Physical laws and the numerical values of fundamental constants

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Data which indicate that the existence of nuclei and atoms and also of stars and galaxies is very sensitive to the numerical values of the fundamental constants, i.e., the coupling constants of the four interactions and the constants G, $\hbar$, c, m_p, etc., are reviewed. The variation of some constants by a few tens of percent with the other constants remaining unchanged would lead to the disappearance of stable ground states. Possible interpretations of this fact and its connection to some aspects of unified field theory are analyzed.

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INTRODUCTION

In this paper, we analyze the part played by the numerical values of the fundamental physical constants—the dimensionless coupling constants of the four interactions and also the dimensional physical constants G, $\hbar$, c, m_p, m_e, etc., in the physical picture of the world.

Intuitively, it appears natural that a relatively small change (within an order of magnitude) in the numerical values of the fundamental constants would not destroy the basic features of the physical picture but merely change some quantitative characteristics. For example, nuclei and atoms would change their sizes and masses, and the stars and other cosmic objects would also somewhat change their quantitative characteristics, etc. In reality, our analysis will demonstrate that a variation of one of the fundamental constants with the others (as well as all the physical laws) remaining unchanged would have a drastic qualitative consequence—the impossibility of the existence of stable bound ground states of nuclei, atoms, stars and galaxies.[1)] In other words, a trivial assertion has reigned hitherto, namely, the set of numerical values of the fundamental constants is sufficient for the existence of ground states. Less trivial is the assertion (which makes up a major part of this paper) that this set is necessary for the existence of ground states. One can advance a postulate which we shall call the principle of "effectiveness" (or "appropriateness"). This takes the form that our basic physical laws, together with the numerical values of the fundamental constants are not only sufficient but also necessary for the existence of ground states. In other words, if something is changed in physics (the values of a fundamental constant within an order of magnitude or one of the internal quantum numbers is eliminated, for example, the isotopic spin), this must result in not merely slight quantitative changes in the physical picture but rather in the destruction of its foundations, i.e., ground states could not exist. One can say that the physical laws (including the numerical values of the fundamental constants) are subject to a harmony that ensures the existence of ground states.[2)] The expression "principle of effectiveness" emphasizes the necessity of the given set of numerical values of the fundamental constants for the existence of ground states. It is possible that the expression does not reflect all aspects of the interconnection between the fundamental constants and ground states.

In English literature, this situation is characterized by the widely used anthropic principle. In our view, this expression emphasizes too strongly the interconnection between the numerical values of the fundamental constants and complex (biological) forms of matter whereas in reality the interconnection occurs already at lower levels—at the nuclear and atomic. On the other hand, it does not appear to be justified to take as the basis of a physical principle a concept such as biological form of matter which, from the point of view of physics, is not entirely definite.

Although the conclusions concerning the interconnection between ground states and the numerical values of the fundamental constants are based on comparatively simple arguments, they appear paradoxical, which is due to the unusual nature of the approach developed here.

[1)]In what follows, these objects will be called ground states.
[2)]In what follows, (see Secs. 1, 3-6, and the Conclusions) this principle is illustrated and its definition made more precise.

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As a rule, when problems in physics are solved, the fundamental constants, like the number of dimensions of physical space, are assumed to be unchanged. Such a method is fully justified, since it is in excellent agreement with experiment. In the new approach, our procedure is to change either one of the fundamental constants, or the dimension N of space,[3)] or one of the internal quantum numbers, etc.

Such an approach may appear entirely meaningless (in our Universe, the fundamental constants and the physical laws do not change in space-time; see Sec. 2). However, the critical nature of the existence of ground states makes it possible to bring forward serious arguments in favor of the principle of effectiveness. It is possible that the further development of this principle could have definite heuristic consequences.

We note further an important fact: important characteristics of ground states (see Refs. 1–3 and the Appendix) can be represented in terms of the fundamental constants.

We introduce the dimensionless constants of the four known interactions:

$$\alpha_e = \frac{e^2}{\hbar c} \tag{1}$$

for the electromagnetic interaction,

$$\alpha_g = \frac{Gm^2}{\hbar c} \tag{2}$$

for the gravitational interaction,

$$\alpha_w = \frac{g_F m^2 c}{\hbar^3} \tag{3}$$

for the weak interaction ($g_F = 10^{-49}$ erg $\cdot$ cm^3 is Fermi's constant), and

$$\alpha_s = \frac{g_s^2}{\hbar c} \tag{4}$$

for the strong interaction.

In the standard constructions, when the interactions are treated as isolated from one another, the values of the masses are free parameters.[4)] Usually, one takes $m = m_p$. We emphasize that at the present time one can represent the characteristics of the ground states in terms of the fundamental constants only for objects which owe their existence to electromagnetic and gravitational interactions.

No bound states due to the weak interaction have been detected. In the framework of the possible unification of the interactions (see Sec. 6), such bound states could arise at an energy $mc^2 \sim 100$ GeV. However, strictly speaking, it is then no longer possible to speak of an isolated weak interaction.

The situation with regard to the strong interaction is more complicated. It is now clear that the nuclear interaction (which recently was identified with the strong interaction) is the analog of van der Waals forces acting between quarks. The nuclear interaction is characterized by an effective coupling constant $\alpha_s \sim 1$. In the framework of the most popular model of the strong interaction—quantum chromodynamics—the coupling constant of the interaction between color charges is $\alpha_s = \alpha_s(r) = \alpha_s(q^2)$, where r and q^2 are the distance or the square of the four-momentum transfer characteristic of the given state. In the framework of quantum chromodynamics with four flavors and three colors when $q^2 \gg (m_p c)^2$ (see Ref. 4)

$$\alpha_s(q^2) \sim \frac{1.5}{\ln(q^2/q_0^2)}. \tag{5}$$

It follows from numerous experimental data[4,5] that $\alpha_s \sim 0.2$–0.3 at $q^2 \sim 10$ (GeV)^2. If we use (5) to make a rough estimate of the fundamental constant q_0^2, we obtain $q_0^2 \sim 5 \cdot 10^{-2}$ (GeV)^2. It follows that $\alpha_s \sim 1$ at nuclear distances $r \gtrsim (\hbar/m_\pi c)$. Therefore, at such distances perturbation theory cannot be used, and it is necessary to take into account forces which prevent the existence of free quarks (the confinement problem). To characterize the bound states due to nuclear forces, we can, at least at the present level of our understanding, use various phenomenological parameters such as the widths and depths of the potential wells of the binding energy, etc. The coupling constant g_s is also a phenomenological parameter of this kind.

