Interaction of electrons with the electromagnetic field in free electron lasers

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The experimental and theoretical work on free-electron lasers is reviewed. Different amplification schemes and electron scattering mechanisms, the relation between the single-particle scattering of free electrons in high-energy beams and stimulated scattering by collective oscillations in a dense electron-beam plasma (comparatively low-energy) are discussed. Various physical approaches used to describe the processes in free-electron lasers are discussed. Stimulated multiphoton emission processes in undulators and stimulated Compton scattering processes are examined. The relation between the quantum and classical properties of these phenomena is discussed. The nonlinear behavior of the gain in a free-electron laser under saturation conditions is described. The possibility of optimizing the gain in a Compton laser in a scheme with noncollinear propagation of electrons and electromagnetic waves is discussed.

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CONTENTS

1. INTRODUCTION

Free-electron lasers (FEL) have recently been attracting a great deal of attention. This is apparently related to the early successes in experiments on FEL and to hopes of creating, in this manner, radiation sources that are tunable over a wide range of frequencies: up to ultraviolet and soft x-ray regions. The number of papers on FEL, published at the present time, is very large. These papers are primarily theoretical. The number of experiments on FEL is significantly lower and the basic results of existing experimental work will be described below. As far as the theoretical studies of FEL are concerned, as a rule, they are concerned either with proposing modifications of known schemes, new principles for amplification, and so on, or understanding the physics of the processes occurring in FEL.

The theoretical methods used are very diverse. The theory of FEL, both in the classical and quantum approaches, is constructed using a numerical solution of the equations and with the help of analytical methods, in the weak and strong field approximations, based on a single-particle description of electrons and starting from the theory of the physics of plasma, and so on. In view of the variety of methods and approaches used, it is useful to try to summarize to some extent the ideas on which FEL are based and to give a unified theoretical description of the physics of processes leading to amplification and saturation in FEL. This paper is concerned with these problems and it is useful to begin by describing existing experiments.

One of the more popular schemes for FEL is a laser based on relativistic electrons propagating along the axis (Oz) of an undulator, whose field is stationary and depends periodically on the longitudinal coordinate z. Spontaneous emission by electrons in an undulator was examined back in 1947 by Ginzburg[1] and in 1951 by Motz.[2] Subsequently, the phenomenon was repeatedly observed experimentally and it was widely studied theoretically, this being illustrated, for example, in Ref. 3. Stimulated undulator emission (or absorption) arises when an external electromagnetic wave, which can be amplified or absorbed, propagates along the undulator axis parallel to the electron beam. In the nonrelativistic energy range, devices based on stimulated undulator emission are known as ubitrons and they apparently are among the most powerful sources of radiation in the centimeter and millimeter wavelength range.[4,5]

The transition to relativistic electron energies is undoubtedly related to some qualitatively new characteristics of stimulated undulator emission and, primarily, to the possibility of greatly increasing the lasing frequency compared to that of the ubitron. One of the first experiments on stimulated undulator emission using relativistic electrons was described in Ref. 6. However, the energy of the electron beam used in that work was not very high (~700 keV). According to the interpretation given in Ref. 6, amplification of a wave propagating toward the beam was observed, while in the ultrarelativistic case the most intense emission is forward emission by a relativistic electron and it is in this case that it is possible to attain a high lasing frequency. Amplification was achieved in Ref. 6 with a single pas-

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sage of radiation through the undulator.

Amplification of a test wave in a relativistic undulator with an electron energy of $\varepsilon = 28$ MeV and radiation frequency $\omega = 2\cdot 10^{14}$ s^{-1} ($\lambda = 10.6$ μm) was observed in Ref. 7. In the next paper by this group,[8] under similar conditions, lasing was detected, i.e., the first free-electron laser was created. The scheme of the experiment in Ref. 8 is shown in Fig. 1. The energy of electrons in the beam was 43 MeV. Both the electron beam and the amplified electromagnetic wave represented a sequence of pulses with duration $\sim 3\cdot 10^{-12}$ s (length of the train $\sim 10^{-1}$ cm). The lasing process consisted of amplification of a train of electromagnetic radiation in the region of localization of an electron bunch within the time for the bunch to pass through the undulator. Due to the mirrors, the train of electromagnetic radiation was confined in the resonator before the arrival of the next electron bunch, which approached the inlet to the undulator simultaneously with the electromagnetic pulse, after which amplification was repeated.

The density of electrons in the beam, estimated from the magnitude of the current $J_{max} = 2.6$ A,[8] with a beam diameter $d \sim 0.3$ cm, is $N_e = 5\cdot 10^{10}$ cm^{-3}. The period of the magnet used in Ref. 8 was $\lambda_0 = 3.2$ cm with an overall magnet length of $L = 5$ m and helical magnetic field intensity $B_0 = 2.4\cdot 10^3$ G.

The lasing frequency in Ref. 8 was $\omega = 5.5\cdot 10^{14}$ s^{-1} ($\lambda = 3.4\cdot 10^{-4}$ cm^{-1}). The maximum radiation power was 7 kW outside the resonator and 500 kW inside the resonator. This permits estimating the field intensity in the undulator: for a caustic with transverse size $d \sim 0.3$ cm, $E_0 \sim 3\cdot 10^4$ V/cm.

The creation of a somewhat different type of FEL was reported in Ref. 9. In this experiment, the electron energy was much lower than in Ref. 8, $\varepsilon = 1.2$ MeV, but the magnitude of the current was much higher, $I = 25$ kA. Generation was achieved at a wavelength $\lambda = 0.5$ mm with the periodicity pitch of $\lambda_0 = 8$ mm. The radiation power, attaining in the experiment[9] a magnitude of $P = 1$ MW, was much higher than the output power of the FEL in Ref. 8.

The lasing mechanism, according to the interpretation given in Ref. 9, consisted of stimulated Raman scattering of equivalent photons, corresponding to the periodic magnetic field, by longitudinal plasma oscillations of the dense electron beam plasma with the emission of photons of the generated radiation.

A similar type of experiment was described in Ref. 10, in which instead of an undulator, a powerful pump wave propagating toward the electron beam was used. The emission mechanism was, apparently, stimulated Raman scattering of the pump by plasma waves in the beam. The electron energy in Ref. 10 was not high ($\sim$600 keV) with a comparatively small current $J = 4.2$ kA. In view of the low energy, conversion of the pump frequency in Ref. 10 was low: the frequency increased by approximately a factor of 3.

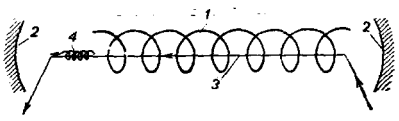

FIG. 1. Diagram of the experiment. 1) spiral magnet; 2) mirrors; 3) electron beam; 4) amplified wave.

The term FEL is used in this review, as a rule, in the narrow sense of the word to describe lasers based on an undulator. It should be noted that many other ideas exist for using electron beams to create lasers based on free–free transitions, which are also often considered as FEL. Some of the suggestions along these lines are: a) Compton laser,[11] in which the electron and the amplified wave interact not with the magnetic field of the undulator, but with the pump wave, propagating toward the electron beam (see Section 6); b) lasers based on the Cherenkov effect with electrons propagating in a waveguide filled with a dielectric medium[12-14]; c) lasers based on the Smith–Purcell effect,[15,16] i.e. on generation with propagation of electrons above the surface of a diffraction grating; d) lasers based on propagation of electrons in a corrugated waveguide,[17,18] and so on. Without stopping to consider the details of all these mechanisms for scattering electrons, which are examined, for example, in detail in Ref. 19, we note that there is a great similarity between them. Generally, if we keep in mind scattering of electrons by periodic structures in the field of the amplified wave, then, apparently, the specific mechanism for realizing the periodic structure is not very significant. For this reason, in particular, many of the conclusions arrived at below for FEL based on an undulator are also valid in reality for other mechanisms for scattering electrons by periodic structures.

The relation between the beam density and the electron energy is much more significant than the specific scattering mechanism.

A comparison of the experiments described in Ref. 6, 9, 10 and 8 indicates that there are two different complementary trends in the development of FEL. Experiments in the first group are concerned with using high-current electron beams in order to obtain high FEL power with a comparatively low lasing frequency. The experiment in Ref. 8, on the other hand, is concerned with achieving high lasing frequencies due to the use of high-energy electron beams, but with comparatively low density. Both the advantages and disadvantages of each of these approaches are clear. It is also evident that the creation of new FEL both of the first and second types is of great interest both from the physical point of view and for applications.

The physical difference between FEL based on high-current low-energy beams and those based on weak-current high-energy beams lies in the fact that in the former case collective effects in the beam plasma can play an important role, while in the latter case the interaction between the electrons in the field is fundamentally of a single-particle nature. From here follows also the difference in the theoretical approaches. In order to construct a theory of FEL based on dense beams, it is necessary to use the equations for the medium: Boltzmann's equation or the Navier-Stokes

equation.[20-23] On the other hand, the theory of the type of FEL in Ref. 8 must be constructed based on single-particle equations of motion for the electron. Using this approximation, both the mechanism of amplification in FEL of the type in Ref. 8 and the nature of the multiphoton processes occurring in them, the physical nature of the saturation and the nature of the gain itself in the saturation regime can be understood. With this formulation of the problem (i.e. applicable to FEL of the type in Ref. 8), we examine the problems formulated.

