Ionization waves in low-temperature plasmas

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The ionizational instability, which results in the excitation of moving and standing striations, is one of the most common instabilities in low-temperature plasmas. Phenomena analogous to striations are encountered not only in a variety of fields in physics but also in related sciences, such as chemistry, biology, and ecology. Striations are thus of general scientific interest. This paper reviews experimental and theoretical research on striations published for the most part over the past decade and thus after the 1968 publication of the reviews by Nedospasov, Pekarek, and Oleson and Cooper. New and more comprehensive scaling laws are used for a systematic description of the regions in which moving and stationary striations exist in inert and molecular gases. From the standpoint of the general theory of oscillations these striations may be thought of as a particular case of a very distinctive wave process with "unusual" properties. It is thus useful to describe the striations by an oscillation approach involving the methods of wave theory. The kinetic theory of striations has been developed markedly in recent years, but the usefulness of the hydrodynamic theory of striations, modified appropriately, has not yet been exhausted, at least with regard to a qualitative explanation of the experimental properties of striations. The linear and nonlinear hydrodynamic theories for the formation of moving and stationary striations at intermediate and high currents, incorporating such factors as the deviation from a Maxwellian electron distribution function, oscillations in the density of metastable atoms, and wave reflection from boundaries, yield several results: the conditions for the spontaneous excitation of striations, the shape of the striations, an explanation for the existence of a striation-free region, an explanation for the excitation of one or several types of striations, confirmation of Novak's rule stating that the potential drop over the length of a striation remains constant, etc. The nonlinearity of the system proves important in the study of such questions as the asynchronous suppression of striations and the excitation of stationary striations in inert and molecular gases.

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CONTENTS

INTRODUCTION

The plasma in the positive column of a gas discharge is rarely in a quiet state. There are usually a variety of instabilities, which lead to a stratification of the positive column. The most common instability of a low-temperature plasma, which leads to the excitation of striations, is the "ionizational instability," and this instability is the subject of the present review. A distinction is made between two types of striations which are of an ionizational nature: moving and stationary. Moving striations are waves of the electron temperature and density which are propagating and growing, as a rule, in the direction from the cathode toward the anode. A distinctive feature of these waves is that their phase velocity is directed opposite to their group velocity (primarily in inert gases). The stationary striations are a special type of wave with zero frequency and a nonzero wave number.

Study of stationary and moving striations began already in the last century.[1-3] Many papers have been published on the subject, with a rich store of experimental data, much of which is covered in existing reviews.[4-10] The derivation of a meaningful theory for the stratification of the positive column of a gas discharge

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is by comparison a recent development. One of the first papers along this line was that by Druyvesteyn.[11]

In the history of research on striations we can identify several periods in which there were practical motivations. One was the beginning of the widespread use of gas-discharge tubes. The interest in striations has risen considerably in recent years because of the use of gas-discharge tubes in laser technology.[10,12-17] The working conditions in the corresponding lasers generally correspond to regions in which striations are excited.

The appearance of striations degrades the operation of discharge devices, and a major effort is accordingly made to prevent their appearance. Success here requires that we learn as much as possible about the nature of the striations and the cause of their appearance. Natural striations arise in a positive column without any external source of periodic perturbations; these natural striations are spontaneous oscillations in a system with distributed parameters. From the standpoint of the general theory of oscillations, the striations may be thought of as a distinctive class of waves whose group velocity is directed opposite to their phase velocity. That such waves could exist in principle was pointed out by Lamb,[18] who invented some mechanical models of one-dimensional media exhibiting this property. Mandel'shtam[19,20] found a natural example of media of this type. It turns out that when elastic waves with a frequency comparable to the lattice vibration frequency propagate in a crystal their group and phase velocities are in opposite directions (these are the "optical" branches of the acoustic wave spectrum of a crystal lattice, which were studied by Born).

The striations are not only of applied but also of general scientific interest. Phenomena analogous to striations are encountered in a variety of fields in physics (in hydrodynamics,[21] geophysics,[22] and semiconductor physics,[23-25] among others) and in related sciences (chemistry, biology, and ecology).[26-30] Consequently, the results obtained in research on striations may find widespread applications.

1. NATURE AND BASIC PROPERTIES OF STRIATIONS

a) Moving striations in inert gases

The general properties of moving striations have been described in detail in other reviews,[4-6] so we will focus primarily on recent results here.

Moving striations exist over broad ranges of pressure ($10^{-3}-10^{2}$ torr) and current ($10^{-4}-10^{1}$ A) in inert and molecular gases, in the vapor of certain metals, and in various mixtures. The striation phase velocity varies over a broad range ($v_{ph}\sim 10^{3}-10^{6}$ cm/s) and is generally directed from the anode toward the cathode in inert gases. The group velocity, v_{gr}, which is comparable in magnitude to the phase velocity, is usually in the opposite direction, and it is for this reason that Pekarek[5] has called these waves "backward waves." There are cases, however, in which the group velocity is in the same direction as the phase velocity, and these waves are called "forward waves."[5]

Since the excitation of striations is accompanied by oscillations in the intensity of the spontaneous lateral emission,[6] moving striations can be observed most easily by placing two photodiodes at different points along the axis of the discharge tube. When the signals from these photodiodes are fed to a dual-trace oscilloscope, approximately sinusoidal oscillations are observed on the screen near the threshold for the existence of striations. The length and phase velocity of the striations can be determined from the change in the phase shift of the oscillations which occurs when one of the photodiodes is moved with respect to the other.

Extensive experiments have shown that the characteristics of striations obey scaling laws quite well.[31-34] The scaling laws proposed by Holm[35] were the first to be tested. Pupp showed that the product of the striation frequency, the tube radius, and the molecular mass of the gas ($fR\mu$) was a function of pR/U_1. Nedospasov[4] gave the corresponding dependence for He, Ne, Ar, and Kr. Holm's scaling laws, however, do not cover stepwise excitation and ionization, which are important for striation formation.

Not until 1969 were more general scaling laws formulated, on the basis of the Boltzmann equation.[33] These scaling laws can also be used to describe a variety of processes involving binary collisions of particles. An important distinction between these scaling laws, called "B-invariant scaling laws" by Pfau *et al.*,[33] and those which had been proposed previously by Holm is that the ratio i/R is introduced as an external scaling parameter (in addition to the parameter pR which had been used previously). In addition to the external scaling parameters in the positive column, according to Pfau *et al.*,[33] there are some internal scaling parameters, such as λ/R, fR, v_{ph}, v_{gr}, ER, T, f/p, and λp. The use of the B-invariant scaling laws substantially simplifies theoretical and experimental study of striations. Figure 1 shows experimental results obtained for neon in Ref. 33 on the basis of the scaling parameters λp, f/p, Ψ/p, pR, and i/R (Ψ is the spatial growth rate of the striations). It can be seen from this figure that the length of the striations usually increases approximately linearly with increasing frequency, while the $\Psi(f)$ dependence is resonant in nature. Similar results are observed in other gases.

1) *Ionization—diffusion nature of striations.* It has now been firmly established that the striations are of an ionization–diffusion nature. A governing role in the appearance of striations was assigned to ionization and diffusion in papers already in the 1920s and 1930s (Refs. 11 and 36–39). Work by Rother,[40,41] Wojaczek,[42] and Pekarek[5,9,43-45] was important in identifying the mechanism for the appearance and growth of a moving ionization wave.

Using the ambipolar diffusion equation for ions and a phenomenological equation for the electron temperature, and also considering the temperature dependence of the ionization rate, Pekarek showed that the introduction of a pulsed perturbation in the positive column led to an oscillatory process: a so-called stratification wave, with a phase velocity directed from the anode

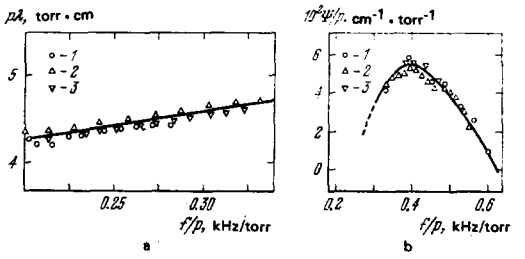

FIG. 1. Dispersion curves for Ne obtained in tubes of various radii,[33] plotted in terms of the scaling parameters. R (cm): 1) 0.75; 2) 1.25; 3) 2.02. $pR = 2$ torr·cm. i/R (A/cm): a) $1.67 \cdot 10^{-3}$; b) $3.3 \cdot 10^{-3}$.

toward the cathode and a group velocity in the opposite direction. The model proposed by Pekarek, however, suffered from several important shortcomings, which are pointed out accurately in the review by Nedospasov.[4] Nevertheless, the Pekarek model has been adopted quite widely because it is relatively simple and graphic.[46-54]

Klyarfel'd[55] was apparently the first to offer a qualitative explanation for the existence of striations. He asserted that over the length of a striation the electrons acquire an energy sufficient to ionize atoms. Then, as a result of the ionization, a group of slow electrons is formed; moving in the electric field, they also acquire energy; etc. This explanation was not supported experimentally, however, until quite recently. A few recent experimental papers[56-58] report measurements of the electron energy distribution which are consistent with Klyarfel'd's interpretation of the existence of striations. The most graphic results are those of Ref. 56, where the perturbation of the electron distribution function is plotted as a function of the phase of the wave for small-amplitude striations in neon. It follows from these figures that there is a group of electrons with an energy which increases with changing phase, up to a level near the ionization of excitation potential (depending on the type of striation). This "hump" on the distribution function moves toward higher energies. Then this group of electrons expends its energy on ionization or excitation, and the distribution function acquires a low-energy hump. Then the process repeats itself periodically. This picture is observed for all types of striations at low currents. Near the current representing the upper boundary for the existence of striations (the Pupp boundary) the distribution function is monotonic, and here the existence of striations can be explainted on the basis of hydrodynamic arguments, by analogy with Refs. 5, 42, and 59, for example.

