was achieved in a light pipe. Also a part of that study was the first measurement of the fraction of the energy carried by the exact conjugate component in the reflected wave; it turned out to be 1 within the experimental error of 15%. In 1972, Nosach, Popovichev, Ragul'skiĭ, and Faĭzullov[21] experimentally developed a two-pass arrangement for PC cancellation of the optical inhomogeneities of a laser amplifier. All the energy stored in a ruby laser amplifier of poor optical quality was used for a 400 $\times$amplification (in two passes) of a beam which, as a result, retained an ideal diffraction-level divergence.

Phase-conjugation cancellation of the distortions of an amplifier was proposed independently in a theoretical paper by Anan'ev.[22] As the PC method he suggested four-wave mixing by virtue of the thermal nonlinearity of a medium.[23]

Among the studies[24-27] in 1973–76, we wish to single out the following for special mention: Yariv[26] proposed a three-wave forward field-conjugation arrangement in which a reference wave at the doubled frequency is used to cancel mode dispersion in optical light pipes.

In an important study by Hellwarth[27] in 1977, the four-wave principle of PC was rediscovered in nonlinear-optics terms. In the first place, that study raised the possibility of using for phase conjugation by four-wave mixing (PC/FWM) any nonlinear medium, including media which had not previously been considered for dynamic holography. In addition, that study strongly motivated further research on PC, primarily outside the USSR. The first of the experiments on PC/FWM stimulated by Ref. 27 were carried out[28,29] in 1977.

In Refs. 30 and 31, PC/FWM was first achieved for a beam focused into an unbounded medium, rather than in a light pipe. Further research on PC was greatly stimulated by Wang and Giuliano[32] (see also their 1975 patent application[33]), who again took up the question of PC self-guidance through optically inhomogeneous media. Experimental implementation of three-wave backward PC was first reported in Refs. 34 and 35.

The theory of coupled waves was first applied to PC/FWM by Yariv[36] and, for saturable absorbers, by Abrams and Lind.[37] The theory of PC/FWM also developed rapidly in 1976–1978 (Refs. 38–47). Bergmann *et al.*[48] were the first to achieve PC/FWM with the beam from a pulsed CO_2 laser, and they were the first to achieve in-resonator PC/FWM.

Experiments were carried out on PC accompanying stimulated Raman scattering of light,[49,50] PC by a photon-echo method,[51] and PC accompanying superluminescence.[52]

In 1980, PC by means of a reflecting surface was proposed[53] and achieved experimentally[54] in studies by Pilipetskiĭ, Sudarkin, Shkunov, *et al.* It has recently been suggested[114] that this method might be used for the conjugation of acoustic waves.

Detailed studies of a Brillouin-scattering-PC method for cancelling the distortions in high-power laser beams have been carried out by Basov, Subarev, and their colleagues[56] and by Kormer and his colleagues.[57] Their studies also dealt with the problem of laser controlled fusion. An experimental implementation of PC self-guidance or self-focusing was reported in Ref. 58. In a study by Ilyukhin *et al.*,[59] stimulated-Brillouin PC focusing of light into a small spot on a target was first used for research in the physics of laser plasmas.

Since research on PC is still being actively pursued, we cannot mention all the studies, or even the most important of them. A collection edited by Bespalov[60] gives a fairly comprehensive bibliography of the papers on PC published up to 1979. That collection also contains references to the earlier work on PC in the rf range, although the rf work had little effect on developments in the optical range. Unfortunately, the work on PC by means of static and dynamic holography over the years 1950–1970 is not included in the bibliography in Ref. 60.

2. THEORY OF PHASE CONJUGATION IN STIMULATED SCATTERING (PCSS)

a) Physical mechanism for PCSS

We will begin by recalling the basic ideas of stimulated scattering (see Refs. 61–63, for example). We assume that two light waves, an exciting wave $E_L(\mathbf{r})e^{-i\omega_L t}$ and a signal wave $E_S e^{-i\omega_S t}$, are propagating in a medium, and we assume that the difference between their frequencies, $\Omega \equiv \omega_L - \omega_S$, is approximately equal to the frequency of a natural mode of the medium, Ω_0. The interference term $E_L^* E_S e^{i\Omega t}$ in the intensity of the resultant optical field drives this natural oscillation mode. In the case of stimulated Brillouin scattering, the oscillations are driven by the electrostrictive force $\mathbf{f} \propto \nabla(|E|^2 \partial\varepsilon/\partial p)$. A factor involved in the Raman scattering is the dependence of the interaction energy $\alpha|E|^2$ on the molecular coordinate Q: $\partial\alpha/\partial Q \neq 0$, where α is the polarizability of the molecule. As a result, a traveling space-time dielectric permittivity grating is produced in the volume of the medium. At resonance, $\Omega = \Omega_0$, this grating is phase-shifted by 90°: $\delta\varepsilon(\mathbf{r}, t) = -iAE_L^* E_S e^{i\Omega t}$. Scattering of the exciting light by this grating gives rise to a term $\delta D_S = -iA|E_L(\mathbf{r})|^2 E_S(\mathbf{r}) e^{-i\omega_S t}$ in the electric displacement; this term agrees in frequency and propagation direction with the signal term. The dielectric permittivity for the the signal thus acquires a negative imaginary part. The corresponding gain $g(\mathbf{r})$ (in reciprocal centimeters) is proportional to the local intensity of the wave E_L: $g = G|E_L|^2$.

The stimulated scattering usually develops from a very low level of spontaneous scattering, which serves as a signal-wave source. The threshold for the detection of stimulated scattering is reached at a total gain $\exp(G|E_L|^2 l) \sim \exp(25) \approx 10^{11}$, and the corresponding intensity is called the "threshold intensity." Stimulated scattering is typically observed under saturation conditions, which are reached at an overall gain $\exp(G|E_L|^2 l) \sim e^{30} \approx 10^{13}$. Here G has the dimensions of centimeters per megawatt if $|E_L|^2$ is in megawatts per square centimeter and l (the length of the gain region) in centimeters. When this threshold condition is satisfied, the intensity of the amplified spontaneous noise grows exponentially to a level comparable to that of the scattered wave. For phase conjugation the most common approach is to use stimulated Brillouin back scattering, for which we have the rather large values $G \sim 10^{-1}$–10^{-2} cm/MW, a short pumping time $\tau_S \sim 10^{-8}$–10^{-9} s, and a small frequency shift $\nu_0 = \Omega_0/2\pi c \lesssim 1$–$10^{-2}$ cm^{-1}.

The physical mechanism for PCSS is based on two fundamental properties of stimulated scattering: the large total gain and the relationship between the local gain and the local intensity of the exciting field. Because of the exponential gain, even a modest increase in the effective growth rate, $\Delta g/g \sim 1$, can cause a radical change in the process. Phase conjugation through stimulated scattering occurs when the spatial intensity distribution $|E_L(\mathbf{r})|^2$ and thus the local Stokes-wave gain $g(\mathbf{r}) = G|E_L(\mathbf{r})|^2$ have large spatial inhomogeneities.[20] The exciting radiation with the irregular part of the divergence θ_0 has a speckle structure with a typical transverse dimension of order $\Delta r_\perp \sim \lambda\theta_0^{-1}$ for the inhomogeneities and with a typical distance $\Delta z \sim \lambda\theta_0^{-2}$ (λ is the wavelength) over which the light is diffracted from one inhomogeneity to another. Under the conditions required for phase conjugation, these dimensions must be smaller than the corresponding dimensions of the interaction volume (Fig. 8). The effective gain for this configuration of the scattered field, $E_S(\mathbf{r}, z)$, in each cross section $z = \text{const}$ is determined by an intensity overlap integral[20]:

$$g_{\text{eff}} = G\int |E_L(\mathbf{r}, z)|^2 |E_S(\mathbf{r}, z)|^2 d^2\mathbf{r} \left(\int |E_S(\mathbf{r}, z)|^2 d^2\mathbf{r}\right)^{-1}. \tag{5}$$

The gain is thus largest for that wave $E_S(\mathbf{r})$ whose local maxima everywhere coincide spatially with the maxima of E_L. The scattered wave with a profile $|E_S(\mathbf{r}, z)|$

$= \mathrm{const}\cdot |_{\mathrm{L}}(\mathbf{r}, z)$, whose local maxima correspond to gain maxima in each cross section z, has an effective gain g_{eff} roughly twice as large as that for any other E_{S} configuration, which is uncorrelated with E_{L}. In the course of diffraction, both fields $E_{\mathrm{L}}(\mathbf{r}, z)$ and $E_{\mathrm{S}}(\mathbf{r}, z)$ undergo structural changes. A natural possibility for keeping the two nonuniform fields matched as they propagate in opposite directions lies in the circumstance that the scattered field $E_{\mathrm{S}}(\mathbf{r}, z)$ is the conjugate of $E_{\mathrm{L}}(\mathbf{r}, z)$; i.e., $E_{\mathrm{S}}(\mathbf{r}, z) \propto E_{\mathrm{L}}^*(\mathbf{r}, z)$. The field $E_{\mathrm{S}}(\mathbf{r}, z)$ is the time-reversed solution with respect to the field $E_{\mathrm{L}}(\mathbf{r}, z)$, and the interference peaks of these two fields coincide throughout the volume.

The name "specklon" might be applied to this type of speckle-inhomogeneous field configuration, which is matched with the inhomogeneity of the medium. An important characteristic of the specklon is that its fine-scale structural inhomogeneities diffract in accordance with the laws describing free propagation in a homogeneous medium, and its spatial evolution reduces to a change in the smooth envelope. The term "mode," introduced in the pioneering papers by Sidorovich,[64,39] is also used for a similar concept of speckle-inhomogeneous solutions with an envelope which varies exponentially along the longitudinal coordinate. At the moment we are in the middle of a lively discussion regarding the relative convenience of the terms "mode," "specklon," and any others.

Among the initial spontaneous scattered-field nucleation centers there are both the conjugate (reversed) configuration and a large number of uncorrelated waves. When the PC configuration $E_{\mathrm{S}} \propto E_{\mathrm{L}}^*$ acquires a total gain of e^{30}, an uncorrelated wave traveling the same distance is amplified by a factor of only e^{15}. It is this discrimination against the uncorrelated solutions which is responsible for the fact that only the PC component of the scattered field has survived at the exit from the nonlinear medium.

The physical mechanism for the PCSS is thus a selective amplification of the conjugate configuration of the scattered field, which occurs because the local maxima of the speckle structure of this configuration coincide with local maxima of the gain throughout the interaction volume.

b) Concept of the specklon

An extremely important aspect of the physical mechanism for PCSS is the rapid mixing of the transverse speckle structure of the two fields in the course of propagation and diffraction (Fig. 8). The speckle-inhomogeneous fields result from the interference of a large number of independent waves; by virtue of the central-limit theorem of probability theory, the speckle field has the Gaussian statistics of a complex random process (cf. Refs. 65 and 66). The scattered wave thus propagates in a medium with a nonuniform gain profile. The diffraction of the highly directional fields of this type,

$$\begin{aligned} E_{\mathrm{L}}^{\mathrm{real}}(\mathbf{r}, z, t) &= 0.5\,[E_{\mathrm{L}}(\mathbf{r}, z)\, e^{-ik_{\mathrm{L}}z - i\omega_{\mathrm{L}}t} + \mathrm{c.c.}], \\ E_{\mathrm{S}}^{\mathrm{real}}(\mathbf{r}, z, t) &= 0.5\,[E_{\mathrm{S}}(\mathbf{r}, z)\, e^{ik_{\mathrm{S}}z - i\omega_{\mathrm{S}}t} + \mathrm{c.c.}], \end{aligned} \tag{6}$$

is described accurately by the parabolic equations which were used for the PCSS problem in Ref. 20:

$$\frac{\partial E_{\mathrm{L}}}{\partial z} - \frac{i}{2k_{\mathrm{L}}} \Delta_{\perp} E_{\mathrm{L}}(\mathbf{r}, z) = 0, \tag{7}$$

$$\frac{\partial E_{\mathrm{S}}}{\partial z} - \frac{i}{2k_{\mathrm{S}}} \Delta_{\perp} E_{\mathrm{S}}(\mathbf{r}, z) = \frac{1}{2} G\, |E_{\mathrm{L}}(\mathbf{r}, z)|^2 E_{\mathrm{S}}(\mathbf{r}, z); \tag{8}$$

here $\mathbf{r} = (x, y)$, $\Delta_{\perp} = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$, $k_{\mathrm{L,S}} = \omega_{\mathrm{L,S}}\sqrt{\varepsilon}/c$ are the wave numbers, and the pump wave E_{L} is assumed to be propagating along the $-z$ direction and the Stokes wave E_{S} along the $+z$ direction. For stimulated Brillouin scattering was have $k_{\mathrm{L}} - k_{\mathrm{S}} \leq 10^{-5} k_{\mathrm{L}}$, so that we may set $k_{\mathrm{L}} = k_{\mathrm{S}} = k$.

