Plasma microwave electronics

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Usp. Fiz. Nauk **133**, 3–32 (January 1981)

The theory of Cherenkov and cyclotron plasma sources of coherent microwave radiation excited by intense electron beams is reviewed systematically. The linear approximation of the theory yields the output frequency spectra, the wave growth rates, and the threshold electron beam currents required for exciting these sources. The general theory is illustrated for some particular devices: the forward-wave plasma Cherenkov source, foward-and backward-wave cyclotron-resonance masers, and foward-and backward-wave Cherenkov sources in a slightly corrugated slow-wave structure. The nonlinear theory of plasma microwave sources leans heavily on a numerical solution of the dynamic equations of the electromagnetic field and the charged particles in the system. The nonlinear operation of a forward-wave plasma Cherenkov source is analyzed. Under optimum conditions, the efficiency of this device can reach 30–35%. The efficiencies of other high-current plasma sources are comparable in magnitude. Experimental progress toward the development of high-power pulsed microwave sources of the Cherenkov and cyclotron types using intense relativistic electron beams is reviewed briefly.

PACS numbers: 52.25.Ps, 52.40.Mj

CONTENTS

1. INTRODUCTION

Our purpose in this review is to discuss systematically the existing theory for plasma sources and amplifiers of coherent electromagnetic radiation which use intense relativistic electron beams. Plasma sources of electromagnetic waves first came under discussion after the publication of papers by Akhiezer and Fainberg[1] and Bohm and Gross[2] (see also Ref. 3), where the two-stream instability was discovered and where it was predicted that the energy associated with the directional motion of electron beams could be converted into the energy of electromagnetic waves in plasmas. Subsequent and extensive experimental work completely confirmed the predictions of Refs. 1–3, but the development of plasma sources of coherent electromagnetic radiation proved far from simple. A firm basis for developing such sources has become available only in the past decade, as a result of progress in the technology and physics of intense relativistic electron beams.[4,5] Although the development of plasma sources and amplifiers of electromagnetic radiation is still in a primitive stage, the basic theory of these devices has already been worked out. It is fair to say that the theoretical foundation has been laid for a new branch of plasma physics: high-current relativistic plasma microwave electronics.

The term "intense electron beams" is usually understood as meaning beams with currents above the so-called vacuum limit. In a metal waveguide of radius R and length $L \gg R$, of the type customarily used as resonators in vacuum-tube electronics (Fig. 1), the beam current is limited by the electron space charge. This limiting current is determined in order of magnitude by[5]

$$\omega_{b\,\mathrm{sp}}^2 \approx \frac{c^2}{S}\gamma, \tag{1.1}$$

where $\gamma = [1-(u^2/c^2)]^{-1/2}$ is the relativistic factor of the electron energy, S is the cross-sectional area of the beam, which is smaller than or comparable to that of the waveguide, and $\omega_b = \sqrt{4\pi e^2 n_b/m}$ is the plasma frequency of the beam electrons. An intense beam would thus be understood as a beam for which the condition $\omega_b^2 \gtrsim \omega_{b\,\mathrm{sp}}$ holds. Such a beam can be transported through a waveguide only if the electron space charge is neutralized; neutralization can be arranged by filling the system with a dense plasma[1)] (dense in comparison with the beam). Strictly speaking, therefore, the field of intense-beam microwave electronics must be a field of plasma electronics.

It is possible, however, that the plasma may not have any important effect on the frequencies of the electromagnetic waves excited by the beam. The wavelengths of the electromagnetic waves excited by the beam are shorter than or comparable to the transverse dimensions of the electrodynamic system of the source, i.e., of the resonator; more precisely, $\omega \gtrsim \omega_{\mathrm{co}} = \mu c/R$, where μ specifies the radial wave number of the oscillation mode (the roots of Bessel functions or their derivatives). Ordinarily, $\mu \sim 3-10$. If the plasma fil-

[1)]The beam current may exceed the vacuum limit even in the absence of a plasma, with purely ionic neutralization of the beam charge.[5] The plasma also neutralizes the beam current, allowing the current to be raised well above the vacuum limit.

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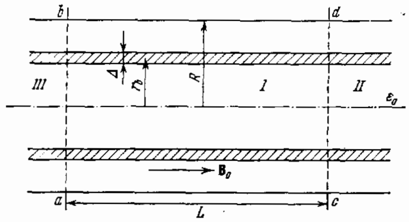

FIG. 1.

ling the resonator has a relatively low density, so that the condition $\omega_p = \sqrt{4\pi e^2 n_p/m} < \mu c/R$ holds, the plasma will not strongly affect the electrodynamics of the resonator, which remains an essentially vacuum device in terms of its electrodynamic properties. Such a plasma may neutralize the beam space charge and allow an intense beam with a current above the vacuum limit to be transported through the resonator. The beam current, however, cannot be much higher than the vacuum limit; specifically, it cannot exceed this limit by a factor greater than $\mu^2 S/\gamma R^2$, as follows from the inequalities $\omega_b^2 < \omega_p^2 < \mu^2 c^2/R^2$. The situation is different in the case of a dense plasma, with $\omega_p > \mu c/R$. A dense plasma will not simply neutralize the beam space charge in the resonator; it will also cause important changes in the entire electrodynamics of the resonator, in particular, in the spectra of the electromagnetic natural modes of the resonator. It is in this case that we are dealing with genuine plasma electronics, which will allow us to use electron beam currents far higher than the vacuum limit, by a factor as high as $\mu^2\gamma S/R^2$, as we shall see below. Furthermore, when ultrarelativistic electron beams are used in such systems it becomes possible to excite efficiently waves with a wavelength far shorter than the transverse dimensions of the resonator, $\lambda \approx R/\gamma^2$. In other words, plasma microwave electronics presents us with the opportunity of developing sources of intense short-wave electromagnetic radiation.

In principle, short waves with $\lambda \approx R/\gamma^2$ could also be excited in vacuum systems. In practice, however, the electrodynamic system of the source must be filled with plasma, and electron beams with a current above the vacuum limit must be used.

The theory of plasma sources and amplifiers of electromagnetic radiation has been constructed by analogy with the theory of plasma instabilities on the basis of a general formalism of the electrodynamics of material media.[6-8] In this sense the theory differs from the theory for the devices of classical vacuum microwave electronics—the theory for the traveling-wave tube, the backward-wave tube, klystrons, "gyrotrons," etc. In vacuum microwave electronics one deals with the interaction of an individual beam electron with the field of the electromagnetic wave in the resonator, and one calculates the work performed by the beam electrons on the field as the beam passes through the resonator. If this work is sufficient to offset the loss of electromagnetic-field energy which results from radiation from the resonator, electromagnetic waves will be excited in the system.[9-12] Generally speaking, this approach is limited to relatively low beam current densities, at which the distortion of the resonator field by the beam is negligible. Obviously, the strong inequality $\omega \approx \mu c/R \gg \omega_b$ must be satisfied; this condition is equivalent to requiring that the beam current be small in comparison with the vacuum limit. For dense, high-current beams, this approach is of course unsuitable, and it is especially poor in the case of plasma electronics, in which plasma-filled resonators are used as the electrodynamic system. In this case the resonator field is distorted by the plasma to the extent that a self-consistent solution must be found for the equations of the electromagnetic field and the equations of motion of the plasma as a material medium; in the case of intense beams, the equations of motion of the beam electrons are also included here. This is the approach taken in plasma electrodynamics in a study of plasma instabilities.[6-8]

In plasma electrodynamics, on the other hand, the customary approach in the theory of the stability of nonequilibrium plasma waveguides is to analyze systems which are unbounded in the longitudinal direction[2)] (Refs. 6–8), while plasma sources and amplifiers of electromagnetic radiation are definitely bounded in the longitudinal direction; there are special devices for injecting the beam and extracting the radiation, and there is also a feedback system. This circumstance must be taken into account in the derivation of a theory of plasma sources of electromagnetic radiation. Consequently, any source or amplifier of electromagnetic radiation is represented as a spatially bounded medium consisting of a plasma and a relativistic electron beam. In a system of this type, which is not at thermodynamic equilibrium, electromagnetic waves can be excited; in other words, small perturbations in such a system may prove unstable and grow with time. The theory for such a system must tell us the conditions for the occurrence of an instability in the system, the frequencies of the electromagnetic waves excited by the beam, the growth rates of these waves, and the threshold electron beam currents required to excite the system. We also need a theory for the nonlinear stage in which the instability reaches saturation, and the theory must tell us how successful the system is in converting beam energy into radiation energy; i.e., it must tell us the efficiency of the source.

The methods which have been developed for analyzing spatially bounded plasma–beam systems are not adequate for solving the source problem as formulated here in the general case of an arbitrary electron beam density. It becomes necessary to restrict the theory to beams for which the density is not very high and for which the current is not much higher than the vacuum limit, so that the frequencies of the waves excited by the beam satisfy $\omega^2 \gg \alpha\omega_b^2$, where α is a small parameter which depends on the nature of the interaction between the beam electrons and the field of the excited wave and which takes on different values for different

[2)] The longitudinal boundedness of the system has been taken into account in several studies of the maximum electron-beam currents in plasma-filled waveguides (see the reviews in Refs. 13 and 14).

types of sources. In solving the self-consistent problem it is thus necessary to consider only the terms which are linear in the beam density, and the problem is thereby simplified considerably. This approximation is equivalent to the one-particle approximation used in conventional vacuum microwave electronics. For this reason, some of the specific results which we will see below are the same as those derived in Refs. 10–12. Nevertheless, again in this case we will follow the general electrodynamic formalism, since it not only is more general and gives us a way to approach the case of truly intense beams but also is in a more canonical form.

The distinction of "truly intense" electron beams is based on a comparison of the magnitude of the beam-induced perturbation of the natural-mode frequency spectrum of the resonator (more precisely, of the width of the excitation band) with the difference between the resonant frequencies of the unperturbed resonator. For the long systems in which we are interested, the minimum difference between resonant frequencies is of order $\Delta\omega \lesssim \pi c/L$, while the perturbation of the frequency spectrum by the electron beam is of order ω_b or even greater. For the intense beams with which we are concerned the conditions $\omega_b \gtrsim c\sqrt{\gamma}/R \gg \pi c/L$ hold, so that the excitation band may span a large number of longitudinal modes of the resonator. As a result, in high-current electronics we are generally dealing with sources which are in principle multimode sources, at least in terms of longitudinal wave numbers.

In this review the events which occur in plasma sources will be described in terms of normal modes which do not, in the linear approximation, interact with each other in the resonator volume but which do undergo mutual conversions at the boundaries of the resonator. We have a classical problem of the linear electrodynamics of spatially bounded media. To solve such a problem we should supplement Maxwell's equations,

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad \mathrm{div}\,\mathbf{D} = 0, \qquad \mathrm{rot}\,\mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \quad \mathrm{div}\,\mathbf{B} = 0 \tag{1.2}$$

with boundary conditions and with the constitutive equation

$$D_i = \hat{\varepsilon}_{ij}E_j, \tag{1.3}$$

where $\hat{\varepsilon}_{ij}$ is the dielectric permittivity tensor operator. The electrodynamic boundary conditions are usually found directly from system (1.2), (1.3) by integrating it over a boundary layer which physically is an infinitesimally narrow layer near the interface; this can be done only if constitutive equation (1.3) holds for the entire system. We must therefore deal with the problem of deriving constitutive equation (1.3) from a specific model for the medium.