According to these arguments, the ionization waves result primarily from a dependence of the ionization rate on the electron temperature and density. Because of plasma quasineutrality (the striation wavelength is much larger than the Debye length) the variation of the electron density in the positive column is described by an ambipolar diffusion equation with a source corresponding to the ionization rate. The electron temperature enters this equation through the ionization rate; this temperature is related to the density by the heat-balance equation, which incorporates heat convection, Joule heating, the thermal conductivity of the electrons, and the collisional energy loss. These equations must be supplemented with Ohm's law, which relates the electric field to the current.

Let us consider the basic processes which make possible the propagation of an ionization wave in the positive column. The characteristics of this wave are such that the relative change in the electron density is determined primarily by diffusion to the tube wall and by the temperature dependence of the ionization rate. The corresponding equation for a small deviation of the density from its steady-state value is

$$\frac{\partial N}{\partial t} = \frac{z_T}{\tau_n} U, \tag{1}$$

where N and U are the relative deviations of the electron density and temperature from their steady-state values (n_0, T_0), z_T is the logarithmic derivative of the ionization frequency with respect to the temperature, and τ_n is the electron lifetime with respect to diffusion to the walls.

The electron energy transport by thermal conductivity is basically balanced by the change in Joule heating; i.e.,

$$\gamma \frac{\partial^2 U}{\partial x^2} = -KW, \tag{2}$$

where W is the relative deviation of the field from its steady-state value E_0, γ is the thermal conductivity, $K = v(n_0 T_0)$, and v is the electron drift velocity.

In turn, it follows from the constancy of the current that $W = (T_0/E_0)\partial N/\partial x$, i.e.,

$$\gamma \frac{\partial U}{\partial x} = -K \frac{T_0}{E_0} N. \tag{3}$$

Differentiating (1) with respect to the coordinate, and using (2), we find a closed equation for N:

$$\gamma \frac{\partial^2 N}{\partial t \partial x} = -K \frac{T_0 z_T}{E_0 \tau_n} N.$$

This equation describes a travelling wave with a phase velocity equal in magnitude to the group velocity but opposite in direction.

From (1)–(3) we conclude that the oscillations of the field and the electron temperature are in phase, while that of the electron density is shifted a quarter-wavelength in the direction of the anode (the positive direction of the x axis is from the cathode toward the anode; Fig. 2).

To describe the excitation of ionization waves we must examine in greater detail the events which occur in the plasma of the positive column. It turns out that a necessary condition for the excitation of ionization waves is a dependence of the ionization frequency on the electron density, which results, for example, from stepwise processes. Other processes—ambipolar diffusion, heat conduction, and convection—damp the ionization waves.

2) *Regions in which moving striations exist.* Moving striations have been studied in most detail in pure inert gases: neon,[33,60-64] argon,[64-66] and helium.[64,67,68] Recently there has been much interest in striations in

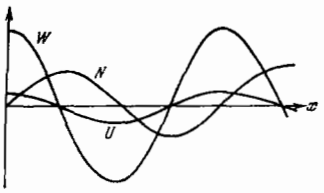

FIG. 2. Profiles of the electron density (N), the electron temperature (U), and the electric field (W) along the positive column.

helium–neon mixtures, which are frequently used in lasers.[10,14,16,17,67-83]

The positive column has similar properties in inert gases and in their mixtures. The striations exist over broad ranges of the pressure and the current below the Pupp boundary. Figure 3 shows the boundaries for the existence of striations in neon according to Ref. 33; a similar picture is observed for argon.[64] For these gases there is a stratification-free region at low pressures and intermediate currents. This region also exists in helium–neon mixtures. The dimensions and configuration of this region are very sensitive to the mixture composition ($\alpha = p_{He}:p_{Ne}$) and to the length of the discharge gap[16,17] (L; Fig. 4). We see from Fig. 4 that the addition of helium to neon expands the stratification-free region and displaces it toward lower pressures and higher currents. At $\alpha = 7$ we can observe a lower boundary for the excitation of striations (curve a-a in Fig. 4a). As α is increased, the lower boundary shifts upward along the pressure scale, and the boundary of the striation-free region (curve b-b) shifts downward. When these boundaries come together, the shape of the striation-free region changes qualitatively. For pure helium the left-hand boundary of the striation-free region is not observed, since it goes into the region in which the discharge is extinguished. With increasing length of the discharge tube the stratification-free region becomes narrower (Figs. 4b and 4c).

Only one type of striation is observed near the Pupp boundary, but at low and intermediate currents we can distinguish four types of striations[5,64,84]: the p, r, s, and s' types. Each has a characteristic voltage drop over the length of the striation, $U_\lambda \approx \lambda E$, which varies only slightly with the discharge conditions (the Novak rule).[85] Zaĭtsev and Savchenko[34] have studied three types of spontaneously excited striations (p, r, and s) in neon and two types (p and s) in argon over the pressure range $pR = 0.45$–36 torr · cm and over the current

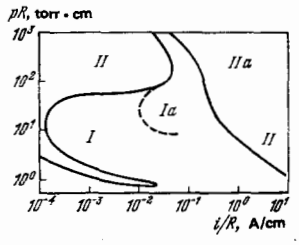

FIG. 3. Boundaries of the regions in which moving striations exist in Ne (Ref. 33). I) Striation exciation region; II) region of quiet discharge; Ia, IIa) regions of contracted column.

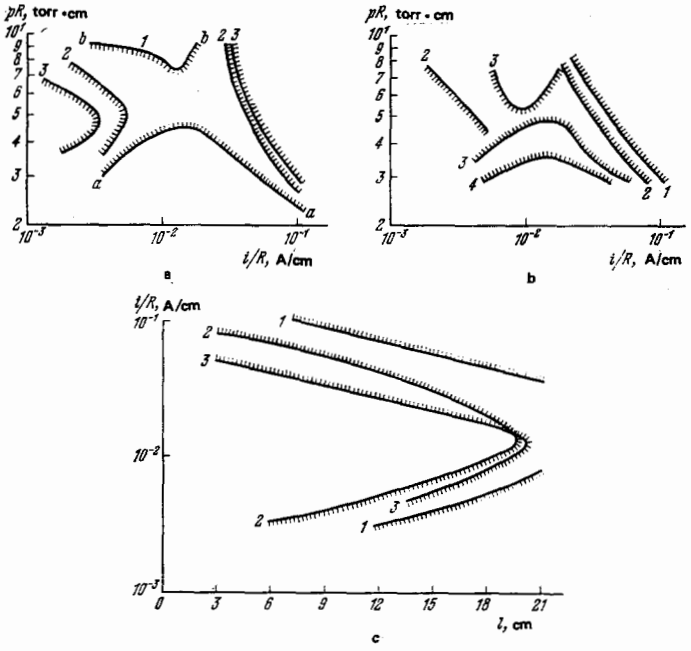

FIG. 4. Boundaries of the regions in which moving striations exist in a helium–neon mixture as a function of the mixture ratio α and the length of the discharge gap, L (the hatching shows the striation excitation region).[16] a: $L = 10$ cm, $\alpha = 7$ (1), 10 (2), and 15 (3). b: $\alpha = 7$, L(cm) = 3 (1), 6 (2), 9 (3), and 15 (4). c: $\alpha = 10$, pR (torr·cm) = 3 (1), 4.5 (2), and 6 (3).

range $i/R = 10^{-2} - 2$ A/cm. As a rule, the different types of moving striations exist independently; for a given value of pR, the striations with lower values of U_λ are observed at lower currents.[34] At the transition from one type of striation to another, two types of spontaneously excited striations may exist simultaneously.[34,86] The properties of the striations of different types change in different ways as functions of the current and the pressure. The frequency and phase velocity of the p and s' striations increase with increasing current, while those of the r and s striations decrease.[84]

3) *Shape of the striations near the self-excitation threshold.* The moving striations observed near the self-excitation boundary can be described approximately as a superposition of two components: a travelling wave (an ionization wave proper) which is growing in the direction from the cathode toward the anode and an in-phase component caused by the modulation of the discharge current. This effect can be seen in Fig. 5, which shows the most typical distribution of the intensity (I) of the spontaneous lateral emission at the fundamental frequency of the striations.[1)] This distribution can be found by setting

$$N = Ae^{\Psi x}\cos(\Omega t + Kx) + C\cos(\Omega t + \varphi), \quad I \sim \sqrt{\overline{N^2}}\,. \tag{4}$$

The in-phase component leads to a spatial modulation of the emission with a period equal to the length of the moving wave. The relative amplitude of this modulation falls off with distance from the cathode. At a cer-

[1)] Analogous distributions are given in Refs. 68–70 and 87.

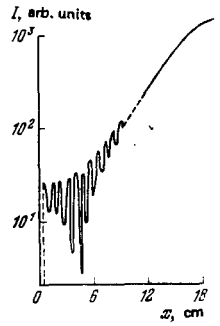

FIG. 5. Profile of the intensity of the spontaneous lateral emission along the x coordinate for a helium–neon mixture.[63]

tain distance from the cathode the intensity I goes through a minimum, at a position which is determined by the spatial growth rate Ψ (Ref. 87). It follows from (4) that the minimum occurs at the coordinate $x_0 \approx (1/\Psi)\ln(C/A)$; here $x_0 > 0$ if $C > A$. Since the ratio C/A is determined solely by the boundary conditions at $x = 0$, the position of the minimum can be used to draw certain conclusions about these boundary conditions.

At sufficiently low currents (to the left of the striation-free region) the distribution has a different shape (Fig. 6). This shape can be explained qualitatively by describing the striations as a sum of three components: two travelling waves, with wave numbers differing by an order of magnitude, and an in-phase component:

$$N = Ae^{\Psi x}\cos(\Omega t + Kx) + Be^{\Psi_1 x}\cos(\Omega t + K_1 x + \varphi_1) + C\cos(\Omega t + \varphi_2). \quad (5)$$

As we shall see below, this description is completely justified.