But let us return to the electromagnetic and gravitational interactions, in the framework of which the ground states can be analyzed very transparently. The electromagnetic interaction governs the stability of atoms. For the hydrogen atom, the binding energy ε_H and the radius r_H are

$$\varepsilon_H \sim \alpha_e^2 m_e c^2, \tag{6}$$

$$r_H = \alpha_e^{-1} \frac{\hbar}{m_e c}. \tag{7}$$

The binding energy and sizes of molecules and complex atoms are ultimately governed by ε_H and r_H. However, because of the diversity and complexity of these systems, numerical coefficients, which sometimes have values of several orders, occur in the expressions for the binding energies and sizes.

The typical mass of the ground state of the gravitational interaction for a star of the main sequence is

$$M_s \sim \alpha_g^{-3/2} m_p. \tag{8}$$

The solar mass $M_\odot \sim 2 \cdot 10^{33} g$ is approximately half this expression. It is here helpful to make two reservations: 1) The relation (8) is not an empirical approximation of the mass of stars. There are deep physical reasons for this relation. 2) In reality, there exists a definite spread (by about one or two orders in both directions) from the value M_s determined by the relation (8) (see the Appendix). In order of magnitude, this value of M_s corresponds to the mass of white dwarfs and neutron stars.

Similarly, the luminosity of typical stars of the main sequence is

$$L \sim \frac{(m_e c^2)^2}{\hbar} \alpha_g^{-1/2}. \tag{9}$$

One can also express in terms of the fundamental con-

[3)] The number of dimensions $N = 3$ can also be regarded as one of the fundamental constants.

[4)] For the values of the masses in unified theory see Sec. 6.

stants the radii of stars of the main sequence, white dwarfs, and pulsars, and also the typical characteristics of galaxies.[6] For expressions of some characteristics of the Universe in terms of the fundamental constants, see Sec. 3 below.

1. STABILITY OF MICROSCOPIC SYSTEMS AND NUMERICAL VALUES OF THE FUNDAMENTAL PHYSICAL CONSTANTS[5)]

a) The deuteron

The condition of stability of the deuteron is

$$V_d > V_0 = \frac{\hbar^2}{m_p r_N^2}, \tag{10}$$

where V_d is the depth of the potential well in the deuteron, $r_N \sim 2 \cdot 10^{-13}$ cm is the range of the nuclear forces, and $V_0 \sim 25$ MeV. Since the deuteron binding energy is $\varepsilon_d \sim 2.2$ MeV, it follows that $V_d \sim 30$ MeV and the deuteron is a stable (but "fragile") system. However, if, for example, the value of $\hbar$ is increased by more than 15% (or m_p is decreased by 30%) while the values of the other constants are kept unchanged, the inequality (10) will not hold. The deuteron would cease to exist as a stable system. A similar effect would result from a decrease in the effective coupling constant g_s. The parameters of the deuteron cannot at present be calculated with arbitrary rigor. However, for rough estimates in the potential approximation we can set $V_d \propto \alpha_s \propto g_s^2$. In this approximation, it is sufficient to reduce g_s by 10–15% in order to reverse the sign of the inequality (10). A development of this nature would have catastrophic consequences, since the nucleosynthesis chain contains a link involving the deuteron—in such a case, there would be nuclei with atomic numbers $A > 1$.

A change of the constants in the opposite direction would have consequences almost as serious. The point is that the He^2 nucleus "almost" exists. Although the binding energy of such a nucleus is negative, it is very small (~0.01 MeV). Therefore, if α_s were to increase by approximately 10%, the system of two protons would satisfy the inequality $V_{pp} > V_0$, and a stable biproton would exist. As was noted in Ref. 8, this would lead to the existence of the reaction

$$p + p \to He^2 + \gamma, \tag{11}$$

which is governed by the electromagnetic interaction, in contrast to the standard reaction of thermonuclear synthesis ($p + p \to d + e^+ + \nu$, which proceeds through the weak interaction). The reaction (11) would proceed so rapidly that all hydrogen would be burnt up in the early stages in the expansion of the Universe.

b) The α particle

The binding energy of the α particle is $\varepsilon_\alpha \sim 7$ MeV; the Coulomb repulsion of the protons corresponds to an energy somewhat less than 1 MeV. Therefore, if one takes a hypothetical elementary charge of value $e' > 3e$, the α particle would cease to exist, and this would lead to the absence of nuclei with $A \geq 4$.

c) Complex nuclei

The condition of stability of nuclei against fission is

$$\left(\frac{Ze'}{e}\right)^2 \frac{1}{A} < 50. \tag{12}$$

If $e' \gtrsim 3e$, then all nuclei with $Z \geq 6$ would decay.

d) Atoms

It is clear that if $e' \gtrsim 10e$ (or, accordingly, $\alpha_e' \gtrsim 100\alpha$), then atoms would not exist as stable bound states. Nor is it possible (without imaginable catastrophic consequences) to decrease the charge e (or α_e) without limit. The point is that the temperature or interstellar gas in the Galaxy is $\gtrsim 100$ °K. Any body immersed for a sufficient time in such a gas cannot have a temperature less than the temperature of the gas. Therefore, if $\alpha_e' \lesssim \alpha_e/10$, neutral atoms could not exist in the Galaxy.

The galaxies are "embedded" in the intergalactic space, which has the temperature ~3 °K of the microwave background. Therefore, if $\alpha_e' \lesssim \alpha_e/100$, there would again be no atoms in the entire Universe.