In the absence of amplification ($G = 0$), Eq. (8) is the same as the complex-conjugate equation in (7), so that the solution $E_{\mathrm{L}}(\mathbf{r}, z)$ for the pump field can always be related to the conjugate field $E_{\mathrm{S}}(\mathbf{r}, z) = \mathrm{const}\cdot E_{\mathrm{L}}^*(\mathbf{r}, z)$, which is the exact solution of Eq. (8) for $G = 0$ and $k_{\mathrm{L}} = k_{\mathrm{S}}$. If the gain $G\,|E_{\mathrm{L}}(\mathbf{r}, z)|^2 k_{\mathrm{L}} = k_{\mathrm{S}}$ is nonuniform in the transverse direction, the conjugate wave cannot be an *exact* solution of Eq. (8). For the problems in which we are interested, however, the effect of the inhomogeneities is in a sense *averaged out* by virtue of the small longitudinal scale dimensions of the changes in these inhomogeneities, and a conjugate wave $E_{\mathrm{S}}(\mathbf{r}, z) = s(z) E_{\mathrm{L}}^*(\mathbf{r}, z)$ remains an approximate solution of Eq. (8). To illustrate the situation, we write the right side of Eq. (8) as

$$|E_{\mathrm{L}}(\mathbf{r}, z)|^2 E_{\mathrm{S}}(\mathbf{r}, z) \equiv \langle E_{\mathrm{L}} E_{\mathrm{L}}^* \rangle E_{\mathrm{S}} + \langle E_{\mathrm{L}} E_{\mathrm{S}} \rangle E_{\mathrm{L}}^* + \langle E_{\mathrm{L}}^* E_{\mathrm{S}} \rangle E_{\mathrm{L}} + F(\mathbf{r}, z); \tag{9}$$

the angle brackets denote an average over the ensemble of realizations of the speckle inhomogeneities. This averaging is equivalent to averaging over the transverse cross section. If the scattered field is strongly correlated with the pump field, $E_{\mathrm{S}}(\mathbf{r}, z) = s(z) E_{\mathrm{L}}^*(\mathbf{r}, z)$, then we have $\langle E_{\mathrm{L}} E_{\mathrm{S}} \rangle = s(z) \langle |E_{\mathrm{L}}|^2 \rangle$. For counterpropagating fields E_{L} and E_{S}, the correlation function $\langle E_{\mathrm{L}}^* E_{\mathrm{S}} \rangle$ is essentially always zero. If we ignore the randomly modulated term $F(\mathbf{r}, z)$ we find from (9)

$$E_{\mathrm{L}}^*(\mathbf{r}, z)\frac{ds(z)}{dz} = g s(z) E_{\mathrm{L}}^*(\mathbf{r}, z), \qquad |s(z)|^2 = \mathrm{const}\cdot e^{2gz}; \tag{10}$$

here $g = G\langle |E_{\mathrm{L}}|^2 \rangle$ is the spatial average of the gain (the intensity gain, per centimeter). The conjugate wave is thus amplified with a *doubled* gain. This result—the Gaussian 2 in the argument of the exponential function for the conjugate solution—was first derived in Refs. 38 and 39. The legitimacy of ignoring the spatially nonresonant term $F(\mathbf{r}, z)$ from (9) was established in Refs. 39, 43, and 67; we will return to this point in the following subsection.

In contrast to the general problem of light propagation in inhomogeneous media, the inhomogeneities of the medium in the present case are induced by the interference pattern of the speckle-inhomogeneous field $|E_{\mathrm{L}}(\mathbf{r}, z)|^2$. When light is diffracted in such media, there exists a special wave $E_{\mathrm{S}}(\mathbf{r}, z)$ which we will refer to as a *specklon.* The spatial structure of this wave E_{S} is matched with the inhomogeneities of the medium: $E_{\mathrm{S}}(\mathbf{r}, z) = s(z) E_{\mathrm{L}}^*(\mathbf{r}, z)$. The diffraction of the specklon by the inhomogeneities does not distrupt this matching. As a result, the specklon is in spatial resonance with the inhomogeneities of the medium, $|E_{\mathrm{L}}(\mathbf{r}, z)|^2$. This spatial resonance intensifies the interaction of the specklon with the inhomogeneities (specifically, it doubles the gain for PCSS). In the case of copropagation of the specklon E_S and the inhomogeneous recording field E_L, these two fields are related by $E_S(\mathbf{r},z) = s(z)E_L(\mathbf{r},z)$; i.e., the structure is reproduced without complex conjugation. The concept of the specklon and the methods of the speckle-mode theory have proved extremely successful in problems in volume holography[64,68,70] amplification in saturable media,[71,72,117] phase conjugation by four-wave mixing,[73-75] and many other problems involving the interaction of speckle-inhomogeneous fields.

c) Theory of spiral distortions (or noise)

In the speckle-inhomogeneous pump field $E_L(\mathbf{r},z)$ (which accordingly has Gaussian statistics) the variations in the gain, $\delta g(\mathbf{r},z) = G[|E_L(\mathbf{r},z)|^2 - \langle |E_L|^2\rangle]$, are of the order of the gain itself, $g = G\langle |E_L|^2\rangle$. In a transverse cross section, the regions with the high gain $G|E_L(\mathbf{r})|^2$ are randomly distributed spots. Since the energy flux of the laser field is conserved, these spots trace out curved, spiral current tubes in the course of the propagation. These tubes are interlaced with each other in a complicated way. With respect to the Stokes field E_S, such a spiral is a curvilinear amplifying microwaveguide. The transverse scale dimension of the speckle spots is determined by the uncertainty relation: $\Delta r_\perp \sim \lambda/\Delta\theta_L$, where λ is the wavelength and $\Delta\theta_L$ is the irregular part of the pump divergence. The typical longitudinal length of one bend of the spiral (the Fresnel length l_{Fr}) is thus $l_{Fr} \sim \Delta r_\perp/\Delta\theta_L \sim \lambda/(\Delta\theta_L)^2$.

The gain over the length of one bend of the spiral is $\exp(gl_{Fr})$. If this gain is large enough, $gl_{Fr} \gtrsim 1$, the scattered wave is "trapped" in the core of the spiral microwaveguide. At the edges of the microwaveguide the scattered field is much weaker than in its core; the relationship $E_S(r,z) = sE_L^*(r,z)$ is disrupted after amplification over a distance no greater than the length of a single bend of the spiral[43] l_{Fr}.

The most favorable case for PCSS is the opposite one: $gl_{Fr} \ll 1$. In this case the relative *amplitude* of the distortion over the length of one bend is a small quantity of order gl_{Fr}. By virtue of the random nature of these distortions, their buildup from the various layers, each of thickness l_{Fr}, occurs in an incoherent way, i.e., in terms of the intensity. Over a distance $L \ll l_{Fr}$ the relative *intensity* of the accumulated distortions is thus $(L/l_{Fr})(gl_{Fr})^2 = RL$, where we have introduced the extinction coefficient $R = g^2 l_{Fr}$ (where R is in reciprocal centimeters), which is a measure of the rate at which the distortions are excited as the specklon propagates through the inhomogeneous-amplification medium. The quantitative calculations of this coefficient[43,67,76] have been based on the effect of the spatially nonresonant term $F(\mathbf{r},z)$ in (9), which is discarded in the first-approximation treatment of the specklon. As a result we find

$$R = \frac{\pi |g|^2}{k\Delta\theta_{\rm eff}^2}, \tag{11a}$$

$$\frac{1}{\Delta\theta_{\rm eff}^2} = \int\int\int j(\theta_1)\, j(\theta_2)\, j(\theta_3)\, \delta[(\theta_1-\theta_2)(\theta_1-\theta_3)]\, d^2\theta_1 d^2\theta_2 d^2\theta_3, \tag{11b}$$

where $j(\theta_x, \theta_y)$ is the normalized angular power spectrum of the pump $E_L(\mathbf{r},z)$. We have deliberately introduced the magnitude symbol for g in (11a), since this result is also correct when the wave E_L induces refraction inhomogeneities in a medium (in a volume phase hologram, for example, in which $g = i\varkappa|E_L|^2$). In order of magnitude, $\Delta\theta_{\rm eff}$ agrees with the width of the E_L angular spectrum. For a parabolic angular spectrum $j(\theta) \propto [1-(\theta^2/\theta_0^2)]\,\mathrm{sign}[1-(\theta^2/\theta_0^2)]$, evaluation of the integral in (11b) yields

$$\frac{1}{\Delta\theta_{\rm eff}^2} = \frac{8}{3\pi}\frac{1}{\theta_0^2} \approx \frac{1{,}7}{\beta_{1/2}^2}. \tag{11c}$$

For a Gaussian angular spectrum, $j(\theta) \propto \exp(-\theta^2/\theta_1^2)$, the following result can be derived[67]:

$$\frac{1}{\Delta\theta_{\rm eff}^2} = \frac{1}{4\theta_1^2} \approx \frac{0{,}69}{\beta_{1/2}^2}. \tag{11d}$$

In both cases the answer is also expressed in terms of $\beta_{1/2}$ (the full angular width at the level of half the maximum intensity or FWHM).

In the PCSS problem the distortions propagate through an amplifying medium. Since their structure is uncorrelated with the inhomogeneities of the pump, the rate at which they are amplified is determined by the spatial average of the gain, $g = G\langle |E_L|^2\rangle$. We recall that the specklon itself is amplified at a doubled gain, $2G\langle |E_L|^2\rangle$, during PCSS. We write the field $E_S(\mathbf{r},z)$ as

$$E_S(\mathbf{r},z) = \frac{s(z)E_L^*(\mathbf{r},z)}{\sqrt{\langle |E_L|^2\rangle}} + n(\mathbf{r},z), \tag{12}$$

where $n(\mathbf{r},z)$ are the distortions (noise). For an unambiguous determination, we impose the condition that the noise is uncorrelated with the specklon: $\langle n(\mathbf{r},z)E_L(\mathbf{r},z)\rangle = 0$. We can then find the following results for the specklon intensity $S(z) = |s(z)|^2$ and the noise intensity $N(z) = \langle |n|^2\rangle$:

$$\frac{dS(z)}{dz} = 2gS(z), \tag{13}$$

$$\frac{dN(z)}{dz} = gN(z) + RS(z) \tag{14}$$

[Eq. (13) ignores the inverse effect of the noise $n(\mathbf{r},z)$ on the specklon]. The solution of Eq. (13) is $S(z) = S_0\exp(2gz)$; then the solution of Eq. (14) is

$$N(z) = N_0\exp(gz) + S_0\frac{R}{(2g-g)}(e^{2gz} - e^{gz}). \tag{15a}$$

At large values $gz \gg 1$, at which there is a discrimination against the initial noise N_0, the relative number of spiral distortions reaches a steady-state level:

$$\frac{N(z)}{S(z)} = \frac{R}{g} = \frac{\pi g}{k\Delta\theta_{\rm eff}^2}. \tag{15b}$$

This steady-state level results from an equilibrium between the excitation of specklon noise (the extinction coefficient R, in reciprocal centimeters) and the fact that the amplification of the distortions lags behind the amplification of the specklon (the discrimination $2g-g$, in reciprocal centimeters).

The structure of the spiral noise was also studied in Refs. 43, 67, and 76. It was shown that the angular spectrum of the noise is confined to roughly the same solid angle as that in which the transverse pump field is propagating. In this regard the spiral noise is quite different from the noise generated by the amplification of uncorrelated spontaneous nucleation centers. The latter noise fills a far larger solid angle, set by the geometry of the amplifying region (more on this point below in Subsection 3c).

It was shown in Ref. 43 that the inverse effect of the spiral noise $n(\mathbf{r},z)$ on the specklon increases the specklon gain. This conclusion corresponds to the tendency for an emphasis of the scattered field in the cores of the spiral microwaveguides. Here is the result for the corrected gain for the PC component:

$$g_{\mathrm{PC}}=2g\left(1+\frac{\pi g}{2k\Delta\theta_{\mathrm{eff}}^2}\right). \tag{15c}$$

The case of pronounced contraction into spirals (stimulated-scattering trapping), corresponding to $gl_{\mathrm{Fr}}\gtrsim 1$, was studied theoretically in Ref. 77.

To summarize this subsection of the review we might say that the relative level of the spiral noise, N/S, is low, and the PC specklon exists and provides a good conjugation quality if the spiral parameter is low: $gl_{\mathrm{Fr}}=\pi g/k\Delta\theta_{\mathrm{Fr}}^2\leq 1$. In other words, under this condition, the specklon $E_{\mathrm{S}}(\mathbf{r},z)=\exp(gz)E_{\mathrm{L}}^{*}{}^{*}(\mathbf{r},z)$ is in fact a solution of Eq. (8), as follows from (15b).

d) Spectral and angular shifts and boundaries of the interval in which the specklon exists

Phase conjugation accompanying stimulated scattering is observed not only in stimulated Brillouin scattering but also in several other types of stimulated scattering. In this subsection we are interested in stimulated Raman scattering, distinguished by a relatively large frequency shift $\alpha=(\omega_{\mathrm{L}}-\omega_{\mathrm{S}})/\omega_{\mathrm{S}}\sim 0.1\text{–}0.3$. At first glance, PCSS with such a large frequency shift would seem impossible: Even fields $E_{\mathrm{L}}^{*}(r,z)$ and $E_{\mathrm{S}}(r,z)$ of different frequencies ω_{L} and ω_{S} which coincide in some cross section $z=\mathrm{const}$ are completely mismatched at a distance $\Delta z\sim\lambda/\alpha(\Delta\theta_{\mathrm{L}})^2$ because of the difference in diffraction laws.

In the approximation of a parabolic equation (7), (8), with $G=0$, this difference reduces to a simple change in the scale of one field along the z axis with respect to that of the other by a factor of $k_{\mathrm{L}}/k_{\mathrm{S}}$. Consequently, over a sufficiently long interaction distance L ($L\gtrsim\Delta z$) diffraction prevents a coincidence of the speckle structures of the free fields. On the other hand, for the discrimination mechanism for PCSS to operate the fields must be matched over the entire distance.

The way out of this contradiction lies in the circumstance that the Stokes wave E_{S} propagates in a medium with a very nonuniform gain. If the inhomogeneities are pronounced, the medium imposes on the field E_{S} the propagation law dictated by the pump field, and there is a capture into a spatial resonance with the gain profile. What is the condition for this capture?[41] The PC solution which has a spatial resonance is amplified by a factor of e in comparison with the nonresonant configurations over the discrimination length $l_{\mathrm{disc}}=(2g-g)^{-1}=g^{-1}$. If the mismatch due to the frequency shift is small over this distance, i.e., if $l_{\mathrm{disc}}\lesssim\Delta z$, the conjugate Stokes-wave configuration manages to adjust its structure continuously to keep up with the structure of the pump.