With increasing distance from the self-excitation boundary the spatial growth of the travelling component of the striations intensifies and becomes nonlinear. At a certain distance from the cathode the amplitude of the travelling component reaches saturation. After saturation is reached, the striations have a definitely nonsinusoidal shape. It should be noted that the generation of the striations is single-mode generation (with harmonics) over a broad current range. This circumstance can apparently be attributed to a strong mode competition in the positive column as a result of the nonlinear interaction among modes. Well away from the excitation boundary the single-mode generation gives way to multimode generation and becomes turbulent.

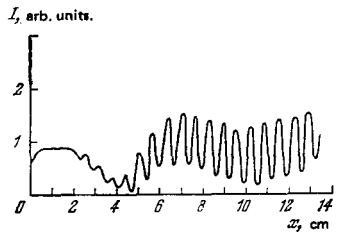

FIG. 6. Profile of the intensity of the sponaneous lateral emission along the x coordinate for a helium-neon mixture at low currents.

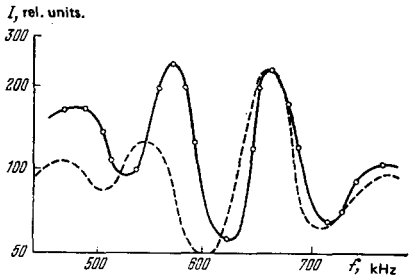

FIG. 7. Frequency dependence of the intensity of the noise component of the discharge current near the excitation threshold for a helium–neon mixture. $\alpha = 7$, $pR = 0.4$ torr·cm, $i = 2.5$ mA, $R = 0.15$ cm, $L = 11$ cm. The dashed curve shows the calculation for the intensity of stimulated striations for the same parameter values.

4) *Self-excited-oscillator properties of striations.* Study of the striation properties show that the positive column is a distributed self-excited-oscillator system with a pronounced nonlinearity. One of the main arguments on which this assertion is based is the mode structure of the striations. Near the excitation boundary, before the conditions for self-excitation have been satisfied, there is an increase in the noise level in the positive column. The amplified noise has the resonant structure shown in Fig. 7. The distance between adjacent peaks, Δf, is a characteristic of a given tube. In the low-frequency part of the spectrum we can also observe an amplified noise at the frequency Δf, which implies pronounced nonlinear properties of the positive column. As the self-excitation boundary is approached, the intensity of each noise component increases. An analogous resonant structure of the noise was observed in Ref. 76.

The stimulated striations also have a mode structure, which can be seen near the self-excitation boundary. The frequency dependence of the amplitude of the stimulated striations is of the same form as that for the noise (Fig. 7). With increasing distance from the self-excitation boundary, the relative width of the peaks increases, and the peaks fade away. If the external agent has a high amplitude, the peaks become asymmetric (Fig. 8), acquiring a shape similar to that of the resonance curves for an oscillator circuit with a nonlinear capacitance.[88]

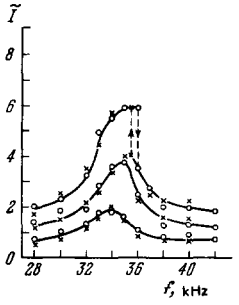

FIG. 8. Dependence of the amplitude of the stimulated current oscillations on the frequency of the external agent near the frequency of the natural striations.[51]

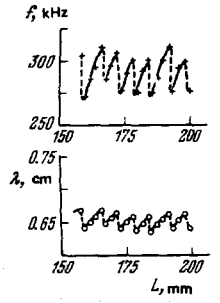

FIG. 9. Dependence of the frequency and wavelength of the moving striations on the length of the positive column.[70] R = 1.75 mn, α= 10, p = 3 torr.

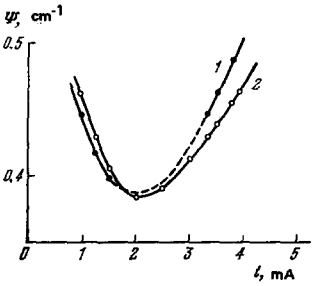

FIG. 11. Dependence of the striation growth rate Ψ on the discharge current. α = 15, pR = 0.3 torr·cm. 1) L = 10.2 cm; 2) L = 14.1 cm. The dashed curve corresponds to stimulated striations.

The self-excited striations also exhibit a mode structure. It was mentioned in Refs. 6 and 73, for example, that a change in the length of the positive column leads to an abrupt change in the frequency and the wavelength of the striations, with a period roughly equal to the average wavelength (Fig. 9). The frequency and wavelength of the striations change discontinuously as the discharge current is changed (see Fig. 10 and Refs. 6, 9, 67, 73, and 89). At a discontinuity, more structure appears in the spectrum of the discharge current and of the lateral spontaneous emission. The spectral components are separated by a frequency interval Δf which corresponds to the distance between the peaks in the noise spectrum in Fig. 7 and to a shift of the striation frequency at the time of the discontinuity. The quantity Δf depends on the discharge current, the length of the discharge gap, and the gas composition.[2)] In a given tube, for example, a change in the length from 18 to 38 cm causes an inversely proportional change in Δf from 57 to 25 kHz.

In distributed self-excited oscillators the boundary loss causes the self-excitation conditions to become dependent on the length of the system. The same effect is observed upon the excitation of striations, as can be seen in the dependence of the dimensions of the striation-free region on the length of the positive column (Fig. 4c). The reason for this dependence is that the boundary growth rate of the striations decreases with increasing length.

Figure 11 shows the striation growth rate as a function of the current (in the stratification-free region,

FIG. 10. Dependence of the frequency of the moving striations on the discharge current.[77] α = 6, p = 2.8 torr, R = 0.9 mm, L = 20 cm.

the growth rate was measured for stimulated striations, and the results are shown by the dashed part of the curve). Interestingly, the growth rates are different near the left-hand and right-hand boundaries of the striation-free region: A lower growth rate is required for self-excitation of striations on the left than on the right.

Any self-excited oscillator must have feedback. In the case of striations the feedback question is a complicated one. It has been asserted in many papers that the feedback occurs through the external circuit, i.e., through the discharge power supply.[46,83] However, many attempts to suppress striations by inserting a variety of filters in the power-supply circuit have resulted in only a negligible shift of the self-excitation boundary.[90] The implication is that there may be an internal feedback in the discharge which leads to the existence of regular striations. This possibility was pointed out, in particular, by Pekarek,[5] but the question of the nature of this internal feedback was left open. A theoretical case for the existence of internal feedback was made in Refs. 91 and 92.

Other indications that the striations are of the nature of a self-excited oscillator are such effects as the synchronization of striations by an external agent,[93] the mutual synchronization of striations in tubes with two discharge gaps,[77] the asynchronous suppression of striations by an external agent,[51,86,94] the asynchronous suppression of striations of one type by striations of another type,[86,94] etc.

An external agent at frequencies near the frequency of spontaneous striations causes a synchronization of these striations; i.e., the frequency of the external agent is imposed on the striations. The width of the synchronous region depends on the amplitude of the external agent. This synchronization effect is used, in particular, to find the dispersion characteristics of striations in their excitation region.[31]

Asynchronous suppression of striations can be seen in a decrease in their amplitude in the case of an external agent at a frequency far from resonance. This external agent may be, for example, a modulation of the discharge current, an external rf field, relaxation oscillations in the discharge, or striations of a different type. Beginning at a certain amplitude of the external agent there is a complete suppression of the stria-

[2)] For pure helium the frequency jumps are not observed as the discharge current is changed.[67]

tions because of a "synchronization by extinction" similar to that which occurs in lumped self-excited oscillators.[95,96] The asynchronous suppression of striations accompanying the insertion of an oscillator in the discharge circuit was first observed by two groups simultaneously: Zaĭtsev *et al.* (see Ref. 97) and Ohe and Takeda.[51] Ohe and Takeda[51] described the dependence of the striation amplitude on the amplitude of the external agent for various frequencies. This dependence is nonmonotonic, so they adopted as the threshold for the striation suppression that amplitude of the external agent which was found by extrapolating the descending part of the dependence.

More careful experiments on this asynchronous suppression were carried out by Zaĭtsev, Il'inskiĭ, and Savchenko.[86,94] At a certain amplitude of the external agent they obtained a complete suppression of moving striations over a broad frequency range. Figure 12 shows the dependence of the threshold value of this amplitude (in arbitrary units) on the frequency of the external agent. Zaĭtsev *et al.* also studied the suppression of striations of one type by striations of a different type.[86,94] They observed an important distinction from the case of external modulation of the discharge current.

The asynchronous suppression can be seen most clearly in the expansion of the striation-free region. Experiments of this type were carried out for a helium–neon mixture by Zakharchenko and Privalov, who measured the boundaries of the striation-free region as a function of the amplitude of an rf field applied to the discharge (Fig. 13).

b) Stationary striations

1) *Regions of existence.* Under certain discharge conditions a homogenously emitting positive column will transform into alternating dark and bright regions (layers), which sometimes differ in color. These are stationary striations. In molecular gases they are usually visible over a substantial part of the positive column, while in inert gases they are rapidly damped and are difficult to observe.

Stationary striations were studied experimentally and described far earlier than moving striations, since they could be observed without any special stroboscopic apparatus. Nevertheless, there has been essentially no

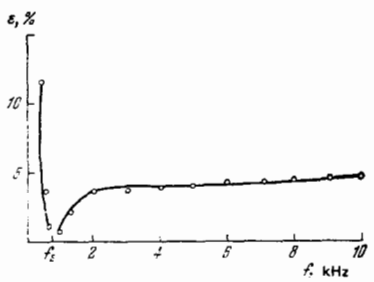

FIG. 12. Experimental dependence of the threshold value of the modulation frequency.[94] $pR = 1.7$ torr·cm, $R = 1.5$ cm, $i/R = 6.67 \cdot 10^{-3}$ A/cm.

