Quarks in high-energy interactions of hadrons, photons, and leptons with nuclei

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The present state of the theory of the interactions of high-energy hadrons, photons, and leptons with nuclei is reviewed. Emphasis is placed on evidence for the quark-parton structure of hadrons in inelastic collisions with nuclei and on effects resulting from an increase in the secondary-particle formation lengths. The discussion is based on the space-time picture of strong interactions which follows from the multiperipheral nature of high-energy inelastic reactions.

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CONTENTS

1. INTRODUCTION

The interactions of high-energy particles with nuclei constitute a branch of the physics of elementary particles which has been developing rapidly in recent years. The first of the two goals of this review is to describe the present state of the physics of the interactions of hadrons, photons, and leptons with nuclei. Study of this problem is a traditional part of particle physics. We will discuss why this field has attracted interest at the present time; the relationship between experimental data and our understanding of the mechanism for the interactions of high-energy particles with nuclei; the new information on the mechanism for the interactions of hadrons with other hadrons, photons, and leptons which comes from a study of collisions with nuclei; and the most promising approaches for future developments in this field of high-energy physics. In pursuing this goal we will be closely following the present ideas regarding the quark–parton structure of hadrons and the multiperipheral nature of high-energy inelastic interactions of hadrons.

The second goal of this review is to identify, to the extent possible, the sources of the modern theory of hadron–nucleus, photon–nucleus, and lepton–nucleus interactions. Many of the recent "discoveries" have actually been rediscoveries of results which were first established more than a quarter of a century ago and which had been forgotten. It will be worth our while to review the history of the question and to recall these classic studies.

Nuclear targets have been and continue to be widely used in high-energy physics. Until comparatively recently, however, there was little interest in what was actually happening inside the nucleus. Nuclei are extended targets with thicknesses ranging up to several interaction lengths. It has been assumed that the interactions of secondary particles inside such a thick target would only distort the picture of the fundamental interaction of the incident particle with a nucleon of the nucleus.

There is no basis for this point of view. The events which occur inside a nucleus are actually similar to the events which occur in thick targets. The interesting point, however, is not this similarity but the fact that at high energies the interactions of secondary particles in a nucleus become fundamentally different from those which occur in a thick, dense target. Since the average particle multiplicity in a single elementary interaction event increases with increasing energy, there are also increases in the number of interactions in a thick target and in the multiplicity of slow particles of cascade origin. In interactions with nuclei, this increase actually occurs at low energies, but it stops somewhere before an energy of 10–20 GeV. It is as if the secondary particles interact only slightly in a nucleus, in contrast with the situation in a thick target. We can formulate this distinction in more detail.

Interactions with a thick target may be described by the equation

$$w_{\mathrm{abs}} = 1 - \exp(-\nu_{\mathrm{abs}}) \tag{1.1}$$

for the probability for the absorption of the incident particle and by the cascade transport equation

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$$\frac{d}{dt}\left(\frac{dN_s(\varepsilon,t)}{d\varepsilon}\right)=-\frac{dN_s(\varepsilon,t)}{d\varepsilon}+\int_{\varepsilon}^{E}d\omega\,\frac{dN_s(\omega,t)}{d\omega}\,\frac{dN_s(\omega\to\varepsilon)}{d\varepsilon} \quad (1.2)$$

for the secondary-particle distribution $dN_s(\varepsilon, t)/d\varepsilon$ at the depth t, reckoned from the point at which the incident particle is absorbed. The target thickness ν_{abs} and t here are expressed in units of the absorption length

$$l_{abs}=\frac{1}{\sigma_{abs}\rho}, \quad (1.3)$$

and ρ is the target density. The boundary condition is

$$\frac{dN_s(\varepsilon,0)}{d\varepsilon}=\frac{dN_s(E\to\varepsilon)}{d\varepsilon}, \quad (1.4)$$

where $dN_s(E-\varepsilon)/d\varepsilon$ is the energy distribution of the secondary particles in a collision of a primary particle of energy E with one nucleus of the target material. The dependence of $R(\varepsilon)=[dN_s(\varepsilon,t)/d\varepsilon]/[dN_s(E-\varepsilon)/d\varepsilon]$ on t and E is obvious: The fast particles are absorbed, and we have $R(\varepsilon\approx E)=\exp(-t)<1$, while for the slow particles $R(\varepsilon)$ increases with increases in both t and E, because of cascade buildup.

Let us assume that the target is compressed, at a constant ν_{abs}, until its density ρ increases to the nuclear density ρ_A. Equation (1.1) remains valid, with slight corrections, at nuclear densities. The experimental dependence of $R(\varepsilon)$ for nuclei on the nuclear thickness is qualitatively the same as for thick targets. Quantitatively, on the other hand, the simple cascade model described by Eq. (1.2) runs into a sharp contradiction with experiment at an energy of only a few tens of GeV. In particular, $R(\varepsilon)$ is completely independent of the energy for slow secondary particles.

The explanation for this result is quite simple: It was implicitly assumed in (1.2) that the secondary-particle formation lengths l_f are much shorter than the absorption length l_{abs}. It might appear that the only characteristic dimension at high energies would be the wavelength $\lambda=1/k$, which would be small, but the actual situation is directly the opposite of the naive classical picture: In the production of high-energy particles, it is found that

$$l_f\approx\frac{k}{m^2}\approx\frac{1}{\lambda m^2}. \quad (1.5)$$

In the case of nuclei, the value of l_f for fast secondary particles is comparable to or even greater than l_{abs} and the nuclear radius R_A. The particles do not interact inside the nucleus because they are formed outside it. This circumstance was first pointed out by Kancheli[1] back in 1973, for the problem of multiple production in nuclei. The large thickness of nuclei becomes an important advantage: The nature of the intranuclear interactions is a unique source of information on the space-time evolution of the multiple-production process. The nucleus is a miniature bubble chamber.

A situation of this type, with increasing characteristic longitudinal dimensions and with $l_f>l_{abs}$ is not new. A large formation length first appeared, to the best of my knowledge, in a study by Frank[2] back in 1942. Frank noted that an accelerated electron emits radiation not from a point but from a line segment

$$\Delta z\approx\frac{\lambda}{1-v_e}\approx\left(\frac{E_e}{m_e}\right)^2\lambda, \quad (1.6)$$

where λ is the photon wavelength, and E_e, m_e, and v_e are the energy, mass, and velocity of the electron. The dramatic effect of the formation lengths on bremsstrahlung and pair production in a medium was discovered by Ter-Mikaélyan[3] and by Landau and Pomeranchuk[4] in 1953 in studies of coherent and incoherent processes, respectively. The Landau–Pomeranchuk effect is particularly reminiscent of particle production processes in nuclei.

The role played by large longitudinal distances in the field of hadron physics itself was first pointed out by Pomeranchuk and Feĭnberg[5] in 1953. They noted that in the "coherent diffractive production" of particles in nuclei, $hA\to h^*A$, the important longitudinal distances are

$$L\approx\frac{E}{m^{*2}-m^2}, \quad (1.7)$$

which exceed the size of the nucleus, $L>R_A$, in the case of coherent production (see also Feinberg's 1941 paper,[6] which preceded Ref. 5). The formal analogy between (1.5) and (1.6), on the one hand, and (1.7), on the other, can be pursued extensively but there is a fundamental distinction between the Landau–Pomeranchuk effect and diffractive dissociation, on the one hand, and *multiple* production, on the other. In the former case we are dealing with the decay time of a state with an excitation energy much smaller than E, and expressions (1.6) and (1.7) are purely kinematic in meaning. In the latter case, the production of the large number of particles consumes essentially all the energy E of the colliding particles. The fact that the formation length in (1.5) for an individual secondary particle is determined only by the momentum of this particle, k, and is independent of E is a far from trivial property of the dynamics of strong interactions. The corresponding space-time picture of the interaction was formulated by Gribov[7] in 1969.

The large dimensions of nuclei make collisions with nuclei sensitive also to the hadron structure. This point can be demonstrated by dN and dA interactions. In the former case the incident deuteron interacts with a nucleon by means of only one of its own nucleons. The production of secondary particles is therefore the same in dN and NN collisions, although the deuteron differs from a nucleon in structure. For heavy nuclei the total cross sections for dA and NA interactions are approximately equal to each other and to the geometric cross section $2\pi R_A^2$, while the production processes are different: Both nucleons are absorbed in essentially all dA collisions, and the secondary-particle multiplicity turns out to be proportional to the number of constituent particles in the incident system. This situation is carried over to the case of hadron-nucleus collisions. According to the present understanding, fast hadrons are quark–parton systems, and their interactions with nuclei present a unique possibility for testing the ideas regarding the quark–parton structure of hadrons. *This possibility is particularly promising* in connection with the success of quantum chromodynamics— the theory of colored quarks and gluons—in interpreting deep inelastic scattering and the physics of charmed particles.[8] Any information on how the hadrons are constructed from the quarks and gluons, and how this composite structure of the hadrons affects their strong interactions at high energies, will be valuable for a future theory of hadrons.

Many models have recently been proposed for multiple production in nuclei, and several of these derive from the familiar concepts of quantum field theory. It would not be possible to discuss in detail all these models in a single review. The discussion below will be subjective in the sense that we will give preference to those models which are based on the multiperipheral picture of strong interactions. These models, and essentially only these models, lead to a consistent description of all the basic features of the collisions of hadrons, photons, and leptons with nuclei. Furthermore, as we will see below, many of the nonfield models simply fail on a detailed comparison with experiment.

Emphasis will be placed on two central problems: How is the number of intranuclear interactions suppressed as the energy is increased in each particular model, and how is the hadron structure manifested in interactions with nuclei? There has been no common approach to these problems even by authors who have worked from the multiperipheral picture of strong interactions. Most of the differences arise in the evaluation of the role played by cascade processes in nuclei (at one extreme, a role of these processes is completely denied) and in the opinions regarding the nature of multiple inelastic interactions of the incident particle in the nucleus. The sources of the differences lie among the several unresolved problems of the theory of hadron-hadron interactions, and a study of collisions with nuclei might contribute to their solution and progress in a theory of hadron-hadron interactions.

We felt it important to draw as complete as possible a picture of the physics of high-energy interaction with nuclei, but several questions have nevertheless been omitted. The bibliography at the end of this review can be used to make up for the omissions. This review will be theoretical; experimental data will be cited primarily to illustrate an agreement or disagreement of theory with experiment. There is a comprehensive compilation of the experimental data in the review by Gulamov *et al.*[9]

High-energy interactions with nuclei have been the subject of previous reviews published in this journal by Barashenkov *et al.*[10] and Nikitin *et al.*[11] These interactions have also been the subject of a recent monograph by Nikitin and Rozental'[12] published by the Soviet publishing house Atomizdat. Barashenkov *et al.*[10] discuss the simple cascade model in detail. This model gives a good description of experimental data at energies up to ~10 GeV, including many characteristics of the fragmentation of the target nucleus. We will not be discussing this energy range here. Although the review in Ref. 11 and the monograph in Ref. 12 come close to the present review, the actual overlap is only slight in terms of the particular problems discussed, and, especially, in terms of the conclusions. For example, the hydrodynamic models, whose formulation and discussion receive much space in Refs. 11 and 12, appear to have been refuted by the latest experimental results on interactions with nuclei.

In Section 2, a brief introduction to the kinematics and definitions of the pertinent quantities will be followed by a summary of the basic experimental data on the interactions of hadrons, photons, and leptons with nuclei. Various examples will be used in Section 3 to illustrate how the large characteristic interaction times and large characteristic longitudinal distances arise at high energies and how the increase in the formation lengths alters the picture of the intranuclear interaction.

In Section 4 we will discuss the total cross sections and diffractive dissociation. This is a classical field of study by comparison with some others, but it is still actively growing. There has been important progress in our understanding of the Gribov corrections for so-called inelastic screening[13] and their relationship with the quark–parton internal structure of hadrons.

Section 5 will be a discussion of photoproduction and deep inelastic scattering of leptons by nuclei, primarily from the standpoint of the manifestation of the hadronic properties of real and virtual photons. A study of the interactions with nuclei turns out to be very useful for reaching an understanding of the space–time picture of the interactions of photons and for a critical analysis of the applicability of the vector dominance model to deep inelastic scattering. It will be seen that the inelastic interactions of a virtual photon are similar to the interactions of a single quark.

The discussion of multiple production begins in Section 6. This section analyzes the experimental data on the single-particle distributions and gives summaries of the most important models for hadron–nucleus interactions, which have been proposed primarily to describe inclusive production in nuclei. The structure of the total cross sections is discussed in terms of production processes: This structure is intimately related to the interpretation of the nature of the inelastic interactions of the primary particle and the secondary particles in a nucleus. The various approaches which have been taken to suppress the number of intranuclear interactions with increasing energy will be compared. From a comparison with experiment, the additive quark model[14] emerges as the phenomenologically most successive model; this model was first applied to interactions with nuclei by Anisovich.[15] The various quark-counting rules which arise within the framework of this model are compared with experiment. The particular importance of multiple production in the deep inelastic scattering of leptons is emphasized. Multiple production furnishes an unambiguous test of various pictures of the interactions of the constituent quarks in a nucleus.

Correlation effects in multiple production in nuclei are discussed in Section 7. Several of the models which give a satisfactory description of the single-particle distributions run into contradictions with experiment in terms of the correlations. There is a particularly interesting dependence of the secondary-particle distributions (and of the correlations between them) on N_g, the number of recoil protons ("gray" tracks; see Subsection 2a). In the quark model, the selection of events with $N_g \gg \langle N_g \rangle$ singles out events in which *all* the constituent quarks of the incident hadron collide with the nucleus. A study of such collisions, which are not found in hadron–hadron interactions, yields new information on the behavior of quarks in strong interactions. The experimental data strongly suggest additivity of the production of secondary particles by the different constituent quarks. It is not clear whether these suggestions are compatible with the ideas of a topological dual decomposition, which have recently been the subject of wide discussion.[16] It is concluded that the N_g dependence of the rapidity correlations and of the multiplicity distributions refutes the coherent-tube model and the hydrodynamic models, leaving only the multiperipheral models as possibilities.

Section 8 deals with "hard" processes: the production of massive lepton pairs and ψ particles and the production of particles with large transverse momenta. The corresponding cross sections are small, and they are naturally proportional to the number of nucleons in the target nucleus, A. The A^1 law breaks down in the production of large-p_T particles,[17] apparently because of a high-energy rescattering of quark–partons, although we still lack a quantitative explanation for the Cronin effect.

In the final section, Section 9, we will summarize the results, list the most important unresolved problems, and list the experiments which hold the most promise in the light of our present theoretical understanding of the mechanism for the interactions of high-energy particles with nuclei.

2. SUMMARY OF THE PRINCIPAL EXPERIMENTAL DATA

This summary is intended primarily for orientation in this review; the experimental data will be discussed in more detail in the comparison with theory in the corresponding places.

a) Kinematics and definitions

All the secondary particles except recoil protons are relativistic in hadron–nucleon interactions. In accordance with the tradition established in experiments with photographic emulsions, the secondary particles from collisions with nuclei are divided into groups on the basis of the degree of ionization, i.e., on the basis of their velocity: light tracks, with $v \geqslant 0.7$, corresponding to shower particles or the secondary particles proper; gray tracks, with $0.3 < v < 0.7$, corresponding primarily to protons ejected from the nucleus; and black tracks, with $v \leqslant 0.3$, corresponding primarily to evaporation products (here and below, we are using a system of units with $\hbar = c = 1$). The corresponding multiplicities are denoted by N_s, N_g, and N_b. In hadron–proton interactions, $\langle N_s \rangle_N$ is related to the charged-particle multiplicity $\langle N_{ch} \rangle$ by $\langle N_s \rangle_N \approx \langle N_{ch} \rangle - 0.5$ (Refs. 9 and 18).

We will use the standard kinematic variables: the rapidity y, the pseudorapidity η, and the Feynman variable x:

$$y = \frac{1}{2} \ln \frac{\varepsilon + k_z}{\varepsilon - k_z}, \tag{2.1}$$

$$\eta = -\ln \operatorname{tg} \frac{\theta_L}{2}, \tag{2.2}$$

$$x = \frac{k_z}{p} \approx \frac{\varepsilon}{E}; \tag{2.3}$$

here ε, k_z, and θ_L are the laboratory values of the energy, the longitudinal momentum, and the angle at which the secondary particle is emitted; and E and p are the energy and momentum of the incident particle. The corresponding variables in the center-of-mass (c.m.) system of the incident particle and the nucleon of the target nucleus are y^*, η^*, and x .

For a comparison of multiple production in nuclei with that at nucleons it is convenient to use the relative quantities

$$R = \frac{\langle N_s \rangle_A}{\langle N_s \rangle_N}, \tag{2.4}$$

$$R_i = \frac{(dN_s/di)_A}{(dN_s/di)_N}, \qquad i = y, \eta, x. \tag{2.5}$$

Other variables discussed are the Wroblewski ratio[19]

$$\frac{D}{\langle N_s \rangle} = \frac{\sqrt{\langle N_s^2 \rangle - \langle N_s \rangle^2}}{\langle N_s \rangle} \tag{2.6}$$

the two-particle and three-particle rapidity correlations (or pseudorapidity correlations)

$$R_2(y_1, y_2) = [(1, 2) - (1)(2)][(1)(2)]^{-1}, \tag{2.7}$$

$$R_3(y_1, y_2, y_3) = [(1, 2, 3) - (1, 2)(3) - (2)(3, 1) - (3)(1, 2) + 2(1)(2)(3)][(1)(2)(3)]^{-1}, \tag{2.8}$$

$$(i) = \frac{dN_s}{dy_i}, \quad (i, j) = \frac{d^2 N_s}{dy_i\, dy_j}, \quad (i, j, k) = \frac{d^3 N_s}{dy_i\, dy_j\, dy_k}, \tag{2.9}$$

and the azimuthal asymmetry

$$A_\varphi = \frac{N(\varphi > \pi/2) - N(\varphi < \pi/2)}{N(\varphi > \pi/2) + N(\varphi < \pi/2)}, \tag{2.10}$$

where φ is the azimuthal angle between the directions in which the two particles are emitted.

We will introduce yet another useful quantity. Equation (1.1) may be written as

$$w_{abs} = \sum_{n=1}^{\infty} \nu_{abs}^n \exp(-\nu_{abs}) \frac{1}{n!} = \sum_n w_n. \tag{2.11}$$

The quantity $w_n = (\nu_{abs}^L)^n \exp(-\nu_{abs})/n!$ might be interpreted naively as the probability for an n-fold inelastic absorption of the incident particle. Although this interpretation is not literally correct, the quantity

$$\bar{\nu} = \langle n \rangle = \sum_n n w_n \cdot \frac{1}{w_{abs}} \tag{2.12}$$

is still a convenient measure of the target thickness.

b) Total cross sections and absorption cross sections

In the case of charged particles, only the absorption cross sections σ_{abs}^A can be measured by the usual beam attenuation method. The observed cross sections σ_{abs}^A

can be described reasonably well by a generalization of Eq. (1.1) which was apparently first proposed by Bethe[20] in 1940:

$$\sigma_{abs}^{A}=\int_{0}^{\infty} db\cdot 2\pi b\{1-\exp[-\nu_{abs}(b)]\}, \qquad (2.13)$$

where b is the impact parameter, $\nu_{abs}(b)=\sigma_{abs}^{N}T(b)$, and $T(b)$ is the nuclear profile function,

$$T(b)=\int_{-\infty}^{+\infty} dz\rho_A(z,b). \qquad (2.14)$$

At large values of σ_{abs}^{N}, such that $\nu_{abs}(b)\gg 1$, expression (2.13) yields $\sigma_{abs}^{A}=\pi R_A^2\sim A^{2/3}$ (a black nucleus), while with $\nu_{abs}(b)\ll 1$ we have $\sigma_{abs}^{A}=A\sigma_{abs}^{N}$. Experimentally, the exponent α in the parametrization $\sigma_{abs}^{N}\approx\sigma_0 A^{\alpha}$ is in fact approximately $\alpha=2/3$ at large values of σ_{abs}^{N} (Fig. 1).

A generalization of (2.12) to the case of nuclei is

$$\bar{\nu}=\frac{A\sigma_{abs}^{N}}{\sigma_{abs}^{A}}. \qquad (2.15)$$

The total cross sections σ_{tot}^{A} have been measured for neutrons and K_L mesons[23,24]: $\alpha(K^LA)=0.84$, $\alpha(nA)=0.77$.

For photons we would have $\sigma_{abs}^{\gamma N}T(b)\ll 1$, but experimentally it has been found that[25] $\alpha(\gamma A)\approx 0.9$ (Fig. 2). This situation is interpreted in the following manner: At high energies, the photons initially convert into a hadron system of the vector-meson type, and it is these mesons which interact with the nucleus.[26] In deep inelastic scattering, i.e., for virtual photons, experiments yield $\alpha(\gamma^* A)\approx 1$. Experiments also reveal, however, some important systematic deviations of $\alpha(\gamma^* A)$ from unity[27,28] (Fig. 2).

c) Average secondary-particle multiplicities

For primary hadrons at energies above ~50 GeV the following approximate universal dependence holds[29] (Fig. 3):

$$R\approx a+b\bar{\nu}, \qquad (2.16)$$

where $a\approx 0.4$, $b\approx 0.6-0.7$. For neutrinos and virtual photons, expression (2.15) yields $\bar{\nu}\equiv 1$. Nevertheless, experimentally we find $R_{\gamma^* A}>1$ (Ref. 31) and $R_{\nu A}>1$ (Ref. 32). No experimental data are available on photoproduction in nuclei.