In this review we are adopting the model of a cold electron plasma and a monoenergetic electron beam. In this model it is a straightforward matter to determine the dielectric permittivity tensor operator and to write constitutive equation (1.3) explicitly. This can be done by solving the linearized Vlasov kinetic equation for the plasma and beam electrons:

$$\frac{\partial \delta f}{\partial t} + \mathbf{v}\frac{\partial \delta f}{\partial \mathbf{r}} - \frac{\Omega}{\gamma(v)}\frac{\partial \delta f}{\partial \varphi} = e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vB}]\right\}\frac{\partial f_0}{\partial \mathbf{p}}. \tag{1.4}$$

Here $\Omega = eB_0/mc$, where B_0 is the external longitudinal magnetic field which prevents transverse expansion of the plasma and beam, so that the following condition holds[3)]:

$$\Omega^2 \gg \omega_b^2\gamma. \tag{1.5}$$

The beam electrons are assumed to be monoenergetic, and their equilibrium momentum distribution is written

$$f_{0b} = \frac{n_b}{2\pi p_{\perp 0}}\delta(p_\perp - p_{\perp 0})\,\delta(p_\parallel - p_{\parallel 0}), \tag{1.6}$$

where $p_{\perp 0} = m\gamma u_\perp$, $p_{\parallel 0} = m\gamma u_\parallel$ and $u_\parallel$ are the velocity components of the beam electrons respectively transverse and longitudinal with respect to B_0. It will be assumed that $u_\perp^2 \ll c^2$, since only under this condition can a relativistic beam be intense.[5] The equilibrium distribution of plasma electrons is similar in form if thermal motion is ignored:

$$f_{0p} = n_p\delta(\mathbf{p}). \tag{1.7}$$

Before we solve Eq. (1.4) with the equilibrium distribution in (1.6) or (1.7) we should examine the structure of those electrodynamic systems which will be discussed below in the particular examples of plasma sources of electromagnetic radiation. One of the most common systems used in practice for this purpose is a smooth metal waveguide of length L and radius R between boundaries ***ab*** and ***cd***, filled with a plasma (region I in Fig. 1). Boundary ***ab*** is a metal grid or thin foil which is transparent to the electron beam but opaque (reflecting) for the radiation. Region III is a pure vacuum, from which the unperturbed electron beam is introduced. Region II is a simplified model of a radiating horn consisting of a smooth waveguide of radius R filled with a dielectric with dielectric permittivity ε_0. Boundaries ***ab*** and ***cd*** play an important role in the excitation of the electromagnetic radiation. The waves which are being amplified—natural modes of the longitudinally unbounded system—are reflected and transformed at these boundaries. This transformation gives rise to a feedback: a mechanism for information transfer from one boundary to the other, which is a necessary attribute of any source of electromagnetic radiation. Not just any type of feedback will be sufficient for excitation, however; the system must not be at equilibrium, and in the case under consideration here the electron beam current must exceed a certain threshold value (or "starting current"). These threshold currents will be determined below for some specific plasma sources of electromagnetic radiation. The key element of the source is region I, which is the region in which the electron beam interacts with the electromagnetic field.

Let us determine the specific form of constitutive equation (1.3) for this model of the medium. For a sys-

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3) Generally speaking, this condition is overly stringent and is necessary only if there is absolutely no neutralization of the beam charge.

tem which is unbounded in the longitudinal direction we should seek a solution of the system of equations consisting of the field equations and the kinetic equations in the following form in the cylindrical coordinates (r, φ, z):

$$f(r)\exp(-i\omega t + ik_z z + il\varphi), \tag{1.8}$$

where ω is the frequency, and k_z and l are the longitudinal and azimuthal wave numbers. Substituting solutions of type (1.8) into system (1.2), (1.4), and using the general methods (which are described, for example, in Refs. 6 and 7), we can easily calculate in the limit $u_\perp^2 \ll c^2$ the dielectric permittivity tensor operator $\hat{\varepsilon}_{ij}$ which we are seeking. We will write this tensor out explicitly here, since it plays a governing role throughout high-current microwave electronics but has never before been written out completely in the literature:

$$\left.\begin{aligned}
&\varepsilon_{rr} = \varepsilon_{\varphi\varphi} = 1 - \frac{\omega_p^2}{\omega^2 - \Omega^2} + \varepsilon_\perp^b,\\
&\varepsilon_{r\varphi} = -\varepsilon_{\varphi r} = -i\frac{\omega_p^2\Omega}{\omega(\omega^2-\Omega^2)} - ig^b,\\
&\hat{\varepsilon}_{\varphi z} = -\frac{\gamma u_\parallel}{\Omega}\left(\frac{\Omega g^b}{\gamma(\omega - k_z u_\parallel)}\frac{\partial}{\partial r} - \frac{l}{r}g^b\right),\\
&\hat{\varepsilon}_{z\varphi} = \frac{\gamma u_\parallel}{\Omega}\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\Omega g^b}{\gamma(\omega - k_z u_\parallel)} + \frac{l}{r}g^b\right),\\
&\hat{\varepsilon}_{rz} = i\frac{\gamma u_\parallel}{\Omega}\left(-g^b\frac{\partial}{\partial r} + \frac{l}{r}\frac{\Omega g^b}{\gamma(\omega - k_z u_\parallel)}\right),\\
&\hat{\varepsilon}_{zr} = i\frac{\gamma u_\parallel}{\Omega}\left(-\frac{1}{r}\frac{\partial}{\partial r} r g^b - \frac{l}{r}\frac{\Omega g^b}{\gamma(\omega - k_z u_\parallel)}\right),\\
&\hat{\varepsilon}_{zz} = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{\gamma\gamma_\parallel^2(\omega - k_z u_\parallel)^2} + \frac{1}{r}\frac{\partial}{\partial r} r\varepsilon_\parallel^b\frac{\partial}{\partial r} -\\
&\qquad - \frac{l^2}{r^2}\varepsilon_\parallel^b + \frac{l}{r}\frac{\Omega}{\gamma(\omega - k_z u_\parallel)}\left(\frac{\partial}{\partial r}\varepsilon_\parallel - \varepsilon_\parallel\frac{\partial}{\partial r}\right).
\end{aligned}\right\} \tag{1.9}$$

Here

$$\left.\begin{aligned}
&\varepsilon_\perp^b = \varepsilon_\perp^+ + \varepsilon_\perp^-, \quad g^b = \varepsilon_\perp^+ - \varepsilon_\perp^-, \quad \varepsilon_\parallel^b = \varepsilon_\parallel^+ - \varepsilon_\parallel^-,\\
&\varepsilon_\perp^\pm = -\frac{\omega_b^2}{2\gamma\omega^2}\left[\frac{\omega - k_z u_\parallel}{\omega - k_z u_\parallel \mp (\Omega/\gamma)} + \right.\\
&\qquad\left. + \frac{u_\perp^2}{2}\left(k_z^2 - \frac{\omega^2}{c^2}\right)\frac{1}{[\omega - k_z u_\parallel \mp (\Omega/\gamma)^2]}\right],\\
&\varepsilon_\parallel^\pm = \frac{\omega_b^2}{2\omega^2}\left[\frac{u_\parallel^2}{\Omega}\frac{1}{\omega - k_z u_\parallel \mp (\Omega/\gamma)} + \frac{u_\perp^2}{\omega}\frac{(\omega \pm \Omega/\gamma)^2 - (\omega^2 u_\parallel^2/c^2)}{[\omega - k_z u_\parallel \mp (\Omega/\gamma)]^2}\right].
\end{aligned}\right\} \tag{1.10}$$

We turn now to the boundary conditions to supplement field equations (1.2), (1.3) in the solution of plasma-electronics problems. Along with the conditions that the fields must be bounded at the waveguide axis are the obvious boundary conditions that the tangential components of the electric field must vanish at the lateral metal walls of the waveguide and at boundary ab

$$\begin{aligned}
&E_z|_{r=R} = E_\varphi|_{r=R} = 0,\\
&E_r|_{z=0} = E_\varphi|_{z=0} = 0.
\end{aligned} \tag{1.11}$$

The other boundary conditions at the plasma-beam interfaces and also at boundary cd, which separates the beam-wave interaction volume (I) from the volume from which the excited radiation is extracted (II), are found through a direct integration of constitutive equation (1.3) near these boundaries. These boundary conditions are quite lengthy, and we will not write them out here (see Ref. 6 for more details). We simply note that the radial profile of the plasma density in the waveguide is assumed homogeneous. The beam, on the other hand, is hollow or annular, confined to a narrow region of thickness $\Delta \ll r_b$ near $r = r_b < R$. However, if the radial profile of the beam electron density, $n_b(r)$ is to be assumed sharply defined, we must require

$$\Delta \gg \frac{u_\perp \gamma}{\Omega}. \tag{1.12}$$

Equivalently we must require that the beam thickness be much greater than the Larmor radius of the electrons in the external longitudinal magnetic field.

Finally, we note that these boundary conditions are not sufficient for solving the source problem in which we are interested here. The missing boundary conditions can be supplied by specifying that there are no perturbations of the beam at the surface ab and that boundaries ab and cd are transparent to the beam electrons. In the case of a monoenergetic beam these conditions are written

$$\begin{aligned}
&\rho_b|_{z=0} = \mathbf{j}_b|_{z=0} = 0,\\
&\{\rho_b\}|_{z=L} = \{\mathbf{j}_b\}|_{z=L} = 0,
\end{aligned} \tag{1.13}$$

where ρ_b and $\mathbf{j}_b$ are the rf densities of the beam charge and current, respectively. An obvious condition for the source problem is also the requirement that there be no perturbations in regions II and III which are incident on region I. It is easy to see that this requirement is equivalent to the radiation condition.

Now that we have discussed the general formulation of the source problem in the electrodynamics of material media, we can proceed to analyze its solution. We first note, however, that in the model adopted above both the plasma and the beam were assumed monoenergetic; i.e., the thermal (energy) spread of the beam and plasma electrons was ignored completely. This approximation is legitimate if the plasma dimensions are much greater than the Debye screening length, and this condition is met with a wide margin in practice. For an electron beam this condition means that the Debye length is small in the proper frame of reference in comparison with the beam thickness. This requirement sets a lower limit on the beam current[15]:

$$J_b > J_{\rm low} \approx 8.5\frac{r_b}{\Delta}\frac{\gamma\Delta\mathscr{E}}{mc^2}\ (\text{kA}); \tag{1.14}$$

where $\Delta\mathscr{E}$ is the energy spread of the beam electrons in their proper frame. Inequality (1.14), which is one of the conditions which must be met if the electron beam is to be judged intense, imposes some extremely stringent restrictions on the energy characteristics of the beam, requiring that the electrons have an extremely small energy spread. For example, at an electron energy $\mathscr{E} = 1$ MeV (i.e., $\gamma = 3$) and with $r_b/\Delta = 3$, an energy spread of only 10 keV (i.e., $\Delta\mathscr{E}/\mathscr{E} \approx 1\%$) leads to the value $J_{\rm low} \approx 2$ kA.

Actually, inequality (1.14) is overly stringent. For the source to operate it is sufficient to require that the excitation bandwidth be greater than the frequency spread which stems from the thermal spread of the beam electrons. This last requirement will be written out explicitly below in the study of the specific sources of electromagnetic radiation.

2. LINEAR THEORY OF PLASMA SOURCES. FREQUENCIES OF THE EXCITED WAVES AND THRESHOLD CURRENTS

As mentioned earlier, plasma sources of electromagnetic radiation operate by virtue of the Cherenkov and cyclotron two-stream instabilities, which can be summarized as the induced excitation of electromagnetic waves by beam electrons in a resonant interaction of these electrons with the natural modes of the resonator, i.e., under the condition

$$\omega_0(k_z) = k_z u_\parallel + s\frac{\Omega}{\gamma}. \qquad (2.1)$$

Here $\omega_0(k_z)$ is the frequency of an electromagnetic natural mode of the resonator in the absence of the beam; in the case $s=0$, the resonance is a "Cherenkov" resonance, while in the case $s\neq 0$ the resonance is a "cyclotron" resonance. The first case corresponds to the Cherenkov radiation of natural modes of the resonator under conditions such that the longitudinal electron velocity is equal to the wave phase velocity; the second case corresponds to the cyclotron radiation of the electron as it revolves in the longitudinal magnetic field. The second case can occur, therefore, only if the transverse component of the electron velocity is nonzero, i.e., only if $u_\perp \neq 0$. If $u_\perp^2 \ll c^2$, the cyclotron radiation occurs primarily at the harmonics with $s = \pm 1$, which correspond to the normal and anomalous Doppler effects.