The quantitative criterion for the densities and energies at which it is necessary to take into account collective effects, under typical conditions, is determined by the parameter[24,25] $\varkappa = \omega_b t \gamma^{-3/2}$, where $\omega_b = \sqrt{4\pi e^2 N_e/m}$ is the plasma frequency of the beam, t is the time of flight of an electron through the undulator, and $\gamma = \varepsilon/mc^2$ is the relativistic factor. In the rest system of the beam, this parameter equals $\omega_b' t'$ (where the primed quantities correspond to a moving system of coordinates). This means that the plasma frequency ω_b', characterizing the maximum increment for the development of instabilities in the plasma, is equal to the inverse interaction time t'^{-1}. It should also be noted that the characteristic time for the development of amplification in FEL can be determined not by the length of the undulator, but by some other factors. For large amplification in FEL, the effective time t_{eff} equals the inverse growth increment of the field in the FEL $1/cg$, where g is the gain per unit length. Finally, as will be shown in Secs. 2 and 6, the nature of the amplification depends considerably on the relation between the number of periods in the undulator $N = L/\lambda_0$ and the energy spread of the electrons in the beam, determined by the parameter $\zeta = N\Delta\varepsilon/\varepsilon$, where $\Delta\varepsilon$ is the width of the electron distribution function $f(\varepsilon)$. For $\zeta < 1$, the effective interaction time t_{eff}, entering into the definition of the parameter $\varkappa$, equals $t/\zeta = (\lambda_0/c)\varepsilon/\Delta\varepsilon$.[26] Taking into account all the possibilities examined, the parameter $\varkappa$, separating regions of single-particle and collective interactions with electrons, can be represented in the form

$$\varkappa = \omega_b \gamma^{-3/2} \min\left\{t, \frac{1}{cg}, \frac{\lambda_0}{c}\frac{\varepsilon}{\Delta\varepsilon}\right\}.$$

If the parameter $\varkappa$ is large, $\varkappa > 1$, then collective effects in the beam plasma can play a significant role and the amplification mechanism is the stimulated Raman scattering of photons, equivalent to the magnetic field of the undulator (see Sec. 2) or of the pump wave by plasma oscillations in the beam. For these reasons, devices corresponding to the region $\varkappa > 1$ are not free-electron lasers in the strict sense of the word (both with respect to the frequency range and the collective nature of the amplification). Such devices are in many ways similar to cyclotron masers,[27] based on the use of the cyclotron instability in the electron plasma.

On the other hand, for $\varkappa < 1$, it is in principle possible to attain a high lasing frequency and amplification is achieved via the mechanism of single-particle scattering of free electrons. Both of these criteria indicate that it is devices that fall into the region $\varkappa < 1$ that most deserve to be called "free-electron lasers."

The criterion $\varkappa < 1$ is almost always satisfied at high electron energies, when $\gamma \gg 1$. In particular, this criterion is well satisfied under the conditions of the experiment described in Ref. 8.

The FEL based on an undulator is most similar to a Compton laser (see Sec. 6 below). Apparently, the first theoretical work along these lines on lasers based on relativistic free electrons is the paper by Pantell *et al.*,[11] in which the idea of using stimulated Compton scattering for creating a laser was formulated and the gain was estimated under certain conditions in second order quantum perturbation theory.

The next step in the theory of FEL was the work by Madey,[28,29] in which the method of equivalent photons relative to the magnetic field of the undulator was used (see Sec. 2). In these papers, an expression was obtained for the lasing frequency and the gain in a weak field was found for the case of a Gaussian electron energy distribution and a Gaussian frequency distribution for the equivalent photons.

The induced emission of a relativistic electron in an undulator was first analyzed, apparently, in Ref. 30 with the use of classical equations of motion.

The results, close to or equivalent to the results of Madey's work,[28,29] were rederived by many workers using different methods.[31-37] In most of these works,[31,34-37] the electron motion is described purely classically. Attempts to construct a quantum theory are contained in Refs. 32, 33, and 38. A direct quantum mechanical calculation of the gain in FEL (in the weak signal approximation), in which a transformation to the rest system of the beam and the method of equivalent photons are not used, and which is much simpler than the procedure used by Madey,[28,29] is given in Refs. 39–41. In Refs. 42–44, the results of the linear theory are generalized to the case when the fixed field approximation is not valid, i.e. the gain per pass of the electrons through the magnet is not small. Such a situation is typical for low-energy electron lasers.[6,9,10] However, already at electron energies ~10–10² MeV, the gain per pass under real conditions does not exceed several percent. This permits using the fixed field approximation in calculating the gain per pass in FEL.

The effects of nonlinearity in FEL were qualitatively discussed in Refs. 36 and 37. The work in Ref. 45, concerned with this problem, apparently, is wrong, since the chain of equations is terminated in it without justification. Numerical solutions of the classical equations of motion of an electron in an undulator in a strong field are contained in Refs. 36, 43, 46–48. The multiphoton processes and saturation of gain in FEL were described analytically using quantum theory in Refs. 49 and 50. Subsequently, some of the results of these papers were rederived using different methods and confirmed in Refs. 51 and 52. The analytical expressions for the gain found in Refs. 49 and 50 were obtained in Refs. 53 and 54 starting from the classical equations of motion for an electron.

Concluding the review of the literature of FEL, a number of papers proposing to optimize the amplifica-

tion conditions in FEL should be noted. The gain can be increased by introducing a dielectric medium into the undulator,[55] by applying an additional longitudinal magnetic field,[56] by using an undulator with a variable step or (and) amplitude of the field intensity B_0.[57-61] A two-step undulator–Compton laser scheme, in which first, as in the usual FEL, radiation is generated at an intermediate frequency, which is then again scattered by the same electron beam, which leads to amplification and generation at a high frequency, was proposed in Ref. 62. An FEL, in which two coupled undulators were used, was discussed in Refs. 63 and 64. One of the possible methods for optimizing the gain in a Compton laser is to choose the most advantageous geometry for the experiment (see Sec. 6 below).[65,66] Some practically important estimates of the gain for existing accelerators were made in Ref. 67.

It should be noted that the number of published works on FEL is increasing very rapidly. This trend can be observed by comparing the review contained in the present paper with the review in Ref. 68. In what follows, we will be concerned primarily with the physical interpretation of amplification in FEL, the physics of multiphoton processes and saturation, and amplification in a Compton laser. The entire analysis that follows is based on the single-particle description and for this reason concerns high-energy electron lasers, for which $\gamma \gg 1$ and $\varkappa \ll 1$. The opposite case, $\varkappa > 1$, was analyzed in many of the original papers cited above, as well as in the review in Ref. 69.

2. PHYSICAL INTERPRETATION OF AMPLIFICATION

Several approaches to interpreting the phenomena in an FEL are known. In the work by Pantell *et al.*,[11] the results of a quantum electrodynamic calculation in lowest (second) order perturbation theory are used. The approach formulated by Madey[28,29] is based on the analogy with processes in FEL with stimulated Compton (Thompson) scattering by an electron at rest. The idea of the method is based on transforming to the center of mass system of the unperturbed relativistic electron beam. In so doing, the potential of the magnetic field, moving with respect to the electron at rest with a velocity close to the velocity of light, is transformed into an expression close to the potential of a plane wave with frequency $\Omega = q_0/\sqrt{1-v_0^2} = \gamma q_0$, where $q_0 = 2\pi/\lambda_0$, v_0 is the velocity of the electron beam, $\gamma = \varepsilon/m$, ε and m are the energy and mass of an electron, $\varepsilon \gg m$, and $h = c = 1$.

This permits replacing, in the center of mass system of an electron, the potential of the electromagnetic field of the moving undulator by the potential of an equivalent plane wave with frequency Ω. The frequency of the amplified electromagnetic wave is Doppler shifted as a result of this transformation and becomes equal to $\omega' = \omega\sqrt{(1-v_0)/(1+v_0)} \approx \frac{\omega}{2}\sqrt{1-v_0^2} = \omega/2\gamma$. The interaction of the electron with two waves with frequencies Ω and ω' can be viewed as stimulated Thompson scattering of a photon with frequency Ω by an electron at rest. Such a process, evidently, is possible if $\omega' \approx \Omega$. This condition determines the resonance frequency ω_{res}, near which (for $\omega \approx \omega_{res}$) stimulated emission or absorption of photons of the amplified wave is possible:

$$\omega_{res} = 2q_0\left(\frac{\varepsilon}{m}\right)^2 \tag{1}$$

(from what follows, it will be evident why the term "resonant frequency" is justified). The use of the method of equivalent photons relative to the field of the moving undulator[70] also permits finding the equivalent-photon density and the gain in an FEL.[28,29]

Another approach to describing FEL is based on interpreting the processes occurring in them in terms of stimulated bremsstrahlung emission and absorption tion.[39-41] Such an interpretation is possible since a stationary magnetic field can be viewed as a particular realization of an external potential, scattering electrons. The main property of the undulator is that due to the spatial periodicity of the magnetic field, both energy and momentum are conserved[39]:

$$\varepsilon' - \varepsilon = \mp\omega, \quad p' - p = \mp(\omega + q_0), \tag{2}$$

where ε' and p' are the energy and momentum of an electron after scattering (we assume that the electron momentum is oriented strictly along the axis of the undulator Oz).