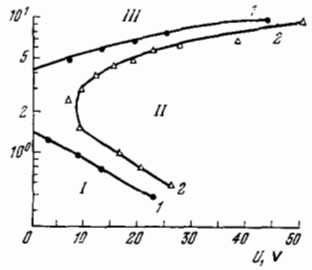

FIG. 13. Dependence of the size of the striation-free region in a helium–neon mixture, with $\alpha = 10$, on the peak voltage from the rf oscillator. 1) $L = 6$ cm; 2) $L = 8$ cm.

systematic classification of the experimental data available. The coverage of stationary striations in the reviews by Nedospasov, Pekarek, and Oleson and Cooper[4-6] was quite brief.

We do know that stationary striations are observed over broad ranges of discharge conditions and have an appreciable amplitude in molecular gases (hydrogen, nitrogen, air, etc.) and in mixtures of inert gases and molecular gases.[55,98-102] Stationary striations may also exist in discharges in pure inert gases,[43,50,103-107] but here they are slightly different from the stationary striations in molecular gases. While a large number of stationary striations of identical amplitude (or with an amplitude which falls off slightly toward the anode) may form in molecular gases, in inert gases one usually observes several (from two to six) stationary striations on the cathode side of the positive column which are rapidly damped in the direction toward the anode.

Stationary striations may be observed either in the presence of moving striations or in their absence. An understanding of the regions in which stationary striations exist and of their properties is of practical importance because lasers using molecular gases and their mixtures have recently found widespread use. Kagan and Mitrofanov[108] report that a stratified positive column in a molecular gas exhibits a macroscopic instability under certain conditions. The instability takes the form of either a slow spontaneous motion of the stratified positive column toward the anode (at a velocity on the order of 1 cm/min) or slow longitudinal oscillations of the column as a whole. The instability causes slow, synchronized changes in the plasma potential in each cross section of the column. These changes in potential in turn cause oscillations of the electron density and temperature and thus oscillations of the laser output power.

The regions in which stationary striations exist are complicated in shape (as they are for the moving striations). These regions are reported for hydrogen and oxygen in Ref. 5 (for example); Fig. 14 shows the regions for nitrogen according to Ref. 99. Comparison of the regions in which stationary and moving striations exist show that the stationary striations are generally observed at lower pressures and lower currents.

The stratification pattern of the positive column is apparently most distinctive in the case of hydrogen.

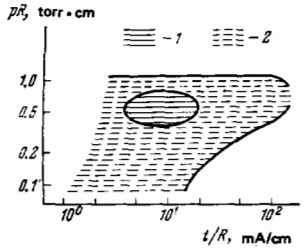

FIG. 14. Region in which stationary striations exist.[99] 1) Sharp stationary striations; 2) "blurry" stationary striation.

For hydrogen there are typically at least two types of stationary striations, which differ markedly in properties, in particular, their color; they have been labelled "red" and "blue" striations. According to Ref. 11, the blue striations are observed at relatively low values of pR. The visual shapes of the triations in hydrogen are also different (ranging from flat disks to cones), and they may even have a double structure. In nitrogen the sharpest stationary striations exist at currents of the order of 10 mA and $pR \sim 0.5$ torr·cm. Here the stationary striations are usually orange disks which are slightly convex toward the cathode.[99]

Research by Klyarfel'd[55] has substantially extended our understanding of the discharge properties for which stationary striations may be observed. According to Ref. 55, stationary striations may exist in hydrogen (for example) over a range of gas pressures amounting to four orders of magnitude (up to 1 atm) and over a range of current densities amounting to eight orders of magnitude (up to 300 A/cm²), in discharge tubes of a broad range of diameters, down to capillaries with a diameter of the order of 0.1 mm.

There has been essentially no systematic study of the regions in which stationary striations exist in inert gases. An exception to this statement is represented by Ružička's report[109] of some data on neon (Fig. 15). We see from this figure that stationary striations occur in neon primarily in a region in which there are no moving striations (at currents in the range 1-10 mA and at pressures in the range 0.1–1 torr). Karelina and Klyarfel'd[105] have observed damped stationary striations in helium, neon, and argon in roughly the same current and pressure ranges; these striations have also been observed[106,107] at lower pressures (down to 10^2 torr) and higher currents (up to 100 mA).

The discrepancies in the results reported on the regions in which stationary striations exist can be attributed to at least two circumstances: First, the formation of stationary striations is aided considerably by the presence of even a very small molecular or other impurity. Second, as shown in Ref. 55, the excitation of stationary striations, particularly in inert gases, is strongly affected by the boundary conditions at the electrodes and by various inhomogeneities in the column: a narrowing or flaring of the discharge tube, probes held at any potential, an imposed transverse magnetic field, etc.[109-112]

2) *Basic characteristics of stationary striations.* From the point of view of determining the nature of stationary striations we are obviously interested in the ranges which their properties (their length λ and the spatial damping factor Ψ) may occupy, depending on the discharge conditions. The dependence of λ on the pressure and the current in molecular gases has been the subject of many studies, but almost no information is available on the values of Ψ. The length of the sharp stationary striations in molecular gases has been measured quite thoroughly. Generalizing from the data, we can draw the following conclusions:

a) The length of the stationary striations is of the order of the tube radius and falls off with increasing current, approaching a constant limit.[99-102,113]

b) The length of the stationary striations falls off with increasing pressure and with decreasing radius in accordance with the empirical law[100] $\lambda(pR)^m/R = \text{const}$, where m is less than unity and depends on the nature of the gas.

In inert gases the stationary striations are usually much longer than in molecular gases, and their length is several times their diameter, especially at low pressures. For example, in neon at $p = 0.05$ torr at a current of 200 mA in a tube 7.5 cm in diameter, the stationary striations reach a length of 20 cm, while in hydrogen with roughly the same discharge conditions the length is 8 cm. In inert gases, in contrast with molecular gases, the length of the stationary striations increases with increasing current. There is also an increase in their spatial damping factor (see Fig. 16, which shows data obtained by Zaĭtsev and Savchenko). A molecular-gas admixture in the inert gas causes the stationary striations to become shorter; the effect is greater, the larger the admixture.[114,115] In most gases

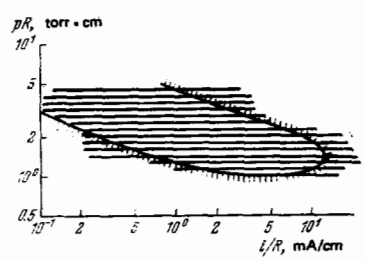

FIG. 15. Region in which stationary striations exist in neon.[109] The curve with the hatching is the boundary for the excitation of moving striations.

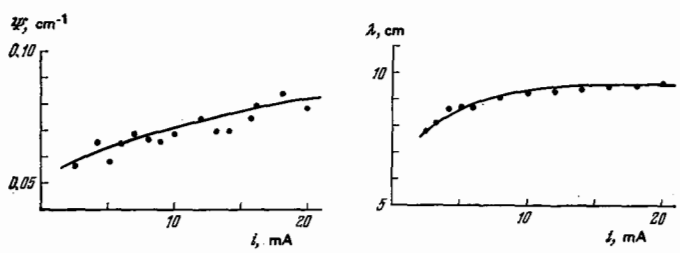

FIG. 16. Dependence of the spatial growth rate Ψ and the length λ of stationary striations in neon on the current i. $pR = 1.6$ torr·cm, $R = 1.825$ cm (data furnished by I. A. Savchenko).

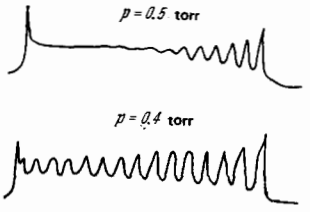

FIG. 17. Shape of the stationary striations in nitrogen.[49,50] The curves are labeled with the gas pressure. L = 39.4 cm, i = 10 mA (the cathode is on the right).

the length of the stationary striations is slightly less than that of the moving striations.

The pressure dependence of the damping factor for the stationary striations was studied in Refs. 49 and 50 for nitrogen at a current of 10 mA (Fig. 17). It was found that the damping of the stationary striations increases substantially as the pressure is raised from 0.4 to 0.6 torr.

Many investigators[49,50,55,106,107] have mentioned that the striation nearest the cathode is slightly different from all the others; it is not only sharper but also the longest.

If the damping of the striations is slight, they exhibit a mode nature as the length of the discharge tube is varied: The number of striations changes aburptly by one at a certain point[116] (Fig. 18).

The stratified structure of the positive column which can be seen in its inhomogeneous emission, both visually and with photomultipliers,[109,112] should also be reflected in an inhomogeneity of properties of the column such as the potential, the field, and the electron temperature and density. The first probe measurements along this line were carried out in hydrogen by Paul[117] and, later, by Boyd and Twiddy.[8] Similar measurements have been carried out by a microwave method.[113] The use of the probe method to study stationary striations has the disadvantage that a change in the probe potential may lead to changes in the properties of the striations themselves, as has already been pointed out.[105] It is for this reason that the results of Refs. 113 and 117 are slightly different. The experimental results show that the electron density and temperature and also the intensity of the lateral spontaneous emission vary along a striation, and there is a phase shift between the variations in these properties. The phase shift between the density and the temperature variations, for example, ranges from $(3/4)\pi$ to π, according to different investigators.

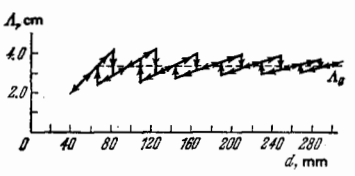

FIG. 18. Dependence of the length of the stationary striations on the distance between the electrodes (rf discharge in neon).[116] p = 1.8 torr, R = 1.55 cm, $j = 3\cdot10^{-3}$ A/cm^2.