These elementary radiation mechanisms take the form of second-order poles in the contribution of the beam to the dielectric permittivity tensor (1.9) under resonant conditions (2.1). The pole corresponding to the Cherenkov resonance figures only in the element $\hat{\varepsilon}_{zz}$, while the poles corresponding to the cyclotron resonance figure in all elements of the tensor $\hat{\varepsilon}_{ij}$. Then, in particular, we can find the order of magnitude of the parameter α in the condition given above to specify a low-density beam, $\omega^2 \gg \alpha\omega_b^2$. For a Cherenkov resonance we find $\alpha \sim 1/\gamma^3$, while for a cyclotron resonance we find $\alpha \sim u_\perp^2/c^2$. In the limit $\omega^2 \gg \alpha\omega_b^2$ it would obviously be sufficient to consider only those beam terms which contain poles of second order in the tensor $\hat{\varepsilon}_{ij}$ in the solution of the problem of the preceding section, of the excitation of electromagnetic waves by an electron beam. In the linear approximation, this problem is an electrodynamic eigenvalue problem, from which we are to determine the frequency (ω) spectrum of electromagnetic waves of the system. If the condition $\mathrm{Im}\,\omega > 0$ holds for some of the eigenfrequencies, the system is unstable with respect to small electromagnetic perturbations, which will grow with time and which will be radiated as electromagnetic waves into region II. The equality $\mathrm{Im}\,\omega = 0$ determines the threshold conditions for excitation of the system.

A general method for solving source problems was derived in Ref. 16 and refined in Ref. 17 (see also Refs. 18 and 19). Below we will use this method for specific plasma sources of electromagnetic radiation. For completeness, we will review the basic features of this method.

The procedure for finding the excitation eigenfrequencies for the system of Fig. 1 can be summarized as follows. The solution of field equations (1.2) and (1.3) in regions[4] I and II should be sought in form (1.8), as mentioned above. Substituting these solutions into the field equations, and using the radial boundary conditions, we find the characteristic equations $D^{\mathrm{I,II}}(\omega, k_z) = 0$, which determine the normal-mode spectra in longitudinally unbounded systems having the radial structures of regions I and II, respectively. We denote the solutions of the equations $D^{\mathrm{I,II}}(\omega, k_z) = 0$ by $k_{zn}^{\mathrm{I,II}}(\omega)$, and we denote the corresponding eigenfunctions by $\varphi_n^{\mathrm{I,II}}$, $n = 1, 2, \ldots, N^{\mathrm{I,II}}$, where $N^{\mathrm{I,II}}$ is the number of normal modes in regions I and II. Then a general solution of the field equations in regions I and II can be written

$$E^{\mathrm{I,II}},\ B^{\mathrm{I,II}} = \sum_{n=1}^{N^{\mathrm{I,II}}} A_n^{\mathrm{I,II}} \varphi_n^{\mathrm{I,II}}(r) \exp[-i\omega t + il\varphi + ik_{zn}^{\mathrm{I,II}}(\omega) z], \qquad (2.2)$$

where $A_n^{\mathrm{I,II}}$ are arbitrary constants. Up to this point we have assumed that regions I and II are unbounded in the longitudinal direction. The boundedness of these regions can be taken into account by substituting (2.2) into the longitudinal boundary conditions at $z = 0,\ L$. Eliminating the constants $A_n^{\mathrm{I,II}}$ from the resulting algebraic relations, we finally find the dispersion relation which we have been seeking. This dispersion relation determines the complex eigenvalues ω, whose real parts are the frequencies of the excited waves and whose imaginary parts are the growth rates of these waves. Equating the imaginary parts of ω to zero, we find the threshold currents for the excitation of the source, as mentioned above. This essentially exhausts the linear part of the general theory of plasma sources. The method described above for solving the problem is valid only if $\varphi_n^{\mathrm{I}}(r) \equiv \varphi_n^{(\mathrm{II})}(r)$, i.e., only if the problem can be reduced to a one-dimensional problem. This is evidently the case for the simple system of Fig. 1. For more complicated systems the calculations become formidable, and there is hardly any point in going through them here.

It is easy to show that in the high-frequency limit $\omega^2 \gg \alpha\omega_b^2$ in which we are interested the characteristic equations in regions I and II can be written in a common form with an accuracy to terms linear in the beam density:

$$D_0(\omega, k_z) = \frac{A\omega_b^2}{[\omega - k_z u_\parallel - s(\Omega/\gamma)]^2}. \qquad (2.3)$$

The quantity $A \sim \alpha$ depends on the particular type of course and will be discussed in more detail below. The equation $D_0(\omega, k_z) = 0$ is the dispersion relation for determining the natural mode spectrum in the given region in the absence of a beam; the region is treated as unbounded in the longitudinal direction.

In region I, in which there is a resonant interaction of the electron beam with the wave, and the wave is amplified, Eq. (2.3) in the absence of a beam, $D_0(\omega, k_z) = 0$, is quadratic in k_z and determines two conjugate branches of natural modes, $\pm k_0(\omega)$. Under resonance condition (2.1) the electron beam perturbs the $k_z = k_0(\omega)$ branch but has essentially no effect on $k_z = -k_0(\omega)$. Accordingly, all four solutions of wave equation (2.3) can be written as follows:

$$k_{zi} = k_0(\omega) + \delta k_i \quad (i = 1, 2, 3), \quad k_{z4} = -k_0(\omega), \qquad (2.4)$$

where $|\delta k_i| \ll |k_0(\omega)|$. Obviously, $k_0(\omega)$ may be either positive or negative. A wave with $k_0(\omega) > 0$ is called a

[4]We are not interested in the solutions in region III, since there are no perturbations of any type in that region.

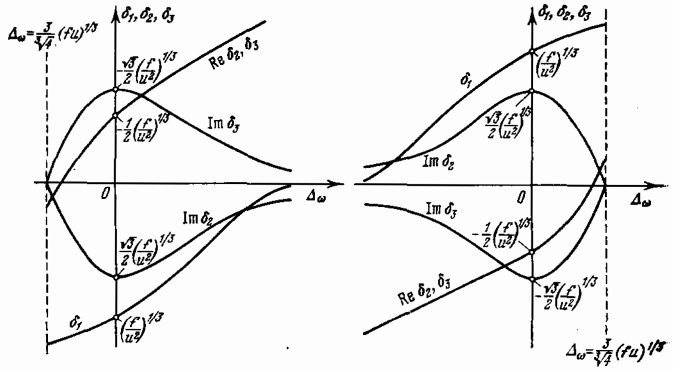

FIG. 2.

"forward" (or "comoving") wave, while a wave with $k_0(\omega)<0$ is a "backward" (or "oppositely directed") wave.[5] From (2.3) we find a cubic equation for δk_i:

$$\delta k\,(\Delta_\omega - \delta k u_\parallel)^2 = \frac{A\omega_b^2}{\partial D_0/\partial k_z} \equiv f, \tag{2.5}$$

where $\Delta_\omega = \omega - k_0(\omega)u_\parallel - s\Omega/\gamma$ is the difference between the radiation frequency and resonant frequency (2.1) (the "detuning") and f is a quantity proportional to the electron density in the beam (and thus proportional to the beam current density).

Equation (2.5) has complex roots corresponding to wave amplification in region I under the following conditions:

$$\frac{\sqrt[3]{4}\,\Delta_\omega}{3(|f|\,u_\parallel)^{1/3}} \begin{cases} <1 & \text{for } f>0, \\ >-1 & \text{for } f<0. \end{cases} \tag{2.6}$$

Obviously, these are the only values of the detuning in which we are interested in our study of the excitation of electromagnetic waves in the system. The quantity $|f|$, and thus also the beam current, must exceed certain threshold or "starting" values if the amplification is to overcome the damping of the waves which results from the escape of radiation from the system. One of the basic problems in the theory of sources is precisely this problem of determining the threshold values of $|f|$ and of the electron beam current for excitation of the source.

We will not write out the analytic expressions for the roots of Eq. (2.5); their behavior as a function of the detuning is shown qualitatively in Fig. 2a for $f<0$ and in Fig. 2b for $f>0$. We will restrict the analysis to the opposite limiting cases of small and large detunings Δ_ω. In the limit $|\Delta_\omega| \ll (|f|\,u_\parallel)^{1/3}$ (small detunings) we have

$$\delta k_1 = \left(\frac{f}{u_\parallel^2}\right)^{1/3}, \quad \delta k_{2,3} = \frac{-1 \pm i\sqrt{3}}{2}\left(\frac{f}{u_\parallel^2}\right)^{1/3}, \tag{2.7}$$

while in the opposite limit, $|\Delta_\omega| \gg (|f|\,u_\parallel)^{1/3}$ (large detunings), we have

$$\delta k_1 \approx \frac{f}{\Delta_\omega^2}, \quad \delta k_{2,3} = \frac{\Delta_\omega}{u_\parallel}\left[1 \mp i\left(\frac{f u_\parallel}{\Delta_\omega^3}\right)^{1/2}\right]. \tag{2.8}$$

[5]Here we are not considering media with a negative dispersion, in which the group and phase velocities of the waves are in opposite directions, so that the wave excited by the beam with $k_0<0$ is a forward wave, while that with $k_0>0$ is a backward wave.

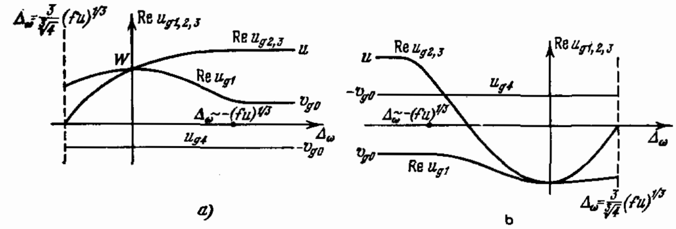

FIG. 3.

Now it is a simple matter to find the group velocities of all four waves: $u_{gi} = (\mathrm{Re}\,\partial k_{zi}(\omega)/\partial\omega)^{-1}$, $i=1,2,3,4$. In the small detuning limit we have

$$u_{gi} = w = \frac{3u_\parallel v_{g0}}{u_\parallel + 2v_{g0}} \quad (i=1,\,2,\,3), \qquad u_{g4} = -v_{g0}, \tag{2.9}$$

while in the large-detuning limit we have

$$u_{g1} = v_{g0}, \quad u_{g2,3} = u_\parallel, \quad u_{g4} = -v_{g0}; \tag{2.10}$$

here $v_{g0} = (-\partial D_0/\partial k_z)/(\partial D_0/\partial\omega)$ is the group velocity of the resonant wave in the absence of the beam. Figure 3a shows the relationship between the wave group velocities u_{gi} and an arbitrary detuning Δ_ω under the conditions $v_{g0}>0$ and $f<0$; Fig. 3b shows the corresponding results for $v_{g0}<0$ and $f>0$.

It can be seen from (2.7)–(2.10) that in the limit of small detuning Δ_ω, in which the growth rates reach their maximum values, the group velocities of all three resonant waves which are interacting with the beam ($i=1,2,3$) are of the same sign (this is the sign of v_{g0}) and are in fact equal in magnitude. This means that for wave excitation at a small detuning the necessary feedback in the system can be arranged only by a resonant wave which is reflected from radiating region II, $k_{z4} = -k_0(\omega)$, with a group velocity $u_{g4} = -v_{g0}$. In other words, there must be a finite reflection of the wave being amplified by the beam at $z=L$. This is precisely what happens when the beam excites a forward wave with a positive group velocity $v_{0g}>0$ (intersection point 1 in Fig. 4). For intense beams we have $\delta k_i L \gg 1$, so that it is sufficient to consider only two of the four waves: the resonant wave which is amplified by the beam and the nonresonant wave which is reflected from the $z=L$ cross section and which introduces feedback in the source. As a result, we find the following dispersion relation for determining the ω fre-

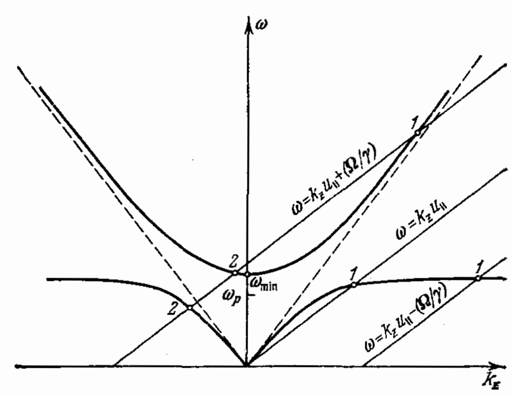

FIG. 4.

quency spectrum:

$$\exp\left[i\left(k_a-k_4\right)L\right]=\frac{3}{\varkappa}, \tag{2.11}$$

where k_a is either k_2 or k_3, depending on the sign of f; and $\varkappa$ is the reflection coefficient at $z=L$ for the wave being amplified.

