Superconductors with excess quasiparticles

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This review presents a systematic kinetic theory of nonequilibrium phenomena in superconductors with excess quasiparticles created by electromagnetic or tunnel injection. The energy distributions of excess quasiparticles and of nonequilibrium phonons, dependence of the order parameter on the power and frequency (or intensity) of the electromagnetic field, magnetic properties of nonequilibrium superconductors, *I-V* curves of superconductor-insulator-superconductor junctions, and other properties are described in detail. The stability of superconducting states far from thermodynamic equilibrium is investigated and it is shown that characteristic instabilities leading to the formation of nonuniform states of a new type or phase transitions of the first kind are inherent to superconductors with excess quasiparticles. The results are compared with experimental data.

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INTRODUCTION

The study of superconductors in a state far from equilibrium is attracting increasing interest. One of the reasons for this is the high sensitivity of the order parameter to external perturbations. Another reason is the richness of phenomena in nonequilibrium superconductors, resulting from coupling of three subsystems: Cooper pairs, quasiparticles, and phonons, i.e., subsystems that reflect the dynamic and kinetic properties.

The general problem of simultaneously describing the dynamic and kinetic properties was solved by Keldysh.[1] Using Keldysh's technique,[1] it was possible to obtain a system of equations describing the exciton and photon insulator (semi-conductor in a strong electromagnetic field)[2] and the superconductor.[3] Gor'kov and Éliashberg[4] developed another method with the help of which Éliashberg obtained a system of equations for a superconductor.[5]

Nonequilibrium states are usually created by the action on the superconductor of an electromagnetic or ultrasonic field of frequency ω, tunnel injection under a voltage V, a flux of charged particles, and other methods. Nonequilibrium states can be arbitrarily separated into two types. The first type arises if the frequency ω is less than 2Δ.[1)] In this case, the field does not create quasiparticles, but changes their energy distribution.

The results of detailed investigations of states of this type, in which stimulation of superconductivity is observed, is presented in Ref. 6.

In the present review, we examine nonequilibrium states of the second type with excess (in comparison with thermal) quasiparticles created by fields with $\omega > 2\Delta$ (or $V > 2\Delta$).

Creation of quasiparticles in superconductors usually leads to a decrease in Δ (however, see Ref. 7 and section 2, wherein hypothetical mechanisms leading to an increase in T_c are examined). Nevertheless, the study of nonequilibrium states of the second kind is of great scientific and applied interest.

The point is that most superconducting devices operate under nonequilibrium conditions (low resistance tunnel junctions, magnet windings in accelerators, thermonuclear reactors, and others) or make use of the nonequilibrium nature of the state (Josephson effect devices, radiation receivers, fast particles detectors, phonon generators, and others). The study of nonequilibrium processes with excess quasiparticles (EQ) provides rich information concerning the parameters of superconductors,[8] which is often not accessible by other methods.

States with EQ (referred to simply as nonequilibrium states in what follows) can in their turn be separated in two groups.

In the first group, the nonequilibrium state is characterized by the quasiparticle distribution function (QDF), which differs from the equilibrium function, but is symmetrical relative to the variable $\xi = (p^2/2m) - E_F$ (E_F is the Fermi energy). This means that the population of the electron-like quasiparticle branch ($\xi > 0$) and the hole-like branch ($\xi < 0$) coincide. Such a state arises with electromagnetic and ultrasonic pumping, as well as with symmetrical tunnel injection (superconductor-insulator-superconductor).

If the intensity of the sources is not large, so that the number of EQ is small in comparison with the thermal number, then the observed energy distribution of excess quasiparticles almost coincides with the thermal distribution. The action of the sources leads to an increase in the effective temperature.[8]

A considerably different situation occurs with intense pumping, when the distribution function for excess quasiparticles (DFEQ) is comparable to unity, while the order parameter decreases to zero. In this case, the DFEQ differs from the thermal function, while its form plays a decisive role and leads to such phenomena as phase transitions of the first kind, a new type of nonuniform state, and others examined in section 1 of this review.

Excess quasiparticles can lead to even more radical changes if their energy distribution is inverted.[3] Such a situation is possible if $2\Delta > \omega_D$ (ω_D is the Debye frequency) and, apparently, can be realized in superconductors with a repulsive interaction. Section 2 of this review is concerned with this problem.

In the second group of nonequilibrium states, the deviation of the QDF from the equilibrium form is an odd function of ξ, i.e., the populations of the electron-like and the hole-like branches do not coincide. Such a situation is referred to as an unbalance in the population of the spectral branches and arises, for example, with tunnel injection in a S–N junction (superconductor-normal metal).

Let us discuss briefly the physical phenomena that result from the appearance of an odd, relative to the Fermi momentum, segment in the QDF under nonequilibrium conditions.

It is well-known that the basic properties super-conductors, ideal diamagnetism (Meissner effect) and perfect conductivity, result from the formation of Cooper pairs of two electrons and their Bose condensation.[9] As a result, the spectrum of single-particle excitations contains a gap Δ, while the wave function of the excitations with momentum p is a superposition of the wave functions of an electron $\varphi^{+}_{p\sigma}$ and a hole $\varphi_{p\sigma}$ (Ref. 10):

$$\psi_{p\sigma} = u\varphi_{p\sigma} + v\varphi_{-p,\,-\sigma}, \tag{1}$$

where

$$u^2 = \frac{1}{2}\left(1+\frac{\xi}{\varepsilon}\right), \quad v^2 = \frac{1}{2}\left(1-\frac{\xi}{\varepsilon}\right), \quad \varepsilon = \sqrt{\xi^2+\Delta^2}.$$

As a result, the effective charge e^* for electronic excitations equals

$$e^* = eu^2 = \frac{e}{2}\left(1+\frac{\xi}{\varepsilon}\right) < 0, \tag{2}$$

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1) Here and in what follows we will consider $\hbar$, c, and e equal to unity.

i.e., it depends on the momentum of the excitation and, generally speaking, is less than the electronic charge e (the charge of a Cooper pair equals $2e$). However, it is evident from expression (2) that the sum of the effective charges of two electronic excitations with momenta p_1 and p_2, symmetrical relative to the momentum p_F, $p_1 - p_F = -(p_2 - p_F)$, exactly equals the electronic charge e. In a state with an even distribution function, the probabilities for populating states with momenta p_1 and p_2 are equal, so that as a sum they act as a single excitation with charge e and do not lead to any singularities.

However, under nonequilibrium conditions when such an odd QDF appears, the difference between the charge of a quasiparticle e^* and the electron charge e leads to the appearance of a total excitation charge,[11] which, due to the condition of electrical neutrality, is compensated by an induced charge in the Cooper pair condensate. In this case, the chemical potentials of the pairs μ_p and quasiparticles μ_q become unequal. As a result, it is possible for a static longitudinal electric field to penetrate into a superconductor,[12] which permits a consistent description of the transformation of the ohmic current into a superconducting current at the boundary between a normal metal and a superconductor. Due to the nonuniformity of the chemical potential μ_p, the total force acting on the superconducting condensate in this case equals zero. The behavior of superconductors with weak coupling is also essentially described by this effect.[12]

The appearance of a total excited charge, compensated by the charge of the condensate pairs, also leads to a new type of weakly damped collective excitations that have a sonic character, which was observed experimentally (see Ref. 12).

The fact that there is a difference between the chemical potentials of pairs μ_p and of quasiparticles μ_q leads to a new class of thermoelectric phenomena in superconductors.

The possibility of observing thermoelectricity in superconductors was first substantiated by V. L. Ginzburg,[13] who showed that in order for such a phenomenon to occur nonuniform or anisotropic systems are necessary. At the present time, this problem is being studied intensively both experimentally and theoretically.[14]

In Ref. 15, it was shown theoretically and experimentally that the unbalance in the populations of the electron-like and hole-like branches can be created by the simultaneous existence of a temperature gradient and a superconducting current. This phenomenon was investigated theoretically in greater detail in Ref. 95.

In section 1 of the present review, we have examined the theory that must explain the following basic experimental facts:

1. the properties of the energy distribution of nonequilibrium quasiparticles and phonons;

2. instability and hysteresis phenomena;

3. mechanisms for phase transitions of the first kind, superconductor–normal metal and superconductor with an order parameter Δ_1—superconductor with an order parameter $\Delta_3 \neq \Delta_1$;

4. nature and structure of nonuniform states in superconductors with optical and tunnel pumping of quasiparticles.

The purpose of this review is to present a consistent description of these phenomena from a unified point of view. The review by no means claims completeness of the bibliography, since it depends primarily on work published recently.

Research in this area began after the appearance of the work by Testardi[16] in 1971, in which the author reported on an experimental investigation of the resistance of superconducting films irradiated by pulses of laser light. Testardi came to the conclusion that the observed superconductor–normal metal resistive transition is due not to simple heating, but rather to excess quasiparticles created by sources.

Later, Sai-Halasz and others,[17] Hu and others,[18] Golovashkin, Motulevich, and others[19] discovered that the resistance of a superconductor irradiated by light arises smoothly, beginning with some critical value of the power β_c, and reaches the normal value over a quite wide range of β (and not discontinuously, as in the equilibrium case). The authors of Ref. 17–19 suggested the possibility of the existence of a new type of nonequilibrium intermediate state, in which the conducting and normal phases coexist.

Subsequent investigations of superconductors with optical[20-22] and especially with tunnel injection[23-27] confirm the existence of a nonuniform state (in particular, with two gaps[25,26]) in nonequilibrium superconductors, as well as an entire series of other phenomena, such as phase transitions of the first kind,[23,24] broadening of I-V characteristics, hysteresis, and other phenomena.[8]

It should be noted that these phenomena are observed in thin films (300–3000 Å). The small thickness is necessary in order to remove nonequilibrium phonons before the phonons reach equilibrium with the lattice and quasiparticles. In the latter case, external action simply leads to heating of the superconductor. The interpretation of these new phenomena has been widely discussed (see Refs. 8, 28, 29) and their close connection to the form of the energy distribution of nonequilibrium quasiparticles has been understood.

Gradually, a concensus was formed (see, for example, Ref. 31) that the thermal distribution function $n_T = (e^{\varepsilon/T}+1)^{-1}$, as well as the heuristic quasiparticle distribution function of the form

$$n_{OS} = (e^{(\varepsilon-\mu^*)/T}+1)^{-1}, \quad n_{T^*} = (e^{\varepsilon/T^*}+1)^{-1}, \tag{3}$$

proposed by Owen and Scalapino (OS)[30] $\mu^* > 0$, $n_{OS}(0) > n_T(0)$ and by Parker,[31] respectively, are not adequate for explaining the phenomena in the vicinity of phase transitions.

The kinetic approach (i.e. finding n/ξ with the help of the solution of kinetic equations) was used by Vardanyan and Ivlev, who found the corrections to n_{OS} for weak optical pumping.

The case of strong optical pumping, in which n attained a considerable magnitude, was studied in Refs. 33 and 34. With the help of a solution of the kinetic equation, it was found that $n(\xi)$ differs noticeably from the equilibrium and heuristic functions (3). The properties of the nonequilibrium $n(\xi)$ have a general nature and stem from the effect of the order parameter (through the coherent factors entering into the probability) on the recombination and scattering processes for quasiparticles as they interact with phonons and on the creation of quasiparticles.

The effect of relaxation processes is most clearly manifested near the phase transition point $\Delta=0$, when an increase in recombination $[\sim 1+(\Delta^2/\varepsilon\varepsilon')]$ leads to a decrease in the number of quasiparticles near $\xi=0$. The quasiparticles appear as if they were overheated in comparison to the thermal distribution $[n(0)<n_T(0)]$. In its turn, this leads to the existence of a gap for pump powers above some critical value ρ_c (corresponding to the order parameter becoming zero). More precisely, the dependence of Δ on β and T in some interval $\beta_c<\beta<\beta_m$ becomes multivalued; in particular, there are three solutions $\Delta_1=0$, Δ_2 an increasing function of β, Δ_3 a decreasing function of β.[34]

In the other limiting case (case of a narrow-band source,[33] $\omega-2\Delta\ll\Delta$) the order parameter controls the rate of creation of quasiparticles, since the creation rate is proportional to $\omega-2\Delta$.

In this case, DFEQ increases with energy,[33,36] so that near threshold[37,38] there are three solutions for $\Delta(\omega)$, one of which increases with ω (or with V).

The concepts of the multivalued dependence of Δ on β and ω play a determining role in the theory being presented. An important problem is the problem concerning the stability of uniform states of superconductors with non-equilibrium quasiparticles. Stability relative to fast (in comparison to the relaxation times) excitations was investigated in Refs. 34, 39–44. It turned out that the stability criteria are closely related to the sign of the quantity

$$N_S=1+2\int_0^\infty \frac{dn}{d\varepsilon}\,d\xi, \tag{4}$$

which has the meaning of the number of superconducting electrons.[48] Positive N_S correspond to stability and negative N_S correspond to instability.[2)]

In the early work,[41,42] the OS functions,[30] which led to negative values of N_S under certain conditions (which would formally be equivalent to a negative diffusion coefficient for quasiparticles), were used. On this basis, a hypothesis was proposed concerning instability,[41,42] which was referred to as the diffusion hypothesis. It would arise with a finite value of the wave vector and would lead to spatial modulation of the order parameter. We note that due to (4) the superconductor would simultaneously become a paramagnet.

However, as can be seen from (4), the sign of N_S is sensitive to the form of $n(\xi)$. If the DFEQ obtained from a solution to the kinetic equation, is used to calculate N_S, then N_S turns out to be positive.[34] Mathematically, this is related to the slower, in comparison to n_{OS}, changes in the function n for small ξ [the property $n(0)<n_T(0)<n_{OS}(0)$]. This result was confirmed in subsequent work.[44,45,29]

The positiveness of N_S indicates that diffusion of quasiparticles stabilizes the excitation, and nonequilibrium super-superconductors remain diamagnets.

Stability relative to slow excitations (of the order of and less than the relaxation time for energy) was studied in Refs. 33, 37, 38, 43, and 44. Two instabilities, characteristic of the non-equilibrium state, the so-called threshold instability (characteristic of a narrow-band source) and the coherent instability (characteristic of a broad-band source and stemming from coherent factors) were discovered.

The threshold instability, quantitatively predicted in 1974 in Ref. 33, was studied in detail in Refs. 37, 38, and 46. The coherent instability was found and investigated in Ref. 43, and it was studied by a somewhat different method in the important work by Eckern, Shmid, Smutz, and Schon.[44]

The threshold and coherent instabilities are closely related to the multivalue nature of $\Delta(\beta)$ and have a maximum decrement in growth for uniform excitations, i.e. the situation has a definite analogy with the Van-der-Waals theory of real gases. Indeed, as shown in Refs. 43 and 47, depending on the conditions, transitions of the first kind between phases, overheating and overcooling phenomena (i.e., hysteresis), and the formation of a nonuniform state are possible. In this model (the so-called layer model), the inhomogeneous state consists of an alternating sequence of regions with different values of the order parameter, separated by transition layers with a length equal to the greater one of the quantities ξ_0 or L (ξ, is the coherence length and L is the diffusion length for quasiparticles). In the case of the coherent instability, the specimen separates into regions with superconducting and normal phases,[43] while for the threshold instability, the specimen separates into regions with different finite gaps.[37,38]

According to the layer model, the nonuniform state can be stationary only for one value of β_0 (or ω_0), for which the energies of the phases become equal. If $\beta\neq\beta_0$, then the phase separation boundary moves with a velocity proportional to $\beta-\beta_0$.[43] With electromagnetic pumping (in particular, optical), if special measures are not taken for maintaining $\beta=\beta_0$,[3)] the nonuniform state can only be a nonstationary state,[43] and the state is observed experimentally under pulsed irradiation.[16-22]

A different situation occurs in superconductors with tunnel injection, when by fixing the current through the tunnel junction, it is possible to control the relative size of the phases (in the Van-der-Waals gas, this is the volume) and to stabilize the nonuniform state. This

[2)]Under the condition that $D=\int_0^\infty(1-2n/\varepsilon^3)d\xi$ is positive (see 3, subsection d of section 1 for greater detail).

[3)]Setting $\beta=\beta_0$ is in principle possible with the help of feedback systems.

circumstance was pointed out in Ref. 47; it was later confirmed experimentally for large voltages $V \gg \Delta$ (Ref. 27) and voltages near threshold $V = 2\Delta$.[25,26]

As a whole, the concepts that have been developed have led to a good description, in many cases quantitative, of the phenomena occurring in superconductors with electromagnetic and tunnel injection.

1. EXCESS QUASIPARTICLES IN SUPERCONDUCTORS WITH ATTRACTION BETWEEN ELECTRONS

a) Basic equations

In order to describe nonequilibrium processes in superconductors, it is necessary to have a system of equations that describes simultaneously the dynamic and kinetic properties.[1,4] In the weak coupling approximation (small interaction constant), this system reduces to an equation for the order parameter and a kinetic equation for the quasiparticle and phonon distribution functions.

1) *Equation for the order parameter.* The stationary equation for the order parameter $\Delta(\mathbf{r})$ has the form[4,10]

$$\Delta(\mathbf{r}) = -\lambda \int \int_{-\omega_D}^{\omega} d\xi \frac{d\Omega}{4\pi} u_{\mathbf{p}}(\mathbf{r}) v_{\mathbf{p}}^{*}(\mathbf{r}) (1 - 2n_{\mathbf{p}}), \tag{5}$$

where λ is the electron-phonon interaction constant ($\lambda < 0$), $n_{\mathbf{p}}$ is the quasiparticle distribution function, which satisfies the kinetic equation (9) (see below); ω_D is the Debye frequency; here, and in what follows, $\Delta(\mathbf{r})$ denotes the modulus of the order parameter, $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ are functions that satisfy the Bogolyubov equations.[10] In order to describe nonstationary processes, it is necessary to have, generally speaking, a time-dependent equation for Δ. However, in most cases, it is possible to limit ourselves to Eq. (5), since the time for establishing a stationary state in a system of Cooper pairs is substantially less than the time for establishing such a state in a system of quasiparticles.

In the spatially uniform case, Eq. (5) takes the form

$$\Delta = -\lambda \int_{\omega_D}^{\omega_D} \frac{d\xi\,(1 - 2n(\xi))}{2\varepsilon} \Delta, \quad n(\xi) = \int \frac{d\Omega}{4\pi} n_{\mathbf{p}}. \tag{6}$$

The spatial variation of $\Delta(\mathbf{r})$ in a nonequilibrium superconductor is described by an equation similar to the Ginzburg-Landau equation:

$$\xi_0^2 \left(\frac{\partial}{\partial \mathbf{r}} - 2ie\mathbf{A}\right)^2 \Delta = +\frac{\partial U}{\partial \Delta}, \tag{7}$$

$$U(\Delta) = +\int^{\Delta} \Delta' \, d\Delta' \left(\frac{1}{|\lambda|} - \int_0^{\omega_D} d\xi \frac{1 - 2n(\xi, \mathbf{r})}{\sqrt{\xi^2 + \Delta'^2}}\right), \tag{8}$$

which is obtained[43] by expanding the functions $u_{\mathbf{p}}$ and $v_{\mathbf{p}}$ in powers of Δ, assuming that Δ is small and that Δ varies slowly with the coherence length ξ_0; here, ξ_0 is a functional of the QDF of a normal metal $n_0(\xi)$:

$$\xi_0^2 = \frac{p_0^2}{3m^2} \int_0^{\infty} \frac{d\xi}{\xi} \left(\frac{d}{d\xi} \frac{2n_0 - 1}{\xi}\right),$$

and A is the vector potential.