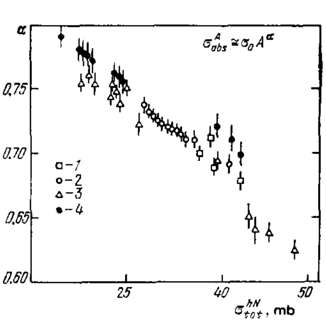

FIG. 1. Dependence of the exponent α in the parametrization $\sigma_{abs}^{A}\approx\sigma_0 A^{\alpha}$ on σ_{tot}^{hN}. 1—Allardyce *et al.*; 2—Bobchenko *et al.*; 3—Gorin *et al.*; 4—Carroll *et al.*[21]

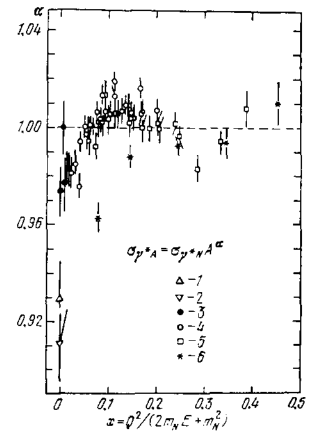

FIG. 2. Dependence of the exponent α in the cross section for electroproduction and photoproduction on the scaling variable x. 1—Caldwell *et al.*; 2—Heynen *et al.*[25]; 3—Eickmeyer *et al.*; 4—Stein *et al.*; 5—Ditzler *et al.*; 6—May *et al.*[27]

Experiments yield $R\approx 2.5$ for pPb interactions (Fig. 3). At ~100 GeV, the simple cascade model would predict a value of R several times larger.[33]

That the average multiplicity in interactions with nuclei was small was emphasized a very long time ago, immediately after the very first cosmic-ray experiments in the early 1950s. At the time, though, it was concluded on an inadequate statistical basis that the cascade model contradicted experiment.[34]

d) Nuclear inclusive distributions

The relative inclusive distributions R_η and R_y specify which intervals of the pseudorapidity η or the rapidity y make a contribution to the multiplicity in reactions with nuclei which is greater than that for reactions with nucleons. For primary hadrons the following facts have been established:

a. Values $R_\eta<1$ are found in the beam fragmentation region, and they fall off with increasing $\bar{\nu}$ (Refs. 35–37 and 40; see Figs. 4–7 of the present paper).

b. In the nuclear fragmentation region, R_η increases with increasing $\bar{\nu}$ approximately in accordance with[40] (Fig. 7)

$$R_\eta=1+S(\eta)(\bar{\nu}-1), \quad S(\eta)\approx 2.5-3. \qquad (2.17)$$

The production of particles in interactions with nuclei

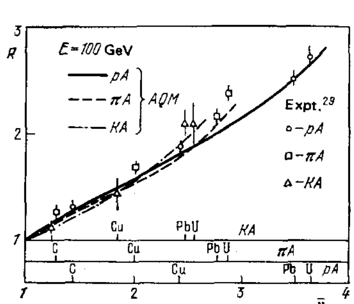

FIG. 3. Dependence of the relative multiplicity $R=\langle N_s\rangle_A/\langle N_s\rangle_N$ on the nuclear thickness $\bar{\nu}$ in KA, πA, and pA collisions at 100 GeV (Ref. 29).

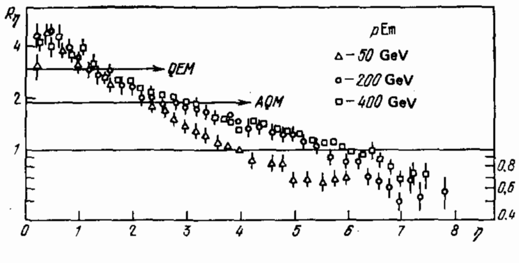

FIG. 4. Dependence on the energy E and on the pseudorapidity η of the relative distribution R_η in interactions of protons with the nuclei of a photographic emulsion [E= 50 GeV, E= 200 GeV (Ref. 36), and E= 400 GeV (Ref. 35)]. Here $R_\eta = \bar{\nu} = 3$ corresponds to the eikonal model (QEM), and $R_\eta = \langle\nu\rangle$ is the height of the plateau expected at high energies in the quark model (AQM).[30]

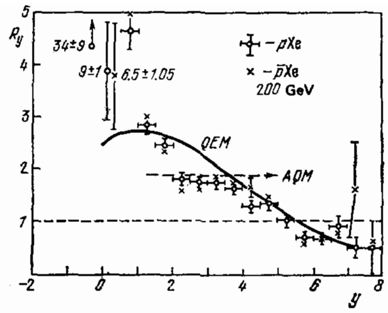

FIG. 6. The relative R_y distributions in pXe and $\bar{p}$Xe interactions at 200 GeV (Ref. 38). Arrow—Height of the plateau predicted in the additive quark model[30]; solid curve—predictions of the eikonal model.[39] The numbers on the points at the far left are the values of R_y.

is similar to that in thick targets in this regard. There are, on the other hand, some qualitative differences:

c. R_η is independent of the energy in the nuclear fragmentation region[9,35,36,40] (Fig. 4).

d. There are indications that a plateau appears in R_η at high energies and separates the beam fragmentation region from the cascade region[35-38] (Figs. 4–6).

At energies $E \gtrsim 100$ GeV the simple cascade model predicts values of R_η for $\eta \approx 0$ which are tens of times larger than the experimental values.[33] On the other hand, many of the models which ignore cascades altogether predict $R_\eta = \bar{\nu}$ at intermediate and small values of η; i.e., they predict $S(\eta) = 1$ (see Ref. 39 and Subsections 6b and 6d of Section 6 below for more details).

Extremely little experimental information is available on leptoproduction in nuclei, but that which is available clearly indicates $R_\eta \approx 1$ at large pseudorapidities and $R_\eta > 1$ at $\eta \approx 0$; it also indicates that all the excess multiplicity stems from the cascade region of small values of η (Refs. 31 and 32; see Fig. 8 of the present paper).

e) Multiplicity distributions

The Wroblewski ratio $(D/\langle N_s\rangle)_A$ is found to be essentially independent of the nuclear dimensions and of the energy E (Refs. 9, 29, and 40–42; see Figs. 9 and 10 of the present paper). Approximate Koba–Nielsen–Olesen scaling[46] is also found for nuclear targets; the KNO functions for hadron-nucleon interactions are approximately the same as those for hadron-nucleus interactions.[47-50]

f) Correlations between secondary particles

The average number of intranuclear interactions increases with an increase in the nuclear dimensions and also with an increase in N_g, the number of gray tracks for the given nucleus. Experimentally, the curves of $\langle N_s\rangle_A$ and $dN_s/d\eta$ against $\bar{\nu}$ and N_g are similar (cf. Fig. 11 and Figs. 4 and 5). There is one fundamental difference: The Wroblewski ratio $(D/\langle N_s\rangle)_A$ is independent of A (Figs. 9 and 10), but it falls off with increasing N_g (in Fig. 10, N_p is the number of gray tracks observed in the experiments of Ref. 37, without any correction for the efficiency of the apparatus). This decrease in $(D/\langle N_s\rangle)_A$ with increasing N_g has also been observed, although less accurately, in emulsion experiments (see the review in Ref. 9) and in πNe interactions at 10.5 GeV (Ref. 50). The rapidity correlation $R_2^A(0,0)$ in the pionization region, $\eta_1^*, \eta_2^* \approx 0$, is also independent of the atomic number of the nucleus, but it falls off with increasing N_g (Refs. 51–53; see Fig. 12 of the present paper).

g) Nuclear fragmentation

The characteristics of nuclear fragmentation— the multiplicities of black and gray tracks, N_b and N_g; the yields of various isotopes; etc.—become almost independent of the energy at $E \gtrsim 10$–20 GeV (Refs. 54–56).

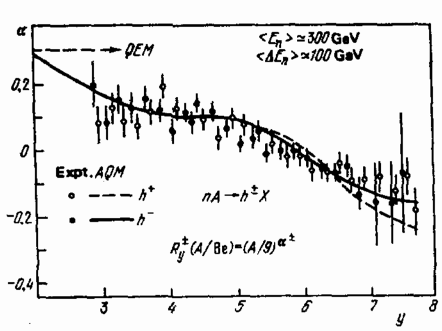

FIG. 5. Dependence of the exponent α in the parametrization $R_y = A^\alpha$ on the rapidity y in nA interactions at $\langle E_n\rangle \approx 300$ GeV (Ref. 37). Curves—Calculations from the quark model (AQM)[30]; the value $\alpha = 1$ corresponds to the eikonal model (QEM).

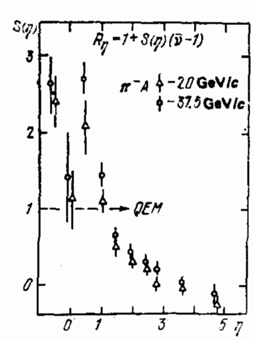

FIG. 7. Dependence of the slope $S(\eta)$ on the pseudorapidity η (Ref. 40). The eikonal model corresponds to $s(\eta) = 1$ (QEM).

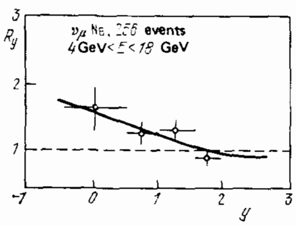

FIG. 8. Dependence of the ratio of the distributions in ν_μNe and ν_μN interactions on the rapidity y (Ref. 32). Curve—Calculation incorporating cascade interactions.[41]

This independence means that the average number of intranuclear interactions is independent of the energy.

h) Historical comments

Most of the general properties of particle production in nuclei discussed here were first noted in cosmic-ray experiments. The results of these experiments have been reviewed in detail by Feĭnberg.[57] The results did not immediately win the respect they deserved, because of the specific shortcomings of cosmic-ray experiments: the low statistical base, the uncertainty regarding the energy, and the uncertainty regarding the nature of the primary particle. The conclusions which were reached from the data at the time are perhaps evidence more of daring than of solid analysis. Nevertheless, many of these conclusions have survived, in their general form, to the present. It is instructive to look back at the reviews in Refs. 34 and 57 and the monograph in Ref. 54 and to compare the conclusions reached there with those of the present review.

3. SCALE TIMES, SCALE LONGITUDINAL DISTANCES, AND FORMATION LENGTHS AT HIGH ENERGIES

a) The Landau-Pomeranchuk effect[4]

The emission of soft photons is described by the classical equation

$$\frac{dn}{d^3\mathbf{k}} \sim \left| \int [d\mathbf{r}\mathbf{n}] \exp\{i\omega[t-\mathbf{n}\mathbf{r}(t)]\}\right|^2, \tag{3.1}$$

where ω is the photon frequency, $\mathbf{n}=\mathbf{k}/\omega$, and the integration is carried out along the classical trajectory of the electron. For double scattering, $\mathbf{v}_1-\mathbf{v}_2-\mathbf{v}_3$, we have

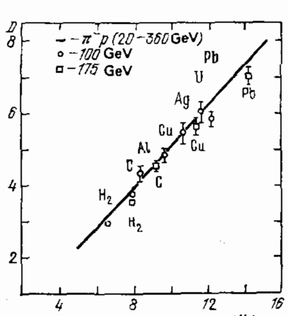

FIG. 9. Comparison of the dependence of the dispersion D on $\langle N_s\rangle$ in π^-p and π^-A interactions.[29] The points are labeled with the symbol for the particular nuclei.

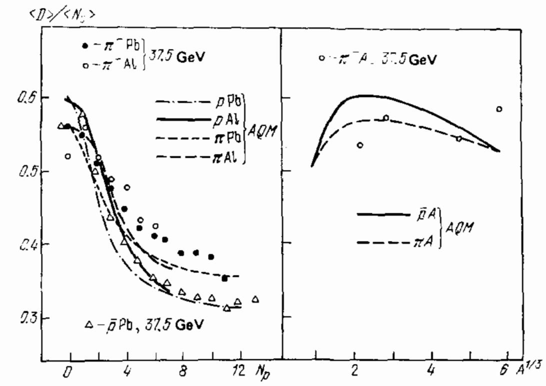

FIG. 10. Dependence of the Wroblewski ratio $D/\langle N_s\rangle$ on the number of observed gray tracks N_p and on the atomic number of the nucleus, A, in π^-A and $\bar{p}$A interactions at 37.5 GeV (Ref. 40). Curves—Calculated from the additive quark model.[44,45]

$$\frac{dn}{d^3\mathbf{k}} \sim \left| \left(\frac{[\mathbf{v}_1\mathbf{n}]}{1-\mathbf{v}_1\mathbf{n}} - \frac{[\mathbf{v}_2\mathbf{n}]}{1-\mathbf{v}_2\mathbf{n}} \right) + \left(\frac{[\mathbf{v}_2\mathbf{n}]}{1-\mathbf{v}_2\mathbf{n}} - \frac{[\mathbf{v}_3\mathbf{n}]}{1-\mathbf{v}_3\mathbf{n}} \right) \exp \frac{il_{12}}{l_f} \right|^2, \tag{3.2}$$

where $\mathbf{v}_i$ is the electron velocity and where we have introduced the formation length

$$l_f = \frac{1}{\omega(1-\mathbf{n}\mathbf{v}_2)}. \tag{3.3}$$

If the distance l_{12} between the two scattering points is large, $l_{12} \gg l_f$, then we find the classical picture, of a summation of intensities in an amorphous medium after an average is taken over l_{12} [the terms in the parentheses in (3.2) are the emission amplitudes at the individual centers]. If $l_{12} \gg l_f$, however, then the terms containing $\bar{\mathbf{v}}_2$ in (3.2) cancel out; only the emission by the initial and final electrons is retained. The emission by the intermediate electron simply does not have room to develop over distances much smaller than the formation length. Following Feĭnberg, we may say that the electron "shakes off" its field after the first scattering event, and until this field is reestablished the scattering in the external field occurs without emission.[58,59]

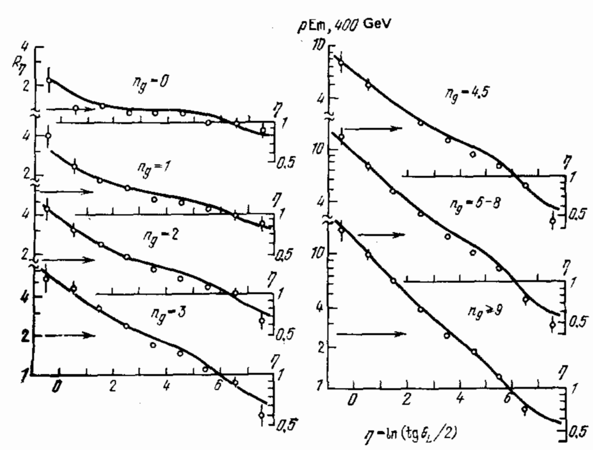

FIG. 11. Dependence of the relative distribution R_η on the number of gray tracks, N_g, in the interactions of protons with emulsion nuclei at 400 GeV (Ref. 35). Curves—Calculated from the additive quark model. The heights of the plateau in R_η expected in the additive quark model at higher energies for the given value of N_g are shown.

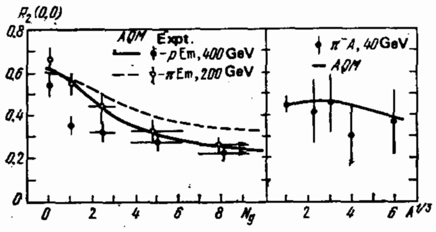

FIG. 12. Dependence of the rapidity correlation in the pionization region, $R_2^A(0,0)$ on N_g, the number of gray tracks, for emulsion nuclei[51,52] and on the atomic number of the nucleus.[53] Curves—Calculated from the additive quark model.[44,45]

A similar situation arises in hadron interactions. Migdal[60] has offered an elegant exact solution of the Landau–Pomeranchuk problem,[4] and Galitsky and Gurevich[61] have published a paper on the effect which is to be commended for its physical transparency.

The simple equation (3.3) has some radical implications regarding, for example, transition radiation. For situations of practical interest in the optical range, l_f is comparable in magnitude to the dimensions of laboratories, and it is not possible to raise the low intensity of optical transition radiation by increasing the number of radiators.[62]

b) Scale longitudinal distances in strong interactions. The Gribov–Ioffe–Pomeranchuk method

The possibility of a hadron analog of the Landau–Pomeranchuk effect has been analyzed in detail by Feinberg.[57,58] Feinberg's arguments showed that large formation lengths are not related to perturbation theory. Nevertheless, the appeal to electrodynamics is not rigorous. A more rigorous approach was proposed by Gribov, Ioffe, and Pomeranchuk[63] in 1965.

The absorption part of the amplitude for forward hadron–hadron scattering, $k+p\to k+p$, can be written in terms of a current commutator:

$$\mathrm{Im}\,F(k,p)=\frac{1}{2}\int d^4x\exp(ikx)\langle p|\,[I(x),I(0)]\,|p\rangle. \tag{3.4}$$

We will treat (3.4) as a function of the square mass k^2 of the incident particle.

At high energies,

$$kx=\varepsilon(t-z)+\left(\frac{k^2}{2\varepsilon}\right)z. \tag{3.5}$$

The 4-vector x in (3.4) may be interpreted as the distance between the point at which the incident particle is absorbed and the point at which the final particle is produced. If $\mathrm{Im}F(k,p)$ depends strongly on k^2, then the longitudinal distances which are important in elastic scattering increase substantially with increasing energy:

$$\Delta z\approx\frac{\varepsilon}{m^2}. \tag{3.6}$$

The external masses are fixed in the scattering of hadrons. The departure from the mass shell during bremsstrahlung cannot be used to monitor the dependence of $F(k,p)$ on k^2, as was first proposed by Gribov *et al.*[63] Ioffe noted that the method of Ref. 63 could be used in the deep inelastic scattering of leptons. In this case the square mass of the virtual photon, Q^2, may change. Ioffe showed rigorously that the scaling behavior of the total cross section for electroproduction implies[64]

$$\Delta z\approx\frac{\varepsilon}{Q^2}. \tag{3.7}$$

This was the first rigorous argument for the hadronic behavior of photons. The distance in (3.7) is the distance from the target at which the photon transforms into a hadron system.

c) Multiple elastic rescattering in the multiperipheral approach

The multiperipheral model is based on the experimentally motivated hypothesis that the hadron amplitudes fall off rapidly upon departure from the mass shell. Large longitudinal distances naturally arise in the multiperipheral model (see Subsection 3b). In the simplest version of the multiperipheral model, elastic scattering is described by the diagram in Fig. 13a. Following Gribov, we may interpret this diagram as follows[7]: The incident hadron, decaying in a sequential manner, forms a parton fluctuation, and it is actually a slow parton of this fluctuation which interacts with the target.[1)] The time in which this interaction occurs is short: $\tau\sim 1/m$. On the other hand, the time over which decay is possible is long, by virtue of the uncertainty principle:

$$\Delta t\approx\frac{1}{\Delta E}\approx\frac{E}{m^2+\langle\mathbf{k}_\perp^2\rangle}, \tag{3.8}$$

since we have $\Delta E\approx(m^2+\mathbf{k}_\perp^2)/E$ for relativistic particles[7,66] in the decay $1\to 2+3$. It is the time for this decay which figures in (3.4).

We will be using this space–time picture drawn by Gribov in several places below.[7] It is based on the short-range nature of the parton interaction in terms of the rapidity. The total cross section is determined primarily by the formation and interaction of fluctuations containing a slow parton, and the times and longitudinal distances in (3.6) and (3.8) are actually the formation times of these fluctuations. It is important to note that the hadron does not convert into a parton fluctuation exclusively just before the interaction with the target; the probability for this sort of "guessing" of

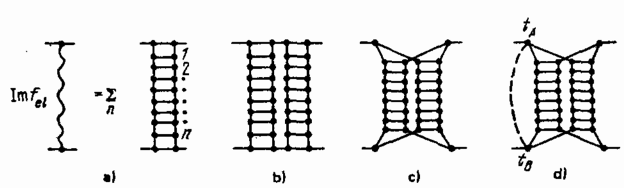

FIG. 13. a—Multiperipheral diagrams for the elastic scattering amplitude; b—planar amplitude for double scattering; c—nonplanar Mandelstam amplitude for double scattering; d—time-ordered Mandelstam diagram. The dashed curve represents a spurion.

[1)] It is useful to recall the well-known Weizsacker–Williams equivalent-photon method in electrodynamics[65]: The equivalent photons here are partons.

the appropriate conversion time would vanish with increasing energy. The slow partons are present in the hadron; they replace each other over times $\sim 1/m$ with a probability of approximately unity.[7,66,67]

In addition to single scattering there may be double scattering. The associated increase in the longitudinal distances involved leads to a severe restriction on the nature of the double scattering. In the high-energy limit the planar diagram of Fig. 13b evidently makes a vanishing contribution, since two successive fluctuations with lifetimes $\sim E/m^2$ could not form[2)] in a time $\sim 1/m$. There is the possibility, however, of two parallel fluctuations, which would lead to the diagram of Fig. 13c, which is nonplanar. In the high-energy limit a hadron may in general be represented as a superposition of parton fluctuations through the introduction of a parton wave function.[7,66,69] In each of these fluctuations, a slow parton or several partons are continuously present with a probability approaching unity, so that there is the possibility of a simultaneous interaction of two slow partons with the target.

