Anomalies in the rotational spectra of deformed nuclei

I. M. Pavlichenkov

I. V. Kurchatov Institute of Atomic Energy, Academy of Sciences of the USSR, Moscow
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A review is given of the phenomenon of backbending which is observed in the rotational spectra of deformed nuclei. The phenomenon is interpreted in terms of superconducting-type pairing correlations. This model was previously successful in the description of the low-lying rotational states. Analysis of high-spin rotational states in the backbending region has established a similarity between pairing correlations in nuclei and in superconductors of small linear dimensions. In view of this, low-lying quasiparticle terms of the subshell with high angular momentum j on the Fermi surface assume particular importance (through the dependence of quasiparticle energy on the nuclear rotational frequency). The importance of quasiparticle terms in the analysis of the structure of deformed nuclei is noted. The review covers the period up to the beginning of 1980.

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1. INTRODUCTION

Rotational motion is a unique collective excitation of the nucleus that has attracted the attention of experimenters and theorists for almost 30 years. Nuclear rotational excitation was predicted by Bohr and Mottelson in 1952 and was discovered with the aid of Coulomb excitation in 1953.[1] Rotational spectra have now been recorded for a large number of nuclei, ranging from the lightest to the transuranic elements.

Nuclear rotational states form a regular spin sequence and are grouped in rotational bands. Each band is characterized by strong (of the order of 100 single-particle transitions) $E2$ transitions between neighboring levels, and a specific dependence of excitation energy on the nuclear spin I (we shall take $\hbar=1$)

$$\mathscr{E}=\frac{I(I+1)}{2\mathscr{J}}, \tag{1}$$

where $\mathscr{J}$ is the nuclear moment of inertia. Rotational excitations are relatively pure even for high spin values, i.e. they contain only a small admixture of states of different origin but the same spin. The energies and transition probabilities within a band are described by a small set of collective parameters such as the moment of inertia, the quadrupole moment, and so on, which vary smoothly from nucleus to nucleus and, in a given nucleus, from one band to another. The collective parameters begin to depend on level spin as the energy of rotational excitation increases. This behavior of collective terms can serve as a very useful source of information on the structure of the nucleus.

Early studies of the energy and spin of rotational states in a band relied on Coulomb excitation. This method is being rapidly developed for modern heavy-ion accelerators. Xe^{126} and Pb^{208} ions are being used in Coulomb-excitation experiments in which rotational levels up to $I=24$ can be excited. This particularly pure method of exciting rotational bands is used for the nuclei of transuranic elements.

The rapid development of research on the rotational states of nuclei began with the discovery of a new method whereby rotational bands are excited in heavy-ion reactions of the form (HI, xn). This method was first introduced in 1963 when Morinaga and Gugelot[2] used the (α, $2n$) and (α, $4n$) reactions with 52-MeV α-particles. They succeeded in producing rotational states with spins up to 10–12. In 1964, the Stephens group in Berkeley[3] used B^{11}, N^{14}, and F^{19} ions for the excitation of rotational states. Because of the use of heavier ions, and of the Ge(Li) detector to record the resulting γ rays, they were able to raise the spin upper limit to 18–20. In 1967, this group began[4] to use Ar^{40} ion beams with an energy of about 160 MeV. These heavier ions enabled them not only to increase the spin but also to measure

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the lifetime of rotational states. By 1968, all the basic methods for studying the nuclear rotational states in heavy-ion reactions had been developed.

The (HI, xn) reaction proceeds in three stages. In the first stage, the heavy-ion energy exceeds the height of the Coulomb barrier, and a compound nucleus is produced with angular momentum up to 80 and excitation energy up to 200 MeV (all the numbers given below refer to reactions involving Ar^{40} ions on nuclei with mass number $A \sim 120$). Neutrons are evaporated from the heated nucleus during the second stage. This is the most probable process for the nuclei of rare-earth elements. In the case of lighter nuclei with a lower Coulomb barrier, the emission of protons and α-particles is a competing process. The fission channel which suppresses the yield of high-spin excited states becomes important in the region of transuranic elements.

Each neutron carries away little angular momentum (1.5 on average). After the evaporation of neutrons, therefore, the nucleus remains in an excited state with an angular momentum of about 60 and energy of about 30 MeV. In the third stage of the reaction, the nucleus is de-excited from this state through three γ-ray cascades. The first to occur is the statistical cascade of mainly $E1$ transitions with an average energy of 10 MeV, which takes the nucleus to excited states, usually referred to as the yrast levels. These are levels with the lowest energy for given spin. The statistical cascade carries a small amount of angular momentum, so that the yrast states have spins of about 35 and energy of about 10 MeV. Next to proceed is the yrast cascade, consisting of γ rays due to $E2$ transitions between levels within the yrast band. This cascade carries off both energy and angular momentum.

At spins of about 20 and energies of about 5 MeV, yrast levels transform into levels in the ground-state band. The third cascade of γ rays due to $E2$ transitions in this band begins at this point. The time interval between the formation of the compound nucleus and the end of the population of levels in the ground-state band is, on average, 10 ps. We have outlined the most probable decay mode of the compound nucleus (through the emission of γ rays). Other decay modes take the nucleus to levels in the ground-state band through states in side bands. The population of the latter reduces the intensity of high-spin transitions and impedes the observation of the upper levels in the ground-state band. High spin yrast levels much be populated to avoid this difficulty. However, increasing the energy or the mass of the bombarding ions does not always have the desired result. The production of high-spin states may be prevented by the structure of the nuclear spectrum near the point where the yrast band crosses the ground-state band. Thus, in the (C^{13}, $5n$) reaction,[5] the maximum spin obtained in the ground-state band of Er^{160} is 24, whereas the maximum spin obtained in Er^{158} at the same energy of the C^{13} ions is 30. This record spin value was achieved in the ground-state band. The yrast band of Dy^{152} has been investigated[6] up to the level with spin of 37 and energy of 12.7 MeV with the aid of the (S^{32}, $4n$) reaction. The mechanism responsible for the population of rotational levels has not so far been adequately investigated.

In view of the foregoing, it is interesting to consider a new type of nuclear reaction, namely, (HI; α, xn), which can be used to excite the rotational states of rare-earth nuclei.[7] This is a direct reaction because the α-particle is emitted in the forward direction with an energy of 35–41 MeV for an incident-ion energy of 75–151 MeV (B^{10}, C^{12}, N^{14}, F^{19}, Ne^{20}). The striking feature of the γ-ray spectrum emitted by the nuclei produced in this reaction is that the intensities due to $E2$ transitions are practically constant for states with $10 \leq I \leq 20$, which suggests that the population of the bands is different from that in the case of the (HI, xn) reaction.

The observed γ-ray spectrum consists of a continuum due to statistical and yrast cascades, and discrete lines on the continuum, which correspond to transitions within the ground-state band. The continuous spectrum contains information on yrast levels with spins $I > 30$. A description and interpretation of the spectrum can be found in the review by Lieder and Ryde.[8]

The third γ-ray cascade carries information that can be used to establish level energies and spins in the band. The angular momentum of the compound nucleus is oriented in the plane perpendicular to the incident beam, which produces angular anisotropy in the emitted radiation that amounts to 0.8–0.9 for the upper levels of the ground-state band. The level spins and transition multipolarities can be determined by recording the angular distribution of the emitted γ rays.

A special method has been developed for measuring the lifetimes of rotational states, which amount to $\lesssim 1$ ps. Ar^{40} nuclei and heavier ions have sufficient momentum to knock out compound nuclei from the thin target into the vacuum, where their velocity may be up to 0.02c. The γ rays emitted by these nuclei will therefore experience Doppler shifts. The absorber is placed in the path of the recoil nuclei, so that if a nucleus emits a γ ray after entering the absorber, the γ ray will not exhibit the Doppler shift. By measuring the fraction of unshifted γ rays and by moving the absorber, i.e., by measuring the time of flight of the recoil nuclei, one can construct the decay curve and hence determine the level lifetime.

The advent of the new generation of heavy-ion accelerators has meant that it is now possible to use the reverse reaction in which the Mg^{24} target is lighter than the incident Xe^{136} ion.[9] This has resulted in more accurate measurements of the lifetimes of rotational states. The reaction produces a well-collimated beam of Dy^{156} recoil nuclei with velocities of 0.07c, so that the lifetime in the ground-state band can be determined with higher precision, and the lifetimes of levels in side bands can also be measured. Such measurements are exceedingly important since they produce information on the variation of the quadrupole moment within ground-state and side bands.

In early work on the excitation of rotational states in heavy-ion reactions, the sequence of γ transitions in a cascade was established in accordance with the spin dependence given by (1) and by the empirical rule describing the reduction in the transition intensity with increasing spin. The increase in the sensitivity of Ge(Li) detectors led, in 1970, to the development of a new method of determining the sequence of γ transitions, which was based on the γ-γ coincidence technique. The coincidence spectra give the sequence of γ transitions in time, and this can then be used to deduce the level scheme for the excited bands.

In 1971, Johnson[10] used this method with the (α, xn) reaction and discovered a considerable departure of the transition energies within the ground state band of Dy^{160} from the $I(I+1)$ rule. In the course of the next few years, similar anomalies were discovered in the rotational spectra of a large number of rare-earth nuclei. The essence of the phenomenon is that the γ-transition energies do not increase monotonically with increasing I within the spin range 12–16, but remain constant or may even fall, which is equivalent to a sharp increase in the nuclear moment of inertia in these states. The latter fact is used as a convenient way of visualizing the effect, since it is not a simple matter to identify the anomaly in the rotational spectrum by plotting the function $\mathscr{E}(I)$ (Fig. 1). Bohr and Mottelson suggested that one should plot the moment of inertia in a band against the square of the nuclear angular frequency Ω. This function can be deduced from experimental values of rotational energies in a band of an even-even nucleus by using the formulas

$$\Omega = \frac{d\mathscr{E}}{dI}, \qquad \mathscr{J} = \frac{1}{2}\left[\frac{d\mathscr{E}}{dI(I+1)}\right]^{-1}.$$

As can be seen from the figure, the result is the characteristic S-shaped curve (hence the designation "backbending").

These rotational anomalies can be explained in terms of an exceedingly simple phenomenon, namely, band crossing. Let us consider the crossing of two rotational bands, 1 and 2, with constant moments of inertia $\mathscr{J}_1 < \mathscr{J}_2$. At the crossing point, $\Omega_1 > \Omega_2$, so that the function $\mathscr{J}(\Omega)$ exhibits the characteristic S-shape shown in Fig. 2 for the lower parts of the crossing bands. The interaction between the bands gives rise to their separation, and the function $\mathscr{J}(\Omega)$ is smoothed out. Backbending vanishes as the interaction increases.

