Bose condensation of moving rotons

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If the initial roton distribution in helium has a resultant momentum above a certain critical value, it will relax to a state with a Bose condensate of particles with a nonzero momentum. In this state, the velocity of the normal component of the liquid will be equal to the Landau critical velocity.

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It is generally assumed that the particles obeying Bose–Einstein statistics can be divided into two groups. The first group consists of particles like He^4 atoms, whose number is conserved. Bose–Einstein condensation can occur in a system of such particles. The other group consists of particles such as photons, phonons, and other elementary excitations. The number of such particles (or "quasiparticles") is determined by the conditions for thermodynamic equilibrium, and Bose–Einstein condensation does not occur in a system of such particles. In this note we wish to show this picture is not correct: A type of Bose–Einstein condensation can occur in a system of quasiparticles, to a state with a finite (rather than zero) momentum.

We start with the usual theory for Bose–Einstein condensation. The equilibrium distribution function for bosons is

$$n(\mathbf{p})=\left[\exp\left(\frac{\varepsilon-\mu}{T}\right)-1\right]^{-1}, \tag{1}$$

where μ is the chemical potential. We see that the condition $\mu<0$ must hold, for otherwise the function n would be negative in a certain momentum range, and we cannot accept this. In a system in which the number of particles is conserved, μ is determined from the normalization condition, which states that the number density of the particles is equal to some "initial" value N:

$$N=\int n(\mathbf{p})\frac{d^3p}{(2\pi\hbar)^3}. \tag{2}$$

It is easy to see, however, that the left-hand side of (2) is a monotonically decreasing function of $|\mu|$, which reaches its maximum value $N_c(T)$ at $\mu=0$. For $N>N_c$, Eq. (2) has no solutions for μ. This contradiction was resolved by Einstein,[1] who showed that at $N>N_c$ some of the particles are in a state with zero momentum, while the others are distributed in accordance with the distribution in (1) with $\mu=0$. For $N>N_c(T)$, the distribution function is thus

$$n(\mathbf{p})=N_0\delta(\mathbf{p})+\left[\exp\left(\frac{\varepsilon}{T}\right)-1\right]^{-1}, \tag{3}$$

and Eq. (2) now determines the "condensate density" N_0. If the number of particles in the system is not conserved, on the other hand, the chemical potential is always zero; Eq. (2) determines the equilibrium particle density; and there is no basis for condensation.

This is the situation for the elementary excitations in liquid helium. The situation changes, however, if we incorporate in the calculation the possibility that the gas of excitations can move as a whole. (It is important to note here that we are talking about excitations in a liquid—not, say, photons in vacuum. For particles in vacuum, motion as a whole is equivalent to a simple transformation to a different Galilean frame of reference and cannot affect the statistical properties of the system.) Let us consider a liquid in a frame of reference in which the superfluid part of the liquid is at rest. Then the equilibrium distribution function of the excitation is

$$n(\mathbf{p})=\left[\exp\left(\frac{\varepsilon-\mathbf{pu}}{T}\right)-1\right]^{-1}, \tag{4}$$

and the velocity of the normal part of the liquid, $\mathbf{u}$, is determined by the initial distribution of excitations. Denoting by $\mathscr{P}$ their initial momentum per unit volume, we should find $\mathbf{u}$ from

$$\vec{\mathscr{P}}=\int \mathbf{p}n(\mathbf{p})\frac{d^3p}{(2\pi\hbar)^3}. \tag{5}$$

It is important to note that there is a rigorous momentum-conservation law for elementary excitations in a homogeneous liquid. Our discussion thus does not apply, for example, to excitations in a solid, where quasimomentum conservation is disrupted by flipping.

Since (4) is positive, the velocity must satisfy the Landau superfluidity condition[2]

$$\varepsilon-\mathbf{pu}\geqslant 0 \quad \text{or} \quad u\leqslant v_c, \tag{6}$$

where v_c is the minimum value of the ratio $\varepsilon(p)/p$ on the spectral curve. In Fig. 1, v_c is the slope of the dashed line, which is drawn tangent to the $\varepsilon(p)$ curve and through the origin. Equation (5) can, by definition, be written as $\vec{\mathscr{P}}=\rho_n\mathbf{u}$, where $\rho_n(T,u)$ is the density of the normal part of the liquid, determined by the Landau method. It is now clear that $\mathscr{P}_c=\rho_n(T,v_c)v_c$ is the maximum value which the right-hand side of (5)

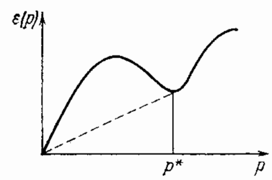

FIG. 1. Spectrum of elementary excitations in liquid He^4. Roton condensation occurs into a state with momentum p^*.

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can take on. For $\mathscr{P} > \mathscr{P}_c$, this equation has no solutions for $\mathbf{u}$. The quantity $\mathscr{P}$ depends on the initial conditions and has an arbitrary value. What happens if $\mathscr{P} > \mathscr{P}_c$? The same reasoning which led Einstein to distribution (3) shows that in this case there is a condensation of rotons with a momentum $\mathbf{p}=\mathbf{p}^*$ (p^* is the abscissa of the point of tangency in Fig. 1, and the direction of $\mathbf{p}^*$ is determined by the initial conditions).[1)] In other words, the distribution function is now

$$n(\mathbf{p}) = N_0\delta(\mathbf{p}-\mathbf{p}^*) + \left[\exp\left(\frac{\varepsilon(p)-\mathbf{p}\mathbf{v}_c}{T}\right)-1\right]^{-1}, \qquad (7)$$

and the condensate density N_0 is determined from (5). When condensation occurs, for any $\mathscr{P} > \mathscr{P}_c$, the excitations move with a velocity precisely equal to v_c with respect to the superfluid part of the liquid. This means, in particular, that this condensation disrupts the superfluidity. As a liquid flows through a tube at $v_s > v_c$, energy is dissipated; i.e., a force arises from the mutual friction between the tube and the superfluid part of the liquid. Actually, there is no dissipation only if the normal part of the liquid is at rest with respect to the tube. This situation could not arise in "supercritical" flow, since the relative velocity of the normal and superfluid parts of the liquid must be equal to v_c.

These arguments are completely rigorous for the spectrum in Fig. 1. In real helium, the rotons and phonons are of course accompanied by a spectral branch associated with eddy rings, for which the superfluidity condition is always disrupted (at least in an unbounded liquid). Consequently, the applicability of these arguments to the problem of the critical velocities in helium requires further study.

[1)]The part of the spectral curve of elementary excitations in helium near the minimum is called the "roton" part.

[1]A. Einstein, Collected Scientific Works (Russ. transl., Vol. 3, Nauka, Moscow, 1966, Article #63).
[2]L. D. Landau, Sobranie trudov (Collected Works), Vol. 1, Nauka, Moscow, 1969, Article #44.

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