Thus, the existence of stable neutral atoms and complex nuclei leads to the restriction $e/3 \lesssim e' < 3e$.[6)]

e) Helium production and connection between the coupling constants

The theory of cosmic nucleosynthesis has interesting consequences, showing that the genesis and existence of light nuclei depend critically on the fundamental constants (Ref. 9).[7)]

In the framework of modern ideas, deuterons and α particles are formed basically during the initial stages in the expansion of the Universe, while the heavier nuclei are formed during the evolution of stars.

Let us consider first the production of deuterons (α particles are then produced in d + p and d + d reactions). During the cosmological expansion, the main synthesis reaction is

$$n + p \to d + \gamma, \tag{13}$$

which hardly occurs at all in stars because of the absence of free neutrons.

Briefly, the production of deuterons in the reaction (13) can be described as follows. During the earliest stages in the expansion of the Universe ($T \gtrsim m_p c^2/k$, hadronic era), the neutron density is determined by nuclear transformations. However, at a temperature $T \lesssim \Delta c^2/k$ ($\Delta = m_n - m_p$, the leptonic era), the weak interactions begin to play the decisive part in establishing the composition of the hadronic plasma, doing this through the reactions $\bar{\nu} + p \to n + e^+, e^- + p \to n + \nu$. The equilibrium composition must correspond approximately to an exponential: $n_n/n_p \sim \exp(-kT_f/\Delta c^2)$, where n_n and n_p are, respectively, the neutron and proton densities, and T_f is the temperature at which the reaction

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5)See also Ref. 7.

6)A stronger bound on e (or rather, the constant α_e) follows in some variants of unified field theory (see Sec. 6).

7)For a complete exposition of the theory of nucleosynthesis, see Refs. 10 and 11.

rate matches the expansion rate of the Universe, which is determined by the Hubble constant H. At $T\sim T_f$, the composition of the hadronic plasma is "frozen".

We can estimate the reaction rate $v_r = n\sigma_W c$ [$n\sim(kT/\hbar c)^3$ is the particle density and $\sigma_W \sim g_F^2 E^2/(\hbar c)^4$ is the weak interaction cross section]. Setting $E=kT$, we obtain

$$v_r = g_F^2 c\,(kT)^5\,(\hbar c)^{-7}. \tag{14}$$

Using the standard expression for the Hubble constant $H\sim\sqrt{G(kT)^4/c^2(\hbar c)^3}$ and setting $v_r\sim H$, we obtain

$$T_f \sim G^{1/6} g_F^{-2/3} h^{11/6} c^{7/6} k^{-1}.$$

The relative (by mass) helium concentration is $Y=2n_n/(n_p+n_n)$ (see, for example, Ref. 10). Observations show that $Y\sim 0.2$–0.25, and, therefore, $n_n/n_p\sim 0.3$. Therefore, we must have $kT_f\sim c^2\Delta\sim m_e c^2$. This is possible if

$$\alpha_W \sim \alpha_g^{1/4}\left(\frac{m_p}{m_e}\right)^{3/2}. \tag{16}$$

The relation (16) is in fact satisfied, although there are no "deep" theoretical reasons for it.

Let us consider the consequences of a violation of (16). Suppose

$$\alpha_W \ll \alpha_g^{1/4}\left(\frac{m_p}{m_e}\right)^{3/2}. \tag{17}$$

In this case, the weak interaction would be so weak that the reaction (13) would occur very effectively and all nucleons would be transformed into helium; there would be no hydrogen in the Universe.

In the opposite case

$$\alpha_W \gg \alpha_g^{1/4}\left(\frac{m_p}{m_e}\right)^{3/2}, \tag{18}$$

helium would not be produced during the cosmological expansion. Note further that if the inequalities (17) or (18) were to hold, the structure of stars would be considerably different from that of the stars in our Universe. The point is that the fundamental reaction of thermonuclear synthesis in stars $(p+p\to d+e^+ +\nu)$ is determined by the constant α_W and, therefore, the structure of stars is governed by the relationship between the constants α_W and α_g. However, actual calculations of the structure of stars for different values of α_W have not been made.

f) Formation of complex nuclei with $Z\leqslant 4$ (Ref. 12; see also Ref. 13)

The simplest fusion of two α particles is very ineffective, since the reaction

$$2He^4 \to Be^8 \tag{19}$$

leads to the formation of the unstable isotope Be^8. It has therefore been suggested that the main channel for the production of complex elements is the triple fusion

$$3He^4 \to C^{12}. \tag{20}$$

However, if this reaction takes place with formation of the ground state of the nucleus C^{12}, its rate is low, since $3m_{He} = m_C + (\Delta E^*/c^2)$, and $\Delta E^*\sim 7.7$ MeV. Therefore, the triple fusion (20) can occur effectively if the nucleus C^{12} has an excited level with energy ΔE^*. Then the reaction

$$3He^4 \to (C^{12})^* \tag{21}$$

is very effective. When the hypothesis of the decisive part played by the triple fusion (21) was advanced, an excited level with energy $\Delta E^*\sim 7.7$ MeV was not known. However, confidence in the need for this level was so great that the suggestion was made for a search for it by means of accelerators that could detect it. It is not difficult to imagine the consequences of a shift (or absence) of this level. All elements with $Z>2$ would have negligible relative abundance. In the opposite hypothetical variant—the existence of the stable isotope Be^8—the reaction $2He^4\to Be^8$ would occur very vigorously. The existence of main sequence stars would terminate with the helium cycle.

We are not in a position to calculate rigorously the structure of nuclear levels. However, it is clear that it is governed by the effective coupling constant α_s. Therefore, a slight change of this constant, which would slightly alter the structure of the levels of C^{12}, would mean the absence of complex elements in the Universe.

2. INTERPRETATIONS OF THE CRITICAL NATURE OF THE EXISTENCE OF STABLE BOUND SYSTEMS

Thus, the existence of ground states, which play an extremely important part in the Universe, depends very critically on the numerical values of the fundamental constants.[8)] Various alternative interpretations can be confronted with this experimental fact.

The simplest interpretation is as follows.

a) It is meaningless to contemplate physical laws outside the Universe; b) we were very "lucky" that the Universe is constructed in such a way that the existence of ground states is possible. Although this interpretation cannot be ruled out on logical grounds, it does not appear very justified.