2. THEORY OF MOVING AND STATIONARY STRIATIONS

a) Basic theoretical premises

The best-developed theory of striations is the linear hydrodynamic theory, which uses a dispersion relation to explain the possible growth of moving striations. The development of this theory began in papers by Prudkovskaya[118] and Rother[40,41] and was continued by Wojaczek,[42] Nedospasov,[4,119,120] and Tsendin.[59]

As mentioned earlier, the starting points in the hydrodynamic theory of striations for high currents are the ambipolar diffusion equation for the electron density n, the heat-balance equation for the electrons, and Ohm's law. It is assumed that the density of metastable atoms, m, can keep up with changes in the electron density. At intermediate currents (this region is defined below) the time for establishing the metastable-atom density is comparable to that for the electron density; i.e., the relationship between them ceases to be algebraic.

We write these equations in the form[4,59]

$$\left.\begin{aligned}
&\frac{\partial n}{\partial t}-\nabla\left(D_a\nabla n\right)-\gamma_1\nabla\left(n\nabla T\right)-n\nu\mu_i\nabla\left(\frac{1}{\mu_e}\right)=nZ\left(n,\,m,\,T\right),\\
&\frac{\partial m}{\partial t}-D_m\Delta m=nP\left(n,\,m,\,T\right),\\
&\frac{3}{2}\frac{\partial}{\partial t}\left(nT\right)-\zeta\nu n\nabla T-\gamma\nabla\left(\mu_e nT\nabla T\right)+n\nu E+nH\left(n,\,T\right)=0,\\
&j=-en\nu=en\mu_e\left(E-\frac{T}{n}\nabla n-\gamma_1\nabla T\right),
\end{aligned}\right\}\qquad(6)$$

where μ_e and μ_i are the electron and ion mobilities; $D_a=\mu_i T$ is the ambipolar diffusion coefficient; T is the electron temperature in electron volts; D_m is the diffusion coefficient of the metastable atoms; E is the longitudinal component of the electric field; ν is the electron drift velocity; j is the current density; ζ,γ,γ_1 are kinetic coefficients which depend on the electron energy distribution; e is the electron charge; $H(n,T)$ is the energy lost by an electron in the course of a collision; and the functions $Z(n,m,T)=Z_{0\infty}+(m/n_g)Z_{m\infty}$ and $P(n,m,T)=Z_{0m}-(m/n_g)(Z_{m0}+Z_{m\infty})$ describe the formation of ions and of metastable atoms (Z_{0m}, Z_{m0}, $Z_{0\infty}$, and $Z_{m\infty}$ are the frequencies of excitation, decay, direct ionization, and stepwise ionization; and n_g is the density of neutral atoms in their ground state). Volume recombination of electrons is usually ignored in research on striations.

The frequencies of processes in which ions and metastable atoms are formed depend exponentially on the electron temperature and are sensitive to the shape of the tail of the distribution function. A non-Maxwellian tail leads to a dependence of these frequencies on the electron density.[42]

In the first equation in (6) there are, in addition to the term describing the ambipolar diffusion, a thermodiffusion term and a term which results from the temperature dependence of the electron mobility. The diffusion of metastable atoms is incorporated in the second equation in (6). These terms, however, have negligible

effects on all the processes discussed below. The third equation—the heat-balance equation—incorporates the convection and thermal conductivity of the electron gas and also the energy lost by electrons in collisions with atoms. Because of the high mobility of electrons ($\mu_e \gg \mu_i$) at the frequencies characteristic of striations we can ignore the term $\partial(nT)/\partial t$ in this equation.[119]

The stability of the positive column is usually analyzed in the one-dimensional approximation. The wall effects are taken into account by introducing diffusive lifetimes for electrons, $\tau_n = (R/\beta)^2/D_a$, and for metastable atoms, $\tau_m = (R/\beta)^2/D_m (\beta \approx 2.4)$, these linearized dimensional effects[59] has shown that the differences from the results of the one-dimensional theory reduce to a negligible increase in the effective thermal conductivity of the electron gas and to a negligible dependence of the stepwise-ionization frequency on the tube radius.

Without taking into account the inconsequential terms which we have just discussed, Eqs. (6) take the following form in the one-dimensional approximation:

$$\left.\begin{aligned} &\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}\left(D_a \frac{\partial n}{\partial x}\right) = n\left(Z(n, m, T) - \frac{1}{\tau_n}\right), \\ &\frac{\partial m}{\partial t} = nP(n, m, T) - \frac{m}{\tau_m}, \\ &\zeta v \frac{\partial T}{\partial x} + \gamma \frac{\mu_e}{n} \frac{\partial}{\partial x}\left(nT \frac{\partial T}{\partial x}\right) = vE + H(n, T), \\ &j = -env = en\mu_e\left(E - \frac{T}{n}\frac{\partial n}{\partial x} - \gamma_1 \frac{\partial T}{\partial x}\right). \end{aligned}\right\} \quad (7)$$

It follows from the second equation in system (7) and the form of the function $P(n, m, T)$ that the lifetime of the metastable atoms is determined not only by diffusion to the walls but also by the disappearance of these atoms with in the volume as a result of deexcitation and stepwise ionization. We can thus introduce an effective lifetime of the metastable atoms, $\tau_{m\,eff} = [(1/\tau_m) + (n_0/n_g)(Z_{m0} + Z_{m\infty})]^{-1}$, which depends significantly on the discharge current. Specifically, τ_{meff} decreases with increasing current, varying over a broad range. In this connection, the overall current range in which striations can exist is conveniently partitioned into three regions: 1) high currents, $\tau_n \gg \tau_{m\,eff}$; 2) intermediate currents, $\tau_n \lesssim \tau_{m\,eff} \ll \tau_m$; 3) low currents, $\tau_{m\,eff} \sim \tau_m$. The mechanism for the instability of the positive column has distinctive features in each of these regions. At high and intermediate currents the density of metastable atoms is near the saturation level and is only a weak function of the current. At low currents the density of metastable atoms increases with increasing current.

The hydrodynamic approximation in the theory of striations presupposes that a local electron energy distribution is established in the positive column as a result of elastic collisions of electrons. If electron–electron collisions are predominant over electron–atom collisions, the "body" of the distribution function is Maxwellian up to energies below the excitation potential U_a. At energies of the order of and higher than U_a there may be important deviations from a Maxwellian distribution.[108,121] The reason is that at energies of the order of or higher than U_a electron–electron collisions become less effective, and inelastic and elastic electron–atom collisions and the interaction of electrons with the electric field come into play. If the average electron energy is significantly lower than the atomic excitation potential U_a, most of the electrons will have an energy $U < U_a$, and the distribution function will be approximately Maxwellian.[42] In this case the distribution of high-energy electrons has a minor effect on the thermodynamic properties. These high-energy electrons do, on the other hand, play a major role in the excitation and ionization of atoms and thus in the excitation of the ionization wave. The deviation from a Maxwellian shape in the tail of the distribution function gives rise to a nonlinear dependence of the ionization rate on the electron density, even when stepwise processes are ignored. Alanakyan[122,123] has used kinetic equations with only direct ionization to derive a simple expression for determining the quantity $\partial Z/\partial n$ from the measured distribution function for short wavelengths ($\lambda \ll 2\pi T_0/E_0$). Analyzing the stability of the positive column for the case in which there is a perturbation of the electron distribution function, Alanakyan showed that this perturbation, if small, would lead to no effects other than the appearance of an n dependence of Z.

It can be concluded from the results found by Alanakyan[122,123] and Wojaczek[42] that the hydrodynamic approach is valid in the case of a local electron energy distribution if we take into account the additional dependence of the rates at which electrons and metastable atoms appear and disappear on the electron density—a dependence which results from the distortion of the tail of the distribution function.

Since the frequency of electron–electron collisions falls off with decreasing current, the hydrodynamic approximation should fail at low currents. According to estimates graciously furnished us by Tsendin, the current range in which the hydrodynamic approximation is not valid is determined by the condition $(i/R)(p/E)^3 < BRp$, where $B = 7 \cdot 10^{-3}$ for Ne and $1 \cdot 10^{-3}$ for Ar. Even in this region, however (within a reasonable distance from this boundary), the hydrodynamic equations are still valuable for a study of striations, as will be shown below. They yield a rather comprehensive qualitative—and frequently quantitative—picture of nearly all known phenomena by comparatively simple methods. Among these phenomena are the growth and excitation of various types of striations, both moving and stationary; the steady-state generation of striations; the nonlinear interaction of striations of various types; and the synchronization of striations by an external agent.

There has recently been an acceleration of work based on the kinetic theory of striations, in which the plasma is described by the ambipolar diffusion equation for the electron energy distribution.

The first papers to use kinetic equations to explain the striations were by Vlasov,[124] who derived periodic solutions of the equations of a collisionless plasma (the Vlasov equations). Vlasov's papers have been justly criticized by Klimontovich[125] and Klyarfel'd[55]; this criticism is included in the review by Nedospasov.[4]

A kinetic approach is a fundamental necessity at low electron densities, at which electron–electron collisions are of minor importance, and a local energy distribution cannot be established over either time or space. This case has been studied by a Czechoslovak group.[84,126-129] In the absence of electron–electron collisions and neglecting the energy loss by electrons in elastic collisions with atoms, the perturbations of the distribution function turn out to be spatially nonlocal. Solution of the Boltzmann equations for perturbations of the electric field with a wavelength λ has shown that nonlocal effects are most apparent at the "resonant" wavelengths $\lambda_0 = U_a/qE$, where $q = 1, 2, \ldots$ As a result, there is an additional spatial phase shift between the perturbations of the electron density and the electric field at these resonant wavelengths. Taking into account these effects in a study of the stability of the positive column should lead to discontinuities in the dispersion curves.[127] Since the highest growth rate corresponds to resonant wavelengths, we believe that an explanation has been found for the empirical law that the characteristic potential is constant for each type of striation (the Novak rule).[5] The results of Refs. 84 and 126–129 raise doubts since there is no experimental confirmation of any kind of the existence of sharp changes in the nature of the dependence of the frequency and the damping of ionization waves near the optimum wave numbers, by analogy with the results in Ref. 127.