Since we have the small parameter $\alpha(\Delta\theta_{\mathrm{L}})^2/\lambda g$ available, we can examine this question quantitatively, following Ref. 41. To solve this problem it is convenient to use a Fourier expansion in the angular components. In this representation the solution of Eq. (7) for the pump is

$$E_{\mathrm{L}}(\mathbf{r},z)=\sum_{\mathbf{q}}C_{\mathrm{L}}(\mathbf{q})\exp\left(i\mathbf{q}\mathbf{r}-\frac{i\mathbf{q}^2}{2k_{\mathrm{L}}}\right), \tag{16}$$

where $\theta=\mathbf{q}/k_{\mathrm{L}}\equiv(\theta_x,\theta_y)$ is the angle made with the z axis. With the goal of finding solutions for the Stokes wave which are correlated with the inhomogeneities of the pump *throughout the volume*, we write $E_{\mathrm{S}}(\mathbf{r},z)$ in the form

$$E_{\mathrm{S}}(\mathbf{r},z)=\sum_{\mathbf{q}}C_{\mathrm{S}}(\mathbf{q},z)\exp\left(i\mathbf{q}\mathbf{r}-\frac{i\mathbf{q}^2}{2k_{\mathrm{L}}}\right), \tag{17}$$

i.e., as an expansion in elementary waves of the conjugate pump. We wish to emphasize that with $|k_{\mathrm{L}}-k_{\mathrm{S}}|\neq 0$ the exponential factor in (17), which involves k_{L}, does not satisfy Eq. (8) without its right side. An identity transformation of Eq. (8) with the help of (17) yields

$$\frac{dC_{\mathrm{S}}(\mathbf{q},z)}{dz}+i\frac{\alpha\mathbf{q}^2}{2k}C_{\mathrm{S}}(\mathbf{q},z)=\frac{1}{2}G\sum_{\mathbf{q}_1\mathbf{q}_2\mathbf{q}_3}C_{\mathrm{L}}^{*}(\mathbf{q}_1)C_{\mathrm{L}}(\mathbf{q}_2)C_{\mathrm{S}}(\mathbf{q}_3,z)\,e^{i(\mathbf{q}^2-\mathbf{q}_1^2-\mathbf{q}_2^2-\mathbf{q}_3^2)z/2k}\,\delta^{(2)}(\mathbf{q}-\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3), \tag{18}$$

where $\alpha=(k_{\mathrm{L}}-k_{\mathrm{S}})/k_{\mathrm{S}}$ and where we have set $k_{\mathrm{L}}=k$ to simplify the notation. When the helical parameter is small we can ignore the spatially nonresonant terms $F(\mathbf{r},z)$ in (9); in the approximation of specklon theory we can then write

$$\frac{dC_{\mathrm{S}}(\mathbf{q},z)}{dz}-\left(i\frac{\alpha\mathbf{q}^2}{2k}-\frac{1}{2}g\right)C_{\mathrm{S}}(\mathbf{q},z)=\frac{1}{2}GDC_{\mathrm{L}}^{*}(-\mathbf{q}), \tag{19}$$

where

$$D(z)=\langle C_{\mathrm{L}}\mid C_{\mathrm{S}}\rangle=\sum_{\mathbf{q}}C_{\mathrm{L}}(-\mathbf{q})C_{\mathrm{S}}(\mathbf{q},z). \tag{20}$$

Making use of the standard analysis theorems, we see that expression (20) corresponds to an overlap integral of the type $\int E_{\mathrm{L}}E_{\mathrm{S}}\,d^2z$, and for fields which are statistically uniform over the transverse cross section the quantity $\langle C_{\mathrm{L}}|C_{\mathrm{S}}\rangle$ is equivalent to an average over an ensemble. A search for a solution in the form in (17), where it is specifically k_{L} appearing in the exponential functions, leads to the appearance of the "inconvenient" term $\propto\alpha\mathbf{q}^2$ on the left side of (19). The reason this term arises is that the elementary exponential waves in (17) do not satisfy the laws for free diffraction of the Stokes field. On the other hand, there is the advantage that Eq. (19) has z-independent coefficients.

Of primary interest is the following solution of Eq. (19):

$$C_{\mathrm{S}}^{0}(\mathbf{q},z)=e^{\mu z}\frac{\frac{1}{2}GD_0}{\mu+(i\alpha\mathbf{q}^2/2k)-(g/2)}C_{\mathrm{L}}^{*}(-\mathbf{q}), \tag{21}$$

which becomes the exact PC wave with $\mu=g$ in the limit $\alpha\to 0$. Multiplying (21) by $C_{\mathrm{L}}(-\mathbf{q})$ and summing over $\mathbf{q}$ (i.e., taking an average over the ensemble), we find, with $D_0\neq 0$, an equation for μ. We will not write out this equation explicitly (see Ref. 41), but we will write its solution as a power series in the small parameter α:

$$\mu=g-i\frac{\alpha q^2}{2k}-\frac{1}{2}g\left(\frac{\alpha k}{g}\right)^2[\overline{(\theta^2)^2}-(\overline{\theta^2})^2]+O(\alpha^3). \tag{22}$$

The superior bar denotes an average over the angular spectrum of the pump intensity; e.g.,

$$\overline{\theta^2}=\int j_L(\theta)(\theta_x^2+\theta_y^2)\,d\theta_x\,d\theta_y.$$

It can be seen from (21) that the solution which we found is not exactly the conjugate of the pump. With $\alpha k(\Delta\theta_L)^2/g\ll 1$ we see from (21) and (22) that

$$E_S(\mathbf{r},z)\approx \text{const}\cdot\exp(\mu z)\left[E_L^*(\mathbf{r},z)+i\frac{\alpha}{kg}\Delta_\perp E_L^*\right]$$
$$\approx \text{const}\cdot\exp(\mu z)E_L^*\left(\mathbf{r},z-\frac{\alpha}{g}\right)+O(\alpha^2). \tag{23}$$

As a quantitative measure of the deviation from the exact conjugate it is convenient to introduce the concept of the conjugation fraction H, a measure of the degree of overlap of the fields E_L^* and E_S. Following Ref. 41, we define this fraction as

$$H=\frac{\left|\int E_L(\mathbf{r},z)E_S(\mathbf{r},z)\,d^2\mathbf{r}\right|^2}{\int|E_L|^2d^2\mathbf{r}\int|E_S|^2d^2\mathbf{r}},\quad 0\leqslant H\leqslant 1. \tag{24}$$

When H is zero, there is a total absence of correlation; on the contrary, we have $H=1$ when $E_S(\mathbf{r})=\text{const}\cdot E_L^*(\mathbf{r})$.

The conjugate fraction for a specklon of the type in (23) differs only slightly from unity:

$$H=1-\frac{1}{2}\left(\frac{\alpha k}{|g|}\right)^2[\overline{(\theta^2)^2}-(\overline{\theta^2})^2]. \tag{25}$$

The reason why the growth rate $2\,\mathrm{Re}\,\mu$ from (22) is smaller than the simple value $2g$ is the incomplete correlation of the interacting fields (25). Curiously, for real g the difference between the fields E_L^* and E_S reduces, within the indicated accuracy, to a spatial shift $\delta z=\alpha/g$ along the z axis.

This analysis is thus shows that for a pump which is a statistically uniform pump in the in the interaction volume the condition for the occurrence of conjugation in the case of stimulated scattering becomes

$$\alpha k\sqrt{\overline{\theta^4}-(\overline{\theta^2})^2}\leqslant g\leqslant k\theta_{\text{eff}}^2, \tag{26}$$

where the inequality on the right in (26) expresses the condition that the spiral noise is low. Up to this point we have been discussing the relation $E_S(\mathbf{r},z)=sE_L^*$ for slow amplitudes exclusively. We recall that the complete complex amplitude differs from the slow amplitude by a factor $\exp(-ik_Lz)$ for the pump and by a factor $\exp(ik_Sz)$ for the scattered wave. When the fast factors are taken into account, the relation $E_S=sE_L^*$ obviously does not hold, since s is not a constant and instead depends strongly on x, in accordance with $\exp[i(k_L-k_S)z]$. The important point, however, is that the transverse field structure is still conjugated.

We now leave the phase conjugation accompanying stimulated Raman scattering to take up a problem of considerable practical interest: phase conjugation accompanying stimulated scattering with an angular shift, i.e., a rotation. Specifically, we consider the case in which upon the reproduction of the transverse structure there must be a rotation of the entire field as a whole:

$$E_S(\mathbf{r},z)\overset{?}{=}E_L^*(\mathbf{r},z)\exp\left(ik\boldsymbol{\psi}\mathbf{r}-\frac{1}{2}ik\psi^2z\right). \tag{27}$$

For simplicity we assume $|\mathbf{k}_L|=|\mathbf{k}_S|=k$. The vector $\boldsymbol{\psi}=\mathbf{e}_x\psi_x+\mathbf{e}_y\psi_y$ lies in a plane perpendicular to the central propagation direction $\mathbf{e}_z$ and is a measure of the magnitude and direction of the rotation angle. If E_L is a solitary plane wave, then a field of the type in (27) satisfies the free parabolic equation (8). If, on the other hand, the speckled field E_L is formed by angular components with a width of order $\Delta\theta_L$, a relation of the type in (27) in free space is maintained over only a bounded interval Δz. In the approximation of a parabolic equation, this mismatch reduces to a simple shift $\Delta\mathbf{r}=\boldsymbol{\psi}z$ in the transverse plane. Hence we find the restriction $\Delta z\sim a/|\psi|=\lambda\Delta\theta_L|\psi|$ where $a=\lambda/\Delta\theta_L$ is the transverse dimension of the speckle inhomogeneities. If, on the other hand, the rotated phase-conjugate wave propagates in the presence of a speckle-inhomogeneous pump, then at a sufficiently high gain there may be a capture into a spatial resonance. After some calculations[78] corresponding to those in the problem of phase conjugation accompanying stimulated Raman scattering, the following approximate PC solution can be found:

$$E_S(\mathbf{r},z)=\text{const}\cdot\exp(\mu z)E_L^*(z,\mathbf{r}+\boldsymbol{\psi}/g)\,e^{ik\boldsymbol{\psi}\mathbf{r}-ik\psi^2z/2}+O(\psi^2), \tag{28}$$
$$\mu=g-ik\overline{(\boldsymbol{\psi}\boldsymbol{\theta})}-\frac{k^2}{g}[\overline{(\boldsymbol{\psi}\boldsymbol{\theta})^2}-(\overline{\boldsymbol{\psi}\boldsymbol{\theta}})^2].$$

As in the case of stimulated Raman scattering, the noise in (28), which reduces in this approximation to a transverse shift $\Delta\mathbf{r}=\boldsymbol{\psi}/g$, gives rise to a slight mismatch between the scattered and exciting fields and thus causes a decrease in the growth rate. In this case the condition for the occurrence of specklon PC is

$$k[\overline{(\boldsymbol{\psi}\boldsymbol{\theta})^2}-(\overline{\boldsymbol{\psi}\boldsymbol{\theta}})^2]\leqslant g\leqslant k\Delta\theta_{\text{eff}}^2. \tag{29}$$

If the inequality at the right in (29) holds with a sufficient margin, $g/(k\Delta\theta^2)\ll 1$, the permissible angle $\Delta\psi$ at which a specklon will still exist is $\Delta\psi\approx(g/k\Delta\theta^2)\Delta\theta\ll\Delta\theta$.

We thus see that a solution matched with the pump—the specklon—is quite stable with respect to both frequency and angular shifts.

e) Onset of transverse coherence in PCSS

In most cases, PCSS is excited from spontaneous random nucleation centers which are uncorrelated in spatial position. Of all possible initial configurations of the scattered field, those are amplified best for which the local maxima coincide with maxima of the speckle structure of the pump. It is not difficult to see, however, that this condition is satisfied not only for the exactly conjugate wave but also for a wave of the type

$$E_S(\mathbf{r})=f(\mathbf{r})E_L^*(\mathbf{r}). \tag{30}$$

where $f(\mathbf{r},z)$ is an envelope, which is smooth in comparison with the scale dimension of the speckle structure. The PC component of the field excited from spontaneous sources will most likely not have a smooth initial envelope but instead a random, very choppy envelope. In the course of the propagation, however, the envelope becomes smoother, and its transverse scale dimension increases; i.e., there is an *onset of transverse coherence of the envelope.* For a quantitative look at this process,[78] it is convenient to write the initial envelope $f(\mathbf{r}, z=0)$ as an expansion in angular components:

$$f(\mathbf{r}, z=0) = \int \tilde{f}(\boldsymbol{\psi}, z=0) \exp(ik\boldsymbol{\psi}\mathbf{r})\, d^2\boldsymbol{\psi}. \quad (31)$$

The growth rate is largest for the PC component with $\psi = 0$ according to Eq. (28) of the preceding subsection. Over a sufficiently large distance z, the components within a relatively small angle $\Delta\psi \sim (2k^2\Delta\theta_L^2 z/g)^{-1/2}$, determined from the condition $2\,\mathrm{Re}[\mu(\psi) - \mu(0)]z \lesssim 1$, have approximately the same gain. It follows that the transverse scale dimension of the inhomogeneities of the amplified PC envelope is given in order of magnitude by

$$\Delta r_{\mathrm{corr}} \sim \sqrt{\frac{2\Delta\theta_L^2 z}{g}}. \quad (32)$$

Using relations like the Zernike–van Cittert theorem, one can derive an explicit expression for the correlation function[78] $\langle f^*(\mathbf{r}_1, z) f(\mathbf{r}_2, z)\rangle$. To estimate Δr_{corr} at the exit from the active medium we should note that the onset of coherence occurs only in the region of exponential amplification, where the total growth rate is $2gz = 25$. From (32) we thus find

$$\Delta r_{\mathrm{corr}} = \frac{4}{\sqrt{25}}\, z \sqrt{\Delta\theta_L^2} = 0.8 z \Delta\theta_L, \quad (33)$$

where z is the length of the exponential-amplification region. Near the threshold for the stimulated scattering, z is the same as the interaction length. The coefficient in (33) was calculated under the assumption that Δr_{corr} is the correlation radius at the level of e^{-1} of the value of the correlation function at its maximum, and $\Delta\theta_L$ is the total angular width, according to the same criterion, for a Gaussian spectrum.

The condition for exact phase conjugation is $\Delta r_{\mathrm{corr}} \gtrsim d$, where d is the diameter of the pump beam as it enters the medium.