To calculate $\varkappa$ we need to specify a model for the radiating device, i.e., for region II. The model adopted in Fig. 1 is far from perfect,[6)] but numerical calculations would be required for the real radiating devices. Accordingly, $|\varkappa|$ is generally determined experimentally from "cold" measurements (i.e., without the beam). If we assume that $|\varkappa|$ is known, then the entire source problem reduces, according to dispersion relation (2.11), to an analysis of region I alone. It is in this region that the beam undergoes the resonant interaction with the electromagnetic wave.

All the further equations are conveniently written in terms of ω_0 and k_{z0}, which are the real solutions of the system of equations

$$D_0(\omega,k_z)=0,$$
$$\omega-k_z u_\parallel-s\frac{\Omega}{\gamma}=0,$$

whose existence is necessary for the occurrence of the two-stream instability in the source. This system of equations is obviously equivalent to resonance condition (2.1). Using (2.4), (2.7) and (2.9), we find from (2.11) the real and imaginary parts of the frequency ($\omega\to\omega+i\delta\omega$):

$$\begin{aligned}\omega&=\omega_0-\min_n v_{g0}\left[k_{z0}-\frac{\pi n}{L}+\frac{1}{4}\left(\frac{|f|}{u_\parallel^2}\right)^{1/3}+\frac{1}{2L}\arg\varkappa\right],\\ \delta\omega&=\left[\frac{\sqrt{3}}{2}\left(\frac{|f|}{u_\parallel^2}\right)^{1/3}-\frac{1}{L}\ln\frac{3}{|\varkappa|}\right]\left(\frac{1}{w}+\frac{1}{v_{g0}}\right)^{-1}.\end{aligned} \tag{2.12}$$

It is not difficult to see that the integer n in (2.12) is the length of the resonator divided by the half-wavelength.

From the condition $\delta\omega=0$ we find the relation we have been seeking for the threshold beam current for excitation of the source with a forward wave[7)]:

$$|f_{\text{cr}}|=\frac{8}{3\sqrt{3}}\frac{u_\parallel^2}{L^3}\left(\ln\frac{3}{|\varkappa|}\right)^3. \tag{2.13}$$

The situation is completely different for excitation by the beam of a backward wave with a negative group velocity $v_{g0}<0$ (intersection points 2 in Fig. 4). Such waves may be excited in the limit of a large detuning Δ_ω even if there is absolutely no reflection from the radiating device or, in other words, if the radiator is perfectly matched with the resonator. In this case the feedback in the system is provided by the resonant waves themselves, which carry energy in opposite directions, according to (2.10). As a result we find the dispersion relation

$$\begin{aligned}&\sum_{i=1}^{3}\alpha_i\exp\left(ik_{zi}L\right)=0,\\ &\alpha_{1,2}=\pm\frac{\delta k_{2,1}\delta k_3}{(\delta k_2-\delta k_1)(\delta k_3-\delta k_{1,2})},\quad \alpha_3=\frac{\delta k_1\delta k_2}{(\delta k_3-\delta k_1)(\delta k_3-\delta k_2)},\end{aligned} \tag{2.14}$$

where the δk_i are given by (2.8). Using (2.8) and (2.10) we find from (2.14) the real and imaginary parts of the frequency ($\omega\to\omega+i\delta\omega$):

$$\begin{aligned}\omega&=\omega_0+\frac{\pi}{2}\frac{v_{g0}}{L}(4n-1)\left(1+\frac{u_\parallel}{|v_{g0}|}\right)^{-1},\\ \delta\omega&=\frac{u_\parallel}{L}\left(\frac{|f|}{|f_{\text{th}}|}-1\right)\left(1+\frac{u_\parallel}{|v_{g0}|}\right)^{-1}.\end{aligned} \tag{2.15}$$

Here $n=1,2,\ldots$ is the index of the longitudinal electromagnetic mode excited by the beam, and f_{th} determines the threshold beam current for excitation of the source with the backward wave. For the case $n=1$ the expression for f_{th} is[9] [cf. (2.13)]

$$|f_{\text{th}}|=8\frac{u_\parallel^2}{L^3}. \tag{2.16}$$

For the $n=2$ mode the quantity $|f_{\text{th}}|$ is nearly six times the value in (2.16). This means that the threshold currents for excitation of the fundamental ($n=1$) longitudinal mode and the second harmonic ($n=2$) of the backward wave differ by a factor of about six, while the difference in the case of the forward wave is a factor no greater than two. In backward-wave sources it is thus a relatively simple matter to arrange single-mode operation in terms of the longitudinal wave numbers. In forward-wave sources it is a considerably more complicated matter to arrange these conditions. Furthermore, forward-wave sources are generally multimode sources in the case of intense beams. A unit change in the number n in (2.12) changes the frequency ω by $\Delta\omega=\pi v_{g0}/L$. On the other hand, the width of the excitation band is of order $|\delta k_i|u_\parallel\gg\Delta\omega$ in high-current systems [this condition was used in the derivation of (2.11)]. It follows that in the case of high-current beams a forward-wave source must be a multimode source with respect to the longitudinal wave numbers n. Fainberg and Shapiro[20] have suggested using electron beams premodulated at the resonant frequency to overcome this difficulty, but experiments have shown[21] that this approach causes a substantial contraction of the excitation band.

3. SPECIFIC TYPES OF ELECTROMAGNETIC WAVE SOURCES

We will now apply this general theory to some specific types of plasma sources of electromagnetic radiation. In other words, we will write out Eqs. (2.12), (2.13), (2.15), and (2.16) explicitly for the several plasma microwave sources which have been used most extensively in experiments on high-current electronics.

a) Cyclotron-resonance maser for H-wave excitation. This maser[11, 22, 23] is a smooth metal waveguide in a longitudinal magnetic field which satisfies the conditions

$$\Omega\sim\frac{c\gamma}{R}\sim\omega\gamma\gg\omega_p,\quad \omega_b\sqrt{\gamma}. \tag{3.1}$$

[6)]The reflection coefficient $\varkappa$ for this model of the radiator is

$$\varkappa=\left[k_0^{\text{II}}(\omega)-\varepsilon_0k_0(\omega)\right]\left[k_0^{\text{II}}(\omega)+\varepsilon_0k_0(\omega)\right]^{-1};$$

where $k_0(\omega)$ corresponds to the forward wave in region I, i.e., $D_0(\omega,k_0)=0$ and $k_0^{\text{II}}(\omega)=\sqrt{\varepsilon_0(\omega^2/c^2)-(\mu_{ls}^2/R^2)}>0$, where μ_{ls} are the roots of the Bessel function, $J_l(\mu_{ls})=0$, or of its derivative, $J_l'(\mu_{ls})=0$, for the E and H waves, respectively. In the case of H waves, the quantity ε_0 in the expression for x should be replaced by unity. The reader is referred to §48 of Ref. 8, for example, regarding the field configuration of E and H waves in a plasma-filled waveguide.

[7)]Strictly speaking, the analysis above of the excitation of the forward wave is valid only under the condition $n\gg1$.

In a system of this type an annular beam can excite an H wave only at the cyclotron resonance [condition (2.1)] corresponding to the normal Doppler effect with $s=1$ (Fig. 4). The dispersion relation written in the form of (2.3) gives us

$$D_0(\omega, k_z)=k_\perp^2+k_z^2-\frac{\omega^2}{c^2}, \quad A=\frac{k_\perp^2 u_\perp^2 \Delta r_b}{2\gamma c^2 R^2} G. \tag{3.2}$$

Here G is a geometric factor which determines the efficiency of the interaction of the beam electrons with the field of the H wave; this factor is given by

$$G=\frac{J_{l+1}^2(k_\perp r_b)}{J_l^2(k_\perp R)-J_{l-1}(k_\perp R)\,J_{l+1}(k_\perp R)}.$$

Also, $k_\perp=\mu_{ls}/R$, where μ_{ls} are the roots of the derivative of the Bessel function, $J'_{ls}(\mu_{ls})=0$. From resonance condition (2.1) we find the frequencies of the electromagnetic waves excited by the beam:

$$\omega_{01,2}=\sqrt{k_\perp^2c^2+k_z^2c^2}=\frac{\gamma_\parallel^2\Omega}{\gamma}\left(1\pm\frac{u_\parallel}{c}\sqrt{1-\frac{k_\perp^2c^2\gamma^2}{\Omega^2\gamma_\parallel^2}}\right). \tag{3.3}$$

The group velocities and longitudinal wave numbers of these waves are

$$v_{g01,2}=c\frac{k_{z01,2}c}{\omega_{01,2}}, \quad k_{z01,2}=\frac{1}{u_\parallel}\left(\omega_{01,2}-\frac{\Omega}{\gamma}\right). \tag{3.4}$$

It is easy to see that under the condition $\gamma_\parallel > k_\perp c\gamma/\Omega > 1$ both roots satisfy $k_{z\,01,2}>0$; i.e., only forward waves can be excited in the system. If, on the other hand $k_\perp c\gamma/\Omega<1$, then $k_{z01}>0$ and $k_{z02}<0$, so that one of the waves excited by the beam is a forward wave, and the other is a backward wave. We also see a method for radial selection of modes. For example, if we impose the inequalities

$$3.8>\frac{R\gamma_\parallel\Omega}{c\gamma}>1.8, \tag{3.5}$$

then a single radial mode, with a minimum value $\mu_{11}=1.8$, will be excited in the system (this is the first asymmetric mode, H_{11}).

We will now use the general theory to determine the threshold currents for excitation of the source in the cases of the forward and backward cyclotron waves. For excitation of the forward wave, according to (2.13), the threshold beam current is

$$J_{\text{th}}\approx 55\frac{u_\parallel^4}{u_\perp^2}\frac{R^2|k_{z01}|u_\parallel\gamma}{L^2k_\perp^2cG}\left(\ln\frac{3}{|\varkappa|}\right)^3\ (\text{kA}). \tag{3.6}$$

Since $k_{z01}\sim n$, the minimum threshold current corresponds to the fundamental longitudinal mode with $n=1$ [we recall, however, that Eq. (3.4), like (2.13), is applicable only at large values of n, strictly speaking]. The threshold current for excitation of the $n=2$ mode differs from the minimum threshold current by a factor of only two, and this result again demonstrates that a real high-current forward-wave source will always be a multimode source in terms of the longitudinal wave numbers, unless special measures are taken.

With regard to the transverse modes, on the other hand, there is yet another possibility for mode selection [in addition to the absolute selection method described above, which involves the imposition of inequality (3.5)]. This other method would involve putting the beam at the maxima of the function G, at which the work performed by the beam electrons on the field of the excited H wave is maximized and at which the threshold current is correspondingly minimized.

More promising for longitudinal-mode selection is a source using a backward H wave. According to (2.16) the threshold current for the excitation of a source of this type, with a perfectly matched radiator, is given by the following expression for the fundamental longitudinal mode with $n=1$:

$$J_{\text{th}}\approx 275\frac{u_\parallel^4}{u_\perp^2}\frac{u_\parallel}{c}\frac{R^2\gamma}{L^2G}\frac{|k_{z02}|}{k_\perp^2}\ (\text{kA}). \tag{3.7}$$

Everything we said above about the selection of radial modes obviously remains completely valid for this case of the excitation of a backward cyclotron wave. On the other hand, the selection of longitudinal modes becomes much simpler in the case of the backward wave, because the threshold currents for the excitation of the $n=1$ fundamental mode and the $n=2$ mode differ by a factor of nearly six, as mentioned above.