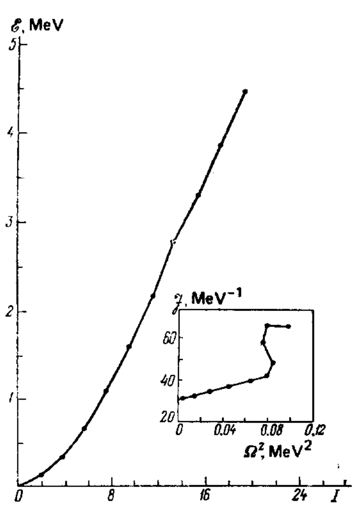

FIG. 1. Anomaly in the rotational spectrum of Er^{162} (taken from Ref. 11).

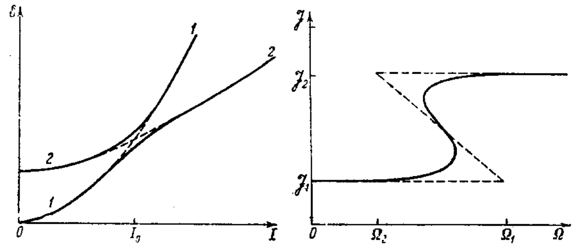

FIG. 2. Moment of inertia as a function of rotational frequency in the lowest band formed by two crossing interacting (solid lines) and noninteracting (broken lines) bands.

Thus, the rotational band of Er^{162} (Fig. 1) actually consists of two crossing bands. The upper parts of these bands have not been detected because of the particular way in which rotational states are excited in the (HI, xn) reaction. The probability of an $E2$ transition is proportional to the fifth power of the transition energy, so that lower-lying levels are preferentially populated. The upper parts are populated only when the angle between the crossing bands is small, for example, in Gd^{156}, Dy^{156}, and Er^{164}, in which levels corresponding to the upper and lower parts of the crossing bands have been found.

The above anomalies in the rotational spectra of nuclei are thus a consequence of band crossing, which has also been seen in molecular spectra.[14] As an illustration, Fig. 3 shows the function $\mathscr{J}(\Omega^2)$ for the lowest-lying band formed as a result of the crossing of two bands corresponding to different electronic terms of the AgH molecule (the experimental data are taken from Ref. 15). The result resembles the S-shaped spin dependence of the nuclear moment of inertia.

The band crossing the nuclear ground-state band is usually referred to as the superband (SB). This band is not always of the same origin in all the nuclei in which rotational anomalies have been observed. In soft nuclei in the transition region, the SB is more likely to correspond to an excited state in which the shape of the nucleus has undergone a sharp change. This appears to

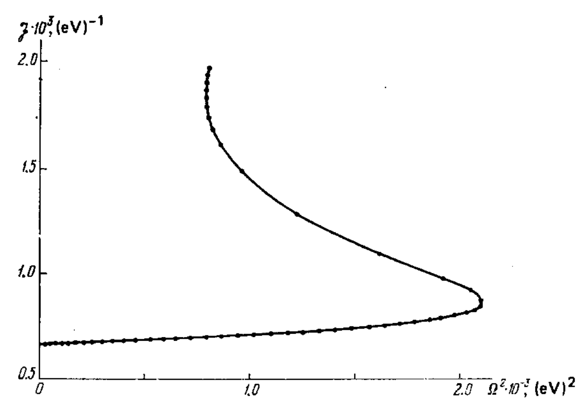

FIG. 3. Anomaly in the rotational spectrum of the AgH molecule.

be indicated by the strong attenuation of $E2$ transitions in the region of the anomaly, which has been seen in Ba[16] and Ce[17], and in certain isotopes of platinum and mercury (shape isomerism). The microscopic phenomenon responsible for the rotational anomalies in deformed nuclei resembles gapless superconductivity in a superconductor that is small in comparison with the size of the Cooper pair in a magnetic field. The superband in such nuclei has its origin in the specific two-quasiparticle excitation, which is a consequence of the effect of Coriolis forces in the rotating nucleus. The angular momentum of this excitation lies along the nuclear axis of rotation, which produces an overlap between the ground- and excited-state wave functions. The interaction between the SB and the ground-state band, determined from the ratio of reduced $E2$ probabilities near the band crossing point, is lower by roughly two orders of magnitude than the interaction between the ground-state band and the β-vibrational and γ-vibrational or two-quasiparticle bands in the same nuclei.[18] It is precisely the fact that this interaction is small that is responsible for the S-shaped dependence of the moment of inertia on Ω^2.

2. PROPERTIES OF THE ROTATIONAL BANDS OF AXIALLY DEFORMED NUCLEI

A. Rotational levels with $I<10$

We begin with the adiabatic approximation in which the motion of the nucleus can be divided into internal motion, described by the coordinates q of the nucleons in the frame x', y', z' attached to the nucleus, and rotation. The latter is described by the three Euler angles θ, φ, and ψ that define the orientation of the primed frame relative to the laboratory frame x, y, z. In the adiabatic approximation, the nuclear wave function is

$$\Psi_{IM\lambda} = \Phi_{IMK}(\theta\varphi\psi)\,\psi_{\lambda K}(q),$$

where M is the z component of nuclear angular momentum. The internal state $\psi_{\lambda K}$ of the nucleus is characterized by the component K of the resultant angular momentum of the nucleons along the symmetry axis z' of the nucleus, the parity π, and certain other quantum numbers. Each internal state has its own rotational band.

The axial symmetry leads to invariance under 180° rotation about any axis perpendicular to the nuclear symmetry axis. Consider a rotation $\mathcal{R}_{x'}$ around the x' axis. The internal state of the even-even nucleus with $K=0$ corresponds to eigenvalues of the operator $\mathcal{R}_{x'}$ equal to ± 1. On the other hand, the rotation which we are considering is equivalent to inversion of the symmetry axis ($\theta \to \pi-\theta, \varphi \to \pi+\varphi$), and hence

$$\mathcal{R}_x \Phi_{IM} = (-1)^I \Phi_{IMK}.$$

Consequently, states that are even under the $\mathcal{R}$ transformation (e states) correspond to the rotational band with even spins, whereas odd states (o states) correspond to odd spins. Single-quasiparticle excitations in an odd nucleus are characterized, in addition to K and π, by the three further asymptotic quantum numbers, N, n_z, and Λ. In the rotational band based on this excitation, the e states correspond to even values of $I - \frac{1}{2}$, whereas the o states correspond to odd values of this difference. In a band of an even-even nucleus, based on the two-quasiparticle excitation, states with opposite $\mathcal{R}$-parity correspond to even spins, whereas those with the same parity correspond to odd spins. We note that the eigenvalue σ of the operator $\mathcal{R}_{x'}$ is sometimes referred to as the *signature*.

The adiabatic property of rotation is violated already at the beginning of the band. However, the departure from the adiabatic approximation is small if the nuclear spin is low. For example, the energy in the rotational band of an even-even nucleus with $K=0$ can be satisfactorily described by

$$\mathcal{E} = \mathcal{A}I(I+1) - \mathcal{B}I^2(I+1)^2, \tag{2}$$

if $I<10$. For the rare-earth nuclei, $\mathcal{A} \sim 10$ keV and $\mathcal{B} \sim 10$ eV. The magnitude of these constants is quite sensitive to nuclear structure.

In 1959, Belyaev and Migdal showed that superconducting-type pairing correlations had a very considerable effect on the moment of inertia ($\mathcal{A}=1/2\mathcal{J}$). In a deformed nucleus, such pairs consist of nucleons with opposite components m of the angular momentum along the symmetry axis. In even-even nuclei, pairing correlations lead to the characteristic spectrum of excitations with a gap of 2Δ (where Δ is the correlation energy of the pair). Pairing correlations produce a reduction in the nuclear moment of inertia by a factor of roughly two as compared with the rigid-body value. They have a greater influence on $\mathcal{B}$. To understand the essence of this, let us consider the forces acting on a nucleon in a rotating deformed nucleus.

Above all, a nucleon interacts with the deformation. For a nucleon on the Fermi surface, the energy of this interaction is $V_\beta \sim \beta\varepsilon_F$, where β is the nuclear deformation parameter which is of the order of $A^{-1/3}$ for a well-deformed nucleus, A is the number of nucleons in the nucleus, and ε_F is the Fermi energy. In addition, the nucleon experiences the Coriolis force. The energy associated with the Coriolis interaction is $V_c = (\mathbf{I}\cdot\mathbf{j})/\mathcal{J} \sim \Omega j_F$, where $\mathbf{j}$ is the single-particle angular momentum of the nucleon and $j_F \sim A^{1/3}$ is the maximum value of this angular momentum on the Fermi surface. Henceforth, we shall consider rotational bands for spins $I < I_c$, so that the pairing correlations will still be unaffected by rotation.

The critical spin $I_c \sim A^{2/3}$ (see below) corresponds to the rotational frequency $\Omega \sim \varepsilon_F/A$ and the Coriolis energy $V_c \sim \varepsilon_F A^{-2/3}$. The Coriolis force is more important in the nucleus than in terrestrial applications or even in molecular spectra. In deformed nuclei, the Coriolis energy is of the order of the pairing correlation energy Δ. The parameter

$$\alpha_\Delta = \frac{V_c}{\Delta} \sim \frac{j_F\Omega}{\Delta} \tag{3}$$

is of the order of unity for $I \sim I_c$. The interaction between rotation and deformation is characterized by the parameter

$$\alpha_\beta = \frac{V_c}{V_\beta} \sim \frac{j_F\Omega}{\beta\varepsilon_F}, \tag{4}$$

which is lower by a factor of $A^{1/3}$ than α_Δ in the case of well-deformed nuclei. This parameter is of the order of unity for spin $I_B \sim A$, for which one would expect a change in nuclear deformation due to rotation. Since α_B is small, one can neglect the dependence of the self-consistent nuclear field on rotation if $I < I_c$. The interaction between rotation and vibration degrees of freedom is described by the parameter

$$\alpha_\omega = \frac{V_c}{\omega} \sim \frac{\langle j\rangle \Omega}{\omega} \sim \frac{\Omega}{\omega},$$

where ω is the frequency of β or γ oscillations, which is of the order of Δ. It is readily seen that α_ω is also lower by a factor of $A^{1/3}$ than α_Δ.