Let us examine the experimental consequences of phase conjugation with an imcompletely coherent envelope. We assume that the phase plate creating the speckle structure from the originally plane pump wave does not lie far from the entrance to the active medium (or its image). A scattered field of the type in (30) then corrects its small-scale inhomogeneities after passing back through the phase plate in the opposite direction, but it retains the smooth modulation of the envelope, $f(\mathbf{r})$. Accordingly, the reconstructed Stokes wave produces a bright core in the far zone, with a solid angle greater than the diffraction limit by a factor of $(d/r_{\mathrm{corr}})^2$. In many problems, where ideal diffraction-level focusing is not required, this incompletely coherent PC may be completely satisfactory for applications.

This discussion has shown that there are actually three selective processes which occur during PCSS and which are responsible for the high quality of the phase conjugation. First, the spontaneous nucleating configurations which are uncorrelated with the pump—the vast majority of the configurations—are amplified at half the gain for the PC field and are thus discriminated against. Second, the spiral noise continuously excited by a specklon is also discriminated against because it lags behind the amplification. Finally, the discrimination against PC solutions with a small angle ψ causes a gradual smoothing of the envelope of the PC component, to the point at which it becomes flat over the entire beam cross section.

f) Phase conjugation of depolarized radiation

The active rods of solid-state amplifiers can introduce not only phase distortions but also polarization distortions in the beam being amplified. We are accordingly interested in the polarization properties of phase conjugation accompanying stimulated scattering.

A theory for polarization effects in phase conjugation accompanying stimulated Brillouin scattering was derived in Ref. 45, and we will follow that derivation here.

System of equations (7), (8) should be written in the following vector form for phase conjugation accompanying stimulated Brillouin scattering:

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k}\Delta_\perp\right)\mathbf{E}_L^*(\mathbf{r}, z) = 0, \quad \left(\frac{\partial}{\partial z} - \frac{i}{2k}\Delta_\perp\right)\mathbf{E}_S(\mathbf{r}, z) = \frac{1}{2} G(\mathbf{E}_L^*\mathbf{E}_S)\mathbf{E}_L, \quad (34)$$

where the conditions $(\mathbf{E}_L\mathbf{e}_z) = (\mathbf{E}_S\mathbf{e}_z) = 0$ follow from the transverse nature of the waves. The vector structure of the right side of Eq. (34) for a Stokes wave can be found from the following arguments: In stimulated Brillouin scattering, an interference of the field $\mathbf{E}_L^*$ and $\mathbf{E}_S$ excites a scalar parameter in the medium—a density perturbation $\delta\rho \propto (\mathbf{E}_L^*\mathbf{E}_S)$—while the scattering of the pump $\mathbf{E}_L$ by the dielectric-permittivity grating $\delta\varepsilon = (\partial\varepsilon/\partial\rho)\delta\rho$ leads to an amplification of the Stokes field.

If the pump is completely polarized, i.e., if the polarization unit vector $\mathbf{e}_L$ is constant over the cross section,

$$\mathbf{E}_L(\mathbf{r}, z) = \mathbf{e}_L E_L(\mathbf{r}, z), \quad (35)$$

then it is easy to see from (34) that the conjugate Stokes wave with the highest gain is

$$\mathbf{E}_S(\mathbf{r}, z) = \mathrm{const}\cdot\mathbf{e}_L E_L^*(\mathbf{r}, z). \quad (36)$$

In other words, for a perfectly polarized pump, (35), there is a phase conjugation of the spatial structure, and the polarization unit vector is reproduced (without complex conjugation!). In terms of the polarization, therefore, a stimulated-scattering mirror is similar to an ordinary mirror; for example, a right-hand polarized incident wave $(\mathbf{e}_x - i\mathbf{e}_y)\exp(-ikz)$, reflected from a scalar-perturbation grating $\delta\varepsilon(\mathbf{r}, z)$, is converted into a left-hand polarized reflected wave, $(\mathbf{e}_x - i\mathbf{e}_y)\exp(ikz)$.

This situation is considerably more complicated if the pump field is "depolarized," by which we mean that the polarization vector is not uniform over the beam cross section (in contrast with the more commonly considered case in which this vector varies with time; see §50 in Ref. 79, for example).

Any transverse field $\mathbf{E}_L(\mathbf{r})$ can be expanded in an arbitrary basis of mutually orthogonal $[(\mathbf{e}_1\mathbf{e}_2^*)=0]$ complex unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$:

$$\mathbf{E}_L(\mathbf{r}, z) = \mathbf{e}_1 E_1(\mathbf{r}, z) - \mathbf{e}_2 E_2(\mathbf{r}, z). \tag{37}$$

It is convenient to choose the unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ in such a manner that the speckle structures of the fields E_1 and E_2 are not correlated. In this case we can introduce the degree of polarization p by means of the definitions

$$\begin{gathered}\langle EE^*\rangle = I, \quad \langle E_1^*(\mathbf{r}, z) E_2(\mathbf{r}, z)\rangle = 0, \\ \langle |E_1|^2\rangle = \frac{1}{2} I(1+p), \quad \langle |E_2|^2\rangle = \frac{1}{2} I(1-p), \quad 0 \leqslant p \leqslant 1.\end{gathered} \tag{38}$$

In the general case of complex unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, with $0<p<1$, we have a partial elliptic polarization of a monochromatic wave.

In accordance with this discussion, it is convenient to write the scattered field as an expansion in the same unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ (rather than in their conjugates):

$$\mathbf{E}_S(\mathbf{r}, z) = \mathbf{e}_1 S_1(\mathbf{r}, z) - \mathbf{e}_2 S_2(\mathbf{r}, z). \tag{39}$$

Now applying the algorithm of the theory of specklons, (9), to (34), we find the system of equations

$$\begin{aligned}\left(\frac{\partial}{\partial z} - \frac{i}{2k}\Delta_\perp\right) S_1 &= \frac{1}{2} G\left[\langle |E_1|^2\rangle S_1(\mathbf{r}, z) + \langle S_1 E_1\rangle E_1^*(\mathbf{r}, z) - \langle S_2 E_1\rangle E_2^*(\mathbf{r}, z)\right], \\ \left(\frac{\partial}{\partial z} - \frac{i}{2k}\Delta_\perp\right) S_2 &= \frac{1}{2} G\left[\langle S_1 E_2\rangle E_2^*(\mathbf{r}, z) + \langle |E_2|^2\rangle S_2(\mathbf{r}, z) - \langle S_2 E_2\rangle E_2^*(\mathbf{r}, z)\right],\end{aligned} \tag{40}$$

where we have used $\langle E_1^* E_2\rangle = 0$. We first consider Stokes waves which are not correlated in terms of spatial structure with either of the pump components. For such waves we have $\langle S_1E_1\rangle = \langle S_2E_1\rangle = \langle S_1E_2\rangle = \langle S_2E_2\rangle = 0$, and the z dependence, of the type $\mathbf{e}\exp(\mu z)$, accordingly corresponds to the following gain values:

$$\mathbf{e} = \mathbf{e}_1, \quad \mu = \frac{1}{4} g(1+p), \quad \mathbf{e} = \mathbf{e}_2, \quad \mu = \frac{1}{4} g(1-p), \tag{41}$$

where $g = GI$. In other words, a given component $\mathbf{e}_{1,2}$ of an uncorrelated Stokes wave senses only the spatial average of the intensity of the pump component of the same polarization.

Among the correlated solutions, that with the highest gain is the specklon

$$\mathbf{E}_S(\mathbf{r}, z) = \mathbf{M}_1(\mathbf{r}, z) = \mathbf{e}_1 E_1^*(\mathbf{r}, z)\exp\left[\frac{g(1+p)z}{2}\right], \tag{42}$$

which is a solution of Eq. (40). There is a corresponding correlated solution of Eqs. (40) for the second polarization component:

$$\mathbf{E}_S(\mathbf{r}, z) = \mathbf{M}_2(\mathbf{r}, z) = \mathbf{e}_2 E_2^*(\mathbf{r}, z)\exp\left[\frac{g(1-p)z}{2}\right]. \tag{43}$$

All these solutions in (41)–(43) have a spatially uniform polarization ($\mathbf{e}_1$ or $\mathbf{e}_2$) and do not interact with the "foreign" polarization component of the pump.

The first equation in (40) actually contains a nontrivial term of the type $E_2^*\langle S_2E_1\rangle$, responsible for the interaction of different polarization components. This term stems from the coherent scattering of the pump component $\mathbf{e}_1E_1(\mathbf{r}, z)$ by that part of the scalar hypersonic grating $\delta\varepsilon$ which is recorded by the interference of "foreign" polarization components, $\mathbf{e}_2^*E_2^*(\mathbf{r}, z)$ with $\mathbf{e}_2S_2(\mathbf{r}, z)$. There is a similar term, $E_1^*\langle S_1E_2\rangle$, in the second equation in (40). For solutions (41)–(43), however, these terms are irrelevant, since the conditions $\langle S_1E_2\rangle = \langle S_2E_1\rangle = 0$ hold for them.

There are, however, two other correlated solutions of system (40), for which these conditions do not hold. In the solutions, a given polarization component of the Stokes field turns out to be correlated with the spatial structure of the orthogonal component of the pump:

$$\mathbf{E}_S(\mathbf{r}, z) = \mathbf{M}_3(\mathbf{r}, z) = [E_1^*(\mathbf{r}, z)(1-p)\,\mathbf{e}_2 - E_2^*(\mathbf{r}, z)(1-p)\,\mathbf{e}_1]\, e^{\frac{1}{2}gz}, \tag{44}$$

$$\mathbf{E}_S(\mathbf{r}, z) = \mathbf{M}_4(\mathbf{r}, z) = (E_1^*\mathbf{e}_2 - E_2^*)\,\mathbf{e}_1 e^{0z} = [\mathbf{E}_L^*\mathbf{e}_z]. \tag{45}$$

A curious property of solution (44) is that its gain does not depend on the degree of polarization of the pump (p) for a given total pump intensity $I = \langle \mathbf{E}_L^*\mathbf{E}_L\rangle$. For this solution, the weakening effect of the depolarization is cancelled exactly by an increase in the gain because of the spatial correlation. The wave in (45), on the other hand, is strictly orthogonal with respect to the pump at all points in space, so that it does not record gratings of any sort at all, and it is not amplified. Figure 9 is a plot of the gains for solutions (41)–(45) as functions of the degree of polarization of the pump (p) for I = const. Experimental results on phase conjugation accompanying the stimulated Brillouin scattering of depolarized fields were discussed in Subsection 2c; on the whole, the experimental results agree well with the theory outlined above.

An important conclusion which follows from the theory of the stimulated scattering of depolarized beams—and which is confirmed by experiment—is the nonlinear polarization selection: The gain is highest for that specklon whose polarization is the same as that of the more intense component of the exciting beam. By virtue of the exponential dependence of the exit intensity on the growth rate ($e^{gz} \sim e^{25}$), this specklon is predominant in the scattered radiation, so that the scattered field is completely polarized (with $p_{exc} \neq 0$). Another example of nonlinear selection will be discussed in the following subsection.

g) Nonlinear selection of nonmonochromatic radiation

In the case of a nonmonochromatic exciting beam, the ratio of the spectral width of the beam, $\Delta\omega$, to th half-

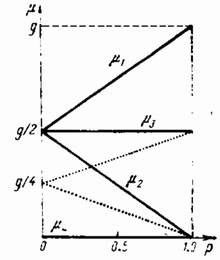

FIG. 9. The gain values $\mu_{1,2,3,4}$ for the specklon (solid lines) and those for uncorrelated waves of various polarizations (dotted lines) as functions of the degree of polarization of the pump p.

width of the spontaneous-scattering line, Γ, is an important parameter. If the width of the pump line is much less than Γ, the stimulated scattering can be described in terms of an interaction of monochromatic waves. We now assume that the pump $E_L(t)$ consists of two monochromatic components, with frequencies ω_1 and ω_2, so that $\omega_1 - \omega_2 = \Delta\omega$, and the Stokes radiation consists of harmonics with corresponding frequency shifts, $\omega_1 - \Omega$ and $\omega_2 - \Omega$:

$$E_L(t) = e^{-i\omega_1 t}(E_{1_L} + e^{i\Delta\omega t}E_{2_L}),$$
$$E_S(t) = e^{-i(\omega_1-\Omega)t}(E_{1_S} + e^{i\Delta\omega t}E_{2_S}). \tag{46}$$

The excitation of the hypersonic wave $\delta\rho(t) = \tilde{\rho}(t)e^{i\Omega t} + \tilde{\rho}^*(t)e^{-i\Omega t}$ is described by the truncated equation

$$\frac{d\tilde{\rho}(t)}{dt} + \Gamma\tilde{\rho}(t) = -iAE_L^*E_S e^{-i\Omega t} \equiv -iA(E^*_{1_L} + e^{-i\Delta\omega t}E^*_{2_L})(E_{1_S} + e^{i\Delta\omega t}E_{2_S}). \tag{47}$$

It follows from the solution of this equation,

$$\tilde{\rho}(t) = -\frac{iA}{\Gamma}\left[(E^*_{1_L}E_{1_S} + E^*_{2_L}E_{2_S}) + \frac{E^*_{1_L}E_{2_S}e^{i\Delta\omega t}}{1-(i\Delta\omega/\Gamma)} + \frac{E^*_{2_L}E_{1_S}e^{-i\Delta\omega t}}{1-(i\Delta\omega/\Gamma)}\right], \tag{48}$$

that for $\Delta\omega \gg \Gamma$ each spectral component of the pump effectively records density perturbations with only "its own" Stokes wave. For $\Delta\omega \gg \Gamma$, the situation is thus analogous to the polarization problem, in which the density perturbations are excited only by an interference of matching polarizations. A distinctive feature of the temporal problem is that the number of independent temporal components, N, may be, instead of two as in the polarization problem, a far higher number: $N \sim \Delta\omega/\Gamma$.