The conservation laws (2) determine both the momentum of the scattered electron p' and one of the parameters characterizing the radiation or the incident electron beam. For example, more exact values of the frequencies ω_e and ω_a, at which emission and absorption of a photon with fixed energy is possible, or corresponding values of the energy $\varepsilon_{e,a}$ at a fixed frequency ω follow from Eqs. (2):

$$\omega_{e,a} = \omega_{res}\left(1 \mp \frac{\omega_{res}}{\varepsilon}\right), \quad \varepsilon_{e,a} = \varepsilon_0 \pm \frac{\omega}{2}, \quad \varepsilon_0 \equiv m\sqrt{\frac{\omega}{2q_0}}. \tag{3}$$

For high magnetic field intensity B_0, it is necessary to take into account in equations of the type (3) the shift in the mass of an electron in a strong magnetic field $\delta m^2 = e^2B_0^2/q_0^2$. In this case, for example, the expression for the wavelengths of the emitted and absorbed photons is written in the form

$$\lambda_{e,a} = \frac{\lambda_0}{2\gamma^2}\left(1 + \frac{r_0B_0^2}{mq_0^2}\right)\left(1 \pm \frac{\omega_{res}}{\varepsilon}\right), \tag{4}$$

where $r_0 = e^2/m$ is the classical radius of the electron.

For a circularly polarized magnetic field of the undulator, the mass shift is the only effect arising for $\delta m^2 \gtrsim m^2$. In the case that the magnetic field is linearly polarized, for $\delta m^2 \gtrsim m^2$, together with this, there also arises the possibility of amplification at harmonics of ω_{res} and the gain changes considerably.[71,72]

The cross sections for stimulated emission σ_e and absorption σ_a of a photon can be found with the help of a direct quantum electrodynamic calculation[39] using second-order perturbation theory: first-order in the magnetic field potential $\mathbf{A}_H$ and first-order in the potential of the electromagnetic wave $\mathbf{A}_{EM}$,

$$\mathbf{A}_H = \frac{B_0}{\sqrt{2}\,q_0}(\mathbf{a}e^{iq_0z} + \text{c.c.}), \quad \mathbf{A}_{EM} = \frac{E_0}{\sqrt{2}\,\omega}(\mathbf{e}e^{i\omega(z-t)} + \text{c.c.}), \tag{5}$$

where $\mathbf{a}$ and $\mathbf{e}$ are unit polarization vectors.

The cross sections $\sigma_{e,a}$, generally speaking, must be averaged over the electron energy distribution function

$f(\varepsilon)$. It is easy to verify that $\bar{\sigma}_{ea} \propto f(\varepsilon_{e,a})$. The gain is determined by the total photon emission cross section, $\bar{\sigma}_T = \bar{\sigma}_e - \bar{\sigma}_a \propto f(\varepsilon_e) - f(\varepsilon_a)$. Expanding the arguments of the distribution functions with respect to the small energy difference $\varepsilon_e - \varepsilon_a$, we find as a result of such a calculation the gain per pass:

$$G = N_e L \bar{\sigma}_T = \frac{\sqrt{2}\,\pi^2 e^4 B_0^2 \,|\, ea \,|^2 N_e L}{m\omega^{1/2} q_0^{5/2}} \frac{df}{d\varepsilon}\Big|_{\varepsilon=\varepsilon_0}. \tag{6}$$

This equation can be interpreted in terms of a population inversion: amplification occurs if the energy ε_0 (3) is such that $df/d\varepsilon > 0$, i.e. if the value of ε_0 corresponds to the increasing part of the distribution function $f(\varepsilon)$.

The contours of the spectral lines for stimulated photon emission $[\propto \bar{\sigma}_e(\omega)]$ and absorption $[\propto \bar{\sigma}_a(\omega)]$, according to relations (3), are slightly shifted relative to the contour of the spontaneous emission line in different directions.

It follows from here that the gain G, proportional to the difference $\bar{\sigma}_e(\omega) - \bar{\sigma}_a(\omega)$, is determined by the derivative of the contour of the spontaneous emission line, whose intensity is proportional to $f(\varepsilon)$. This relation between stimulated and spontaneous emission is in excellent agreement with the experimental results (Fig. 2).[7]

If the electron beam is sufficiently monoenergetic, then the magnitude of the gain can be determined not by the electron energy spread, but by the finite length of the magnet L. Equation (6) is inapplicable, if $\Delta\varepsilon < \varepsilon/N$, where $N = L/\lambda_0$ is the number of periods of the magnet.

In this case, the gain with $e = a^*$ has the form[30-37,39]

$$G = \frac{2\sqrt{2}\,\pi N_e e^4 B_0^2 q_0^{1/2} t^3}{m^3 \omega^{3/2}} \frac{d}{du} \frac{\sin^2 u}{u^2}, \tag{7}$$

where $u = -m^2\omega t\Delta/2\varepsilon^3$, $\Delta = \varepsilon - m\sqrt{\omega/2q_0}$ is the detuning of the resonance, and $t \approx L$ is the duration of the interaction.

We note that the spectral width of the function $G(\Delta)$ in this approximation equals $\Gamma_t = 2\varepsilon^3/m^2\omega t$, i.e. it is determined by the inverse duration of the interaction.

Finally, one more interpretation of amplification in FEL, developed in many papers, is based on using the classical equations of an electron in the fields (5), which, according to Refs. 35–37, 42, 43, 47, 48, and 54, can be reduced to the equation of a simple pendulum[73] for the phase $\varphi \equiv (\omega + q_0)z - \omega t$:

$$\frac{d^2\varphi}{d\mu^2} = \sin\varphi, \tag{8}$$

where $\mu = (et/\varepsilon)\sqrt{2E_0B_0}$ is the dimensionless time.

The initial conditions for Eq. (8) follow from the definition of the phase φ and the parameter μ and have the form

$$\varphi(\mu = 0) = \varphi_0, \quad \frac{d\varphi}{d\mu}\Big|_{\mu=0} = \frac{\Delta}{\Delta_m}, \tag{9}$$

where

$$\Delta_m = \frac{e\varepsilon}{m}\sqrt{\frac{E_0B_0}{q_0\omega}} \tag{10}$$

is a quantity which characterizes the scale of the detunings and corresponds to the field-dependent width of the resonance curve $G(\Delta)$ for strong fields (see Sec. 5 below) and φ_0 is the initial phase.

As is well known,[73] the equation of a simple pendulum (8) has a first integral that expresses the conservation of energy, which taking into account the initial conditions (9), can be written in the form

$$\left(\frac{d\varphi}{d\mu}\right)^2 + 2(\cos\varphi - \cos\varphi_0) = \frac{\Delta^2}{\Delta_m^2}. \tag{11}$$

Here, $(d\varphi/d\mu)^2$, $2\cos\varphi$, and $2\cos\varphi_0 + (\Delta^2/\Delta_m^2)$ are the effective kinetic, potential, and total energies of the pendulum (Fig. 3).

The energy emitted by the electron per pass is defined as the work performed by the field of the electromagnetic wave[74]:

$$\Delta\mathscr{E}(t) = -e\int^t dt\,\mathbf{E}_{EM}\mathbf{v}_\perp, \tag{12}$$

where $\mathbf{E}_{EM}$ is the intensity of the electric field corresponding to the potential $\mathbf{A}_{EM}$ (5). The classical velocity of the electron in directions perpendicular to the undulator axis, $\mathbf{v}_\perp$, determining $\Delta\mathscr{E}$ (12), can be found explicitly, if the fact is taken into account that since the potentials $\mathbf{A}_H$ and $\mathbf{E}_{EM}$ (5) are transverse and depend only on z and t, there exists an integral of motion $\mathbf{p}_\perp + (e/c)(\mathbf{A}_H + \mathbf{A}_{EM}) = \text{const}$. Finding from here explicit expressions for $\mathbf{v}_\perp$, substituting them into Eq. (12), integrating, and taking into account the definition of the phase φ, we find that the following relation exists between the emitted energy $\Delta\mathscr{E}$ and the rate of change of phase $d\varphi/d\mu$ (Refs. 43, 53, and 54):

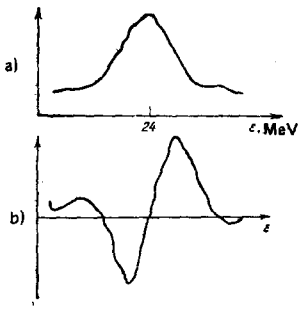

FIG. 2. Experimentally measured spectral intensity of spontaneous emission (a) and gain G (b) in a free-electron laser as a function of the emission frequency.[7]

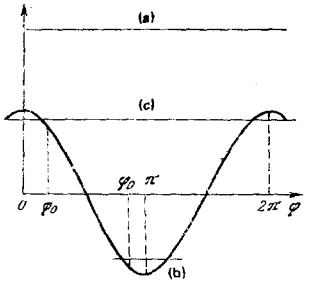

FIG. 3. Effective potential energy of a pendulum as a function of phase φ. The horizontal lines represent the total energy level: a) for $|\Delta| > \Delta_m$ (weak-signal approximation); b) for $|\Delta| < \Delta_m$ and for initial phase φ_0 close to π; c) for $|\Delta| < \Delta_m$ and for values of φ_0 close to 0 and 2π.

$$\Delta\mathscr{E} = \Delta - \Delta_m \frac{d\varphi}{d\mu}. \tag{13}$$

Since the longitudinal size of the electron bunches in the experiment in Ref. 8 (~3 mm) greatly exceeded the wavelength of light (~10^{-4} cm), all physical quantities (such as, for example, the emitted energy $\Delta\mathscr{E}$) must be averaged over the initial phase φ_0.