The case that the fluctuations actually form before the target can be put on a more formal basis. In the Feynman diagram of Fig. 13c, considered in the laboratory frame, we introduce a time ordering:

$$\theta(t_B - t_A) = \frac{i}{2\pi}\int \frac{d\omega}{\omega + i0}\exp[-i\omega(t_B - t_A)]. \tag{3.9}$$

For this purpose we introduce a "spurion,"[70] over whose energy ω the integration must be carried out (Fig. 13d). A lengthy but straightforward analysis of the structure of the singularities in the resulting Feynman integrals shows that with the time ordering $t_B < t_A$ the diagram in Fig. 13d makes a vanishing contribution,[71] and the amplitudes decrease substantially upon departure from the mass shell. Caneschi *et al.*[72] analyzed perturbation-theory diagrams which do not decrease with departure of the particles from the mass shell and reached the erroneous conclusion that there is no time ordering in the nonplanar Mandelstam diagram of Fig. 12c.

d) Formation lengths in multiple production from the parton standpoint

In the Landau–Pomeranchuk effect we are actually dealing with the time required for the decay of a virtual electron into an electron and a photon. There is nothing which would reduce the probability for a rescattering of the virtual photon in the external field during this decay time, but such scattering events do not increase the number of bremsstrahlung photons.

Equation (3.3) may be rewritten as

$$l_f = \frac{E_e}{m_e}\frac{1}{\omega_0}, \tag{3.10}$$

where ω_0 is the photon frequency in the rest frame of the radiating electron; i.e., a large value of l_f corresponds to the Lorentz dilatation of the time required for the emission of a photon by a slow electron, which is of order $T \approx 1/\omega_0$. Analogously, (1.5) may be interpreted as the Lorentz-transformed Yukawa relation for hadrons: $T \sim 1/m$. This is as far as the formal analogy can be pursued, however.

In bremsstrahlung we are dealing with the decay of a "slightly virtual" electron, $e^* \to e\gamma$, and in this case only a small fraction of the electron's energy is expended on the decay. In hadron interactions, on the other hand, we are dealing with the production of a large number of particles and the complete dissipation of the energy of the colliding hadrons. The fact that the formation length for an individual secondary particle depends only on its momentum k, not on the energy E of the incident particle, is a nontrivial property of (specifically) strong interactions.

Let us examine the formation of the final hadrons as it appears in Gribov's space–time picture.[7] A slow parton of a fluctuation interacts with the target, thereby leaving the fluctuation and generating a first secondary hadron (a slow parton is equivalent to a hadron). The coherence of the fluctuation is disrupted by the inelastic interaction, with the result that the next parton in terms of the rapidity decays over a time $\tau_2 \approx \varepsilon_2/m^2 \approx 2/m$, and a second hadron is produced. The departure of the second parton from the fluctuation makes the next parton (again, in terms of the rapidity) unstable; etc. Each parton of the initial fluctuation is "hadronized," decaying right up to the appearance of the slow parton, in the time in (1.5). The perturbation propagates upward along the rapidity scale until the fastest parton of the fluctuation is hadronized. In any frame of reference the slow secondary particles are produced first, and then the fast particles, in order of increasing energy.

An important point here is that the fast partons of the initial fluctuation continue to have a small interaction cross section right up to the time of hadronization. The only secondary particles which can interact inside the nucleus are those for which the condition $l_f < R_A$ holds, as was first pointed by Kancheli.[1] The similar picture in inelastic interactions has been discussed in several papers by Bjorken.[73]

e) Formation lengths in the multiperipheral formalism

Following Ref. 74, we will now supply a more formal basis for the intuitive picture of the formation of secondary particles drawn in Subsection 3d. Let us consider the rescattering of secondary particles in multiple production in collisions involving deuterons, as described by the diagram in Fig. 14a. The total cross section for an inelastic interaction with rescattering, σ_{Re}, is described by Fig. 14b, which is formally similar to the three–pomeron diagram of Fig. 14c, with some effective three-pomeron vertex G_{eff}. If G_{eff} were the same as the usual three–pomeron vertex G_{PPP}, which is very small, the diagram in Fig. 14a would have a negligible effect on the cross section. The particles of the internal lines in G_{PPP}, however, are far from the mass shell, while in the problem of scattering by *spatially separated* centers the particles b

[2)]This result should be credited to Mandelstam.[68] The discussion in the present review, in space–time terms, was developed by Gribov[66] and Ansel'm.

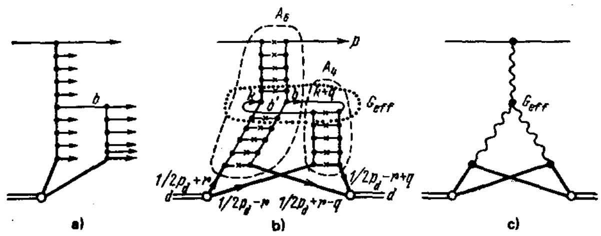

FIG. 14. a—Diagram for an inelastic collision with a deuteron accompanied by rescattering; b—diagram for the cross section for an interactions with rescattering; c—its three-pomeron representation.

and b' in Fig. 14b are nearly real. More precisely, either b or b' is simply on the mass shell, while the Green's function of the second particle is $(1/2)(\mathbf{kq})$, where $\mathbf{q}$ in this scalar product is the relative momentum of the nucleons in the deuteron. If $|\mathbf{kq}|<m^2$, the amplitude A_6 in Fig. 14b is the amplitude which appears in the Kancheli-Mueller optical theorem[75] for the inclusive cross section for the production of the particle b. Using Gribov's technique,[13] we easily find the following expression for $\sigma_{\rm Re}$ (Ref. 74):

$$\sigma_{\rm Re}\approx\int d^3\mathbf{k}\,\frac{d\sigma_b}{d^3\mathbf{k}}\,\sigma_{\rm abs}^{\rm bN}\int\frac{d^3q}{(2\pi)^3}\,\frac{\rho_D(\mathbf{q}^2)}{(q_z+i\varepsilon)},\qquad(3.11)$$

where the range of the integration over $\mathbf{k}$ and $\mathbf{q}$ is restricted by the requirement that the intermediate particles lie near the mass shell, $|k_zq_z|\lesssim m^2$, and where $\rho_D(\mathbf{q}^2)$ is the deuteron form factor. In the coordinate representation, expression (3.11) becomes

$$\sigma_{\rm Re}\approx\sigma_{\rm abs}^{\rm hd}\int d\varepsilon\,\frac{dn_b}{d\varepsilon}\,\sigma_{\rm abs}^{\rm bN}\int d\mathbf{r}\,|\psi_d(r)|^2\theta\left(r-\frac{\varepsilon}{m^2}\right).\qquad(3.12)$$

We wish to emphasize that (3.12) requires absolutely no information about the Regge structure of either the cross section for the production of particle b at the nucleons or the cross section for bN scattering; directly measurable quantities appear in this expression. Expression (3.12) itself has a simple probabilistic meaning: For secondary particles with a formation length

$$l_f\approx\frac{\varepsilon}{m^2}<R_d\qquad(3.13)$$

the inelastic interaction with the second nucleon has the simple geometric probability

$$w_{\rm Re}\approx\sigma_{\rm abs}^{\rm bN}\left\langle\frac{1}{4\pi r_d^2}\right\rangle,\qquad(3.14)$$

while for particles with $l_f>R_d$ the rescattering probability is negligibly small. The θ function in (3.12) should not be taken literally; it simply reflects the nature of the transition between the regions $l_f(\varepsilon)\gtrless R_d$.

A simple picture thus emerges for the rescattering of the secondary particles, as described by Kancheli[1] in 1973 (see also the earlier paper by Kancheli and Matinyan[76]): Secondary hadrons with an energy ε have formation lengths $l_f(\varepsilon)\approx\varepsilon/\mu_0^2$. At distances greater than $l_f(\varepsilon)$, the motion of the secondary particles can be assumed classical (Gottfried and Low[77] have also proved this assertion in wave-packet terms); in other words, the probabilistic treatment is applicable. The effect of regions outside the formation zone, where, in contrast, the detailed structure of the amplitude and interference effects are important, can be ignored. A generalization of (1.2) incorporating the formation lengths is[78]

$$\frac{dN_s(\varepsilon,t)}{d\varepsilon}=\frac{dN_s(E\to\varepsilon)}{d\varepsilon}\theta(t-l_f(\varepsilon))\exp[-(t-l_f(\varepsilon))]$$
$$+\int_0^{t-l_f(\varepsilon)}d\tau\exp[-(t-\tau-l_f(\varepsilon))]\int_\varepsilon^E d\omega\,\frac{dN_s(\tau,\omega)}{d\omega}\,\frac{dN_s(\omega\to\varepsilon)}{d\varepsilon}.\qquad(3.15)$$

In the limit $l_f(\varepsilon)\ll 1$, expression (3.15) converts into the integral form in (1.2). Secondary particles with $l_f(\varepsilon)>t$ do not interact in the target at all, and this is why the number of interactions within the target nucleus is suppressed and becomes independent of the energy. According to the conclusion stated above, μ_0^2 is determined by the rate at which the amplitude decreased as the external particles depart from the mass shell. A natural scale dimension here is the mean square transverse mass of the "direct" particles $\mu_0^2\approx\mu^2+\langle\mathbf{k}_T^2\rangle\approx m_\rho^2$.

Equation (3.15) ignores the multipomeron interactions described by $G_{\rm PPP}$. The numerical effect of these interactions could be important only at energies of the order of the mass of the universe. Furthermore, at finite energies their effect is similar to the cascade effect. Nevertheless, at high energies, multipomeron interactions can lead to several specific effects (Subsection 6e). It is inconsistent to describe the entire effect of the nucleus on the spectra by means of the tree disgrams in Fig. 15 alone, as has been done by Shwimmer[79] and Gedalin,[80] because of the inequality $G_{\rm PPP}\ll G_{\rm eff}$ and because it is incorrect to consider multipomeron interactions if cascades are ignored.

f) Do the secondary particles interact collectively?

As Barashenkov *et al.* have noted,[10,81] if all the secondary particles were produced from a single point then the fast particles would interact collectively in the nucleus, as if they constituted a single particle. In fact, a secondary particle with a momentum k would be singled out from the beam at an impact parameter $\Delta b\gtrsim 1/\mu_\pi$ and could be considered independent only over longitudinal distances

$$L\gtrsim\frac{k}{\langle k_T\rangle}\,\frac{1}{\mu_\pi}.\qquad(3.16)$$

Such collective interactions suppress the development of the cascade, and calculations from the cascade model as refined in this manner with experiment at energies up to several tens of GeV (Refs. 10 and 81).

Collective interactions are not possible within the framework of the field multiperipheral approach. As Feĭnberg and Chernavskiĭ[57] and Gribov[7] have shown,

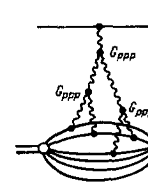

FIG. 15. Multipomeron tree diagrams for inclusive distributions in interactions with nuclei.

the points at which the secondary particles are produced are distributed in a random-walk manner over the impact-parameter plane, so that all the secondary particles are already spread over impact parameters when they are produced and therefore interact independently.

g) Does the nuclear medium affect the formation lengths?

In the Landau-Pomeranchuk problem the absorption of an electron would lead to a term of the type $-t/l_{abs}$ in the argument of the exponential function in the intergrand in (3.1). For formation lengths $l_f < l_{abs}$ the entire discussion in Subsection 3a remains valid. The emission of photons, on the other hand, for which the length in (3.3) is much greater than l_{abs}, acquires an additional small factor $(l_{abs}/l_f)^2$. Valanju *et al.*[82] made a literal extension of the Landau–Pomeranchuk problem with absorption to the case of hadrons. In this manner, the production within the nucleus of all secondary particles with formation lengths (1.5) greater than the absorption length of the incident particle was suppressed, and a satisfactory qualitative description was found for experimental data on inclusive production in nuclei. From the standpoint of the picture under consideration here, this approach is inapplicable in many regards. First, the basic assumption that the secondary particles are emitted by a primary particle which is propagating continuously in the nucleus after an arbitrary number of inelastic absorption events is incorrect. Second, the Lee model, which Valanju *et al.* studied as a prototype, actually describes diffractive dissociation, rather than multiple production. Third, in this approach the production amplitudes do not decrease with departure from the mass shell, although this decrease is important for an analysis of the space-time picture of the interaction (see Subsections 3b, 3d, and 3e).

Bialkovski *et al.*[83] have discussed another possibility. When there is absorption it is possible to introduce the formal length

$$L = \frac{1}{\omega(1-\mathbf{nv}) + (i/l_{abs})}, \tag{3.17}$$

which is equal to the usual formation length at small values $l_f < l_{abs}$ but which has the upper limit l_{abs} on its modulus at larger values. Bialkovski *et al.*[83] interpret (3.17) as an upper limit set on the formation lengths by the absorption length l_{abs}. The necessary group-theory justification for this approach has not been worked out. The resulting suppression of the intranuclear cascade is not strong enough, and at high energies the model overestimates the secondary-particle multiplicities.[84]

h) The nuclear Cherenkov effect

Dremin[85] has recently pointed out that at extremely high energies ($E \gtrsim 1$ TeV) the real part of the refractive index in nuclear matter satisfies $n = 1 + (2\pi/k^2)\rho_A \,\mathrm{Re}f > 1$ for all hadrons, so that in proton-nucleus collisions the condition for the Cherenkov radiation of pions with momenta of tens of GeV is satisfied:

$$n v_p v_\pi > 1. \tag{3.18}$$

Formally, the Cherenkov radiation is the consequence of a pole at

$$1 - n(\mathbf{nv}) = 0 \tag{3.19}$$

in the analog of amplitude (3.2) (see, for example, the textbook[86] by Landau and Lifshitz). For hadrons, however, absorption is important even in the pertinent energy range: $\varkappa = \mathrm{Im}f/\mathrm{Re}f \gtrsim 20$. Consequently, condition (3.19) does not hold strictly; the integrated intensity falls off by a factor of at least $\varkappa$, and the radiation is not concentrated at the angle θ_c, where $\cos\theta_c = 1/n$, but spans the angular cone $\Delta\theta \sim \sqrt{\varkappa\theta_c}$. Because of this suppression, the emission of a large number of Cherenkov particles, which would form a characteristic Cherenkov ring, becomes extremely improbable.

The production angles of the Cherenkov particles are large, as are the transverse momenta of these particles. The process is thus a hard process, and we should perhaps study the emission of gluons by quarks. There is some uncertainty regarding the choice of a refractive index for quarks and regarding the incorporation of edge effects in the emission resulting from the confinement of colored gluons. Even in this case the emission is still suppressed by the finite nuclear dimensions, and the formation of a genuine ring structure is again improbable. Dremin[85] has, however, reached the optimistic conclusion that the Cherenkov radiation could be predominant over ordinary production processes at transverse momenta above 30 GeV and at c.m. angles $\approx 60°$.

I. B. Khriplovich and the present author jointly made this comment regarding the role of absorption in the Cherenkov emission of hadrons.

4. TOTAL CROSS SECTIONS AND DIFFRACTION PROCESSES

a) Absorption, elastic scattering, and diffractive dissociation

For an elementary particle which is incident on a black nucleus there are two processes: total absorption, at impact parameters $b \le R_A$, and the diffractive elastic scattering which results from this absorption. For a composite particle such as the deuteron the absorption cross section begins to depend on the internucleon distance b_d in the impact-parameter plane:

$$\sigma_{abs} = \pi(R_A + b_d)^2. \tag{4.1}$$

As a result, the fraction of states with small values of b_d in the transmitted wave $|f\rangle$ is greater than in the deuteron. After an expansion in the system of eigenfunctions of the np system,

$$|f\rangle = S_d|d\rangle + \sum_i S_i|i\rangle \tag{4.2}$$

we find an admixture of continuum states, $|i\rangle$, in the final state. In other words, the elastic scattering is accompanied by a fundamentally new process: diffractive dissociation,[5,87]

$$dA \to (np)A. \tag{4.3}$$

The nucleus remains in its ground state, and the process is coherent, if $q_L = (m_{np}^2 - m_d^2)/2E$ is small, specifically, if $q_L R_A < 1$. Correspondingly, the characteristic longitudinal distances are large[5,87]:

$$L \approx \frac{1}{q_L} > R_A. \qquad (4.4)$$

b) The eigenstate method and the Glauber formalism

The deuteron is a diagonal state in the spectrum of masses but not in the spectrum of the scattering operator. The diagonal states or eigenstates of the scattering operator are those states which are only absorbed and scattered elastically. In the case of the deuteron there is no particular need for a special introduction of scattering eigenfunctions, although there would be no difficulty in doing this.[87] In the case of hadrons the eigenfunction system is not known, but the eigenstate formalism itself proves very useful.

Let us assume that for the state vector $|A\rangle$ of the incident particle we have the following expansion in eigenfunctions of the scattering operator:

$$|A\rangle = \sum_i c_i |i\rangle. \qquad (4.5)$$

This expansion is meaningful to the extent that the lifetimes of the parton fluctuations (which will be identified below with scattering eigenstates) are long in comparison with the actual time of the interaction with the target (see Subsection 3c).

For a fixed impact parameter the matrix element of the T matrix satisfies

$$\langle A| \operatorname{Im} T |A\rangle = \sum_i |c_i|^2 t_i = \langle t\rangle. \qquad (4.6)$$

Diffractive dissociation corresponds to the final-state component

$$|D\rangle = \operatorname{Im} T\,|A\rangle - \langle t\rangle\,|A\rangle. \qquad (4.7)$$

From this we find

$$\frac{d\sigma_D}{d^2b} = \langle D\,|\,D\rangle = \langle t^2\rangle - \langle t\rangle^2. \qquad (4.8)$$

Here we have followed a recent paper,[88] although an essentially similar formalism was used in Refs. 5, 87, and 89.

To generalize (4.6) to the case of nuclei we need the amplitudes for the interaction of the eigenstates with the nucleus. For short wavelengths the semiclassical approach is valid, and we find the following for the probability that an eigenstate will pass through the nucleus *without* undergoing an interaction:

$$W = \exp[-\sigma_{tot}^N T(b)] = \exp[-\nu(b)]. \qquad (4.9)$$

To make the transformation to a quantum scattering theory we identify W with $|S(b)|^2$. At high energies the scattering amplitudes are purely imaginary, so that

$$S(b) = \sqrt{W} = \exp\left[-\tfrac{1}{2}\nu(b)\right], \qquad (4.10)$$

i.e.,[90]

$$\sigma_{tot}^A = 2\int_0^\infty db\cdot 2\pi b\left\{1 - \left\langle \exp\left[-\tfrac{1}{2}\nu(b)\right]\right\rangle\right\}. \qquad (4.11)$$

Expression (4.10) is the same as the expression given for $S(b)$ by the familiar Glauber–Sitenko formalism.[91,92] The two approaches are actually the same. In the limit $A \gg 1$ the Glauber formalism transforms into the optical model with the potential[93]

$$V(\mathbf{r}) = -4\pi f(0)\,\rho_A(\mathbf{r}). \qquad (4.12)$$

In the case of diffractive dissociation, $f(0)$ should be replaced by the matrix $f_{ik}(0)$ of diffraction-process amplitudes, and the following equation should be solved[94]:

$$(\nabla^2 + k_i^2)\,|i,\mathbf{r}\rangle = -\sum_j 4\pi f_{ij}(0)\,\rho_A(\mathbf{r})\,|j,\mathbf{r}\rangle. \qquad (4.13)$$

Under condition (4.4) the scattering eigenstates are simply those states which diagonalize the matrix $f_{ik}(0)$.

The potential approach is not rigorous at high energies. The Glauber formalism was given a group-theory basis by Gribov[13] in 1969, and Gurvits and Marinov[95] have developed a general formulation of the Gribov formalism and have published a detailed discussion of its relationship with the potential approach. Gribov has shown that the amplitude for scattering by a nucleus is the sum of all the multiple-scattering diagrams in Fig. 15. The role of diffractive excitation, which leads to the multichannel potential in (4.13), can be discussed particularly clearly in terms of diagrams. Equation (4.11) corresponds to the exact sum of all the diagrams in Fig. 16, incorporating all possible intermediate states. If we replace all the diffractively produced systems by a single effective state, i.e., if we approximate the matrix f_{ik} by a 2×2 matrix, then we can write closed expressions for the amplitude for scattering by a nucleus directly in terms of f_{ik}. This has been done by Shabel'skiĭ.[96]

c) Inelastic screening in the total cross sections and the absorption cross sections

The simple optical model yields

$$\sigma_{tot}^A = 2\int_0^\infty db\cdot 2\pi b\left\{1 - \exp\left[-\tfrac{1}{2}\sigma_{tot}^N T(b)\right]\right\}. \qquad (4.14)$$

In the eigenstate method we have $\sigma_{tot}^N T(b) = \langle \nu(b)\rangle$. Since

$$\left\langle \exp\left[-\tfrac{1}{2}\nu(b)\right]\right\rangle \geqslant \exp\left[-\tfrac{1}{2}\langle\nu(b)\rangle\right], \qquad (4.15)$$

the incorporation of diffractive dissociation (or "inelastic screening") reduces the total cross sections to values below those given by (4.14). This circumstance was first demonstrated by Gribov,[13] who studied the inelastic corrections to the total cross sections for the

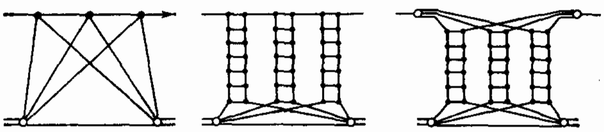

FIG. 16. a—The Gribov–Glauber diagram for the amplitude for elastic scattering by a nucleus; b—the corresponding planar amplitudes for m-pomeron exchange; c—nonplanar diagrams for m-fold scattering by a nucleus, treated as equivalent to planar diagrams (part b).

interaction with the deuteron.