2) *Kinetic equation for quasiparticles.* The kinetic equation for the QDF has the form

$$\frac{\partial n(\mathbf{p}, \mathbf{r}, t)}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial \varepsilon}{\partial \mathbf{r}} = \left(\frac{\partial n}{\partial t}\right)_{\mathrm{im}} + \left(\frac{\partial n}{\partial t}\right)_{\mathrm{f}} + \left(\frac{\partial n}{\partial t}\right)_{\mathrm{ee}} + Q, \tag{9}$$

where $\varepsilon = \sqrt{\xi^2 + \Delta^2}$ is the quasiparticle energy, Q is a source of quasiparticles, and $(\partial n/\partial t)_{\mathrm{f}}$, $(\partial n/\partial t)_{\mathrm{ee}}$, and $(\partial n/\partial t)_{\mathrm{im}}$ are the collision integrals describing collisions between quasiparticles and phonons, quasiparticles and impurities, respectively, and in addition, the last integral can be written in the form

$$\left(\frac{\partial n}{\partial t}\right)_{\mathrm{im}} = -\frac{|\xi|}{\varepsilon \tau_0} (n_{\mathbf{p}}(\mathbf{r}, t) - n(\xi, \mathbf{r}, t)); \tag{10}$$

where τ_0 is the relaxation time for collisions with impurities.

The equation for the QDF integrated over the angles $n(\xi, \mathbf{r}, t)$ is obtained from (9) in the usual manner, using the smallness of τ_0/τ_{f} (τ_{f} is the relaxation time for collisions with phonons):

$$\frac{\partial n(\xi, \mathbf{r}, t)}{\partial t} + \frac{|\xi|}{\varepsilon} \frac{L^2}{\tau_{\mathrm{f}}} \left[\frac{\partial}{\partial r_1}\left(-\frac{\partial n}{\partial r_1} + \frac{\partial \varepsilon}{\partial r_1}\frac{\partial n}{\partial \varepsilon}\right) + \frac{\partial \varepsilon}{\partial r_1}\frac{\partial}{\partial \varepsilon}\left(\frac{\partial n}{\partial r_1} - \frac{\partial \varepsilon}{\partial r_1}\frac{\partial n}{\partial \varepsilon}\right)\right] = \left(\frac{\partial n}{\partial t}\right)_{\mathrm{f}} + \left(\frac{\partial n}{\partial t}\right)_{\mathrm{ee}} + Q. \tag{11}$$

The integral describing collisions with phonons,[5] written taking into account the fact that $n(\xi)$ is even, has the form

$$\left(\frac{\partial n}{\partial t}\right)_{t} = S^{+} - S^{-} - S^{\mathrm{R}}, \tag{12}$$

$$S^{+} = \frac{1}{\tau_{\mathrm{f}}} \int_{\xi}^{\omega_D + \xi} \frac{d\xi'}{\Delta_0^{k+2}} (\varepsilon' - \varepsilon)^{k+1} \left(1 - \eta \frac{\Delta^2}{\varepsilon\varepsilon'}\right) [(1 - n) n' (1 + N_{\varepsilon' - \varepsilon}) - n(1 - n') N_{\varepsilon' - \varepsilon}], \tag{13}$$

$$S^{-} = \frac{1}{\tau_{\mathrm{f}}} \int_0^{\xi} \frac{d\xi'}{\Delta_0^{k+2}} (\varepsilon - \varepsilon')^{k+1} \left(1 - \eta \frac{\Delta^2}{\varepsilon\varepsilon'}\right) [n(1 - n')(1 + N_{\varepsilon - \varepsilon'}) - (1 - n) n' N_{\varepsilon - \varepsilon'}], \tag{14}$$

$$S^{\mathrm{R}} = \frac{1}{\tau_{\mathrm{f}}} \int_0^{\omega_D - \xi} \frac{d\xi'}{\Delta_0^{k+2}} (\varepsilon + \varepsilon')^{k+1} \left(1 + \eta \frac{\Delta^2}{\varepsilon\varepsilon'}\right) [nn'(1 + N_{\varepsilon + \varepsilon'}) - (1 - n)(1 - n') N_{\varepsilon - \varepsilon'}], \tag{15}$$

$$\frac{1}{\tau_{\mathrm{f}}} = \frac{\pi|\lambda|}{2} \Delta_0 \left(\frac{\Delta_0}{\omega_D}\right)^{k+1}, \quad n(\xi) \equiv n', \quad \varepsilon(\xi') = \varepsilon'. \tag{16}$$

Here, N_ε is the phonon distribution function, k is an exponent showing the dependence of the matrix element for the electron-phonon interaction on q ($M_q^2 \sim q^k$); η is a parameter that takes on the value +1 for a superconductor[5] and −1 for an exciton or a photon insulator.[2] The first two terms in (12) describe the energy relaxation of quasiparticles (S^+ denotes arrival, and S^- denotes departure), while the third term S^R corresponds to recombination of quasiparticles with phonon emission. The coherence factors $1 \pm (\eta\Delta^2/\varepsilon\varepsilon')$ arise due to the coherence of the interaction of quasiparticles with phonons and play an extremely important role in the phenomena examined in what follows.

The form of the collision integrals $(\partial n/\partial t)_{\mathrm{ee}}$ was found in Ref. 5. As shown in Refs. 5 and 32, electron-electron collisions have a weak effect on the form of the QDF for small values of ξ due to the smallness of C_1, the ratio of the electron-electron interaction constant and the electron-phonon interaction constant (see Table I and Ref. 29). However, they lead to renormalization of the quasiparticle source, which is substantial at high energies.

Integration of (11) with respect to ξ gives the conservation law for the number of quasiparticles $\bar{n}(\mathbf{r}, t)$:

TABLE I. Values of the parameter C_1 for some superconductors.

Parameter	Metal					
	Pb	In	Sn	Nb	Al	Zn
$C_1=\frac{(\hbar\omega_D)^2}{E_F T_c}$	0.0141	0.034	0.096	0.133	1.14	1.12

$$\frac{\partial\bar{n}}{\partial t}+L^2\frac{\partial}{\partial r_1}\int\frac{\xi}{\varepsilon}\left(\frac{\partial n}{\partial r_1}-\frac{\partial\varepsilon}{\partial r_1}\frac{\partial n}{\partial\varepsilon}\right)d\xi+\int S^R\,d\xi=\bar{Q}, \tag{17}$$

$$\bar{n}(\mathbf{r},t)=\int_0^\infty\frac{d\xi}{\Delta_0}n(\xi,\mathbf{r},t),\quad \bar{Q}=\int_0^\infty\frac{d\xi}{\Delta_0}Q(\xi). \tag{18}$$

3) *Sources of nonequilibrium quasiparticles.* Nonequilibrium states, examined in this review, are formed by the creation of quasiparticles by external sources (electromagnetic and ultrasonic fields with $\omega > 2\Delta$, tunnel injection with $V > 2\Delta$, ionizing particles, and others). Although the results obtained below are applicable to a certain extent to all the sources mentioned, we will examine in detail only two: Q_ω, an electromagnetic pumping source, and Q_V, symmetrical tunnel injection. According to Ref. 5, Q_ω can be represented in the form

$$Q_\omega=Q_\omega^{(1)}+Q_\omega^{(2)}+Q_\omega^{(3)}, \tag{19}$$

$$\begin{gathered}Q_\omega^{(1)}=\alpha_\omega\rho^\omega(\omega-\varepsilon)(1-n_\varepsilon-n_{\omega-\varepsilon}),\\ Q_\omega^{(2)}=\alpha_\omega\rho^\omega(\varepsilon-\omega)(n_{\varepsilon-\omega}-n_\varepsilon)=Q_{-\omega}^{(3)},\end{gathered} \tag{20}$$

$$\rho^\omega(\omega-\varepsilon)=\frac{(\omega-\varepsilon)-(\Delta^2/\varepsilon)^{1+(k_\omega/2)}}{\sqrt{(\omega-\varepsilon)^2-\Delta^2}}\theta(\omega-\varepsilon-\Delta), \tag{21}$$

$$\alpha_\omega=\frac{E^2e^2L^2r}{2m\omega^2\pi(1+\omega^2\tau_0^2)}, \tag{22}$$

$$L^2=\frac{v_0^2\tau_0\tau_f}{3},\quad r=r_e r_f,\quad r_e=\left(1+\frac{c_1\omega^3}{6\cdot 2^{2/3}E_F\omega_D^2}\right), \tag{23}$$

where $k_\omega=+1$, α_ω is the electromagnetic injection parameter, L is the quasiparticle diffusion length, v_0 is the electron velocity on the Fermi surface, r_e is the multiplication factor for quasiparticles due to electron-electron collisions[5,32] and r_f is the multiplication factor due to reabsorption of phonons.[21,22]

It should be noted that expression (22) corresponds to absorption by free electrons. In some superconductors, other mechanisms, for example, interband absorption, can make the dominant contribution. Due to the fact that the thickness of the film d usually exceeds the inverse absorption coefficient, its explicit form is not important and it can be replaced by $1/d$.[21,22]

A source that creates quasiparticles is strictly described by the quantity $Q_\omega^{(1)}$, while $Q_\omega^{(2,3)}$ describe the redistribution of existing quasiparticles with respect to energy, since

$$\int_0^\infty d\xi\,(Q^{(2)}+Q^{(3)})=0. \tag{24}$$

For tunnel injection through a symmetrical SiS-junction expression (19), in which the substitutions $\omega=V$, $k_V=-1$, and $\alpha_\omega=\alpha_V$, should be made, is applicable to Q_V (Ref. 50):

$$\alpha_V=\frac{2\sigma_N\tau_f}{de^2N(0)}, \tag{25}$$

where σ_N is the conductivity of the junction in the normal state, d is the thickness of the film, and $N(0)$ is the electron state density. An important difference between Q_V and Q_ω is the absence of the coherence factor Δ^2/ε in Q_V.

Sources can be separated into two classes independent of their nature:[33] broad-band sources, which create quasiparticles in a wide energy interval $\Delta<\varepsilon<\bar{\varepsilon}$ and are realized with optical pumping with $\omega\gg 2\Delta$, tunnel injection with $V\gg 2\Delta$, ionization by high-energy charged particles, and other means; narrow-band sources, which create quasiparticles in a small energy interval $\varepsilon-2\Delta\ll\Delta$. This classification turned out to be very convenient for understanding physical phenomena and for the analytical solution of the equations in the limiting cases of broad-band and narrow-band sources. The point is that in these cases it is possible to separate out the effect of the order parameter on the formation of the QDF.

Indeed, in the case of a broad-band source, Δ drops out of the expression for the source Q, i.e., $Q=\alpha_\omega\theta(\omega-\varepsilon)$, and it affects only the relaxation processes. Taking into account the fact that the "density" of the source action turns out to be low,[33] the kinetic equation can be viewed as homogeneous (it can be assumed that $Q=0$) and it is possible to find the connection between $n(\xi)$ and Q from the conservation law for the number of quasiparticles (17).

In the case of a narrow-band source, the order parameter has a determining influence on the source. We note that $Q^{(2)}$ and $Q^{(3)}$ do not play an essential role in the phenomena being examined for $\omega>2\Delta$.

For tunnel injection, it is necessary to add an equation for the external circuit:

$$\mathscr{E}=V+\bar{I}R,\quad \bar{I}=\int d^2r\,I(V,\Delta), \tag{26}$$

$$I(V,\Delta)=I_1+I_2,\quad I_1=\alpha_V\int_\Delta^{V-\Delta}d\varepsilon\,\rho(\varepsilon)Q_V^{(1)},\quad I_2=2\alpha_V\int_\Delta^\infty\rho(\varepsilon)Q_V^{(3)}\,d\varepsilon, \tag{27}$$

where $\mathscr{E}$ is the EMF, R is the external resistance, I is the current through the junction, and $\rho(\varepsilon)=\sqrt{\varepsilon^2-\Delta^2}$.

4. *Kinetic equation for phonons.* The phonon distribution function $N(\varepsilon)$ can differ from the equilibrium function

$$N_T(\varepsilon)=(e^{\varepsilon/T}-1)^{-1} \tag{28}$$

and is found from the following kinetic equation[28]:

$$\begin{aligned}\frac{\partial N(\varepsilon)}{\partial t}+\frac{N-N_T}{\tau_{es}}=&\frac{1}{\tau_B}\int_\Delta^{\varepsilon-\Delta}\frac{d\varepsilon'}{\Delta_0}\rho(\varepsilon')\rho(\varepsilon-\varepsilon')[n_{\varepsilon'}n_{\varepsilon-\varepsilon'}\\&-N_\varepsilon(1-n_{\varepsilon'}-n_{\varepsilon-\varepsilon'})]\left[1+\frac{\Delta^2}{\varepsilon'(\varepsilon-\varepsilon')}\right]\\&+\frac{2}{\tau_B}\int_\Delta^\infty\frac{d\varepsilon'}{\Delta_0}\rho(\varepsilon')\rho(\varepsilon+\varepsilon')[n_{\varepsilon+\varepsilon'}(1-n_{\varepsilon'})\\&-N_\varepsilon(n_{\varepsilon'}-n_{\varepsilon+\varepsilon'})]\left[1-\frac{\Delta^2}{\varepsilon'(\varepsilon+\varepsilon')}\right],\end{aligned} \tag{29}$$

$$\tau_{es}=\frac{4d}{S},\quad \tau_B^{-1}=\frac{\pi|\lambda|}{2}\frac{\Delta_0\omega_D}{E_F}, \tag{30}$$

where s is the speed of sound. The second term in (29) takes into account the loss of phonons from the film into the substrate, and the third and fourth terms taken into account the creation of phonons by recombination and scattering of quasiparticles, respectively. Some-times, the system (11) and (29) is approximated by a system of equations for the average values $\bar{n}$ and $\bar{N}$ (the Rotworf and Taylor equations[52]). However, these equations are inadequate for describing the phenomena being examined, since they do not take into account the properties of the energy distribution of quasiparticles and phonons.

5) *Kinetics of quasiparticles in superconductors.* The energy distribution of excess quasiparticles is formed by a source and relaxation processes. The action of the source $Q^{(\alpha)}$ reduces to breaking up a Cooper pair and creating two quasiparticles. A broad-band source creates quasiparticles with high energies $\bar{\varepsilon} \gg \Delta$. The quasiparticle created gives up its energy to the electron gas and the lattice. In the first case, the energy of the quasiparticle is expended on increasing the energy of existing quasiparticles and the creation of new quasiparticles by impact ionization through the gap 2Δ. Multiplication of quasiparticles, characterized by a multiplication factor r_e, proceeds in this manner.

In the second case, the energy goes into emission of nonequilibrium phonons. Quasiparticles finally reach the bottom of the band and recombine with the emission of a phonon ($\Delta \ll \omega_{ph}$), forming a Cooper pair once again. Nonequilibrium phonons can leave the film within a time τ_{es} or be reabsorbed within a time τ_B, creating a pair of quasiparticles (or increasing their energy), thereby leading to multiplication with a factor r_t. The relative probability for the loss and reabsorption of phonons is characterized by the parameter $\gamma = \tau_{es}/\tau_B$, whose upper bound is determined by interphonon interaction.

The kinetics of quasiparticles in superconductors has a definite analogy with the kinetics of electrons in semiconductors, with the formation of electronhole pairs in the latter.[53] Indeed, 2Δ plays the role of the width of the forbidden band E_g and the electrons and holes play the role of quasiparticles. However, nonequilibrium processes in superconductors have important differences:

1. The gap in superconductors 2Δ is unusually sensitive to the concentration and energy distribution of quasiparticles and can vary all the way to zero.

2. Due to the smallness of the ratio $2\Delta/\omega_D$, scattering and recombination processes are single photon processes and proceed approximately at the same rate.[33] This means that the recombination and scattering times are approximately equal, so that the QDF must differ from the quasiequilibrium function (occurring under the condition that the recombination time is large in comparison with the scattering time).

3. The order parameter has a strong effect on relaxation and creation processes for quasiparticles.

b) Uniform states of superconductors with a broad-band source of quasiparticles

In this section, starting from the system of equations (5), (9), and (29), we will obtain the distribution functions for quasiparticles and phonons, as well as the order parameter as a function of the pump power, temperature, and the parameter γ.

1) *Heuristic models for the quasiparticle distribution functions.* Heuristic functions $n(\xi)$ (not involving the solution of the kinetic equation) were proposed in Refs. 30 and 31. The function (3) was examined in the well-known work of Owen and Scalapino (OS),[30] which played a considerable stimulating role, and it was assumed that T^* coincides with the temperature of the lattice, while the chemical potential of the quasiparticles $\mu^* > 0$ is related to the concentration of quasiparticles. Parker[31] proposed a modified thermal model, in which the function (3) with $\mu^* = 0$ was used and the effective temperature T^* was determined from balance considerations. Finally, in later works[29] for thick films ($\gamma \gg 1$), the use of a distribution function of the form

$$n = \exp[-(\varepsilon - \mu^*)/T^*], \tag{31}$$

was proposed, in which both parameters μ^* and T^* were found from conservation laws for the number of quasiparticles and energy.