The possibility of a nonmonotonic wave-number dependence of the temporal growth rate was also mentioned by Alanakyan,[130] who suggested that the processes having the greatest effect on the distribution function were both elastic and inelastic collisions of electrons with atoms and the interaction of electrons with the electric field. The growth rate reaches maxima at wavelengths λ_0 satisfying the condition $\lambda_0 = (U_i - U_a)/E(0, 8+q)$, where $q = 0, 1, \ldots$; and U_i is the ionization potential. The sharpness and height of these resonances, however, were not determined by Alanakyan. In Ref. 131 a kinetic approach was taken to explain experiments on stimulated striations in argon at low pressures (0.005–0.03 torr). Justification for this approach comes from the circumstance that the distortion of the tail of the distribution function becomes important at such low electron densities. The distinction between the hydrodynamic and kinetic results found in this paper, however, is simply a quantitative distinction. For example, kinetic-theory calculations yield phase shifts between the electric field, the ionization rate (the intensity of the lateral spontaneous emission), and the electron density which are in better agreement with experiment. This result again demonstrates that it is valid and worthwhile to use the simpler hydrodynamic theory to derive a qualitative picture of events.

The hydrodynamic approximation is apparently not suitable for calculating the shape of the sharp striations, since the shape of the electron distribution function varies substantially over the length of a striation.[57,58] At the head of a striation there is a particularly pronounced deformation of the distribution function, and there is a group of fast electrons. These effects were considered by Nedospasov and Petrov,[132] who used kinetic equations to study the structure of the potential drop at the boundary between two striations and found the distribution of the electron density and average energy over the length of a discontinuity and also the distribution function. The calculated results are in satisfactory agreement with experiment.[58]

So far the kinetic theory has not had any major successes, because of its complexity. At the same time, as we have shown in this review, the capabilities of the simpler hydrodynamic theory are far from exhausted. One purpose of this review is to summarize a definite stage in the development of striation theory and to show that it is still not necessary to discard the hydrodynamic approximation completely.

b) Conditions for the spontaneous excitation of moving striations at intermediate and high currents

The theory of Refs. 91, 92, and 133–139 can be used to determine the conditions for spontaneous excitation of striations. This theory differs from that customarily used to study plasma stability[140] in that it incorporates the reflection of waves at the boundaries. Fundamentally different results are found in many cases. The reason for this difference is as follows: In the ordinary linear theory of stability, a perturbation which is harmonic along a coordinate is specified, and the wave frequency and damping rate are calculated. If the damping rate turns out to be negative, it is concluded that the plasma is unstable and that waves are excited. On the other hand, we know[141] that if a system is convectively unstable then this instability means only that a wave may grow—not that it is excited. A correct resolution of the excitation question requires a consideration of the absorption and reflection of waves at the plasma boundaries. When these effects are taken into account, the wave number becomes complex near the excitation boundary, so that the damping rate may change substantially. It is the reflected wave which creates the internal feedback which is required for self-excitation (see Subsection a4 of Section 1). In some cases the reflection causes only a slight quantitative difference between the condition for excitation and the condition for growth. This is this case for striations in narrow tubes, which will be discussed below. Admittedly, even in this case the wave reflection leads to the requirement that the spatial growth rate remain finite at the excitation boundary, as it does experimentally, and as it does not in the "ordinary" theory. For striations in wide tubes the difference is a fundamental one: Although waves with a nonzero frequency can grow (the drift–temperature instability),[59] they cannot be excited spontaneously.

Particularly noteworthy among the papers which use the hydrodynamic approach (without consideration of boundaries) to study the properties of striations are the papers by Wojaczek.[42] These papers have reported a detailed study, both theoretical and experimental, of the conditions for the growth of ionization waves in argon at high currents (near the Pupp boundary). In calculating the steady-state properties of the positive col-

umn, Wojaczek considered stepwise ionization and a deviation from a Maxwellian electron energy distribution. He showed that a distortion of the tail of the distribution function leads to a very nonlinear dependence of the ionization rate on the electron density. The calculated results were in complete agreement with experiments carried out over broad ranges of the pressure and the current. Since Wojaczek calculated the stability of the positive column for high currents, it was fair to assume that the lifetime of the metastable atoms was short and that their density could keep up with changes in the electrondensity over time and space. A necessary condition for the instability turned out to be that the ionization rate should increase more rapidly than in proportion to the first power of the density; i.e., $\partial Z/\partial n$ had to be positive. This situation is provided for primarily by a deviation from a Maxwellian tail of the electron distribution function. As the current is raised, the distribution function becomes more nearly Maxwellian, weakening the n dependence of Z and leading to an upper limit of the striation-excitation region on the current scale (the Pupp boundary). Wojaczek found a quantitative correspondence between the calculated properties of the ionization waves and the experimental results.

If the discharge power supply has a finite internal resistance, Eqs. (7) must be supplemented with Ohm's law for the closed circuit:

$$\mathcal{E} = V_1 + V_c , \tag{8}$$

where $V_1 = jSR_1$ is the voltage drop across the resistance in the external circuit (R_1), which includes the electrode regions; $V_c = \int_0^L E\,dx$ is the voltage drop in the positive column; $S \approx 1.36R^2$ is the effective cross section of the tube; and L and R are the length and radius of the tube.

To study the self-excitation conditions and to determine the boundaries of the region in which striations exist, we can linearize Eqs. (7) and (8) with respect to small deviations from the steady state (corresponding to a positive column which is homogeneous over its length). In terms of the dimensionless variables $\tau = \varepsilon^2 t/\tau_n$, $\xi = \varepsilon\beta x/R$ ($\varepsilon = E_0R/\beta T_0$, $\beta \approx 2.4$), these linearized equations are[134]

$$\left.\begin{aligned}
&\frac{\partial N}{\partial \tau} - \frac{\partial^2 N}{\partial \xi^2} = \eta_m M + \tilde{\eta}_T U,\\
&\frac{\partial M}{\partial \tau} = (q + JP_n)\,N - JM + JP_T U,\\
&\alpha\frac{\partial U}{\partial \xi} - \gamma\frac{\partial^2 U}{\partial \xi^2} - \frac{\partial N}{\partial \xi} + 2(N - \tilde{j}) + h_T U = 0,\\
&\tilde{j} = \frac{R_0}{(R_1+R_0)l}\left[\int_0^l N\,d\xi - N(l) + N(0) - \gamma_i\,(U(l) - U(0))\right].
\end{aligned}\right\} \tag{9}$$

Here N, M, U, and $\tilde{j}$ are the relative deviations of the electron density, the density of metastable atoms, the electron temperature, and the current from their steady-state values; $R_0 = E_0L/j_0S$ is the dc resistance of the positive column; and $J = \tau_n/\varepsilon^2\tau_{m\ eff}$, $q = \tau_n/\varepsilon^2\tau_m$, $\alpha = \zeta - \gamma_1$, $\eta_m = (1/\varepsilon^2)(m_0/n_e)Z_{m\infty}/(Z_{0\infty} + (m_0/n_e)Z_{m\infty})$, $\eta_T = (1/\varepsilon^2)(Z_T - 1)$, $h_T = (T_0/H_0)\partial H/\partial T\big|_0$. Here Z_T, P_n, and P_T are parameters determined by the derivatives of the frequencies of excitation, deexcitation, direct ionization, and stepwise ionization with respect to the electron density and the electron temperature.

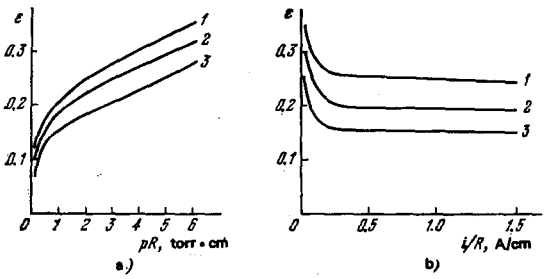

FIG. 19. Dependence of the parameter ε on the pressure and the current.[32] a: i/R (A/cm) = 0.064 (1); 0.13 (2); 4.5 (3). b: pR (torr·cm) = 2.2 (1); 2.4 (2); 0.6 (3).

The parameters ε, J, q, η_m, $\tilde{\eta}_T$, P_T, P_n, and h_T depend in a complicated way on the current and the gas pressure. For example, the dependence of ε on the current and the pressure is shown in Fig. 19, according to the data of Ref. 42. This figure shows that ε is small and a weak function of the current at currents which are not very low ($i/R \gtrsim 0.1$ A/cm) and pressures which are not very high ($pR \lesssim 5$ torr·cm).

The values of these parameters can be estimated in the following manner.[3)] At intermediate and high currents, direct ionization can be ignored in comparison with stepwise ionization. Also ignoring the deexcitation of atoms, we find

$$J = \frac{1}{\varepsilon^2}\frac{n_0}{m_0} \approx \frac{1}{1.36\varepsilon^2\mu_e m_0 ER}\frac{i}{R},$$
$$q = \frac{T_g}{\varepsilon^2 T_0}, \quad \eta_m = \frac{1}{\varepsilon^2},$$
$$Z_T = \frac{T_0}{Z_{m\infty}}\frac{\partial Z_{m\infty}}{\partial T}\Big|_0 \approx \frac{U_1 - U_a}{T_e},$$
$$P_T = \frac{T_0}{Z_{0m}}\frac{\partial Z_{0m}}{\partial T}\Big|_0 - Z_T \approx \frac{2U_a - U_1}{T_e},$$

where T_g is the gas temperature. The parameter $P_n = (n_0/Z_{0m})\partial Z_{0m}/\partial n\,|_0$ depends on the ratio of the frequency of elastic electron–electron and electron–atom collisions. If electron–electron collisions are frequent (if the current is high), then $P_n \sim 1$. If, on the other hand, the frequency of electron–electron collisions is low (and the current is low), then $P_n \to 0$. At intermediate currents, $0 < P_n < 1$.