Threshold currents (3.6) and (3.7) should evidently exceed threshold current (1.14) which corresponds to neglecting the thermal velocity spread of the beam electrons. We have already called attention to the fact that condition (1.4) is very stringent. The necessary condition for neglecting the thermal spread is $|\delta k|u_\parallel > (\Omega/\gamma)\sqrt{(\Delta\mathscr{E})/mc^2}$, which reduces to

$$J_b > J_{\text{low}}\approx\frac{40\gamma|k_{z0}|k_\perp R^2c^2\Delta\mathscr{E}}{u_\perp^2mc^2G}\times\begin{cases}\sqrt{\frac{\Delta\mathscr{E}}{mc^2}}\ (\text{kA}) & \text{for the forward wave}\\ \frac{|k_{z0}|}{k_\perp}\ (\text{kA}) & \text{for the backward wave,}\end{cases} \tag{3.8}$$

where $\Delta\mathscr{E}$ is the energy spread of the beam electrons in their proper frame of reference. On the other hand, the beam currents cannot be arbitrarily large. Neglecting the terms with first-order poles in the elements of the dielectric permittivity tensor (1.10)—these poles play a stabilizing role with respect to the cyclotron instability—places an upper limit on the beam current: $J_b < J_{b\,\max}\approx J_{\text{th}}(u_\perp^2\Omega L/c^2u_\parallel\gamma)^3$. The condition $J_{\max}>J_{\text{th}}$ must obviously hold.

It follows from (3.3) that $\omega_{01}\sim\gamma^2\omega_{02}\sim\gamma^2k_\perp c$. Accordingly, a cyclotron-resonance maser with a relativistic electron beam could in principle be used to excite short-wavelength radiation with a wavelength $\lambda\sim R/\gamma^2 \ll R$. Obviously, the threshold current for excitation of the source at a high frequency must be lower than that for a low frequency. Comparison of currents (3.6) and (3.7) leads to the following conditions for the excitation of microwave radiation:

$$0.2\gamma^2\left(\ln\frac{3}{|\varkappa_1|}\right)^3<1, \tag{3.9}$$

if a low-frequency backward wave can be excited in the system ($k_\perp c<\Omega$) or

$$\gamma^{2/3}\ln\frac{3}{|\varkappa_1|}<\ln\frac{3}{|\varkappa_2|}, \tag{3.10}$$

if a backward wave cannot be excited ($\gamma_\parallel > k_\perp c\gamma/\Omega > 1$). The subscripts 1 and 2 in (3.9) and (3.10) refer to the high and low frequencies, respectively (Fig. 4). It is easy to see that at large values of γ condition (3.10) can be satisfied much more easily than (3.9)

To complete this subsection we note that the density of the plasma filling the waveguide has appeared in none of these equations. This circumstance is a consequence of inequalities (3.1), under which the most effective resonance is the cyclotron resonance between the beam electrons and the electromagnetic wave. In this sense the theory for a plasma source differs in no way from the theory for a vacuum source (the discussion here is being carried out in terms of the electrodynamics of material media, in contrast with Refs. 11 and 22). We note, however, that the plasma may play an extremely important role even under these conditions: Because of the neutralization of the electron space charge of the beam by the plasma it is possible to raise the current in a cyclotron source substantially, especially for small values of the ratio $u_\perp^2/u_\parallel^2$. It can be seen from (3.6) and (3.7) that as this ratio is reduced the threshold currents for the excitation of cyclotron sources increase dramatically and may exceed the vacuum limit, which for this beam geometry is[5]

$$J_{\rm sp} \approx 17 \frac{(\gamma_\parallel^{2/3}-1)^{3/2}}{\frac{\Delta}{r_{\rm b}}+2\ln(R/r_0)} \frac{\gamma}{\gamma_\parallel} \text{ (kA)}. \qquad (3.11)$$

In vacuum sources, on the other hand, the threshold currents should be only a small fraction of the vacuum limit in (3.11). We must recall here that the dense plasma may shield the cyclotron radiation excited by the beam. If this shielding is to be avoided, the plasma density in the source must be limited to the value satisfying the condition[8]

$$\omega_{p\,\max}^2 < \frac{\Omega^2}{\gamma^2}(1+\gamma).$$

Finally, we will use (3.3)–(3.7) to find the characteristics of high-current plasma cyclotron sources using forward and backward waves in the centimeter wavelength range. Since such sources are well adapted for the excitation of high radial modes, we adopt $\mu_{ls}=\mu_{13}=8.5$; i.e., we consider the excitation of an H_{13} wave in a resonator with $|\varkappa|\approx 10^{-1}$, radius $R\approx 4$ cm, and length $L=12$ cm by a 1-MeV electron beam with $u_\perp/u_\parallel\approx 0.3$. The average beam radius, which coincides with a maximum of the function $J_2(\mu_{13}r/R)$, is chosen to be $r_b\approx 1.8$ cm, and the beam thickness is $\Delta\approx 0.6$ cm. At a magnetic field $B_0\approx 6$ kOe the frequency of the forward cyclotron wave excited by the beam turns out to be of order $\omega_1\approx 1.5\times 10^{11}$ s^{-1} ($\lambda_1\approx 1.3$ cm), while the frequency for the case of a backward wave is $\omega_2\approx 10^{11}$ s^{-1} ($\lambda_2\approx 2$ cm). The mode of the forward wave which is excited in this case is of order $n\approx 10$, and the corresponding threshold current is $J_{\rm th}\approx 30$ kA. For the backward wave, on the other hand, the threshold beam current for the excitation of a matched source is $J_{\rm th}\approx 10$ kA. In this case it is the fundamental longitudinal mode with $n=1$ which is excited. These threshold currents are above the longitudinal vacuum limit, which is of order $J_{\rm sp}\approx 3$ kA for this beam geometry, according to (3.11)

b) *Cyclotron-resonance maser for E-wave excitation.* This maser[11] is structurally the same as the H-wave source discussed above, and conditions (3.1) must be satisfied again in this case. Furthermore, Eqs. (3.2) and (3.3) remain valid, if we understand μ_{ls} as the roots of the Bessel function, $J_l(\mu_{ls})=0$, and

$$A=\frac{u_\parallel^2 u_\perp^2}{c^4}\left(\frac{k_\perp^2 c^2}{\omega^2}\right)^2 k_\perp^2 \frac{r_{\rm b}\Delta}{R^2} G, \quad G=\frac{J_{l-1}^2(k_\perp r_{\rm b})}{J_{l+1}^2(k_\perp R)}. \qquad (3.12)$$

It follows that the threshold currents for the excitation of cyclotron-resonance masers using forward and backward E waves differ from (3.6) and (3.7) by a factor A_E/A_H, where A_E and A_H are given by Eqs. (3.12) and (3.2), respectively. Therefore, the entire analysis above for H waves remains valid for E-wave sources. There are some quantitative changes, because of the changes in the quantities μ_{ls}, which in this case are equal to the roots of the Bessel functions themselves, rather than of their derivatives. As a result, there are quantitative changes in inequality (3.5), which is the condition for the selection of a single radial mode with excitation of the fundamental symmetric mode, $\mu_{01}=2.4$ (the E_{01} mode):

$$3.8 > \frac{R\gamma_\parallel\Omega}{c\gamma} > 2.4. \qquad (3.13)$$

Also associated with the excitation of the E wave is the circumstance that the minimum threshold current is reached when the beam is put at the maxima of the function $J_{l-1}(\mu_{ls}r/R)$, in which case the work performed by the electrons on the field of the E wave is maximized. There are also changes in inequalities (3.9) and (3.10); they become far more difficult to satisfy for excitation of a high-frequency E wave.

Finally, we note that we have discussed the excitation of H and E waves which are traveling in the azimuthal direction [see (1.8)]. All the equations above, however, are easily generalized to the case in which the beam excites waves which are standing waves in the azimuthal direction. For this purpose, the obvious replacement

$$J_{l\pm 1}^2(k_\perp r_{\rm b}) \to \frac{1}{2}\left[J_{l\pm 1}^2(k_\perp r_{\rm b})+J_{l\mp 1}^2(k_\perp r_{\rm b})\right]$$

must be made in the equations.

c) *Plasma Cherenkov source working at a low-frequency E wave.* This device[23, 24] is a smooth cylindrical waveguide which is completely filled with a strongly magnetized plasma whose properties satisfy the conditions.

$$\Omega \gg \omega_p \sim \frac{c\gamma}{R} \sim \omega \gg \frac{\omega_b}{\gamma^{3/2}}. \qquad (3.14)$$

Under these conditions, both purely Cherenkov excitation ($s=0$) of a forward plasma wave and cyclotron excitation of forward and backward plasma waves may occur through the normal Doppler effect ($s=1$) and the anomalous Doppler effect ($s=-1$), respectively (see the lower curves in Fig. 4). It is easy to see that in this limit of a strongly magnetized plasma the waves excited in the plasma at the cyclotron resonance are to a large extent electrostatic waves. Such waves are trapped in the plasma and are essentially not radiated out of it. From the standpoint of the excitation of electromagnetic waves, therefore, these particular waves are of no special interest, and we will discuss them no further.[8)]

[8)]See Ref. 25 regarding the particular features of the excitation of electrostatic plasma waves by an electron bean in the case of the anomalous Doppler effect.

In the Cherenkov interaction of a beam with a forward wave in the plasma, the dispersion relation, written in form (2.1), gives us

$$D_0(\omega, k_z) = k_\perp^2 + \left(1 - \frac{\omega_p^2}{\omega^2}\right)\left(k_z^2 - \frac{\omega^2}{c^2}\right),$$
$$A = 2\frac{\Delta r_b}{R^2}\frac{k_z^2\omega_p^2}{\gamma\gamma_\parallel^4}\frac{J_l^2(k_\perp r_b)}{J_{l+1}^2(k_\perp R)}; \qquad (3.15)$$

here $k_\perp = \mu_{ls}/R$, where μ_{ls} are the roots of the Bessel function, $J_l(\mu_{ls}) = 0$. From resonance condition (2.1) we find the frequency of the wave excited by the beam to be

$$\omega_0 = \sqrt{\omega_p^2 - k_\perp^2 u_\parallel^2 \gamma_\parallel^2}. \qquad (3.16)$$

It follows in particular that under the condition

$$3.8 > \frac{\omega_p R}{u_\parallel \gamma_\parallel} > 2.4 \qquad (3.17)$$

only a single radial mode can be excited in the plasma, with $\mu_{01} = 2.4$ (this is a fundamental axisymmetric E_{01} mode).

From (3.16) we easily find the group velocity and the longitudinal wave number of the forward wave in the plasma:

$$v_{g0} = \frac{u_\parallel}{(u_\parallel^2/c^2) + (\omega_p^2/k^2 u_\parallel \gamma_\parallel^4)}, \quad k_{z0} = \frac{\omega_0}{u_\parallel}. \qquad (3.18)$$

The threshold beam current for Cherenkov excitation of a source using such a wave is found from general expression (2.13):

$$J_{th} \approx 13.5\gamma\gamma_\parallel^6 \frac{J_{l+1}^2(k_\perp R)}{J_l^2(k_\perp r_b)}\frac{k_\perp^2 R^2}{k_{z0}^3 L^3}\frac{u_\parallel^3}{c^3}\left(\ln\frac{3}{|\varkappa|}\right)^3 \quad \text{(kA)}. \qquad (3.19)$$

In this case the threshold current falls off with increasing n, in proportion to n^{-3}; when the electron beam is put at the maximum of the function $J_l(k_\perp r)$, and the work performed by the beam electrons on the field of the E wave in the plasma is maximized, J_{th} reaches its minimum value.

Finally, we note that the condition under which we can ignore a thermal velocity spread of the beam electrons, $\delta k > (k_{z0}/\gamma^2)\sqrt{\Delta\mathscr{E}/mc^2}$, leads to a restriction on the beam current; only above the following value do Eqs. (3.15)–(3.19) hold [cf. (1.14)]:

$$J_{low} \approx 8.5\mu_{ls}^2\gamma\left(\frac{\Delta\mathscr{E}}{mc^2}\right)^{3/2}\frac{J_{l+1}^2(\mu_{ls})}{J_l^2(\mu_{ls} r_b/R)}. \qquad (3.20)$$

Obviously, threshold current (3.19) is meaningless if it lies below this lower limit.