The inelastic correction to the cross sections for heavy nuclei was found by Karmanov and Kondratyuk,[97] who solved (4.13) by perturbation theory in terms of the off-diagonal elements $f_{ij}(0)$. Even in perturbation theory it is necessary to adopt some additional assumption regarding the diagonal transitions, $f_{ii}(0)$, which are not small. Karmanov and Kondratyuk assumed

$$\sigma_{h^*N} \equiv \sigma_{hN}. \tag{4.16}$$

This approach leads to the inelastic correction[97]

$$\Delta\sigma_{\text{tot}}^{A} = -4\pi \int db \cdot 2\pi b \left(\frac{d\sigma_D}{dt}\right)_{t=0} \times T(b)^2 \exp\left[-\frac{1}{2}\sigma_{\text{tot}}^{N} T(b)\right], \tag{4.17}$$

where $(d\sigma_D/dt)_{t=0}$ is the total differential cross section for diffractive excitation at $t=0$.

The application of perturbation theory to the actual process of diffractive dissociation by nuclei leads to meaningless results, as will be discussed in detail in Subsection 4d. Despite this internal contradiction of perturbation theory, the Karmanov–Kondratyuk equation, Eq. (4.17), agrees well with experiment (Figs. 17 and 18), and it is not possible to derive a quantitative description of the available data on the $K_L A$ (Ref. 22) and nA (Refs. 23 and 24) total cross sections without incorporating inelastic screening.

This circumstance is explained in the following manner.[99] According to (4.11) and (4.14), $\Delta\sigma_{\text{tot}}^{A}$ may be written

$$\Delta\sigma_{\text{tot}}^{A} = -2\int_0^\infty db \cdot 2\pi b \left\{\left\langle \exp\left[-\frac{1}{2}\nu(b)\right]\right\rangle - \exp\left[-\frac{1}{2}\langle\nu(b)\rangle\right]\right\}. \tag{4.18}$$

A series expansion of the integrand yields

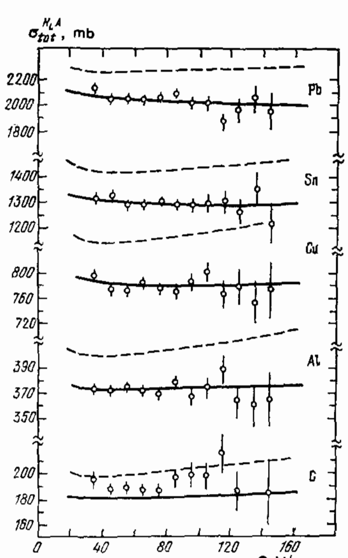

FIG. 17. Dependence of the total cross section for the $K_L A$ interaction on the energy and on the atomic number of the nucleus. Dashed curves —Calculations[22] from the single Glauber model; solid curves—calculations incorporating an inelastic correction.[98]

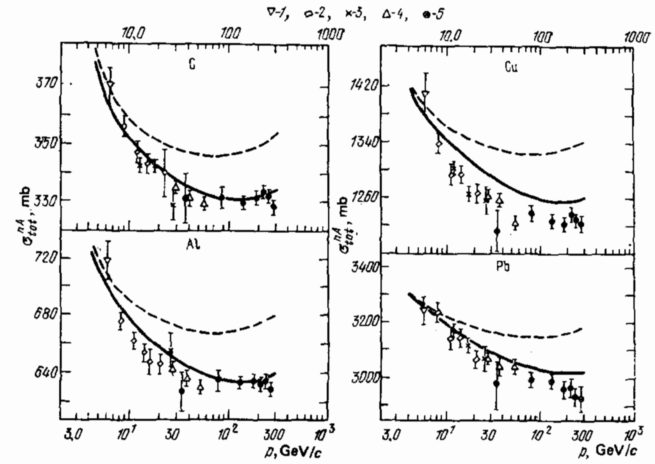

FIG. 18. Dependence of the total cross section for the nA interaction on the energy and the atomic number of the nucleus. 1—Parker *et al.*; 2—Engler *et al.*; McCorriston *et al.*, 4—Babaev *et al.*[23]; 5—Murthy *et al.*[24] Solid curves—Calculated from the simple Glauber model; dashed curves—calculations incorporating inelastic shadowing.[24]

$$\left\langle \exp\left[-\frac{1}{2}\nu(b)\right]\right\rangle - \exp\left[-\frac{1}{2}\langle\nu(b)\rangle\right] \approx \exp\left[-\frac{1}{2}\langle\nu(b)\rangle\right]\frac{T(b)^2}{8}(\langle\sigma^2\rangle - \langle\sigma\rangle^2). \tag{4.19}$$

Comparison of (4.19) with (4.8) leads to Eq. (4.16).

The fluctuations of the cross sections, $\Delta\sigma_i = \sigma_i - \langle\sigma\rangle$, are of the order of $\langle\sigma\rangle$. At small impact parameters, at which $\nu_i(b)$ is greater than unity, expansion (4.19) is not legitimate, but the contribution of this region to the inelastic screening is suppressed by the exponential factor $\exp[-(1/2)\langle\nu(b)\rangle]$. This suppression is the reason why the inconsistent Karmanov–Kondratyuk approximation yields a satisfactory quantitative estimate of the inelastic correction.

In the two-channel approximation with $f_{11} = f_{22}$ it follows from (4.13) that

$$f_{a,b} = f_{11} \pm f_{12}, \tag{4.20}$$

so that the cross section is small for one of the eigenstates, $|a\rangle$ or $|b\rangle$. In the multichannel problem, the smallest of the cross sections σ_i may in fact turn out to be zero. Recalling the analogy between the parton model and the Weizsacker–Williams method, we conclude that a passive state of this type, which contains no slow partons, is extremely natural. In theories with decreasing cross sections, in which the intersection of the vacuum trajectory satisfies $\alpha_P(0) < 1$, the weight of the active state would generally fall off with increasing energy, in proportion to E^{α_P} raised to some power between the zeroth and the first.[100] It has been pointed out that the weight of the passive state $P = |c_0|^2$ also increases with increasing energy in theories with increasing total cross sections, approaching the asymptotic value from below.[101] The incorporation of passive states in the quark model has been discussed in some recent papers.[90,98,102]

Experimentally, σ_{tot}^{pN} and $\sigma_{\text{tot}}^{K_L N}$ increase with increasing energy, causing σ_{tot}^{A} also to increase for light nuclei.

For a black nucleus, however, it follows from (4.11) that[90,103]

$$\sigma_{tot}^{A}=(1-P)\cdot 2\pi R_{A}^{2}, \qquad (4.21)$$

i.e., we would expect σ_{tot}^{A} to *decrease* with increasing energy. We note that (4.21) corresponds to a relative inelastic correction which does not fall off with increasing A,

$$\frac{\Delta\sigma_{tot}^{A}}{\sigma_{tot}^{A}}\approx -P, \qquad (4.22)$$

while the relative value of (4.17) does fall off with increasing A, because of the factor $\exp[-\sigma_{tot}^{N}T(b)/2]$ in the integrand. It is natural to find a difference between (4.17) and (4.21), since expansion (4.19) is not valid in the presence of a passive state for heavy nuclei.

The experimental data on σ_{tot}^{nA} and $\sigma_{tot}^{K_L A}$ in Figs. 17 and 18 confirm that a change of this type occurs in the energy dependence of the cross sections when we go from light to heavy nuclei. The data on σ_{tot}^{nA} indicate that approximation (4.17) underestimates the size of the inelastic correction for heavy nuclei and also indicate that σ_{tot}^{A} increases more rapidly with increasing energy than as described by (4.17) (Fig. 18). Here we may be seeing the first direct evidence of passive states.[99] A continuation of precise measurements of σ_{tot}^{nA} up to the Tevatron energy range would be very important.

The value of P remains an open question. Fialkowski and Miettinen[104] were the first to point out that the fluctuations in the cross sections σ_i should be assumed large in order to describe the diffractive dissociation in proton–proton scattering. Miettinen and Pumplin[88] found the value $P\approx 0.05$ in an analysis of diffractive dissociation in pp scattering. Levin and Ryskin[105] found $P\approx e^{-2}\approx 0.14$ as a solution of the bootstrap equation for the cross section for the interaction of a parton system with a nucleus. They attributed this value of P to constituent quarks. Analysis of the total cross sections yields the estimates $P=0.2-0.4$ for constituent quarks.[88,98]

In the derivation of (4.11) and (4.16), all the diffraction amplitudes have been assumed to be purely imaginary. This assumption is correct in the limit $E\to\infty$ for the excitation of a state with any finite mass m^*, since the contributions of the nonvacuum trajectories vanish.[106] This is not the case in the production of large masses, with $m^{*2}/2m_N E=\text{const}$. Masses $m^{*2}/2m_N E\lesssim 1/R_A m_N$ contribute to inelastic screening. As was first pointed by Anisovich *et al.*[107] the production of large masses as the result of several nonvacuum exchanges leads to a negative-shadowing contribution to σ_{tot}^{A}. The relative size of this contribution does not fall off with increasing energy. In scattering by deuterons, the negative-shadowing effect is of the order of 10% of the inelastic correction.[108] The effect has not been estimated for heavy nuclei, although we would expect it to be suppressed by the large nuclear radius.

From (4.14) we find

$$\sigma_{in}^{A}=\sigma_{tot}^{A}-\sigma_{el}^{A}=\int db\cdot 2\pi b\{1-\exp[-\sigma_{tot}^{N}T(b)]\}, \qquad (4.23)$$

which is not the same as (2.13). The reason is that in an interaction with nuclei there is a specific process: the quasielastic scattering $hA-hA^*$ accompanied by excitation of the nucleus but not by the production of any new particles. For the cross section for quasielastic scattering, σ_Q^A, it is a simple matter to find[109,110]

$$\sigma_Q^A=\int db\cdot 2\pi b\{\exp[-\sigma_{abs}^{N}T(b)]-\exp[-\sigma_{tot}^{N}T(b)]\}. \qquad (4.24)$$

An expression for σ_Q^A in the form of an expansion in σ_{el}^N is given in the review by Tarasov.[109] In scattering by heavy nuclei the strong Coulomb field rules out separate measurements of σ_Q^A and σ_{el}^N; only the beam-loss cross section $\sigma_{abs}^A=\sigma_{tot}^A-\sigma_Q^A-\sigma_{el}^A$ can be measured. This cross section is equal to (2.13) by virtue of (4.23) and (4.24). This distinction between σ_{abs}^A and σ_{in}^A has not always been handled correctly.[111]

If the inelastic correction is important in the total cross sections, then (2.13) gives a good description of experiments even without inelastic corrections. A rather involved derivation leads to the following expression[110] for $\Delta\sigma_{abs}^A$:

$$\Delta\sigma_{abs}^{A}=-4\pi\int db\cdot 2\pi b\left(\frac{d\sigma_D}{dt}\right)_{t=0}T(b)^2\exp[-\sigma_{abs}^{N}T(b)]\times\left(1-\frac{\beta\sigma_{el}^{N}}{\sigma_{tot}^{N}}\right)^2. \qquad (4.25)$$

The coefficient β is model-dependent, but it lies in the range $1\leqslant\beta\leqslant 2$. In the case of geometric scaling we would have $\beta=1$, while if the slope of the diffraction cone is only a weak function of the cross section we would have $\beta=2$. Calculations from (4.25) yield $\Delta\sigma_{abs}^A=(0.2-0.3)\Delta\sigma_{tot}^A$. The statistical errors in the measurement of σ_{abs}^A are of the same order of magnitude, while the systematic errors are even larger,[21,112] so that it is not possible to check (4.25).

d) Diffractive dissociation by nuclei and interaction cross sections of unstable particles

In the two-channel approximation, the following expression can be derived from (4.13) by perturbation theory:

$$t_{12}^{A}(b)=2f_{12}^{N}(0)\left\{\exp\left[-\frac{1}{2}\sigma_1 T(b)\right]-\exp\left[-\frac{1}{2}\sigma_2^* T(b)\right]\right\}(\sigma_2^*-\sigma_1)^{-1}. \qquad (4.26)$$

From diffractive dissociation by nuclei it would also be possible to determine the cross sections σ_2^* for the interaction of diffractively produced systems with nucleons. When Kölbig and Margolis[113] pointed this out, they stimulated several experiments on diffractive dissociation by nuclei. Tarasov[109] has reviewed in detail the early work on the theory of diffractive dissociation by nuclei. He discussed the basic experimental data in the same paper. The results were disheartening: The experimental value of σ_2^* for multiparticle systems turned out to be smaller than or comparable to $\sigma_{\pi N}$ or σ_{NN}, in contrast with the naive expectation that it would be equal to the sum of the cross sections over all the particles of the system.[114,117] Figure 19 shows a typical recent determination of σ_2^* for the πN and $N\pi\pi$ systems from diffractive dissociation of nucleons in collisions with deuterons.[118]

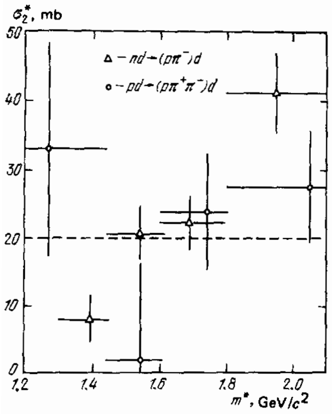

FIG. 19. Cross sections for the $(p\pi^-)N$ and $(p\pi^+\pi^-)N$ interactions found through an analysis of diffractive dissociation by deuterons by the Kölbig-Margolis method.[113]

In one way or another, the different interpretations of this phenomenon reduce to the inapplicability of the two-channel approximation, (4.26). The situation can be clarified through the example of the following three-channel problem, which was proposed by Czyz[119] and which has been discussed elsewhere from the standpoint of diffractive dissociation[120]: We assume that the scattering eigenstates $|1\rangle$, $|2\rangle$, and $|3\rangle$ have the interaction cross sections $\sigma_1=\sigma_0$, $\sigma_2=2\sigma_0$, and $\sigma_3=3\sigma_0$. We further assume that the physical states, which we will denote arbitrarily by $|\pi\rangle$, $|3\pi\rangle$, and $|5\pi\rangle$, are related to the eigenstates by the expansions

$$\left.\begin{aligned} |\pi\rangle &= \frac{1}{2}|1\rangle+\frac{1}{\sqrt{2}}|2\rangle+\frac{1}{2}|3\rangle,\\ |3\pi\rangle &= -\frac{1}{2}|1\rangle+\frac{1}{\sqrt{2}}|2\rangle-\frac{1}{2}|3\rangle,\\ |5\pi\rangle &= -\frac{1}{\sqrt{2}}|1\rangle+\frac{1}{\sqrt{2}}|3\rangle. \end{aligned}\right\} \qquad (4.27)$$

From (2.24) we have

$$\sigma_{\pi N}=\sigma_{(3\pi)N}=2\sigma_0,\quad \sigma_{(5\pi)N}=2\sigma_0, \qquad (4.28)$$

and for the amplitudes for diffractive dissociation we find

$$t^A_{\pi\to 3\pi}\sim 2\exp\left[-\sigma_0 T(b)\right]-\exp\left[-\frac{1}{2}\sigma_0 T(b)\right]-\exp\left[-\frac{3}{2}\sigma_0 T(b)\right],$$
$$t^A_{\pi\to 5\pi}\sim\exp\left[-\frac{1}{2}\sigma_0 T(b)\right]-\exp\left[-\frac{3}{2}\sigma_0 T(b)\right]. \qquad (4.29)$$

These expressions have nothing in common with (4.26) if the cross sections are given by (4.28). Equation (4.29) for $t^A_{\pi\to 5\pi}$ is particularly noteworthy; it corresponds, in terms of Eq. (4.26), to the initial propagation in the nucleus of a particle with an interaction cross section $\sigma_1=\sigma_0=(1/2)\sigma_{\pi N}$, which subsequently transforms into a particle with an interaction cross section $\sigma_2^*=3\sigma_0=(3/2)\times\sigma_{(5\pi)N}$. Consequently, if σ_1 is also treated as a free parameter in an analysis of diffractive dissociation by nuclei, then the values found for σ_1 will be different from the physical cross section for a beam particle.[121] In an attempt to describe $t^A_{\pi\to 3\pi}$ by the Kölbig-Margolis formula, (4.26), the result would be[120] $\sigma_2^*<0$!

The physical reason for the effect, as has been emphasized repeatedly by Feĭnberg,[59] is that the system which is produced through the diffractive dissociation is not a directly observable state, say the $|3\pi\rangle$ state.

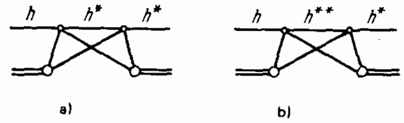

FIG. 20. Diagonal and off-diagonal shadowing in diffractive dissociation by deuterons.

The $|3\pi\rangle$ component separates from the excited system only after it has passed through the nucleus; during the passage, the off-diagonal transitions of the $\pi-5\pi-3\pi$ are extremely important. The Kölbig-Margolis two-channel approximation and perturbation theory ignore these transitions. The description of the multichannel problem in an effective two-channel approximation can, as we have seen in example (4.29), lead to a three-channel problem and, in general, to $\sigma_2^*<0$! To pursue this point we will consider the case of diffractive dissociation by the deuteron.

The inelastic correction to the total cross section is quadratic in f_{hh^*} and is of definite sign [see Eq. (4.17)]. The inelastic correction to the amplitude for the diffractive dissociation $hd\to h^*d$ contains the shadowing terms $f_{hh}f_{hh^*}$ or $f_{hh^*}f_{h^*h^*}$ (see the diagram in Fig. 20a) and also terms of the type $f_{hh^{**}}f_{h^{**}h^*}$ (Fig. 20b), whose sign is model-dependent. In the simple parton model of Ref. 122, for example, we have $\mathrm{Im} f_{hh^*}<0$ for all the off-diagonal transitions, so that the inelastic off-diagonal corrections have a negative-shadowing sign. The relative contribution of the diagram in Fig. 20b increases with increasing mass of the product system, and the shadowing gives way to a negative shadowing[122] (Fig. 21). The possibility of a distinction between shadowing in diffractive dissociation by the deuteron and the process described by the Glauber model was first pointed out by Levin *et al.*[123]

In general, the amplitude for coherent diffractive dissociation by a nucleus, $aA\to bA$, is

$$t^A_{a\to b}(\mathbf{b})=\sum_i a_i b_i^*\exp\left[-\frac{1}{2}\nu_i(b)\right], \qquad (4.30)$$

where a_i and b_i are the coefficients in the expansions $|a\rangle=\sum_i a_i|i\rangle$ and $|b\rangle=\sum_i b_i|i\rangle$ in terms of the system

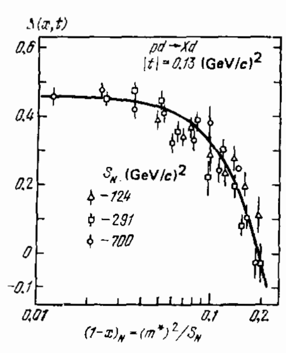

FIG. 21. Dependence of the shadowing component $\Delta(x,t)$ in the cross section for the dissociation $pd\to Xd$ on the mass of the product system, m^*. The decrease and the change in sign of $\Delta(x,t)$ with increasing m^* correspond to a transition from shadowing to negative. Curve—calculation from the parton model of Ref. 122.

of scattering eigenfunctions. Some examples of multichannel problems in which a fit of (4.30) by the Kölbig–Margolis formula, (4.26), leads to nonphysical values $\sigma_2^* < 0$ are described in a recent paper by Czyz and Zielinski.[124]

Noncoherent dissociation accompanied by breakup of the nucleus could also be used to determine σ_2^*. At low energies, $E < R_A(m^{*2} - m^2)$, the noncoherent dissociation is described by the classical probabilistic picture. Perturbation theory predicts[113]

$$\left(\frac{d\sigma_D}{dt}\right)_A = \left(\frac{d\sigma_D}{dt}\right)_N \int db \cdot 2\pi b \times \{\exp[-\sigma_1 T(b)] - \exp[-\sigma_2^* T(b)]\}(\sigma_2^* - \sigma_1)^{-1}. \quad (4.31)$$

At high energies $E > R_A(m^{*2} - m^2)$, the coherence along the coordinate z of the interactions with different nucleons of the nucleus is important. The interactions at various impact parameters are again noncoherent. In this case, perturbation theory yields[125] (for $t = 0$)

$$\left(\frac{d\sigma_D}{dt}\right)_A = \left(\frac{d\sigma_D}{dt}\right)_N \int db \cdot 2\pi b \times \left\{\left(\sigma_1 \exp\left[-\frac{1}{2}\sigma_1 T(b)\right] - \sigma_2^* \exp\left[-\frac{1}{2}\sigma_2^* T(b)\right]\right)(\sigma_2^* - \sigma_1)^{-1}\right\}^2. \quad (4.32)$$

In the approximation of a single inelastic interaction, the multichannel generalization of (4.32) is[110] [the notation is the same as in (4.30)]

$$\left(\frac{d\sigma_D}{dt}\right)_A = \frac{1}{k^2}\int db \cdot 2\pi b T(b) \left|\sum a_i b_i^* f_i(\mathbf{q}) \exp\left[-\frac{1}{2}\sigma_i T(\mathbf{b})\right]\right|^2. \quad (4.33)$$

The exact formula, incorporating the multiple inelastic interactions, is quite complicated.[110] It is nevertheless possible to make the general assertion that the values found for σ_2^* for a given system from coherent dissociation with the help of (4.26) may be very different from the values found from noncoherent dissociation with the help of (4.31) or (4.32) (Ref. 21). For example, in an experiment on the coherent dissociation $pA - (p\pi^+\pi^-)A$ for the mass intervals $m^* = 1.4$–1.6, 1.6–1.8, and 1.8–2.2 GeV, the cross sections σ_2^* were found to be

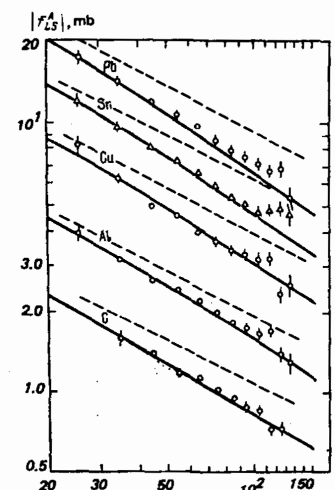

FIG. 22. Dependence of the regeneration amplitude f_{LS}^A on the energy and the atomic number of the nucleus.[130] Dashed curves—Calculations in the Glauber model without inelastic screening[130]; solid curves—calculations incorporating inelastic screening.[98]

24, 25, and 18 mb, respectively[126]; these values should be compared with the values $\sigma_2^* = 0.8$ and 7 mb ($\sigma_2^* < 24$, 32, and 10 mb at a 95% confidence level) found from noncoherent dissociation.[127]

In summary, diffractive dissociation cannot be used to determine σ_2^* with the help of the two-channel equations, (4.26) and (4.31) or (4.32). Furthermore, it would hardly be possible to determine the parameters of the multichannel problem directly from experimental data. Some progress has recently been noted, however, in the construction of quark–parton models for the scattering eigenfunctions.[105,128]

Several experimental studies of nondiffractive noncoherent processes of the types $\pi A \to \rho A^*$, $NA \to \Delta A^*$ have been reported. At relatively low energies, $E < R_A(m^{*2} - m^2)$, their cross sections are described by Eq. (4.31), and the cross sections found in this manner for the ρN, ΔN, etc., interactions are actually physical cross sections (see, for example, the reviews in Ref. 129 for citations of the experimental papers).

e) $K_L \to K_S$ regeneration in nuclei

The regeneration of K_S mesons is a striking example of a coherent nondiffractive process. Experiments by the Telegdi group[130] revealed that neither the regeneration amplitude f_{LS}^A nor its phase shift φ_{LS}^A (nor its energy dependence) agreed with the predictions of the simple optical approximation (Fig. 22). For example, $\langle\langle\alpha_\omega(0)\rangle\rangle = 0.39 \pm 0.01$ was found[130] for the effective ω trajectory in f_{LS}^A, while data on KN and $\bar{K}$N interactions yield[131] $\alpha_\omega(0) = 0.44 \pm 0.01$.