The fact that the classical equations of motion of an electron in FEL reduce to the equation of a simple pendulum reflects, in particular, the analogy with radioelectronic devices of the travelling-wave tube type. It is well known that in the fixed field approximation the equations for a travelling wave tube also have a form similar to Eq. (7).[75] However, the physical meaning of the parameters μ, Δ, and Δ_m, entering into Eqs. (8)–(13), are different for FEL and travelling wave tubes. For this reason, the physical results following from these equations are different for FEL and travelling wave tubes.

The weak signal approximation in FEL corresponds to small values of the parameters μ and Δ_m: $\mu, \Delta_m \to 0$ (but the ratio μ/Δ_m in this case can be arbitrary). The motion of the electron with respect to phase φ in this case is infinite, since $|\Delta| \gg \Delta_m$ and the effective total energy of the pendulum is much greater than its potential energy. For this reason, the term $2(\cos\varphi - \cos\varphi_0)$ in Eq. (11) can be taken into account by iterating.

The zeroth-order solution with respect to μ, Δ_m

$$\varphi^{(0)} = \varphi_0 + \Delta \frac{\mu}{\Delta_m} \tag{14}$$

is cancelled out on substitution into (13). The first-order solution

$$\frac{d\varphi^{(1)}}{d\mu} = \frac{\Delta_m}{\Delta}\left[\cos\varphi_0 - \cos\left(\varphi_0 + \mu\frac{\Delta}{\Delta_m}\right)\right] \tag{15}$$

vanishes after averaging with respect to φ_0. It is only in second-order in iterating Eq. (11) with respect to μ, Δ_m that a nonvanishing average rate of change of phase is obtained:

$$\overline{\frac{d\varphi^{(2)}}{d\mu}} = \left(\frac{\Delta_m}{\Delta}\right)^3\left(\cos\frac{\mu\Delta}{\Delta_m} - 1 + \frac{1}{2}\frac{\mu\Delta}{\Delta_m}\sin\frac{\mu\Delta}{\Delta_m}\right). \tag{16}$$

Substituting this expression into (13) and calculating the gain $G = 4\pi N_e \Delta\overline{\mathscr{E}}/E_0^2$ again leads to Eq. (7).

Within the scope of the classical description, amplification in FEL is often interpreted as the result of spatially periodic bunching of the beam as it passes through the undulator and subsequent coherent emission by the modulated beam.[76] It should be emphasized that modulation of the beam, in this case, arises automatically. Initially, at the inlet to the undulator, the beam is uniform (on the scale of the order of the wavelength of the radiation ~10^{-4} cm).

The quality of FEL, as any amplifying system, in addition to the gain G, is also characterized by the efficiency. The latter is defined as the ratio of the energy $\Delta\overline{\mathscr{E}}$ emitted by an electron per pass to the initial electron energy ε. In the weak signal approximation, starting from Eqs. (13) and (16) or directly from (7), it is easy to verify that the efficiency, corresponding to the detuning Δ for which the gain $G(\Delta)$ is maximum, $\Delta \sim \Gamma_f = 2\Delta_m/\mu$, equals $\mu^4/64\pi N$. As the field E_0 increases, the efficiency increases $\propto E_0^2$. As will be evident from what follows, the parameter μ is a saturation parameter, which is attained for $\mu \sim 1$. In this case, the efficiency attains the value $1/64\pi N$, which is determined by the inverse number of periods of the undulator and is the maximum possible efficiency in the weak signal approximation. Numerically, the efficiency of FEL is low, since $N \gg 1$ must be satisfied. These results were obtained in Ref. 35.

The study of the limits of applicability of equations (6), (7), and (16) requires going beyond calculations in lowest order quantum perturbation theory and beyond the weak signal approximation in the classical approach.

It should be noted that at first glance the agreement of the results of classical and quantum calculations indicates the complete equivalence of the approximation of single-photon transitions in quantum mechanics and the weak signal approximation in classical theory. Based on this, it is sometimes stated that FEL based on an undulator and a Compton laser are single and double quantum devices.[77] Sometimes, this assertion is not clearly stated, but in fact is used in deriving the gain with the help of calculations in the single-(double-) quantum approximation.[11,28,29,35,39] However, as analysis shows,[49,50] in reality the single-quantum approximation and the weak-signal approximation in classical theory are not equivalent. The conditions for applicability of these approximations are appreciably different and, as a rule, amplification in FEL always has a multiquantum character. In order to analyze completely the relation between the quantum and classical descriptions of processes in FEL and, in particular, in order to describe multiphoton transitions, it is necessary to start from the quantum approach, whose basic results are presented below in Sec. 3. In order to find the gain in FEL (taking nonlinearity into account), as the result in Refs. 49 and 50 shows, it is possible to start from quantum as well as classical equations of motion for the electron (Sec. 4). Each of these approaches reveals some new physical characteristics of the amplification of a strong wave in FEL and, for this reason, the quantum and classical descriptions of FEL in a strong field compliment one another.

3. QUANTUM DESCRIPTION OF MULTIPHOTON PROCESSES AND SATURATION

Neglecting small spin corrections,[70] let us begin with the Klein–Gordon equation in fields $\mathbf{A}_H$ and $\mathbf{A}_{EM}$ (5). The quadratic terms $e^2\mathbf{A}_H^2$ and $e^2\mathbf{A}_{EM}^2$ in this equation for circularly polarized fields $\mathbf{e} = \mathbf{a}^* = 1/\sqrt{2}(\mathbf{x} - i\mathbf{y})$ are constant and determine the shift in the electron mass, which we will assume is taken into account in m; $\mathbf{x}$ and $\mathbf{y}$ are unit vectors along the Ox and Oy axes. Examining only one-dimensional electron motion along the Oz axis, we start, therefore, with the equation

$$\left\{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} + 2e^2\mathbf{A}_H\mathbf{A}_{EM} + m^2\right\}\Psi = 0. \tag{17}$$

The initial condition for Eq. (17) has the form $\Psi(t=0) \propto e^{ipz}$: according to Refs. 49 and 50, it may be as-

sumed that the interaction is turned on instantaneously at $t=0$, since under real conditions the switching time Δt, in order of magnitude, equals the time for the electron to traverse a distance λ_0, much less than the period of characteristic oscillations in the system $\sim L/v_0 \approx L$. In examining the initial problem, it is not necessary to take into account formally the spatial finiteness of the region of interaction, taking it into account actually by fixing the finite duration of the interaction $t \approx L$. Under these conditions, due to the law of conservation of momentum, the state of the electron with momentum p is related only to states with momentum $p \pm (\omega + q_0)$.

For this reason, all states coupled with one another can be enumerated by the discrete integer index $n = 0, \pm 1, \pm 2, \ldots$, so that

$$p_n = p - n(\omega - q_0), \qquad \varepsilon_n = \sqrt{p_n^2 + m^2}, \tag{18}$$

where p is the initial momentum of the electron.

Due to the conservation laws, the problem of transitions into the continuum reduces to an equivalent problem of resonant excitation of a system of discrete levels ε_n (18). Excitation of the system to a level ε_n corresponds to absorption (for $n > 0$) or emission (for $n < 0$) of $|n|$ photons.

It follows from (18) that the wave function $\Psi(z, t)$ at an arbitrary time is a superposition

$$\Psi = \sum_n a_n(t) \exp\{i(p_n z - (\varepsilon + n\omega)t)\}, \tag{19}$$

where $\varepsilon \equiv \varepsilon_0$ is the initial energy of the electron.

The coefficients $a_n(t)$ are slowly varying functions of time. For this reason, their second derivatives $\ddot{a}_n$ can be dropped in the equations for $a_n(t)$.[50]

The energy ε_n can be expanded in a series in powers of n, which gives

$$i\dot{a}_n - \left[n\left(\frac{m^2\omega}{2\varepsilon^3} - q_0\right) - n^2\frac{m^2\omega^2}{2\varepsilon^3}\right]a_n = \frac{e^2E_0B_0}{2q_0\omega\varepsilon}(a_{n-1} - a_{n-1}) \tag{20}$$

with the initial condition $a_n(0) = \delta_{n,0}$.

It is evident that Eqs. (20) are analogous to the quantum mechanical equations that describe excitation of an anharmonic oscillator by a resonant field.[78] The frequency ω_{res} (1) plays the role of a characteristic frequency of the system, relative to which the electromagnetic field is resonant.