The models in Refs. 30 and 31 give a satisfactory description of the experimental data for small pump powers and become inapplicable near a phase transition, when the distribution function of excess quasiparticles attains magnitudes comparable to unity.[32,33] The reason is as follows. As is well-known, functions of the form (3) are a good approximation if the recombination time for quasiparticles is much larger than the energy relaxation time. However, in superconductors, recombination and relaxation processes for $n \sim 1$ proceed at the same rate (see subsection 5 in section 1). For this reason, the relaxation and recombination times for quasiparticles are of the same order of magnitude,[54,38,33,55] so that the QDF can differ considerably from the quasiequilibrium function and in order to find $n(\xi)$ it is necessary to solve the kinetic equation. Such solutions were first found for high pump levels in Refs. 33 and 34. In order to demonstrate clearly the properties of $n(\xi)$, it is useful to examine the very simple $k = -1, \gamma = 0, T = 0$ model, for which an exact solution exists.[34]

2) *$k = -1, \gamma = 0, T = 0$ model.* Substituting $k = -1$ and $N_\varepsilon = 0$ into (11), we obtain the following equation for $n(\xi)$:

$$-(1-n)\int_{\xi}^{\omega_D} d\xi'\, n(\xi')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)+n\int_0^{\xi} d\xi'(1-n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right)+n\int_0^{\omega_D} d\xi' n'\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)=0 \tag{32}$$

[we recall that $n(\xi)$ is an even function of ξ and ξ is assumed to be positive] and the normalization condition is

$$\frac{1}{\Delta_0^2}\int_0^\infty d\xi\int_0^\infty d\xi' n(\xi)\, n(\xi')\left(1+\frac{\Delta^2}{\varepsilon\varepsilon'}\right)=\beta^0,\qquad \beta^0=\frac{4\alpha_\omega\omega}{\pi\lambda\Delta_0^2}. \tag{33}$$

We note that in the model being examined Δ enters into the equation only as a result of the coherent factors. For $\Delta = 0$, the function $n_0(\xi)$[33] is an exact solution of (32):

$$n_0(\xi) = \frac{1}{2}\left(1 - \frac{\xi}{\sqrt{\xi^2+a^2}}\right), \tag{34}$$

$$a = 2\Delta_0\bar{n} = 2\int_0^\infty n_0(\xi)\,d\xi, \tag{35}$$

representing a monotonically decreasing function of ξ, attaining the value $1/2$ for $\xi = 0$. It is interesting to note that for very high values of ξ/Δ_0, n_0 is close to the thermal value $n_T(\xi)$.[56] Substitution of $n_0(\xi)$ into the equation for the gap (6) allows finding the critical concentration of quasiparticles $a_c = 2\Delta_0\bar{n}_c$ [and the dimensionless critical power from (33)], for which $\Delta = 0$:[33]

$$a_c = \Delta_0, \quad \beta_c^0 = \frac{1}{4}, \quad \bar{n}_c = \frac{1}{2}. \tag{36}$$

Near a phase transition point ($\Delta/\Delta_0 \ll 1$), it is possible to look for a solution of (32) in the form

$$n(\xi) = n_0(\xi) + n_1, \quad n_1 \ll n_0, \tag{37}$$

where the small correction n_1, proportional to Δ, satisfies the equation

$$n_1\Gamma(\xi) - P(\xi)\int_\xi^{\omega_D} n_1(\xi')\,d\xi' = \psi(\xi), \tag{38}$$

$$\Gamma(\xi) = a + \int_0^{\xi'} d\xi' P(\xi'), \qquad P(\xi) = 1 - 2n_0(\xi), \tag{39}$$

$$\psi(\xi) = -\frac{\Delta^2}{\varepsilon}\left[\int_\xi^{\omega_D} \frac{d\xi' n_0'}{\varepsilon'} - n_0\int_0^\xi \frac{d\xi'(1-2n_0')}{\varepsilon'}\right]. \tag{40}$$

The correction n_1 stems from the presence of coherent factors. There is an exact solution of (38):

$$n_1 = \frac{\psi(\xi)}{\Gamma(\xi)} + \frac{P}{\Gamma^2}\int_\xi^{\omega_D} \psi(\xi')\,d\xi', \tag{41}$$

and in addition, the functions Γ, P, and ψ are obtained in explicit form:

$$P = \frac{\xi}{\sqrt{\xi^2+a^2}}, \quad \Gamma(\xi) = \sqrt{\xi^2+a^2},$$

$$\psi(\xi) = -\frac{\Delta^2}{2\varepsilon}\left(\ln\frac{a+\Delta}{\xi+\varepsilon} + \frac{\xi}{\sqrt{\xi^2+a^2}}\ln\frac{\varepsilon+\sqrt{\xi^2+a^2}}{a+\Delta}\right). \tag{42}$$

Small values of $\xi \sim \Delta$, where the coherent addition $n_1(\xi)$ is negative, make the main contribution to the equation for the gap[34]:

$$n_1(\xi) = -\frac{\Delta^2}{\varepsilon}\left(\frac{1}{2\Delta_0}\ln\frac{\Delta_0}{\Delta}\right), \quad \xi \ll \Delta_0. \tag{43}$$

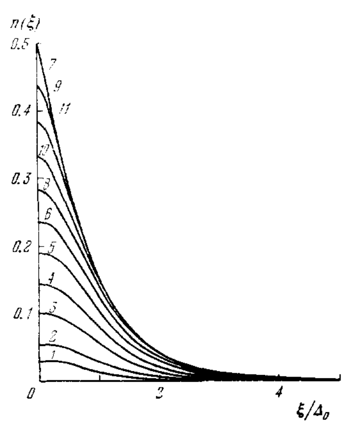

FIG. 1. Energy dependence of the quasiparticle distribution function for $T = 0$ and $\gamma = 0.15$. The values of the order parameter Δ/Δ_0 (and, correspondingly β) are 0.95 (1), 0.8 (3), 0.6 (5), 0.4 (8), 0.5 (11), and 0.01 (7).

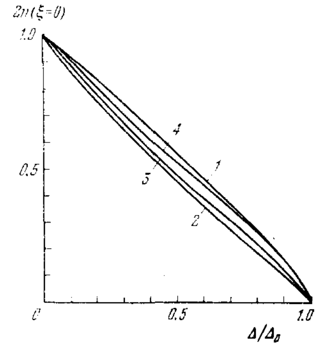

FIG. 2. Dependence of $2n$ ($\xi = 0$) on the order parameter. 1—according to the BCS theory: 2—nonequilibrium distribution function with $\gamma = 0$ and $T/\Delta_0 = 0$; 3—$\gamma = 0.15$ and $T/\Delta_0 = 0$; 4—$\gamma = 0.15$ and $T/\Delta_0 = 0.44$.

The decrease in $n(\xi)$ for small values of $\xi \sim \Delta$ is due to the increase in the recombination rate for quasiparticles, proportional to $1 + (\Delta^2/\varepsilon\varepsilon')$, especially noticeable for $\xi \to 0$. It follows from (34) and (43) that the distribution function for nonequilibrium quasiparticles has the property

$$n(0) < n_T(0) < \frac{1}{2}, \tag{44}$$

i.e., the number of nonequilibrium quasiparticles decreases for small values of ξ in comparison with thermal quasiparticles, so that the distribution becomes "overheated" (see below, Fig. 2).

This important property of $n(\xi)$ is retained in the general case and leads to the existence of a finite gap for $\beta > \beta_c$. One other general property of $n_1(\xi)$ consists of its localized nature for $\xi \sim \Delta$ and, therefore, the smallness of the integral

$$\frac{1}{\Delta_0}\int_0^\infty n_1(\xi)\,d\xi = -\left(\frac{\Delta}{2\Delta_0}\ln\frac{\Delta_0}{\Delta}\right)^2. \tag{45}$$

For this reason, in order to find corrections linear in Δ, we can drop the integral terms in the equation for n_1.

3) *Energy distribution of nonequilibrium quasiparticles created by a broad-band source.* The results do not change qualitatively if a more realistic matrix element, $M_q^2 \sim q$, $k = 1$, and the finiteness of the temperature and phonon reabsorption effects are taken into account. In this case, the additional factors $(\varepsilon \pm \varepsilon')^{k+1}$ and terms with N_ε appear in the kinetic equation.

In Refs. 34 and 56, it was shown that in the general case $n(\xi)$ can be represented for $\Delta/\Delta_0 \ll 1$ in the form

$$n(\xi) = n(\varepsilon) + n_1(\xi), \tag{46}$$

where $n(\varepsilon)$ satisfies the kinetic equation (11) with the coherent factors equal to zero ($\eta = 0$), while the correction n_1 is proportional to the coherent factors. It turned out that the function $n(\varepsilon)$ has two important properties:

1. The expansion of $n(\varepsilon)$ and its integrals involves only powers of Δ^2 [as for $n_T(\varepsilon)$].

2. The function $n(\varepsilon)$ for $\varepsilon = 0$ equals $1/2$ and $n(\varepsilon) < n(\varepsilon = 0) = 1/2$. The coherent correction n_1 can be found

with the use of property (45). As shown in Refs. 34 and 56, n_1 can be represented in the form

$$n_1 = -\eta \frac{\Delta^2}{\pi\varepsilon}\varphi_k, \tag{47}$$

$$\varphi_k = \frac{\pi}{a_{k+1}}\int_0^\infty \frac{d\xi}{\varepsilon}\varphi(\xi, 0)\,\varepsilon^{k+1}, \tag{48}$$

$$a_i = \int_0^\infty \xi^i n_0(\xi)\,d\xi,$$

$$\varphi(\xi, \xi') = n(\xi)\,n(\xi') - N_{\varepsilon+\varepsilon'}(1 - n(\xi) - n(\xi')). \tag{49}$$

It is easy to see that for $k = -1, N = 0$ (47) changes into (43). The following properties of n_1 follow from (47)–(49):

1. The sign of the coherent correction n_1 is determined by the sign of $\eta = \pm 1$ in the coherent factors. In superconductors $\eta = 1$, $n_1 < 0$; in an exciton insulator, $\eta = -1, n_1 > 0$.

2. The correction n_1 is proportional to the function $\varphi(\xi, \xi')$ [see (49)], which is a measure of the degree to which the system is in nonequilibrium. The function $\varphi(\xi, \xi')$ represents a recombination collision integral, vanishing at equilibrium. The maximum is attained for $T = 0$, $\gamma = 0$ and for $k = 1$ equals

$$\varphi_m = \frac{\pi a_1}{2a_3} = \frac{0.71}{\Delta_0}. \tag{50}$$

It is evident that with increasing T and film thickness (i.e., γ) and corresponding decrease in β_c, N_ε increases and correspondingly φ decreases, approaching zero while remaining positive. Physically, this is related to the fact that $\varphi(\xi, \xi')$ is proportional to the resulting recombination rate for quasiparticles. For $\beta > 0$, the number of recombining particles is always greater than the number of created particles, due to the reabsorption of nonequilibrium phonons and $\varphi > 0$. Thus, the property of overheating (44) follows from the properties of the functions $n(\varepsilon)$ and n_1.

The properties and form of $n(\xi)$ described above were completely verified with the help of numerical solution of the system of equations (6), (12), and (29) on a computer.[4),57,58,45] In these works, the functions $n(\xi)$ and N_ε were found for a wide range of values of β (and, correspondingly, Δ), T and γ. Analysis of the results shows that $n(\xi)$ is a monotonically decreasing function of ξ ($k = 0, 1$), does not exceed $1/2$, and is localized in the energy range $\xi \sim \Delta_0$. Fig. 1 shows the typical behavior of $n(x = \xi/\Delta_0)$ for different values of β, $T = 0$, $\gamma = 0.15$, and $\omega/\Delta_0 = 10$.

In order to demonstrate the behavior of the distribution function for $\xi = 0$, Fig. 2 shows the values of $n(0)$ as a function of Δ as well as the values of the thermal QDF $n_T(0)$ [taken with the same γ and temperature $T(\Delta)$ according to BCS] for comparison. As is evident from Fig. 2, $n(0)$ for arbitrary γ and T is less than $n_T(0)$ in accordance with the property (44). It is interesting to approximate $n(\xi)$ with the help of the quasiequilibrium QDF $n_F = \{\exp[(\varepsilon - \mu^*)/T^*] + 1\}^{-1}$, treating μ^* and T^* as

[4)] Numerical calculations were also carried out in Ref. 59 for weak pumping and fixed Δ.

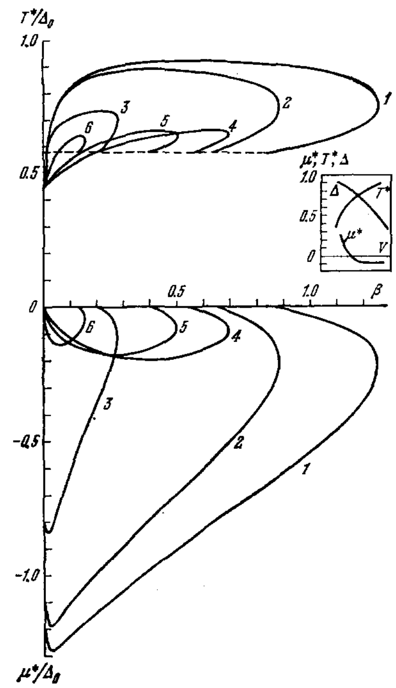

FIG. 3. Effective temperature T^* and the effective chemical potential μ^* as a function of the source power β. Values of the parameters are as follows: 1—$\gamma = 0$, $T = 0$; 2—$\gamma = 0.02$, $T = 0$; 3—$\gamma = 0.15$, $T = 0$; 4—$\gamma = 0$, $T/\Delta_0 = 0.44$; 5—$\gamma = 0.02$, $T/\Delta_0 = 0.44$; 6—$\gamma = 0.15$, $T/\Delta = 0.44$.

adjustable parameters.[5)] Above all, it should be noted that for arbitrary values of μ^* and T^* it is not possible to superpose $n(\xi)$ and $n_F(\xi)$ over the entire energy interval (especially large disagreement for $\gamma \to 0, T \to 0$).

If the approximation is carried out according to the minimum of the deviations $(n - n_F)$ along the entire energy interval, then it is possible to find, to within 5–10%, values of μ^* and T^* for each set of β, γ, and T.[45] The behavior of μ^* and T^* is shown in Fig. 3 as a function of β for different values of T and γ.

As can be seen, both parameters differ from zero (in contrast to the assumptions in Refs. 30 and 31), and they strongly depend on the pump power; the effective chemical potential is always negative. Finally, in the range $\beta_c < \beta < \beta_m$, T^* and μ^* are double-valued functions of β.

It is interesting to note that the effective temperature T^* exceeds T_c (the ratio reaches $T^*/T_c \approx 1.55$), and at the same time, the order parameter differs from zero ($\Delta/\Delta_0 \geq 0.3$).[6)] This means that T^* loses the significance of temperature and clearly indicates (together with $\mu^* < 0$) overheating of quasiparticles.

[5)] The fact that this is no more than an approximation is clearly evident from the following example. For $T = 0$ and $\gamma = 0$, $n(\xi)$ is determined from (12), when not one of the terms S^+, S^-, and S^R vanishes separately (as should occur in equilibrium).

[6)] Overheating of quasiparticles was observed experimentally in Ref. 61 (see below).

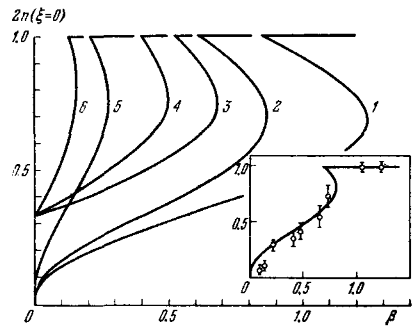

FIG. 4. Coefficient of ultrasonic absorption $\alpha_s/\alpha_N = 2h(0)$ as a function of the optical pumping power β. The values of the parameters γ and T/Δ_0 are as follows: 1—0, 0; 2—0.02, 0; 3—0, 0.44; 4—0.02, 0.44; 5—0.15, 0; 6—0.15, 0.44. The inset shows the experimental results (o) for ultrasonic absorption in a tin film 3,000 Å thickness at $T = 1.33$ K.[60] The solid line corresponds to $2n(0)$ at $\gamma = 1$ and $T = 0$.

4) *Comparison with experiment.* The form of the QDF was determined experimentally in Refs. 24, 60, and 61. Lambert and others[60] determined the value of $n(\xi=0)$ by measuring the ratio of the ultrasonic absorption coefficient in the superconducting α_S and normal α_N states

$$\frac{\alpha_S}{\alpha_N} = 2n(0). \tag{51}$$

Fig. 4 compares the computed values of $2n(0)$ with the experimental values. It is evident that for $\gamma \approx 1$ there is good agreement, and in addition, $n(\xi) < 1/2$ everywhere. Unfortunately, the authors of Ref. 60 do not present data on $2n(0)$ in the interval $\Delta/\Delta_0 < 0.3$, where, apparently, the nonuniform state arises.

Important data on the form of $n(\xi)$ in thin aluminum films with tunnel injection of quasiparticles were obtained in Ref. 61. The authors analyzed their data with the help of a function of the form $n_F(\xi)$. They showed that for a high voltage across the junction (satisfying the criterion for a broad-band source), μ^* and T^* differ from zero and depend on V (see Fig. 3). In addition, the chemical potential is negative, $\mu^* < 0$, while T^* exceeds T_c, so that the ratio T^*/T_c attains 1.5, while $\Delta/\Delta_0 \geq 0.3$. These data agree well with the theoretical results[34,45] (see subsection 3) and indicate overheating of quasiparticles and satisfaction of the property (44).

5) *Order parameter.* The order parameter is sensitive to the energy distribution for quasiparticles, especially for $\xi \approx 0$. Overheating of quasiparticles leads, as was first shown in Ref. 34, to a state with $\Delta \neq 0$ for $\beta > \beta_c$. It is convenient to represent the equation for the gap (6) in the form

$$\Delta\left(\frac{1}{|\lambda|} - \int_0^{\omega_D} \frac{1-2n(\varepsilon)}{\varepsilon}\, d\xi + 2\int_0^{\omega_D} \frac{n_1}{\varepsilon}\, d\xi\right) = 0. \tag{52}$$

Let us first find the equation for the gap with the help of the functions n_0 (34) and n_1 (43) (the $k = -1$ model). Substituting n_0 and n_1 into (52), we obtain

$$\frac{\Delta}{\Delta_0}\left(\ln\frac{\Delta_0}{\Delta} - \frac{2}{\pi}\cdot 0{,}915\right) = \frac{2}{\pi}\,\frac{a - a_c}{a_c}. \tag{53}$$

It is easy to see that for small values of Δ for $\beta > \beta_c^0$,

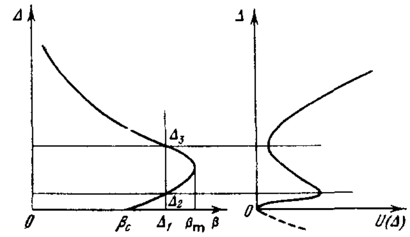

FIG. 5. a) The order parameter Δ as a function of the pump power β; b) energy $U(\Delta)$ of a nonequilibrium superconductor as a function of Δ for pump power $\beta > \beta_c$.

we obtain a solution according to which $\Delta = \Delta_2$ increases with increasing β. For $\Delta > \Delta_m$, we have an additional solution $\Delta_3 = \Delta$, decreasing with increasing β and coinciding with the increasing solution for $\beta = \beta_m$ (Fig. 5a). Thus, there is a region in which $\Delta(\beta)$ takes on three values (together with $\Delta_1 = 0$). It should be noted that for a different sign in the coherent factors the increasing solution is absent, and $\Delta(\beta)$ is a single-valued function.[43] The multivaluedness of $\Delta(\beta)$ is in general retained in a nonequilibrium superconductor. Indeed, using the properties of $n(\varepsilon)$, let us find the contribution from the first two terms in (52) ($k = 1$) for small Δ:

$$\frac{1}{|\lambda|} - \int_0^{\omega_D} \frac{1-2n(\varepsilon)}{\varepsilon}\, d\xi = \delta + \alpha\Delta^2, \tag{54}$$

where $\delta = (a_1 - a_{1c})/a_{1c}$, $a_1 = \int_0^\infty n_0(\xi)\xi d\xi$, and α is a numerical coefficient of the order of unity (see below). Finally, substituting n_1 into (52), we obtain an equation for the order parameter of a nonequilibrium superconductor,[34,56] ($k = 1$):

$$\frac{\partial U}{\partial \Delta} = \Delta(\delta + \alpha\Delta^2 - \Delta\varphi) = 0, \quad U(\Delta) = +\Delta^2\left(\frac{\delta}{2} - \frac{\varphi\Delta}{3} + \frac{\alpha\Delta^2}{4}\right) \tag{55}$$

(here, Δ and φ are written in dimensionless form $\Delta \equiv \varphi_m$, $\varphi \equiv \varphi/\varphi_m$). This result was verified in subsequent works (see, for example, Ref. 44). Eq. (55) differs from the Ginzburg-Landau equation by the presence of a term linear in Δ with the nonequilibrium coefficient φ (and in the form of the coefficients δ and α). The solution of (55) has the form

$$\Delta_1 = 0, \quad \Delta_{2,3} = \frac{\varphi}{2\alpha}\left(1 \pm \sqrt{1 - \frac{4\alpha\delta}{\varphi^2}}\right), \tag{56}$$

i.e., for $\varphi \neq 0$, instead of $\Delta_1 = 0$, there are two solutions: one solution corresponds to Δ_3, decreasing with increasing β, and the second, increasing with β, appears

TABLE II. Values of the parameters β_c, $\bar{\delta}$, and $\bar{\alpha}$.