Pursuing this analogy, we note that the nonmonochromatic pump field in the factored form

$$E_L(\mathbf{r}, z, t) = e_L(t)\,E(\mathbf{r}, z), \tag{49}$$

along with a Stokes wave of the type

$$E_S(\mathbf{r}, z, t) = ce_L(t)\,E^*(\mathbf{r}, z), \tag{50}$$

in which the *temporal* structure is *reproduced without conjugation*, while the *spatial* structure *is conjugated,* causes an effective excitation of a hypersonic wave:

$$\delta\rho(\mathbf{r}, z, t) = -iAc\,[E^*(\mathbf{r}, z)]^2 \int\limits_{-\infty}^{t} |e_L(t')|^2 e^{-\Gamma(t-t')}\,dt'. \tag{51}$$

The scattering of the pump by these density perturbations causes an amplification of the Stokes wave of the type in (50) with a doubled gain. We wish to emphasize that no fast phase oscillations of the function $e_L(t)$ will affect the efficiency at which a Stokes wave of the type in (50) is excited, according to (51).

An arbitrary field $E_L(\mathbf{r}, t)$ incoherent in both space and time, can always be written (and, moreover, in an infinite number of ways) as an expansion

$$E_L(\mathbf{r}, t) = \sum_i \alpha_i e_i(t)\,\mathscr{E}_i(\mathbf{r}). \tag{52}$$

where $e_i(t)$ are functions which are normalized and orthogonal on an interval $T \lesssim \Gamma^{-1}$ such that $\Delta\omega T \gg 1$. This expansion can be rendered unambiguous, as in the polarization problem, by requiring that the structures of $\mathscr{E}_i(\mathbf{r})$ be spatially orthogonal. They may be regarded as normalized by incorporating the other factors in the coefficient α_i. Such expansions are known in the literature as Karhunen-Loève expansions.[109] Expansions in factorized functions of the type in (49) were used in Refs. 105–108 and 110 for the problem of phase conjutation accompanying the stimulated scattering of nonmonochromatic radiation. The biorthogonal expansion in (52) was first used in Ref. 107.

Ay analogy with the polarization problem, we seek a solution for the Stokes field in the same case, $\Delta\omega \gg \Gamma$, in the form

$$E_S(\mathbf{r}, z, t) = \sum_i f_{ik}(z)\,e_i(t)\,\mathscr{E}^*_k(\mathbf{r}, z). \tag{53}$$

Making use of the biorthogonality of the expansion basis when corrections of order $\Gamma/\Delta\omega$ are ignored, and treating $\mathscr{E}_k(\mathbf{r}, z)$ as speckle fields with a Gaussian statistics, we find the system of equations[2)]

$$\frac{df_{ik}}{dz} = \frac{1}{2}G(|\alpha_i|^2 f_{ik} + \alpha_i\alpha_k^* f_{ki}). \tag{54}$$

Denoting by N the number of terms in expansion (52), we see that the dimensionality of the matrix f_{ik} is $N \times N$, and we see that system (54) has N^2 linearly independent solutions. Remarkably, all N^2 eigenfunction solutions [i.e., of the form $\exp(\mu z)$] of this system can be found and written out explicitly. First, there are N solutions of the type

$$f_{ik} = \delta_{im}\delta_{km}, \quad \mu_m = G|\alpha_m|^2, \quad m = 1, \ldots, N. \tag{55}$$

In addition, there are $N(N-1)/2$ "intricate" solutions with a matrix

$$f_{ii} = 0, \quad f_{ik} = \frac{\alpha_i}{\alpha_k} f_{ki} \quad \text{for} \quad i \neq k, \quad \mu_{ik} \equiv \mu_{ki} = \frac{1}{2}G(|\alpha_i|^2 - |\alpha_k|^2), \tag{56}$$

and there are an equal number of unamplified solutions with the matrix

$$f_{ii} = 0, \quad f_{ik} = -\left(\frac{\alpha_k}{\alpha_i}\right)^* f_{ki} \quad \text{for} \quad i \neq k, \quad \mu = 0. \tag{57}$$

The case $N = 2$ is precisely the same, in terms both of the system of equations and the system of modes, as the polarization problem[45] discussed in the preceding subsection. For a pump consisting of two monochromatic components, the same solution for $N = 2$ was given in Ref. 110.

A basic conclusion to be drawn from these solutions is that under the condition $\Delta\omega \gg \Gamma$ and near the threshold for stimulated scattering the nonlinear selection should lead to a predominance in the scattered field of a factorized PC component of the type $\mathscr{E}^*_{i0}(\mathbf{r})e_{i_0}(t)$, for which the weight factor $|\alpha_{i_0}|^2$ is largest in the pump. This conclusion was first reached in Refs. 108 and 107 by a slightly different approach. The importance of this conclusion can be illustrated[107] in the problem of the PC of a weak signal incident on a high-power amplifier. The amplifier raises the power level of the signal directed into the nonlinear medium, and it simultaneously adds to it amplified spontaneous radiation, whose total power level may even exceed that of the amplified signal by a large factor. Nevertheless, even after amplification the signal has a factorized form, and the noise

2) In this section, a repeated index does not imply a summation.

is distributed among many components of expansion (52). Consequently, the nonlinear selection can make it possible to find conditions under which the noise is not reflected, while the signal undergoes a spatial phase conjugation.

Unfortunately, the efficiency of this selection is high only if saturation of the pump is ignored. In the model under consideration here, each factorized Stokes mode of the type in (55) acquires energy only from the corresponding component of the pump. Accordingly, with a significant saturation the distribution of the quantities $|\alpha_i|^2$ in the medium will become uniform, and the selection will be weakened. This tendency has been detected experimentally[83,84] for the case $N=2$ (in the polarization problem).

h) Effect of saturation on phase conjugation

In the overwhelming majority of experiments, stimulated scattering occurs with a significant depletion of the pump because of rescattering into the Stokes wave. Actually, the range of pump intensities in which the stimulated scattering can be detected, but in which saturation has not yet set in, is extremely narrow: $20 < G|E_L|^2 \leq 25$. Furthermore, achievement of a high reflection coefficient in stimulated-scattering phase conjugation automatically implies deep saturation. Saturation accompanying stimulated backscattering has been analyzed theoretically by Tang[111] without reference to phase conjugation. The onset of saturation was studied in Refs. 112 and 113 and also in the collection in Ref. 60. We turn now to the results of those studies.

When pump depletion is taken into account, the original system of equations consists of Eqs. (7) and (8), where Eq. (7) is modified to the form

$$\frac{\partial E_L}{\partial z} + \frac{i}{2k}\Delta_\perp E_L(\mathbf{r}, z) = \frac{1}{2}G|E_S(\mathbf{r}, z)|^2 E_L(\mathbf{r}, z). \tag{7a}$$

At the entrance to the medium ($z=L$) the pump field has a pronounced speckle structure. In the course of the depletion of a *nonuniform* Stokes wave, not only the average pump intensity but also the structure of the pump varies along the z direction. In other words, we cannot assume, as we did in the nonlinear case, that the speckle structure of the pump is diffracted in accordance with the laws of a homogeneous medium. The Stokes wave is excited from the spontaneous noise and thus has Gaussian statistics. By virtue of the interaction with the pump, the Stokes wave not only is amplified but also undergoes a change in structure. An important point for the discussion below is that when the spiral parameter in (15) is small it is reasonable to assume that the fields retain Gaussian statistics throughout the volume. In this case we can apply the methods of specklon theory to them, and Eqs. (7a) and (8) can be transformed to

$$\frac{dI_S}{dz} = \frac{dI_L}{dz} = GI_L I_S(1+H), \tag{58}$$

$$\frac{dH}{dz} = G(I_L + I_S)H(1-H), \tag{59}$$

$$I_{L,S} = \langle |E_{L,S}(\mathbf{r}, z)|^2 \rangle, \quad H(z) = \frac{|\langle E_L E_S^* \rangle|^2}{I_L I_S}. \tag{60}$$

We wish to emphasize that $H(z)$ is the same as the phase-conjugation fraction, understood in its usual sense, only at the exit cross section, $z=L$. In all other cross sections, $H(z)$ is the coefficient of the overlap of the Stokes wave not with the original pump field but with the actual field in the given cross section. With $H \equiv 1$, Eqs. (58) describe stimulated scattering of mutually conjugate fields, and the effective interaction coefficient is $2G$ (the Gaussian 2). The spatial resonance is lost at $H=0$, and there is an energy transfer between uncorrelated waves, i.e., an interaction with the coefficient G. It follows from Eq. (59) that dH/dz vanishes for both $H=0$ (the fields remain uncorrelated over the entire interaction length) and $H=1$ (exact phase conjugation in all cross sections).

Equation (58) actually represents a system of two equations, for I_L and I_S, which has the exact integral

$$I_L(z) - I_S(z) = \text{const} = I_L(L) - I_S(L) = I_L(L)(1-R), \tag{61}$$

where we have introduced the notation R for the (intensity) reflection coefficient. We are thus left with only two equations, e.g., those for $L_L(z)$ and $H(z)$. Introducing

$$y(z) = GI_L(L)\int_z^L [1+H(z')]\,dz'. \tag{62a}$$

we find a solution in the form

$$I_L(z) = I_L(L)\frac{1-R}{1-R\exp[-y(1-R)]}, \quad I_S(z) = I_L(z) - I_L(L)(1-R), \tag{62b}$$
$$H(z)[1+H(z)]^{-2} = \text{const}\cdot I_L(z) I_S(z).$$

The constant in (62b) is found from the boundary condition on $H(z=0)$, which gives the magnitude of the projection in the $z=0$ cross section of the Stokes signal onto that pump field which reaches this cross section. In a typical situation involving phase conjugation from spontaneous noise we would have $H(0) \ll 1$. Kochemasov and Nikolaev[113] have reported a detailed study of the behavior of $H_{L,S}(z)$ and $H(z)$ as functions of z for various values of the dimensionless parameters of the problem, $GLI_L(L)$ and $H(0)$. The point of most interest is the relationship between the quality of the phase conjugation [i.e., the value of $H(L)$] and the reflection coefficient R. This relationship can be found directly from (62) (see Refs. 113 and 60):

$$H(L) = 1 + \frac{b^2}{4} - \sqrt{b^2 + \frac{b^4}{4}}, \quad b = \sqrt{\frac{I_S(0)}{I_L(L)}\frac{1-R}{R}\frac{1}{H(0)}}, \tag{63}$$

so that we have $H(L) \approx 1-b$ at $b \ll 1$ and $H(L) \approx b^{-2}$ at $b \gg 1$. The condition for high-quality phase conjugation, i.e., the condition for an adequate discrimination against the uncorrelated noise which is being amplified, takes the form $b \ll 1$ under saturation conditions. For comparison with the condition for discrimination in the linear case (see Subsection 3c below) we note that we have $H(z=0) \approx (\theta_{diff}/\theta_{noise})^2$, where the diffraction angle is $\theta_{diff} = d/\lambda$, and θ_{noise} is the angle of the noise being amplified. Furthermore, we have $I_S(0) \approx I_L^{thr}\exp(-30)(\theta_{noise}/\theta_{diff})^2$, since $I_S(0)$ receives contributions from the entire noise angular spectrum which is being amplified, and the fraction of the scattering into the precisely conjugate configuration (i.e., into the solid angle θ_{diff}^2) is typically about e^{-30}. Finally, we have $I_L(0) \approx I_L^{thr} \approx I_L(L)(1-R)$, so that the discrimination condition takes the following form, when saturation is taken into account:

$$1-H \approx b \approx e^{-15}\left(\frac{\theta_{\text{noise}}}{\theta_{\text{diff}}}\right)^2 \sqrt{\frac{(1-R)^2}{R}}. \tag{64}$$

The factor $e^{-15}(\theta_{\text{noise}}/\theta_{\text{diff}})^2$ is a small parameter which determines the discrimination quality in the linear case (without saturation). In a crude approximation, the PC configuration is amplified by a factor $\exp(2gz) = \exp(30)$, while the noise is amplified by a factor $\exp(gz) \approx \exp(15)$; on the other hand, the nucleating intensity of the noise is higher by a factor $(\theta_{\text{noise}}/\theta_{\text{diff}})^2$.

Expressions (63) and (64) constitute the primary result of this subsection and lead to the following conclusion: As the reflection coefficient increases, i.e., as we move to a progressively deeper saturation, the degree of the discrimination against the noise increases,[114] while the discrimination condition itself is relaxed by a factor $(1-R)$. According to this approach, saturation thus improves the quality of the phase conjugation.

3. EXPERIMENTAL RESEARCH ON PCSS

a) Methods for measuring the conjugation quality

In most of the experimental strategies for achieving phase conjugation accompanying stimulated scattering (PCSS), a phase plate is used to produce the speckle inhomogeneities of the pump field. This is usually a glass plate etched in hydrofluoric acid to produce random inhomogeneities of the thickness. As a light wave passes through such a plate it acquires a nonuniform phase $2\pi(n-1)\delta h(\mathbf{r})/\lambda$, where n is the refractive index of the glass, λ is the wavelength in air, and $\delta h(\mathbf{r})$ is the local thickness variation. If the transverse scale dimension of the thickness inhomogeneities is $a_\perp$, we find from geometric optics that the local deflection angle is $\delta\theta(\mathbf{r}) = (n-1)\nabla h \sim (n-1)\delta h/a_\perp$. The quantity $\langle|\delta h|\rangle$ is usually chosen such that the nonuniform phase shift is of the order of 2π or slightly larger. In the opposite case, i.e., with $|\delta h|(n-1) \lesssim \lambda$, the regular (undeflected) wave may retain a significant amplitude as it traverses the plate. At $\delta h(n-1) \sim \lambda$, the divergence introduced by the phase plate is of order $\delta\theta \sim \lambda/a_\perp$. The phase plates ordinarily used introduce an irregular ("gray") divergence $\delta\theta \sim 10^{-2}$ rad in the beam. To produce the speckle-inhomogeneous pump in the interaction volume is the first of the functions performed by the phase plate.