It is clear from the foregoing that the coefficient $\mathscr{B}$ is due to pairing correlations. One can readily obtain an estimate for this quantity, which depends on the reduction in Δ with rotation: $\mathscr{B} \sim \varepsilon_F A^{-3}$. The contribution to $\mathscr{B}$ due to the interaction between rotation and vibration is lower by a factor of $A^{2/3}$. Nuclear rotational bands are thus seen to differ from molecular rotational bands for which the coefficient $\mathscr{B}$ in the expansion given by (2) is connected with the interaction between rotation and vibration. This phenomenon was discovered in the 1960's by the present author and Grin'.[20] Much later, calculations of Marshalek,[21] Zelevinskiĭ and Shtokman,[22] and Mikhailov *et al.*[23] confirmed this result.

B. High-spin states and anomalies of rotational bands

The effect of the Coriolis force in the nucleus is analogous to the effect of a magnetic field that removes pairing correlation in a superconductor (Meissner effect). Consequently, by increasing the nuclear rotation frequency, i.e., by moving upward within the rotational band, one can eventually reach a level above which there is no pairing correlation. The spin I_c of this level is the phase transition point from the macroscopic point of view.

If the rotational bands of the nucleus are represented by the Regge trajectories $\mathscr{E}(I)$, the spin I_c will correspond to the crossing point between the ground-state band ($\Delta \neq 0$, moment of inertia $\mathscr{J}$) and the band based on the normal state with energy $\rho_F\Delta^2/2$, where ρ_F is the level density at the Fermi surface. The pairing correlation energy is zero within this band and corresponds to a rigid-body moment of inertia $\mathscr{J}_T > \mathscr{J}$. The spin I_c can be estimated very approximately from the formula[24]

$$\frac{I_c(I_c+1)}{\mathscr{J}} = \frac{I_c(I_c+1)}{\mathscr{J}_T} + \rho_F\Delta^2.$$

Since $\Delta \sim \varepsilon_F A^{-2/3}$ and $\mathscr{J} \sim \mathscr{J}_T \sim A^{5/3}/\varepsilon_F$, we find that $I_c \sim A^{2/3}$.

There have been many attempts to calculate I since the publication of the paper of Mottelson and Valatin.[24] The calculations reported by Krumlinde,[26] which were based on the method of Chan and Valatin,[25] show that the spin I_{cn} at which neutron pairing correlations vanish, lies in the range $16 < I_{cn} < 20$ for rare-earth nuclei, and in the range $20 < I_{cn} < 25$ for actinides. The corresponding figure for proton correlations is $I_{cp} \approx 1.5 I_{cn}$. These results must be regarded as only preliminary because the Chan-Valatin method does not take into account the change in quasiparticles in the rotating nucleus. On the other hand, perturbation theory calculations have shown that this is one of the most important effects. Modern numerical solutions of the Hartree-Fock-Bogolyubov equations yield the correct solution of this problem. However, no systematic calculations of I_c have been carried out so far.

Experimenters have begun to use the Bohr-Mottelson method described in the Introduction as a way of detecting the phase transition in the rotational band. It is not surprising, therefore, that the rotational anomalies discovered by Johnson were initially interpreted as being due to the disappearance of pairing under the influence of rotation. These anomalies lie somewhat below the critical spin I_c calculated theoretically.

However, theorists have found a way of reducing I_c, and it would appear that the phase transition point *has* been found in the nucleus, but we are still some way off the final success of the theory. In fact, evidence has gradually accumulated, indicating that certain facts are either difficult or impossible to fit into the phase transition picture. They include the following:

1. The nuclear moment of inertia beyond the region of the anomaly is lower than the rigid-body value. This is not a very strong argument because one can imagine that the phase transition has taken place only in the neutron system.

2. The upper parts of crossing bands have been found in Gd^{154} and Dy^{156}. This is a much stronger argument against the phase transition, but it can be answered by pointing out that band crossing in these nuclei has nothing to do with the phase transition observed in most other nuclei.

3. Electromagnetic transitions between normal and superfluid nuclear states must be weaker by a factor of $\exp(-\rho_F\Delta) \sim 10^{-2}$. However, no evidence of hindered $E2$ transitions has been found in the region of the anomalies, although the accuracy of these measurements was low.

4. The strongest argument against the phase transition was put forward as a result of the experiments performed by the Stephens group.[27] It is well known that an odd particle reduces the phase volume and weakens the pairing correlation in the nucleus. The phase transition in the rotational band of an odd nucleus should, therefore, be observed for lower spin values than in an even nucleus. However, the rotational anomalies in the bands of the neutron-odd Er^{157} and Er^{159} were not detected for the same spins for which they were observed in the ground-state band of the neighboring even-even nuclei (Fig. 4).

Experiments with odd nuclei have turned out to be decisive for the understanding of the rotational anomaly. They have resulted in the identification of the origin of the phenomenon. It turns out that the odd proton in $Ho^{157,159,161}$ has no effect on the anomalies. The backbending of the moment of inertia in bands based on single-particle excitations of these nuclei is observed for the same rotational frequencies as in the neigh-

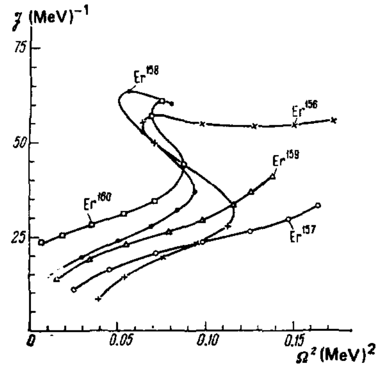

FIG. 4. Moments of inertia of the rotational bands of the even and odd isotopes of Er.

boring even-even Er nuclei. The $Er^{157,159}$ bands are based on the excited state of the odd neutron from the $i_{13/2}$ subshell. These are the so-called decoupled bands which are highly distorted by the Coriolis interaction. Unfortunately, it has not been possible to excite other high-spin bands in these nuclei. However, in odd isotopes of tungsten, normal bands are excited in addition to decoupled bands. For example, in the bands of W^{175} based on the $1/2^-[521]$ state, the backbending of the moment of inertia was found at somewhat lower frequencies than in W^{174}, whereas, in the decoupled band based on the $7/2^+[633]$ state in the $i_{13/2}$ subshell, there is no backbending.[28]

An analogous picture is observed in W^{179}, in which two bands have been excited, namely, the decoupled band on the $9/2^+[624]$ state from the $i_{13/2}$ subshell and the ordinary $7/2^-[514]$ band.[29] It is interesting that, in the case of W^{173}, the anomaly has not been observed either in the $7/2^+[633]$ decoupled band or the $1/2^-[521]$ ordinary band.[28] Nor has it been seen in the ground-state band of the even-even nucleus W^{172}.

The decoupled bands are based on states belonging to a subshell with high angular momentum j. The Coriolis interaction is at a maximum in these states. For the rare-earth nuclei, this is the $i_{13/2}$ subshell for neutrons and the $h_{9/2}$ subshell for protons. The latter is responsible for the rotational anomalies in Os isotopes. The rotational bands of Os^{183} and Re^{181} were investigated in Ref. 30. It was found that, in the band based on the state from the $i_{13/2}$ subshell (probably, $9/2^+[624]$) of Os^{183}, the backbending of the moment of inertia is observed at approximately the same rotational frequency as in the neighboring Os^{182}. The same occurs in bands based on the $5/2^+[402]$ and $9/2^-[514]$ states of Re^{181}. However, the anomaly was not found in the decoupled band from the $h_{9/2}$ subshell of this nucleus. One could therefore formulate the following empirical rule: the rotational anomaly does not occur in the decoupled band of an odd nucleus belonging to the subshell with high angular momentum j that is responsible for the phenomenon. We shall see later that this is not a rigorous rule. However, the connection between the rotational anomaly and decoupled bands can be regarded as experimentally confirmed with a reasonable degree of reliability.

C. Decoupled bands

Let us consider in greater detail the interaction between an odd nucleon, on the one hand, and deformation and rotation, on the other. We shall use for this purpose the widely used model of a particle coupled to an axially deformed nuclear core. The Hamiltonian for this system is

$$H = H_{sph} + \frac{(\mathbf{I}-\mathbf{j})^2}{2\mathcal{J}} + V, \tag{5}$$

where H_{sph} is the spherically symmetric mean field and V is the energy of interaction between the nucleon and the deformation, which is proportional to β. In the strong coupling approximation, in which the Coriolis interaction is small in comparison with V, the nuclear energy is given by

$$\mathcal{E}_{IK} = \frac{[I(I+1)-K^2]}{2\mathcal{J}} + \varepsilon_{jK},$$

where ε_{jK} is the energy of a nucleon in the subshell j of the deformed core. It is clear from the scheme shown in Fig. 5a that the component K is conserved in this case.

In the other limiting case in which $V \ll (\mathbf{I}\cdot\mathbf{j})/\mathcal{J}$, the Coriolis force decouples the angular momentum of the nucleus from the axis of symmetry and tends to align it with the axis of rotation. The result is a change in the addition scheme for the angular momenta (Fig. 5b). The total spin of the nucleus is now $\mathbf{I}=\mathbf{R}+\mathbf{j}$, where $\mathbf{R}$ is the angular momentum of the core. A similar phenomenon is observed in molecular spectra in which the phenomenon is restricted to the decoupling of only the electron spin, in accordance with Hund's rule.[31] The energy levels in the decoupled band are degenerate since the angular momentum $\mathbf{R}$ in the expression for the energy $\mathcal{E} = R(R+1)/2\mathcal{J}$ is determined from the addition rule for the angular momenta. The result is a system of bands (Fig. 6) in which the angular momentum j of the lowest band is either parallel to $\mathbf{R}(I=R+j)$ or antiparallel to it $(I=j-R)$. The first band is called favoured and the second antifavoured. The angular momentum $\mathbf{j}$ occupies an intermediate position in the other bands. The conserved quantity for this coupling scheme is the component α of the nucleon angular momentum along the rotational axis of the nucleus.

Rigorous diagonalization of the Hamiltonian (5) has been carried out by Vogel[32] and by Stephens.[33] The interaction between the nucleon and the deformation distorts the above picture of complete decoupling and the distortion increases with decreasing α. It follows that the favored and antifavored bands of odd nuclei with

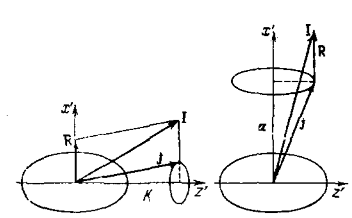

FIG. 5. Two ways of combining the angular momenta of a nucleon and a core.