The coefficients $a_n(t)$ determine the energy $\Delta\mathscr{E}$, emitted by an electron over the time of the interaction t, and the gain per pass G:

$$\Delta\mathscr{E} = -\omega\sum_n n\,|a_n|^2, \qquad G = \frac{4\pi N_0 \Delta\mathscr{E}}{E_0^2}. \tag{21}$$

The system of equations (20) is characterized by four fundamental parameters: the interaction energy of the electron interacting with fields $\mathbf{A}_H$ and $\mathbf{A}_{EM}$ $\mathscr{E}_{int} = e^2E_0B_0/2q_0\omega\varepsilon$, the anharmonic energy $\mathscr{E}_{anh} = m^2\omega^2/2\varepsilon^3$, detuning of the resonance $\Delta = \varepsilon - m\sqrt{\omega/2q_0}$, and the interaction time t. It is convenient to introduce the dimensionless parameters

$$\eta = t\mathscr{E}_{int} \qquad \beta = t\mathscr{E}_{anh} \qquad \rho = \frac{\eta}{\beta} = \frac{\mathscr{E}_{int}}{\mathscr{E}_{anh}}, \tag{22}$$

as well as the saturation parameter $\mu = 2\sqrt{\eta\beta} = 2t\sqrt{\mathscr{E}_{int}\mathscr{E}_{anh}}$, which coincides with the dimensionless time entering into the classical equation for the pendulum (8). As shown in Refs. 49 and 50, the parameters $\sqrt{\rho}$ and η determine the degree to which the electron scattering process in the FEL is a multiphoton process, $n_{max} = \min(\eta, \sqrt{\rho})$, the parameter μ determines the condition for transition to saturation ($\mu \sim 1$), and the patameter β determines small quantum corrections to the gain.

The numerical values of these parameters under the conditions of the experiment in Ref. 8 are: $n_{max} \sim \sqrt{\rho} \sim \eta \sim 10^5$, $\mu \approx 5$, $\beta \sim 10^{-5}$.

The solution of Eqs. (20) in first-order perturbation theory with respect to $\mathscr{E}_{int}$ permits calculating the transition probability amplitudes $a_{\pm 1}$, differing from zero, and with their help the gain, coinciding with (7). The criterion for applicability of perturbation theory is the condition $|a_{\pm 1}| < 1$, which is already violated in very weak fields, since $|a_{\pm 1}|_{max} \sim \eta$ and, for example, under the conditions of the experiment in Ref. 8 $\eta \sim 10^5 \gg 1$. For $\eta \gg 1$, during the excitation process, a large number of photons is absorbed and emitted and a large number of levels $\mathscr{E}_n$ equivalent to a system with a discrete spectrum is excited, $|n| \lesssim n_{max} \gg 1$. This means that the quantum description of FEL, based on calculations in lowest order perturbation theory,[11,28,29] is strictly speaking incorrect. The problem of resonant excitation of a multilevel system in accordance with Eqs. (20) was solved in Refs. 49 and 50 and subsequently in Refs. 51 and 42 for two characteristic relations between the parameters:

$$\sqrt{\rho} \gg \eta \gg 1 \quad (\mu \ll 1) \quad \text{and} \quad \eta \gg \sqrt{\rho} \gg 1 \quad (\mu \gg 1).$$

For $\mu < 1$, but $\eta \gg 1$, the explicit form of the distribution of the electrons over the energy levels ε_n after passage through the undulator to lowest order in the small parameter β (22) is determined by the expression[79]

$$a_n^2 = J_n^2\left(2\frac{\rho\omega}{\Delta}\sin\frac{\beta\Delta}{\omega}\right) = J_n^2\left(\frac{e^2E_0B_0\lambda_0^2}{4\pi^2\hbar\omega\Delta}\sin\frac{m^2c^4\omega t\Delta}{\varepsilon^3}\right). \tag{23}$$

For a characteristic detuning $\Delta \sim \Gamma_t$, corresponding to the width of the curve $G(\Delta)$ (7), the argument of the Bessel function equals, in order of magnitude, η and we have verified that for $\eta \gg 1$ and $\mu < 1$ the degree to which the scattering process is a multiphoton process is $n_{max} = \eta \gg 1$ and perturbation theory cannot be used to calculate the probabilities of multiphoton transitions $|a_n|^2$.

However, the gain, found with the help of Eqs. (20) and (21) by summing with respect to n, in the approximation $\eta \gg 1$ and $\mu < 1$, turns out to be equal to the gain in the weak signal approximation (7).[50] This means that the parameter characterizing the nonlinearity of the gain is the classical parameter μ and not the quantum parameter η. The reason that the nonlinearity parameters for the scattering amplitudes $a_n(t)$ and for the gain G differ so greatly lies in the appreciable cancelling of the higher order contributions in the sum over n (21), determining $\Delta\mathscr{E}$ and G. This effect, evidently, is of the same nature as the compensating effect in multiphoton bremsstrahlung absorption of a strong wave in the

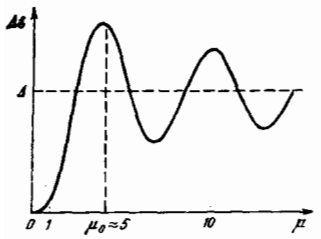

FIG. 4. The energy emitted by an electron in FEL per pass as a function of the saturation parameter μ. $\mu \approx 5$ corresponds to the conditions of the experiment in Ref. 8. This curve also represents the dependence of the gain G on the interaction time t or magnet length L.

presence of scattering of an electron by a Coulomb potential.[80]

The probabilities for multiphoton transitions $|a_n|^2$ (23) determine, for example, the moments of the number of emitted photons, i.e. the average values of powers of the emitted energy. It is easy to see that, for example,

$$\overline{\Delta\mathscr{E}^2} = \sum_n (n\hbar\omega)^2 |a_n|^2 = \frac{1}{2}\left(\frac{e^2 E_0 B_0 \lambda_e^2}{4\pi^2 \Delta}\right)^2 \sin^2 \frac{m^2 c^4 \omega t \Delta}{\varepsilon^3}.$$

For $\Delta \sim \Gamma_t \overline{\Delta\mathscr{E}^2} \sim (32/\mu^4)\, \overline{\Delta\mathscr{E}^2} \gg \overline{\Delta\mathscr{E}^2}$, where $\Delta\mathscr{E}$ is defined by Eqs. (13) and (16). Such a large difference between $\overline{\Delta\mathscr{E}^2}$ and $\overline{\Delta\mathscr{E}^2}$ again reflects strong cancelling in the sign alternating sum over n, defining $\overline{\Delta\mathscr{E}}$ (21).

The analogy between the cancelling of the probabilities for multiphoton transitions in the theory of FEL and the elimination of the well-known infrared divergence of the cross section for spontaneous bremsstrahlung of soft photons[70] was pointed out in Ref. 81. Equations (7) and (23) were obtained in Ref. 81 by examining the interaction of a classical electron current with the field of a quantized electromagnetic wave, amplified in an undulator.

For strong fields, when the saturation parameter μ is large ($\mu > 1$), the energy $\Delta\mathscr{E}$ emitted by an electron, obtained with the help of the solutions of Eqs. (20),[49,50] has the form

$$\Delta\mathscr{E} = \Delta\left\{1 - \sqrt{\frac{2}{\pi\mu}}\left[\cos\mu\left(1 - \frac{\Delta^2}{16\Delta_m^2}\right) - \sin\mu\left(1 - \frac{\Delta^2}{16\Delta_m^2}\right)\right]\right\}. \quad (24)$$

A graph of $\Delta\mathscr{E}(\mu)$ is shown in Fig. 4. As the field E_0 (or the interaction time t, i.e. the length of the undulator L) increases, the emitted energy reaches a saturation level equal to Δ, undergoing in this case amplitude damped oscillations. The condition for applicability of Eq. (24), aside from the assumption that $\mu > 1$, is a restriction on the magnitude of the detuning $|\Delta| < \Delta_m$, where Δ_m is defined by Eq. (10). The physical consequences of Eq. (24) will be discussed in Sec. 5. For now, we only note that both expression (24) itself and the conditions for its applicability do not depend on Planck's constant, which indicates the classical nature of the gain saturation. In Sec. 4, we discuss how and for what specific reasons saturation arises within the scope of the classical description of FEL.