γ	T/Δ_0	β_c	$\bar{\delta}$	$\bar{\alpha}$
0.0	0	0.84	3.42	6.52
0.02	0	0.61	3.28	6.4
0.15	0	0.21	2.67	5.87
1.00	0	0.037	1.23	3.77
0.0	0.3	0.77	2.99	5.62
0.0	0.44	0.52	2.76	6.21
0.0	0.5	0.31	2.77	8.28
0.0	0.55	0.083	3.58	23.1
0.02	0.44	0.39	2.52	6.15
0.15	0.44	0.13	1.83	5.04

$\bar{\delta} = \frac{\varphi}{c\beta_c}$, $\bar{\alpha} = \frac{\alpha}{\beta_c}$

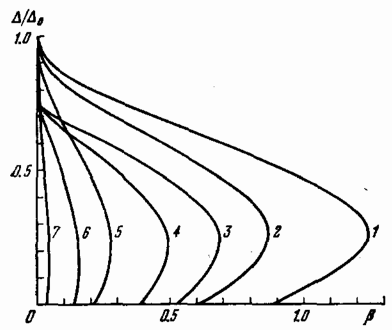

FIG. 6. Order parameter as a function of the source power β. The values of the parameters γ and T are: 1—0, 0; 2—0.02, 0; 3—0, 0.44; 4—0.02, 0.44; 5—0.15, 0; 6—0.15, 0.44; 7—1.0, 0.

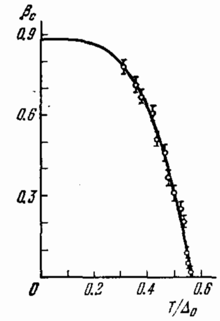

FIG. 7. Temperature dependence of the critical power β_c.

for $\delta > 0$, or $\beta > \beta_c$. Both solutions coalesce for $\delta = \delta_m = \varphi^2/4\alpha$, so that the interval in which $\Delta(\beta)$ is double-valued equals $\varphi^2/4\alpha$. The coefficients $\delta = c(\beta - \beta_c)\beta_c$, α, and φ in general depend on T, γ, and k. They were found in Refs. 45, 57, and 58 and are presented in Table II. In particular, for $\gamma = 0$, $T = 0$, and $k = 1$, it was found that $\varphi = 1$, $\alpha = 1.9$, $c = 0.37$, and $\beta = 0.84$;[57,58] for $\gamma = 0$, $T \to T_c, k = 1$, it was found[34] that

$$\varphi = \beta\zeta_3, \quad \delta = \frac{T - T_c}{T_c} + 2\beta\zeta_2, \quad \zeta_i \sim 1, \quad \alpha = \frac{7\zeta(3)\Delta_0^2}{4\pi^2 T_c^2}.$$

It is useful to examine also the temperature dependence of Δ for $\beta \neq 0$.[34] It is easy to see that for

$$T > T_c(1 - 2\beta\zeta_2)$$

the function $\Delta(T)$ becomes double-valued.

The behavior of the order parameter near $\Delta = 0$ is determined by the derivative

$$\frac{d\Delta}{d\delta} = \frac{1}{\varphi}. \tag{57}$$

According to subsection 3), φ always remains positive, which leads to Δ increasing with β for arbitrarily small values of φ (and n_1). This sensitivity of Δ to n_1 is explained by the fact that n_1 enters into the equation for the gap alongside the small function $1 - 2n(\varepsilon), n(0) = 1/2$. Numerical calculations enabled the authors of Refs. 45, 58 to obtain $\Delta(\beta)$ over a wide range of values of β and γ, T. The functions $\Delta(\beta)$ for $k = 1$, $\omega/\Delta_0 = 10$ are presented in Fig. 6. It is evident that for all values of T, γ there exists a branch of the solution $\Delta(\beta)$ which increases with β and which appears for intensities exceeding a critical value $\beta_c(\gamma, T)$. As T and γ increase, the region in which $\Delta(\beta)$ is double-valued decreases, but the slope $d\Delta/d\beta$ always remains positive. The experimentally observed functions $\Delta(\beta)$, in general, agree well with the theoretical ones, although the observation of $\Delta(\beta)$ in the multivalued region is difficult due to the transition of the superconductor into the nonuniform state. Such measurements were carried out for intense pumping in a series of investigations. For example, the function $\Delta(V)$ in the previously mentioned article[61] (see Fig. 3) agrees well with the theoretical function up to $\Delta/\Delta_0 = 0.3$; for larger values of β (or V), no data are presented, and this, apparently, is related to the appearance of a transition of the first kind or a transition into the nonuniform state. It is possible to extract the function $\Delta(\beta)$, which agrees well with the theoretical one (see Fig. 4), from Ref. 60. The data are presented for values $\Delta/\Delta_0 \leq 0.3$. The results of Ref. 24, where a transition of the first kind was observed for $\Delta/\Delta_0 = 0.6$ disagree somewhat.[7)] These results were discussed critically in Ref. 62.

6) *Critical power*. As shown in Ref. 33 (§ 7), for a certain critical value of the pump power W_c, the order parameter vanishes. The relation between W_c and the dimensionless parameter β_c depends on the absorption mechanism. If the fact that the thickness d usually exceeds the inverse absorption coefficient in superconducting films is taken into account, then the absorption coefficient can be replaced by $1/d$. Keeping this in mind, we obtain for W_c[21,22]

$$W_c = \beta_c(T)\frac{\bar{n}_0\hbar\omega}{4\tau_t r k_1^2}\frac{d}{1+\gamma}, \tag{58}$$

where β_c is given by (17) with $\Delta = 0, \bar{n}_0 = p_0\Delta_0 m/\pi^2\hbar^3$, r is the multiplication factor for quasiparticles (see subsection 3), and k_1^2 is the coefficient of reflection for the electromagnetic field.

The temperature dependence of the critical power is given by the universal function $\beta_c(T)$. For $T = 0$, it reaches a maximum value equal to 0.84.[57] With increasing T, β_c decreases and for $T \to T_c$, it is described by the expression[34] $\beta_c(T) = A(T_c - T)$. The dependence of β_c on T and γ was found over the entire temperature range in Ref. 45 and is presented in Fig. 7. This figure also shows the experimental data obtained in the papers by Golovaskin, Motulevich, and others,[20-22] which agree well with theoretical results.

Substitution of the characteristic values of the parameters into (58) gives values of W_c that coincide with experimental ones (10^3–10^4 W/cm^2 for lead), although the magnitude of the multiplication factor r introduces some uncertainty.

7) *Generation of nonequilibrium phonons*. Nonequilibrium phonons, leaving the film, can be directly observed and are a good indicator of the nonequilibrium state of the system. The energy distribution of phonons was studied theoretically in Refs. 28, 45, 51, and 63. The function N_ϵ for weak pumping was found nu-

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7) In the recent work by Imai,[90] it was shown that if the voltage across the junction is comparatively low, then the order parameter on the upper branch $\Delta(\beta)$ attains a value $\Delta(\beta_c)/\Delta_0 = 0.7$ for $\beta = \beta_c$.

merically in Ref. 28, and it was shown that N_ϵ has a maximum for $\varepsilon \approx 2\Delta$. N_ϵ was evaluated using a computer in Ref. 45 over a wide range of values of β, T, and γ.

Figure 8 shows the typical dependence of the number of phonons $\psi_z = z^2 N_z$ ($z = \varepsilon/\Delta$, z^2 is a quantity that is proportional to the density of phonon states) as a function of z for $T = 0$, $\gamma = 0.15$ The function ψ_z has a maximum approximately for $z = 2$ for any β. A characteristic property is the presence of a discontinuity in $\psi(z)$ for $z = 2$, stemming from switching on of the recombination mechanism for generating phonons (for $z < 2$, phonons are generated only as a result of the energy relaxation of quasiparticles). The maximum of the function $\psi(z)$ increases monotonically with β, in contrast to $n(\xi)$, which depends uniquely on Δ (and not β).

The phonon distribution function has quite a narrow maximum near $z = 2$ with a width ~0.5 and is shown in Fig. 8. There is experimental proof,[64-67] that the phonon distribution differs strongly from the thermal distribution, while the function $\psi(z)$ has a maximum for $z \approx 2$.

8) *Magnetic properties of nonequilibrium superconductors.* The magnetic properties for small **q** are closely related to the number of superconducting electrons N_S.[48] In equilibrium superconductors for $T \to T_c$, $N_S \sim \Delta^2, N_S > 0$,[48] which corresponds to a diamagnetic contribution to the current.

In the nonequilibrium case, the sign of N_S remained controversial for a long time. In the early work,[41,42] wherein the distribution function from the OS model was used,[30] under certain conditions negative values of N_S were obtained.

Calculations with the help of nonequilibrium quasiparticle distribution functions, satisfying the kinetic equation, lead to positive N_S.[34] Indeed, we obtain N_S up to terms of order Δ^2, using the general properties of $n(\varepsilon)$:

$$N_S = 1 + 2\int_0^\infty \frac{dn_0(\xi)}{d\xi}\, d\xi + 2\int_0^\infty \frac{dn_1}{d\varepsilon}\, d\xi = 2\int_0^\infty \frac{dn_1}{d\varepsilon}\, d\xi. \tag{59}$$

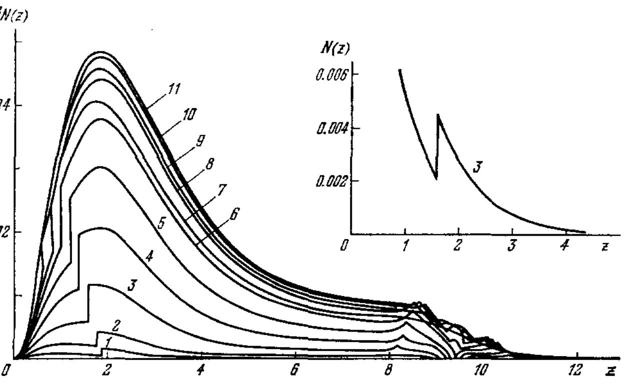

FIG. 8. Number of phonons $z^2N(z)$ as a function of the frequency $z = \omega/\Delta_0$ at $T = 0$ and $\gamma = 0.15$. The values of the pump power β and the order parameter Δ/Δ_0 are: 1—0.0218, 0.95; 3—0.379, 0.8; 5—0.134, 0.6; 7—0.213, 0.01; 8—0.233, 0.4; 11—0.269, 0.2.

In view of the fact that $n_0(0) = 1/2$, the first two terms in (59) are compensated and N_S is determined only by the coherent correction n_1. Substituting n_1 into (59), we finally obtain[34,56]

$$N_S = \Delta\varphi, \tag{60}$$

i.e., N_S is positive and is linear in Δ. Similar results were obtained later in Refs. 44 and 29, as well as in numerical calculations.[45] Thus, superconductors with a broad-band source of quasiparticles remain diamagnets.

We note that the sign of N_S is directly related to the signs in the coherent factors, for example, in the exciton insulator N_S would be negative.[2,34]

The effect of a magnetic field **H** and the current state on the nature of the phase transition in a superconductor with a broad-band source of quasiparticles was studied in Ref. 69. If the film is thin ($d \ll \tilde{\lambda}, \xi_0$; $\tilde{\lambda}$ is the London length) and the magnetic field is parallel to the film, then the function $\Delta(\beta)$ retains its multivalued nature, while the critical value β_c decreases with increasing magnetic field:

$$\beta_c(H) = \beta_c(0) - \frac{1}{24}\left(\frac{H}{H_\lambda}\right)^2\left(\frac{d}{\tilde{\lambda}}\right)^2, \quad H_\lambda^2 = 4\pi N(0)\Delta_0^2\zeta, \tag{61}$$

where H_λ^2 is the "critical" magnetic field, and ζ is of the order of unity. The free energy of a nonequilibrium superconductor in a magnetic field or in a current state can be represented in the form ($T = 0$)[43]

$$\frac{F_S - F_N}{\Delta_0^2 N(0)} = |\Delta|^2\left(\delta - \frac{3}{2}\varphi\Delta + \alpha\Delta^2 + \frac{\overline{v^2}}{2}\right) + \frac{h^2}{8\pi};$$

here, the term $\overline{v}^2/2$ is added to F_S, where **v** is the velocity of pairs in dimensionless units $\sqrt{3\Delta_0^2/mv_0^2}$ and $h^2/8\pi$ is the energy of the magnetic field (h in units of $\sqrt{\Delta_0^2 N(0)/0.71}$). The velocity **v** is related to the superconducting current $\mathbf{J}_S$ and the number of superconducting electrons N_S by the relation

$$\mathbf{J}_S = e\mathbf{v}N_S.$$

Varying (62) and calculating the mean square velocity averaged over the thickness (see Ref. 68), we obtain an expression for Δ (for $d/\tilde{\lambda} \ll 1$):

$$\Delta = \frac{\varphi}{2\alpha}\left(1 \pm \sqrt{1 - \frac{4\alpha\delta(H)}{\varphi^2}}\right), \quad \delta(H) = \delta + \frac{1}{24}\left(\frac{Hd}{H_\lambda\tilde{\lambda}}\right)^2. \tag{62}$$

From here, it is evident that the function $\Delta(\beta)$ has two solution branches, appearing for $\beta > \beta_c(H)$, given by (61).

Similar results are obtained for the current state. The critical current (for $T = 0$) decreases with increasing pumping and vanishes for $\delta = \delta_m = \varphi/4\alpha$.

9) *I–V curves for tunnel junctions.* The change in Δ due to injection of quasiparticles in tunnel junctions with quite small resistance can lead to radical changes in their $I-V$ curves.

The effect of weak injection on the magnitude of the gap was studied theoretically in Refs. 70 and 51. Elesin and Levchenko examined the case of strong injection in a SiS-junction, sufficient to make Δ vanish. For small injection parameters α_V, they found that the necessary critical voltage

$$\frac{V_c}{\Delta_0} = \frac{1}{2\alpha_V} \gg 1 \tag{63}$$

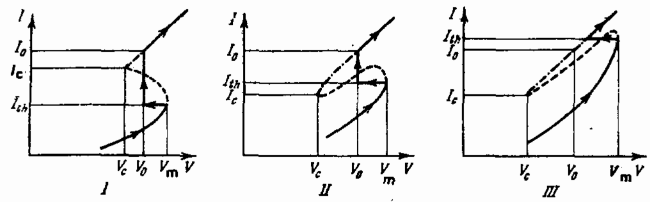

FIG. 9. Three types of I-V curves for superconductor–insulator–superconductor junctions: I, II, III. The dashed line corresponds to the unstable branch.

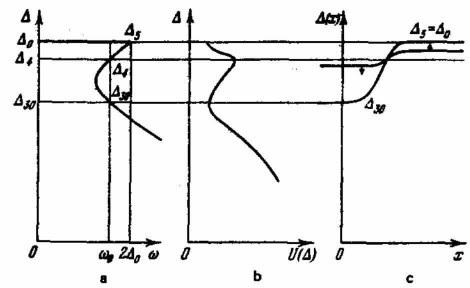

FIG. 10. a) Order parameter Δ as a function of the frequency ω of a narrow-band source of quasiparticles; b) energy of the system $U(\Delta)$ as a function of the order parameter for $\omega=\omega_0$; c) order parameter as a function of the coordinate (layer solution).

is large in comparison with Δ, so that tunnel injection under these conditions is equivalent to the action of a broad-band source. (We note that the magnitude of V_c decreases with increasing temperature.) For this reason, for $V>V_c$, the function $\Delta(V)$ becomes multivalued, i.e., three solutions appear. The multivaluedness of $\Delta(V)$, naturally, leads to multivaluedness of the $I-V$ curves $I(V)$. Depending on the parameters α_V, γ, and T, there can exist three types of $I-V$ curves, illustrated in Fig. 9. They can be classified by the relations between the characteristic values of the currents I_c, I_0, and I_{th} (Ref. 46):

$$I_c=I(V_c,\ \Delta=0),\quad I_0=I(V_0,\ \Delta=0),\quad I_{th}=I(V_m,\ \Delta_m), \tag{64}$$

where V_0 is the voltage at which the energies of the normal and superconducting phases are equals. $I-V$ curves of the first type satisfy the condition $I_0>I_c>I_{th}$, those of the second type satisfy $I_0>I_{th}>I_c$, while those of the third type satisfy $I_{th}>I_0>I_c$. As shown in Ref. 46 (see below, subsection 5, section 1), $I-V$ curves of the I- and II-type lead to a transition into the nonuniform state (with $\Delta\neq 0$ and $\Delta=0$), while $I-V$ curves of the III-type lead to a transition of the first kind into the normal state. The conditions for realizing a given type were obtained in Ref. 46. We note that an increase in T and γ can cause a transition from type III into type II.

c) Uniform states of superconductors with a narrow-band quasiparticle source

1) *Concentration of quasiparticles and order parameter near the injection threshold* A narrow-band source quasiparticles in a narrow energy interval $\omega-2\Delta$, so that the quasiparticles appear localized near $\varepsilon\sim\Delta$. The localized nature of $n(\xi)$ permits finding from (6) a universal relation between Δ and $\bar{n}$, the concentration of quasiparticles (see, for example, Ref. 56):

$$\Delta=\Delta_0(1-2\bar{n}). \tag{65}$$

In order to obtain a closed system for Δ and $\bar{n}$ for small α_ω, it is enough to make use of the normalization condition (18). Taking into account the localized nature of $n(\xi)$, we obtain the relation[33] (for $T=0$)

$$\bar{n}^2=\alpha_\omega\frac{\omega-2\Delta}{2\Delta_0}\,\theta(\omega-2\Delta), \tag{66}$$

expressing the equality of the recombination rate of quasiparticles (left side) and the rate at which quasiparticles are created by the electromagnetic field [in view of the smallness of $\alpha_\omega n(\xi)$ in Q they were set equal to zero]. It is important to note that the creation rate is proportional to the interval $\omega-2\Delta$ and strongly depends on Δ, i.e., in the case of a narrow-band source, the order parameter determines the rate at which quasiparticles are created. This circumstance leads, as shown in Ref. 33, to the characteristic threshold instability (see subsection 2, section 2) and the multivalued dependence of Δ on ω.[37,38] Indeed, eliminating Δ from (65) and (66) we obtain an equation for $\bar{n}$[8)]

$$\bar{n}^2=\alpha_\omega(\delta+2\bar{n})\,\theta(\delta+2\bar{n}),\quad \delta=\frac{\omega-2\Delta_0}{2\Delta_0},$$

the solution of which has the form[37]

$$\left.\begin{array}{c}\bar{n}_1=0,\quad \delta<-\alpha_\omega,\\ \bar{n}_1=0,\ \ \bar{n}_{2,3}=\alpha_\omega\left(1\mp\sqrt{1+\frac{\delta}{\alpha_\omega}}\right),\quad -\alpha_\omega<\delta<0,\\ \bar{n}=\alpha_\omega\left(1+\sqrt{1+\frac{\delta}{\alpha_\omega}}\right),\quad \delta>0.\end{array}\right\} \tag{67}$$

It follows from (67) that in the region $-\alpha_\omega<\delta<0$, there are three solutions for $\bar{n}$ and correspondingly for Δ $[\Delta_S=\Delta_0,\ \Delta_{4,3}=\Delta_0(1-2\bar{n}_{2,3})]$ (Fig. 10). We note that the multivalued nature of the function $\Delta(\omega)$ arises for any α_ω (there is no intensity threshold), but the width of the region of multivaluedness is proportional to α_ω.