Finally, let us find the characteristics of a Cherenkov source using an E wave in the plasma for the centimeter wavelength range. It is simple to show that for a plasma density $n_b \approx 3\cdot 10^{12}$ cm^{-3} and for $R = 2.5$ cm a 1-MeV electron beam (with $u_\perp \approx 0$ and $u_\parallel \approx 2.8\cdot 10^{10}$ cm/s) will excite the E_{01} fundamental radial mode with a frequency $\omega = 6\cdot 10^{10}$ s^{-1}. In a relatively low-Q resonator with $|\varkappa| \approx 0.3$ and $L \approx 20$ cm the threshold current for $r_b \approx 1$ cm is of order $J_{th} \approx 6$ kA, according to (3.19). This value is not much higher than the vacuum limit for a beam of this geometry in a vacuum waveguide. The longitudinal wave number of the excited wave is $n \approx 12$.

Special note should be taken of the fact that the threshold current in (3.19) increases very rapidly as the beam electrons become more relativistic, in proportion to γ^7, while the vacuum limit on the current increases in proportion to γ, according to (3.11). Accordingly, by increasing γ it is possible to use progressively more intense electron beams in plasma sources. Also noting that the plasma density at which the instability sets in increases at large values of γ, according to (3.16), as does the maximum frequency of the excited radiation, $\omega \approx c\gamma/R$, we clearly see the advantages of sources using plasma waves for exciting short-wavelength radiation in the millimeter and, perhaps, submillimeter ranges.

d) Cherenkov source with a slightly corrugated slow-wave structure. This device[10,28] is a metal waveguide whose lateral surface is specified by the equation $R(z) = R_0 + h\cos k_0 z$, where R_0 is the mean radius, h is the corrugation depth, and $2\pi/k_0$ is the corrugation period. If cyclotron excitation of high-frequency E and H waves and also the Cherenkov excitation of a low-frequency plasma E wave are to be prevented, the following conditions must be satisfied:

$$\Omega \gg \frac{c\gamma}{R} > \omega_p, \quad k_0 u, \quad \omega_b\sqrt{\gamma}. \qquad (3.21)$$

For simplicity we assume that the beam is rectilinear ($u_\perp \approx 0$) and fills the waveguide completely. Under these conditions, a high-frequency E wave may be excited in the system; its spectrum will differ only slightly from that of E waves in a vacuum waveguide, since the waveguide corrugation is slight, $h^2 \ll R_0^2$. The mechanism for wave excitation in such a system can be thought of as the induced transition radiation of an electron which is moving in a periodically inhomogeneous medium, so it might appear that the theory for such sources would be different from that outlined above. However, it is simple to show that this is not the case. If we transform to a curvilinear coordinate system in which the corrugated waveguide surface becomes flat, then we find that the motion of the beam electrons in this system is oscillatory with a frequency $k_0 u$. The problem is thus reduced to the problem of the induced radiation of accelerated electrons—a problem which was discussed above in the analysis of cyclotron-resonance masers. There is another way to approach the mechanism for the interaction of the electron beam with the corrugated waveguide. In any periodic structure the electromagnetic field is a superposition of Brillouin waves, among which there are slow waves with a phase velocity below the speed of light. The two-stream instability in the periodic structure can thus be interpreted as the induced Cherenkov radiation of the beam electrons, as it was above[9)] in the analysis of the plasma Cherenkov source. Which of these interpretations will prove more

[9)]In corrugated systems there can in principle be cyclotron excitation of slow waves by rectilinear electron beams ($u_\perp = 0$) in the case of the anomalous Doppler effect ($s = -1$), in addition to the Cherenkov excitation. It is easily shown, however, that in the limit of strong magnetic fields, under which conditions (3.21) hold, the beam wave with the anomalous Doppler effect can be coupled only with a high Brillouin harmonic of the field with a very low amplitude (h^n/R^n, where $n \gg 1$). This type of wave excitation should accordingly be very inefficient.

successful is difficult to say, and in fact this question is not really pertinent. The only important circumstance is that the formalism of the theory remains the same and reduces completely to Eqs. (2.11)–(2.16). Incidentally, this assertion holds for other types of microwaves sources, such as undulators, ubitrons, free-electron lasers, scattrons, etc.[10-12, 27]

As mentioned earlier, the theory for sources with a slightly corrugated slow-wave structure is completely analogous to the theory of cyclotron-resonance masers for the case of the excitation of the E wave. In Eqs. (2.1), (2.3), (3.3), (3.4), and (3.13) we should replace Ω/γ by $k_0 u$ and take A from the expression[10)]

$$A = -\frac{1}{4}\frac{h^2k_0^2}{\gamma^3}\frac{\omega^2}{c^2}\omega_b^2\left(1-\frac{k_\perp^2c^2}{k_0^2u^2\gamma^2}\right)\left[1-\frac{I_1^2(x)}{I_0^2(x)}\right]; \tag{3.22}$$

here $k_\perp = \mu_{0s}/R_0$, where μ_{0s} are the roots of the Bessel function, $J_0(\mu_{0s})=0$; $I_0(x)$ and $I_1(x)$ are modified Bessel functions of argument $x=\omega R_0/u\gamma$; and for simplicity we are restricting the analysis to axisymmetric E modes.

Now, following general equations (2.13) and (2.16), we can easily determine the threshold beam currents for excitation of the resonator with the forward and backward waves. For the forward wave we have

$$J_{th} \approx \frac{55\gamma^3}{h^2k_0^2}\frac{u^3v_{g0}R_0^2}{c^3\omega_{01}L^3}\left(1-\frac{k_\perp^2c^2}{k_0^2\gamma^2u^2}\right)^{-1}\left[1-\frac{I_1^2(x_1)}{I_0^2(x_1)}\right]^{-1}\left(\ln\frac{3}{|\varkappa_1|}\right)^3. \tag{3.23}$$

Analogously, for the backward wave we find

$$J_{th} \approx \frac{275\gamma^3}{h^2k_0^2}\frac{u^3v_{g0}R_0^2}{c^3\omega_{02}L^3}\left(1-\frac{k_\perp^2c^2}{k_0^2\gamma^2u^2}\right)^{-1}\left[1-\frac{I_1^2(x_2)}{I_0^2(x_2)}\right]^{-1}; \tag{3.24}$$

here $x_{1,2}=\omega_{01,2}R_0/u\gamma$. If $\gamma > k_\perp/k_0 > 1$, then both the waves excited by the beam are forward waves, but if $k_\perp < k_0$, then the wave ω_{01} is a forward wave, while ω_{02} is a backward wave.

We could pursue this analysis of specific microwave sources. The analysis would be completely similar to that above and would actually reduce to putting general equations (2.12), (2.13), and (2.15), (2.16) in explicit form. We have restricted the analysis here to the most common types of Cherenkov and cyclotron sources.

4. NONLINEAR THEORY OF THE PLASMA SOURCE OF ELECTROMAGNETIC WAVES AND ITS EFFICIENCY

In the preceding sections we have been dealing with the linear theory of plasma sources and amplifiers of electromagnetic waves, which operate by means of the two-stream instability. The linear approximation reveals that excitation occurs in the electrodynamic system [there is a growth of the field $E(t)$ which is unbounded in time] only if the beam current is above the threshold. There are several other important questions, however, which require a nonlinear theory: How long will the field in the system continue to increase? What will be the consequence of the field increase? What will the efficiency and output power of the source be? How can this efficiency and power be maximized?

In this section we will outline such a nonlinear theory for the case of the Cherenkov mechanism for the two-stream instability, i.e., for the interaction of a rectilinear ($u_\perp=0$), monoenergetic, annular electron beam with a forward, axisymmetric, plasma electromagnetic E wave in a smooth metal waveguide which is completely filled with plasma and immersed in a strong longitudinal magnetic field. For the solution of this problem, the starting point consists of Maxwell's equations for the E waves (this rectilinear electron beam can interact with only these waves) and the Vlasov kinetic equations for the plasma and beam electrons.[7,8]

As shown in Refs. 29–31, the plasma can be treated in the linear approximation if the electron density in the beam is low. Furthermore, since the beam is narrow, it is natural to assume that the radial structure of the fields in the waveguide will not be perturbed by the beam. We should thus seek a solution of the field equations, e.g., for the field component E_z, in the following form:

$$E_z(z,\ r,\ t) = E(z,\ t)\,J_0\left(\mu_{0s}\frac{r}{R}\right)\cos\left[\omega t - k_z z + \alpha(z,\ t)\right]; \tag{4.1}$$

here ω and k_z are related by the dispersion relation $D_0(\omega, k_z)=0$, which determines the spectra in the system without a beam. Strictly speaking, in this problem, as in any nonlinear process, solution (4.1) must contain harmonics of integral multiple frequencies. As shown in Refs. 29–31, however, these harmonics can be ignored if the electron beam has a low density.

Since the beam is assumed to perturb the system only slightly, the following inequalities hold for the amplitude $E(z,t)$ and the phase $\alpha(z,t)$:

$$\left|\frac{1}{\omega}\frac{\partial E}{\partial t}\right|,\quad \left|\frac{1}{k_z}\frac{\partial E}{\partial z}\right| \ll E,\quad \left|\frac{1}{\omega}\frac{\partial \alpha}{\partial t}\right|,\quad \left|\frac{1}{k_z}\frac{\partial \alpha}{\partial z}\right| \ll 1. \tag{4.2}$$

Substituting (4.1), (4.2) into the basic equations, and taking the average over the wavelength involved, $\lambda = 2\pi/k_z$, we find equations for the slowly varying functions[24] E and α, which are written as follows in terms of dimensionless variables:

$$\left.\begin{aligned}
&\varepsilon\left(\frac{\partial}{\partial x}+\frac{1}{q}\frac{\partial}{\partial \tau}\right)\alpha = \frac{\nu}{N}\sum_{p=1}^{N}\tilde{v}_p\sin(\tau - x_p + \alpha),\\
&\left(\frac{\partial}{\partial x}+\frac{1}{q}\frac{\partial}{\partial \tau}\right)\varepsilon = -\frac{\nu}{N}\sum_{p=1}^{N}\tilde{v}_p\cos(\tau - x_p + \alpha),\\
&\frac{dx_p}{d\tau} = \tilde{v}_p,\quad \frac{d\tilde{v}_p}{d\tau} = \tilde{\gamma}_p\varepsilon\cos(\tau - x_p + \alpha).
\end{aligned}\right| \tag{4.3}$$

Here

$$\varepsilon = -\frac{eE(z,\ t)\,\gamma^{-3}}{m\omega u}J_0(k_\perp r_b),\quad q = \frac{v_{g0}}{u},\quad \tau = \omega t,\quad x = k_z z,$$

are the dimensionless electric field, the unperturbed group velocity of the electromagnetic wave, the time, and the coordinate, respectively; u is the unperturbed longitudinal velocity of the beam electrons. The value of ν is given by $\nu = |\delta k/k_z|^3$, where δk is the linear growth rate of wave (2.7) equal to

$$|\delta k| = k_z\left[\frac{\Delta r_b}{R^2}\frac{J_0^2(k_\perp r_b)}{J_1^2(k_\perp R)}\frac{\omega_b^2}{\gamma^3k_\perp^2u^2}\right]^{1/3}. \tag{4.4}$$

System (4.3) is written in a form convenient for numerical integration by the method of finite-size particles,[32] which is the method most frequently used in

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[10)]Obviously, this is true only if the frequencies of the excited waves $\omega_{01,2}$, lie far from the opaque zones of the corrugated waveguide (the Bragg-reflection zones)[28] and if the corrugation depth is small, $h^1 \ll R^2$.

problems of this type for simulating the dynamics of the two-stream instability. Consequently, we understand $x_p = k_z z_p$ and $\tilde{v}_p = v_p/u$ in this system to be the dimensionless coordinate and velocity of particle p; $\gamma_p = [1-(v_p^2/c^2)]^{3/2}[1-(u^2/c^2)]^{-3/2}$; and N is the number of finite-size particles per wavelength $\lambda = 2\pi/k_z$. As in the linear theory, the requirement that the perturbations in the plasma-filled waveguide caused by the electron beam be small reduces to the requirement that the parameter ν be small.[11)]

System (4.3) describes the self-consistent interaction of a monoenergetic rectilinear electron beam and a quasimonochromatic plasma wave (4.1). To go over to the source problem, we must supplement this system of equations with boundary conditions; these boundary conditions are found from the following considerations. In addition to wave (4.1), there is a nonresonant wave in the resonator which is propagating opposite to the beam and which provides feedback in the system. If we assume that, on the average, the electron beam does not interact with this wave we find the feedback equation

$$\varepsilon(0, \tau) = |\varkappa| \, \varepsilon\left(\tilde{L}, \tau - \frac{\tilde{L}}{q}\right), \qquad (4.5)$$

which is one of the boundary conditions which we need. Here $\varkappa$ is the reflection coefficient for wave (4.1) at the $z = L$ boundary, and $\tilde{L} = k_z L$ is the dimensionless length of the resonator. System (4.3) should also be supplemented with the condition that the electrons enter the resonator at the $z = 0$ boundary:

$$\tilde{v}_p|_{x_p=0} = \frac{\omega}{k_z u} = 1. \qquad (4.6)$$

Equations (4.3), (4.5), and (4.6) permit a comprehensive study of the nonlinear stage of the operation of a plasma Cherenkov source, and the solution of these equations describes all the processes which occur in a single-mode plasma source, in particular, the processes of setting up the oscillations. In the present review, however, we will not discuss the setting-up processes; we will limit the discussion to the steady-state solution, i.e., to the steady-state operation of the source. There is always a steady state if the electron beam current exceeds the threshold current (steady-state linear solutions exist only if the beam current is equal to the threshold current).