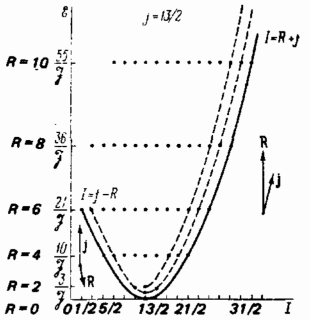

FIG. 6. Decoupled bands for the $j = 13/2$ subshell.

moderate spins ($I \lesssim 20$) can exist when the Coriolis interaction is strong. The necessary condition for this is that the nuclear deformation should be as small as possible and the odd nucleon should be in a state with large j and small component of the angular momentum along the nuclear symmetry axis. Such nuclei are encountered among the lanthanum isotopes ($\beta \lesssim 0.2$), in which protons begin to fill the $h_{9/2}$ subshell, or among the light isotopes of gadolinium, dysprosium, and europium ($\beta \approx 0.2$), in which neutrons begin to fill the $i_{13/2}$ subshell. The favored bands of odd isotopes of these elements have, in fact, been seen in the experiments performed by the Stephens group.[34] The separations between neighboring levels in these bands are the same as the corresponding separations in the ground-state band of the even-even nucleus. Analysis of the favored bands of odd nuclei in terms of the model involving a particle coupled to an axially deformed core has been given by Pyatov *et al.*[35]

The decoupled bands are observed in some intermediate nuclei between rare-earth elements and lead. For example, the favored band of Ir^{187} is based on the particle state of a proton from the $h_{9/2}$ subshell. In Hg^{193}, it is based on the hole state of a neutron from the $i_{13/2}$ subshell and, in Au^{195}, on the hole state of a proton from the $h_{11/2}$ subshell. Meyer-ter-Vehn[36] and Faessler[37] have analyzed these bands in terms of a model involving a particle coupled to a nonaxial core.

The favored band may exist in even nuclei as well. The angular momentum j is then equal to the angular momentum of the two-quasiparticle excitation. The favored band in even-even nuclei of rare-earth elements must, therefore, begin with spin $I = 12$ and energy $2\Delta \approx 2$ MeV. The rotational anomalies are observed for such spins and energies. Stephens and Simon[38] have therefore suggested that the superband is a favored band based on a two-quasiparticle excitation. The Stephens-Simon model is very simple and, therefore, exceedingly attractive to experimenters who have put forward experiments based upon this model. However, the model has some weak points as well. Despite the fact that the favored bands exist only in nuclei with small deformations, in which there are levels with large j and small K near the Fermi surface, the rotational anomalies are observed in Hf and W isotopes in which the $i_{13/2}$ subshell is completely filled. The favored bands have not been seen in odd isotopes of these elements. From the theoretical point of view, this model is essentially a single-particle approach to the two particles coupled to the core, and does not take into account the difference between the quasiparticles in the rotating and resting nuclei.

3. MICROSCOPIC THEORIES OF ANOMALY IN ROTATIONAL SPECTRA

A. Methods of describing rotational excitations in deformed nuclei

We shall now try to approach the problem of the rotational anomaly from a more general point of view. Rotation is a collective excitation of the nucleus and there is a number of microscopic models that can be used to describe it. All such models rely on the description of a nucleus as a system of nucleons moving in a self-consistent potential and interacting with one another through the residual interaction.

The forced rotation model, put forward in the early 1950's by Inglis,[39] is the most widely used. Here, the nucleus rotates classically with an angular velocity Ω. To calculate the rotational energy, we must transform to the rotating coordinate frame. The Hamiltonian for the axially deformed nucleus in this system is

$$H' = H - \Omega R_{x'}\,, \tag{6}$$

where H is the Hamiltonian for the nucleus at rest and $R_{x'}$ is the component of the angular momentum of the nucleus along the x' axis which is perpendicular to the symmetry axis (z'). The eigenvalue $\mathscr{E}'(\Omega)$ of the Hamiltonian given by (6) is the energy of the nucleus in the rotating coordinate frame, and can be used to determine its energy in the laboratory frame, namely, $\mathscr{E} = \mathscr{E}' - \Omega(\partial\mathscr{E}'/\partial\Omega)$, for given average angular momentum

$$\langle R_{x'}\rangle = -\frac{\partial\mathscr{E}'}{\partial\Omega} = \sqrt{I(I+1) - \langle R_{z'}^2\rangle}\,. \tag{7}$$

Thus, in the forced rotation model, the angular momentum of the nucleus is not a conserved quantity. The eigenfunction of H' is therefore a superposition of wave functions with different angular momenta.

For a relatively long period of time, the condition for the validity of the Inglis model was considered to be the quasiclassical nature of the rotation, i.e., high I. However, the generalized density matrix method developed by Belyaev,[40] in which rotation is described in such a way that the conservation of angular momentum is taken into account, has led to a more rigorous criterion for the validity of this model. It turns out that an important condition is the requirement that the nucleus be axial. Although it is possible, at least in principle, to consider a Hamiltonian such as that given by (6) even in the case of a nonaxial nucleus, Belyaev[41] has shown that, in the limit, the quantized rotation of the nucleus becomes identical for large values of I with the forced rotation model, but only for axial deformation. For an axial nucleus, the range of validity of the model is not restricted to the condition $I \gg 1$, since it is valid for small I by virtue of the fact that the rotation is adiabatic.

B. HFB equations for a rotating nucleus and methods for their solution

A simple pairing interaction for which the correlation energy Δ is constant throughout the nucleus (homogeneous pairing) is usually employed in the case of deformed nuclei. Numerical calculations have shown that the degree of pairing inhomogeneity in the ground state of a deformed nucleus does not exceed 5–7%, which agrees with the estimate $1/(\rho_F\Delta)\sim A^{-1/3}$. In this approximation, the Hamiltonian is

$$H=\sum_{\lambda}(\varepsilon_\lambda-\varepsilon_F)a_\lambda^+a_\lambda - \frac{G}{4}\sum_{\lambda,\lambda'}\gamma_\lambda\gamma_{\lambda'}a_\lambda^+a_{\bar\lambda}^+a_{\bar\lambda'}a_{\lambda'},\ \gamma_{\bar\lambda}=-\gamma_\lambda,\ |\gamma_\lambda|=1; \quad (8)$$

where a_λ^+ and a_λ are the creation and annihilation operators for nucleons in the state λ in the self-consistent field h with energy ε_λ, $\bar\lambda$ is the state conjugate in time, and G is the pairing interaction constant.

The HFB approximation[42] is commonly used to calculate the energy of a rotating nucleus in the forced rotation model. This is done by introducing the creation and annihilation operators α_ν^+ and α_ν for quasiparticles through the transformation

$$\psi(\mathbf{r})=\sum_\nu\{u_\nu(\mathbf{r})\alpha_\nu-\mathcal{R}_{y'}^{-1}v_\nu(\mathbf{r})\alpha_\nu^+\},$$

where $\psi(r)$ is the nucleon annihilation operator in second quantization and $\mathcal{R}_{y'}=\exp(-i\pi j_{y'})$ is the operator representing the rotation of the proper coordinate frame. The transformation amplitudes u and v are then determined from the equations

$$\begin{cases}(h-\varepsilon_F-\Omega j_{x'})u_\nu(\mathbf{r})-\Delta v_\nu(\mathbf{r})=E_\nu u_\nu(\mathbf{r}),\\(h-\varepsilon_F+\Omega j_{x'})v_\nu(\mathbf{r})+\Delta u_\nu(\mathbf{r})=-E_\nu v_\nu(\mathbf{r}),\end{cases} \quad (9)$$

where E_ν is the quasiparticle energy and $j_{x'}$ is the nucleon angular momentum operator.

The collective parameters of the nucleus, such as the Fermi energy ε_F, the deformation, the correlation energy Δ, and the angular momentum, are self-consistent quantities, i.e., they are determined from equations containing the amplitudes u and v. The fact that α_β in (4) is small for rotation frequency $\Omega\lesssim\varepsilon_F/A$ enables us to neglect changes in the deformation and ε_F under the influence of rotation because these quantities are determined in an energy range much greater than Δ. The function $\Delta(\Omega)$ is found from the equation

$$\Delta=\frac{G}{4}\sum_\nu(1-2n_\nu)\frac{\partial E_\nu}{\partial\Delta}, \quad (10)$$

which can be used[43] to determine the energy of the nucleus in the rotating coordinate frame

$$\mathscr{E}'=\frac{1}{2}\sum_\lambda\varepsilon_\lambda+\frac{\Delta^2}{G}-\frac{1}{2}\sum_\nu(1-2n_\nu)E_\nu, \quad (11)$$

where n_ν is the quasiparticle occupation number. In the ensuing analysis, we shall require the expression for the expectation value of the angular momentum of the quasiparticle excitation along the nuclear axis of rotation:

$$\langle j_{x'}\rangle_\nu=\int[u_\nu^*(\mathbf{r})j_{x'}u_\nu(\mathbf{r})+v_\nu^*(\mathbf{r})j_{x'}v_\nu(\mathbf{r})]\,d\mathbf{r}=-\frac{\partial E_\nu}{\partial\Omega}. \quad (12)$$

The total angular momentum of the nucleus is

$$I_{x'}=-\frac{\partial\mathscr{E}'}{\partial\Omega}=\frac{1}{2}\sum_\nu(1-2n_\nu)\frac{\partial E_\nu}{\partial\Omega}. \quad (13)$$

The function $\Omega(I)$ can be determined from this with the aid of (7).

The equations given by (9) are invariant under the transformations $\mathcal{R}_{x'}$ and $T\mathcal{R}_{y'}$ (T is the time reversal operator). The former invariance enables us to characterize the state of a particle by the signature σ, which, as noted by Goodman,[44] reduces the dimensionality of the basis space in the diagonalization of (9) by a factor of two. The invariance under the transformation $T\mathcal{R}_{y'}$ leads to the following expression for the quasiparticle energies:

$$E_{\nu,\sigma}=-E_{\nu,-\sigma}. \quad (14)$$

Since the model involving a superfluid nucleus with the Hamiltonian given by (8) provides a good description of the low-lying rotational states, it is natural to try to extend it to higher angular momenta in order to explain the above rotational anomalies. This is an exceedingly complicated problem because the parameter α_Δ in (3) is small for $I<10$, but is of the order of unity for high spins. The nonlinear HFB equations must be solved without introducing perturbation theory. Some qualitative results can be obtained from a rigorous solution of these equations for some model systems.[45–47]

The absence of a small parameter in the case of high-spin states forces us to seek numerical methods for the solution of the HFB equations. In 1973, Mang and Ring[48] found a solution of these equations, which had some similarity with the favored band of the Stephens-Simon model. The crossing of this band by the ground-state band corresponds to the vanishing of the energy of the two-quasiparticle excitation at the rotational frequency for which pairing correlation is still nonzero. The calculations of Mang and Ring were repeated by Goodman,[49] but not until 1976. Mang and Ring used the Kumar-Baranger Hamiltonian with homogeneous pairing and quadrupole-quadrupole interaction in the case of Er^{162} and $Yb^{166,168,170}$. Similar calculations were performed by Faessler[50] for Er^{162}. Goodman used the residual interaction of a more general form with the Reid potential. In recent calculations by Mang and Ring,[51] the residual Skyrme interaction was used for Yb^{162} and Er^{164}. These calculations are in good agreement with each other and, qualitatively, with experiment, although their complexity restricts the number of nuclei that could be treated. Moreover, none of the calculations has succeeded in reproducing the observed S-shaped dependence of the moment of inertia.