The existence of three solutions is related to the effect of Δ on the quasiparticle creation rate. The solution $\bar{n}_1=0$ corresponds to the case when $\Delta=\Delta_0$ and the source is switched off ($\omega<2\Delta$). If $\bar{n}\neq 0$, (Δ_6, Δ_5), then Δ decreases and the source creates quasiparticles and in addition, solutions $\bar{n}_3$ that increase as well as decrease with ω are possible.

Qualitatively similar results are also obtained for tunnel injection ($\omega-V$). Only now, the solution $\bar{n}_3=\sqrt{\pi\alpha_V/4}$ is constant, while $\bar{n}_2=(2\Delta_0-V)/4\Delta_0$ depends "universally" on V (and does not depend on α_V)(Fig. 11), This difference is related to the absence of coherence factors in the source for tunnel injection. As is well-known, the latter circumstance leads in the nonequilibrium theory, which does not take into account the effect of injected quasiparticles, to a discontinuity in the current at $V=2\Delta$. As shown in Ref. 37, in describing the junction, the effect of the injection of quasiparticles cannot be neglected even for small values of α_V, and

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8) We assume that the use of the same letter δ for the difference of the source powers $c(\beta-\beta_c)$ and the frequency of the source $(\omega-2_0)$ will not lead to any misunderstandings.

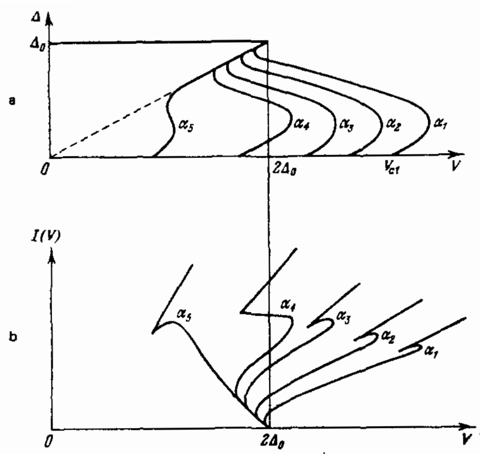

FIG. 11. a) Order parameter Δ as a function of the voltage across the junction for different values of the injection parameters α_i $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5$; b) voltage dependence of the tunnel junction current $I(V)$ for different α_i.

therefore, $n(\xi)$ in the source cannot be neglected. Taking injection into account makes the switching on of the source smoother for $V \geqslant 2\Delta$ (the problem of smoothing the $I-V$ curves for tunnel junctions due to the nonequilibrium situation was qualitatively discussed in Ref. 72). We emphasize that the solution $\bar{n}_2$ appears due to the smooth manner in which the source is switched on. In the opposite case, strictly speaking, this solution would not have existed. Indeed, in the recent work by Shon and Tremblay,[38] in which the authors independently arrived at a qualitatively similar conclusion concerning the existence of three solutions for $\Delta(V)$, it was also proposed that the discontinuity of the source is smeared out by some interaction [they solved graphically the equation for Δ for $T \to T_c$ and do not take into account the finiteness of $n(\xi)$ in Q].

The case of finite temperatures was examined in Ref. 46 (analytically for $T \to T_c$ and numerically for finite T). The effect of stimulating Δ due to the redistribution of the energy of quasiparticles by the field (the terms $Q_V^{(2)}$ and $Q_V^{(3)}$, the Eliashberg mechanism[5]) was also taken into account in this case.

Let us present the final results for Δ in the range of voltages $-\tilde{\alpha}_V < \delta(T) < 0$ (Ref. 46):

$$\Delta_5 = \tilde{\Delta}(T), \quad \Delta_4 = \frac{V}{2}, \quad \Delta_3 = \Delta_0(T),$$
$$\tilde{\Delta}(T) = \Delta_0(T) + \tilde{\alpha}_V, \quad \delta(T) = \frac{V - 2\tilde{\Delta}(T)}{2\Delta_0(T)}, \tag{68}$$

where $\Delta_0(T)$ is the gap of an equilibrium superconductor at temperature T, $\tilde{\Delta}(T)$ is the gap taking into account the stimularion effect, and $\tilde{\alpha}_V$ is the renormalized injection parameter.

2) *Distribution functions for quasiparticles in a superconductor with a narrow-band source.* We will restrict ourselves to the case $\gamma = 0$. The kinetic equation for a narrow-band source can be greatly simplified if we take into account the fact that relaxation processes are strongly suppressed in comparison with recombination processes due to the factors $(\varepsilon - \varepsilon')^2$ and the coherent factors $1 - (\Delta^2/\varepsilon\varepsilon')$. For $T = 0$, when it is possible to neglect the terms $Q^{(2,3)}$, describing the redistribution of quasiparticles with respect to energy, in the source, we arrive at the equation[33,36]

$$n_z \bar{n} = \alpha_\omega (\tilde{\delta})^{k_\omega/2} \frac{1 - n_z - n_{1-z}}{\sqrt{1-z}} \theta(z) \theta(1-z), \tag{69}$$

where

$$\tilde{\delta} = \frac{\omega - 2\Delta}{2\Delta}, \qquad z = \frac{\varepsilon - \Delta}{\Delta\tilde{\delta}}, \qquad n_z \equiv n(z).$$

For tunnel injection, α_ω must be replaced by α_V, $\sqrt{\bar{\delta}}$ by $\bar{\delta}^{-1/2}$ due to the absence of the coherent factor (see section 3).

An exact solution of (69) was found by Aronov and Spivak[36]:

$$n_z = A \frac{\sqrt{z}\, \theta(z) \theta(1-z)}{\sqrt{z(1-z)} + A(\sqrt{z} + \sqrt{1-z})}, \tag{70}$$

with the coefficient A satisfying the following normalization condition

$$A_{\omega, V}^2 \int_0^1 \frac{dz}{\sqrt{z(1-z)} + A_{\omega, V}(\sqrt{z} + \sqrt{1-z})} = \frac{\alpha_{\omega, V}}{4} (\tilde{\delta})^{\frac{k_{\omega, V}}{2}}.$$

It is easy to see that $n(z)$ differs from zero in the interval $0 < z < 1$ and has the values $n(0) = 0$ and $n(1) = 1$, i.e. in the case of a narrow-band source the distribution function differs sharply from the equilibrium function, has a positive derivative, and can attain the value unity. The coefficients A_ω and A_V are obtained from the normalization condition and turn out to differ considerably. Thus, for small α_ω, $A_\omega \sim \sqrt{\alpha_\omega}$ is small, while A_V for $\bar{\delta} \to 0$ can be large. For this reason, the QDF for electromagnetic pumping differs from zero in a narrow interval near $z = 1$, while for tunnel injection n_z is of the order of unity in almost the entire interval $0 < z < 1$ (Ref. 37), and it is this fact that eliminates the discontinuity in switching on the source.

An exact solution has also been found for n_z with $T \to T_c$ and it can be represented in the form

$$n = n_T + n_2 + \hat{n},$$

where n_2 a smooth function, stemming from the action of $Q^{(2)}$, was found in Ref. 78, while the narrow part of n is given by the expression[46]

$$\hat{n} = A \frac{\psi_T(z) \sqrt{z}\, \theta(z) \theta(1-z)}{\sqrt{z(1-z)} + A(\sqrt{z} + \sqrt{1-z})},$$
$$\psi_T(z) = 1 - n_T(z) - n_T(1-z). \tag{71}$$

The correction term $\hat{n}$ has the properties of (70).

3) *Threshold electromagnetic absorption and $I-V$ curves for tunnel junctions at threshold voltage.* Electromagnetic absorption for $\omega = 2\Delta$ is controlled by the order parameter, which takes on three values for a given value of ω. For this reason, the frequency dependence of the absorbed power Q_ω will be S-shaped:[37]

$$Q_\omega = \begin{cases} 0, & \delta < -\alpha_\omega, \\ 2\alpha_\omega(\delta + 2\alpha_\omega \mp \sqrt{\alpha_\omega^2 + \delta\alpha_\omega}), & -\alpha_\omega < \delta < 0. \end{cases}$$

An estimate shows that this effect becomes noticeable for powers of the order of 1–10 W/cm². The multivaluedness of $\Delta(V)$ for $V \approx 2\Delta$ must lead to multivaluedness of the function $I(V)$. Indeed, in Refs. 37 and 38, it was shown that the $I-V$ curve for a SiS-junction is S-shaped near $V = 2\Delta$ (see Fig. 10b). The current $I(V)$ can be represented by two terms, I_1 and I_2 [see (25)], of which I_1 corresponds to a current involving breaking up of Cooper pairs, while I_2 corresponds to a current of available quasiparticles. The negative slope of the decreasing branch of the $I-V$ curve is due to the current I_1, which has the following simple form for $T \to T_c$ (Ref. 46):

$$I_1 = \alpha_V \frac{7\zeta(3)}{8\pi^2} \frac{\Delta_0^2}{T_0^2} \bar{n}_T \begin{cases} 0, & \delta(T) < 0, \\ (2\tilde{\Delta}(T) - V)/2\Delta_0, & -\tilde{\alpha}_V < \delta(T) < 0, \\ \tilde{\alpha}_V, & -\tilde{\alpha}_V < \delta(T). \end{cases} \tag{72}$$

The current I_2 is small for $T \to 0$, but attains a significant magnitude for $T \to T_c$ (Ref. 46):

$$I_2(V, \Delta) = 2\alpha_V \int_0^\infty d\varepsilon \rho(\varepsilon) \rho(\varepsilon + V) [n_T(\varepsilon) - n_T(\varepsilon + V)], \tag{73}$$

and creates a "pedestal" for the total current. It is I_2 that makes the largest contribution to the observed threshold current, which can be determined as $I_2[V = 2\tilde{\Delta}(T),\ \Delta = \tilde{\Delta}]$. I_2 increases with increasing temperature, attaining a maximum value near T_c, $I_2 = \alpha_V \tilde{\Delta}(T)$, and then decreases due to decreasing $\tilde{\Delta}(T)$. This temperature behavior of $I_2 \equiv I_{th}$ agrees with that observed experimentally.[25]

Fig. 11b shows the $I-V$ curve over the entire range of voltages, obtained with the help of analytical[37,47] and numerical[46] calculations. The second segment of the S-shaped $I-V$ curve stems from coherent mechanisms (see subdivision 9, subsection b, in section 1). Qualitative considerations concerning the $I-V$ curve with a negative slope for $V = 2\Delta$ were used in Ref. 26 and in Ref. 74, an S-shaped $I-V$ curve was observed in short tin junctions. Usually, it is difficult to observe an S-shaped $I-V$ curve due to the transition of the superconductor into the nonuniform state.

d) Instabilities in superconductors with excess quasiparticles

1) *Types of instabilities.* Research on instabilities has shown that three types of instabilities are characteristic of nonequilibrium superconductors: the so-called threshold, coherent, and diffusion instabilities. The threshold and coherent instabilities are realized in superconductors and explain available experimental data. The conditions under which the diffusion instability can arise apparently are not satisfied in superconductors with electromagnetic and tunnel injection.[9)]

2) *Threshold instability.* In Ref. 33, in 1974, an instability was predicted in a superconductor with a narrow-band source of quasiparticles, leading to the transition out of the state with $\Delta = \Delta_0$ into the state with $\Delta < \Delta_0$. The instability arises for a frequency ω close to the threshold for pair creation in the following manner. Assume that initially $\Delta = \Delta_0$, $\bar{n} = 0$, and $\omega \leq 2\Delta_0$. Then, fluctuations in $\bar{n}$ lead to a decrease in Δ [see (65)] and, therefore, to an increase in the width of the source action, proportional to $\omega - 2\Delta$ [see (66)], which in its turn increases $\bar{n}$, and so on. A similar instability can arise in superconductors with tunnel injection[37,38] for $V \leq 2\Delta$, which was discovered experimentally in Refs. 25 and 26.

The theory of threshold instability in superconductors with electromagnetic and tunnel pumping was examined in Refs. 37, 38, and 46. We will restrict ourselves to the case $T = 0, L \gg \xi_0$ and we will follow Ref. 37. The basic equations for the concentration of quasiparticles

$$\frac{\partial \bar{n}}{\partial t} - L^2 \frac{\partial^2 \bar{n}}{\partial \mathbf{r}^2} = \frac{\partial U(\bar{n}, \delta)}{\partial \bar{n}}, \tag{74}$$

$$U_{\omega, V} = -\int^{\bar{n}} d\bar{n} (2\bar{n}'^2 - \bar{Q}_{\omega, V}), \tag{75}$$

can be obtained as follows. Taking into account Eq. (17), the localized nature of $n(\xi)$ as well as the properties of the distribution functions of a narrow-band source $n(\varepsilon = \Delta) = 0$ $n(\varepsilon = \omega - \Delta) = 1$ [it can be shown that these properties are retained in the nonuniform state, if the kinetic equation is written in the form $\varepsilon(r) = \text{const}$], we arrive at equation (74).

Let us examine the stability of the system for fixed ω (or V) relative to small perturbations

$$\bar{n}(r, t) = \bar{n} + \bar{n}' \exp(\tilde{\gamma} t + i\mathbf{q}\mathbf{r}). \tag{76}$$

Substituting $\bar{n}(\mathbf{r}, t)$ with $\mathbf{q} = 0$ into (74), we obtain the decrement $\tilde{\gamma}$ (in dimensionless form):

$$\tilde{\gamma} = +\frac{\partial^2 U(\bar{n}, \delta)}{\partial \bar{n}^2}, \tag{77}$$

which, as could be expected, is related to the second derivative of the energy with respect to $\bar{n}$. With the help of expression for U (75) and the equation $\partial U/\partial \bar{n} = 0$, it is convenient to represent $\tilde{\gamma}$ in the form

$$\tilde{\gamma} = -\frac{2\alpha_\omega}{\bar{n}_\delta}, \quad \bar{n}_\delta = \frac{\partial \bar{n}}{\partial \delta}. \tag{78}$$

From here, it is evident that the decreasing branch of of $\bar{n}_2$ is unstable, while the $\bar{n}_1 = 0$ and $n = n_3$ branches are stable. Starting from the general properties of the equation $\partial \bar{n}/\partial t = \partial U/\partial \bar{n}$, it can be shown[75] that if $\bar{n} > n_2(\delta)$, then the solution $\bar{n}(t)$ changes into $\bar{n}_3$, and into $\bar{n} = 0$ for $\bar{n} < \bar{n}_2(\delta)$. This corresponds to the qualitative picture of the threshold instability described at the beginning of this section.

It is useful to analyze the relation of the stability of the system to properties of the energy U, keeping in mind (76). It is easy to verify that U has three extrema $\bar{n}_1, \bar{n}_2, \bar{n}_3$ $(\Delta_5, \Delta_4, \Delta_3)$ (corresponding to stationary and uniform solutions), and in addition $\bar{n}_1$ and $\bar{n}_3$ correspond to minima and $\bar{n}_2$ to a maximum (see Fig. 10). The unstable region obtained above corresponds to the region near the maximum of U. It is evident that different energies can correspond to the stable solutions. For this reason, one of the states (Δ_5 or Δ_3, depending on δ) will be metastable, while the other is actually stable. For some $\delta = \delta_0$, the energy of the states $U(\Delta_5) = U(\Delta_3)$ become equal. As shown in Ref. 44, the time for a transition out of the metastable state into the stable state for $\delta \neq \delta_0$ is very long in the absence of large fluctuations and nucleation centers for the stable phase. For this reason, if δ is increased, then the state with Δ_5 remains up to $\delta = 0$ as a metastable state, and then

[9)] We are not familiar with any work in which the possibility of realizing the diffusion instability in superconductors was proved.

makes a transition of the first kind into the state with Δ_3. When δ decreases, beginning with $\delta = 0$, the state with Δ_3 persists up to $\delta = -\alpha_\omega$, after which it jumps into $\Delta = \Delta_5$, i.e., there is hysteresis. In the presence of a priming region for another phase or large fluctuations, a transition of the first kind occurs at $\delta = \delta_0$ (if we neglect the time for filling up the specimen with the new phase, which will be discussed in what follows).

Let us consider nonuniform excitations with $\mathbf{q} \neq 0$. In this case the decrement[36]

$$\tilde{\gamma} = -\frac{2d_\omega}{\bar{n}_\delta} - \mathbf{q}^2 L^2 \tag{79}$$

decreases with increasing $\mathbf{q}^2$, i.e., diffusion of quasiparticles stabilizes the perturbation.

Threshold instability in superconductors with tunnel injection. An important difference relative to magnetic pumping is the possibility of fixing the current[47] (some difference between U_V and U_ω does not lead to any qualitative changes). In a regime with a given current, Eq. (24) must be added and it is necessary to take into account changes in voltage

$$V(t) = V + \tilde{V} \exp(\tilde{\gamma} t).$$

In this case, the decrement $\tilde{\gamma}$ has the form[37]

$$\tilde{\gamma} = -\alpha_V (\bar{n}_\delta)^{-1} \cdot \frac{1 + R\,(\mathrm{d}I/\mathrm{d}V)}{1 + R I_V}, \qquad I_V = \left(\frac{\partial I}{\partial V}\right)_{\bar{n}}, \quad I_{\bar{n}} = \left(\frac{\partial I}{\partial \bar{n}}\right)_V, \quad \frac{\mathrm{d}I}{\mathrm{d}V} = I_V + \bar{n}_\delta I_{\bar{n}}. \tag{80}$$

If $R > |dI/dV|^{-1}$, then the uniform solution $\bar{n}_2$ becomes stable, since $I_V > 0$, $dI/dV < 0$, and $\tilde{\gamma} < 0$. However, the state $\bar{n}_2$ on the decreasing branch $dI/dV < 0$ is unstable relative to nonuniform perturbations that do not change the total current $\bar{I}$. In this case, the decrement becomes positive for sufficiently small $\mathbf{q}$ [see (79)] and nonuniform excitations begin to develop. A clear picture of the process of the development of the instability can be obtained with the help of Fig. 9. Assume that a nonuniform fluctuation $\Delta(x)$ arises in a superconductor in the state with $\Delta = \Delta_4(\bar{n}_2)$ (see Fig. 10c). Due to the threshold instability (see subsection 2), in the state with $\Delta(x) < \Delta_4$, the order parameter will decrease to Δ_3, and in the state with $\Delta > \Delta_4$ it will increase to Δ_5, i.e. stratification will occur into regions with different values of Δ (Δ_3 and Δ_5). At the same time, the total current will not change. This phenomenon of the specimen separating into layers or filaments in many ways is similar to the phenomenon of filamentation in semiconductors with S-shaped $I-V$ curves.[77]

3) *Coherent instability.* The instability specific to a superconductor with a broad-band quasiparticle source was predicted in Ref. 43 and named the coherent instability, since it is related to the coherence of the interaction between quasiparticles and phonons. The instability arises in the state with $\Delta = \Delta_2$, corresponding to the increasing solution (see subdivision 5, subsection b of section 1). The coherent instability was investigated in Refs. 43 and 44 and, apparently, is the reason for the transition into the nonuniform state of superconductors with optical and tunnel (for high V) pumping.