Figure 5 shows, in relative units, the steady-state field profile of $\varepsilon(x)$ for the case of the reflection coefficient $|\varkappa| = 0.5$. We see that the amplitude of the field $\varepsilon(x)$ reaches a maximum at a certain point in the resonator and then falls off toward the right boundary of the resonator. At the point at which $\varepsilon(x)$ reaches its maximum amplitude, the beam electrons are captured in the wave field; then the electrons begin to acquire energy from the wave, and the field decreases (see Refs. 29–31 and 24 for more details on this capture phenomenon).

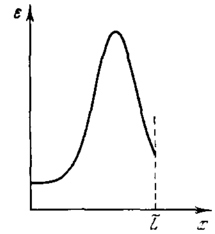

FIG. 5.

If, however, we choose the resonator length L and reflection coefficient $|\varkappa|$ appropriately, we can arrange conditions such that the maximum field amplitude is equal to the capture amplitude and is reached at the exit from the resonator, i.e., at $z = L$ (Fig. 6). This situation is evidently the most favorable for maximizing the excitation efficiency and the output power.

Figure 7 shows the optimum length of the resonator (curve 1) calculated as a function of the reflection coefficient $|\varkappa|$. In resonators with parameters corresponding to this curve, a field approximately equal to the capture field is established at the exit. Also shown in this figure is the threshold functional dependence of L on $|\varkappa|$ (curve 2) found by equating the beam current to the threshold current for excitation of the source [in dimensionless units, the equation of this curve is $\tilde{L}\nu^{1/3} = \ln^3|\varkappa| \cdot 2/3\sqrt{3}$, according to (2.13)]. The optimum curve lies above the threshold curve, as expected. Curves 1 and 2 are not continued into the region of larger values of $\tilde{L}\nu^{1/3}$ (above the dashed line) since in this region the excitation is multimode excitation in terms of the longitudinal wave numbers, so that we cannot seek a solution in the form (4.1). In this region the problem should be solved by the methods of the quasilinear theory.[5]

To conclude this section we will find the efficiency of the steady-state plasma source. By definition, the energy flux of the electromagnetic field (the radiation power) in a cylindrical waveguide is

$$P_z = \frac{c}{4\pi}(1 - |\varkappa|^2) \int_0^R r\,dr \int_0^{2\pi} d\varphi\, [\mathbf{EB}]_z \,|_{z=L}. \qquad (4.7)$$

The efficiency of the source should thus be taken as the ratio

$$\eta = \frac{P_z}{2\pi\Delta r_b n_b m c^2 u\,(\gamma - 1)}. \qquad (4.8)$$

For a nonrelativistic electron beam, system (4.3) can be reduced to a "universal" steady-state system, i.e., one which is independent of the parameters of the system, through an appropriate choice of dimensionless variables.[31] Taking this approach, we find a general analytic expression for the amplitude of the capture

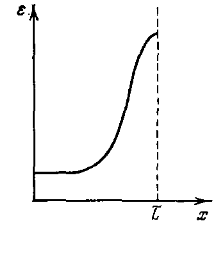

FIG. 6.

[11)]Inequalities (4.2), which allow us to ignore the second derivatives of the amplitude E and phase α in the field equations, hold if $\nu^{1/2} \ll 1$. This condition furthermore allows us to ignore in the field (4.1) the integral harmonics $\sim n(\omega t - k_z z)$ for $n \geqslant 2$.

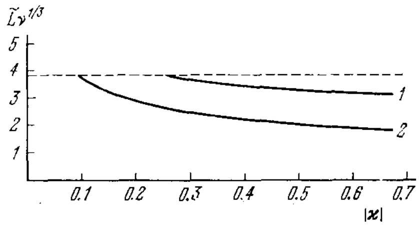

FIG. 7.

field, E_{max}:

$$\frac{eE_{max}}{k_z} J_0(k_\perp r_b) \approx 2.34 m u^2 \nu^{2/3}. \tag{4.9}$$

Substitution of (4.9) into (4.8) leads to the following expression for the efficiency of a nonrelativistic Cherenkov plasma source:

$$\eta \approx 2.75 (1 - |\varkappa|^2) \nu^{1/3}. \tag{4.10}$$

For a relativistic electron beam, Eqs. (4.3) can also be put in a universal form if the motion of the beam in the frame of reference of wave (4.1) is nonrelativistic.[29] In this case the amplitude of the capture field for a beam with $\gamma \gg 1$ is given by

$$\frac{eE_{max}}{k_z} J_0(k_\perp r_b) \approx 2.34 m u^2 \gamma^3 \nu^{2/3}, \tag{4.11}$$

and the source efficiency is

$$\eta \approx 1.37 (1 - |\varkappa|^2) \gamma^2 \nu^{1/3}. \tag{4.12}$$

It can be seen from (4.12) that an increase in the beam current is accompanied by an increase in the source efficiency also, and at $\gamma^2\nu^{1/3} \gtrsim 1$, in which case the beam current is above the vacuum limit, (3.11), the efficiency can formally be even greater than unity, according to (4.12). In this case, however, Eq. (4.12) is not applicable. The assumption that the motion of the beam electrons is nonrelativistic in the frame of reference of the wave reduces to the assumption that the relative change in the electron energy is small, as is easily shown[12)]:

$$\frac{\Delta\gamma}{\gamma} \sim \frac{u^2}{c^2}\gamma\frac{\Delta u}{u} \sim \gamma^2\nu^{1/3} \ll 1. \tag{4.13}$$

This parameter is the same as the efficiency in (4.12).

For arbitrary relativistic beams, i.e., for arbitrary values of the parameters $\gamma^2\nu^{1/3}$, it is not possible to find universal expressions for the efficiency of the plasma sources. In this case, Eqs. (4.3) must be solved numerically, and the efficiency must be determined from (4.8). Calculations of this type have been carried out for the following parameters: $R = 4.1$ cm, $r_b = 2$ cm, $\Delta = 0.1$ cm, $\omega_p = 12 \cdot 10^{10}$ s^{-1}, $u = 2.81 \cdot 10^{10}$ cm/s($\gamma = 3$), $\omega_b = 5 \cdot 10^{10}$ s^{-1}, $k_z = 3.93$ cm^{-1}, and $\omega = 11 \cdot 10^{10}$ s^{-1} ($\nu^{1/3} = 0.046 \ll 1$, $\gamma^2\nu^{1/3} \approx 0.42$). The results of the calculations show that the optimum length of the resonator for $\varkappa \approx 0.23$ is 19.6 cm ($\tilde{L}\nu^{1/3} \approx 3.5$), while the efficiency calculated from (4.8) is $\eta \approx 16\%$. The power radiated from the source, $P_z = \eta J_b (mc^2/e)(\gamma - 1)$, is $P_z \approx 0.55 \cdot 10^9$ W for these parameters. The beam current is of the order of the vacuum limit.

The efficiencies of the other plasma sources discussed in the preceding section could be calculated in a similar way. These calculations may be omitted, however, since the efficiency of sources of high-frequency waves in which the plasma does not play a governing role could hardly be very different from the efficiencies calculated in Refs. 9–11 for the vacuum case, which are 20–30%.

5. EXPERIMENTAL PROGRESS IN INTENSE-BEAM PLASMA MICROWAVE ELECTRONICS

Although this review has concentrated on a description of the present state of the theory of plasma microwave electronics, we will devote this last section to a brief account of the recent experimental progress in this field. We will limit this review of experimental work to a discussion of microwave sources which use intense relativistic beams and which operate by the Cherenkov and cyclotron mechanisms, for which the theory was outlined above and for which the fundamental arrangement is shown in Fig. 1.

Immediately after the appearance of the first intense relativistic electron accelerators, in both the USA and USSR, attempts were made to harness intense electron beams to excite intense microwave pulses. The first of these experiments, carried out in the 1970's, must be judged unsuccessful, however, since the excitation efficiency was very low—less than 1%. These experiments suffered from the disadvantage that the parameters of the sources had not been completely optimized, and this circumstance was responsible for the low efficiency. We will not discuss this early work below; we will discuss only the work which led to relatively efficient sources of electromagnetic radiation.

A 1973 experiment carried out in the Lebedev Physics Institute, Moscow, in collaboration with the Gor'kiĭ Radiophysics Institute should be acknowledged as the first successful experiment on microwave emission.[33] The theory had been used to design a Cherenkov source with a slow-wave structure in the form of a corrugated waveguide for the E_{01} mode (this is the fundamental radial mode of an axially symmetric E wave) with a wavelength $\lambda \approx 3.1$ cm. The length of the slow-wave structure was $L \approx 12$ cm; the inner radius was 1.6 cm; the corrugation period was 1.6 cm; and the corrugation depth was 0.4 cm. The entire system was immersed in a strong magnetic field of order 3–5 kG and evacuated to a pressure of $2 \cdot 10^{-5}$ torr, to prevent a plasma from forming during the beam injection, for a time $\tau \approx 30$ ns. An electron beam with a current up to 8 kA (the vacuum limit) and an energy of 670 keV was passed through the structure. The radiation reached a maximum at a current $J_b \approx 5$ kA, in comparison with the threshold current $J_{th} \approx 3$ kA. The efficiency of this source reached $\eta \approx 15\%$ at an output power $P_z \approx 400$ MW with a radiation pulse length $\tau_u \approx 15$ ns. The width of the output line was $\lesssim 5\%$. Finally, the observation that the nature of the excitation was basically independent of the magnetic field over the range $B_0 \approx 3$–5 kG confirmed that a vacuum Cherenkov slow-wave source was in fact operating (at $B_0 < 3$ kG, there was still a rather intense loss of electrons because of the expansion caused by the beam

12) If $\Delta\gamma \ll \gamma$, then the condition $\tilde{\gamma}_p = 1$ holds in system (4.3).

space charge). This experiment was repeated in the USA in 1974 (Ref. 35); a radiation power of 500 MW, corresponding to an efficiency of 17%, was achieved in a source of the same type, with an electron beam having approximately the same properties.

A recent increase[34] of the magnetic field to 18 kG in this source has made it possible to produce a radiation power of 1000 MW at an efficiency ≈30%.