At high currents the density of metastable atoms manages to keep up with changes in the electron density, and M can be eliminated from Eqs. (7). Ionization waves of only one type (the type studied by Wojaczek) are excited in this region, and we will refer to these waves as waves of type n (this type of striation apparently corresponds to type s in Pekarek's classification). At intermediate currents, the effective lifetime of the metastable atoms is longer than or comparable to the electron lifetime, and yet another type of ionization wave may arise. This type is excited by a phase shift between the oscillations of the metastable atoms and the electrons. This type of wave, which we will call

[3)] In estimating the parameters Z_T and P_T we used data graciously furnished by L. D. Tsendin.

type m, can also be excited at low currents. Tsendin[59] has shown that at low currents ($\tau_{m\ eff} \sim \tau_m$) the wave of type m should grow, while that of type n should be damped. At high currents ($\tau_{m\ eff} \ll \tau_n$) the wave of type m should be damped. (Damping of the type-n wave at low currents was also mentioned back in Nedospasov's review.)[4] Tsendin studied the m and n types separately; this is a legitimate procedure at low and high currents, where these waves have very different frequencies (although the validity of the hydrodynamic equations is doubtful at low currents).

Pekarek *et al.*[142] studied the stability of waves of both types at intermediate and high currents using a numerical solution of the dispersion relation taking metastable atoms into account. At intermediate currents they used the Druyvesteyn equation for the electron energy in calculating the kinetic coefficients, but they ignored the effect of the distortion of the tail of the distribution function on the rates at which electrons and metastable atoms appear and disappear. The use of the Druyvesteyn equation led to an additional term in (7), representing the energy transport along the electron density gradient. (In the Maxwellian case, the coefficient of this term vanishes.) This additional term has a significant effect on the growth of the wave of type n, so that the dependence of P on the electron density n becomes inconsequential. The instability of the m wave at intermediate currents is due primarily to stepwise ionization, and its growth rate is less sensitive to variations in the kinetic coefficients.

Ohe and Takeda[143] carried out an experimental study of the dispersion characteristics for waves of two types in the striation-free region by means of stimulated excitation. The experimental results were compared with theory through a numerical solution of the dispersion relation corresponding to system (9). An important and typical methodological error in the theoretical part of their work was that real values of the wave numbers were substituted into the dispersion relation, and complex values were found for the frequencies—just the opposite of the situation experimentally, where a study was made of stimulated striations, which were excited by an external force which was sinusoidal in time. For moving striations, the dependence of the spatial growth rate on the wavelength is typically a resonant dependence. In the approach mentioned above, Ohe and Takeda found the wavelength dependence of the temporal growth rate. In general, the maxima of these curves may correspond to different wavelengths; in fact, one curve may not have a maximum at all, while the other does (this was apparently the case for the p wave in Ref. 143).

All these papers dealt with only limited current intervals; the instability of the positive column upon a continuous variation of the current was not studied. In particular, the existence of the striation-free region observed experimentally at intermediate currents in narrow discharge tubes[33,67] was not covered adequately.

By noting that the parameter ε is small (correspondingly, the tubes are narrow) we can make an approximate study of the striation excitation conditions over the entire current range and find an explanation for the existence of the striation-free region. Here we will simply summarize the results of such a study, which was reported in Refs. 91, 92, and 134–136.

The solution of Eqs. (9) is the sum of four waves and an in-phase component which results from the modulation of the discharge current. The wave numbers of these waves are the roots of the corresponding fourth-degree dispersion relation.

To determine the conditions for spontaneous excitation of striations we must find the wave numbers and use the boundary conditions. We find a characteristic equation for the complex frequency p. The condition for spontaneous excitation of striations is equivalent to the inequality $\mathrm{Re}\ p \geq 0$. Analysis of the dispersion relation shows that one of its roots has a small real part and a large imaginary part at a purely imaginary value of p (of the order of the striation frequency). This root corresponds to the fundamental ionization wave (the striation proper).

The real and imaginary parts of the second root are comparable to the real part of the first. The wave corresponding to this root was called the "oppositely directed ionization wave" in Refs. 91 and 92. The waves corresponding to the two other roots of the dispersion relation are damped over a distance much shorter than the length of the fundamental ionization wave and can thus be incorporated through the boundary conditions.

We denote the wave number of the fundamental (forward) wave by $k = \psi + i\varkappa$, while that of the oppositely directed wave is $k_1 = \psi_1 + i\varkappa_1$, where ψ is the spatial growth rate of the forward wave, and ψ_1 is that of the oppositely directed wave. The rate ψ is a resonant function of $\varkappa$, reaching a maximum at a certain value of $\varkappa$ which corresponds to the striation wavelength.

The conditions at the boundaries of the positive column can be found by examining the electron and ion currents and the heat flux across the electrode sheath by analogy with Refs. 144 and 145. At small values of N and U, the boundary conditions at the anode and cathode are similar and can be written

$$N + \beta_{1,2} \frac{\partial N}{\partial \xi} = 0, \quad U + \alpha_{1,2} \frac{\partial U}{\partial \xi} = 0, \tag{10}$$

where subscript 1 refers to the cathode and 2 to the anode. It follows from our estimates that the particular form of the boundary conditions is not of fundamental importance if the discharge tube is long enough. It is sufficient that the wave absorption at the boundaries be slight.

Using (10), we can find the eigenvalues $\varkappa_m$ and the self-excitation condition, $\psi \geq \psi_0(\psi_1, V_1/T_0, \beta_1, \beta_2)$, from the characteristic equation. We note that the condition for spontaneous excitation of striations is not the same as the condition for their spatial growth ($\psi > 0$), because of the wave reflection at the boundaries.

Calculations show that the limiting value of the striation growth rate, ψ_0, increases with decreasing length of the positive column (l), with increasing damping of the oppositely directed wave (ψ_1), and with increasing

ratio $R_1 l/R_0 = V_1/T_0$. At $V_1/T_0 \gg e^{\psi_1 l}$ (corresponding to the use of a current source as a power supply for the discharge) and with $\beta_1 = \beta_2 = 0$, the limiting value of ψ approaches ψ_1; i.e., the growth of the forward wave should offset the damping of the oppositely directed wave. In this case the feedback is internal, provided by the oppositely directed wave. If a voltage source is used as the power supply (if V_1 is small), then the reflection from the boundaries is slight; the feedback occurs primarily through the external circuit through the modulation of the discharge current; and the threshold value of ψ_0 is small.

At high currents, at which the density of metastable atoms manages to keep up with the changes in the electron density, the dependence of ψ_0 on V_1/T_0 is as shown in Fig. 20 we see that as V_1/T_0 decreases the quantity ψ_0 approaches a limiting value $\psi_{0\min} = (1/l)\ln[\varkappa_1 l\sqrt{1+(\varkappa_1^2/2)}]$, where $\varkappa_1 = h_T/\gamma\varkappa$; as V_1/T_0 increases, it approaches $\psi_1 = 2$. The ratio V_1/T_0 depends on the current and is usually of the order of $10-10^3$.

The relationship between the amplitudes of the forward and oppositely directed waves and the in-phase component also depends on the ratio $V_1/T_0 e^{\psi_1 l}$. If $V_1/T_0 \ll e^{\psi_1 l}$, then the amplitude of the oppositely directed wave is small in comparison with that of the forward wave and the in-phase component. In this case the shape of the density oscillations is as described in (4); it follows from boundary conditions (10) that $C/A = |1+\beta_1 k_1|$. If C is to be greater than A, as it is experimentally, we must have $\beta_1 \neq 0$. If $\exp(\psi_1 l) \leq V_1/T_0$, then the amplitude of the oppositely directed wave can be comparable to that of the forward wave and the in-phase component. Then the shape of the density oscillation is described by (5).

To find the values of $\varkappa$, $\varkappa_1$, ψ, and ψ_1 from the dispersion relation we can make use of the small parameter ε. It turns out that in the striation-excitation region the root corresponding to the forward ionization wave is large ($\sim\varepsilon^{-0.5}$), while the second root, which corresponds to the oppositely directed wave, is of order unity. The growth rate of the forward wave, $\varkappa$, depends on the frequency $\omega = \mathrm{Im}\, p$ and can have two maxima for certain values of the parameters. One maximum is at low frequencies, $\omega \sim \varepsilon^{-1}$, and one is at high frequencies, $\omega \sim \varepsilon^{-1.5}$. In the first region, the dispersion law and the expression for ψ are

$$\gamma\omega\varkappa^3 = JP_T\eta_m, \tag{11}$$

$$\psi = \frac{1}{3}\left(\frac{q+JP_n}{JP_T}\gamma\varkappa^2 - \frac{\gamma}{P_T\eta_m}\varkappa^4 - \frac{\tilde{\eta}_T}{\gamma\varkappa^2}\right). \tag{12}$$

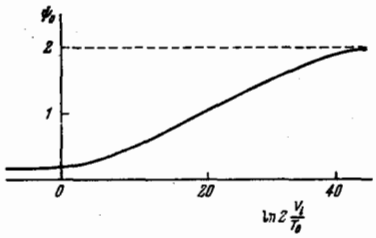

FIG. 20. Dependence of the threshold spatial growth rate on V_1/T_0.

It can be seen from (12) that ψ depends on $\varkappa$ in a resonant manner, reaching a maximum $\psi_{\max}$ at some $\varkappa = \varkappa_0$. It is this value which determines the striation wavelength: $\lambda_0 = 2\pi/\varkappa_0$. Expanding (11) around the value $\varkappa = \varkappa_0$, we find an approximate (linear) equation for the dispersion law:

$$\omega = \frac{3JP_T\eta_m}{2\pi\gamma\varkappa_0^3}\left(\lambda - \frac{2}{3}\lambda_0\right).$$

It follows that the striation group velocity should be three times the phase velocity in this case and in the opposite direction.