As Bertocchi and Treleani[132] pointed out first, the production of large masses is important in the exchange of not only a pomeron but also a ω trajectory. The phenomenological introduction of such transitions leads to a quantitative description of the measured amplitudes[131,98] f_{LS}^A. The role played by inelastic corrections can be understood best in terms of eigenstates. The quantity $\mathrm{Im} f_{LS}^A$ is given by the difference between the cross sections for the K^0A and $\bar{K}^0A$ interactions,[98]

$$\Delta\sigma_A = \int db\, 2\pi b T(b) \sum_i |c_i|^2 \Delta\sigma_i \exp\left[-\frac{1}{2}\sigma_i T(b)\right]. \quad (4.34)$$

In the quark–gluon theory of strong interactions, the pomeron is coupled with a purely gluon fluctuation,[133] while the ω reggeon is associated with fluctuations in which the slow parton is a valence quark. In the simple quark model, the $\Delta\sigma_i$ are determined by this valence quark and are independent of the number of soft gluons. Furthermore, a fluctuation with a slow valence quark corresponds to a quark–gluon ladder in which there is always a source of gluons: the slowed quark itself. In other words, this is an active state. Accordingly,

$$\sum_i |c_i|^2 \Delta\sigma_i \exp\left[-\frac{1}{2}\sigma_i T(b)\right] = \Delta\sigma_N \cdot \left\langle \exp\left[-\frac{1}{2}\sigma_i T(b)\right]\right\rangle', \quad (4.35)$$

where the average $\langle\ldots\rangle'$ is taken over only the active states. Since $\sigma_{tot}^{KN} = (1 - P)\langle\sigma\rangle'$, then $\langle\sigma\rangle' > \sigma_{tot}^{KN}$. We therefore find the numerical result $\langle\exp[-\sigma_i T(b)/2]\rangle' < \langle\exp[-\sigma_{tot}^{KN} T(b)/2]\rangle$ and $|f_{LS}^A|$ is smaller than the prediction of the simple optical model.

Since the weight of the passive state increases with increasing energy, $\langle\sigma\rangle'$ increases with increasing energy more rapidly than σ_{tot}. The result is an additional renormalization of $\langle\langle\alpha_\omega(0)\rangle\rangle$. Here

$$\exp\left[-\frac{1}{2}\langle\sigma\rangle' T(b)\right] \approx \exp\left[-\frac{1}{2}\langle\sigma(E=E_0)\rangle' T(b)\right]\left(\frac{E}{E_0}\right)^{-\delta}, \quad (4.36)$$

where

$$\delta \approx \left(\frac{d\sigma_{tot}^{KN}}{d\ln E} + \sigma_{tot}^{KN}\frac{dP}{d\ln E}\right)\frac{T(b)}{2}. \quad (4.37)$$

In a theory with increasing total cross sections the fact that the secondary trajectories are not universal is not surprising. The important point is that the second term in (4.37), which is proportional to $dP/d\ln E$, accounts for roughly half of the observed renormalization of $\langle\langle\alpha;(0)\rangle\rangle$ (Ref. 98).

5. PHOTOPRODUCTION, ELECTROPRODUCTION, AND NEUTRINO REACTIONS IN NUCLEI

a) Hadronic properties of photons

By virtue of the uncertainty principle, a high-energy photon of energy E could transform into a hadron system with mass m at a distance $\approx E/m^2$ from a nucleus, and it would then be this hadron system which interacted with the nucleus (Subsection 3b). This type of photon–nucleus interaction was first formulated in 1954 by Pomeranchuk for the case of the photoproduction of $\pi^+\pi^-$ pairs in interactions with nuclei.[134] A simple generalization of that work leads to the conclusion that shadowing also occurs in the total cross sections for photoproduction.

Pomeranchuk's work was forgotten, and work on the question started over again a decade later, with papers by Bell,[135] Stodolsky,[136] and Gribov.[26] Adler's relation[137] relates the differential cross section for the neutrino reaction $\nu N \to \mu X$ in collinear kinematics with $|q|^2 \lesssim m_\pi^2$ with the cross section for the interaction of a pion, $\pi N \to X$:

$$\frac{d^2\sigma_\nu}{dq_0\,dq^2} = K(q_0, q^2)\,\sigma_{tot}^{\pi N}(E_\pi = q_0); \quad (5.1)$$

here $K(q_0, q^2)$ is a kinematic factor.[137] Bell noted that in the case of scattering by nuclei it follows from (5.1) that $d\sigma_{\nu A} \sim \sigma_{tot}^{\pi A} \sim A^{0.75}$ in this kinematic region.[135] The cross section for the νA interaction is shadowed, although $\sigma_{tot}^{\nu N} T(b) \ll 1$. Stodolsky noted that (5.1) corresponds to a π dominance of the neutrino interaction at $|q^2| \lesssim m_\pi^2$ and extended this interpretation to photons, formulating a vector dominance model for the interactions of high-energy photons.[136]

In the simplest case the vector dominance model predicts

$$\sigma_{\gamma^*N} = \Gamma_{\gamma\rho}\frac{m_\rho^2}{m_\rho^2+Q^2}\sigma_{\rho N}\frac{m_\rho^2}{m_\rho^2+Q^2}\Gamma_{\gamma\rho}, \quad (5.2)$$

where $Q^2 = -q^2$ (Fig. 23a). Gribov derived a general dispersion relation incorporating all the hadron states V to which the photon could undergo transitions and also the off-diagonal transitions $V_i \to V_j$ (Ref. 26):

$$f_{\gamma^*N} = \int \frac{dM^2 M^2}{M^2+Q^2}\frac{dM'^2 M'^2}{M'^2+Q^2}\Gamma_{\gamma^*V} f_{VV'}\Gamma_{\gamma^*V'}. \quad (5.3)$$

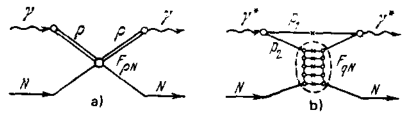

FIG. 23. a—Amplitude for the photon-hadron interaction in the vector dominance model; b—multiperipheral diagram for deep inelastic scattering.

The vertices Γ_{γ^*V} in (5.3) can, in principle, be measured in the annihilation $e^+e^- \to \gamma^* \to V \to$ hadrons. If only vector mesons are used as the states V in (5.3), then the latter corresponds to the so-called expanded or generalized vector dominance model.

b) Bjorken's paradox and criticism of vector dominance

For hadrons, the inelastic screening is quantitatively small in the total cross sections. Correspondingly, by analogy with this diagonal approximation we also restrict (5.3) in a similar manner:

$$\sigma_{\gamma^*A} \sim \int \frac{dM^2 M^4}{(Q^2+M^2)^2}\sigma_{VA}\sigma_{e^+e^-}(M^2). \quad (5.4)$$

Using the scaling behavior $\sigma_{e^+e^-}(M^2)$ in (5.4),

$$\sigma_{e^+e^-}(M^2) \sim \frac{1}{M^2}, \quad (5.5)$$

and assuming

$$\sigma_{VA} \sim \pi R_A^2, \quad (5.6)$$

as for all hadrons, we find[138]

$$\sigma_{\gamma^*A} \sim \alpha\pi R_A^2 \ln\frac{E}{Q^2 R_A} \quad (5.7)$$

in sharp contradiction of the scaling behavior

$$\sigma_{\gamma^*A} \sim \frac{1}{Q^2}F\left(\frac{E}{Q^2}\right). \quad (5.8)$$

If, within the framework of the vector dominance model, we persist in identifying the states V in (5.3) with intermediate vector states in the annihilation reaction, the paradox can be resolved only if (see Ref. 138 and the reviews in Refs. 139 and 140)

$$\sigma_{VA} \sim \frac{1}{M_V^2}. \quad (5.9)$$

At large values of Q^2, (5.4) is determined primarily by masses $M^2 \sim Q^2$, and (5.9) corresponds to the complete disappearance of nuclear shadowing in electroproduction at [141-143,139] $Q^2 \gg m_\rho^2$.

Field-theory approaches lead to a different solution.[144,145] In both the scaling model and the more recently developed quantum chromodynamics parton models, the amplitude for deep inelastic photon–hadron scattering is determined primarily by multiperipheral diagrams (Fig. 23b; see, for example, Ref. 146). The amplitude F_{qN} in this diagram is small except when the quark is near the mass shell, $|p_2^2| \lesssim m^2$. This condition leads to a sharp asymmetry of the $q\bar{q}$ pair: $\varepsilon_2 \approx Em^2/Q^2$. Only the slow quark of the pair interacts with the target.[144,145] In terms of its hadronic properties, the virtual photon turns out to be similar to a single (virtual) quark. This result strongly distinguishes the parton model from the vector dominance model, in which the vector states V are symmetric $q\bar{q}$ pairs. This asymmetry leads to an additional cutoff in (5.4) in terms of the transverse momenta of the quarks of the pair: $|\mathbf{p}_T|^2 \leq m^2$. This cutoff eliminates the contradiction with scaling.[145]

To see what this resolution of Bjorken's paradox implies in the vector dominance model, we consider Gribov's dispersion representation for the amplitude corresponding to the multiperipheral diagram in Fig. 20b. This representation, derived by Brodsky *et al.*,[141] is

$$f_{\gamma^* N} \sim \int \frac{dM^2}{Q^2+M^2} \frac{dM'^2}{Q^2+M'^2} \Big\{ \delta(M^2-M'^2) \Big[M^2+\theta(M^2-\Lambda^2)\Big(M^2-\Lambda^2 \ln \frac{eM^2}{\Lambda^2}\Big)\Big] - \frac{\Lambda^2}{(M^2-M'^2)^2} [M^2\theta(M'^2-M^2-\Lambda^2)+M'^2\theta(M^2-M'^2-\Lambda^2)] \Big\}, \tag{5.10}$$

where Λ is a parameter which determines the rate at which the amplitude decreases with departure of the quark from the mass shell. The diagonal term in (5.10) corresponds precisely to the nonscaling cross section in (5.7). Taken separately, the off-diagonal term leads to precisely the same nonscaling contribution, but with the opposite sign. After the cancellation, (5.10) naturally corresponds to the scaling cross section.

This sort of exact cancellation of diagonal and off-diagonal amplitudes, which is necessary in order to save the generalized vector dominance model, is extremely artificial and is inapplicable from the standpoint of hadron physics, for a simple reason: The intermediate states M in electroproduction are not the same as the states of the same mass in the annihilation reaction. Over the lifetime of the hadron fluctuation of a photon, ν/Q^2, only the slow quark of the $q\bar{q}$ pair into which the photon transforms has time to undergo hadronization.[145] Consequently, as was pointed out by Frankfurt, for states with a large mass we cannot use the factorization of the integrand in (5.3) in terms of σ_{VA} and the vertex $\Gamma_{\gamma^* V}$ measured in e^+e^- annihilation.

c) Shadowing and negative shadowing in deep inelastic scattering

At $x = Q^2/2mE \leq (m_\pi/m_N)A^{-1/3}$, according to Subsection 5b, the distances E/Q^2 are greater than the nuclear radius, and the scattering is diffractive. In terms of structure functions, the shadowing which arises means that

$$F_2^A(x) < AF_2^N(x). \tag{5.11}$$

At $x \geq (m_\pi/m_N)$, the distances E/Q^2 are smaller than the internucleon distances; the scattering is noncoherent; and $F_2^A(x) = AF_2^N(x)$. At the same time, there is the sum rule[145]

$$\int_0^1 dx\, F_2^A(x) = A \int_0^1 dx\, F_2^N(x), \tag{5.12}$$

which means that the momentum of the quarks in the nucleus is equal to the sum of the momenta of the quarks in the nucleons making up the nucleus. Equations (5.11) and (5.12) are compatible only if there is a *negative-shadowing region* in which[145]

$$F_2^A(x) > AF_2^N(x). \tag{5.13}$$

The actual mechanism for the occurrence of negative shadowing—the coalescence of the parton clouds of the nucleons in the nucleus[1]—has been discussed in detail elsewhere.[145] Negative shadowing is expected at $x \approx m_\pi/m_N$. Experimental data[27] on the exponent α in the parametrization $F_2^A(x) = A^\alpha F_2^N(x)$ are shown in Fig. 2. These data describe a slight tendency toward a negative shadowing ($\alpha > 1$) at $x \sim 0.1$ and a transition to shadowing ($\alpha < 1$) at smaller x.

The case of quantum chromodynamics is particularly interesting. In this case, the virtual masses in Fig. 23b are small in the valence-quark region and increase as the virtual photon is approached: $p^2 \sim Q^2$. In the limit $Q^2 \to \infty$, the virtual quarks and the gluons will interact strongly with the nucleus. The transverse-momentum cutoff discussed in Subsection 5b is only weak, so that both shadowing and negative shadowing should disappear in the limit $Q^2 \to \infty$. More formally, the shadowing corresponds to small nonplanar diagrams of order $1/\ln Q^2$. The asymmetry of the $q\bar{q}$ pair into which the photon converts, which was discussed above, is retained but weakened, and the sharp distinction between the parton model and the generalized vector dominance model is erased. A version of the parton model corresponding to the vector dominance model with the cross sections in (5.9) has been discussed by Brodsky *et al.*[141,144] It would be very important to analyze the problem quantitatively in quantum chromodynamics.

d) Relationship between photoproduction and electroproduction

In photoproduction a substantial nuclear shadowing of the total cross sections, $\sigma_{\gamma A} \sim A^{0.9}$, is observed at $Q^2 = 0$ (Fig. 2). The rate at which this shadowing disappears with increasing Q^2 is surprising and has yet to be explained. At small values of Q^2 in electroproduction, it is not possible to cite any scale value which is

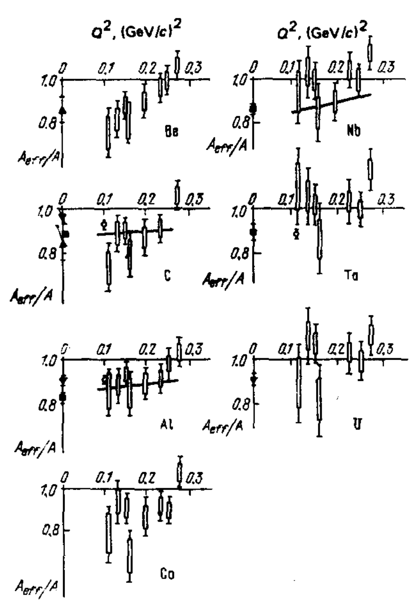

FIG. 24. Q_2 dependence of the degree of shadowing in the cross section for electroproduction in nuclei (Ref. 147). Curves—Calculations in the vector dominance model.[148]

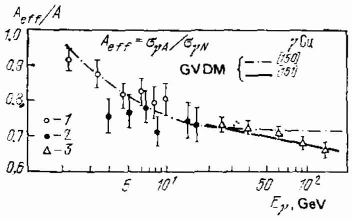

FIG. 25. Comparison of the shadowing in the cross section for photoabsorption by the copper nucleus with calculations from the vector dominance model. 1—Michalowski *et al.*; 2—Caldwell *et al.*[25]; 3—Caldwell *et al.*[149]

sharply different from the masses of the vector mesons, while experimentally the shadowing disappears in the interval $Q^2 \leq 0.2$ GeV (Fig. 24).

The simple vector dominance model with ρ, ω, and φ dominance overestimates the degree of shadowing, although, admittedly, this model describes only 80% of the total cross section for photoproduction at nucleons (see the recent reviews in Refs. 139 and 140 for a detailed discussion). The contradiction with experiment can be partially resolved by arbitrarily assuming that the other 20% of the cross section corresponds to an unshadowed "point" component of the photon.[139, 140, 144] There is better agreement with the new data of Ref. 149 on $\sigma_{\gamma N}$ at energies up to 150 GeV, which indicate an increase in the degree of shadowing with increasing energy (Fig. 25). Also shown in this figure are two versions of the description of $A_{eff}=\sigma_{\gamma A}/\sigma_{\gamma N}$ for a copper nucleus within the framework of the generalized vector dominance model.[150, 151] In general, the question of the applicability of the vector dominance model to the subtle aspects of photoproduction remains open.

6. INCLUSIVE PARTICLE PRODUCTION IN NUCLEI

a) How many times is the incident particle absorbed inelastically in a nucleus?

In retaining the analog of Eq. (1.1) for the total nuclear cross sections [see (4.9)–(4.11) in Subsection 4b] and in replacing (1.2) by (3.14) we have implicitly assumed that the rescattering of the secondary particles within the nucleus does not affect the total cross section for absorption of the incident particle. The basis of this assumption is the probabilistic nature of the rescattering of the secondary particles in production in extended targets (Subsections 3d and 3e).

The Regge form of the elastic-scattering amplitude in hadron–hadron collisions is dictated (through the unitarity condition) by the dominance of multiperipheral particle-production processes.[7] Strictly speaking, in hadron–nucleus interactions also we should first find all the important production processes and then reconstruct the elastic amplitude as the solution of the unitarity condition. This approach has not been fully implemented, although the interaction picture drawn in Subsections 3d and 3e does incorporate the basic intranuclear interactions.

The solution of this problem "the other way around" has been widely discussed in the literature. This solution ultimately reduces to the interpretation of

$$w_n = \frac{1}{\sigma_{abs}^{A}} \int db \cdot 2\pi b\, [\nu_{abs}(b)]^n \exp[-\nu_{abs}(b)] \frac{1}{n!} \tag{6.1}$$

in the expansion of Eq. (2.13) as [in accordance with (2.11)] the probabilities for the n-fold inelastic interaction of the incident particle. This interpretation is incorrect, since by the very definition of an inelastic interaction the incident particle must disappear in the first and only interaction. Citation of the leading-particle effect is groundless since, for incident deuterons, for example, the final states do not contain leading deuterons in any significant number.

We turn now to a more "rigorous" derivation of (6.1) (Refs. 39, 96, 152, and 153). In the normalization $\mathrm{Im}\, f_{hN} = \sigma_{tot}^{N}$ we write (4.14) as the series

$$\frac{if_{hA}}{2} = \int db \cdot 2\pi b \sum_{m=1}^{\infty} [\tfrac{1}{2}\, if_{hN} T(b)]^m \frac{1}{m!}, \tag{6.2}$$

interpreting the terms of the series as the amplitudes for m-pomeron exchange (Fig. 15). Production processes correspond to jumps in f_{hA} at the energy cut, which we write as a series in the jumps in f_{hN} (in the "cut" pomerons). The contribution of m-pomeron exchange with k "cut" pomerons is

$$\sigma_{m,k} = \frac{1}{m!} \int db \cdot 2\pi b C_m^k [\sigma_{tot}^{N} T(b)]^k \left\{\left[\frac{if_{hN}}{2} + \left(\frac{if_{hN}}{2}\right)^*\right] T(b)\right\}^{m-k}. \tag{6.3}$$

Here the "cut" pomerons correspond to σ_{tot}^{N}, the pomerons to the right of the cut pomerons correspond to $if_{hN}/2$, and those to the left correspond to $(if_{hN}/2)^*$ $= if_{hN}/2$ (scattering in the initial and final states, respectively). Substituting $(\sigma_{tot}^{N})^k = \sum_n C_k^n (\sigma_{abs}^{N})^n (\sigma_{el})^{k-n}$ into (6.3), and summing over k, we find the cross section for an n-fold inelastic interaction in m-pomeron exchange:

$$\sigma_{m,n}^{in} = \frac{1}{m!} \int db \cdot 2\pi b C_m^n [\sigma_{abs}^{N} T(b)]^n [-\sigma_{abs}^{N} T(b)]^{m-n}. \tag{6.4}$$

When we sum over m, we find a result which is precisely the same as (6.1). The interaction picture corresponding to (6.1) is essentially a cascading of the leading particle in the complete absence of intranuclear interactions of any of the other secondary particles.