The next interpretation reduces to the assumption that the fundamental constants change throughout the Universe in space-time. We live in a space-time region in which a favorable combination of the constants is realized. It is well known that Dirac[14] suggested a variation of some fundamental constants in connection with the extreme smallness of the ratio α_g/α_e. Dirac's paper stimulated numerous experimental investigations (see, for example, Refs. 15–18), which demonstrated with high accuracy the absence of data indicating any change of the fundamental constants during the expansion of the Universe. Without giving here a detailed review of this question (see Refs. 17 and 18), we shall restrict ourselves to some results. For example, $|\dot\alpha_e/\alpha_e|<10^{-17}$ year^{-1}, $|\dot g_s/g_s|<5\cdot10^{-9}$ year^{-1}, $|\dot g_F/g_F|<2\cdot10^{-12}$ year^{-1}, $|\dot e/e|<10^{-12}$ year^{-1}, $|\dot G/G|<10^{-10}$ year^{-1} $|(\hbar c)^{\cdot}/\hbar c|<10^{-12}$ year^{-1}).[9)] If the possible variation of

[8)]In the following sections, we shall give an additional number of arguments for the uniqueness of the set of numerical values of the fundamental constants.

[9)]Note that the most exact measurements of the limits of variation in time of some fundamental constants were obtained by studying the natural nuclear reactor at Oklo in Gabon, which became critical about one billion years ago.[15,18]

$\hbar$ is represented in the form

$$\hbar = \hbar_0 (1+z)^{-n}, \tag{22}$$

(where z is the cosmological red-shift parameter), then $n = 0.004^{+0.031}_{-0.027}$.

Thus, the fundamental constants keep constant values with high accuracy. It follows that Dirac's hypothesis is not in agreement with experimental data. There remains only the final alternative: Either there exists a multimode of universes with their own physical laws and combinations of fundamental constants, or our Universe has passed through a number of cycles, the combination of fundamental constants changing at the beginnings of these cycles. In the present cycle, we have a combination of constants favorable for the existence of ground states. The hypothesis of the existence of many universes was advanced independently on the basis of the arguments presented above,[7,9] and also on the basis of some cosmological arguments[19,20] (see Sec. 3).

Although this hypothesis appears surprising, it does not contradict the modern picture of the world. At present, there are no physical arguments for the uniqueness of our Universe. This assertion is true, however, with one reservation. If the universes do not interact with one another, there are no physical problems. However, if interaction is possible, one encounters the problem of their coexistence with different sets of fundamental constants. To elucidate the resulting problem, let us divide the fundamental constants into three classes: the dimensionless constants α, the quantum numbers of the elementary particles e, m_p, m_e, etc., and the dimensional fundamental constants G, $\hbar$, and c. It is trivial to imagine the interaction of objects with the same physical laws but with different fundamental constants of the first two classes. However, the encounter of objects with two different values of c would contradict the theory of relativity, and with two different values of G general relativity. It is not easy (and may be impossible) to construct a noncontradictory quantum mechanics with two different values of $\hbar$. At the present level of knowledge, this problem ceases to be acute if the constants change when the Universe passes through a singularity. We must here end our discussion of this difficult but little considered question.[10)]

Although the hypothesis of the existence of many universes appears natural, it contradicts the motto deeply rooted since the time of Newton: hypotheses should not be invented. And if this rule is to be broken, it has become accepted to put forward a method of direct verification of the hypothesis. In the present case, these principles are not well maintained. We do not know how we could communicate with the other universes (and, possibly, may never know). Nevertheless, some indirect arguments can be put forward in favor of this hypothesis on the basis of the principle of effectiveness. This will be discussed below.

3. SOME COSMOLOGICAL PROBLEMS (REFS. 19 AND 20)

One of the fundamental questions of cosmology is whether our Universe is open or closed. This problem has been intensively discussed during the last decades. In the framework of the Friedmann model, the main way in which one can attempt to solve this problem is based on a measurement of the mean density ρ_0[11)] of matter in the Universe. If $\rho_0 > \rho_{0c}$ [$\rho_{0c} = (\frac{3}{8}\pi)H_0^2/G \sim 10^{-29}$ g/cm^3], the Universe is closed, but if $\rho_0 < \rho_{0c}$, it is open. The existing observational data (see, for example, Refs. 10 and 11) indicate that $\rho_0 \sim 0.1\rho_{0c}$. Because of the existence of significant hidden (from observation) mass, it is possible that $\rho_0 \sim \rho_{0c}$. At present it is only possible to assert that to within an order of magnitude

$$\rho_0 \sim \rho_{0c}. \tag{23}$$

Here, one must immediately ask why the unique coincidence (23) is realized among the infinite number of possibilities open to nature.

In Ref. 20, Hawking attempted to give an interpretation. His argument reduces to the conclusion that if the value of ρ_0 differed significantly from ρ_{0c}, anisotropic perturbations would have to develop in the Universe. It therefore appears probable that the Universe must be anisotropic. However, observations reveal a high degree of isotropy of the Universe. Such a Universe can exist only if the relation (23) is satisfied. But then we encounter another question; Why is precisely this case realized? The following answer is given. If $\rho_0 \ll \rho_{0c}$, the expansion of parts of the Universe with respect to each other would occur too rapidly for the formation of stable objects such as galaxies to be possible. If $\rho_0 \gg \rho_{0c}$, one can calculate the lifetime t_u of the Universe, which is too short for the development in it of highly organized matter.

Using this approach, which is an extension of the principle of effectiveness, one can also interpret other impressive semiempirical relations.[19]

It is well known that the lifetime t_s of a main sequence star is

$$t_s \sim t_u. \tag{24}$$

Indeed,

$$t_s \sim \eta\frac{M_s c^2}{L} \sim \eta\frac{\alpha_g^{-3/2} m_p c^2}{L} \sim \frac{1}{H_0}. \tag{25}$$

Here, $\eta \sim 10^{-3}$ is the fraction of the rest energy of the star transformed into radiation in the process of thermonuclear reactions and L is the mean luminosity of the star; see (8) and (9).