3.1) *Kinetic approach. Optical pumping.* In order to describe coherent instabilities, we will investigate the equation for $n(\xi, \mathbf{r}, t)$ (and not $\bar{n}$), since the number of quasiparticles changes in a narrow energy range $\xi \sim \Delta \ll \Delta_0$. If the perturbed values of n and Δ

$$n(\xi, \mathbf{r}, t) = n(\xi) + \tilde{n}(\xi) \exp(\tilde{\gamma} t + i\mathbf{q}\mathbf{r}), \qquad \Delta(\mathbf{r}, t) = \Delta + \tilde{\Delta} \exp(\tilde{\gamma} t + i\mathbf{q}\mathbf{r}) \tag{81}$$

are substituted into the equation for the gap (7) and the kinetic equation (11) and the result is solved for $\tilde{n}$, then, using this solution, it is possible to obtain the following expression for $\tilde{\gamma}$ (Ref. 43) [$n(\xi)$ and Δ are the stationary uniform solutions studied in section 2]:

$$\tilde{\gamma} = -\frac{J_0 + \mathbf{q}^2 L^2 N_S + \mathbf{q}^2 \xi_0^2 (\Gamma^2 + \mathbf{q}^2 L^2)}{D + \mathbf{q}^2 \xi_0^2}, \tag{82}$$

where N_S is given by (4), J_0 and Γ are functionals of $n(\xi)$ (see Ref. 43),

$$D = \Delta^2 \int_0^\infty \frac{1 - 2n}{\varepsilon^3}\, \mathrm{d}\xi. \tag{83}$$

Let us first examine uniform perturbations with $\mathbf{q} = 0$. Calculation of J_0 and D with the functions $n(\xi)$ found earlier (subdivisions 2 and 3 of subsection b in section 1), permits representing $\tilde{\gamma}$ in the form

$$\tilde{\gamma} = (\Delta_\delta)^{-1}. \tag{84}$$

It is evident from (84) that the increasing solution Δ_2 is unstable, while Δ_3 is stable (stability of the solution $\Delta_1 = 0$ is discussed below). It is easy to show that for $\Delta(t=0) < \Delta_2(\delta)$ the system goes over into the normal state, while for $\Delta(t=0) > \Delta_2(\delta)$ it goes over into the superconducting state.

The physical significance of the instability consists of the following. Let the gap in the state on the increasing branch with Δ_2 decrease somewhat due to fluctuations. Then, the probability for recombination of quasiparticles $\sim 1 + (\Delta^2/\varepsilon\varepsilon')$ will decrease and, therefore, the number of quasiparticles (near $\xi \sim \Delta$) will increase. As a result of the equation for Δ, this leads to the subsequent decrease in the gap and so on. It is interesting to note that with the transition into the normal state (for fixed β) an additional number of quasiparticles δn is liberated due to the decrease in the recombination rate. It is useful to relate the criteria for stability to the properties of $U(\Delta)$ (55), having obtained from (84) and the equation $\partial U/\partial \Delta = 0$ the relation

$$\tilde{\gamma} = -\Delta \left(\frac{\partial^2 U}{\partial \Delta^2}\right)^{-1}. \tag{85}$$

The energy $U(\Delta)$ has a structure similar to $U(\bar{n})$ (see Fig. 5b), i.e., there exist three extrema Δ_1, Δ_2, and Δ_3; Δ_1 and Δ_3 correspond to a minimum in $U(\Delta)$, while Δ_2 correspond to a maximum. For $\delta = \delta_0$, the energies are equal: $U(\Delta_{10}) = U(\Delta_{30})$. It can be shown (in analogy to subdivision 2 of subsection d of section 1) that depending on the conditions, the system can undergo a transition of the first kind with $\delta = \delta_0$ or with $\delta = \delta_m$, $\delta = 0$, accompanied by hysteresis.

Let us now examine the nonuniform perturbations with $\mathbf{q} \neq 0$. In the limit $qL \gg 1 (uL \gg \xi_0)$ (82) changes into the expression[43]

$$\tilde{\gamma} = -\mathbf{q}^2 L^2 \frac{N_S}{D}, \tag{86}$$

which coincides (to within D) with the expressions obtained in Refs. 34, 39–42, concerned with studying the stability of relatively fast perturbations (for times less than the relaxation times).

Thus, the sign of N_s and D determines the stability of the system relative to fast perturbations and, in the general case, determines the sign of the contribution to $\tilde{\gamma}$ in front of the term q^2L^2. The quantity N_s was discussed in section 2, and it was shown that N_s remains a positive quantity up to $\Delta=0$, where $N_s=0$.

Thus, diffusion of quasiparticles leads to stabilization of the instability.[43] A similar conclusion was arrived at in a recent publication[44] with the help of a somewhat different method.

The coherent instability can be realized in superconductors with tunnel injection for $V\gg\Delta$.[47] Similarly to the case of the threshold instability, it can be shown that for a given current, stratification occurs into regions with normal ($\Delta_1=0$) and superconducting phases (Δ_3). The picture of the development of stratification is similar to that examined in subdivision 2 of subsection d.

Important results on the stability of superconductors, part of which have already been mentioned, were obtained by Eckern, Shmid, Smutz, and Schön.[44] The authors, using the general properties of $n(\xi)$, established in Ref. 34 (see subdivisions 2 and 3, subsection b of section 1), formulated a model (EMS) that is very convenient for describing nonstationary and nonuniform processes for $T\to T_c$. They propose writing the collision integral for quasiparticles and phonons in the form

$$\left(\frac{\partial n}{\partial t}\right)_t=\frac{1}{\tau_I}\left[n(\xi)-n_T-\frac{\Delta^2B}{\pi\epsilon T_c}\right],\quad B\sim\beta, \tag{87}$$

separating out the contribution of n_1 from the coherent factors, proportional to β. Using these equations, they obtained a closed equation for Δ for $T\to T_c$.

In the same way it is possible to obtain a closed equation for Δ over a wider temperature range.

3.2) *Closed equation for* Δ. If the kinetic equation (11) is solved for the spatially nonuniform case (see below), while $n(\xi, r)$ is substituted into the equation for the gap (7), then it can be represented in the following form:

$$\hat{\Pi}(\xi_0, L)\,\Delta=+\frac{\partial U(\Delta,\delta)}{\partial\Delta}, \tag{88}$$

where $\hat{\pi}(\xi_0, L)$ is a differential operator depending on ξ_0 and L (see below) and $U(\Delta,\delta)$ is the energy of the system. The time derivatives of Δ (necessary for studying stability) can be of two types. First of all, the term $(1/\tau_\Delta)\partial\Delta/\partial t$ that takes into account the change in Δ with a relaxation time of the parameter of the order of τ_Δ.[44] This term is important only for analyzing the stability of the normal state. Second, there is the derivative taking into account the change in Δ due to changes in the quasiparticle distribution function. In the general case, the corresponding term is nonlinear with respect to $\partial/\partial t$. However, in order to investigate instability, we are interested in small $\tilde{\gamma}$ (near the stability boundary), i.e., only in terms that are linear in $\partial/\partial t$. In this approximation, from (82) and (88), we obtain an equation for Δ

$$-\left(\frac{1}{\tau_\Delta}+D\right)\frac{\partial\Delta}{\partial t}+\hat{\Pi}\Delta=\frac{\partial U}{\partial\Delta}. \tag{89}$$

For $\gamma=0, T=0, D=\pi\Delta a_1/a_2$, Eq. (89) coincides with Ref. 46, and for $\gamma=0, T\to T_c, D=\pi\Delta/4T_c$ it coincides with the equation in Ref. 44.

With the help of (89), it is possible to obtain a criterion for stability that coincides with that found in subdivision 3 of this section, as well as to investigate the stability of the normal state. Substituting $\Delta(r,t)=\tilde{\Delta}\exp(\tilde{\gamma}t+i\mathbf{q}\cdot\mathbf{r})$ into (89) and setting $\Delta=0$, $\xi_0\ll L$, we find

$$\tilde{\gamma}=-\delta-q^2\xi_0^2, \tag{90}$$

i.e., for $\delta>0$ the normal state $\Delta_1=0$ is stable. Diffusion of the order parameter (second term) leads to additional stabilization. This result was obtained in Ref. 78, where the dependence of the critical power β_c on q^2 was investigated at the stability threshold ($\tilde{\gamma}=0$). For small values of $q\xi_0$, we obtain from (90) in accordance with Ref. 78 a decrease in $\beta_c(\mathbf{q})$

$$2\sqrt{\beta_c(\mathbf{q})}=1-\frac{1}{3}\left(\frac{qv_0}{\Delta_0}\right)^2,$$

which indicates stability relative to nonuniform perturbations. The decrease in $\beta_c(\mathbf{q})$ as a function of $\mathbf{q}$ was proved in Ref. 78 for arbitrary $\xi_0^2q^2$. We note that the situation being examined differs from the situation involving the formation of nonuniform states in equilibrium superconductors with magnetic impurities.[82] Indeed, the value of the effective magnetic field (analog of β_c), for which Δ vanishes, increases with increasing $\mathbf{q}$. With the help of (89), it is possible to study the stability of nonuniform solutions

$$\Delta(\mathbf{r},t)=\Delta(\mathbf{r})+\tilde{\Delta}(\mathbf{r})\exp(\tilde{\gamma}t),\quad \tilde{\delta}(t)=\delta+\delta' e^{\tilde{\gamma}t}. \tag{91}$$

In this case for $\tilde{\Delta}(r)$ we obtain the equation

$$\left(\hat{\Pi}-\frac{\partial^2U}{\partial\Delta^2}\right)\tilde{\Delta}(\mathbf{r})-\frac{\partial^2U}{\partial\Delta\,\partial\delta}\hat{\delta}'=-\tilde{\gamma}D, \tag{92}$$

of which the decrement $\tilde{\gamma}$ is an eigenvalue.

4) *Diffusion instability.* The diffusion instability (DI), based on the possible anamalous diffusion of quasiparticles, was predicted in Refs. 41 and 42. It is assumed that the diffusion of quasiparticles occurs from a region with a large value of the order parameter Δ (and therefore, a small concentration of quasiparticles $\bar{n}$) into a region with small values Δ (and large $\bar{n}$).

The instability develops in the following manner. Let Δ decrease in some region of space. Then, under the action of a gradient in Δ, there arises a flux of quasiparticles further decreasing Δ and so on. The conditions for the appearance of the diffusion instability are determined by the sign of $N_s(4)$ which is a functional of $n(\xi)$. As already noted, the functions in the OS model satisfying the condition $n(0)>n_T(0)$ (overcooled) lead to $N_s<0$ and satisfaction of the conditions for the appearance of DI. However, in superconductors, with optical and tunnel pumping, the quasiparticle distribution is overheated and $N_s>0$. Generally speaking, in nonequilibrium systems with an order parameter the sign of N_s is closely related to the sign of the coherent factors[56,79] ($\eta=\pm1$). For this reason, in systems in which the sign of η is opposite to that of a superconductor, N_s can become negative and DI can arise (according to Ref. 44, the NEMS model). An example is the exciton insulator,[80] in which the signs of the coherent factors are opposite to the signs in a superconductor.[2] The possibility for the realization of DI in an exciton insulator was pointed out in Ref. 79, wherein the conditions for the appearance of the instability and the values of the wave vector with which spatial changes in $\Delta(r)$ are modulated, were found.

The possibility of $N_s<0$ in the case of phonon pumping were mentioned in Ref. 44. DI was also studied in Ref. 81. Smith[81] obtained criteria for DI in the form $\partial n/\partial\Delta>0$. It is easy to see that Smith's criterion agrees with the analysis performed above, since for a superconductor $\partial n/\partial\Delta<0$ (see Ref. 47), while for an exciton insulator $\partial n/\partial\Delta>0$.

e) Nonuniform states in superconductors with excess quasiparticles

The coherent, threshold, and diffusion instabilities can lead to a transition into a nonuniform state. These states must be described by nonlinear stationary equations: for a broad-band source by (7), (11), and (89), and for a narrow-band source, by Eq. (74). As shown in Refs. 43 and 44, the nonlinear equations (89) and (74) admit different types of solutions for $\Delta(r)$: Oscillatory, soliton, monotonic, and a special case of monotonic, the stratified solution. In order to choose a solution among the possible stable nonuniform solutions, it is necessary to investigate their stability using Eq. (92). Equations of this type has been studied in a number of papers.[77,83,44] The object is to find $\tilde{\gamma}$ by substituting the nonuniform solution $\Delta(r)$ being studied into U. If there is at least one $\tilde{\gamma}>0$, then the nonuniform state is unstable. It is easy to verify that for the stratified solution $\tilde{\gamma}=0$ and the solutions are stable for any regime. If $\delta\neq\delta_0$, then, as shown in Refs. 83 and 44, the oscillatory and soliton solutions $\Delta(r)$ are unstable; the stable solutions are only the monotonic solutions with the additional conditions

$$\frac{d\bar{I}}{dV}<0,\quad R>\left|\frac{d\bar{I}}{dV}\right|, \tag{93}$$

satisfied for a given current.

1) *Structure of the nonuniform state of a superconductor with a broad-band source in the absence of diffusion of quasiparticles.* The problem is most simply understood in the limiting case $L\ll\xi_0$, when it is possible to neglect the diffusion of quasiparticles. In this case, terms with L^2 can be dropped from equations (11) and $n(\xi,r)$ can be assumed to depend parametrically on r via Δ. Equation (7) for Δ takes the form (for $T=0,\gamma=0$)[43]

$$\xi_0^2\frac{d^2\Delta}{dr^2}=-\frac{\partial}{\partial\Delta}\left(-U(\Delta,\delta)\right), \tag{94}$$

$$U(\Delta,\delta)=+\Delta^2\left(\frac{\delta}{2}-\frac{\Delta}{3}+\alpha\frac{\Delta^2}{4}\right). \tag{95}$$

Equation (94) coincides with the equation of motion of particles with a coordinate Δ in a field $-U$. Equations of this type arise in plasma theory,[84] and in the theory of superconductors with a negative differential conductivity.[76,77] The potential $U(\Delta)$ for different values of the parameter δ is illustrated in Fig. 5b. As δ varies from 0 to δ_m, the potential has three extrema for $\Delta=\Delta_1$, Δ_2, and Δ_3, corresponding to the three possible uniform states for given pumping δ. It is simplest to elucidate the nature of the distribution of Δ in the one-dimensional case:

$$\frac{d^2\Delta}{dx^2}=\frac{\partial U}{\partial\Delta},\quad \left.\frac{d\Delta}{dx}\right|_{x=\pm l/2},\quad x=\frac{x}{\xi_0}, \tag{96}$$

where l is the length of the film. The first integral in (96) is given by the equation

$$\frac{1}{2}\left(\frac{d\Delta}{dx}\right)^2=C+U(\Delta),\quad C=-U(\Delta^{(2)})=-U(\Delta^{(3)}),$$

where C is a constant and $\Delta^{(i)}$ ($i=1,2,3,4$) denote the roots of the equation

$$C+U(\Delta^{(i)})=0.$$

Analysis of the trajectories leads to the following possible spatially nonuniform solutions for Δ (Ref. 43):

1) oscillatory from $\Delta^{(2)}$ to $\Delta^{(3)}$;

2) narrow layers with $\Delta\neq0$ or with $\Delta=0$ (soliton solutions);

3) monotonic solutions.

According to the analysis in Refs. 77 and 83 [see (93)], the only stable solutions are those with a monotonic variation of Δ.

Among the monotonic solutions, there is a singular (connecting two singular points Δ_{10} and $\Delta_{30}=\Delta$) solution, which is a called a stratified solution. The stratified solution is realized for a particular value of $\delta=\delta_0$, corresponding to phases with equal energies:

$$U(\Delta_{10})=U(\Delta_{30}). \tag{98}$$

From (98), taking into account (95), we find δ_0 and Δ_{30} (Ref. 43):

$$\delta_0=\frac{2}{9\alpha},\quad \Delta_{30}=\frac{2}{3\alpha}. \tag{99}$$

The layered solution represents a layer of width ξ_0, separating uniform regions with $\Delta_1=0$ and $\Delta=\Delta_{30}$:

$$\Delta(x)=\frac{\Delta_{30}}{1+\exp\left[-\sqrt{\delta_0}(x+x_0)\right]}, \tag{100}$$

where x_0 is the position of the transition boundary (compare Ref. 49). It is important to note that the boundary conditions do not fix the position x_0. Monotonic solutions can also be found for $\delta\neq\delta_0$. For large L/ξ_0, they are expressed in terms of elementary functions.[46] The results obtained are valid for optical and tunnel injection ($V\gg2\Delta_0$). Using the solutions $\Delta(x)$ that have been found, it is possible to calculate the $I-V$ curves of a nonuniform SiS-junction. The voltage dependence of the total current $\bar{I}$ is illustrated in Fig. 12 and has the following properties:

$$\left.\frac{d\bar{I}}{dV}\right|_{V\leqslant V_c}>0,\quad \left.\frac{d\bar{I}}{dV}\right|_{V\geqslant V_0}\to-\infty, \tag{101}$$

where V_0 is the voltage (corresponding to δ_0) at which the energies of the two phases are equal. It is evident from Fig. 12 and (101) that $\bar{I}$ at first decreases with V decreasing below V_c, and then sharply increases for $V\to V_0$, approaching a vertical asymptote.

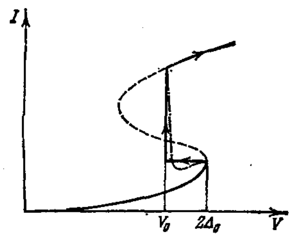

FIG. 12. I-V curve for a SIS junction. The dashed line corresponds to the I-V curve in the uniform state; the dot-dash line corresponds to the nonuniform unstable state; the solid line corresponds to the nonuniform state stable and observed experimentally.