4. CLASSICAL INTERPRETATION OF SATURATION

It is well known,[65] that the solutions of the pendulum equation (8) in general can be written in an implicit form in terms of elliptic integrals. However, it is impossible to find a general explicit form for the solutions $\varphi(\mu, \varphi_0)$ and to average with respect to φ_0 analytically. It was shown in Refs. 53 and 54 that, asymptotically, in a strong field $\mu > 1$ not all electrons make the same contribution to the average rate of change in phase: for $\mu > 1$, $|\Delta| < \Delta_m$ electrons whose initial phase φ_0 is close to the value $\varphi_0 = \pi$, corresponding to the stable position of equilibrium of the pendulum, make the main contribution to $\overline{d\varphi/d\mu}$. As μ increases, the interval of values of the initial phase $\delta\varphi_0$, making an appreciable contribution to $\overline{d\varphi/d\mu}$, decreases, which is what leads to the decrease in the amplitude of the oscillations, i.e. to damping of $\overline{d\varphi/d\mu}$ and to saturation of $\Delta\overline{\mathscr{E}}(\mu)$. For $|\Delta| < \Delta_m$ and $|\varphi_0 - \pi| < 1$, the pendulum equation (8) simplifies, transforming into the equation of a classical anharmonic oscillator with small anharmonicity for the phase shift relative to the equilibrium position $x = \varphi - \pi$:

$$\frac{d^2x}{d\mu^2} - x - \frac{x^3}{6} = 0. \quad (25)$$

Taking into account the initial conditions $x(0) = x_0 \equiv \varphi_0 - \pi$, and $\dot{x}(0) = \Delta/\Delta_m$ and the corrections to the oscillation frequency due to small anharmonicity,[73] the solution of Eq. (25) has the form

$$x(\mu) = x_0 \cos\left[\mu\left(1 - \frac{\Delta^2}{16\Delta_m^2} - \frac{x_0^2}{16}\right)\right] - \frac{\Delta}{\Delta_m} \sin\left[\mu\left(1 - \frac{\Delta^2}{16\Delta_m^2} - \frac{x_0^2}{16}\right)\right]. \quad (26)$$

Solution (26) permits finding the contribution of the small interval values of x_0, $\Delta x_0 < 1$, near the point $x_0 = 0$ (or $\varphi_0 = \pi$), to the average rate of change in phase $\overline{d\varphi/d\mu}$ or to the difference $\overline{\Delta\mathscr{E}} - \Delta$ (13):

$$(\overline{\Delta\mathscr{E}} - \Delta)_n = -\frac{\Delta}{2\pi} \operatorname{Re}\left(e^{i\mu[1-(\Delta^2/16\Delta_m^2)]} \int_{-\Delta x_0/2}^{\Delta x_0/2} dx_0 e^{-i\mu x_0^2/16}\right). \quad (27)$$

The characteristic interval of values of x_0, which concontributes to the integral (27), is $\delta x_0 \sim 1/\sqrt{\mu} < 1$. For $\delta x_0 < \Delta x_0$, the limits of integration in (27) can be replaced by $\mp\infty$, which again leads to Eq. (24) for $\Delta\overline{\mathscr{E}}$. From estimates of the range of values of the initial phase δx_0, contributing to the integral (27), it follows that as μ increases this interval becomes narrower. For $|x_0| > \delta x_0$, the difference $\Delta\overline{\mathscr{E}} - \Delta$ as a function of x_0 oscillates rapidly, as a result of which the contribution of the corresponding regions to $\Delta\overline{\mathscr{E}} - \Delta$ vanishes. As shown in Ref. 54, for $\mu > 1$, other regions of values of the initial phase φ_0, corresponding to unstable equilibrium positions $\varphi_0 = 0, 2\pi$, also do not contribute to $\Delta\overline{\mathscr{E}} - \Delta$. For this reason, for $\mu > 1$, Eq. (27) represents not only an estimate of the partial contribution of electrons close to the bottom of the potential well to $\Delta\overline{\mathscr{E}} - \Delta$, but it also determines the total average energy emitted by the electron beam as a whole, in the limit $\Delta x_0 \to \infty$, going over into expression (24).

5. ASYMPTOTIC PROPERTIES OF FEL IN A STRONG FIELD

Thus, in the saturation regime $\mu > 1$ and $|\Delta| < \Delta_m$, the energy $\Delta\mathscr{E}$, emitted by electrons per pass through the resonator, is determined by expression (24). This re-

sult leads to a number of interesting consequences concerning the nature of amplification in FEL when the saturation parameter μ is large.

If the gain G (21) is viewed as a function of the interaction time t (or magnet length L), then it is characterized by the same curve as the emitted energy $\Delta\mathscr{E}$ (Fig. 4). As the length L increases, the gain first increases and then, oscillating, attains a constant value.

For very large detuning $|\Delta| > \Delta_m$, independent of the magnitude of the parameter μ, the weak signal approximation is valid and Eq. (24) is replaced by expression (7), which in this case describes the decrease in gain G with increasing $|\Delta|$. Therefore, the quantity Δ_m, defined by Eq. (10), is the asymptotic spectral width of the gain Γ for a high field $\mu > 1$ (in the weak signal approximation $\mu < 1$, $\Gamma = \Gamma = 2\Delta_m|\mu$). Depending on the interaction time t, the spectral width of the gain Γ first decreases with increasing t (in region $\mu < 1$), and then assumes a constant value Δ_m (Fig. 5). On the other hand, depending on the field intensity of the wave E_0, the spectral width Γ remains constant, while $\mu < 1$, and for $\mu > 1$ increases as $\sqrt{E_0}$.

The function $G(\Delta)$ for fixed μ is illustrated qualitatively in Fig. 6.

The frequency ω, for which the maximum value of the gain is attained, for $\mu > 1$ decreases with increasing E_0:

$$\omega = \omega_{res}\left(1 - 2\frac{e}{m}\sqrt{\frac{E_0 B_0}{q_0\omega}}\right). \tag{28}$$

The shift in the frequency ω relative to ω_{res}, estimated under the conditions of the experiment described in Ref. 8, in order of magnitude equals $10^{-3}\,\omega_{res}$. The maximum (relative to frequency ω or detuning Δ) values of the emitted energy and gain for $\mu > 1$ are equal to $\Delta\mathscr{E}$ $(\Delta = \Delta_m)$ and $G(\Delta = \Delta_m)$.

The dependences of these quantities on the intensity E_0, following from Eqs. (10), (21), and (24), are illustrated in Fig. 7.

As E_0 increases, $\Delta\mathscr{E}_{max}$, oscillating, increases on the average as $\sqrt{E_0}$, while G_{max}, oscillating, decreases as $E_0^{-3/2}$. The decrease in the gain G_{max} with increasing E_0 could be the mechanism that determines the stationary lasing conditions in FEL: if G_{max} decreases to the loss level, then further increase in E_0 ceases. The horizontal dot-dash lines in Fig. 7b characterize the loss level under different conditions. The oscillatory dependence $G_{max}(E_0)$ can be the reason for the fact that for small losses (see Fig. 7b, 2) lasing, generally speaking, can occur in several ranges of values of the field intensity E_0, where G_{max} is greater than the loss

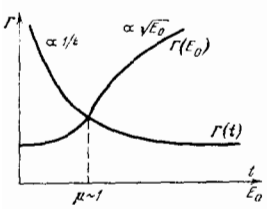

FIG. 5. The spectral width Γ of the gain as a function of the interaction time t and field intensity E_0.

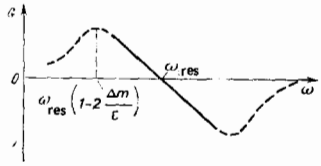

FIG. 6. Spectral dependence of the gain $G(\Delta)$ in the saturation regime $\mu > 1$.

level. The efficiency equals $\Delta\mathscr{E}(\mu)/\varepsilon$ and behaves like $\Delta\mathscr{E}(\mu)$. For detuning Δ corresponding to the maximum gain, $\Delta \approx \Delta_m$, under saturation conditions $\mu > 1$, the efficiency in order of magnitude equals

$$\frac{\mu}{4\pi N}\left[1 - \sqrt{\frac{2}{\pi\mu}}(\cos\mu + \sin\mu)\right], \tag{29}$$

where, as previously, N is the number of undulator periods.

Just as $\Delta\mathscr{E}_{max}$, the efficiency (29) with increasing field E_0, oscillating, increases on the average $\propto\sqrt{E_0}$ (see Fig. 7a). The decrease in efficiency noted in Refs. 35 and 43, after passing through a maximum, is, in reality, only a temporary drop, related to the oscillations in $\Delta\mathscr{E}_{max}(\mu)$, which then again increases, as long as E_0 (or μ) does not stop increasing due to a decrease in G_{max}.

The conditions of the experiment described in Ref. 8 correspond to a saturation parameter $\mu \approx 5$, which corresponds to the beginning of the saturation region (Fig. 4), where the weak signal approximation is no longer valid, but the numerical difference between estimates using Eqs. (7) and (24) are not yet too large. From this point of view, in order to check the theoretical predictions, it is undoubtedly interesting to perform experiments in a region with greater saturation.

As already noted, the pendulum equation (8) was solved, in application to the theory of FEL, numerical-

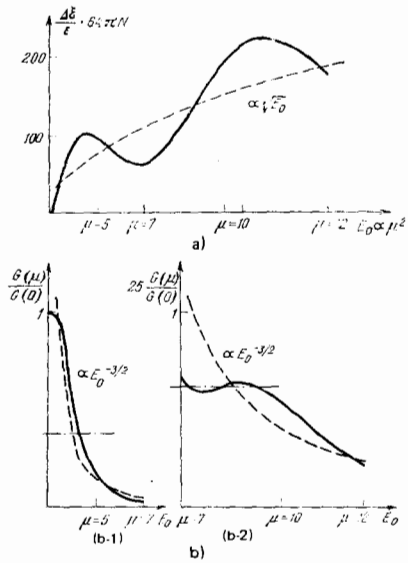

FIG. 7. The dependence on the field intensity E_0 of maximum (along the spectrum) gain G_{max} (b) and emitted energy $\Delta\mathscr{E}_{max}$ (a) (for $\Delta \approx \Delta_m$).

ly in Refs. 36, 43, 46–48. The results of numerical calculations agree very well with the theoretical equations, presented above, from Refs. 49 and 50. Graphs of $G(\Delta)$ for different values of the saturation parameter μ were constructed numerically in Ref. 46. These curves are similar to the ones shown in Fig. 4. The maximum values of the gain, found from these curves, do not contradict the dependence $E_0^{-3/2}$, although the number of points satisfying the conditions of an asymptotically strong field, in numerical calculations,[47] was not very large (2–3). The position of the maximum on the curves $G(\Delta)$ also, with this reservation, agrees well with that determined by (10) and Eq. (28). Here, it should be noted that the dependences described by Eqs. (24), (28), and (29), were not computed directly in Refs. 46–48 in the course of solving the problem numerically, although this would have undoubtedly been interesting.