A second function performed by the same phase plate is to analyze the conjugation quality. Let us assume that a plane pump wave with a diffraction-level divergence is incident on the phase plate, while the reflected field contains only the conjugate component and the part of the light which is uncorrelated with the conjugate component. In order to determine the relative intensity of the phase conjugation it is necessary to spatially separate these two components. During the return pass through the phase plate the conjugate component is converted into a plane wave and thus produces a bright diffraction-level core in the far zone. As the uncorrelated waves, on the other hand, pass through the phase plate they produce a beam with a divergence $\gtrsim \delta\theta$. Optical elements with phase inhomogeneities have been used in holographic research on phase conjugation for a rather long time now (see Refs. 5 and 6, for example). However, only the method of illuminating a phase plate with a diffraction-quality plane wave, first used in Ref. 20, allows spatial localization of the exactly conjugate component and thus quantitative measurements of the characteristics of this component.

The analyzing function of the phase plate can thus be used (and is in fact used) in any method for producing a conjugate wave.

The energy of the exactly conjugate component can be measured in the far zone beyond the phase plate by placing there a diaphragm with an aperture corresponding to the diffraction-level divergence of the beam. This method was used in Refs. 32, 46, and 51. It has the advantage that the measurements are easy to interpret. A disadvantage is the need to place a diaphragm of comparatively small dimensions exactly in the conjugate laser beam. If the apparatus is to be made less sensitive to fluctuations in the propagation of this beam, the size of the aperture in the diaphragm must be increased, at the cost of measurement accuracy.

The mirror-wedge method, now quite popular, was used in Ref. 20. In this method one compares the energies of the laser pump and of the reflected wave and also the brightnesses of the pump and the scattered wave in the far zone. The energies are measured, in the same units, with two calorimeters. It is important that the calorimeter for the reflected wave receive light in a rather broad solid angle. Photographic materials are used to measure the brightnesses of the two beams: the pump, with a diffraction-level divergence, and the scattered wave reconstructed by the phase plate. The limited dynamic range of the photographic materials is countered by splitting each of the beams into a fan of spatially similar beams and then reducing the intensity by a constant factor (a halving of the intensity is the most convenient). For this purpose, a mirror wedge is used in this method. The wedge is formed by two half-silvered plane mirrors (Fig. 10a) or by one total-reflection mirror and one half-silvered mirror (Fig. 10b).

As a result, somewhere among the different beams in the fan there are necessarily some whose energy falls

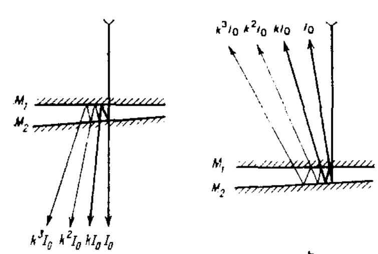

FIG. 10. The mirror wedge, which splits a beam into a fan of beams of progressively greater attenuation. The attenuation factor k is determined by the reflection coefficients R_1 and R_2 of mirrors M_1 and M_2: $k = R_1R_2$.

in the working range of the particular photographic material. It is thus possible to compare quantitatively the brightnesses and angular distributions of any beams in a single laser shot. If the entire recording is made on a single photographic plate, the variations in the properties of the photographic material and in the development conditions will have absolutely no effect on the measurement accuracy. Furthermore, this approach automatically yields a blackening curve for the material.

The wave reconstructed by the phase plate contains a diffraction-level bright core in the far zone. If the scattered wave consisted exclusively of the conjugate component (the conjugate of the pump), the brightness ratio of the scattered and pump waves, B_S/B_L, at the centers of the diffraction-level cores would be equal to the energy ratio W_S/W_L measured by the calorimeters. Any distortion of the scattered field in comparison with the exact conjugate would cause the ratio B_S/B_L to become lower than $W1/W_L$.
come lower than W_S/W_L. The conjugate fraction H, expressed analytically by (24), is

$$H = \frac{W_L}{W_S}\frac{B_S}{B_L}. \tag{65}$$

b) Investigation of the conjugate component

1). *Discovery of* PCSS. Figure 11 is a diagram of the experimental apparatus in which phase self-conjugation was observed in Ref. 20. The beam from the ruby pump laser ($\tau = 110$ ns, $W \approx 0.14$ J) had a diffraction-level divergence along the vertical coordinate, $\sim 10^{-4}$ rad, and a slightly poorer divergence along the horizontal coordinate. This beam was measured by the mirror-wedge method. A corresponding photograph is shown in Fig. 12a (the "left eye"). Transmission of the beam through the phase plate resulted in the appearance of a large, irregular divergence $\delta\theta \approx 3 \cdot 10^{-2}$ rad. This divergence is illustrated in Fig. 12c (the "hairy face") by the angular distribution of the pump beam after it has passed through the phase plate, has been reflected by an ordinary mirror, and has passed back through the phase plate in the opposite direction. In the experiment, an image of the phase plate was produced by a lens with $f = 1$ m at the entrance to a hollow glass light pipe of square cross section (inside dimensions of 4×4 mm; length $L \approx 1$m). As the light propagated through this pipe, the various angular components interfered with each other, giving rise to many maxima and minima. Essentially no energy escaped through the side walls because of the high Fresnel reflection coefficient of these walls at grazing-angle incidence.[3)]

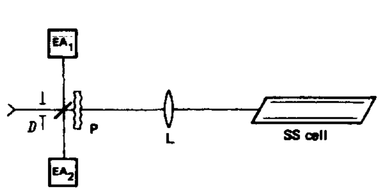

FIG. 11. Arrangement of the first experiment carried out to observe phase conjugation accompanying stimulated scattering (SS). EA_1 and EA_2—Apparatus for measuring the energies and angular distributions of the incident and conjugate beams; P—phase plate; L—lens used to construct an image of the phase plate at the entrance of a light pipe in a cell holding gaseous methane compressed to 130 atm.

FIG. 12. Far-zone angular distributions. a—Incident beam before it reaches the phase plate; c—after a double pass through the phase plate without conjugation; b—the reflected beam reconstructed by the phase plate; d—angular spectrum of the reflected light in the absence of the phase plate.

Compressed methane inside this pipe gave rise to stimulated Brillouin backscattering. The reflection coefficient in these experiments was $W_S/W_L \approx 25\%$ in terms of the energies measured by the calorimeters.

The stimulated-scattering beam passed through the optical elements in reverse order and produced an angular distribution in the far zone beyond the phase plate which reproduced the diffraction-level quality of the laser beam along the vertical coordinate and all the details of the angular structure of this beam along the horizontal coordinate (Fig. 12b; the "right eye"). Comparison of the brightnesses of the angular distributions of the laser beam and of the scattered wave after it traversed the phase plate yielded the ratio $B_S/B_L \approx 0.25$. By comparing this ratio with the ratio $W_S/W_L \approx 25\%$ found from the calorimeter measurements of the energies, we find the conjugate fraction to be $H = 1$. Within the experimental error ($\lesssim 15\%$), therefore, *all* the energy of the reflected light is in the component which is the *exact* conjugate of the pump.

To determine the role played by the spatial inhomogeneity of the pump, the experiment was repeated without the phase plate. In this case, the light incident on the entrance to the light pipe was a nearly ideal outgoing spherical wave formed by a lens from a parallel beam. The intensity inhomogeneities which arose were far more pronounced than those which resulted from the use of the phase plate. The angular distribution of the

[3)]We might note that there are no stringent requirements on the geometry or quality of the light pipe. Its basic role in this case is to provide a large distance over which the nonlinear interaction of the light with the medium can take place. We are therefore using the term "light pipe" rather than "waveguide."

backward-reflected light in this case is shown by the "mouth" in Fig. 12d. Here again we see that the directionality of the reflected light is extremely good, $\delta\theta \sim 5\cdot 10^{-3}$ rad, although there is clearly no phase conjugation. Consequently, the matched PC solution (the specklon) does not exist if the pump inhomogeneities in the interaction volume are too large. A theoretical analysis[39,42] (see also Subsection 2c) shows that a necessary condition for the existence of a specklon is that the gain over the longitudinal diffraction length of the inhomogeneities be small: $\pi g/k\delta\theta_L^2 \lesssim 1$. Since we have $2gL \approx 30$ under these measurement conditions, we find $L/(\lambda/\delta\theta_L^2) \gtrsim 7$, where L is the total interaction length.

2) *Measurement of the amplification of the phase-conjugate component.* The key aspect of the PCSS mechanism is the discrimination against the amplification of the uncorrelated waves in comparison with that of the phase-conjugate component. Although this mechanism is generally accepted for a speckle-modulated pump, for a long time no direct experimental comparison was made of the various configurations in terms of their amplification effectiveness.

Only recently, in Refs. 80 and 81, have the intensities of the uncorrelated and phase-conjugate components been measured during stimulated Brillouin backscattering for the case of excitation *from spontaneous noise.* Some slightly different quantities were later measured in Ref. 82: the gain levels for some *specially prepared uncorrelated configurations* and for phase-conjugate configurations. The results of both these experiments confirm the theoretical conclusions. Let us examine these experiments in more detail.

In Refs. 80 and 81, phase conjugation accompanying stimulated Brillouin scattering was excited by the beam from a ruby laser in a circular light pipe filled with liquid carbon disulfide, CS_2. The energy carried by the conjugate component was measured with a calorimeter. The uncorrelated components over a very broad solid angle ($\Delta O \sim 10^{-1} \times 10^{-1}$ rad^2) were detected with both photographic film and a calorimeter, and they were also subjected to spectral analysis. At a low pump level the conjugate component was missing, and over a broad solid angle there was scattered light for which the spectrum and dependence on the pump intensity corresponded to a spontaneous process. As the pump intensity was raised, stimulated scattering was effectively excited, and the discrete conjugate component appeared with a quite noticeable intensity. Intensity measurements led to calculations of the gain ratio $g_0/g_{\rm not}$ for the conjugate and uncorrelated components; it turned out to be $2.04 \pm 0.06 \leqslant g_0/g_{\rm not} < 2.43 \pm 0.07$, where the higher value corresponds to the assumption that the maximum number of light-pipe modes are excited simultaneously, while the lower value coresponds to the assumption that it is instead the minimum number of modes which are excited simultaneously.[81] We thus have a completely satisfactory correspondence with the theoretical prediction $g_0/g_{\rm not} = 2$. There is some quantitative discrepancy, which can also be attributed to several additional factors: a partial coherence of the spontaneous sources, depolarization of the pump and scattered fields at the walls of the circular light pipe, and an increase in g_0 due to capture into active spiral microwaveguides (see Section 2 and Ref. 43).

In a report to the Tenth Conference on Coherent Nonlinear Optics, held in Kiev in October 1980, Baranova, Zel'dovich, Shkunov, and Yalkovleva suggested a test of the theory by an experiment with a differential-amplification cell. This suggestion was subsequently implemented in Ref. 82, where an auxiliary "differential cell" was used. A speckle-inhomogeneous pump field of moderate intensity was directed into the cell. The stimulated scattering from the spontaneous noise was not excited in this cell, but there was a noticeable amplification of some specially prepared counterpropagating Stokes waves which were incident on this cell. The cell contained a rectangular-section light pipe filled with liquid carbon disulfide, which kept the pump polarization constant. Special measures were taken to arrange statistical homogeneity of the speckle field of the pump in the interaction volume. Measures were also taken to ensure that the gain over the length of a single band of a spiral was small ($g/k\delta\theta_L^2 \leq 0.1$) and to ensure that the amplification was linear.

A specially prepared signal, frequency-shifted so that it would undergo stimulated-Brillouin amplification in the laser pump field, was coupled into the light pipe in the opposite direction. Two types of signals were used: one with the exact conjugate of the transverse structure of the pump and one which was uncorrelated with the pump. In the first case the phase-conjugate signal was produced by threshold-free stimulated-Brillouin conjugation of the pump beam, after it had passed through the differential cell, in a second cell, also filled with carbon disulfide. In the second case, the signal was produced through the stimulated Brillouin scattering of an auxiliary plane wave in an additional cell holding CS_2. The auxiliary wave propagated at an angle with respect to the central direction of the pump. The logarithm of the total Stokes-wave gain in the differential cell turned out to depend linearly on the pump intensity. At a given pump intensity, the gain for the conjugate wave exceeded that for the uncorrelated wave by a factor of 2.0 ± 0.2, in agreement with the theory.[20,38,39,41]

3) *Conjugation of depolarized light.* Close on the heels of a theoretical paper[45] on PCSS of depolarized beams came two simultaneous independent experimental studies, reported in Refs. 83 and 84, on the one hand, and 85 and 86, on the other. The practical aspects of PCSS were studied in Refs. 85 and 86 in connection with the depolarization of light as it passes through the rods of neodymium amplifiers. An arrangement for total spatially polarized phase conjugation was also developed in Refs. 85 and 86. That arrangement made use of the reversible conversion of a depolarized beam into a completely polarized beam with a large angular divergence. We will be discussing the results of Refs. 83 and 84 in more detail here; those results permit a direct quantitative comparison with the theoretical conclusions.

An experimental arrangement was developed in Refs. 83 and 84 in which each of the specklons in (42)–(44)

could be detected separately and its energy measured. The most important part of that arrangement was a special phase plate, which introduced not only phase inhomogeneities but also inhomogeneities of the polarization state in the wave. It furthermore doubled as a device for analyzing the quality of the phase conjugation. The depolarizer was a calcite plate cut parallel to the optic axis and etched in nitric acid. The etching left craters on the surface with an average depth ~8 μm and a transverse dimension ~250 μm; these craters were sufficient for creating a path difference $\gtrsim\lambda$ for the orthogonal polarizations ($n_0 - n_e = 0.16$). The plate was placed in an immersion liquid with a refractive index $n \sim (n_o + n_e)/2$. As a result, the divergence levels of the beams with the orthogonal polarizations were the same beyond the depolarizer.