It is useful to note that Eq. (95) can be obtained by varying the functional Φ,

$$\Phi=\int d^3r\left[\frac{\xi_0^2}{2}\left(\frac{\partial\Delta}{\partial r}\right)^2+U(\Delta)\right]. \tag{102}$$

In the two-dimensional and three-dimensional cases, the same qualitative results are obtained, while the quantity δ_0 increases somewhat.[43]

2) *Structure of the nonuniform state of a superconductor taking into account the diffusion of quasiparticles.* In the general case the system (7.11) is characterized by two parameters ξ_0 and L. It is important to take into account diffusion of quasiparticles since usually in superconductors $L \gg \xi_0$. It was shown in Refs. 43 and 44 that taking into account diffusion of quasiparticles does not qualitatively change the structure of the nonuniform state. The width of the transition layer is replaced by L, and δ_0 changes insignificantly. The considerable mathematical complexity of solving the nonlinear integrodifferential kinetic equation for $n(\xi, \mathbf{r})$ should be noted. Assuming that $\Delta(\mathbf{r})$ varies slowly, an equation was obtained in Ref. 43 for any $\tilde{\eta}=L/\xi_0$:

$$\nabla^2\Delta+\frac{1}{2}(\nabla\Delta)^2k(\Delta)=\frac{\partial\tilde{U}}{\partial\Delta}, \tag{103}$$

where

$$\tilde{U}=\int^{\Delta}\frac{d\Delta'\,\partial U/\partial\Delta'}{1+\tilde{\eta}^2\Delta'},\quad k(\Delta)=\frac{2}{1+\tilde{\eta}^2\Delta}. \tag{104}$$

We note that the numerical coefficient in the function $k(\Delta)$ is corrected compared to that in Ref. 43 (5/4 is replaced by 2). Since the singular solutions of the equation $\partial\tilde{U}/\partial\Delta=0$ coincide with $\partial U/\partial\Delta=0$, while the term with $(\nabla\Delta)^2$ is quadratic (and does not give rise to "true friction"), the results are qualitatively similar to the case of $L \ll \xi_0$.

The stable solution is the stratified solution passing through the singular points $\Delta=0$ and Δ_3. This solution is realized under the following conditions[43]:

$$\frac{\partial\tilde{U}}{\partial\Delta}=0,\qquad \int_0^{\Delta}(1+\tilde{\eta}^2\Delta')\frac{\partial U}{\partial\Delta'}\,d\Delta'=0.$$

From here we obtain for $\tilde{\delta}_0$ the value

$$\tilde{\delta}_0=\frac{15}{64\alpha}, \tag{105}$$

which does not differ significantly from δ_0 (99). Recently, an equation was obtained for Δ, without assuming slow variation of Δ.[44] This can be done due to the fact that the kinetic equation (11) is transformed into a representation in which the diffusion of quasiparticles occurs under the condition that $\varepsilon(\mathbf{r})=\text{const}$ (the importance of this condition was pointed out by Smith[81]).

The kinetic equation (11) in this representation takes the form

$$L^2\frac{\partial^2 n_1}{\partial r^2}=\Gamma n_1-\psi(\mathbf{r}). \tag{106}$$

The solution (106) can be represented in the form

$$n_1(\mathbf{r})=\int K(\mathbf{r}-\mathbf{r}')\,\psi(\Delta(\mathbf{r}'))\,d^3r', \tag{107}$$

where $K(\mathbf{r}-\mathbf{r}')$ is the Green's function for the equation

$$L^2\frac{\partial^2K}{\partial r^2}=\Gamma K+\delta(\mathbf{r}-\mathbf{r}').$$

Substituting $n_1(\mathbf{r})$ into the equation for the gap (7), after several transformations, we obtain (88) with the operator given by

$$\hat{\Pi}(\xi_0, L)\Delta=-[L^2\xi_0^2\nabla^4-L^2\nabla^2(\delta+\alpha\Delta^2)-\xi_0^2\nabla^2]\Delta. \tag{108}$$

Equation (88) coincides with the equation obtained in Ref. 44 for $T \to T_c$ by a somewhat different method.

In the one-dimensional case and for $L \gg \xi_0$, the first integral of (88) has the form

$$\frac{1}{2}\left(\frac{d\Delta}{dx}\right)^2(\delta+3\alpha\Delta^2)^2+\int^{\Delta}d\Delta'(\delta+3\alpha\Delta'^2)\frac{\partial U}{\partial\Delta'}=C. \tag{109}$$

The solution passing through the singular points $\Delta_1=0$ and Δ_3 is realized under the conditions

$$\frac{\partial\tilde{U}}{\partial\Delta}=0,\qquad \int^{\Delta_{30}}d\Delta'(\tilde{\delta}_0+3\alpha\Delta'^2)\frac{\partial U}{\partial\Delta'}=0,$$

from which we find the value $\delta_0=15/64\alpha$,[44] coinciding with (105).

3) *Nonuniform states of a superconductor with a narrow-band source of quasiparticles.* The equation that describes the change in the quasiparticle density is given by (74)

$$L^2\frac{\partial^2\bar{n}}{\partial r^2}=-\frac{\partial U(\bar{n})}{\partial\bar{n}}. \tag{110}$$

This equation and the singularities of $U(\bar{n})$ are similar to the equation for and singularities of $U(\Delta)$ for a broad-band source and for this reason we will not repeat the analysis given in subdivision 1 of this section. We will only present the final results.[37]

The stable nonuniform solution is the layered solution, which separates uniform regions with $n_1=0$ ($\Delta_5=\Delta_0$) and $\bar{n}_3\neq 0$ (Δ_{30}) and is realized at a certain frequency ω_0 or voltage V_0. For the electromagnetic field the layered solution has the form ($T=0$)

$$\bar{n}_0(x)=\begin{cases}\dfrac{3}{(x-x_0-4\sqrt{2/\alpha_\omega})^2}, & x\leqslant x_0,\\ \bar{n}_{30}\,\text{th}^2\left[\sqrt{\dfrac{\alpha_\omega}{2}}(x-x_0)+C_2\right], & x>x_0,\end{cases} \tag{111}$$

where $C_2=1/2$, x_0 is the coordinate of the transition boundary

$$\bar{n}_{30}=\frac{3}{2}\alpha_\omega=\frac{\Delta-\Delta_{30}}{2\Delta_0},\quad \frac{\omega_0}{2\Delta_0}=1-\frac{3}{4}\alpha_\omega. \tag{112}$$

Quasiparticles are created in the phase Δ_{30}, since $\omega_0-2\Delta_{30}=9/2\alpha_\omega>0$. On the other hand, quasiparticles cannot be created in the Δ_0 phase, since the frequency

$\omega_0 < 2\Delta_0$. Qualitatively similar results are also obtained with tunnel injection. In particular, the quantities V_0 and Δ_{30} are given (for $T=0$) by the equations[37]

$$\frac{V_0}{2\Delta_0} = 1 - \sqrt{\frac{\pi\alpha_V}{4}}, \quad \frac{\Delta_0 - \Delta_{30}}{\Delta_0} = \sqrt{\pi\alpha_V}. \tag{113}$$

Since the SiS-junction is symmetrical, in the phase with the smallest Δ_{30} the gap on both sides of the junction equals Δ_{30} and the difference $V_0 - 2\Delta_0 = \Delta_0\sqrt{\pi\alpha_V} > 0$, i.e. in this region quasiparticles are injected; in the region with $\Delta_5 = \Delta_0$ there is no injection, since the voltage V_0 is less than $2\Delta_0$.

It is easy to find the monotonic solutions of (110) for $V = V_0$ and with their help to compute the $I-V$ curve for the nonuniform junction. The voltage dependence of the total current $\bar{I}$ is similar to the dependence for a broad-band source (see subdivision 2 of this subsection; see Fig. 12) and has the properties[46]

$$\left.\frac{d\bar{I}}{dV}\right|_{V\leqslant 2\Delta_0} > 0, \quad \left.\frac{d\bar{I}}{dV}\right|_{V\geqslant V_0} \to -\infty. \tag{114}$$

4) *Nonstationary nonuniform intermediate state.* In superconductors with optical pumping the frequency $\omega\,(R=0!)$ is fixed. For this reason, according to (94), the stationary nonuniform solution exists for a single value of the pump power δ_0. For $\delta \neq \delta_0$, the phase separation boundary will move with a velocity proportional to the difference $\delta - \delta_0$.[43] In particular, for $L \ll \xi_0$, the velocity of motion of the phases equals

$$v_c = \frac{\delta - \delta_0}{\delta_0} \frac{3\xi_0\sqrt{\delta_0}}{\tau_\Delta}. \tag{115}$$

Indeed, we seek a solution of (89) in the form of a traveling wave $\Delta(x - v_c t)$. The equation for $\Delta(x - v_c t)$ will differ from (96) by the term $(v_c\tau_\Delta/\xi_0)(\partial\Delta/\partial x)$, which has the effect of a frictional force. Multiplying the modified equation by $d\Delta/dx$ and integrating along the trajectory of motion, we find the velocity given by (115). For $\delta > \delta_0$, there is a stationary wave, transforming the specimen from the superconducting into the normal phase. On the other hand, for $\delta < \delta_0$, the wave transforms the normal phase into the superconducting phase. In both cases, the final uniform state corresponds to the absolute minimum of the potential $\Phi = -U$.

In the opposite limiting case $L \gg \xi_0$, the phase separation boundary moves with a velocity[43]

$$v_c = \frac{L}{\tau_f} \frac{\delta - \tilde{\delta}_0}{\tilde{\delta}_0} \zeta, \quad \zeta \sim 1. \tag{116}$$

The motion of the boundaries is to some degree similar to the motion of the boundary of a combustion wave in gas mixtures.[85] Indeed, on the phase separation boundary, due to the instability of the state with $\Delta < \Delta_m$ (for $\delta > \delta_0$), a transition into the normal phase occurs accompanied by liberation of excess quasiparticles (see subdivision 2 in subsection b of section 1), which is the analog of a thermal reaction. Excess quasiparticles diffuse into a neighboring region and decrease the gap in it to $\Delta < \Delta_m$. In this region, a transition into the normal phase occurs again and so on, i.e., the "combustion" wave of the superconducting phase travels with a velocity $\sim L/\tau_f$. It is evident that after the passage of some time the sample is filled by a single phase and for this reason the intermediate nonuniform state is observed with optical pumping in the stationary regime. In all the experiments with which we are familiar,[16-22] the nonuniform state is observed (for example, from a smooth increase in resistance) in the pulsed, i.e. nonstationary, regime. In this case, the pulse duration τ is of the order of the filling time. For this reason, it was proposed in Ref. 43 that the observed smooth transition in the resistance (and other phenomena) be interpreted with the help of the model of the nonstationary nonuniform state. According to this model, the resistance appears after the appearance of a region with a normal phase (overlapping the film along the width b), the size of which increases with velocity v_c (116). The total resistance of the film R_1 toward the end of the pulse can be written in the form

$$\frac{R_1}{R_N} = \frac{v_c\tau}{l} = \frac{\tau L(\delta - \tilde{\delta}_0)}{\tau_f l \tilde{\delta}_0}\zeta = \frac{\tau L}{\tau_f l}\frac{\beta - \tilde{\beta}_0}{2\tilde{\delta}_0\beta_c}\zeta, \tag{117}$$

where R_N is the resistance of the film in the normal state, l is the length of the film, and $\beta_0 = (1+\tilde{\delta}_0)^2\beta_c$. It is evident from (117) that the resistance appears for powers exceeding $\tilde{\beta}_0$ (almost coinciding with β_c, since $\tilde{\delta}_0 \ll 1$) and increases linearly with β in agreement with experiment. The temperature dependence of $\beta_c(T)$ and its absolute magnitude were studied in detail experimentally[20-21] and agree well with theoretical results (see subdivision 6 in subsection b of section 1). The observed delay in the voltage pulse can be related to the time $\tau_{c\tau}$ it takes to cover the normal phase of the film along the width: $\tau_{c\tau} \approx b/v_c$.

It should be noted that the nonstationary nonuniform state can be observed with electromagnetic pumping with $\omega \geqslant 2\Delta$, and with tunnel injection at fixed voltage. The effect of the motion of the boundary separating the insulator-metal phases in a nonequilibrium state was recently predicted by Keldysh in Ref. 86.

5) *Stationary nonuniform state in superconductors with tunnel injection.*[37,46] For a given current $(R\to\infty)$, the monotonic distribution of $\Delta(x)$ is stable when the conditions (93) are satisfied. Let us first examine the structure of the nonuniform state with a voltage close to threshold $V = 2\Delta$. When the current through the SiS-junction is increased to a value for which $d\bar{I}/dV < 0$ (on the decreasing branch of $I(V)$ for $V \leqslant 2\Delta_0$), threshold instability arises. As a result of the development of the instability, stratification into phases (filaments) with Δ_5 and Δ_3 occurs. According to (114), the $\bar{I}-V$ curve for a nonuniform junction has a positive slope for $V < 2\Delta_0$, $d\bar{I}/dV > 0$, which in accordance with the criterion (93) indicates an instability of the nonuniform state. For this reason, there is a discontinuous decrease in voltage almost to V_0, at which the nonuniform state becomes stable, since $d\bar{I}/dV|_{V\geqslant V_0} < 0$ (while the voltage is fixed near V_0) and the layered solution that separates the phases with $\Delta_{50} = \Delta_0$ and Δ_{30} is formed. The position of the phase separation boundary x_0 is fixed by the current $\bar{I}$ passing through the junction.

If the current is increased further, then the volume of the phase with Δ_{30} increases (the boundary X_0 moves in the direction of the Δ_5 phase), while the values of Δ_{50} *and* Δ_{30} *in the phases and the voltage across the* junction remain constant and equal to (113). The increase in current continues until the specimen is completely filled with the Δ_{30} phase. Further increase in current is accompanied by an increase in the voltage V and a decrease in Δ_3 (see Fig. 12). For finite temperatures the structure of the nonuniform state remains as before, only the values of the order parameters in the phases Δ_5 and Δ_3 [see (68)] and the voltage V_0 change. It should be noted that for $T \to T_c$, $\Delta_{50} = \tilde{\Delta}_0(T)$ is greater than $\Delta_0(T)$, due to the effect of stimulated superconductivity (see Ref. 46).[73]

The model of the nonuniform state for $V \approx 2\Delta$ examined above agrees well with experimental data on the observation of a double-gap state in superconductors with tunnel injection (for details see below, subdivision 7).[25,26] For high voltages ($V \gg \Delta$), which attain the critical value V_c (63), conditions for the onset of coherent instability are satisfied. In view of the analogy between the properties of states with $V = V_c$ and $V = 2\Delta$, the $I-V$ curves for a nonuniform junction and the structure of the state with $V = V_c$ are similar to the $I-V$ curves with $V = 2\Delta$ (see Fig. 12). In particular, the $I-V$ curve has a vertical segment with $V = V_0 < V_c$, the sample separates into phases with $\Delta_1 = 0$ and Δ_{30}. The expressions for V_0, Δ_1, and Δ_{30} are given by (99). As the current increases, the phase $\Delta_1 = 0$ increases, until the specimen is completely transformed into the normal state. Apparently, such a state, predicted in Ref. 47, was recently observed experimentally by Mitsen.[27] The transition into the nonuniform state for high voltages is possible only for $I-V$ curves of the first and second type (see subdivision 9, subsection b, section 1). For $I-V$ curves of the third type, the system passes into the normal state (therefore, uniform) before the voltage across the junction attains the value of V_0, since $I_{th} > I_0$. For this reason, for $I-V$ curves of the third type, the specimen makes the transition from the uniform superconducting state into the uniform normal state via a transition of the first kind.

6) *Comparison with experiment.* Experiments with tunnel junctions with high voltages ($V \gg \Delta$), where phase transitions of the first kind and transition to the nonuniform state were observed, can be explained with the help of the model with the coherent mechanism (see subdivision 5 of this section). We recall that if the junction $I-V$ curve is of the third type, then a transition of the first kind to the normal state occurs. $I-V$ curves of the first and second kind lead to a transition to the nonuniform state. It is characterized by a vertical segment of the $I-V$ curve of the generator with fixed V_0, by a discontinuity in the voltage $V_m - V_0$ at the transition, by an $I-V$ curve for the detector with two gaps. The type of $I-V$ curve depends on α_V, T, and γ. For example, a sharp change in γ as T passes through the λ-point in helium can transform type 2 into type 3. We assume that this mechanism is related to the phenomena discovered by Iguchi.[23,63] For a temperature above T_λ, he observed a nonuniform state (from the smooth change in resistance). As T decreased below T_λ (and, therefore, γ decreased), the transition to the normal state occurred discontinuously for some critical value of the current.[10)] A similar phase transition of the first kind was observed by Fuch and his coworkers[24] in Sn-I-Sn-I-Pb junctions.

Recently, the transition into the nonuniform state was observed in Sn-I-Sn.[27] The high value of V_0, the nature of the dependence of V_0 on α_V, the temperature dependence of I_{th}, and other characteristics indicate the coherent mechanism for the transition from $I-V$ curves-1 and $I-V$ curves-2.

The threshold mechanism for the nonuniform state apparently was realized in the experiments by Dynes, Narayanamurti, and Carno[25] (Al-I-Al) and especially clearly in those by Gray and Willemsen.[26] In this work, quasiparticles were injected into thin aluminum films ($d \sim 300$ Å) with $V \approx 2\Delta$ (generator), and the double-gap nonuniform state was studied with the use of an additional contact (detector). The authors identified and described the basic properties of the nonuniform state, and the $I-V$ curves of the generator and detector, in particular:

1. The instability and transition to the nonuniform state do not require critical concentration of quasiparticles, i.e., they occur for any α_V and over the entire temperature range.

2. The large gap almost does not change, while the small gap depends on the resistance of the junction according to the law

$$\frac{\Delta - \Delta_3}{\Delta_0} = \sqrt{\pi \alpha_V}.$$

3. The vertical segment of the $I-V$ curve of the generator ($V = V_0$) is related to the growth of the phase with small Δ.

These and other properties [in particular, the temperature dependence of I_{th} (Ref. 25)] are completely described by the threshold model [see subdivisions 3 and 5 of this section, formula (117)] and, in addition, the values of V_0 and Δ_{30} [see (113)] agree quantitatively with experimental data.

7) *Structure of the nonuniform state in the diffusion model.* Recently there have appeared publications[47,44,79] in which the stationary nonuniform state, arising after the completion of the development of DI, was examined. In Ref. 87, a model with a postulated negative diffusion coefficient was examined; in Ref. 44, the NEMS model was examined; and, in Ref. 79 a model of the exciton insulator ($\Delta \ll \omega_D$) was examined.

In these investigations, it was shown that beginning with some threshold power spatial modulation of the order parameter

$$\Delta(\mathbf{r}) = \Delta + \tilde{\Delta} \cos \mathbf{q}_m \mathbf{r}$$

with finite wave vector $\mathbf{q}_m$ and amplitude $\tilde{\Delta}$ occurs. The quantities $\tilde{\Delta}$ and $\mathbf{q}_m$ were computed for specific models,

[10)] In the work by Hida,[87] Iguchi's experiments are interpreted from the point of view of the diffusion instability, assuming that it exists. Hida[87] also noted that according to the model of Ref. 88 the diffusion instability must be absent for $T > 0.6\,T_c$, while the effects of Ref. 23 remain up to T_c.

and in addition $q_m \sim (\xi_0 L)^{-1/2}$ everywhere. The expression presented above is strictly applicable for small distances above threshold. However, it may be expected that for intense pumping the depth of modulation becomes of the order of unity, since regions with the normal phase appear.

2. SUPERCONDUCTORS WITH AN INVERTED QUASIPARTICLE DISTRIBUTION

a) Superconducting pairing in systems with repulsion

Up to this point, we have examined the effect of excess quasiparticles on the state of superconductors that are superconducting and in the equilibrium states due to attraction between electrons. As was shown above, superconductivity in this case is suppressed as the degree of disequilibrium increases.