A comparison of the analytic and numerical results permits estimating more precisely the limits of applicability of the asymptotic equations (24), (28), and (29). Although formally in deriving them, it was assumed that $\mu \ll 1$ and $|\Delta| \ll \Delta_m$, numerical calculations show that the asymptotic equations are already readily applicable for $\mu \gtrsim 2$.

6. COMPTON LASER. NONCOLLINEAR AMPLIFICATION SCHEME

As noted above, the idea of a Compton laser was first proposed by Pantell.[11] Subsequently, the possibilities for amplification in a Compton laser were studied theoretically in a number of papers,[65,66,82-87] but this idea has not been realized experimentally up to now.

According to Ref. 11, in a Compton laser, a low-frequency electromagnetic wave (pump wave with frequency ω_1) must propagate toward a relativistic electron beam. In this case, within the scope of the collinear scheme, examined in Ref. 11, amplification is possible at the frequency ω_2 of the reflected wave, propagating parallel to the electron beam

$$\omega_2 \approx \omega_{2\,\mathrm{res}} = 4\left(\frac{\varepsilon}{m}\right)^2 \omega_1. \tag{30}$$

The frequency $\omega_{2\mathrm{res}}$ is similar to the resonant frequency in the undulator (1) and expression (30), just as Eq. (1), follows from conservation of energy and momentum, characterizing the process of stimulated Compton scattering in second-order perturbation theory. In Ref. 11, the gain in a Compton laser was also found in second-order perturbation theory. The general equations that describe multiphoton transitions and nonlinear amplification in a Compton laser are, to a large extent, similar to the equations examined above for a FEL based on an undulator (20). From here it follows, in particular, that relative to the process of stimulated Compton scattering in this very simple geometry all the considerations and conclusions concerning the gain saturation, the role of multiphoton processes, and nonlinearity parameters presented above are valid.[79] In Eqs. (10), (21), (22), and (24), in this case, E_0 and B_0 are replaced by the intensities of the pump field and the generated radiation field E_1 and E_2, while the parameters q_0 and ω are replaced by ω_1 and ω_2. Over a wide range of values of the parameters determined by the condition $\eta > 1$, the process of stimulated Compton scattering is a multiphoton process. In this case, calculations in lowest order quantum perturbation theory, strictly speaking, are not applicable (although they do give the correct expressions for the gain). The value of the multiphoton parameter $\eta \approx 1$, for example, for $\lambda_1 = 3.2$ cm, $\varepsilon = 50$ MeV, corresponds to intensities $E_1 \approx E_2 \approx 3\cdot 10^3$ V/cm. The conditions for gain saturation in a Compton laser, as before, are determined by the relation $\mu \sim 1$, which corresponds to $E_1 \approx E_2 \approx 10^5$ V/cm. It is interesting to compare these estimates to values of the field intensity for which the multiphoton nature of the absorption process with spontaneous Compton scattering in the field of a single strong wave is manifested.[70,88,89] It is well known that Compton scattering of an electron in the field of a single strong wave with frequency $\omega \ll m$ becomes a multiphoton process for $eA \sim m$, where A is the vector potential of the wave or for $v_E \sim c$, where $v_E = eE_0/m\omega$ is the amplitude of the oscillations of the electron in the field of the wave. Numerically, the condition $v_E \sim c$ for $\omega = 3\cdot 10^{15}$ s^{-1} corresponds to a field intensity $E_0 \sim 5\cdot 10^9$ V/cm. A comparison of the estimates presented shows that in the field of two waves the multiphoton nature of the scattering appears much earlier than in the field of a single strong wave.

The probabilities for multiphoton stimulated Compton scattering in the field of two waves for $\mu < 1$ are determined by Eqs. (23).[71]

The gain in a Compton laser, within the scope of the traditional collinear scheme proposed in Ref. 11, for $\mu < 1$, is determined practically by the same expressions (6) and (7) as the linear gain in FEL based on an undulator.

Thus, for example, for a relatively short length L of the interaction region $(\Delta\varepsilon/\varepsilon)(L/\lambda_1) \ll 1$ (where $\lambda_1 = 2\pi/\omega_1$ is the length of the pump wave), the gain at frequency ω_2 can be written in the form

$$G = \frac{16\pi N_e E_1^2 e^4 L^3 \omega_1^{1/2}}{m^3 \omega_2^{5/2}} \frac{\mathrm{d}}{\mathrm{d}u} \frac{\sin^2 u}{u^2}, \quad u = \frac{t\,(\omega_2 - \omega_{2\,\mathrm{res}})}{2\gamma^2}, \tag{31}$$

where E_1 is the amplitude of the electric field intensity of the pump wave, $c = \hbar = 1$.

Under saturation conditions ($\mu > 1$), all characteristics of the behavior of the gain are determined by the results of Sec. 5. For $E_2 \approx E_1$, in the region $\mu \gg 1$, the gain $G_{\max}$ decreases as $E_1^{1/2} E_2^{-3/2} \approx E_1^{-1}$.

We will examine, in what follows, one of the possible ways of optimizing the amplification conditions in a Compton laser, related to an extension beyond the collinear scheme. This possibility was investigated in Refs. 65 and 66. It should be noted that amplification in a noncollinear scheme was considered in Ref. 86. However, the analysis carried out in Ref. 86 relates only to the nonrelativistic case and does not permit finding the optimal lasing conditions. These problems, just as the analysis of the dependence of the gain on the geometry, are examined below mainly following Refs. 65 and 66.

The vector potentials of two interacting waves, in

general, can be represented in the form

$$\mathbf{A}_{1,2}=\frac{E_{1,2}}{2\omega_{1,2}}\left(\mathbf{e}_{1,2}e^{i(\omega_{1,2}t-\mathbf{k}_{1,2}\mathbf{r})}+\text{c.c.}\right), \tag{32}$$

where $\mathbf{e}_{1,2}$ are unit polarization vectors, $\mathbf{k}_{1,2}$ are the wave vectors, and $|\mathbf{k}_{1,2}|=\omega_{1,2}$.

According to Refs. 65 and 66, the case of a small deviation from a collinear scheme, when the pump wave propagates toward the electron beam, while the amplified wave propagates at a small angle θ relative to the direction of the electron momentum, is of greatest interest. In this case, the relation between the frequencies ω_1 and ω_2 has the form

$$\omega_2=\frac{4\gamma^2\omega_1}{1+\gamma^2\theta^2}. \tag{33}$$

For $\gamma\theta\ll 1$, Eq. (33) goes over into (30), while for $\gamma\theta\gg 1$, the frequency ω_2 is almost independent of the electron energy, but depends on the angle θ: $\omega_2\approx 4\omega_1|\theta^2$.

Just as in the one-dimensional scheme, in the noncollinear geometry, the gain can be determined either by the finite interaction length $l\approx t$ or by the energy spread of the electrons $\Delta\varepsilon$. The parameter ζ, separating these two regions, in the noncollinear scheme with $\theta\ll 1$, according to Refs. 65 and 66, equals

$$\zeta=\frac{\Delta\varepsilon}{\varepsilon}\frac{2\sqrt{2}\omega_2 t}{1+\gamma^2\theta^2}. \tag{34}$$

If $\zeta>1$, then it is necessary to take into account the energy spread of the electrons and the gain has the form[65]

$$G=\frac{4\pi^2e^4N_e tE_1^2 f'(\varepsilon)\varepsilon^3}{m^4\omega_1\omega_2^2}\left(\frac{(\gamma\theta)^2-1}{(\gamma\theta)^2+1}\right)^2. \tag{35}$$

As a function of angle θ, the gain first decreases, vanishing for $\gamma\theta=1$, and then increases for $\gamma\theta=1$, and due to the decrease in frequency ω_2 (33) ($\omega_2\propto\theta^{-2}$).

