G. M. Zaslavskiĭ. *Aspects of the origin of stochasticity in quantum systems*. We define stochasticity as the chaotic (random) motion of classical dynamic systems that arises in the absence of all random forces or parameters.[1] The phenomenon of stochasticity results in mixing of paths in phase space (the term "*K* system" is also used with certain reservations). In Hamiltonian systems, stochasticity arises as a result of breakdown of separatrices and the formation of a stochastic layer around them.[2] From the dynamic standpoint, the basis of the phenomenon of stochasticity is local instability: a small disturbance of initial system conditions results in an exponential increase of the distance D between paths:

$$D(t) = D(0)\, e^{ht}. \tag{1}$$

The average value of the increment h (the Komogorov entropy) is the basic characteristic of stochasticity. Systems with the same h have topologically equivalent dynamics in phase space (the universality property of systems with mixing).

Investigation of the local instability conditions (1) makes no sense in quantum mechanics, which does not have a path concept. The uncertainty relation must result in correlation effects. In quantum mechanics, there is also the steady-state eigenvalue and eigenfunction problem, which has no analog in classical mechanics. There are also several problems of a more subtle nature that arise in attempts to investigate stochasticity in quantum systems.[3]

The following statement of the problem is possible in quantum mechanics: what are the properties of a quantum system that is a K system in the classical limit ($\hbar=0$)? Analysis of this problem has shown[4-8] that the existing level of development and understanding in quantum mechanics is in a certain sense inadequate when we attempt to answer this question.

It is convenient to distinguish two types of problems for analysis of quantum K-system properties: 1) the steady-state problem and determination of the system's energy spectrum; 2) the nonsteady-state problem and determination of system evolution.

Let us consider a classical system of nonlinear oscillators (composed, for example, of N coupled nonlinear oscillators) with violation of the integrals of motion (except for the energy E). Violation occurs at $E > E_s$. Violation of the motion integrals means the appearance of stochasticity at $E > E_s$.[1] The appearance of a finite boundary of stochasticity E_s is possible even at $N \geqslant 2$. Violation of the classical motion integrals simultaneously implies violation of the corresponding quantum numbers. The problem of quantization rules for cases in which the number of integrals of motion in the corresponding classical system is smaller than the number of degrees of freedom was first posed by Einstein.[9] Considerations pertaining to the energy-spectrum structure of very complex systems (heavy excited nuclei, etc.) with violated quantum numbers were advanced in the book by Landau and Smorodinskiĭ:[10] the distribution of levels is random, but the probability $P(E|\Delta E)$ that if E is a level, the level next to it is at a distance ΔE, tends to zero as $\Delta E \to 0$ (the principle of "repulsion" of levels). An analysis of quasiclassical quantization rules for the case in which the motion integrals of the corresponding classical system are violated (i.e., the system is a K system) was made in Refs. 4, 5, and 7. It indicated that the level distribution in quantum K systems is quasistochastic, and that the distribution of distances among the levels $P(E|\Delta E)$ has the asymptotic form

$$P(E \mid \Delta E) \propto |\Delta E|^{\alpha}, \quad \alpha = \frac{\text{const}}{h(E)}. \tag{2}$$

Relation (2) indicates that equivalence of the level distribution and the eigenvalue distribution of an ensemble of random matrices of a certain symmetry type (Dyson[11]) does not hold, since according to Dyson's results, α may assume any of the values 1, 2, 4, which do not depend on any way on the dynamic properties of the system. A numerical confirmation of these results was obtained in Ref. 12.

These results call for a review of several of our conceptions in intramolecular dynamics and concerning the formation of the chemical bond.[3,7] At $E > E_s$, the vibrational spectra of molecules, even those with small numbers of atoms, acquire a quasistochastic structure that should completely change the shapes of absorption lines, etc. In particular, when the motion integrals are violated, the energy absorbed by a molecule should be distributed over different degrees of freedom, and the mixing time should determine the nature of the dissociation process. Other examples are given in Refs. 3 and 7.

Quantum K systems also exhibit new properties in the nonsteady state problems.[6,3] In particular, external time-periodic fields may result in excitation of molecules or an atom and their dissociation or ionization. The excitation mechanism is stochastic. This effect is an interesting case of the Fermi acceleration mechanism brought about on a single atom or on a single molecule.

The averaged dynamics of quantum K systems may differ significantly from the dynamics of classical K systems. The extent of this difference is determined by the dimensionless parameter ζ:[3,7]

$$\zeta = \gamma \frac{\hbar}{I} \omega T = \frac{K}{\varkappa} \quad \left(\frac{I}{\hbar} \gg 1, \quad K \gg 1, \quad \varkappa \gg 1\right), \tag{3}$$

where γ is a dimensionless nonlinearity parameter, I is the action, ω is the system frequency, T is the characteristic time scale on which the action of the system changes significantly, K is the stochasticity parameter ($\ln K = h$), and $\varkappa$ is the number of quanta in the perturbation.

At $\zeta \ll 1$, there is an interval of time during which the correspondence principle is valid and the dynamics of the quantum system is similar to the classical dynamics. Then the quantum corrections come to be of the order of magnitude of unity, and the macroscopic average values differ strongly from the classical values. In particular, a deviation in the values of the diffusion coefficient was found from numerical analysis in Ref. 15.

If $\zeta \gg 1$, the wave packet is dispersed quickly and the quantum dynamics is essentially nonquasiclassical, despite the fact that $I/\hbar \gg 1$ and $\varkappa \gg 1$.

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