Two papers on a Cherenkov source using a relativistic electron beam appeared simultaneously in 1975. The first[36] reported a study carried out in the Khar'kov Physicotechnical Institute on the excitation of 3-cm radiation during the injection of an electron beam with an energy $\mathscr{E} \approx 1$ MeV and pulse length $\tau \approx 30$ ns into a slow-wave structure of length $L=70$ cm, loaded with irises and designed to slow the E_{01} mode at the wavelength $\lambda=3.3$ cm to the beam velocity. The entire system was immersed in a longitudinal magnetic field (which could be adjusted up to 12-kOe) and filled with gas to a pressure in the range $10^{-5}-10^{-2}$ torr. At low pressures ($p_0 \lesssim 10^{-4}$ torr), at which a plasma could not form, and at which the device was operating as a vacuum Cherenkov source, the maximum radiated power was $P_{\mathrm{r}} \approx 200-300$ MW at an output pulse length of 15–20 ns. The vacuum current limit for this source was $J_b \approx 12$ kA; the injection of this current through the vacuum source resulted in an efficiency $\eta \approx 2-3\%$. At the higher gas pressures ($p_0 \approx 10^{-3}$ torr) the beam was capable of ionizing the gas, and the current through the source rose to $J_b \approx 20-22$ kA, i.e., to a level less than twice the vacuum limit. The output power tripled, reaching $P_{\mathrm{r}} \approx 600$ MW at an efficiency $\eta \approx 6-7\%$. At a very high gas pressure ($p_0 \gtrsim 10^{-2}$ torr), the radiation was cut off. Tkach *et al.*[36] attributed this cutoff to the formation of a dense plasma with $\omega_p > \omega$, which shielded the radiation.

In a more recent study,[37] Tkach *et al.* repeated their experiment with an electron energy $\mathscr{E} \approx 0.7$ MeV at a fixed beam injection current $J_b \approx 5$ kA. At low gas pressures, $p_0 < 10^{-4}$ torr, the output power at the wavelength $\lambda \approx 3.3$ cm was $P_{\mathrm{r}} \approx 200-300$ MW. As the gas pressure was raised, the output power rose, reaching $P_{\mathrm{r}} \gtrsim 700$ MW, corresponding to a high efficiency $\eta \approx 22\%$ at $p_0 \approx 10^{-2}$ torr. At $p_0 > 10^{-2}$ torr the excitation was cut off, apparently because of shielding by a dense plasma. Tkach *et al.* explained their results by appealing to the concept of a double resonance, such that both high-frequency and low-frequency (plasma) E waves could be excited in the resonator, with approximately equal frequencies. This assumption is supported by the fact that the maximum radiation power was observed experimentally at $\omega \approx \omega_p$.

The second 1975 study[38] was carried out in the USA with an annular electron beam with an inner radius of 0.8 cm and a thickness of 0.3 cm. The energy of the beam electrons was $\mathscr{E} \approx 450$ keV, and the current reached 7 kA at a pulse length $\tau \approx 50$ ns. The beam was injected into an iris-loaded vacuum ($p_0 < 10^{-5}$ torr) waveguide of length of 1 m, which slowed the radiation at the wavelength $\lambda \approx 10$ cm to the electron velocity. The maximum radiation power was $P_{\mathrm{r}} \approx 600$ MW at a radiation pulse length $\tau_u \approx 30-40$ ns. The radiation efficiency was $\eta \approx 20\%$.

The last experimental study on Cherenkov vacuum sources using rectilinear relativistic electron beams which we will discuss here was carried out in the Lebedev Physics Institute in collaboration with the Institute of Applied Physics of the Academy of Sciences of the USSR, on the excitation of short-wave radiation with $\lambda \approx 8$ mm (Ref. 39). These experiments used a miniature slow-wave structure [a corrugated waveguide with a square cross section (4×4 mm) and a length ≈6 cm] in a vacuum $p_0 \lesssim 2 \cdot 10^{-5}$ torr. The electron beam had an energy $\mathscr{E} \approx 670$ keV and a current up to $J_b \approx 500$ A with a current pulse length $\tau \approx 20$ ns. The E_{11} mode was excited; the output power was 10 MW at an output pulse length $\tau_u \approx 15$ ns, with an efficiency of 3%. This source is inefficient in comparison with the Cherenkov sources described above because of the low beam current, which is just barely above the threshold current. If the beam current could be raised to 1–2 kA we would expect a substantial increase in the efficiency, to at least 10–15%. The output power would correspondingly reach 100 MW. However, this has not been possible, and at present it has not been possible to reach beam current densities much above 10^4 A/cm^2 while keeping the energy spread of the electrons at the low level required for coherent radiation.

Efficient sources using the cyclotron mechanism for the interaction with relativistic electron beams appeared somewhat later. Here again, first-place honors went to the USSR. In work at the Lebedev Physics Institute, published in 1975 (Ref. 40), it was shown that the injection of a relativistic electron beam with $\mathscr{E} \approx 300-400$ keV and a current up to 10 kA into a smooth metal waveguide at an angle with respect to the longitudinal magnetic field resulted in the excitation of radiation at a frequency $\omega \approx \Omega/\gamma$, demonstrating a cyclotron mechanism for the radiation. In addition, an important effect was discovered: When the waveguide was filled with a relatively low-density plasma the output power rose sharply, by a factor of 30–40, while the current carried by the electron beam through the waveguide increased by a factor of only 2–3. This observation implies an increase in the efficiency of the cyclotron excitation when there is a plasma in the system which neutralizes the electron beam. At high plasma densities the excitation was cut off; the radiation was shielded. This system was not optimized, and it would not be appropriate to judge its efficiency. The maximum radiation power at the wavelength $\lambda \approx 3$ cm apparently did not exceed 10 MW, at an efficiency $\eta \approx 1-2\%$.

This shortcoming was subsequently eliminated,[41] and a cyclotron source ("gyrotron") operating at the H_{13} mode with a wavelength $\lambda \approx 3$ cm was designed. This was a smooth waveguide ≈7 cm in diameter and 10–12 cm long, through which an annular electron beam with a mean radius of 1.5 cm and a thickness of 1 cm was passed. The beam was at a maximum of the H_{13} field and excited this mode. The angle between the electron velocity and the magnetic field was 45°. The maximum radiation power in the vacuum version was P_{r}

$=25$ MW at $J_b \approx 0.5$ kA (half the vacuum limit), $\mathscr{E}=350$ keV, $\tau=40$ ns, and $\eta \approx 20\%$. When the system was filled with plasma,[42] it became possible to triple the beam current without changing the excitation efficiency; i.e., the current was raised to $J_b \approx 1.5$ kA, which is 1.5 times the vacuum limit. The output power thus rose to $P_z \approx 70$ MW at $\eta \approx 20\%$. The length of the beam pulse in both the vacuum and plasma cases was about $\tau \approx 30$–40 ns, and the length of the radiation pulse was about $\tau_u \approx 20$–30 ns.

An ultrahigh-power vacuum gyrotron was constructed in the USA in 1975 (Ref. 43). An electron beam with $\mathscr{E} \approx 3.3$ MeV and $J_b \approx 80$ kA, with a pulse length $\tau \approx 70$ ns, was injected into a smooth waveguide at an angle $\approx 7^\circ$ from the strong magnetic field. An output power $P_z \approx 10^9$ W was achieved at the wavelength $\lambda \approx 6$ cm. Although this power figure is extremely impressive, the device was clearly not operating under optimum conditions, since the efficiency was $\eta \approx 1\%$.

The ultrahigh-power gyrotron constructed for the wavelength $\lambda = 10$ cm at the Institute of Nuclear Physics, Tomsk Polytechnical Institute, in 1976 operated under conditions much closer to the optimum.[44] Here an electron beam with an energy of 900–1200 keV, a current up to $J_b \approx 30$ kA, a pulse length $\tau \approx 60$ ns, and a radius $r_b \approx 2$ cm was injected into a smooth waveguide 9.8 cm in radius. The length of the source and the threshold current were calculated for the H_{11} mode for an injection angle determined by the scattering of the beam by a 50-μ titanium foil. Optimum excitation in the vacuum version was observed at $\mathscr{E} = 900$ keV and $J_b = 8$ kA and yielded a power $P_z \approx (1.4\text{–}2) \cdot 10^9$ W. In other words, the efficiency was $\approx 30\%$. When the system was filled with gas to a pressure $p_0 \gtrsim 10^{-2}$ torr, a plasma formed, and the radiation power decreased; the increase in the efficiency and the output power with increasing plasma density at relatively low plasma densities which was observed in Ref. 38 was not observed in these experiments.

These papers at this point essentially exhaust the literature on high-current relativistic Cherenkov and cyclotron sources using rectilinear beams. In all the experiments described above the plasma either neutralized the beam current and charge, thereby increasing the current through the system, or shielded the radiation excited by the beam, but it did not determine the excitation mode or frequency; the mode and frequency were determined by the vacuum electrodynamic system. So far, there have been no systematic experimental studies with intense relativistic beams on purely plasma sources in which a plasma wave is excited. For nonrelativistic beams, such studies have been carried out, and they are reviewed in Refs. 15 and 45. We will not discuss this work here, especially since it also could not and did not lead to the development of purely plasma sources of coherent electromagnetic radiation. The theoretical papers mentioned above show that such a source could operate highly efficiently under conditions such that $\omega_b > u\gamma/R \gtrsim \omega = 2\pi c/\lambda$, i.e., with $\gamma \gtrsim 2\pi R/\lambda$, and this condition can be met in beams with energies above 0.3–0.5 MeV. So far, there have been no systematic experiments on plasma excitation with such beams. The development of purely plasma sources is thus a timely problem.

TABLE I.

Year	1973 [33]	1974 [35]	1975 [36]	1975 [36]	1975 [37]	1975 [37]
Type of source	Slow-wave structure	Slow-wave structure	Slow-wave structure	Slow-wave structure	Slow-wave structure	Slow-wave structure
$\mathscr{E}$, MeV	0.67	0.67	1	1	0.7	0.7
J_b, kA	5	8	12	22	5	5
τ, ns	20	40	30	30	30	30
B_0, kG	2–5	4–10	12	12	10	10
p_0, torr	$2\cdot10^{-5}$	10^{-5}–10^{-4}		10^{-3}–10^{-2}	10^{-4}	10^{-3}–10^{-2}
λ, cm	3.1	3.1	3.3	3.3	3.3	3.3
P_z, MW	400	500	200–300	600	200–300	700
τ_u, ns	15	30	20	20	20	20
η (eff), %	15	17	3	7	7	22
Year	1975 [38]	1978 [39]	1978 [41]	1978 [42]	1975 [43]	1976 [44]
Type of source	Slow-wave structure	Slow-wave structure	Gyrotron	Gyrotron	Gyrotron	Gyrotron
$\mathscr{E}$, MeV	0.45	0.67	0.35	0.35	3.3	0.9
I_b, kA	5	0.5	0.5	1.5	80	8
τ, ns	50	30	40	40	70	60
B_0, kG	10	5	6	4	20	3
p_0, torr	10^{-5}	$2\cdot10^{-5}$	10^{-5}		10^{-5}	10^{-5}
λ, cm	10	0.8	3	3	6	10
P_z, MW	600	10	25	70	10^3	10^3
τ_u, ns	30–40	15	30	30	50	30
η (eff), %	20	3	20	20	1	30

In conclusion we would like to point out that high-current pulsed electron beams are also being used successfully in sources of different types, which use curvilinear beams. Examples are the magnetron, in which the beam as a whole is rotating in the slow-wave structure, and the ubitron, in which the beam is moving along a periodic magnetic field. We will not discuss these sources in detail here, but we would like to point out that important results have also been achieved in these devices; the excitation power and efficiency are as good as those which have been achieved with Cherenkov and cyclotron sources. The experimental progress on high-current microwave electronics is reviewed in Refs. 10–12, 97, 46, and 47 [an entire recent issue (No. 10, 1979) of the journal Izvestiya Vysshykh Uchebnykh Zavedeniǐ, Seriya Fizika was recently devoted to the topic].

We can draw the conclusion that relativistic electronics has now emerged from the purely research stage and has found widespread applications. Also noting the report[48] of successful experiments on the development of microwave sources with a high pulse repetition frequency, we may say that we are seeing the beginnings not only of a new scientific field but also of a new field in energetics: relativistic microwave energetics.

We wish to thank N. I. Karbushev, V. I. Kurilko, M. I. Petelin, N. F. Kovalev, and M. V. Nezlin for discussions which clarified several points in this review.

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Translated by Dave Parsons