The degree of linear polarization of the pump in the nonlinear medium was controlled by sending an initially linearly polarized laser wave to the depolarizer through a Faraday cell, in which the angle (α) through which the polarization vector of the wave was rotated with respect to the axis of the depolarizer could be controlled. The degree of polarization of the beam in the cell turned out to be $p = |\cos 2\alpha|$. The back-scattered light passed through the depolarizer and was sent to a system which measured the energy and angular distribution. Figure 13a shows the experimental values of η_1 and η_2, the fractions of the energy in the specklons $\mathbf{M}_1$ and $\mathbf{M}_2$, respectively, as functions of the degree of polarization of the pump, p (see Subsection 2f for the definition of the specklons $\mathbf{M}_1$ and $\mathbf{M}_2$). Figure 13b shows the reciprocal of the threshold pump intensity as a function of p.

For the completely polarized light ($p=1$), only a single phase-conjugation specklon, $\mathbf{M}_1$, is excited; it contains nearly all the back-scattered energy ($95 \pm 10\%$). The uncorrelated waves were thus suppressed almost completely by the discrimination mechanism. With decreasing p, the gain for the uncorrelated solutions remains lower than that for the conjugate component by a factor of two,[83,84] so that these uncorrelated solutions are not found in the scattered light at any value of p. The same is true of the specklon $\mathbf{M}_4$, for which the gain is zero in general. At $p \lesssim 0.5$, the specklon $\mathbf{M}_3$ is noticeable with a fraction $\eta_3 = 1 - \eta_1 - \eta_2$, and at $p \lesssim 0.2$ the specklon $\mathbf{M}_2$ is noticeable. For a pump beam which is

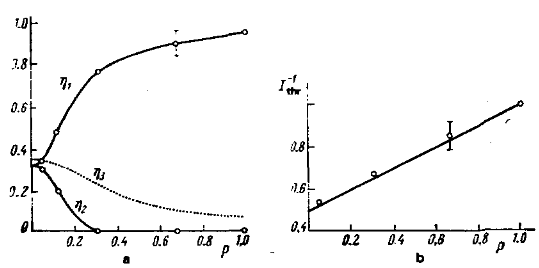

FIG. 13. a—The measured fractions of the energy in specklons μ_1, μ_2, and μ_3 (η_1, η_2, and $\eta_3 = 1 - \eta_1 - \eta_2$) as functions of the degree of polarization p; b—reciprocal of the threshold intensity for stimulated scattering. The points are experimental, and the line is theoretical.

completely depolarized in the interior of the medium ($\alpha = 45°$), all three specklons have the same energy, on the average: $\eta_1 \approx \eta_2 \approx \eta_3 \approx 1/3$. This experimental result agrees well with the theoretical conclusion that the gain levels are the same for all three of the specklons $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$ at $p=0$.

Experimentally, the specklons $\mathbf{M}_2$ and $\mathbf{M}_3$ arise at a higher value of p than would be expected from the simple theoretical expression $\exp(g_i L)$ with g_i from (42)–(44). The apparent reason for the deviation is a saturation of the stimulated scattering: The more intense polarized component $E_1 \mathbf{e}_1$ is depleted more rapidly because of pumping into the specklon $\mathbf{M}_1$; as a result, the effective degree of polarization in the interaction volume decreases.

An important advantage of the method used in Refs. 83 and 84 to detect the pulse shape is that it is possible to measure the stimulated-scattering threshold, where saturation effects have not yet come into play. For this reason, the experimental data on the reciprocal of the threshold power agree well with the theoretical prediction $g_1 \sim (1+p)$ for the specklon $\mathbf{M}_1$.

In summary, the experimental results of Refs. 83 and 84 confirm in detail the specklon picture of PCSS.

4) *Fluctuations in the common speklon phase.* The stimulated emission in the active media of both lasers and stimulated-scattering devices grows from a spontaneous-noise level. For stimulated Brillouin scattering this spontaneous noise corresponds to the scattering of the exciting field by fluctuational hypersonic waves. Because of the exponential nature of the amplification, $\sim\exp(gz)$, the amplified Stokes field comes primarily from a comparatively thin layer of the medium at the beginning of the amplification region, with a thickness $\Delta l \sim g^{-1}$. The correlation scale time of the nucleating fluctuations is $\tau_S \sim \Gamma^{-1}$, where Γ is the width of the scattering line, and τ_S is the same as the decay time of a hypersonic phonon. Typically, $\tau_S \sim 10^{-8}$–10^{-9} s.

The excitation of stimulated scattering from spontaneous noise determines many important aspects of PCSS. One example is the possibility of achieving phase conjugation of extremely weak signals, as will be discussed in detail in Subsection 3d below. Just how weak the signals can be is determined by the level of the amplified spontaneous, uncorrelated noise. Here we will discuss another consequence of the nature of this excitation: fluctuations of the complex amplitude of the phase-conjugate specklons. The amplitude gain for a specklon in the linear regime is given by $\exp[g(\omega)z]$, where $g(\omega)$ basically reproduces the shape of the spontaneous-scattering line with a width Γ. Consequently, as simple arguments show, the correlation time of the fluctuations in the amplified light increases to a level of the order of $\tau_{corr} \sim \Gamma^{-1}\sqrt{2gL} \sim 5/\Gamma$, where $2gL \approx 30$. This increase corresponds to an approximately fivefold contraction of the stimulated-scattering line in comparison with the width of the spontaneous line.

If the length of the Stokes pulse, T_S, is greater than $\tau_S \sim 5/\Gamma$, the complex amplitude of the Stokes wave can undergo $N \sim T_S/\tau_S$ fluctuations. If saturation of the stimulated scattering is also important, the fluctuations in the modulus of the amplitude are suppressed almost completely, while the phase fluctuations remain as before (see Refs. 115 and 116).

Specklon phase fluctuations during stimulated scattering were first observed in Refs. 83 and 84. With a completely depolarized pump, the scattered light contains three specklons: M_1, M_2, and M_3, described by expressions (42)–(44). The amplitudes of these specklons are approximately equal. Since each of the specklons is excited and amplified independently, the phases of these specklons, φ_i, should fluctuate independently, as was found experimentally (see the original paper for more details).

Fluctuations in the phases of two waves scattered with phase conjugation from two different parts of the same cell were observed in Ref. 85. These fluctuations caused variations in the shape of the spot reconstructed by the phase plate in the far zone.

Detailed quantitative studies of the temporal fluctuations in the complex amplitude of specklons were carried out in some studies reported simultaneously.[87-89] The results found in the different studies agree well, despite the differences in measurement methods. For the present discussion we will work from Ref. 88. A laser beam with a plane wavefront is split by half-silvered mirror M into two beams of equal intensity, which are then sent through two identical phase plates into two cells holding the same substance. The phase-conjugated scattered light is reconstructed into plane waves by the plates, and the plane waves are then combined at mirror M to form a common plane wave of fluctuating amplitude. The time evolution of the intensith of this wave is measured by a photodetector and displayed on an oscilloscope screen. Several precautionary measures (described in detail in Ref. 88) were taken to eliminate a possible coupling of the two waves and to eliminate any differences in the excitation conditions for the two waves.

The PCSS thus occurred independently in each cell, and the fluctuations in the phase difference between the reconstructed specklons gave rise to interference fluctuations in the intensity measured by the photodetector. Figure 14 shows some typical oscilloscope traces of the intensity for stimulated scattering in CCl_4 and acetone. With the experimental error, the correlation time for the fluctuations agreed with the theoretical estimate $5/\Gamma$, where Γ is the width of the spontaneous-Brillouin-scattering line in the corresponding substance.

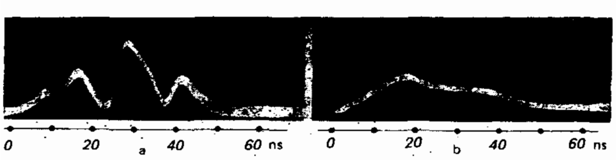

FIG. 14. Some typical oscilloscope traces of the intensity of the beam formed from the two Stokes beams of identical intensity scattered from different cells holding the same substance. a—CCl_4 with $\Gamma \approx 2.5 \cdot 10^9$ s^{-1}; b—acetone with $\Gamma \approx 0.45 \cdot 10^9$ s^{-1}.

c) Angular structure and intensity of the uncorrelated waves

Study of the noise, i.e., uncorrelated, component is important for learning about the physics of PCSS and also in connection with the problem of the quality of phase conjugation. Amplified uncorrelated waves are unavoidably present as a result of the spontaneous nature of the nucleating sources for stimulated scattering.

The intensity of the spontaneous nucleating noise for Brillouin backscattering is, in ergs per square centimeter per second per steradian,

$$P_{sp} = \frac{\omega_L}{\Omega_{Br}} k_B T \cdot \frac{k^2}{(2\pi)^3} \Delta\omega_{AM}. \tag{66}$$

This expression can be derived by making use of the relationship between the gain and the spontaneous-scattering cross section.[63,111] Here $\Omega_{Br} = 2kv_{sound}$ is the frequency shift in the stimulated Brillouin scattering; $k_B T$ is the temperature of the medium, expressed in energy units; k is the wave number; $\Delta\omega_{AM}$ is the width of the effective gain band; and $\Delta\omega_{AM} \simeq \Gamma/\sqrt{gL}$, where Γ is the width of the scattering line (for more details on this last factor, see Subsection 2c). To determine the intensity W_{con} (in ergs per second) of the spontaneous nucleating noise, referred to the exactly conjugate component, we should multiply P_{sp} by the factor $S\Delta O = (2\pi)^2/k^2 \equiv 1/\lambda^2$, which corresponds to the excitation of modes with a single transverse index ($\Delta O = \lambda^2/S$) in a light pipe with a cross-sectional area S (in square centimeters), where λ is the wavelength in the medium. The intensity of the scattered field at the exit from the medium is thus

$$W\left(\frac{\mathrm{erg}}{\mathrm{s}}\right) = \frac{\omega}{\Omega_{Br}} k_B T \frac{\Gamma}{2\pi\sqrt{gL}} \left\{ e^{2gL} + \frac{S}{\lambda^2} \int dO \exp[g(\mathbf{n}) L] \right\}, \tag{67}$$

where the first term corresponds to the amplified phase-conjugate component, and the second corresponds to the uncorrelated noise. Here we are taking into account the circumstance that the gain for the uncorrelated waves may decrease with increasing angle θ $[\cos\theta = (\mathbf{e}_z \mathbf{n})]$, because of, for example, an increase in the loss at the walls of the light pipe. If we introduce the effective solid angle for amplification of the uncorrelated noise by means of the definition $\Delta O_{uncorr} = \int \exp\{[g(\mathbf{n}) - g(\mathbf{e}_z)] \times L\} dO$, we find that the intensity ratio of the phase-conjugate and uncorrelated components—a ratio which should be large for high-quality phase conjugation—is

$$\frac{W_{PC}}{W_{uncorr}} = e^{gL} \frac{\lambda^2}{S\Delta O_{uncorr}} \equiv e^{gL} \left(\frac{\Delta\theta_{\text{диф}}}{\Delta\theta_{uncorr}}\right)^2 \overset{!}{\gg} 1, \tag{68}$$

where $\Delta\theta_{diff} \approx \lambda/\sqrt{S}$ is the diffraction-level divergence corresponding to a transverse dimension $\sim\sqrt{S}$. In actual experiments, we might note, $\Delta\theta_{uncorr}$ may be significantly smaller than would be allowed by the properties of the light pipe, because of structural limitations (lenses and diaphragms) which transmit only a fraction of the uncorrelated waves. The discrimination inequality (68), which tells us that the uncorrelated noise is at a low level, is a very important condition for the achievement of PCSS, along with the condition that the spiral noise be at a low level (15b).

Direct experimental studies of the uncorrelated noise were carried out in Refs. 80, 81, and 90–92. The results of Refs. 80 and 81 are described in Part 1 of Subsection 3b. Amplified uncorrelated radiation was first detected photographically, with measurements of the absolute brightness and with a high spatial resolution, in Refs. 90 and 91. A photograph of the noise shown in Refs. 90 and 91 exhibits the speckle-inhomogeneous structure of the noise field with a high contrast and with spots having an angular dimension ~0.2 mrad, which correspond to a diffraction-level divergence for the end of the light pipe, ~5 mm in diameter. These two circumstances mean that the length of the scattered pulse is of the order of the correlation time of the amplified spontaneous noise and that the noise waves fill the exit end of the light pipe uniformly, on the average. The noise intensity in a unit solid angle was measured both photographically[90,91] and with a calorimeter.[92] In specific experiments, the (intensity) reflection of the pump into noise waves per unit solid angle was $\rho_{\text{noise}}(0) \approx 1.5$ sr^{-1} near the central direction, and the corresponding reflection into the diffraction solid angle was $\rho_{\text{noise}}\Delta\theta^2_{\text{diff}} \approx 5\cdot 10^{-8}$. The measured angular distribution of the noise brightness can be described by the empirical expression $\rho_{\text{noise}}(\beta) = \rho_{\text{noise}}(0)\exp(-0,1l\beta^2/d)$, where β is the angle in the medium (expressed in radians), l is the length of the light pipe, d is its diameter, and the number 0.1 is an extremely good approximation of $n-1$, where n is the relative refractive index of carbon disulfide and quartz, i.e., of the materials making up the interior of the light pipe and its walls. The value of l was varied from 0.33 m to 1 m, d from 0.2 to 0.6 cm, and β from 0 to 0.3 rad. The mechanism responsible for the decay of $\rho_{\text{noise}}(\beta)$ with increasing β is apparently the depolarizing effect of oblique reflections from the walls of the light pipe. The $\rho_{\text{noise}}(\beta)$ distribution is essentially independent of the pump divergence, which was varied from 10 to 40 mrad (in air); it is also independent of the pump intensity under saturation conditions. This result is further evidence that the measured noise was uncorrelated with the pump.