It is clear that one must have $t_s \lesssim t_u$ and a value of t_u sufficiently large to ensure the development of highly developed forms of matter. However, if $t_s \ll t_u$, all the possibilities for the evolution of matter would not be exploited. This argument can be illustrated by reducing the coefficient η by an order of magnitude. In this case,

[10)]Note that the hypothesis of the existence of universes with left-handed particles and universes with right-handed particles was considered earlier in connection with the violation of CP invariance.[21] It was demonstrated that contact between a mirror universe and our Universe could be realized only through the gravitational interaction.

[11)]The subscript 0 is appended to all quantities in our epoch.

$t_s \sim 10^9$ years, and many complex forms of matter would not exist. For example, according to geological data the simplest microscopic organisms arose about 3×10^9 years ago,[22] while the age of the Earth is $\sim 4.5 \times 10^9$ years. If the latter were reduced by 10 times, biological molecules would not have arisen on the Earth. The relation (23) is a manifestation of optimal effectiveness.

If the Universe is closed, then, using the relation (25), and also the expression for the radius of the Universe

$$R_{u\ max} \sim \frac{GM_u}{c^2} \sim \frac{c}{H_0}, \tag{26}$$

we find the mass of the Universe[12)]

$$M_u \sim \alpha_g^{-2} m_p. \tag{27}$$

Unexpected relationships between the fundamental constants can be established on the basis of the conditions needed for the formation of galaxies (see Ref. 10):

$$kT_{min} < \varepsilon_H = \alpha_e^2 m_e c^2, \tag{28}$$

$$kT_{min} < S^{-1} m_p c^2. \tag{29}$$

Here, T_{min} is the minimal temperature attained during the expansion of the Universe. For an open Universe $T_{min} = 0$; for a closed one, T_{min} corresponds to the greatest radius $R_{u,max}$; $S = n_\gamma / n_p$ is the ratio of the mean densities of photons and protons. The condition (28) corresponds to the beginning of hydrogen recombination; the condition (29), to predominance of matter energy over radiation energy.

Using (23) and the standard relations in the framework of the Friedmann model, we obtain

$$kT \sim \left(\frac{\hbar^3 c^5}{Gt^2}\right)^{1/4}. \tag{30}$$

For a closed Universe we have

$$kT_{min} \sim \alpha_g^{1/4} m_p c^2. \tag{31}$$

From (28) and (31) there follows the condition

$$\alpha_g^{1/4} < \alpha_e^2 \frac{m_e}{m_p}. \tag{32}$$

Using (29) and (31), we deduce[9]

$$\alpha_g^{1/4} < S^{-1}. \tag{33}$$

Therefore, if the inequalities (32) and (33) were not satisfied, there would be no galaxies or stars. Both inequalities, (32) and (33), are realized in the Universe ($S_{exp} \sim 10^8$), but neither with a good margin. For example, if α_e were reduced by an order of magnitude, the inequality (32) would not be satisfied. The inequality (33) ceases to hold if S is increased by two orders of magnitude. In the framework of existing physical conceptions, the values of α_g and S are not connected. Therefore, the circumstance that in reality the inequalities (32) and (33) are satisfied can be interpreted in two ways: a) The Universe is open and the relations (32) and (33) are fortuitous; b) the Universe is closed and the principle of effectiveness is satisfied.

4. STABILITY OF BOUND STATES AND THE NUMBER OF DIMENSIONS OF PHYSICAL SPACE

It might appear that a change in the number N of dimensions of space would lead to an unpredictable change in physical laws. In reality, the situation is different. If the properties of space are characterized by the Minkowski metric with arbitrary integral dimension N, the physical laws (at least, for classical physics) in such spaces are to a considerable degree pre-determined. We give the simple but important example of static (nonrelativistic) interactions described by the large class of linear or quasilinear equations for which the superposition principle holds. Then for point sources one can use Gauss's theorem, and the action of a source on another point body at distance r from the source is described by the law

$$F \propto \frac{1}{r^{N-1}}. \tag{34}$$

The stability of a system of two bodies interacting in accordance with the law (34) for arbitrary N was analyzed by Ehrenfest.[23] The results of the analysis are as follows: for $N > 3$, there are no stable bound states. For $N \leq 2$, there exist only finite motions. The number $N = 3$ is distinguished by the fact that for it both stable finite and infinite motions are possible in the corresponding space.[13)] Later,[25,26] similar conclusions were drawn in the framework of quantum mechanics.

Thus, in spaces with dimension $N > 3$ the analogs of planetary systems or atoms cannot exist. It is possible that this circumstance will turn out to be the key to an understanding of the dimension of the space of the Universe, since for $N < 3$ it is hard to imagine the formation of complex forms of matter.

5. TYPES OF INTERACTION AND INTERNAL QUANTUM NUMBERS

At first glance, the question of why there exist four types of interaction appears either idle or premature. Indeed, until we have a unified field theory, which combines all the interactions, each of them is primary and not subject to further justification. However, in the framework of the principle of effectiveness, the posed problem is quite in order: Are all four interactions needed? The answer is clear. All types of interaction are needed for the formation and existence of ground states (see Sec. 1 for a discussion of the part played by the weak interaction). One can also formulate the following question: Do the four studied interactions exhaust all interactions in the Universe? It is very probable that the answer to this question is negative. Indeed, in the framework of our ideas about the four known interactions, regarded in isolation, it is difficult to explain the charge asymmetry of the Universe ($S \neq \infty$ is a very important factor relating to the existence of fundamental constants). However, in the framework of a theory that combines the strong, weak, and electromagnetic interaction (Grand Unification), from which instability of the proton follows,[14)] this

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[12)]On the basis of the relations (26) and (27) it is simple to interpret the well-known[14] empirical relation $H_0 \sim \alpha_g m_p c^2/\hbar$.

[13)]Ehrenfest's paper was published in an inaccessible journal. For a detailed exposition, see Ref. 25.