This "derivation" of (6.1) has been extremely sketchy. Strictly speaking, the "cut" pomeron should be associated with σ_{abs}^{N}, rather than σ_{tot}^{N}. The jumps in f_{hA} corresponding to quasielastic scattering must be taken into account correctly.[96] As was shown in Subsection 4c, quasielastic scattering must be taken into account in order to find a correct expression for σ_{abs}^{A}, but this point is unimportant for the discussion below.

On a purely formal level, this derivation of (6.1) corresponds to the Abramovskiĭ–Gribov–Kancheli (AGK) cut rules[154] and is the basis of the eikonal model which is being developed actively by Shabel'skiĭ,[96] Capella *et al.*,[39, 155] and others.[152, 153, 156–159] That the (unquestionably correct) AGK rules would lead to an unacceptable result, (6.1), is not surprising; the AGK rules are valid only for nonplanar diagrams, while the eikonal Glauber amplitude is planar, as was demonstrated explicitly some time ago by Gribov.[13] For the eikonal amplitudes there may be a cut in only a single pomeron. The correct use of the analog of the AGK rules for planar amplitudes leads to the natural result $w_1 = 1,\ w_{n>1} = 0$.

Expression (6.1) can be retained if we treat the Glauber expansion as a purely mnemonic rule, assuming that the amplitude for the m-fold interaction corresponds not to successive interactions but to a "parallel" interaction of an m-particle component of the incident hadron[39,158,160] (Fig. 15c). This interpretation is also unsatisfactory: The structure of the hadron turns out to depend on the target, and the amplitudes for the interactions of the particles making up the hadron should be assumed equal to the amplitude for the interaction of the hadron itself. Furthermore, there is no direct proof that the Glauber–Gribov planar diagrams are exactly cancelled by nonplanar diagrams, as is assumed in Fig. 15c.

The first attempt to incorporate the effect of the rescattering of the secondary particles on the absorption of the incident particle can be credited to a recent paper by Levin and Ryskin.[105] They used a space–time picture slightly different from that formulated in Subsections 3d and 3e; specifically, they assumed that the fast partons have an interaction cross section which is independent of the distance to the production point but that the probability for the interaction over the lifetime is of the order of unity. It was found that the absorption of the incident particle takes a different form: A passive component arises. The equation for the inclusive spectra, however, retains its cascade form.

Levin and Ryskin discussed the case of decreasing total cross sections and found that the weight of the passive state depends on the size of the nucleus. It is not clear how this result may change in the cases of constant or growing total cross sections and how it depends on the details of the space–time picture of the interaction.

b) Multiple rescattering and the quark model

A natural mechanism for multiple rescattering emerges from the quark model. In deep inelastic scattering at large Q^2 the hadrons are seen as consisting of valence quarks, gluons, and $q\bar{q}$ pairs of the "sea." The parton wave function of the hadrons is additive; i.e., its various parts can be ascribed to different valence quarks.

In inelastic hadron–hadron interactions, the resolution scale is the average transverse momentum of the direct secondary particles: $\langle p_T^2 \rangle \approx m_\rho^2$. It is important to note that

$$R_h^2 \langle p_T^2 \rangle \gg 1, \tag{6.4'}$$

so that this resolution is smaller than the hadron dimensions R_h and permits a resolution of the internal structure of the hadrons.[161] Various estimates of the size of the constituent quarks, R_q, yield[161-163] $R_q^2/R_h^2 \lesssim 1/10$. These results explain the success of the additive quark model and allow us to treat hadrons as light nuclei consisting of spatially separate *constituent* quarks, each having its own system of partons and interacting independently.[161] The independence of the interaction of the constituent quarks does not contradict color confinement; by virtue of (6.4), the constituent quarks form first, and then they recombine into hadrons. By repeating Gribov's analysis[7,66] of the spatial dimensions of parton fluctuations, we easily see that the colored entities are not separated by distances greater than the confinement radius.

The resulting picture of particle production in nuclei may be described as follows.[30,165-167] The probability that ν of the n constituent quarks of the incident hadron will interact inelastically with the nucleus is[3)] (Ref. 167):

$$w_\nu = \frac{1}{\sigma_{abs}^{A}} \int d^2b\, C_n^\nu \exp[-(n-\nu)\sigma_{abs}^{qN} T(b)] \times \{1 - \exp[-\sigma_{abs}^{qN} T(b)]\}^\nu. \tag{6.5}$$

Equation (3.15) can be used to describe the spectra of the particles produced by a single constituent quark.[30,166] The $l_f(\varepsilon)$ in (3.15) are the formation lengths of the constituent quarks; after these distances are traversed the quarks recombine into direct hadrons. Figure 26 shows the structure of the spectra in ν-quark collisions. At asymptotic energies there are three sharply different regions. In the beam fragmentation region, the secondary-particle yields are determined by the average number of spectator quarks. In the central region, $y \gtrsim y_c = \ln(R_A \mu_0^2/\langle k_T \rangle)$, a plateau appears in the plot of R_y [y_c corresponds to $l_f(\varepsilon) = R_A$]. Rapidities $y < y_c$ correspond to a cascade multiplication of secondary particles.

The results which will be discussed below were derived through several approximations in Refs. 30 and

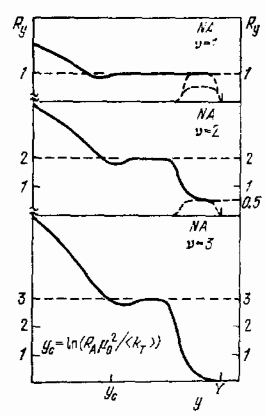

FIG. 26. Qualitative picture of the relative distributions in one-, two-, and three-quark interactions of proton with a nucleus. The decrease in the contribution of the spectators to the fragmentation distribution with increasing ν is shown.

[3)] To avoid any confusion, we emphasize that ν here and below represents the number of inelastic interactions of the leading system. In the additive quark model, ν is the number of constituent quarks which have interacted; in the eikogonal model, it is the number of inelastic interactions of the incident particle. We are using the general notation, since ν enters the distributions in the central region, the multiplicity distributions, and the correlation functions in the same way in the two models (see the discussions below).

165–168. To neglect multipomeron interactions is completely justified. The role of the planar-diagram corrections to the cascade picture remains open.[100,169] There is some uncertainty about just what is to be understood by the term "amplitude of the quark–nucleon interaction." We know that branchings are phenomenologically quite important in the amplitude for the hadron–nucleon interaction. In particular, branchings are required for describing the multiplicity distributions[170,171] and the rapidity correlations.[172,173] In the additive quark model, these branchings should be attributed partially to the amplitude for quark–quark scattering and partially to a small admixture of multiquark interactions.[174] Whether a change may occur in the structure of the branchings in the quark–nucleon amplitude itself in the transition to intranuclear interactions remains an open question. Furthermore, those corrections to the interaction picture drawn above which result from the admixture of the passive components of the constituent quarks have not been determined. All these uncertainties do not, however, change the most important consequences of the additive quark model, which are discussed below.

c) Quark counting rules for beam fragmentation in nuclei

The direct application of Eq. (3.15) to hadrons leads to the prediction that the fast-particle multiplicity is independent of the target nucleus[1,78]:

$$R_y = 1, \quad l_f(\varepsilon) > R_A. \tag{6.6}$$

Until about 1976 it appeared that (6.6) agreed with experiment.[78] More-detailed data obtained since then have revealed $R_y < 1$ at all energies in the beam fragmentation region (Figs. 4–7). Such an absorption of fast particles does not contradict the growth of the formation lengths, and it arises naturally in the quark model[30,165,167]: The number of spectator quarks, which determine the yields of fast beam fragments, falls off with increasing size of the nucleus, regardless of the energy (Fig. 26).

Let us consider, for example, pA interactions. The fast secondary nucleons are formed by the recombination of two spectator quarks with one quark from among those newly produced, while a single spectator quark is sufficient for the formation of a fast pion. These baryons and mesons will predominantly have the values $x_B \approx 2/3$ and $x_M \approx 1/3$. Accordingly,[167]

$$R_x^{p\to B}\left(x_B \approx \frac{2}{3}\right) \approx w_1, \qquad R_x^{p\to M}\left(x_M \approx \frac{1}{3}\right) \approx w_1 + a w_2. \tag{6.7}$$

The coefficient a depends on the particular recombination model, but we would always have $a \approx 1$ (Refs. 30, 167, 175, and 176). The agreement with experiment can be judged from Fig. 27. We emphasize that predictions (6.7) do not depend on the nature of the quark absorption in the nucleus.

Anisovich *et al.*[167] believe that the constituent quarks have a narrow x distribution with a sharp peak at $x_M \approx 1/3$, and they suggest testing (6.7) specifically for $x_B = 2/3$ and $x_M = 1/3$. However, the production of baryons with $x_B = 2/3$ may be affected, if only slightly, both by interactions with a single spectator and interactions without any spectators. Furthermore, for a detailed comparison with experiment it is important to find a description of the nature of the transition from the beam fragmentation region to the central region. A procedure was proposed in Ref. 30 for taking the observed spectral shape into account and for making a smooth transition to small x as the number of spectator quarks is reduced. Let us examine this procedure.

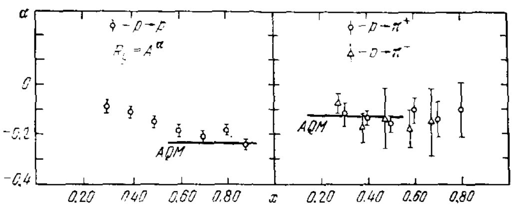

FIG. 27. The x dependence of the exponent α in the parametrization $R_x = A^\alpha$ for proton fragmentation into protons and pions.[177] Shown here are the values of α given by the quark counting rule in (6.7) for $x_\pi \approx 1/3$ and $x_p \approx 2/3$.

The spectrum is expanded in the components $F_s(x)$, which differ in the number (s) of spectator quarks which recombine into the given fragment. For example,

$$\frac{dN^{p\to p}}{dx} = \frac{1}{2} w_1 [F_2(x) + F_1(x)] + \frac{1}{2} w_2 [F_1(x) + F_0(x)] + w_3 F_0(x). \tag{6.8}$$

A reduction of the number of spectators by one may be thought of as yet another scattering event. We denote by $L(x)$ the spectrum of leading particles; then[30]

$$F_{s-1}(x) = \int_x^1 \frac{dz}{z} F_s(z) L\left(\frac{x}{z}\right). \tag{6.9}$$

If the recombination is of a probabilistic nature, then this procedure is exact for mesons.[176] A good approximation of $L(x)$ is $L(x) \approx x$. The distributions of secondary protons minus the diffractive contribution can be described by choosing $F_2(x) \approx x$. For $R_x^{p\to p}$ we then find

$$R_x^{p\to p} = \left[w_1\left(1 + \ln\frac{1}{x}\right) + w_2\left(\ln\frac{1}{x} + \frac{1}{2}\ln^2\frac{1}{x}\right) + w_3\left(\frac{1}{2}\right)\ln^2\frac{1}{x}\right]\left(1 + \ln\frac{1}{x}\right)^{-1}. \tag{6.10}$$

It can be seen from (6.10) that for a lead nucleus, with $w_1 \approx w_2 \approx w_3 \approx 1/3$, the contribution proportional to w_2 increases $R_x^{p\to p}$ $(x_p \approx 2/3)$ by ~20%. Alaverdyan *et al.*[178] have emphasized the importance of incorporating higher-order rescattering in a comparison with experiment.

The fragmentation of protons into Λ^0 hyperons is described well by $F_2(x) \sim x(1-x)$. The x dependence of $R_x^{p\to\Lambda^0}$ and $R_x^{p\to p}$ calculated from (6.9) agrees very well with the experimental data[179] in Fig. 28. The data from Ref. 37 on neutron fragmentation can also be described well, but these data are not very accurate (Fig. 5).

Rules (6.7) and (6.9) refer to direct particles, since the absorption of the decay products is determined by the absorption of the parent resonance.[165] It is possible that decays of the type $N^* \to \Lambda^0 A^0$ can be explained on the basis that the experimental A dependence of the

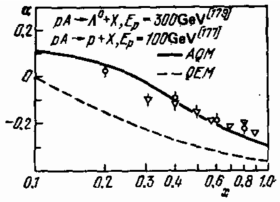

FIG. 28. The exponent α in the parametrization $R_x = A^\alpha$ for the fragmentations $p \to p$ (Ref. 177) and $p \to \Lambda^0$ (Ref. 119). The curves show the values of α calculated in the eikonal model (QEM) and in the additive quark model (AQM).[176] The values of $\alpha_{p \to p}$ and $\alpha_{p \to \Lambda^0}$ are approximately equal.

fragmentation $p - \Lambda^0$ is similar to that of the fragmentation $p - K^0$ (Ref. 179). The absorption of Λ^0 hyperons also turns out to be similar to the absorption of Λ^0 and K^0 (Ref. 179). How the latter fact affects the quark model is not clear, since that model does not apply to unusual fragmentation processes of the type $p - \bar{\Lambda}^0$, which do not involve any spectator quarks.

The rule in (6.7) corresponds to an additive interaction of quarks. It has been noted elsewhere[176] that there would be a universal A dependence for all the fragments if the quarks behaved collectively (for example, if the hadrons interacted through a gluon component common to all the valence quarks[180]). The data discussed here support the additive model, but a better understanding of the fragmentation mechanism requires both more-accurate experimental data and a more detailed analysis, incorporating the production of all resonances.

The concept of an effective absorption cross section σ^*_{bN} for absorption of the newly produced particle is very frequently introduced in analysis of the A dependence of the distributions of particles with $x \approx 1$ with the help of Eq. (4.31). Such an analysis shows that the effective cross sections σ^*_{bN} are smaller than the physical cross sections σ_{bN}. For secondary pions in pA interactions, for example, we find $\sigma^*_{\pi N} \approx (1/2)\sigma_{\pi N}$ (Ref. 181). The absorption of Λ^0 hyperons and protons observed experimentally is weaker than that calculated from (4.31) with the cross sections $\sigma^*_{\Lambda N} = \sigma_{\Lambda N}$ and $\sigma^*_{pN} = \sigma_{pN}$ (Fig. 28). A direct comparison with (6.7) or with the distributions calculated from (6.9) is still correct. Dem'yanov *et al.*[182,183] reached the conclusion $\sigma^*_{pN} \approx (1/3 - 1/4)\sigma_{pN}$ for the secondary protons in pA interactions, but this conclusion is not justified, because of the overly crude approximation used of a nucleus with a uniform density. When the diffuse nuclear boundary is taken into account, a satisfactory description can be found for the distributions for a primary-proton energy of 20 GeV (Ref. 184), even in the eikonal model, with $\sigma^*_{pN} = \sigma_{pN}$ (Ref. 173).

Bialas and Bialas[185] have suggested determining $F_s(x)$ through a direct comparison of the observed A dependence of the distributions in the fragmentation region with that given by Eq. (6.8). More precisely, they introduced independent fragmentation functions for a spectator diquark, a single spectator quark, and for a quark which has interacted inelastically. Analysis of the fragmentation $p - \Lambda^0$ revealed that the fragmentation of a single spectator [the term proportional to w_2 in (6.8)] could be quite important in the case of a heavy nucleus. It must be emphasized, however, that the introduction of independent fragmentation functions is not consistent. Conservation of strangeness and of baryon charge requires that all the $F_s(x)$ in expansion (6.8) be normalized to the same multiplicity. This requirement is met automatically when procedure (6.9) is used.

d) Central pionization region and average multiplicities

In the central plateau, $R_y = \langle\nu\rangle = \sum_\nu \nu w_\nu$, Bialas *et al.* noted[186]

$$(R_y)_{\rm plat} = \langle\nu\rangle = \frac{\bar{\nu}_{hA}}{\bar{\nu}_{qA}}. \tag{6.11}$$

For a black nucleus we have $\sigma^{ha}_{abs} = \pi R_A^2$, regardless of the incident particle, and (6.11) converts into the Anisovich relation,[15]

$$(R_y)_{\rm plat} = \nu_{\max} = \begin{cases} 3, & pA, \\ 2, & \pi A. \end{cases} \tag{6.12}$$

Until recently, the evidence for the appearance of a plateau in $R_{\eta(y)}$ (Figs. 4 and 5) was not convincing. The pseudorapidity distributions were poor in that the sharp boundary between the plateau and the beam fragmentation and the cascade region was erased. The first reliable evidence for a plateau in R_y came from the CERN NA5 experiment[38] (Fig. 6). The height of the plateau agrees well with (6.11). So far, however, this is the only solid piece of evidence available.

Bialas *et al.* noted that the height of the plateau can be used directly to determine $N_q = \nu_{\max}$, the number of constituent quarks in the hadron.[186] Using (2.15), we easily find

$$N_q = \frac{\sigma^{hA}_{abs}}{\sigma^{qA}_{abs}} (R_y)_{\rm plat}. \tag{6.13}$$

Bialas and Bialas[186] analyzed the data obtained by Busza *et al.*[29] on the pA interactions at 100 GeV and reached the conclusion $N_q = 3$ for the proton. In the pseudorapidity distributions which they used, however, there was no indication of a plateau, and the choice of the pseudorapidity for which (6.13) was checked was arbitrary. Anisovich *et al.* noted that it was more convenient to compare the pA and πA interactions with the same nucleus.[167] The resulting analog of the Anisovich relation, (6.12),

$$\left(\frac{R_y^{pA}}{R_y^{\pi A}}\right)_{\rm plat} \frac{\sigma^{pA}_{abs}}{\sigma^{\pi A}_{abs}} = \frac{3}{2}, \tag{6.14}$$

is valid not only in the central region but also (approximately) in the cascade region, so that the quark model can be checked even before the appearance of a plateau in $R_{y(\eta)}$. This comparison was made by Shekhter,[187] who found good agreement with experiment (Fig. 29).

The height of the plateaus in R_y and R_η is extremely important for the theory. In the eikonal models, plateaus are predicted with a height[39,96,152-159,188]

$$(R_y)_{\rm plat} = \bar{\nu}, \tag{6.15}$$

or, equivalently,

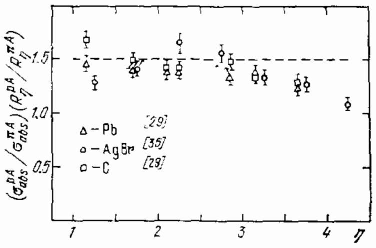

FIG. 29. A test of the Anisovich-Shabel'skiĭ-Shekhter relation, (6.14), for the distributions in the central region and in the nuclear fragmentation region[187] at 200 GeV.

$$\left(\frac{d\sigma^A}{dy}\right)_{\text{plat}} = A\left(\frac{d\sigma^N}{dy}\right)_{\text{plat}}. \quad (6.16)$$

The same result can be found from a quark model if each constituent quark interacts inelastically $\bar{\nu}_{qA}$ times.[189] The situation with regard to the eikonal models may be described in general as follows: They give a satisfactory description of the average multiplicities,[39,96,190] the multiplicity distributions,[39,96,191] and the dependence of the shower-particle multiplicities on the number of recoil nucleons,[192,193] but they fail to describe the single-particle inclusive distributions. Relations (6.15) and (6.16) do not hold experimentally, and there is no evidence of any sort that they will begin to apply at higher energies (Figs. 4–7). At rapidities $y(\eta) \leq 1-2$, experiments show that $R_{y(\eta)}$ begins to increase and becomes substantially larger than $R_{y(\eta)} = \bar{\nu}$. This result demonstrates that cascade effects are occurring— effects which were generally ignored in the derivation of (6.15) (see Subsection 6a). This increase in $R_{y(\eta)}$ cannot be ascribed to Fermi-motion effects,[195] as was asserted in Ref. 194. Kinoshita *et al.*[196] have suggested incorporating cascade interactions of recoil protons. This refinement does not substantially improve agreement with experiment or make the model itself more consistent, since there is nothing which distinguishes the recoil nucleons from all the other secondary particles, which may also interact inside the nucleus.

The dependence of $\langle N_s\rangle_A$ on $\langle N_s\rangle_N$ might be used to estimate the height of the plateau. At asymptotic energies, the beam fragmentation and the cascade make energy-independent contributions to $\langle N_s\rangle_A$, and we have

$$\langle N_s(E)\rangle_A = (R_y)_{\text{plat}} \langle N_s(E)\rangle_N + \text{const.} \quad (6.17)$$

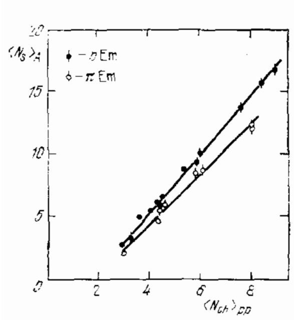

FIG. 30. Compilation of data on the $\langle N_{ch}\rangle_{pp}$ dependence of $\langle N_s\rangle_A$ in interactions of pions and protons with emulsion nuclei.[197] The lines correspond to the asymptotic ratio $R = 0.91\bar{\nu}$.