The integral of $\rho_{\text{noise}}(\beta)$ over solid angle gives us the total energy of the noise radiation. Under the typical expreimental conditions of Refs. 90–92, this total energy was ~13% of the pump energy, in comparison with the roughly 50% of the energy which was reflected in the conjugate component. The conjugate fraction would be formally calculated to be $H = 50/63 \approx 0.79$ here, but in practice the solid angle of most of the elements of laser apparatus is not large, $\Delta\theta^2_{\text{uncorr}} \ll (10^{-1}\ \text{rad})^2$. Only a small fraction of the noise enters this aperture, so that the calculated fraction of the uncorrelated spontaneous noise is of the order of 1%.

d) PCSS of subthreshold signals

Let us assume that we are to conjugate radiation at an intensity below the threshold for stimulated scattering. This difficulty can be overcome by mixing the signal to be conjugated with another wave (a reference wave) whose intensity is above the stimulated-scattering threshold and then arranging PCSS of the resultant field. This idea was suggested and implemented experimentally in Ref. 93; see also Ref. 94. If the time evolutions of the signal and reference waves are not identical, the efficiency of the phase conjugation of the signal will be lowered. In Ref. 95 the subthreshold signal was sent into the nonlinear medium at a time at which PCSS of the reference wave had already been established in the medium. It was shown experimentally and theoretically in that study that when the reference and signal waves have the same frequency the reflection coefficient for the signal wave reaches a steady-state value equal to that for the reference wave in a time $\tau \sim \Gamma^{-1}$, where Γ (in reciprocal seconds) is the Lorentz width of the stimulated-Brillouin-scattering line. In Ref. 96 the subthreshold signal was of approximately the same duration as the reference wave, and the frequency shift of the signal was varied. The theoretical and experimental study of Ref. 96 showed that effective phase-conjugation reflection of the signal occurs in a narrow spectral interval, specifically, within the line of spontaneous Brillouin scattering near the frequency of the reference wave.

The results of Refs. 95 and 96 agree well with each other. Those studies yielded the important result that the scale time for the onset of stimulated Brillouin scattering from spontaneous noise (at a pump level moderately above the threshold for stimulated scattering) is approximately $2gz \approx 25$ times the value of Γ^{-1}. Furthermore, the spectral width of the radiation scattered under stimulated-scattering conditions is smaller than the line width Γ by a factor of about $\sqrt{2gz} \approx 5$. The reason for the more rapid response of the system to a weak signal is as follows. It is convenient to discuss the process as a four-wave mixing. As the subthreshold signal E_3 interferes with the conjugate reference wave $E_2 \equiv E_S \propto E_L^*$, it records a travelling hypersonic grating proportional to $E_3^* E_2$, localized near the entrance end of the cell, where $E_2 = E_S$ reaches a significant intensity. The requirement for this grating to be recorded is that the time for the resonant hypersound to reach locally a steady-state amplitude be $\tau \sim \Gamma^{-1}$. The recorded grating $E_3^*E_2$ is then read out instantaneously by the reference wave $E_1 = E_L$.

The phase conjugation method for weak signals described above is frequently called the "threshold-free arrangement" for PCSS. It has recently been adopted quite extensively in the laboratory.

e) PCSS in focused beams

The light pipe makes it possible to keep the pump intensity at a high level over a long interaction distance L. The threshold for stimulated scattering, set by $2G|E_L|^2L \approx 30$, is thus comparatively low. Another important advantage of arrangements with light pipes is that the various angular components of the pump are mixed well, so that the average pump intensity is uniform in the volume of the medium. Repeated reflection from the walls of the light pipe makes it possible to achieve phase conjugation with a fully coherent envelope.

On the other hand, the simplicity of the arrangement in which the pump is focused into a nonlinear medium without a light pipe is always attractive to the experimentalists (while the mathematical complexity of the problem scares off the theoreticians). For this reason, many experimental studies use PCSS in a focused pump beam. Without going into the fine points of the rather complicated theory,[46,44,47,97] we will estimate the basic parameters.

A large number of speckle inhomogeneities in the interaction volume is a necessary condition for the operation of the PCSS discrimination mechanism. Let us assume that a "gray" divergence $\Delta\theta_0$ of a speckle-inhomogeneous pump beam of diameter D exceeds the diffraction minimum by a factor of ξ; i.e., $\xi = \Delta\theta_0 D/\lambda \gg 1$. Then at the focus of the lens, at a distance f, a focal neck of diameter $d \approx 2f\Delta\theta_0$ and length $L \approx f^2\Delta\theta_0/D$ forms. The corresponding characteristic dimensions of the speckle-inhomogeneities at the neck are considerably smaller: $\Delta r_\perp \sim d/\xi, \Delta z \sim L/\xi$. Since the total gain near the stimulated-scattering threshold over distance L is about e^{30}, condition (15)—under which the spiral noise is low—gives us an estimate of a lower boundary on the divergence for good-quality phase conjugation: $\xi \geqslant 15$.

An upper boundary on the divergence is set by the condition that the uncorrelated noise must be at a low level (Subsection 3c). The point is that the number of angular components with a significant amplification increases in proportion to ξ^2 with increasing gray divergence of the pump. For the most part, noise emitted from an area $S \approx (d/\sqrt{30})^2$ and in the solid angle $(\Delta\theta_{\text{noise}})^2 \approx (D/f\sqrt{30})^2$ is amplified. These values are smaller than the corresponding parameters for the pump, d^2 and $(D/f)^2$ each by a factor of 30. The reason for this contraction lies in the high total gain in the active waveguide formed by the smooth envelope of the pump; a contraction of this sort is typical of this amplification geometry and is analogous to the contraction of the stimulated-scattering line (see Subsection 3b). The number of modes over the area S within the solid angle $(\Delta\theta_{\text{noise}})^2$ is known to be $N \approx S(\Delta\theta_{\text{noise}})^2/\lambda^2$; on this basis the number of uncorrelated waves which are effectively excited can be estimated to be [46]$N_{\text{noise}} \approx (\xi/30)^2$.

At first glance the condition under which the phase-conjugate component would be singled out, and there would be a discrimination against noise, seemingly would have to be $N_{\text{noise}} < \exp(gL) \approx e^{15}$. For focused beams, however, this is not the case.[46] In the absence of saturation the conjugate wave has a smooth envelope which falls off radially with distance from the beam axis[44,47,97]: $E_S(\mathbf{r}) \propto f(\mathbf{r})E_L^*(\mathbf{r})$. As a result, the phase-conjugate specklon is also mismatched with the fine structure of the gain profile.[97] Detailed calculations[47,97] show that the average gain for the specklon in this case is 1.3–1.4 times the gain for uncorrelated waves.[4)] In a focused beam, therefore, the discrimination against an individual noise component is less effective: $1/Q \approx 10^{-3} \approx \exp[-30(1.3-1)/1.3]$. The conjugate wave is thus accompanied by noise—by the uncorrelated components—under the condition $N_{\text{noise}}Q \gtrsim 1$, i.e., $\xi \gtrsim 30/\sqrt{Q} \approx 9 \times 10^2$.

Phase conjugation accompanying stimulated scattering in a focused beam was first detected experimentally in Refs. 30 and 31. A detailed experimental study was carried out in Ref. 46. The threshold beam power increased linearly with increasing ξ; the amplification distance was $L \propto \xi$, and the intensity (in watts per square centimeter) at the neck was proportional to $d^{-2} \propto \xi^{-2}$. The conjugate fraction was measured by a diaphragm method combined with a phase plate; the latter also made it possible to adjust the gray divergence of the pump. Good-quality phase conjugation was observed over the range $10^2 \lesssim \xi \lesssim 10^3$, in agreement with the theory of Ref. 46. (Experiments were not carried out at $\xi \lesssim 10^2$, while at $\xi \gtrsim 10^3$ the conjugate fraction dropped off sharply.) The maximum conjugate fraction, ~80%, was observed in the interval $10^2 \lesssim \xi \lesssim 5\cdot 10^2$.

As it exists today,[47,97] the theory for PCSS ignores saturation effects, which nearly always occur experimentally. There is reason to believe that saturation makes the pump envelope flatter, and this effect would in turn improve the fine structure of the phase conjugation and increase the degree of discrimination. We might add that up to this point there has been no detailed study of how phase conjugation is affected by such factors as the focal length of the lens, the position of the center of the neck in the nonlinear medium, or time variations. Furthermore, there has been no study in the extremely interesting range $\xi \lesssim 50$. The large number of parameters available raises the hope that it will be possible to optimize PCSS in a focused beam. In Refs. 30 and 83, 84, under significantly different conditions, for example, conjugate fractions of 100% were achieved (within an experimental error of the order of 10–20%).

f) Phase conjugation accompanying other amplification mechanisms

Underlying the PCSS discrimination mechanism is the coincidence of the local maxima of the exciting and amplified fields. Clearly, this mechanism should in principle operate regardless of the type of amplification, provided that the local gain increases with increasing local intensity of the speckle-inhomogeneous pump. So far, discrimination phase conjugation has been studied for the following amplification mechanisms (in addition to stimulated Brillouin scattering): stimulated Raman scattering,[41,49,50,98,99] stimulated Rayleigh-wing scattering,[99] stimulated thermal scattering of light caused by absorption,[100,101] amplification of a dye under superluminescence conditions with a speckle-inhomogeneous pump,[52,102] and saturable media.[117]

For stimulated Raman scattering and dye superluminescence, there is typically a large frequency shift, ranging up to a few tenths. This shift prevents the

[4)]Here we are using results from a more recent paper,[97] in which the important conclusion of Refs. 46 and 47 that there is a decrease in the discrimination among gain values was refined quantiatively. A degree of discrimination of 1.3 moreover leads to a better agreement with experiment.[46]

practical use of these two phase-conjugation methods, but they remain of scientific interest. The first systematic studies of phase conjugation accompanying stimulated Raman scattering were reported in Refs. 49 and 50; these studies were followed by a rather large number of other papers (see Refs. 98, 99, and 103, for example). A phase-plate method was used to detect phase conjugation in Ref. 49. A distinctive feature of Refs. 50, 98, and 99 and of other studies by Sokolovskaya's group is the systematic use of amplitude (rather than phase) distorting transparencies, with a subsequent focusing of the beam into a medium without a light pipe. Let us assume that a plane wave is incident on an amplitude transparency. The transmitted light will then retain, in addition to the inhomogeneous component caused by diffraction by the transparency, a significant fraction of its energy in the form of a discrete plane wave travelling in the original direction. When this beam is focused by a lens, a bright focus is produced in the medium from the plane wave, and it is here that the stimulated scattering is initiated. The Stokes field is then amplified between the entrance window of the cell and the focus, i.e., in a part of the medium where the regular and inhomogeneous parts of the pump beam are mixed and interfere. In this region, a discrimination mechanism comes into play: The component which is correlated with the pump is amplified the most (this problem was studied in Ref. 42, with allowance for the presence of a regular part in the pump). Sokolovskaya *et al.*[50,98,99] describe this mechanism in terms of a readout of a volume amplifying hologram by means of a spherical Stokes wave being emitted from a diffraction-quality constriction. The implication is that the hologram is recorded by an interference of the regular and irregular parts of the pump. In those studies, the occurrence of phase conjugation was inferred from the reconstruction of an image of a transparency by the wave at the Stokes frequency. In some extremely interesting experiments,[98] a longitudinal displacement of the resulting image and a change in its transverse scale dimension were detected and measured. These changes resulted from a difference between the propagation laws for the pump and the Stokes wave in air, where the interaction between these waves was cut off. The results emerging from these measurements correspond to a picture of a "detachment" of the Stokes wave from the pump wave at the boundary of the nonlinear medium.

To the best of our knowledge, Kudryavtseva *et al.*[99] have reported that only study of the PCSS of ultrashort light pulses (~25 ps) accompanying Rayleigh-wing stimulated scattering. This type of stimulated scattering has a small frequency shift and a short rise time, so that phase conjugation by this mechanism may find practical applications, including problems involving the phase conjugation of depolarized fields.[104]

Phase conjugation accompanying stimulated thermal scattering was observed and studied in Refs. 100 and 101. The frequency shift in stimulated thermal scattering is extremely small (an advantage for applications), but the rise time of this scattering is unfortunately quite long. A curious aspect of phase conjugation by stimulated thermal scattering is that it is accompanied by a significant spectral broadening of the scattered wave. This broadening seems to result from phase modulation resulting from thermal changes in the refractive index of the medium being heated.[100]

Phase conjugation accompanying superluminescence was observed in Ref. 52 during pumping of a dye by the second harmonic of a neodymium laser ($\lambda = 532$ nm) transmitted through a phase plate. The phase plate was used both to produce the speckle inhomogeneities in the gain in the volume of the dye and to monitor the phase conjugation. The superluminescence radiation in the spectral interval from 545 to 565 nm has a bright diffraction-quality core after the return passage through the phase plate. The conjugate fraction was measured and found to lie between 10% and 20%.

4. CONCLUSION

We have attempted to describe the present level of experimental research on phase conjugation accompanying stimulated scattering. We have not had room here to cover anything like all the most important studies. In particular, we have said almost nothing about the research more closely related to applications (see Refs. 60, 105, and 55, for example).

Studies of lasers with phase-conjugation mirrors are very interesting (see Refs. 117–120 and 60, for example). The resonators of such lasers have many interesting features, e.g., an absence of angular or frequency selection and a reduced sensitivity to inhomogeneities of the amplifying medium.

A long list of papers (about 200 as of January 1982) have been published on the four-wave method for phase conjugation. A discussion of those studies requires a separate review, but we do wish to make the following comments: The four-wave method has the advantage that there is no threshold, the conjugation can be carried out with a simultaneous amplification of the conjugate signal, and the conjugate wave can be controlled. So far, phase conjugation in the far-IR range ($\lambda \gtrsim 2\,\mu$m) has been achieved only by four-wave mixing. Among the disadvantages of four-wave mixing are the severe requirements placed on the wavefront quality of the reference waves and on the optical homogeneity of the nonlinear medium. In contrast with four-wave mixing, phase conjugation by Brillouin scattering is a *self-conjugation method*, i.e., it does not require high-quality auxiliary reference waves and does not impose stringent conditions on the homogeneity of the nonlinear medium.

The problem of phase conjugation in the *UV* range is highly interesting. The first experimental results have already appeared.[121-123]

The material covered in this review suggests that the physics of phase conjugation accompanying stimulated scattering has basically been clarified. There remains a need, however, for extensive systematic studies of this phase conjugation with the ultimate goal of optimizing practical devices making use of phase conjugation.

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