The factor $(\gamma\theta)^2-1/(\gamma\theta)^2+1$ in Eq. (36) stems from interference of the second-order matrix elements, arising from the terms that are quadratic ($e^2\mathbf{A}_1^2$) and linear ($2e\mathbf{pA}$) in the field in the interaction energy in the multidimensional Klein-Gordon equation. For $\gamma\theta\gg 1$, the difference in the gain $G(\theta)$ and $G(\theta=0)$ is determined by the factor $(\gamma\theta)$.[4]

At first glance, the fact that the gain $G(\theta)$ increases ourside the relativistic cone $\gamma\theta>1$ may appear to be unexpected. Actually, the spectral intensity of the spontaneous Compton scattering outside the relativistic cone decreases. In the geometry being examined, it has the form[64]

$$\frac{d\mathscr{E}_{sp}}{d\omega_2 d\Omega_{k_2}}=\frac{1}{2\pi}\frac{e^4tE_1^2\varepsilon^3 f(\varepsilon)}{m^4\omega_1(1+\gamma^2\theta^2)}\left(\frac{(\gamma\theta)^2-1}{(\gamma\theta)^2-1}\right)^2, \tag{36}$$

where $d\Omega_{k2}$ is the element of the solid angle in the direction $\mathbf{k}_2$. As is easily seen, for $\gamma\theta>1$, the spectral intensity (36) decreases as θ^{-2} with increasing angle θ. Aside from the decrease in absolute magnitude of the intensity, in this case, there is also a narrowing of the spectral line corresponding to spontaneous emission. The width of the spectral line (36) is determined by the distribution function $f(\varepsilon)$. However, according to (33), for $\gamma\theta>1$, the dependence of the frequency ω_2 on energy ε decreases. For this reason, the change in energy ε by an amount $\Delta\varepsilon$ corresponds to a decreasing change in the frequency ω_2, $\Delta\omega_2$, i.e. as a function of frequency ω_2 the spontaneous emission line narrows:

$$\Delta\omega_2\approx\frac{8\omega_1}{\gamma^2\theta^4}\frac{\Delta\varepsilon}{\varepsilon}. \tag{37}$$

The gain G for $\zeta>1$ is proportional to the derivative of the spectral intensity of spontaneous emission:

$$G=\frac{16\pi^3N_e}{\varepsilon\omega_2}\frac{d}{d\omega_2}\left(\frac{d\mathscr{E}_{sp}}{d\omega_2 d\Omega_{k_2}}\right). \tag{38}$$

Narrowing of the spontaneous emission line for $\gamma\theta\gg 1$ leads to an increase in the derivative of the spectral intensity with respect to frequency and, therefore, to an increase in gain.

The increase in gain $G(\theta)$, of course, is not unlimited. The limitation is related to the fact that if for some values $\theta>1/\gamma$, $\zeta(\theta)>1$, then for large θ the sign of the inequality changes and the parameter ζ becomes small, $\zeta(\theta)<1$. In this case, it is possible to neglect the energy spread of the electrons, taking into account instead of this, the finiteness of the interaction time, which gives[65]

$$G=\frac{2\pi t^3e^4E_1^2N_e}{\varepsilon^3\omega_1}\left(\frac{\gamma^2\theta^2-1}{\gamma^2\theta^2+1}\right)^2\frac{d}{du}\frac{\sin^2 u}{u^2}, \tag{39}$$

where

$$u=\frac{t}{2\gamma^2}\left[(1+\gamma^2\theta^2)\omega_2-4\gamma^2\omega_1\right].$$

In the general case of arbitrary ζ, the gain G is determined by the smallest of the quantities (35) and (39). It should be kept in mind that the interaction time t itself (or length l) can depend on the angle θ. If d is the transverse size of the electron beam, then $l=d/\theta$. It follows from here that in the region where $\zeta(\theta)>1$, the gain G (35) increases as θ^3: $G\propto l\theta^4=d\theta^3$. On the other hand, for $\zeta(\theta)<1$, the gain decreases as θ^{-3}: $G\propto l^3=d^3/\theta^3$.

The nature of the function $G(\theta)$ is illustrated qualitatively in Fig. 8.

The optimum conditions for generation in a Compton laser are determined as the condition for changing from one of the two mechanisms for determining the gain examined above to the other, i.e. by the relation $\zeta(\theta)\approx\sqrt{2\pi}$. This equality determined the optimum angle

$$\theta_0\approx 2^{2/3}\pi^{1/6}\left(\frac{1}{\gamma^2}\frac{d}{\lambda_1}\frac{\Delta\varepsilon}{\varepsilon}\right)^{1/3}. \tag{40}$$

The maximum gain, attained for $\theta\approx\theta_0$, in order of magnitude equals

$$G_{max}\approx\pi\frac{l_0^3e^4E_1^2N_e}{\varepsilon^3\omega_1}, \tag{41}$$

where $l_0=d/\theta_0$. Substituting l_0 and θ_0 into Eq. (41) transforms the optimum gain G_{max} into the form

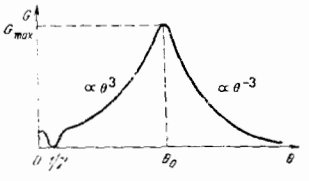

FIG. 8. The dependence of the gain G in a Compton laser on the angle between the directions of propagation of the electron beam and the amplified wave. θ is the optimal angle.

$$G_{\max} = 10^{-16} \frac{\lambda_1^2 I_1 (\mathrm{W/cm^2})\, J\,(\mathrm{A})}{(\Delta\varepsilon/\varepsilon)\,\gamma}, \tag{42}$$

where I_1 is the intensity of the radiation in the pump wave and J is the current in the electron beam.

According to (42), all the dependence of $G_{\max}$ on the parameters of the pump wave is concentrated in the factor $\lambda_1^2 I_1$. This means that when the pump wavelength λ_1 increases, the intensity I_1 necessary for attaining a fixed gain decreases as λ_1. It should also be noted that in the noncollinear scheme the optimum gain (42) depends on the total current J and not on the current density, and does not depend on the transverse size of the electron beam.

We will present an estimate, illustrating the possibility of amplification in a Compton laser in the noncollinear experimental scheme. For electron beam parameters $J_{\max} = 1$ kA, $d = 0.5$ cm, $\gamma = 20$, $\Delta\varepsilon/\varepsilon = 10^{-3}$ and pump wave $\hbar\omega_1 = 0.1$ eV, $E_1 = 5\cdot 10^7$ V/cm, we have $l_0 = 5$ cm, $\theta_0 = 0.2$, $G \sim 1\%$. This result indicates the possibility of attaining significant amplification in the ultraviolet frequency range ($\hbar\omega_2 = 10$ eV) using CO_2 laser radiation as a pump. If it is assumed that the same expression as in the one-dimensional case is retained for the saturation parameter μ in the noncollinear scheme, then the highest attainable field intensity E_2 in the generated radiation for the numerical values of the parameters of a Compton laser presented above, in order of magnitude, equals $E_2 \sim 3\cdot 10^4$ V/cm.

7. CONCLUSIONS

In summarizing the discussion of free-electron lasers, it is useful to point out the basic lines along which it is possible and useful to setup experiments on FEL. Of course, there is great interest in developing classical high-frequency electronics[90] and going on to high density relativistic beams. Here, it is difficult to expect very high electron energies and high frequency conversion factors. However, the power of the existing radiation sources can be very high with the use of high-current electron accelerators.[91] In essence, this group of problems was not discussed in this review, since we concentrated on FEL based on high-energy electrons.

For electron energies of the order of several tens of MeV, it becomes possible to create FEL operating at infrared frequencies. Creation of such lasers could be of great interest for studying the physics of the interaction of radiation with molecules. In this electron energy range, under typical conditions, the amplification mechanism in FEL is the single-particle electron scattering examined in detail above.

It is possible to create FEL both using the traditional undulator scheme and, in principle, utilizing stimulated Compton scattering. In this case, it is necessary to use powerful UHF radiation sources, for example, a magnetron, as the pump. Estimates show that in order to attain acceptable magnitudes of the gain it is necessary to use UHF radiation sources with intensities of the order of 10–10^2 MW/cm^2, which, apparently, is possible in the pulsed regime.

Finally, there could be great interest in the ultraviolet frequency range. In this case, for electron energies of several tens of MeV, it is hardly possible to count on creating a magnetic undulator with the necessary small spacing (10^{-3} – 10^{-4} cm). According to results and estimates of the preceding sections, under these conditions, it may be expected that a Compton laser can be created, if a powerful CO_2 laser, operating in the picosecond regime, is used as a pump.

A completely independent set of problems is opened up by analyzing the possibilities of creating radiation sources by channelling particles in crystals. These problems have recently been attracting increasing interest.[92,93] However, physically, channelling of particles, undoubtedly, differs considerably in nature from amplification with passage of electrons through an undulator or with stimulated Compton scattering, as examined in the present review, although physically, the phenomena occurring here are related to FEL and are of undoubted interest.

Finally, from the point of view of studying the physics of processes in FEL, it is interesting to study experimentally the nonlinear dependences of the gain in the saturation region, discussed in Sec. 5. In this respect, the experiment reported in Ref. 94, in which the dependence of the gain in a FEL based on an undulator on different parameters of the system in the UHF range were studied, is very interesting. In particular, it was discovered that the gain depends nonmonotonically on the length of the undulator, which can be interpreted as the appearance of an oscillatory dependence with the transition to saturation (first oscillation in Fig. 4). Setting up such experiments and, especially in the infrared and optical frequency ranges, is of great interest for studying the physics of amplification in FEL.

On the whole, apparently, it may be stated that the basic directions for development of FEL, which are of greatest interest, are: increasing the lasing frequency, increasing the FEL power, and investigating and using new schemes and principles for amplification.

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