[14)]See Ref. 5 and Sec. 6.

phenomenon can be explained.[27,28] However, an important part is then played by violation of CP invariance,[15)] which leads to an inequality of the direct and inverse processes. It is possible (although this question has not yet been completely solved) that the violation of CP invariance is related to a new superweak interaction.[29,30]

In such an approach, one can interpret the existence of fundamental internal quantum numbers of elementary particles. These numbers are also necessary for the existence of ground states. For example, if the elementary particles had no spin, all the "electrons" in atoms would go over to the ground state, there would be no electromagnetic or gravitational interaction, and so forth.

If the isotopic spin of all hadrons were zero, complex stable nuclei would not exist. The catastrophic consequences of a serious violation of the law of conservation of the baryon or lepton numbers are obvious.[16)] From the point of view of the principle of effectiveness, it is less easy to interpret the existence of quantum numbers for exclusively unstable particles.

We merely mention that strangeness is a necessary element in one of the simplest variants of the composite model of nucleons, ensuring the following very important features of the strong interaction, which are essential for the stability of nuclei and atoms: a) nucleons with charges 0 and e form an isotropic doublet, b) the isotropic relationships are universal at the nuclear and subnuclear levels; c) the baryon charge is conserved.

By no means every composite model interprets these characteristics. For example, the composite model based on an isotropic triplet of nonstrange "quarks" cannot satisfy all three conditions simultaneously.

6. UNIFIED FIELD THEORY AND THE PRINCIPLE OF EFFECTIVENESS

It would seen that the principle of effectiveness contradicts unified field theory, which is based on attempts to establish a deep connection between interactions. At the present stage in the development of unified field theory, one can decisively say that there is no contradiction between the principle of effectiveness and unified theory. In reality, the principle of effectiveness and unified field theory relate basically to different questions. To see this, let us make a brief digression into unified theory (for a discussion of unified theory, see, for example, Refs. 5 and 31–33). The hopes associated with unified field theory are based on three principles: a) the unified interaction is characterized by a single coupling constant, b) the components of the unified field transform in accordance with a single group, c) in the framework of the unified theory it will be possible to eliminate the infinities in the final results (renormalizability) or, which corresponds to the maximal program, to eliminate the infinities from the intermediate calculations as well.

Let us consider in more detail point a), which is the most perspicuous and has a close relationship to the subject of this paper. The only possibility of reducing the constants of all the interactions to a single constant is to set all the constants α equal at some characteristic distance (or, accordingly, a characteristic mass). Let us consider first the unification of the electromagnetic, weak, and gravitational interactions. Since the unique "true" constant—independent of distances or momenta—is the constant α_e (see the Introduction), we must set

$$\alpha_w^1 \sim \alpha_e, \tag{35}$$

$$\alpha_g^1 \sim \alpha_e. \tag{36}$$

Here, α_w^1 and α_g^1 correspond to the characteristic masses. Let m_{we} and m_{gwe} be the values of the masses at which the relations (35) and (36) hold. Then, using (1)–(3) and (35) and (36), we obtain

$$m_{we} \sim \sqrt{\frac{\alpha_e}{\alpha_w}}\, m_p \sim 10^2\ \mathrm{GeV}/c^2, \tag{37}$$

$$m_{gwe} \sim \sqrt{\frac{\alpha_e}{\alpha_g}}\, m_p \sim 10^{19}\ \mathrm{GeV}/c^2. \tag{38}$$

The distance l at which the constants become comparable corresponds to the largest characteristic mass:

$$l \sim \frac{l_0}{\sqrt{\alpha_e}}; \tag{39}$$

$l_0=\sqrt{G\hbar/c^3}$ is the Planck length.

It follows from the simple arguments given here that the principle of effectiveness and unified field theory correspond to different problems. The former can answer questions such as why the ratio α_e/α_g is so large or α_e/α_w satisfies the relation (37). Unified theory does not pretend to solve these questions. This conception takes the relation between the constants to be primary and given. However, the unified theory must predict a numerical factor ~1 in the relations (37) and (38).[17)] Thus, if the ratio α_e/α_g were changed strongly, physics would be significantly simplified in the sense that many ground states would not exist; the unified field might exist, though no one would exist to write down its equations.

The principle of effectiveness predicts that the relationship between the constants has a "fortuitous" nature due to the existence of ground states. If one could detect a deep connection between the fundamental constants that is not due to the principle of effectiveness, this would be a refutation of the principle. Bearing in mind that this assertion is somewhat imprecise, it is helpful to give two examples illustrating the thesis.

a) Unified theory of the strong, weak, and electromagnetic interactions

Let us consider the most popular variant, which combines quantum chromodynamics and the Weinberg–Salam theory on the basis of the group $SU(5)$. This variant

[15)]And also by the expansion of the Universe.[10]

[16)]That is, if the proton lifetime satisfies $t_p \le t_u$.

[17)]This factor depends on the properties of particles with masses m_{we} and m_{gwe}. In the Weinberg–Salam model, the mass of the charged heavy bosons ($W^\pm$) is $m^\pm_{we}=37/\sin\theta_W$ GeV/c^2; the mass of the neutral boson (Z^0) is $m^0_{we}=75/\sin 2\theta_W$ GeV/c^2; θ_W is the mixing angle.[30,31]

was proposed in Ref. 34 (for a review of it and a bibliography, see Refs. 5 and 33). In this model, not only the photon and intermediate bosons with mass m_{we} transmit interactions but also a very massive particle, the so-called leptoquark boson with mass m_{wes}. Exchange of this boson can transform quarks into leptons. Therefore, in the framework of this model the proton is an unstable particle. The value of m_{wes} can be calculated on the basis of the relation[5,35]

$$\ln\frac{m_{wes}}{\mu}=\frac{\pi}{11\alpha_e}\left[1-\frac{8}{3}\,\frac{\alpha_e}{\alpha_s([\mu c]^2)}\right]; \tag{40}$$

$\mu\sim 3$ GeV/c^2. Setting $\alpha_s\sim 0.25$ [see relation (5)], we obtain

$$m_{wes}\,(\mathrm{GeV}/c^2)\sim 3\exp\left(\frac{a}{\alpha_e}\right). \tag{41}$$