Despite these shortcomings, the numerical calculations have reproduced the main effect, namely, the vanishing of the energy of the quasiparticle excitation. This phenomenon resembles gapless superconductivity in superconductors of small size.[52] Qualitative considerations supporting the possibility of this phenomenon in a rotating nucleus have been reported by Lin *et al.* and by Grin'.[53] The physical picture of the phenomenon can be understood in terms of the isolated j-level model,[54] which can be solved analytically and is quite realistic.

C. Isolated j-level model

The strength of the Coriolis force acting on a nucleon in a rotating nucleus is proportional to the single-particle angular momentum j. Nucleons that experience the strongest coupling to rotation are those occupying levels with maximum j near the Fermi surface (for example, $i_{13/2}$ subshells for neutrons and $h_{9/2}$ for protons in the rare-earth nuclei). These levels are distinguished from other states in the shell being filled by their parity. It follows that j is a good quantum number for such states, since the admixture of states with other values of j, due to deformation and rotation, corresponds to transitions to a neighboring shell. Consequently, states in the subshell with maximum j may be looked upon as isolated.

For rotational frequencies $\Omega \sim \varepsilon_F/A$, rotational perturbation theory cannot be used for states in the subshell with maximum j near the Fermi surface. On the other hand, coupling between rotation and nucleons in other levels may be treated in terms of perturbation theory, since they have low values of j or lie well away from the Fermi surface. In the latter case, pairing can be neglected, and the perturbation theory parameter is α_B (4). Thus, the problem of solving the HFB equations in the space of all the single-particle states of the nucleus reduces to the much simpler problem of solving these equations in the space of a single subshell.

We shall now find the solutions of the HFB equations for an isolated j-level, taking the hexadecapole deformation into account. If we suppose that the self-consistent field of the nucleus is in the form of a potential well with infinite walls, the j-level splitting due to second- and fourth-order deformations is

$$\varepsilon_{njlm} = \varepsilon_{njl}\left\{1 + \beta_2\sqrt{\frac{5}{16\pi}}\,\frac{3m^2 - j(j+1)}{j(j+1)} - \frac{9\beta_4}{32\sqrt{\pi}}\left[\frac{35m^4 - 5(6j^2+6j-5)m^2}{(j-1)j(j+1)(j+2)} + 3\right]\right\},$$

where β_2 and β_4 are the quadrupole and hexadecapole deformation parameters and ε_{njl} is the energy of the j-level in a spherical field:

$$\varepsilon_{njl} = \varepsilon_{nl} - \varkappa\left[j(j+1) - l(l+1) - \frac{3}{4}\right], \quad \varepsilon_{nl} = \frac{14.39\chi_{nl}^2}{A^{2/3}}, \tag{15}$$

where χ_{nl} is the root of the corresponding Bessel function and $\varkappa = 0.37$ MeV is the spin-orbit coupling constant. The expression for ε_{njlm} enables us to write the HFB equations (9) for the isolated j-level in the following form:

$$\left\{\begin{array}{l}(\epsilon_j + \delta j_{z'}^2 - 4\delta' j_{z'}^4 - \Omega j_{x'})\,u - \Delta v = Eu,\\ (\epsilon_j + \delta j_{z'}^2 - 4\delta' j_{z'}^4 + \Omega j_{x'})\,v + \Delta u = -Ev.\end{array}\right. \tag{16}$$

The parameters in these equations are as follows:

$$\begin{gathered}\delta = \delta_2\left[1 + 3\sqrt{5}\,\beta_4\,\frac{6j(j+1) - 5}{8\beta_2(j-1)(j+2)}\right],\\ \delta' = \frac{315\beta_4\varepsilon_{njl}}{128\sqrt{\pi}(j-1)j(j+1)(j+2)},\\ \delta_2 = \sqrt{\frac{45}{16\pi}}\,\frac{\beta_2\varepsilon_{njl}}{j(j+1)},\\ \epsilon_j = \varepsilon_{njl} - \varepsilon_F - \frac{1}{3}j(j+1)\left[\delta_2 + \frac{38\delta'(j-1)(j+2)}{35}\right].\end{gathered} \tag{17}$$

The equations given by (16) contain the large parameter $j \sim A^{1/3}$. The quasiclassical approximation must therefore be used to solve them. We shall use the one-dimensional complex realization of the group SU(2) for this purpose.[55] The angular momentum operators in this representation depend on a single complex variable. The set of partial differential equations in (16) therefore reduces to an ordinary differential equation, whose solution can be found by the method described in Ref. 54. It relies on quantization conditions that can be used to determine the quasiparticle excitations in the subshell, depending on the parameters β_2, β_4, ε_F, Ω and the pairing correlation energy Δ. The resulting solutions will be discussed at the beginning of the next Section. The isolated j-level model will be compared with experiment at the end of Section 4.

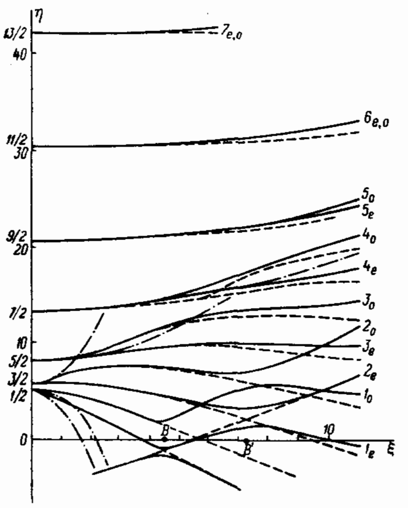

FIG. 7. Quasiparticle levels ($\eta = 4E/3\delta_2$, $\xi = 4\Omega/3\delta_2$) in a rotating nucleus for $j = 13/2$, $\epsilon_j = \beta_4 = 0$, $\Delta/\delta_2 = 5$.[54] Figures on the left represent the asymptotic number K; figures on the right are the values of the quantum number ν of the quasiparticle excitation. Solid line—accurate solutions, broken lines—quasiclassical calculations, dot-dash lines—perturbation theory.

The dependence of the quasiparticle excitations in the $i_{13/2}$ subshell on Ω at constant Δ is shown in Fig. 7. The quantity Δ can itself depend on the rotational frequency although this dependence is relatively weak for $\Omega \lesssim \varepsilon_F/A$. The function $\Delta(\Omega)$ can therefore be found from perturbation theory. We shall use the results of numerical calculations to illustrate the adequacy of perturbation theory in this connection. The maximum rotational frequency observed in Dy^{158} and Er^{164} is 360 keV ($I = 22$). The reduction in Δ relative to the value Δ_0 at $\Omega = 0$ is not more than 18%. The average maximum value for the rare-earth elements is $(\Delta_0 - \Delta)/\Delta_0 = 0.11$.

4. QUASIPARTICLE EXCITATIONS IN THE ROTATING NUCLEUS

We must now analyze the solution of the HFB equations for the rotating nucleus. As already stated, this solution can be used to determine the rotational spectrum of the nucleus and, consequently, the function $\mathcal{J}(\Omega^2)$ which is usually compared with experiment. However, the phenomenon of rotational anomaly is dominated by quasiparticle excitations from the high-j shell near the Fermi surface. As the rotational frequency of the nucleus increases, the energy of the low-lying excitations in this subshell is found to fall, which is characteristic for a small superconductor. This reduction in the energy of the quasiparticle excitation enables us to explain the crossing of the ground-state band by the two-quasiparticle band.

In view of the foregoing, we shall compare experimental data with the energy E_ν of the low-lying quasiparticle excitations as a function of the nuclear rotational frequency Ω. This will distinguish our approach from the traditional method of describing rotational anomalies. We note that this approach to the role of quasiparticle excitations in the backbending phenomenon was used independently in Refs. 56 and 57. We shall show at the end of this section that quasiparticle excitations in the rotating nucleus are of independent interest in the study of the structure of deformed nuclei.

A. Properties of quasiparticle terms

We shall refer to the quasiparticle energy $E_\nu(\Omega)$ as the quasiparticle term. In the qualitative analysis of the behavior of quasiparticle terms, we shall assume that Δ is constant. Quasiparticle excitations in the deformed nucleus can be characterized by quantum numbers $K^\pi[Nn_z\Lambda]$. Neither the component of the angular momentum along the nuclear symmetry axis nor the asymptotic quantum numbers N, n_z, Λ are good quantum numbers in the rotating nucleus. Only two quantities are conserved in this case, namely, the spatial parity π and the $\mathcal{R}_{x'}$-parity represented by the quantum number σ. Other quantum numbers can occasionally be usefully employed as asymptotic quantum numbers corresponding to the quasiparticle in the nonrotating nucleus. Moreover, in accordance with our discussion in the last Section, we can use the quantum number j to characterize the quasiparticle excitation. We shall see below that all quantum numbers in the quasiparticle term are determined by the single-particle level on which the rotational band in the odd nucleus is based.

The characteristic feature of the function $E_\nu(\Omega)$ is that the energy of the lowest quasiparticle excitation is equal to zero for a certain value of the nuclear rotation frequency. We have already noted that a similar phenomenon is observed in superconductors of small linear dimensions. The Coriolis force in the nucleus tends to orient the angular momenta of nucleons in a pair along the rotational axis. The attendant reduction in the correlation energy is compensated by an increase in the energy associated with the Coriolis interaction. The energy of the lowest quasiparticle excitation is therefore given by

$$E_{1e} \approx \Delta - \Omega \langle j_{x'} \rangle.$$

The quantity $\langle j_{x'} \rangle$ increases with increasing j and decreasing K, so that the quasiparticle energy vanishes at lower rotational frequency.

This is essentially the end of the analogy between the phenomena in a rotating nucleus and gapless superconductivity. At the point Ω_0 at which $E_{1e}=0$, the quasiparticle vacuum in the superconductor must be modified. Unless this is done, the quasiparticle energy will become negative for $\Omega > \Omega_0$. The quasiparticle excitation to the right of Ω_0 can be formed with the aid of (14). The particle is then replaced by a hole or vice versa, so that the total number of particles changes by one. Since the properties of odd and even nuclei are fundamentally different, we must not mix the states of these nuclei. The even nucleus can support only two quasiparticle excitations, and we are interested in the change in the sign of this excitation. A change in the sign of a one-quasiparticle excitation in the nucleus does not lead to any physical consequences because of the conservation of the parity of the number of particles.