On the other hand, it is evident from Eq. (6) for the order parameter Δ that formally this equation can have a solution with $\Delta \neq 0$ in the case of a repulsive interaction ($\lambda > 0$), if in some range of energies the QDF $n(\varepsilon)$ exceeds the value 1/2, i.e. if their distribution becomes inverted in the energy range $\varepsilon < \mu$. The solution for Δ has in this case, when $n(\varepsilon) = 1$ for $\varepsilon < \mu$ and $n(\varepsilon) = 0$ for $\varepsilon > \mu$, the following form:

$$\Delta = \frac{2\mu^2}{\tilde{\omega}} e^{-1/\lambda}, \tag{118}$$

where $\tilde{\omega}$ is of the order of the plasma frequency of the system. Thus, with increasing pump power and, therefore, with an increase in the magnitude of μ, the order parameter Δ increases.

As is well-known,[89] the basis for introducing the order parameter Δ, described by Eq. (6), is the analysis of the electron-electron scattering amplitude in the normal state. It turns out that this amplitude contains an imaginary pole at the temperature $T = 0$, which indicates the instability of the normal phase relative to pairing of electrons near the Fermi surface in the case of arbitrarily weak attraction.

Let us examine the analogous question in the case when a part of the electronic states in a layer of μ below the Fermi energy E_F is transferred to the state of a layer of μ above the Fermi energy (Fig. 13). For such an inverted population, there are three discontinuities in the distribution function. It is easy to verify (see Ref. 3) that in this case an imaginary pole arises in the electron-electron scattering amplitude for arbitrarily weak interelectron repulsion which indicates pairing of electrons and the appearance of a gap Δ near the Fermi level E_F (point 2 in Fig. 13). This indicates the fact that in such a superconducting state there are electronic excitations above the gap and hole excitations below the gap.

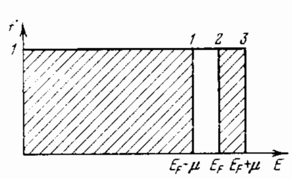

FIG. 13. Electron distribution function of energy.

b) Creation of an inverted quasiparticle distribution

The basic problem here is to obtain an inverted distribution. As shown in section 2, in the case of superconductors with a gap $\Delta < \omega_D$, as, and especially, in the case of a normal metal ($\Delta = 0$), the magnitude of $n(\varepsilon) < 1/2$, i.e., an inverted distribution turns out to be impossible. In order to solve this problem, in Ref. 91 a model was proposed for a stratified semiconductor with a nonrectilinear forbidden band E_g. In order to obtain an inverted distribution the following condition is necessary: $E_g > \omega_D$. In this case, due to the impossibility of single-phonon recombination through the gap E_g, the electron and hole distribution functions will be quasi-Fermi functions, i.e., it is possible to achieve an inverted distribution.

However, the presence of the forbidden band leads to the fact that in the self-consistency equation for ξ the interval E_g near the Fermi level drops out in the course of integration with respect to ξ. As a result, the solution for Δ has the form

$$\Delta = \sqrt{\Delta_0 (\Delta_0 - 2E_g)}, \quad \Delta_0 = \frac{2\mu^2}{\omega_p} \exp\left[-\frac{2}{N(0)(\lambda_0+\lambda_1)}\right], \tag{119}$$

where the parameter Δ_0 corresponds to the magnitude of the superconducting gap for $E_g = 0$ and to the given magnitude of the inverted population μ. $N(0)$ is the density of states near the conduction and valence band edges, which is assumed to be independent of energy, this being valid for a stratified system; the interaction constants λ_0 and λ_1 characterize the interelectronic intraband and interband interactions of a repulsive type.

It is evident from expression (119) that there exists a critical pump power μ_c equal to

$$\mu_c = \sqrt{\omega_p E_g} \exp \frac{1}{N(0)(\lambda_0+\lambda_1)}, \tag{120}$$

and superconducting pairing is realized for $\mu > \mu_c$.

It turns out that pairing with repulsive interaction and simultaneously an inverted quasiparticle distribution are also possible in the metallic model.[92] It follows from an analysis of the system of equations for Δ (6) and the kinetic equation (9) that there exists a nontrivial self-sustaining solution ($\Delta \neq 0$) for the inverted distribution ($n(\varepsilon) > 1/2$) in the layer 2μ under the condition that

$$2\Delta > \omega_D,$$

where $\Delta = (2\mu^2/\omega_p)\exp(-1/N(0)\lambda)$ and λ is the interelectronic interaction constant.

One of the methods for bringing the system from the state with $\Delta = 0, n(\xi) < 1/2$ to a superconducting state ($\Delta \neq 0, n(\xi) > 1/2$) was examined in Ref. 92. It consists of creating a priming gap with the help of a resonant electromagnetic field.[71]

We note that the magnitude of Δ is always less than the degree of inverted population μ. Since the function $n(\varepsilon)$ is smeared out with increasing temperature in the range T near the quasi-Fermi levels of electrons and holes at distances μ from Δ, the temperature for the transition to the normal state is determined not by the magnitude of Δ for $T = 0$, as in the case of attraction, but rather by the magnitude of μ, i.e. such superconductivity must be high-temperature superconductivity.

c) Investigation of the stability of a superconductor with an inverted distribution

A distinguishing feature of superconducting systems with repulsion in the case of an inverted population is the monotonic increase of Δ with increasing pump power, i.e., the magnitude of μ. It is easy to verify that in this case the uniform state is stable without taking into account electrical and magnetic fields.[78] If in some region the parameter Δ decreases as a result of fluctuations, then the concentration of excitations will increase due to diffusion from neighboring regions. Since the magnitude of Δ increases with increasing number of excitations, this diffusion will lead to an increase in Δ. Thus, the fluctuation is damped. This follows formally from the expression for the damping decrement $\tilde{\gamma}$ (86). As shown in Ref. 92, the quantity N_s is negative. In this case, the parameter D [determined by (83)] is also negative, since the basic contribution to it is made by $\xi\sim\Delta$, for which $n>1/2$. Therefore, $\tilde{\gamma}$ is negative, which corresponds to stability relative to fast perturbations. It is easy to verify that stability relative to slow (in comparison with the relaxation time for quasiparticles) perturbations follows from the general expression for $\tilde{\gamma}$ (82).

Superconductivity in the case being examined can exist only in the presence of quasiparticles, which are continuously being excited by a source and which recombine. The recombination process destroys the coherence of the superconducting state. It may appear that it plays the same role as, for example, scattering by a magnetic impurity with a characteristic time τ_s, leading to breaking up of electron pairs. In the latter case, as is well-known, superconductivity exists until $\tau_s\Delta_0>1$ (Δ_s is the parameter in the absence of impurities),[93] i.e., until the pairing time for particles Δ_0^{-1} turns out to be less than the time for pair break-up τ_s. The recombination time τ_R, in contrast to τ_s, characterizes the time for quasiparticles, and not pairs, to leave the coherent state. The recovery time for the coherent state is τ_Q, the time for recovery of the population of the electron-like and the hole-like branches of the excitations.[94] The point is that the pumping source continuously excites particles and holes with amplitude u_p and v_p [see (1)] which are equal to zero or unity (limiting unbalance of the branches). The wave functions of the excitations are tuned to the coherent state within a time τ_Q.

The superconducting state with repulsion will occur if the time τ_Q is less than the recombination time τ_R (for a more detailed discussion see Ref. 94). In the limit $\tau_Q/\tau_R\ll 1$, the unbalance of the branches can be neglected. The time τ_Q can be quite small in the case of large concentrations of a nonmagnetic impurity.[95]

d) Electromagnetic properties

Let us now go on to the problem of the behavior in external electric and magnetic fields of systems with superconducting pairing with repulsive interaction.

We will first examine the problem of the possibility of the existence of undamped currents in the presence of scattering in such a system. For this purpose, it is necessary to find the current in an alternating electric field $E(t)=E_0e^{i\omega t}$ taking into account, for example, elastic scattering by impurities. With the help of Keldysh's technique,[1] an expression is obtained for the current that coincides with the corresponding expression for the equilibrium case,[96] but with the nonequilibrium QDF $n(\xi)$, satisfying the kinetic equation and appearing as a quasi-Fermi function in the case that we are considering. In the static limit, $\omega\to 0$, the expression for the current has the form[3,92]

$$J(t)=E(t)\sigma_N\left(2+\frac{i\pi\Delta}{\omega}\right), \tag{121}$$

where $\sigma_N=2e^2n_0v_F\tilde{l}/3m$ is the normal conductivity and $\tilde{l}$ is the mean free path length.

It is evident from (121) that the system can be described by the two-fluid model of superconductivity. In this case, the first term corresponds to the normal component and stems from scattering of quasiparticles near quasi-Fermi levels $\pm\mu$. On the other hand, the conductivity of the superfluid component (second term) becomes infinite even in the presence of scattering. We note that the sign of the second term is opposite to the sign of the corresponding term in the current of an equilibrium superconductor.

Standard calculations[93] of the linear response to a static magnetic field H lead to the following expression for the current:[3,92]

$$\mathbf{J}_{q=0}=-\frac{e^2n_0}{mc}N_S\mathbf{A}, \tag{122}$$

where rot $\mathbf{A}=\mathbf{H}$. For a quasi-Fermi distribution n, we obtain from (122)

$$\mathbf{J}_{q=0}=\frac{e^2n_0}{mc}\left(\frac{2\mu}{(\mu^2-\Delta^2)^{1/2}}-1\right)\mathbf{A}\approx\frac{e^2n_0}{mc}\mathbf{A},\quad \mu\gg\Delta,$$

i.e., the current equals the current in an equilibrium superconductor, but with opposite sign. This means that the system being examined has an anomalous paramagnetism, while the Meissner effect is absent.

For a more detailed analysis of the behavior of such a system in an arbitrary (not weak) magnetic field, it is possible to obtain Ginzburg–Landau equations for the semiconductor model. From expression (119) for the gap Δ with pump power close to some critical value, when $(\mu-\mu_c)/\mu_c\ll 1$, we obtain

$$\Delta=\sqrt{2}E_g\sqrt{\frac{\mu-\mu_c}{\mu_c}}. \tag{123}$$

Thus, the dependence of the order parameter Δ on μ near μ_c has the same form as the temperature dependence near the critical T_c for an equilibrium superconductor. A procedure similar to that carried out by Gor'kov for the equilibrium case leads to the following system of equations:[97]

$$\left[\frac{1}{2m}\left(\frac{\partial}{\partial\mathbf{r}}-2ie\mathbf{A}\right)^2+\frac{1}{\eta_\mu}\left(\frac{\mu-\mu_c}{\mu_c}-|\psi|^2\right)\right]\psi(\mathbf{r})=0, \tag{124}$$

$$\mathbf{j}(r)=\frac{ie}{m}\left(\psi\frac{\partial\psi^+}{\partial\mathbf{r}}-\psi^+\frac{\partial\psi}{\partial\mathbf{r}}\right)+\frac{4e^2|\psi|^2}{m}\mathbf{A}, \tag{125}$$

$$\eta_\mu=\frac{\mu_c^2+2\mu_cE_g-4E_g^2}{6\mu_c^2E_g},\quad \psi(r)=\sqrt{\frac{\mu_c^2-2E_g^2}{8\mu_c^2}}\frac{\Delta(r)}{E_g}.$$

Equation (124) for Δ is completely analogous to the Ginzburg-Landau equation. It should be noted that due to the positive sign of the coefficient in front of the term with the derivative with respect to r, the uniform state without taking into account the magnetic field is stable. This agrees with the conclusion arrived at on the basis of the criterion (83) for the diffusion instability. The expression for the current (125) has the opposite sign compared to the equilibrium case, which also agrees with the result (122) for the linear response.

In order to clarify the behavior of such a superconductor in a magnetic field, we consider the fact that Eqs. (124) and (125) transform into the equilibrium equations if the penetration depth $\tilde{\lambda}=\sqrt{m\mu_c/16\pi e^2}$ is replaced by the imaginary quantity $i\tilde{\lambda}$. In doing so, a similar substitution must be carried out for all variables expressed in terms of $\tilde{\lambda}$, i.e., the parameter $\varkappa=\tilde{\lambda}/\xi_0$ must be replaced by $i\varkappa$, r must be replaced by $-ir$, and H must be replaced by iH. Such a substitution corresponds to the appearance of trigonometric functions of the coordinate r in the solutions instead of hyperbolic functions in the equilibrium case. As a result, the phase transition in a magnetic field for thin films in the case being examined always turns out to be a phase transition of the second kind instead of the first kind for equilibrium superconductors. For a half-space, the coordinate dependences of the magnetic field and the parameter ψ contain oscillating factors and, in addition, the period of oscillations of the field equals $2\pi\tilde{\lambda}$, while the period of the parameter ψ equals[97] $\pi\tilde{\lambda}$:

$$H=H_0\cos\frac{z}{\tilde{\lambda}}$$
$$\psi=1-\frac{H_0^2}{\tilde{H}^2}\left(1-\frac{\varkappa^2}{\varkappa^2+2}\cos 2\frac{z}{\tilde{\lambda}}\right), \tag{126}$$

where H_0 is the external field at the boundary of the specimen and parallel to its surface, $\tilde{H}=2\sqrt{\pi n/\eta_\mu}(\mu-\mu_c)/\mu_c$, and z is the distance from the boundary. The maxima of the parameter Δ coincide with the maxima of the field H. This is explained by the fact that superconductivity is destroyed directly not by the magnetic field, but by currents that are induced in the superconductor by the magnetic field. The difference in this sense with respect to the equilibrium superconductor, where in regions with maximum values of the field the magnitude of Δ is minimum, is related to the change in sign in the expression for the current (122).

These results are obtained assuming that the magnetic field corrections to the parameter Δ are small. Taking into account these corrections in Maxwell's equations, leads to the appearance of a correction to the field H linear in the coordinate z (Ref. 97):

$$\mathrm{H}=\mathrm{H}_0\left(\cos\frac{z}{\tilde{\lambda}}+\frac{1}{4}\frac{H_0^2}{\tilde{H}^2}\frac{z}{\tilde{\lambda}}\sin\frac{z}{\tilde{\lambda}}\right). \tag{127}$$

This indicates instability relative to magnetic fluctuations (see also Ref. 99). It is evident from expression (127) that the magnetic field increases as the distance from the boundary increases, which in accordance with expression (126) for ψ must lead to the disappearance of superconductivity at some distance from the boundary. Since, on the other hand, in the region of the nonsuperconducting phase far away from the boundary the field H will equal the external field H_0, which is assumed to be small, the nonsuperconducting state is unstable. As a result, the system must transform into some vortical state.

We will now show that for a reason analogous to the change in sign in the electric field dependence of the nondissipative part of the current [see (121)] an electric instability occurs in such superconductors. Using Maxwell's equations and the expression for the current (121), we obtain an equation for the charge density ρ:

$$\frac{\partial^2\rho}{\partial t^2}+8\pi\sigma_N\frac{\partial\rho}{\partial t}-4\pi^2\Delta\sigma_N\rho=0. \tag{128}$$

The decrement $\tilde{\gamma}$ in the solution $\rho\sim\exp(\tilde{\gamma}t)$ is given by the expression

$$\tilde{\gamma}=4\pi\sigma_N\left(-1\pm\sqrt{1+\frac{\Delta}{4\sigma_N}}\right). \tag{129}$$

It follows from (129) that there exists a positive value of $\tilde{\gamma}$ that corresponds to the instability mentioned above.

Thus, for the case of a uniform parameter Δ, an electric field that will accelerate the superconducting condensate to critical velocities must arise spontaneously in the system. As a result, superconductivity should vanish. On the other hand, in the normal state ($\Delta=0$) the electric instability does not occur; for this reason, such a state ($\Delta=0$, $E=0$) is unstable relative to the appearance of superconducting pairing.

Apparently, states with nonuniform Δ and E must be stable. The characteristic scale for the nonuniformity must be determined by the penetration depth l_E of a longitudinal electric field into the superconductor,[12] which, as noted in the introduction, arises as a result of unbalance of the electron-like and hole-like quasiparticle branches. The detailed structure of the nonequilibrium state must be built up taking into account simultaneously both electric and magnetic instabilities.

CONCLUSIONS

At the present time, it may be assumed as being firmly established that the wide range of new phenomena in superconductors with excess quasiparticles is closely related to the properties of the quasiparticle energy distribution. Depending on the form of the quasiparticle distribution function, the following types of nonuniform states are possible in nonequilibrium superconductors.

The first type is characterized by an overheated quasiparticle distribution, multivalued dependence of the order parameter on the source parameters, transition to a nonuniform stratified state due to the development of the coherent or threshold instabilities. Diffusion of quasiparticles inhibits the development of instabilities, while the critical source power β_c decreases with increasing wave vector. This type apparently is realized in superconductors with tunnel and optical pumping.

The second type is characterized by an overcooled quasiparticle distribution, single-valued order parameter, and presence of the diffusion instability, leading to a nonuniform state (diffusion model). Diffusion of quasiparticles leads to the development of an instability, while the critical power decreases with increasing wave vector. This type can be realized in systems with electronhole pairing[79] for $\Delta \ll \omega_D$. Electronhole correlations turn out to be responsible for the metal-semiconductor phase transition[80], for structural,[101] ferroelectric,[102] antiferromagnetic,[103] and ferromagnetic[104] phase transitions.

The third type relates to systems with electronhole pairing for $2\Delta > \omega_D$.[78] In systems of this type, the quasiparticle distribution is quasi-Fermi, the order parameter is a multivalued function, and the nonuniform state is characterized by a nonzero pair vector. The critical power increases with the wave vector. All three types are stable relative to electric and magnetic excitations.

The fourth type can be realized in superconductors with repulsive interaction between electrons and with an inverted quasiparticle distribution. This type is characterized by an order parameter Δ ($2\Delta > \omega_D$), which increases monotonically with the pump power, and by the absence of diffusion and coherent instabilities. Systems of the fourth type are unstable relative to spontaneous excitation in them of magnetic and electric fields, as a result of which an intermediate type nonuniform state can be formed.

Besides systems with purely superconducting and electronhole pairing, there exist systems in which superconducting and dielectric pairing[7] can exist which can lead to the presence of structural or antiferromagnetic phase transitions near the superconducting transition temperature T_c.

Coherent factors in this case can give rise to temperature and frequency singularities in such kinetic phenomena as light absorption,[105] ultrasonic absorption, and the intensity of nuclear magnetic resonance.[106]

In the last reference, an attempt is made to explain from this point of view the experimental results for compounds of the A15 group, which undergo a structural transformation near T_c.

For such systems, the description of the phase transition under nonequilibrium conditions, presented in the present review, must be supplemented by including superconducting and dielectric correlations simultaneously. This can lead to an entirely new series of effects and, for this reason, it is of great interest to perform experiments on such substances for the purpose of studying a phase transition under nonequilibrium conditions.

It should be noted that the study of nonequilibrium states in the usual superconductors cannot be considered as complete. In particular, it is necessary to have a detailed experimental verification of the theory of the nonuniform state with optical pumping and with tunnel injection at high voltages, to determine the multiplication factors for quasiparticles, to study the effect of electron-electron collisions on the energy *distribution of quasiparticles, and so on.*

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