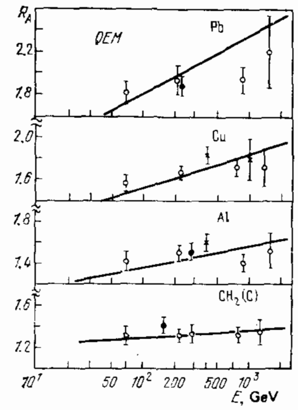

FIG. 31. High-energy behavior of the relative multiplicity R according to data on cosmic-ray interactions from the Tskhra-Tskaro apparatus.[198] Solid curves—Energy dependence of R_A in the eikonal model.[96]

Comparison of (6.17) with experimental data for energies up to 200–400 GeV made in Refs. 189 and 197 yields $\approx 0.91\bar{\nu}$ (Ref. 197; see Fig. 30 in the present paper) for the slope in (6.17). This result is closer to (6.15) than to (6.11), but the data used correspond to energies at which there is no plateau in $R_{y(\eta)}$, and (6.17) cannot hold, so that the meaning of this result is not clear.

At energies in the range 50–200 GeV, experiments yield $R \approx 0.4 + (0.6 - 0.7)\bar{\nu}$ (Fig. 3). In the eikonal model we would have $R = \bar{\nu}$ at high energies; i.e., R should increase with increasing energy. The most comprehensive data on R at energies above accelerator energies have been obtained at the Tskhra-Tskaro cosmic-ray station[198] (Fig. 31), but even these results are not accurate enough to decide whether R increases to $R = \bar{\nu}$, as in the eikonal model, or instead decreases to $R = \langle\nu\rangle$, as in the quark model.

A particularly interesting case is multiple production in interactions with deuterons in which both recoil nucleons participate in the interaction. The relative R_y distribution in such events was recently measured by the Lubatti group[199]; the results are compared in Fig. 32 with the quark-model predictions.[74] In the quark model, some of the rescatterings by the second nucleon involve spectator quarks from the first interaction, while others result from cascade interactions. The ex-

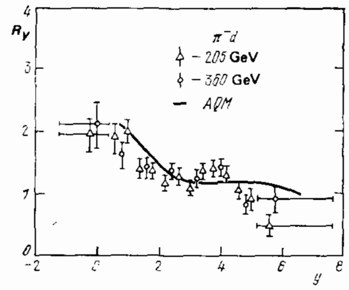

FIG. 32. Relative distribution of R_y in π^-d interactions with rescattering of the product particles by the second nucleon of the deuteron.[199] Curve—Prediction of the additive quark model.[74]

tent to which $(R_y)_{\rm plat}$ exceeds unity gives us the fraction of interactions of spectator quarks, $\approx 20\%$ for primary pions. At $y \approx 0$, the value $R_y \approx 2$ is predicted.[74] In the eikonal models, on the other hand, we would expect $R_y \approx 2$ in the central region also.[74] The experimental results support the predictions of the additive quark model well.

Multipomeron interactions are suppressed by the small value of $G_{\rm PPP}$. When these interactions are taken into account, the parton wave function of the constituent quarks takes the form of a tree with a vertex facing the nucleus (see Fig. 15 and, for example, Ref. 122). The plateau in $R_{y(n)}$ at asymptotic energies is not found when multipomeron interactions are taken into account; a small slope, proportional to $G_{\rm PPP}$, appears. There have been no detailed calculations of the slope or the shape of the distribution in the central region at extremely high energies, at which we have $G_{\rm PPP} \ln E \sim 1$ and at which multipomeron interactions become important.

Again, all the basic consequences of the additive quark model agree well with experiment. It would be extremely interesting to see precise measurements of the distributions in the central part of the TeV region.

e) Multiple production in deep inelastic scattering of leptons by nuclei

This is a particularly interesting case. In the total cross sections for electroproduction in the ranges of E and Q^2 which have been studied, there is essentially no shadowing (Fig. 2; E is the energy of the virtual photon):

$$f_{\gamma^* A}(E, Q^2) = A f_{\gamma^* N}(E, Q^2). \tag{6.18}$$

The formal application of the rules of the eikonal model to (6.18) would mean that (Subsection 6a)

$$\langle N_s \rangle_{\gamma^* A} \equiv \langle N_s \rangle_{\gamma^* N}, \tag{6.19}$$

$$\left(\frac{dN_s}{d\eta}\right)_{\gamma^* A} \equiv \left(\frac{dN_s}{d\eta}\right)_{\gamma^* N}. \tag{6.20}$$

Neither (6.19) nor (6.20) holds experimentally (Figs. 8 and 33–35), and intranuclear interactions of the secondary particles furnish a natural explanation.[41,142]

As was discussed in detail in Subsection 5b, the hadronic properties of the strongly virtual photon are

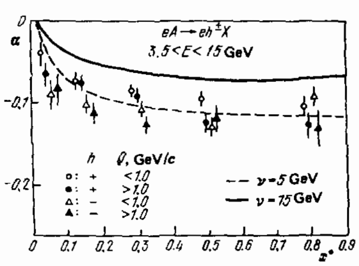

FIG. 33. The exponent α in the parametrization $R_{x^*} = A^{\alpha}$ of the spectrum in the photon fragmentation region in deep inelastic scattering of electrons.[200] Curves—Calculations from the quark model in incorporating formation lengths.[41]

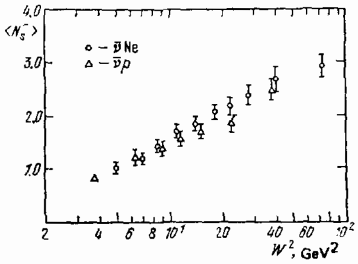

FIG. 34. Comparison of the average multiplicities in $\bar\nu$Ne and $\bar\nu$p interactions[201] ($W^2 = 2m_N E - Q^2$).

similar to those of a single (virtual) quark. The original arguments of Kancheli[1] can thus be applied to the distributions in electroproduction: At energies $E > R_A \mu_0^2$, the distributions in the photon fragmentation region are predicted to be independent of the target [see (6.6)]. At relatively low energies, $E < R_A \mu_0^2$, however, the photon fragments would form still within the nucleus and would be absorbed. This type of nuclear absorption has been observed in a SLAC–MIT experiment.[200] Both the degree of absorption and the x^* dependence of R_{x^*} are described well by Eq. (3.15) (Fig. 33). A similar explanation of these data has been discussed by Nilsson *et al.*[202] and Bialas and Bialas.[203] Formation-length effects were ignored in those papers. Bialas and Bialas[203] assert, in particular, that the absorption of current fragments at high energies makes it possible to measure the cross sections for both absorption and elastic scattering of a quark by nucleons. Because of the increase in the formation lengths, this absorption should generally disappear with increasing energy (Fig. 33).

In diffractive scattering at $\omega - 2m_N E/Q^2 \gtrsim R_A m_N \approx 10A^{1/3}$, cascading occurs over a greater thickness of the nucleus than in noncoherent scattering at $\omega < 10$ (Refs. 41, 142, and 204). A weak increase in R with increasing ω is thus predicted, as shown schematically by the arrows in Fig. 35.

Deep inelastic scattering by nuclei is the most direct source of information about the quark–nucleus interaction.

f) $\tilde{\nu}$ Scaling in particle production in nuclei

Busza *et al.*[29] have pointed out that the experimental

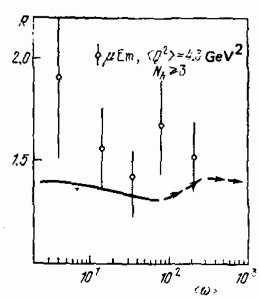

FIG. 35. Dependence of the relative multiplicity R on $\omega = 2m NE/Q^2$ in deep inelastic scattering of muons by emulsion nuclei for events with $N_h \geq 3$ (Ref. 31). Curve—Calculations of Ref. 41; arrows show the increase in R expected at the transition from incoherent to different scattering.

values of R_η and R for πA, pA, and KA interactions have approximately the same $\bar{\nu}$ dependence (Fig. 3). This observation has been widely interpreted as proving that successive inelastic interactions occur only inside the nucleus and only for the primary particle.[29,39] At energies up to hundreds of GeV, approximate $\bar{\nu}$ scaling is found in the quark model also, when cascades are taken into account: With identical values of $\bar{\nu}_\pi$ and $\bar{\nu}_p$ in πA interactions, the values of $\bar{\nu}_q$ and thus the contribution of cascades are higher than in pA interactions, so that the values of $R_\eta^{\pi A}$ and R_η^{pA} are approximately equal, as are $R^{\pi A}$ and R^{pA} (Ref. 30; see Fig. 30 of the present paper). In deep inelastic scattering we would have $\bar{\nu}\equiv 1$, while exact $\bar{\nu}$ scaling would yield $R=R_\eta=1$, in contradiction with experiment (Subsection 6e).

g) Nonmultiperipheral models

A typical model of this group is the coherent-tube model, which has been rediscovered repeatedly over the past five years.[206-209] In this model the incident particle is assumed to interact simultaneously with all N_T of the nucleons in a tube of cross section $S=\sigma_{hN}$. The interaction with the tube is assumed to be equivalent to the interaction with a single nucleon, but at an energy

$$E_{eff} = EN_T. \tag{6.21}$$

This model gives a satisfactory description of the single-particle inclusive distributions and of the average multiplicities.[207,208,210] In it, $(D/\langle N_s\rangle)_A$ and the correlations $R_2^A(0,0)$ are naturally independent of the target nucleus.[211] If we introduce an energy dependence of the length of the tube, $L\sim E/m^2$, we find that there is also a good description of the increase in R with increasing energy at relatively low energies.[212]

However, the model fails in a more detailed comparison with experiment. For example, Eq. (6.21) implies a broadening of the rapidity distributions with increasing N_T or N_g ($N_g\sim N_T$), while experimentally, on the contrary, there is a contraction.[213] Neither $(D/\langle N_s\rangle)_N$ nor $R_2^N(0,0)$ depends on the energy, so that both would be independent of N_g, according to (6.21) (Ref. 211). The experimental data on $(D/\langle N_s\rangle)_A$ and $R_2^A(0,0)$ in Figs. 10 and 11 refute the tube model.

In the hydrodynamic version of the tube model the renormalization of the total energy is supplemented by a renormalization of the initial volume. The latter renormalization leads to an explanation for the contraction of the rapidity distributions with increasing N_g (Ref. 207), but the decrease in $(D/\langle N_s\rangle)_A$ and $R_2^A(0,0)$ with increasing N_g is an argument against even this hydrodynamic version of the tube model.

The cluster model of Kalinkin and Shmonin[214] has also been discussed widely. Here it is assumed that a cluster forms in an hN collision, and as this cluster moves through the nucleus it is excited in successive collisions with the nucleons of the nucleus; it expands and decays into the final particles outside the nucleus. A contradiction of this model[214] by experiment has been demonstrated by Gulamov and Uzhinskiĭ.[215] We might add to their criticism that the initial equations of Ref. 214 for the cluster excitation contradict energy conservation.

The Gottfried model[216] is frequently cited in a discussion of experimental data. In terms of its consequences this model may be described as similar to the eikonal model, but the entity undergoing the repeated interactions in the nucleus is a system corresponding to two-thirds of the total rapidity interval. The consequences of this model do not agree with experiment in terms of either the inclusive distributions or the multiplicities. Nevertheless, Gottfried's paper[216] and, especially, its review in Ref. 217 have played a very important role in attracting attention to interactions with nuclei. The reader is referred to Ref. 219 for a comparison of experimental data with the consequences of Gottfried's model and also with those of another non-field-theory model, proposed by Fishbane and Trefil.[218]

7. CORRELATION PHENOMENA IN MULTIPLE PRODUCTION IN NUCLEI

a) Relationship between the rapidity correlations and the multiplicity distributions

For uncorrelated production of secondary particles we would have $R_2(y_1,y_2)=0$, and the multiplicity distribution would be Gaussian with $D=\sqrt{\langle N_s\rangle}$. The correlations observed experimentally are not small. For charged particles the result $R_{chch}(0,0)\approx 0.6$ is found in all hN interactions, and D depends on $\langle N_s\rangle$ as described by the Wroblewski law,[19]

$$D = a\,(\langle N_s\rangle - b), \tag{7.1}$$

where $a\approx 0.5$–0.6, and b is small, $b\approx 0.5$–1 (see the reviews in Refs. 220 and 221).

Analysis of the dependence of $R_2(y_1,y_2)$ on the rapidities and on the initial energy shows that $R_2(y_1,y_2)$ can be broken up into short-range correlations $R_S(y_1,y_2)$ with a range $\Delta=|y_1-y_2|\approx 2$, and long-range correlations $R_L(y_1,y_2)$, which do not depend (in the central region) on the difference between the rapidities y_1 and y_2. Experimentally, it is found that $R_S(0,0)\approx R_L(0,0)\approx 0.3$ (see the reviews in Refs. 220 and 221 and the papers cited there). Near the boundary of the kinematic region, it is found that $R_2(y_1,y_2)<0$, because of energy conservation.

It is not difficult to derive relations between a and b in (7.1), on the one hand, and the rapidity correlations, on the other:[45]

$$a = \sqrt{R_L(0,0)}, \tag{7.2}$$
$$b = (2\delta\cdot R_L(0,0) - 1 - C\Delta R_S(0,0))\,(2R_L(0,0))^{-1}. \tag{7.3}$$

The coefficient δ in (7.3) is approximately equal to $2\ln 2$ and reflects the fact that the two-particle distribution vanishes near the boundary of the kinematic region; also, $C=(dN_s/dy)_{y=0}$. Equations (7.2) and (7.3) yield correct values for both the slope a and the intercept b.

b) Rapidity correlations in particle production in nuclei

The equations[44,45]

$$R_2^A(0,0) = \frac{\langle\nu^2\rangle-\langle\nu\rangle^2}{\langle\nu\rangle^2} + \frac{R_2^N(0,0)}{\langle\nu\rangle}, \tag{7.4}$$

$$\left(\frac{D}{\langle N_s\rangle}\right)_A^2=\frac{\langle\nu^2\rangle-\langle\nu\rangle^2}{\langle\nu\rangle^2}+\frac{1}{\langle\nu\rangle}\left(\frac{D}{\langle N_s\rangle}\right)_N^2, \tag{7.5}$$

relate the correlations in the central rapidity region and the Wroblewski ratio for nuclei with the distributions in the number of interacting quarks, ν (in the quark model), or with the number of inelastic interactions of the incident particle (in the eikonal model). The dependence of $R_2^A(0,0)$ and $(D/\langle N_s\rangle)_A$ on the atomic number of the nucleus which follows from (7.4) and (7.5) is rather weak[44,45] (Figs. 10 and 12). This result is a consequence of the mixing of peripheral and central collisions; the terms $(\langle\nu^2\rangle-\langle\nu\rangle^2)/\langle\nu\rangle^2$ in (7.4) and (7.5) offset the decrease in the second term, which is proportional to $1/\langle\nu\rangle$.

Calculations from the additive quark model agree well with the available data, but the simple equations in (7.4) and (7.5) may be corrected for the finite value of the energy. In (7.5), for example, we should take into account the decrease in the multiplicity in the beam fragmentation region with increasing ν. In the quark model of Ref. 30, this decrease is offset by cascade production of particles. In the eikonal model, in which there are no cascades, the values of $(D/\langle N_s\rangle)_A$ may turn out to be lower than those predicted by (7.5) (Refs. 39 and 222).

The three-particle correlation function for nuclei is[44]

$$R_3^A(0,0,0)=\frac{\langle\nu^3\rangle-3\langle\nu^2\rangle\langle\nu\rangle+2\langle\nu\rangle^3}{\langle\nu\rangle^3}+3\frac{\langle\nu^2\rangle-\langle\nu\rangle^2}{\langle\nu\rangle^3}R_2^N(0,0)+\frac{R_3^N(0,0,0)}{\langle\nu\rangle^2}. \tag{7.6}$$

The first data on $R_3^A(0,0,)$ were obtained for π^-C interactions at 40 GeV in the Joint Institute for Nuclear Research.[223] The experimental value $R^{---}(0,0,0)=-0.02\pm0.04$ agrees well with the quark model, which predicts[44] $R^{---}(0,0,0)=0.1$. The eikonal model predicts $R^{---}(0,0,0)=0.40$ (Ref. 44).

In production in nuclei, the interactions of the different quarks involve different nucleons of the nucleus, so that the two-particle distributions in the nuclear fragmentation region are not made small by kinematic factors.[45,211] In particular, this is the reason for the positive value[45,211] of $R_2^A(y_1,y_2)$ in the nuclear fragmentation region, although $R_2^N(y_1,y_2)<0$. This prediction agrees with experiment.[51,53,224] The coefficient δ in (7.3) should be replaced by $\delta/2$, and this change reduces b. When the corrections for the change in $R_s(0,0)$ are also taken into account, Eq. (7.3) is found to yield $b_{Em}=b_D-0.5$, in good agreement with the emulsion data given in Ref. 9.

c) Correlation of the production of fast particles with nuclear fragmentation

The number of intranuclear interactions which occur cannot be determined successfully from the size of the nucleus. A more direct measure of this number, N_{int}, is the number of gray tracks. Semiempirical relations between N_g and N_{int} have been derived from the quark model[45] (here N_{int} includes the interactions of both the constituent quarks of the incident particle and cascade interactions); corresponding relations have also been derived from the eikonal model.[18,225] In the latter case it is necessary to assume that a single inelastic interaction of the incident particle in the nucleus produces several fast protons, which are ejected from the nucleus, although cascades are not formally considered.

In the additive quark model, events with $N_g\gg\langle N_g\rangle$ correspond to $\nu=\nu_{max}$. Correspondingly, a plateau is predicted in $R_{y(\eta)}$ with a height independent of the target nucleus[45]:

$$(R_{y(\eta)})_{plat}=\nu_{max}. \tag{7.7}$$

As was mentioned in Subsection 6d, a high energy is required for observation of the plateau, especially in the pseudorapidity distributions. The quark model describes the N_g dependence of R_η well, but an energy of 400 GeV is not sufficient to test (7.7) (Fig. 11). In the eikonal model or in the model of Ref. 189 with eikonal rescatterings of the quarks, the values of ν are not bounded, and R_η in the central region should increase without bound with increasing N_g.

It is clear from a comparison of Figs. 6 and 10 that the limiting shape of the distributions has not yet been established. Attempts[226] to guess the limiting properties of the N_g dependence of the distributions through an extrapolation of the low-energy data are unreliable. Data on the distributions in the TeV range are urgently needed.

While giving a good description of the N_g dependence of the distributions, the additive quark model also predicts the correct N_g dependence of the average multiplicities.[45] The eikonal model overestimates the particle yield in the central region, it ignores cascades. As a result, although this model does not describe the distributions themselves, the multiplicities turn out to agree with experiment. Furthermore, the N_g dependence of the multiplicities is described reasonably well.[192,193]

In photoproduction, the N_g dependence of the distributions must be similar to that in πA interactions, while in electroproduction the distributions are predicted to be independent of N_g in the photon fragmentation region.[41] This behavior of the inclusive distributions has been observed in experiments[31] on deep inelastic scattering of muons by nuclei of a photographic emulsion, but the statistical base (86 events) is too small for definite conclusions. In electroproduction, the multiplicity $\langle N_s\rangle_A$ is larger than $\langle N_s\rangle_N$ solely because of cascades, so that the N_g dependence of R should be much weaker than for hadrons.[41] The situation will presumably become clearer with the advent of data on photoproduction and neutrino reactions in photographic emulsions with better statistics.

d) Dependence of the fast-particle rapidity correlations on nuclear fragmentation

It follows from (7.4) and (7.5) that at $N_g\gg\langle N_g\rangle$ we would have[44,45]

$$R_2^A(0,0)=\frac{R_2^N(0,0)}{\nu_{max}}, \tag{7.8}$$

$$\left(\frac{D}{\langle N_s\rangle}\right)_A=\frac{1}{\sqrt{\nu_{max}}}\left(\frac{D}{\langle N_s\rangle}\right)_N, \tag{7.9}$$

with limiting values which are independent of the target

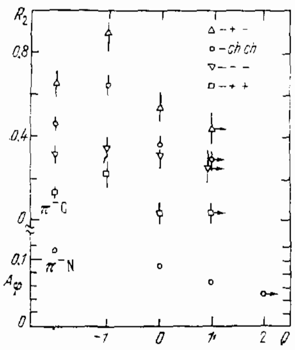

FIG. 36. Dependence of the rapidity correlations and the azimuthal asymmetry on the total charge of the final particles in π^-C interactions at 40 GeV (Ref. 223). Also shown here are the values of A_φ and $R_2^A(0, 0)$ in π^-N interactions or for the total set of π^-C interactions (points at the far left).

nucleus, as in (7.7). The N_g dependence of $(D/\langle N_s\rangle)_A$ and $R_2^A(0,0)$ given by (7.4) and (7.5) agrees well with that observed experimentally (Figs. 10, 12, and 36; in Fig. 36, Q is the charge of the observed secondary particles, i.e., a quantity similar in meaning to N_g-1). In Fig. 10, $(D/\langle N_s\rangle)_A$ actually has a limiting value at large N_p which is approximately the same for all nuclei. According to (7.9), the limiting value of $(D/\langle N_s\rangle)_A$ in pA or $\bar{p}A$ interactions is smaller than in πA interactions, in excellent agreement with experiment (Fig. 10). The same inequality holds for the rapidity correlations (Fig. 12). The correlations in π^-C interactions also fall off roughly by half for all charge combinations at large values of Q (Fig. 36).

The azimuthal correlations are particularly interesting, in that the dispersion of the ν distribution does not contribute[44,227] to A_φ. Accordingly,

$$A_\varphi^A=\frac{A_\varphi^N}{\langle\nu\rangle}. \tag{7.10}$$

At large values of Q, the value of A_φ for $\pi^+\pi^-$ pairs in π^-C interactions falls off roughly by half,[200] in good agreement with the quark model (Fig. 36) (it is difficult to use the data of Ref. 200 on $\pi^+\pi^+$ and $\pi^-\pi^-$ pairs because of the identical-particle effect).