Rotation splits the quasiparticle terms with opposite $\mathcal{R}$-parity. The splitting increases with increasing Ω and decreasing asymptotic quantum number K. This can be understood by noting that the wave function of a nucleon with particular $\mathcal{R}$-parity in the nonrotating nucleus is

$$\psi_{K\sigma,0} = \frac{1}{\sqrt{2}}\left(|jK\rangle \pm e^{i\pi\left(j+\frac{1}{2}\right)}|j,-K\rangle\right).$$

The operator $j_{x'}$ mixes the states $\psi_{1/2e}$ and $\psi_{1/2o}$. Consequently, states with $K=1/2$ split in first-order perturbation theory, those with $K=3/2$ split in the second order, and so on. As the asymptotic quantum number K increases, the term splitting becomes smaller. It therefore decreases as ε_F increases at constant β and Δ, so that states with large K are found near the Fermi surface. Negative hexadecapole deformation will partially compensate this effect. We shall consider this later.

The expectation value of the angular momentum of a quasiparticle excitation along the nuclear axis of rotation is given by (12). It can be shown that $\langle j_{x'} \rangle \sim j$ for the low-lying quasiparticle states if K is small. Thus, the angular momentum of the quasiparticle excitation is favored along the rotational axis for nuclei in which subshells with high j begin to be filled. This limit corresponds to the Stephens-Simon model. It is important to note that the range of validity of (12) is restricted on the side of low values of Ω because we are using the forced rotation model. Moreover, this expression is not valid in the region of the anomalies where states with different $\langle j_{x'} \rangle$ are mixed.

The crossing of quasiparticle terms corresponds to the crossing of rotational bands. It is therefore exceedingly important to understand what happens near these crossing points. It is clear from Fig. 7 (points B and B') that rigorous calculations lead to an interaction between quasiparticle terms with the same $\mathcal{R}$-parity, as expected. For lower-lying terms, this interaction is small because of the slight overlap between the wave functions of the favored states.

In the quasiclassical approximation, the interaction between the terms is exponentially small, and need not be taken into account accurately. The quantization condition from which the spectrum of quasiparticle energies $E_\nu(\Omega)$ is determined has the following form when the exponentially small terms are neglected:

$$\oint p(x,E,\Omega)\,dx = \pi(\nu+\gamma). \tag{18}$$

Hybridization of the terms occurs when they interact, i.e., a term with quantum number ν up to the crossing point is converted into a term with quantum number ν' beyond this point. The quasiclassical condition given by (18) cannot describe this phenomenon because the left-hand side is an adiabatic invariant and each level is characterized by quantum number ν, which is independent of Ω. In the quasiclassical approximation, therefore, the terms can only cross.

B. Crossing of rotational bands

The dependence of quasiparticle energies on the nuclear rotational frequency enables us to understand the reason for the anomaly of the rotational spectrum. We shall consider that the subshell with large j lies near the Fermi surface. In the expression for the energy of the nucleus in the rotating coordinate frame, given by (11), the term $-\frac{1}{2}\sum_\nu E_\nu$ can be conveniently interpreted as the quasiparticle vacuum corresponding to the filling of all the quasiparticle levels with negative energy in the ground state of the even-even nucleus. A band based on this state will be the fundamental band.

Let us now form the lowest two-quasiparticle excitation from the subshell with large j in the even-even nucleus (Fig. 8). To do this, we must transfer two quasiparticles from the higher-lying vacuum states to the lower quasiparticle states. The band based on this state will be the two-quasiparticle superband. Its energy will differ from the ground-state band by the amount $E_{1e}+E_{1o}$ which decreases with increasing rotational frequency. It vanishes for $\Omega=\Omega_0$. At this point, the ground-state band $\mathscr{E}_{g.}(\Omega)$ crosses the superband $\mathscr{E}_2(\Omega)$. Since, in each band, there is a unique relationship between spin and frequency, which is given by (7), the observed bands $\mathscr{E}_{g.}(I)$ and $\mathscr{E}_2(I)$ will cross.

The ground-state band and the superband have different internal structures. The component of the angular momentum of nucleons in the former band will be smaller since the terms in the sum $\sum_\nu(\partial E_\nu/\partial\Omega)$ in (13) tend to cancel out. The angular momentum in the direction of the rotational axis in the second band is largely connected with the two quasiparticles. This is the favored band. The overlap integral between the wave functions of the two bands is small, and this should lead to the observed weak coupling between the bands.

Each two-quasiparticle excitation in the subshell with anomalous parity in the even nucleus corresponds to its own superband. According to the general rule, it contains even or odd spins, depending on whether the $\mathscr{R}$-parities of the quasiparticle excitations are different or the same. This distinguishes the superband from the band based on the usual two-quasiparticle excitation with particular K and even or odd I. The favored part of the superband begins with $I=2j-1$, and its states are characterized by small values of K or, more precisely, $\langle R_{\xi'}\rangle$, since K is not a good quantum number. This is why transitions between the SB levels and the ground-state band should not be forbidden.

Only the lower parts of the crossing bands have been detected for the majority of nuclei for which anomalies of the rotational spectrum have been observed. There are three exceptions. In Dy^{156}, the SB is crossed by the β-band for $I=10$ and by the ground-state band for $I=16$.[9,58] A similar band-crossing picture is observed in Gd^{154}, except that the SB crosses the ground-state band for $I=18$.[59] Three superbands have been seen[60] in Er^{164}. The lowest lying corresponds to the excitation $E_{1e}+E_{1o}$ and is associated with the $i_{13/2}$ subshell; it has been identified for spins in the range 12–24. It crosses the ground-state band for $I=14$. The odd-I superband corresponding to the excitation $E_{1e}+E_{2e}$ and the even-I superband corresponding to the excitation $E_{1e}+E_{2o}$ cross the γ-band for $I\sim14$. The maximum spin recorded in these bands is 21 and 18, respectively. In addition to these superbands, the rare-earth nuclei exhibit negative-parity bands based on excitations from the $i_{13/2}$ subshell and another subshell with a lower j and opposite parity. These bands will be considered below.

Recent experiments[61,62] have established the existence of a second backbending of the moment of inertia in the ground-state band of Er^{158} and Yb^{160} for $I\sim28$. It is possible that this is connected with a superband based on the proton two-quasiparticle excitation from the subshell $h_{9/2}$. The second anomaly in Er^{158} has been explained[63] by the repeated crossing of the ground-state band by the SB.

Consider an odd nucleus in which the odd particle is in the lowest quasiparticle excitation from the subshell with anomalous parity. This means that the $1o$ level with negative energy is free, whereas $1e$ with positive energy is occupied (Fig. 9). The band based on this

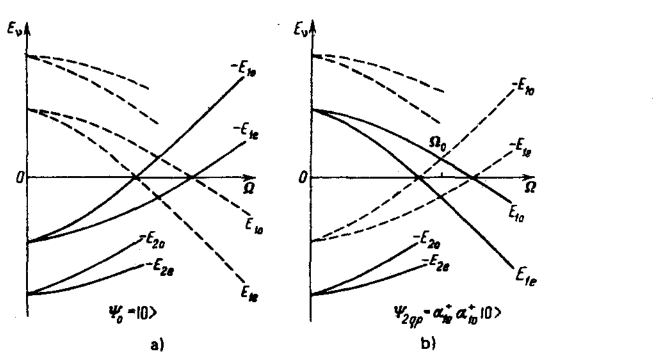

FIG. 8. Filling of quasiparticle levels for the groundstate band (a) and SB (b) in an even-even nucleus.

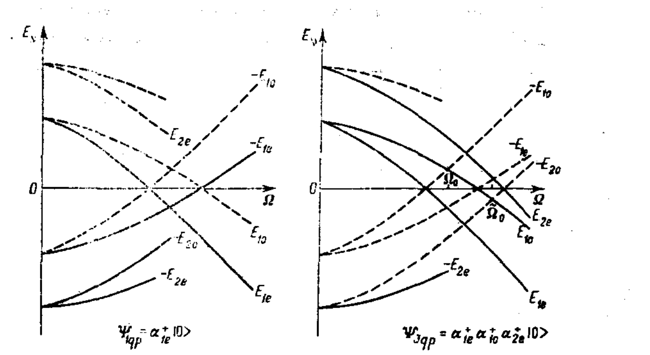

FIG. 9. Filling of quasiparticle levels for crossing bands of an odd nucleus.

state will have even values of $I-1/2$. The band based on the lowest three-particle excitation corresponds to the lowest SB in the odd nucleus. The energy of this SB differs from the energy of the original band by the amount $\mathscr{E}_{1o}+E_{2e}$, which vanishes at frequency $\tilde{\Omega}_0>\Omega_0$. Rotational anomalies in odd nuclei should therefore be observed for high spins.

If the odd particle occupies the state from another subshell, all the negative-energy quasiparticle levels belonging to the subshell with the anomalous parity are filled. The lowest SB can therefore be constructed from the lowest quasiparticle excitations $1e$ and $1o$. Consequently, rotational anomalies in this band should be observed for the same spins as in the even nucleus. This explains the empirical rule stated in the previous Section. Departures from this rule have been observed both in anomalous parity bands and in ordinary bands. For example, in $Yb^{161,163}$, the anomaly in the band based on the excitation from the subshell $i_{13/2}$ has been seen at somewhat higher frequencies than in the neighboring isotopes $Yb^{160,162}$. The same nuclei show the anomaly in the $5/2^-[523]$ band, but the anomaly is not present in the $3/2^-[521]$ band of $Yb^{163,164}$. In $Yb^{165,167}$, the anomaly is absent from the band associated with the subshell $i_{13/2}$, whereas, in the $5/2^-[523]$ band, it has been seen[64] at the same frequencies as in the neighboring $Yb^{164,166}$. However, in $Lu^{167}(Yb^{166}+\text{proton})$, the anomaly has not been seen in the favored band $1/2^-[541]$, which is associated with the proton subshell $h_{9/2}$, whereas, in the $7/2^+[404]$ band, it was observed at the same frequency as in $Yb^{166,165}$. In $Tm^{165}(Yb^{166}-\text{proton})$, the anomalies have been seen in the $7/2^-[523]$ (favored band from the $h_{11/2}$ subshell), $7/2^+[404]$, and $1/2^+[411]$ bands at the same frequencies as in the ground-state band of Er^{164}. However, the anomalies are not present in the $1/2^-[541]$ favored band from the $h_{9/2}$ subshell.[66] Finally, in the $1/2^-[541]$ band (proton subshell $h_{9/2}$) of Ir^{183}, anomalies are present but, in Ir^{185}, they are not.[67] These departures correspond to transition nuclei and nuclei close to them. The most likely explanation is that the odd particle produces an appreciable change in the internal structure of some states.