The constant $a\sim 1/4$, and therefore $m_{wes}\sim 10^{15}$ GeV/c^2. If we set $q^2=(m_{wes}c)^2$ in formula (5), the relationship between the constants takes the form

$$\alpha_{wes}=\alpha_s\,([m_{wes}c]^2)\sim 2.5\alpha_e; \tag{42}$$

α_{wes} is the coupling constant of the unified interaction. One might get the impression that in the relationship (42) there is a "deep physical" connection between the fundamental constants. In reality, the situation is the opposite, in the sense that the very existence of protons in the framework of this model is exceptionally sensitive to the numerical value of α_e. The point is that the proton lifetime t_p depends very strongly on the value of the mass m_{wes} (Refs. 28 and 36). For the decay of the proton p into the lepton l, which proceeds according to the scheme $p\to 3q\to l+\pi$, we have

$$t_p\sim\alpha_e^{-2}\left(\frac{m_{wes}}{m_p}\right)^4\frac{\hbar}{m_pc^2}. \tag{43}$$

It is obvious that we must have the inequality

$$t_p>\frac{1}{H_0}. \tag{44}$$

Using (41), (43), and (44), we obtain

$$\alpha_e<\frac{1}{80}.^{18)} \tag{45}$$

We emphasize the generality of the bound (45). Evidently, any unified theory which includes quantum chromodynamics as the model of the strong interaction and predicts instability of the proton leads to an inequality close to (45). This bound is due in the first place to the exponential dependence $m_{wes}\propto\exp(a/\alpha_e)$ characteristic of quantum chromodynamics [see (5) and (41)]. The nature of the dependence $t_p(m_{wes})$ [see (43)] is fairly general[19)] and corresponds to different models, although the coefficient a depends on the form of the diagrams, the numbers of leptons, quarks (flavors), and so forth (Refs. 5, 33, 36, and 37). However, in any unified theory, $a\sim 1$ ($\alpha_{wes}\sim\alpha_e$!). In the framework of the model based on the group $SU(5)$, the maximal value is $m_{wes}\sim 10^{16}$ GeV/c^2; if this value is adopted, the right-hand side of the inequality (45) is increased by several percent. Thus, if α_e were increased by a factor of approximately 1.6, all protons would be transformed into leptons! This is a good illustration of the principle of effectiveness.

However, we can also present an example which is in a sense the opposite.

b) Unified nonlinear theory

Heisenberg[38] formulated an essentially nonlinear equation that contains a unique constant with the dimensions of length. Heisenberg hoped that this equation would serve as the basis for creating a unified theory and would establish connections between different fundamental constants. He did not succeed in carrying through his program. However, in the framework of this theory it proved possible to obtain the striking relation

$$\alpha_e\sim 0.4\left(\frac{m_\pi}{m_p}\right)^2\sim\frac{1}{120}, \tag{46}$$

which differs only slightly from the empirical value of α_e. The relation (46) is a characteristic example of a dependence between fundamental constants that is not related to the principle of effectiveness. If Heisenberg's program could be carried through to the end and, which is the hardest thing of all, the empirical ratio α_g/α_e could be obtained from such general arguments, this would refute the principle of effectiveness. However, it is well known (see, for example, Ref. 39) that the nonlinear theory[8] encountered serious difficulties (nonrenormalizability, difficulties with the description of the weak interaction, and so forth). Therefore, although Heisenberg's theory played an eminent part in stimulating the creation of a unified theory, the latter developed in a different direction.

CONCLUSIONS

Thus, the existing set of numerical values of the fundamental constants is necessary for the existence of ground states. Of course, this concept of necessity does not correspond to its mathematical content. Essentially, one can now say with confidence that if one of the fundamental constants is changed while the others remain fixed the conditions for the existence of ground states are violated. The existence of ground states is particularly sensitive to the values of the constants α_e, α_g, and α_s.

We can consider a further question: Could we, simultaneously changing two constants, again obtain optimal conditions for the existence of ground states? Such a possibility appears improbable, since the constants occur in many relationships that determine the existence of ground states, and these are relationships that would be violated if a second constant is changed.

But, it is at present hardly possible to show that a simultaneous "fortunate" modification of all constants within one or two orders of magnitude will not ensure the existence of ground states, which, however, might

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18) If it is assumed that the Universe is closed or one requires fulfillment of the inequality $t_p>t_s$, then on the basis of the relations (25) or (27) one obtains the remarkable dependence $\alpha_e\leqslant-(\ln\alpha_g)^{-1}$, which is at the limit of being realized in our Universe.

19) The dependence (43) is a consequence of dimensional arguments relating to the fact that the mass m_{wes} is very large compared with the masses of particles which participate in the reaction $p\to l+\pi$.

have properties different from those in our Universe.

We mention that the creation of a physical picture of universes with sets of fundamental constants different from the set in our Universe is a rather fascinating problem.

The necessity (in the indicated sense) of the set of fundamental constants poses an exceptionally serious question: Why has nature "chosen" precisely this set? As yet, the most probable answer is that there exists a set of universes, and this choice had a "random" nature. The further development of unified field theory will show whether this is the definitive answer. Such an answer would be fully refuted by the complete realization of the final aim of Heisenberg's program, i.e., the connection of all fundamental constants to one another on the basis of a single parameter.

Another possible alternative to the principle of effectiveness would be a theory in the framework of which the numerical values of all the fundamental constants are determined solely by the dimensional constants G, $\hbar$, and c. However, this idea, which was already formulated by Planck, has not found concrete implementation.

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APPENDIX: CONNECTION BETWEEN THE CHARACTERISTICS OF STARS AND THE FUNDAMENTAL CONSTANTS

We shall here restrict ourselves to a brief derivation of the interconnection between the characteristics of stars and the fundamental constants.[20] We shall use the following simplifications: 1) instead of the distributions of the physical variables with respect to the radius of a star (which is assumed to be a sphere), we shall use the mean values, 2) we shall assume that complex nuclei are not present in the star, 3) we shall omit numerical dimensionless coefficients that are ~1, and 4) we ignore rotation of stars and their magnetic field.