With increasing N_g, the dispersion of the ν distribution falls off in any multiple-scattering model, including the eikonal model. Since the values of ν are not bounded, there should be no limiting value of either $R_2^A(0,0)$ or $(D/\langle N_s\rangle)_A$ in the limit $N_g\to\infty$ in the eikonal model. Furthermore, it is not difficult to see that the sign of the inequality between $(D/\langle N_s\rangle)_A$ and $R_2^A(0,0)$ for $\bar{p}A$ and πA interactions for equal values of N_g in the eikonal model is opposite to that predicted by the quark model. The relationship between ν and N_g for emulsion nuclei found in Refs. 18 and 225 corresponds to values $\nu\geq 6$ at $N_g\geq 8$ or $N_p\geq 6$. At such values of ν, Eqs. (7.8) and (7.9) contradict the experimental data in Figs. 10 and 12.

While the eikonal model seems to predict too fast a decrease in the correlations with increasing N_g, the correlations should be completely independent of N_g in the coherent-tube model. As was mentioned in Subsection 6g, the experimentally observed decrease of $R_2^A(0,0)$ and $(D/\langle N_s\rangle)_A$ with increasing N_g refutes all versions of the coherent-tube model.

In deep inelastic scattering of leptons by nuclei we have $\nu\equiv 1$, so that there should be no N_g dependence of the correlations in the central region.[41] This circumstance distinguishes deep inelastic scattering from photoproduction, which should be similar to πA interactions.

An effective boundary of the kinematic region arises at the beginning of the cascade region. This circumstance is related to the minimum in R_y, in particular (Fig. 26). This boundary of the kinematic region leads to a negative contribution to the rapidity correlation near the minimum.[211] According to the estimates in Ref. 74, we have $\delta R_2^A(y_c,y_c)\approx-(0.05-0.10)$ in hd interactions with rescattering. The effect has not been estimated for heavier nuclei. In principle, an analog of Eq. (3.15) could be formulated not only for single-particle but also for multiparticle inclusive distributions. This has recently been done by Verbetskiĭ,[228] but the equations have not yet been solved.

e) Associated multiplicities

According to (6.7), only the single-quark interactions contribute to the fragmentation $pA\to pX$ with $x\sim 1$:

$$\langle\nu\,(p\to p\,(x\sim 1))\rangle\approx 1. \tag{7.11}$$

In the fragmentation $pa\to\pi X$ we have from (6.7)

$$\langle\nu\,(p\to\pi\,(x\sim 1))\rangle=\frac{w_1+2aw_2}{w_1+aw_2}>1. \tag{7.12}$$

This result means that the multiplicity $\langle N_X\rangle$ of the particles X associated with the fragmentation $p\to\pi$ $(x\sim 1)$ is greater than in the fragmentation $p\to p(x\sim 1)$ (Ref. 222). For lead nuclei we have $w_1\approx w_2$, and from (7.12) we find $\langle N_X\rangle_{p\to\pi}\approx 1.5\langle N_X\rangle_{p\to p}$ (for $x\sim 1$).

The production of antiprotons is negligible at any realistic energies, so that slow protons with $x\ll 1$ are produced primarily in two- and three-quark interactions,

$$\langle\nu\,(p\to p\,(x\ll 1))\rangle\approx 3, \tag{7.13}$$

while for pionization pions from lead nuclei we have

$$\langle\nu\,(p\to\pi\,(x\ll 1))\rangle\approx 2. \tag{7.14}$$

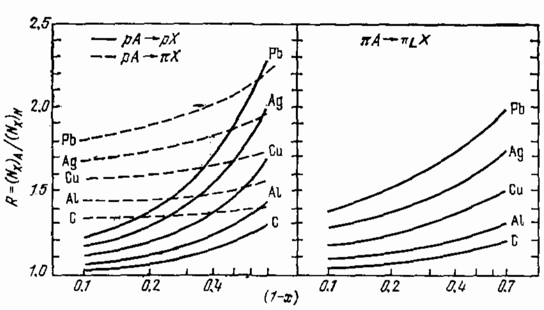

FIG. 37. The dependence of the relative associated multiplicities on the value of x for the fragment particle in the fragmentations $pA\to pX$, $pA\to\pi X$, and $\pi A\to\pi_L X$ for various nuclei.[229]

In other words, the sign of the inequality between $\langle N_X\rangle_{p-p}$ and $\langle N_X\rangle_{p-r}$ changes as x decreases.[229] Figure 37 shows the dependence of the ratio $R=\langle N_X\rangle_A/\langle N_X\rangle_N$ on x of the fragment according to the quark model. Predictions were also generated in Ref. 229 for the distributions as functions of x and for the distributions in the multiplicity N_X. The N_X distributions and the Wroblewski ratio, in particular, turn out to depend on x and on the nature of the fragment only weakly. Similar results for the N_X distributions in the fragmentation p−p have been derived in the eikonal model.[230]

8. "HARD" PROCESSES IN HADRON-NUCLEUS INTERACTIONS

a) Production of lepton pairs, ψ particles, and charmed particles

The additivity of the cross sections for deep inelastic scattering (Subsections 5c and 5d) implies the additivity of the parton densities of the nucleons of the nucleus, at least for $x \gtrsim 0.1$. The "hard" processes which occur over short distances are described by a noncoherent interaction of fast partons from the colliding hadrons. These interactions are the annihilation $q\bar{q}\to l\bar{l}$ in the production of massive lepton pairs[231] (the current situation is reviewed in Ref. 232); the coalescence $q\bar{q}\to\psi$ or the coalescence of gluons $qq\to\chi_1$, with the subsequent decay $\chi_1\to\psi\gamma$ in the production of ψ particles[233]; and the large-angle scattering of quarks and gluons in the production of particles with large transverse momenta.[234] For our purposes, the important point is that the cross sections of the hard processes are proportional to the product of the densities of the incoming partons from the colliding hadrons and should be proportional to A^1 (Refs. 235 and 236).

Figure 38 is a compilation[237] of the data of Ref. 238 on the A dependence of the production of dileptons in pA collisions. In πA collisions a similar dependence of α on $M_{\mu\mu}$ has been observed.[239] According to recent data from CERN, in πA collisions we have $\alpha = 1.03\pm 0.03$ for heavy dileptons.[240] The A^1 law sets in at masses $M_{\mu\mu} \gtrsim 4$ GeV. Interestingly, this mass is the presently accepted boundary of the Drell–Yan continuum: At lower masses Drell–Yan scaling does not hold, and the angular distribution of the muon pairs is not described by[241] $1+\cos^2\theta$. The nature of the transition from $\alpha=2/3$ at $M_{\mu\mu}\approx m_\rho$ to $\alpha=1$ at $M_{\mu\mu}>4$ GeV, as well as the mechanism itself for the production of small masses, remains unknown. At the same time, the production of such masses in nuclei is very interesting, particularly in the nuclear fragmentation region and in the central region, since the dileptons do not interact inside the nucleus.

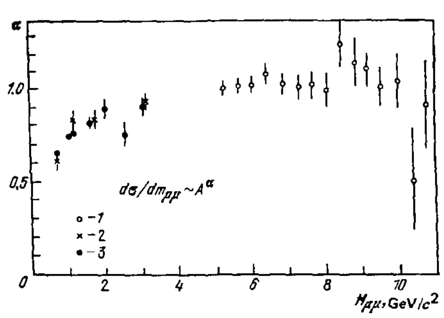

FIG. 38. The exponent α in the cross section for the production of massive lepton pairs by protons.[237] 1—Kaplan *et al.*; 2—Binkley *et al.*; 3—Branson *et al.*[238]

Partons and antipartons can undergo hard rescattering in the nucleus before annihilation. Estimates based on quantum chromodynamics show that the possible effect in the production of dileptons is negligibly small.[242]

In the production of ψ particles, α is approximately equal to unity, but lower: $\alpha_\psi = 0.927\pm 0.030$ (Refs. 239, 243, and 244). For particles containing heavy quarks the characteristic value of the virtual mass is the mass of the heavy quarks (see, for example, Ref. 8). For ψ particles, therefore, the formation lengths are small,[245] $l_\psi \sim k/m_\psi^2$, and α_ψ decreases because of the absorption of ψ particles inside the nucleus. Estimates put the cross section for the ψN interaction at a value of the order of several millibarns.[246] We might also estimate $\sigma_{\psi N}$ from α_ψ, assuming that the mechanisms for the production of ψ particles and dileptons with $M_{\mu\mu}=m_\psi$ are identical:

$$\sigma_{\psi_N} \approx 10\ \text{mb}\cdot(\alpha_{\mu\mu}(M_{\mu\mu}=m_\psi)-\alpha_\psi). \tag{8.1}$$

However, no sufficiently accurate measurements of $\alpha_{\mu\mu}(M_{\mu\mu}=m_\psi)$ have been made.

The A^1 law should also hold in the production of charmed particles in hadron–nucleus interactions. This circumstance has not always been taken into account appropriately in analyses of experimental results from "beam-dump" neutrino experiments.[247]

b) Production of particles with large transverse momenta

The situation with regard to the production of particles with large p_T is quite complicated. There is no systematic theoretical description of the observed cross sections at $p_T \leqslant 4$–5 GeV. Calculations from quantum chromodynamics can be reconciled with experiment, beginning at $p_T \gtrsim 3$ GeV, only by assigning a significant transverse momentum to the partons in the hadrons, thereby increasing the cross section by more than an order of magnitude.[234,248] In this situation it is not a simple matter to interpret the experimental data on the production of particles with large p_T in nuclei. These data are intriguing: While we find $\alpha_{\mu\mu}(M_{\mu\mu}>4\ \text{GeV})=1$ in the production of massive lepton pairs, the exponent α in the parametrization $d\sigma/d^3p\sim A^\alpha$ for large-p_T particles increases with increasing p_T for all particles and becomes greater than unity (Fig. 39). This fact was first observed back in 1974 by Cronin's group,[17] and it has been confirmed in several subsequent experiments, but it still awaits a satisfactory theoretical explanation. Indications of an even faster increase in α with increasing p_T have come from experiments on the production of particle jets with large p_T (Ref. 250; see Fig. 40 of the present paper).

The fact that the exponent has the value $\alpha=1$ in deep inelastic scattering and in the production of massive lepton pairs immediately eliminates all explanations of the Cronin effect which are based on the hypothesis

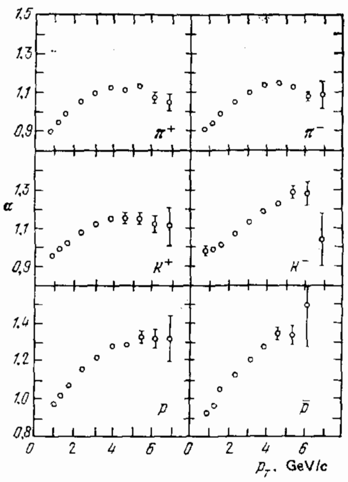

FIG. 39. The exponent α in the cross section for the production of large-p_T particles in proton–nucleus interactions.[17]

that the density of hard partons in the nucleus increases more rapidly than A^1 for some reason or other.[251-253] We are left with models which use the mechanism of repeated rescattering of hard partons. One possibility is a double hard scattering of one parton of the incident particle by partons of the nucleus, with a subsequent fragmentation into an observable hadron. In this case the differential cross section would be

$$d\sigma = A\, d\sigma^{(1)} + A^{4/3} d\sigma^{(2)} + \ldots. \quad (8.2)$$

The Cronin effect can be explained if $d\sigma^{(2)} \sim d\sigma^{(1)}$.

The most certain test of this hypothesis would be in the production of pairs of particles with large p_T (Ref. 166). The idea behind this test is that the term $d\sigma^{(1)}$ in (8.2) corresponds to the production of primarily symmetric pairs, while the term $d\sigma^{(2)}$ corresponds to the production in one arm of a particle (or jet) with large p_T and to the production in a second arm of two particles (or jets) with transverse momenta $\sim p_T/2$ (Ref. 166). Therefore, if pairs of particles are detected we would have

$$\alpha\,(p_{T_1} \approx p_{T_2}) \approx 1, \quad (8.3)$$

for symmetric pairs and

$$\alpha\,(p_{T_1} \gg p_{T_2}) \approx \frac{4}{3} \quad (8.4)$$

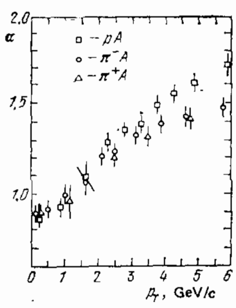

FIG. 40. The exponent α in the cross section for the production of large-p_T particle jets in collisions with nuclei.[250]

for asymmetric pairs. Behavior of this type for the pair exponent α has in fact been observed in FNAL experiments by Lederman's group[254] and on the DAKOR apparatus at the Institute of High-Energy Physics by Sulyaev's group.[255] The data obtained by a third group, who observed large-p_T pairs,[256] do not confirm the effect, but the value of p_T there was lower than in the FNAL experiment,[254] and the pairs were observed at an angle of 110°, rather than 90°.

It can be seen from Fig. 39 that the exponent α is higher for secondary particles which do not contain valence quarks ($\bar{p}$, K^-). There is the possibility that the production of $\bar{p}$ and K^- involves the production of gluons through a large angle. In quantum chromodynamics, the cross sections for gg and gq scattering are larger than those for qq and $q\bar{q}$ scattering (the color charge of the gluons is higher). Consequently, double scattering intensifies the relative yield of large-p_T gluons. Estimates by Krzywicki *et al.*[257] in fact lead to $d\sigma^{(2)} \sim d\sigma^{(1)}$ and to the correct inequalities between the values of α for the different particles. For protons, α is again large, but in this case the result may be related in some way to the scattering of the beam protons themselves through a large angle (this scattering has not yet been explained).

A second explanation of the Cronin effect, but only for jets, may be called the "pseudojet" mechanism. Here it is assumed that two partons from the incident particle are scattered simultaneously by different nucleons of the nucleus, but in the same direction. Zmushko[258] and Takagi[259] have shown that under real experimental conditions the cross sections for the formation of such pseudojets are not small. In these pseudojets, the relative number of fast particles is lower, and the total multiplicity is higher, than in an ordinary jet.[259] Both of these conclusions agree with the data obtained by Bromberg *et al.*[250] If this pseudojet mechanism does operate, then the cross section for the production of four uncorrelated jets with momenta p_T would have to be comparable to the cross section of a single jet with momentum $2p_T$.

In calorimetric observations of large-p_T jets, the calorimeter receives not only the products of the large-p_T jet proper but also background particles from the jet along the beam formed by the fragments of the target and the beam particle. The number of such fragments increases with increasing atomic number of the nucleus, and the momentum of the jet is overestimated in nuclei; the cross section for the production of a jet is also overestimated. Estimates show that this background effect may also explain the difference between the values of α for jets and for single particles as well as the "softening" of the jets in the transition to nuclei. In experiments on the production of particle pairs, this background and also the background from pseudojets would not be present.

For an understanding of the mechanism for the Cronin effect and of the various explanations which have been discussed for it, it would be extremely useful to see a detailed experimental study of correlations in the production in nuclei of particles and jets with large p_T.

9. CONCLUSION

A comparison of this review with the 1977 review published in this journal by Nikitin *et al.*[11] reveals substantial progress, both experimentally and in the theory of the interaction of high-energy particles with nuclei. What appears to be the most realistic model for multiple production in nuclei—the additive quark model—is a new development since 1977. The experimental evidence available, and discussed above, strongly indicates that the constituent quarks behave in an additive manner in inelastic collisions. The simplest mechanism for additivity—a small size of the constituent quarks—was first pointed out by Anisovich.[161] This small size can be associated with the small slope of the pomeron trajectory and with the large transverse momenta of the internal motion of the partons in hadrons.[161,260] These facts should be taken into consideration in the construction of a quantum-chromodynamics theory of hadrons. Progress in this direction so far has been less than modest.

A small value of α_P' corresponds to large scale masses in the pomeron ladder.[261] Gribov has noted that the mass of gluonium,[262] which is large,[263] could serve as this large scale mass in a chromodynamic, purely gluon ladder. According to Ref. 264, on the other hand, the quark–gluon constant is large only at very low energies, ≈ 100 MeV, while energies of the order of 1 GeV correspond to a perturbation-theory region. It is therefore difficult to see how a scale corresponding to energies of the order of 1 GeV will arise in a description of the structure of hadrons from ordinary quarks in chromodynamics.

A new phenomenology of hard processes has arisen within the framework of quantum chromodynamics, involving quark and gluon fragmentation functions. The possibilities of this phenomenology have essentially been exhausted, and there is the unavoidable question of just how the quarks and gluons produced in hard collisions convert into hadrons. This is a problem of large distances, and interactions with nuclear targets may be of much assistance in its solution. In collisions with nuclei, for example, it is a simple matter to study the interactions of all constituent quarks of the incident particles simultaneously, but this would not be possible in interactions with hydrogen.

Returning to recent developments in the physics of the interactions with nuclei, we note that there has been substantial progress toward an understanding of the correlation effects; two or three years ago there was neither detailed experimental evidence nor a theoretical framework for discussing these effects. The data which have now been accumulated on correlations apparently spell the doom of the collective models: the coherent-tube model and the hydrodynamic model.

It is clear that the simple cascade model[10,55] correctly describes the most important features of multiple production in nuclei at relatively low energies. At energies of a few tens of GeV this model starts to diverge sharply from experiment, and this result has been interpreted naively as demonstrating the complete absence of any intranuclear interactions of the secondary particles. A correct account of the formation lengths of the secondary particles corrects the situation, and there is no longer any doubt that suitably modified cascades do occur. Experiments on electroproduction and on neutrino–nucleus interactions are particularly important in this regard.

There has been important progress toward an understanding of the inelastic corrections for diffraction processes in collisions with nuclei. It is now clear that there is no simple way to determine the cross section for the interaction of unstable systems with nuclear matter. There is, however, the possibility of determining the parton structure of hadrons in more detail. An important task here is to reconcile the new phenomenology with quantum chromodynamics.

We do not have a really satisfactory understanding of the relationship between diffractive scattering and production processes. The conventional interpretation of the eikonal diagrams for multiple rescattering in terms of production processes is contradictory and leads to contradictions with experiment. Furthermore, we do not have a complete understanding of the corrections to the cascade picture of the interactions of the secondary particles, as corrected for formation lengths. The nature of the inelastic processes changes at $E > R_A \mu_0^2$, and there might be related changes in the total cross sections by virtue of unitarity. Experimentally, on the other hand, the simple optical model with inelastic corrections works well.

It has been necessary to be selective in the topics covered in this review. Little has been said about interactions with the deuteron; on that subject I recommend the thorough review by Bergstrom and Fredrikson[265] and Shabel'skiĭ's lectures.[266] The fragmentation of the nucleus and the cascade region proper have been completely neglected. Particularly interesting effects here are nuclear scaling and the cumulative production of particles into the rear hemisphere in collisions with nuclei. I suggest the comprehensive reviews by Frankfurt and Strikman (see Ref. 267), Baldin,[268] and Stavinskiĭ[269] and the lectures of Leksin.[270]

Here is a list of the experiments which seem the most promising in the light of the existing theory:

1. Precise measurements of the total cross sections for nA and $K_L A$ interactions (a test of the existence of passive states and of the theory of inelastic screening; see Subsection 4c).

2. A comparison of coherent and noncoherent diffractive dissociation in nuclei (Subsection 4d).

3. Highly accurate measurements of the amplitude and regeneration phase shift at nuclei at energies of hundreds of GeV (a study of parton fluctuations with slow valence quarks; see Subsection 4e).

4. A search for shadowing and negative shadowing in deep inelastic scattering by nuclei at $x \lesssim 0.1$–0.2 (Subsection 5c).

5. Precise measurements of shadowing in the transi-

tional region between photoproduction and electroproduction: Where do the small scale values of Q^2 come from? (Subsection 5d)

6. A search for a plateau in the nucleus/nucleon distribution ratio at TeV energies as a way of measuring the number of inelastic absorption events in a nucleus. Do multipomeron interactions have any visible effects in the central region? (Subsections 6b and 6e)

7. A test of the quark counting rules for fragmentation spectra for various incident particles and their fragments. Measurement of the associated multiplicities and distributions (Subsections 6d and 7e).

8. A precise comparison of the N_g dependence of the inclusive distributions, the Wroblewski ratios, and the rapidity correlations in πA and pA interactions (Subsections 7c and 7d).

9. A comparison, on the basis of a large statistical base, of electroproduction, neutrino production, and photoproduction in nuclei with production by hadrons. A search for the disappearance of the absorption of current fragments at high energies (Subsections 6d, 6e, and 7c).

10. A study of the correlations in the production of particles and particle jets with large transverse momenta in nuclei (Subsection 8b).

New predictions have appeared in the last two or three years on all of these questions, and tests are important for further development of the theory.

To a large extent, this review has been based on lectures[120,166] to the Sixth and Seventh Winter Schools of the Institute of Theoretical and Experimental Physics on the Physics of Elementary Particles. I wish to thank I. Yu. Kobzarev and G. A. Leksin for the opportunity to deliver these lectures. I thank V. B. Gavrilov, V. G. Grishin, L. Enik, O. V. Zhirov, E. M. Levin, S. Otwinowski, M. G. Ryskin, M. S. Fessler, and the late V. M. Shekhter for many discussions. E. L. Feĭnberg offered some particularly useful comments regarding the history of formation lengths and the theory of diffraction processes. These questions are covered in more detail in his review in Ref. 271, which appeared after the present review had been written. V. V. Anisovich, V. N. Gribov, I. M. Dremin, B. Z. Kopeliovich, and Yu. M. Shabel'skiĭ read the manuscript and made many useful comments, for which I am grateful. I also thank L. B. Okun' for some valuable criticism of the first draft of this review.

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