The crossing of the SB by the ground-state band is due to the crossing of the quasiparticle terms with the same signature. Consequently, the interaction between the bands is due to the interaction between the terms. When the HFB equations are rigorously diagonalized, this interaction leads to the hybridization of rotational bands at constant rotational frequency and not of the nuclear angular momentum. This is why there is a large fluctuation in the angular momentum in the crossing region, since different angular momenta appear in different bands at the same frequencies. This was first pointed out by Hamamoto[68] and was subsequently examined in detail by Marshalek,[69] Faessler,[70] and Goodman.[71] It follows that the HFB approximation cannot be used in the hybridization region. New methods that take into account the conservation of angular momentum must be introduced in this region.

Band hybridization does not occur in the isolated j-level model based on the quasiclassical approximation because there is no interaction between the quasiparticle terms. In this sense, the model is more satisfactory, especially since the absence of the interaction has practically no effect on the precision of the calculated energies of levels in crossing bands because the interaction is so small that the perturbed energies differ from the unperturbed energies only for two states on either side of the crossing point.

Nevertheless, the interaction between the bands is important if we are to achieve the correct description of the shape of the $\mathscr{J}(\Omega^2)$ curve in the band hybridization region, and if the backbending phenomenon is to be correctly predicted. This interaction is not constant. Estimates[72] show that the interaction between the ground-state band and the superband oscillates, depending on the degree of filling of the subshell. It is possible that this effect explains the absence of rotational anomalies in nuclei with neutron number $N=98$. However, this problem lies outside the scope of the forced rotation model.

C. Experimental and theoretical values of quasiparticle terms

The dependence of the quasiparticle excitation energy on the nuclear rotational frequency can be deduced from the observed rotational band energies. To do this, let us transform $\mathscr{E}(I)$ into $\mathscr{E}'(\Omega)$ with the aid of the following formulas:

$$\mathscr{E}'(I)=\mathscr{E}(I)-\Omega R_{x'}(I),\quad \Omega(I)=\frac{\mathscr{E}(I)-\mathscr{E}(I-2)}{R_{x'}(I)-R_{x'}(I-2)},$$
$$R_{x'}(I)=\sqrt{I(I+1)-\langle R_{z'}^2\rangle}. \tag{19}$$

Naturally, a given value of I will correspond to different Ω in different bands.

Let us consider an odd nucleus formed from an even-even nucleus with N neutrons (protons) by adding one particle to the state λ in the subshell with anomalous parity. If $\mathscr{E}_{e,o}$ are the level energies in a band with a particular spin sequence (even or odd $I-1/2$), measured from the energy of the state λ, and $\mathscr{E}_{gs}$ is the level energy in the ground-state band of the even-even nucleus, the energy of the quasi-particle excitations can be found from the following expressions:

$$E_{1e}=E_\lambda+\mathscr{E}'_e(\Omega)-\mathscr{E}'_{gs}(\Omega),\quad E_{1o}=E_\lambda+\mathscr{E}'_o(\Omega)-\mathscr{E}'_{gs}(\Omega). \tag{20}$$

The quantities $\mathscr{E}'$ and Ω are given by (19) with $\langle R_{z'}^2\rangle=K^2$, and E_λ is the quasiparticle energy at $\Omega=0$. If the band is based on an excited state of the odd nucleus with $\mathscr{E}_\lambda$ then $E_\lambda=E_{\lambda_0}+\mathscr{E}_\lambda$. The energy of the quasiparticle corresponding to the ground state can be deduced from the nuclear binding energy[73]

$$E_{\lambda_0}=\mathscr{E}(N+1)-\mathscr{E}(N)-\mu(N),\quad \mu(N)=\frac{1}{2}[\mathscr{E}(N+2)-\mathscr{E}(N)].$$

In calculations, it is best to replace $\mu(N)$ by the average over neighboring nuclei with $\Delta N=2$.

Thus, two quasiparticle terms can be extracted from each band of the odd nucleus. The expressions given by (20) contain the energies of the even and odd nuclei. When experimental and theoretical energies of quasiparticle excitations are compared, it is necessary to remember that these nuclei may have different values of collective parameters such as deformation, pairing, and so on. If we neglect this difference, the single-quasiparticle excitations will be identical in the even and odd nuclei.

Multi-quasiparticle excitations can naturally be looked upon, in the first approximation, as superpositions of noninteracting quasiparticles. They can be extracted from the rotational bands of even or odd nuclei, since the number of quasiparticles is even or odd. Thus, the energy of the two-quasiparticle excitation is given by

$$E_{2qp} = \mathscr{E}'(\Omega) - \mathscr{E}'_{gs}(\Omega), \tag{21}$$

where $\mathscr{E}'$ is obtained from the level energies in the SB with particular parity of the spin I.

A. Bohr has pointed out[56] that experiments will also yield the angular momentum of the quasiparticle excitation in the direction of the nuclear axis of rotation

$$\langle j_{x'} \rangle = R_{x'}(\Omega) - R_{x' gs}(\Omega), \tag{22}$$

where $R_{x'}$ is the component of the nuclear angular momentum along the axis of rotation for the band based on the quasi-particle excitation and $R_{x' gs}$ is the corresponding component for the ground-state band of the even-even nucleus.

To compare the theoretical values of the quasiparticle terms calculated in the isolated j-level model with experimental values, we must know Δ_0, ε_F, β_2, and β_4. These model parameters must be determined from experiment. The pairing correlation energy Δ_0 in the ground state is determined from the nuclear binding energies. In calculations, one uses not the Fermi energy but the quantity ε_j (17), which is deduced from the excitation energy E_λ, as follows:

$$\varepsilon_j = -\delta K^2 + 4\delta' K^4 \pm \sqrt{E_\lambda^2 - \Delta_0^2}, \tag{23}$$

where the positive sign corresponds to particle and the negative sign to hole excitation. The expression given by (23) is incorrect when K is not a good quantum number for the excitation λ. The energy ε_F can then be roughly estimated from the occupation of the single-particle levels (cf. the nucleus Dy^{157}). Experimental values of these deformation parameters are used in calculations. If the hexadecapole deformation is not known, theoretical values of β_2 and β_4 are taken. The theory does not, therefore, have any free parameters. The blocking effect and the change in deformation between even and odd nuclei are not taken into account. Single-quasiparticle excitations are therefore ascribed to even nuclei.

The splitting of levels in a subshell, which is determined by δ and δ' in (17), depends on the shape of the self-consistent field in the nucleus. All calculations have been performed for an infinite, rectangular, potential well. The j-level energy ε_{nlj} in a spherically symmetric potential, which appears in δ and δ', can be calculated from (15). We note that the contribution of spin-orbit splitting to ε_{nlj} does not exceed 4%. For the Saxon-Woods potential, the parameters δ and δ' are, on the average, lower by 30% as compared with the rectangular well.

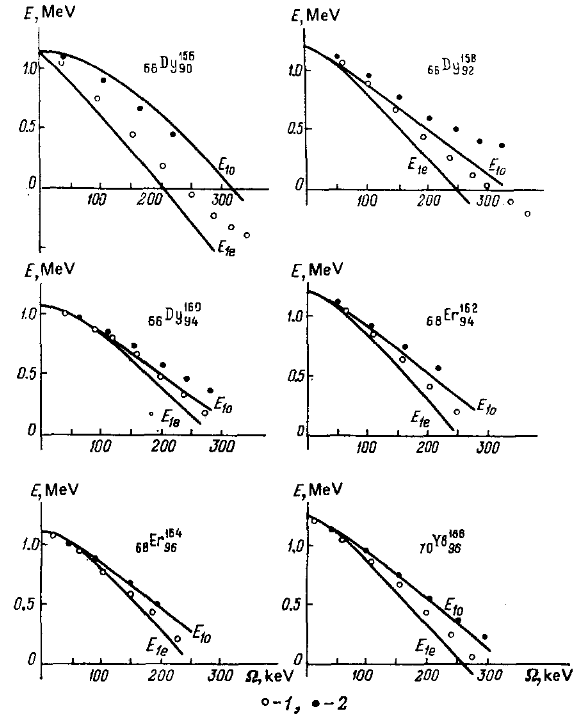

FIG. 10. Single-quasiparticle neutron term $5/2^+(642)$. The experimental points were deduced from the odd-nucleus band: 1—I-1/2-even; 2—I-1/2-odd.

D. Quasiparticle terms and the structure of deformed nuclei of rare-earth elements

Comparison of experimental and theoretical values of quasiparticle terms enables us to understand some of the structural features of deformed nuclei. Let us illustrate this by considering the example of neutron quasiparticle excitations in rare-earth nuclei.

We shall begin with more or less well-known facts. Figure 10 shows theoretical (solid lines) and experimental values of the $5/2^+[642]$ quasiparticle terms in the $i_{13/2}$ subshell. The exception is Dy^{156}, for which there is a $K=3/2$ excitation. Table 1 lists the parameter values, used in the calculation, and a characterization of the base level in the rotational band of the neighboring odd nucleus, used to determine the quasiparticle energy. It is clear from Fig. 10 that the energy E_{1e} of the lowest excitation in $Dy^{156,158}$ does become equal to zero.

This is a direct experimental confirmation of the fact that the nucleus is a superconductor of small size. The curves of Fig. 10 reproduce all the features of quasiparticle terms noted in Subsection A, above. We note

TABLE I.