Under these assumptions, we write down the condition for equilibrium in a star in terms of a proton-electron pair:

$$\frac{GM_s}{R_s} m_p \sim E_k; \tag{47}$$

E_k is the total kinetic energy of the pair. We find further a lower bound $M^{(1)}_{s,\min}$. The value of $M^{(1)}_{s,\min}$ is determined as follows: if $M<M^{(1)}_{s,\min}$, then gravitational forces are insufficient to break up the atomic shells. The value of $M^{(1)}_{s,\min}$ corresponds approximately to the maximal mass of a planet. From the assumption that atomic shells must still exist at $M^{(1)}_{s,\min}$, it follows that

$$E_k \sim \varepsilon_H. \tag{48}$$

In the general case, we can set

$$M_s \sim n m_p R_s^3; \tag{49}$$

R_s is the radius of the star. The mean density is

$$n \sim \frac{1}{r^3}. \tag{50}$$

In the given case $r \sim r_H$, where ε_H and r_H are determined by the relations (6) and (7); r is the mean distance between particles. Substituting (48)–(50) in (47), we obtain

$$M^{(1)}_{s,\min} \sim \left(\frac{\alpha_e}{\alpha_g}\right)^{3/2} m_p. \tag{51}$$

As we have noted, the mass $M^{(1)}_{s,\min}$ corresponds to the greatest mass of planets. The question of whether $M^{(1)}_{s,\min}$ is equal to the minimal mass of existing stars remains open. The problem is that the observation of stars with mass $M_s \sim M^{(1)}_{s,\min}$ is far beyond the capability of existing instruments. First, the luminosity of such stars is low and, second, they radiate basically in the infrared and even, possibly, in the submillimeter range. At present, the minimal observed mass of stars (red dwarfs) is $\sim 0.04 M_\odot$ which is approximately two orders of magnitude greater than the value of M_s determined by (51). There is therefore an alternative: Stars with masses in the interval $M^{(1)}_{s,\min} - 0.04 M_\odot$ exist but are not observed by modern instruments, or they do not exist at all. The second possibility is supported by some estimates of the mean mass of stars based on a realistic model of their formation[41] (see below).

The second lower bound $M^{(2)}_{s,\min}$ is determined by the onset of effective thermonuclear reactions. The value of $M^{(2)}_{s,\min}$ gives approximately the boundary between red dwarfs and main sequence stars. Thermonuclear reactions take place effectively if

$$E_k \sim kT \sim \frac{e^4 m_p}{\hbar^2}. \tag{52}$$

By the uncertainty principle,

$$r \sim \frac{\hbar}{\sqrt{m_e kT}}. \tag{53}$$

It follows from (47), (49), (50), (52), and (53) that

$$M^{(2)}_{s,\min} \sim \left[\frac{\alpha_e}{\alpha_g}\left(\frac{m_p}{m_e}\right)^{1/2}\right]^{3/2} m_p. \tag{54}$$

The maximal value $M_{s,\max}$ can be obtained from the condition of stability of stars against radiation. Radiative instability, which determines $M_{s,\max}$, arises if the radiation pressure significantly exceeds the kinetic pressure. Therefore, instability arises under the condition

$$\frac{(kT)^4}{(\hbar c)^3} > nkT. \tag{55}$$

Let us elucidate qualitatively the reasons for the instability. If the equilibrium is determined by radiation, then

$$r \sim \frac{\hbar c}{kT}. \tag{56}$$

Using (47) and (49), (52), we readily obtain

$$M_{s,\max} \sim \alpha_g^{-3/2} m_p, \tag{57}$$

$$R_s \sim \alpha_g^{-1/2} \frac{\hbar c}{kT}. \tag{58}$$

[20] I have added this Appendix because discussions with well-known Soviet physicists have made clear to me the need for a brief but perspicuous derivation of the connection between the characteristics of stars and the fundamental constants, expecially the relation (8). For a detailed exposition of the stability of stellar configurations, see Ref. 40.

Thus, for a given value of $M_{s,\max}$ the equilibrium configuration corresponds to any radius determined by the mean temperature. In this case, indifferent equilibrium is realized. Such a state leads to the occurrence of radial pulsations of the star, which are sustained by thermonuclear reactions. When M_s is increased, the rate of thermonuclear reactions increases, and this excites the pulsations. If the amplitudes of the pulsations are sufficiently large, the star is either disrupted, or it loses its surface layers, i.e., the mass of the star is decreased.

Detailed estimates lead to the conclusion that the effect becomes important at $M_s \sim 30\alpha_g^{-3/2} m_p$. This value is to be expected to be the real limit to M_s. Observations confirm this conclusion.

We emphasize that the estimates made here refer to the best studied equilibrium states of stars. Some conclusions can also be drawn in the framework of models of star formation. Thus, on the basis of arguments about thermal balance during the evolution of protostars the mass M_s was estimated in Ref. 41. It was again found that $M_s \sim \alpha_g^{-3/2} m_p$. An upper bound on M_s in the process of evolution from the protostar state to the equilibrium state was obtained in Ref. 42 by a consideration of the balance between the pressure due to gravitation and radiation pressure. It was found that $M'_{s,\max} \sim 10\alpha_g^{-3/2} m_p$.

Thus [see (54) and (57)], the masses of main sequence stars are close to the value determined by the relation (8).

Note that for white dwarfs $E_k \sim m_e c^2$ and $r \sim \hbar/m_e c$. Therefore, the mass of a white dwarf is $M_w \sim \alpha_g^{-3/2} m_p$. For neutron stars $E_k \sim m_p c^2$, $r \sim (\hbar/m_p c) - (\hbar/m_\pi c)$, and the mass is $M_n \sim [1 - (m_p/m_\pi)]\alpha_g^{-3/2} m_p$.

Using these values for the masses of stars and the characteristic distances r, one can readily obtain typical values of star radii [see (49)].

For stars of the main sequence

$$R_s \sim \alpha_g^{-1/2} r_H, \tag{59}$$

for white dwarfs

$$R_w \sim \alpha_g^{-1/2} \frac{\hbar}{m_e c}, \tag{60}$$

and for neutron stars

$$R_n \sim \alpha_g^{-1/2} \frac{\hbar}{(m_\pi m_p)^{1/2} c}. \tag{61}$$

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