Nucleus	Δ_0, MeV	β_2	β_4	Band
Dy^{156}	1.14	0.30	0 74	$3/2^+$, 235 keV, Dy^{157}
Dy^{158}	1.00	0.33	0 74	$5/2^+$ [642], 178 keV, Dy^{159}
Dy^{160}	0.97	0.33	0 74	$5/2^+$ [642], gs Dy^{161}
Er^{162}	1.06	0.32	0 74	$5/2^+$ [642], 69 keV, Er^{163}
Er^{164}	1.01	0.31	0 74	$5/2^+$ [642], 47 keV, Er^{165}
Yb^{166}	1.12	0.30	0 75	$5/2^+$ [642], 30 keV Yb^{167}

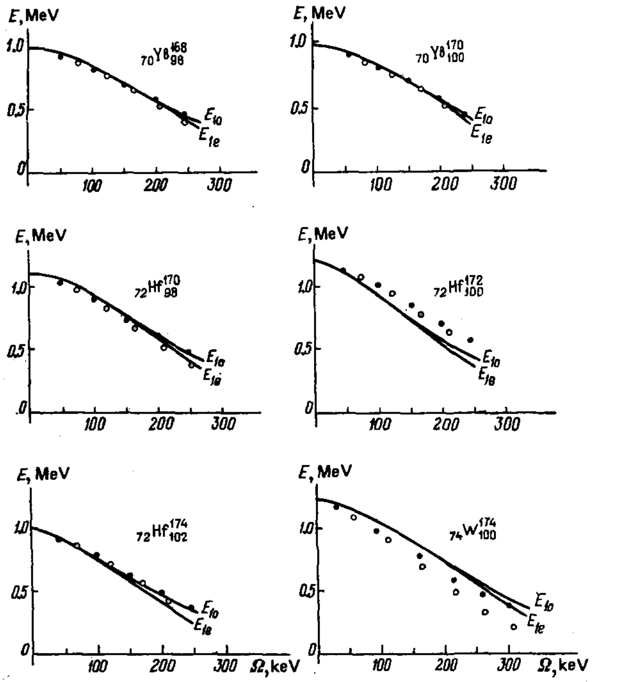

FIG. 11. Single-quasiparticle neutron terms $7/2^+[633]$. The notation is the same as in Fig. 10.

that the agreement between theory and experiment is worse for dysptosium than for the heavier nuclei. The smaller experimental slope of the quasiparticle terms can be explained by the admixture of states from the $d_{3/2}$ subshell of the $N=4$ shell to the $i_{13/2}$ subshell of the $N=6$ shell.

The component of the angular momentum of the quasiparticle excitation along the axis of rotation of these nuclei, determined in accordance with (22), is 40–50% of the magnitude of j, and is greater for the $\mathscr{R}$-even excitations. The theoretical values of this quantity, calculated from (12), are in good agreement with experiment.

The single-particle state $7/2^+[633]$ is observed in heavier nuclei of the rare-earth elements (Fig. 11). If the theoretical curves are calculated as for the excitations with $K=5/2$, assuming $\beta_4=0$, they are found to lie above the experimental points. This discrepancy is due to the negative hexadecapole deformation (Table II).

The hexadecapole deformation of the rare-earth nuclei is positive at the beginning of the region and zero for the Er and Yb isotopes. It becomes negative towards the end of the region. The negative hexadecapole deformation modifies the splitting of the j-level by reducing the energy of states with moderate m. The magnitude of the energy splitting in a subshell is practically unaltered thereby. The contribution of states with low values of m to the energy of the lowest quasiparticle excitation is therefore increased.

TABLE II.

Nucleus	Δ_0, MeV	β_2	β_4	Band
Yb^{168}	0.95	0.26	0 [76]	$7/2^+$ [633], gs, Yb^{169}
Yb^{170}	0.81	0.26	−0.02 [76]	$7/2^+$ [633], 95 keV, Yb^{171}
Hf^{170}	1.02	0.28	−0.01 [77]	$7/2^+$ [633[, gs Hf^{171}
Hf^{172}	0.94	0.29	−0.01 [77]	$7/2^+$ [633], 197 keV, Hf^{173}
Hf^{174}	0.74	0.30	−0.02 [77]	$7/2^+$ [633], 207 keV, Hf^{175}
W^{174}	0.92	0.25	−0.02 [78]	$7/2^+$ [633], 235 keV, W^{175}

The hexadecapole deformation is still greater in nuclei in which the $9/2^+[624]$ state is observed. The slope of the quasiparticle terms and their splitting is not, therefore, very different from that obtained for $K=7/2$.

An appreciable discrepancy between theory and experiment is obtained for $Hf^{172,176}$ and $W^{174,178,180}$. The first and the simplest explanation of this discrepancy is that incorrect deformation parameters were used in the calculations. The experimentally determined hexadecapole deformation for these nuclei is unknown, so that theoretical values of β_2 and β_4, taken from Refs. 77 and 78, had to be used. The second explanation is more interesting. It suggests that even and odd nuclei have different deformations.[1)] This hypothesis is exceedingly simple to verify. All that is needed is to compare the sum of quasiparticle excitations $1e$ and $1o$ with the two-quasiparticle excitation $E_{1_e}+E_{1_o}$ deduced from the SB of the even-even nucleus. When these two experimental values do not agree, the odd particle must modify the nuclear deformation. Still greater discrepancy between theory and experiment has been obtained for the quasiparticle term $11/2^+[615]$ of Os^{186}.

Calculations of the equilibrium deformation of the above nuclei[77,78] show that the isotopes $Hf^{172,176}$ are well-deformed nuclei. The isotopes $W^{178,180}$ lie nearer to the transition region. Finally, W^{174} and Os^{186} are soft to quadrupole deformation. It would appear, therefore, that the concept of static deformation does not apply to these nuclei.

Figure 12 shows the two-quasiparticle excitations for nuclei for which the SB's are well known. When the quasiparticle energies were determined from (21), the result was corrected for the interaction between the bands to ensure that the experimental points corresponded to the noninteracting bands. For comparison, the figure also shows the lowest single-quasiparticle excitations.

There is one further type of band from which two-quasiparticle excitations can be deduced. These are the negative-parity bands with even or odd spin sequence, which have been seen both in deformed and transition nuclei. In the rare-earth nuclei, these bands are based on two-quasiparticle excitations from subshells with different spatial parity: $i_{13/2}$ and $h_{9/2}$ (or $f_{7/2}$) for neutrons and $h_{9/2}$ and $d_{5/2}$ for protons. Alternatively, these rotational bands can be described as octupole bands. For light rare-earth nuclei, octupole vibrations with $K=1,2,3$ lie well above vibrations with $K=0$. This can explain the preferential population of negative-parity bands with odd spin sequences in these nuclei.

The quasiparticle terms enable us to elucidate the

[1)]The difference between the values of Δ_0 for odd and even nuclei is small and cannot explain the discrepancy because the quasiparticle terms are slowly-varying functions of the pairing energy.

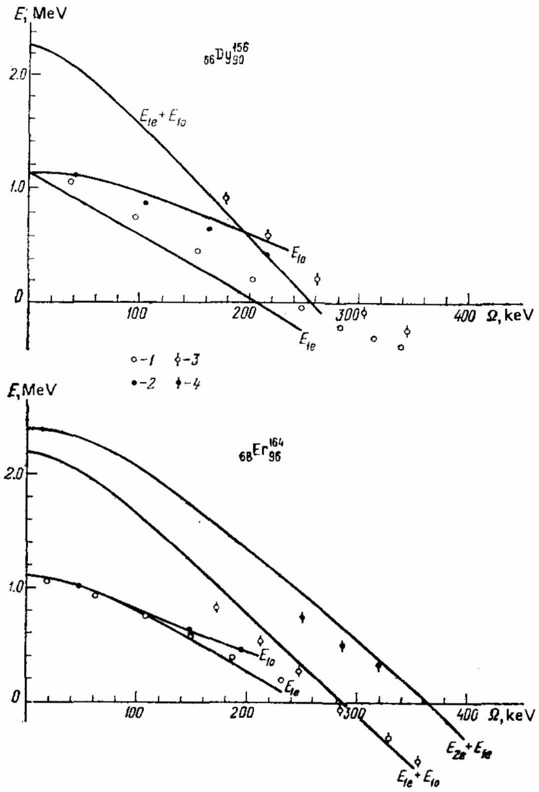

FIG. 12. Two-quasiparticle neutron excitations in Dy^{156} and Er^{164}. The experimental points were determined from bands of (a) odd nucleus: 1—I-1/2-even, 2—I-1/2-odd and (b) even nucleus: 3—lowest SB, I-even, 4—SB with odd I.

nature of the negative-parity bands. For example, the theoretical two-quasiparticle terms in the neighboring neutron subshells $i_{13/2}$ and $h_{9/2}$ of Dy^{156} and Er^{162} have a much greater slope than the experimental terms. Consequently, these bands are based on a collective excitation. The average angular momentum of this type of excitation is much smaller than j_F, which means that the excitation energy is not very dependent on Ω. A completely different picture is observed in Er^{156} which has two bands with odd spins and $\pi=\pm 1$. The corresponding quasiparticle terms are close to the neutron two-quasiparticle excitations from the $i_{13/2}$ subshell and the $i_{13/2}$ $(\pi=+1)$ and $h_{9/2}$ $(\pi=-1)$ subshells. Vogel's calculations[79] support the quasiparticle origin of the negative-parity band of Er^{156}.

5. CONCLUSIONS

During the six years between 1971, when the anomaly of the rotational spectrum was discovered, and 1977, when the second backbending region was found in the ground-state band of Er^{158}, the attention of the majority of nuclear physicists was drawn to rotational excitations of atomic nuclei. The original phase transition hypothesis, which was the starting point of work on high-spin rotational states, has had to be rejected and replaced by the Stephens-Simon hypothesis about the existence of quasiparticle excitations in even nuclei, whose angular momenta lie along the nuclear axis of rotation.

This semiquantitative theory has been widely used by experimenters. On the other hand, theoreticians have sought solutions of the HFB equations corresponding to the Stephens-Simon model. Such solutions were first found by Mang and Ring by numerical integration of these equations. Analytic solutions were found later. It turns out that the Stephens-Simon model is a limiting case of highly favored angular momentum of quasiparticle excitations. Up to this point in time, this approach has led to the elucidation of the main features of the phenomenon (although there are other explanations of the rotational anomaly based on the catastrophe theory[80] or on a phase transition to a more symmetric coupling scheme for the quasiparticle angular momentum[81]). However, there are several unresolved problems to which attention must be drawn. They are:

1) The reason for the departures from the general behavior in certain bands of odd nuclei, which are described in Sec. 4b, remains unclear; if these departures are connected with a change in the deformation or some other collective parameters of the nucleus in the excited state, the anomalies may help in the investigation of this phenomenon.

2) The reason for the appearance of the second backbending in the function $\mathcal{J}(\Omega^2)$ is not clear.

3) The rotational anomalies have not been detected in the actinides although the $j_{15/2}$ subshell begins to be filled in these nuclei; new experimental techniques for the excitation of rotational bands are necessary in this region.

So far, there is no quantitative theory of the S-shaped behavior of the moment of inertia or of the criterion for its existence. Modern theory is unable to describe the probabilities of transitions between high-spin states in a given band or between bands. The nature of rotational anomalies in transition nuclei is still less understood. Because of the softness of these nuclei, rotational and quasiparticle excitations are accompanied by other collective phenomena. Finally, we note that studies of rotational anomalies began with the question whether a phase transition from the superfluid state to the normal state occurs in the rotational band of deformed nuclei. This remains an open question.

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