The values found for ψ, $\varkappa$, and ω correspond to striations of type m. The maximum value of ψ for striations of this type falls off with increasing current, so that $\psi_{\max}$ becomes negative at a certain current $J = J_m$. It can be shown that $J_m > 0$ if P_n is sufficiently small $[P_n \leq 3\,(P_T\tilde{\eta}_T/4\gamma^2\eta_m^2)^{1/3}]$. Under this condition, however, J_m increases with increasing P_n.

In the second region (for striations of type n) the dispersion law and the expression for ψ are

$$\gamma\omega\varkappa = \tilde{\eta}_T \quad \left(\text{i.e., } \omega = \frac{\eta_T}{2\pi\gamma}\lambda\right), \tag{13}$$

$$\psi = \frac{\gamma JP_T\eta_m}{\tilde{\eta}_T^2}\varkappa^2 - \frac{\gamma}{\tilde{\eta}_T}\varkappa^4 - 2 + \frac{\alpha}{\gamma}. \tag{14}$$

The maximum value of ψ is reached at $\varkappa = \varkappa_0 = \sqrt{JP_T\eta_m/2\tilde{\eta}_T}$, and the condition for excitation of striations becomes satisfied at

$$J \geqslant J_n = \frac{2\tilde{\eta}_T}{P_T\eta_m}\sqrt{\left(2 - \frac{\alpha}{\gamma} + \psi_0\right)\tilde{\eta}_T/\gamma}.$$

We see that the parameter P_n does not affect the excitation of striations of type n at intermediate currents; P_T plays the major role. The group velocity of striations of type n should be equal in order of magnitude to the phase velocity and opposite in direction.

As an example, Fig. 21 shows the J dependence of $\psi_{\max}$, $\varkappa$, and ω for striations of types m and n for the following parameter values: $\varepsilon = 0, 1, Z_T = 2, P_T = 3, q = 1$, $P_n = 0.32$. We see that for this particular case there is a striation-free region whose width depends on ψ_0 and thus on the length of the column. Experiments with helium–neon mixtures reveal a behavior of the growth rate ψ in the striation-free region (Fig. 11) in qualitative agreement with Fig. 21. In these experiments, however, no discontinuities were observed in the frequency or wavelength in the striation-free region, apparently because the maxima of the growth rate ψ are

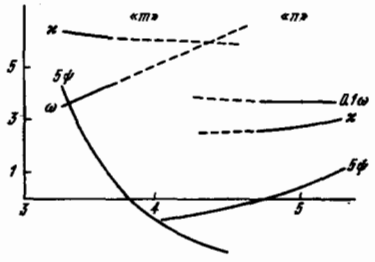

FIG. 21. Dependence of the frequency ω, the wave number $\varkappa$, and the spatial growth rate ψ on the quantity J, which is proportional to the discharge current.[134]

close together (or there is only a single maximum), and the different types of striations are indistinguishable on the basis of their frequencies and wave numbers. This could be the case if the parameter ε were not small. Unfortunately, this case is not amenable to analytic calculations.

As the parameter P_n increases, the striation-free region contracts and eventually disappears. As a result, the excitation of both types of striations becomes possible in a certain range of currents. Which type is actually excited depends on the "past history" of the system, so a hysteresis should be observed as the current is varied up and down. This picture corresponds to the experimental data reported by Zaĭtsev if we identify the striations of type m as type p and those of type n as type s. In the region of the transition from one type to another there should be discontinuities in the frequency and the wave number, according to the calculations. This prediction again agrees with experiments by the Zaitsev group[31] for values $pR \sim 1$ torr·cm (Fig. 22).

These results explain the empirical Novak rule: that the potential drop over the length of the striation remains constant, $U_\lambda = E_0\lambda = 2\pi T_0/\varkappa = \text{const}$. For striations of type m (Fig. 21) the growth rate $\varkappa$ decreases with increasing current. Since T_0 also falls off with increasing current, the value of U_λ should remain approximately constant. For striations of type n, on the other hand, the growth rate $\varkappa$ decreases abruptly by a factor of about two and then increases slightly with increasing current. The value of U_λ should decrease slightly, as is confirmed by the data of Ref. 34.

According to the data of Ref. 31, the transition from one type of striation to another occurs at $i/R = 3\cdot 10^{-2}$ A/cm for $pR = 1$ torr·cm. This value of the current lies below the range of applicability of the hydrodynamic approximation as specified in Subsection 2a (the threshold current for $pR = 1$ torr·cm is $i/R \simeq 5.6\cdot 10^{-2}$ A/cm). As will be seen in the data discussed below, however, there is a fairly good quantitative correspondence between the results of the theory outlined above and the experimental results.

In the comparison of theory with experiment, the kinetic coefficients were taken for a Maxwellian electron energy distribution ($\gamma = 1$). The values of T_0 and E/p were determined from Ref. 147; data on the electron and ion mobility were taken from Refs. 148 and 149; and data on the density of metastable atoms, m_0, and the temperature T_g in Ne were taken from Ref. 150 for approximately the same discharge conditions. The greatest difficulty was in evaluating the parameter P_n. On the other hand, this parameter has only a very slight effect on the wavelengths and frequencies of striations of type m and essentially no effect at all on the corresponding properties of striations of type n. We accordingly adopted the arbitrary value $P_n = 0.5$.

The results calculated from Eqs. (11) and (13) for the frequencies and wavelengths of striations of types m and n in Ne for $pR = 1$ torr·cm are listed in Table I. The wave number was determined from the condition for a maximum of the corresponding spatial growth rate. Also shown in this table are the phase velocities and characteristic potentials and the corresponding experimental values from Ref. 31. We see that the discrepancy between the theoretical and experimental values is less than 20% in all cases, and this discrepancy lies within the error of the equations.

At high currents, the dispersion law, the spatial growth rate, and the optimum wave number (that corresponding to the maximum value of ψ) are determined from[91,92]

$$\gamma\varkappa\omega = \eta_T, \quad \psi = \frac{\gamma\varkappa^2}{\eta_T}\left(\eta_n - \varkappa^2 - 2\left(1 - \frac{\alpha}{2\gamma}\right)\frac{\eta_T}{\gamma\varkappa^2}\right),$$

$$\varkappa_0 = \left[\eta_T\left(1 - \frac{\alpha}{2\gamma} + \psi_0\right)\frac{1}{\gamma}\right]^{1/4},$$

where $\eta_n = P_n\eta_m$, $\eta_T = P_T\eta_m + \tilde{\eta}_T$. The characteristics of the striations in this current range are well known,[4,42,59] and we will discuss them no further.

From the dispersion relation corresponding to system (9) we can also determine the wave number and spatial growth rate of the oppositely directed wave. It follows from the general expressions for these properties that at high currents we would have $\psi_1 = 2$, $\varkappa_1 = \omega h_T/\eta_T = h_T/\gamma\varkappa$. At intermediate currents the quantity ψ_1 reaches a minimum and can be much smaller than two (even negative). In the case of Fig. 6, the value of ψ_1 is apparently quite small, so that it is possible to observe the oppositely directed wave.[4)] If $\psi_1 < 0$, then the limiting value of the growth rate ψ for the direct wave may also be negative. This situation may explain the excitation of striations in helium which are damped in the direction toward the anode.[67]

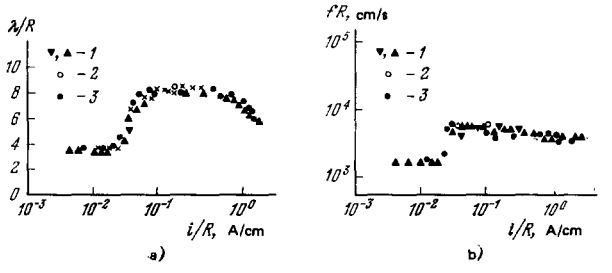

FIG. 22. Dependence of the wavelength λ and the frequency f of the striations on the discharge current in neon.[31] R (cm) = 2.6 (1); 1.5 (2); 0.75 (3).

TABLE I.

Type of striation		$\frac{i}{R}$, A/cm	Rf, cm/s	$\frac{\lambda}{R}$	v_{ph}, m/s	U_λ, V
m	Theory	$2.5\cdot 10^{-2}$	$1.6\cdot 10^3$	2.8	45	7.3
	Experiment		$1.6\cdot 10^3$	3.5	56	9.1
n	Theory		$8.3\cdot 10^3$	5.7	472	15
	Experiment	$3.5\cdot 10^{-2}$	$7.0\cdot 10^3$	7	490	18.3

4) Attempts have been made[151] to observe experimentally the oppositely directed wave at high currents.

c) Some problems in the nonlinear theory of moving striations

The nonlinear theory of striations is still in a primitive state, with extremely few published papers. A few nonlinear effects for moving striations were discussed in Refs. 51–54, 132, and 152–154. Several aspects of the behavior of large-amplitude striations were first explained by Nedospasov,[132] on the basis of a semiphenomenological theory for ionization–diffusion relaxation oscillations. It was shown there that the head of the striation, with the potential drop and the sharp spike in the electron density, results from elementary processes at the end of the preceding striation. Gorelik and Tsendin[153] worked in the hydrodynamic approximation using the Whitham method[155] to find the shape of the saturation ionization waves under growth conditions. Since the wave reflection from the boundaries of the positive column was ignored in those papers, the solution may be quite different from that for excitation conditions. This difference is quite familiar in, for example, microwave electronics.[156]

In a series of papers, Grabec *et al.*[52-54] described numerical simulations based on Pekarek's nonlinear model. Although these simulations are definitely of interest for the general theory of nonlinear waves, they are not particularly pertinent to ionization waves. In the first place, periodic boundary conditions are used, without adequate justification, for the density and temperature perturbations; these boundary conditions do not correspond to the boundary conditions at the ends of the positive column. Second, in this model the conditions for the excitation of waves begin to be satisfied for the lowest-order mode, again in contradiction to the experimental data.

Kogan and Fisun[154] studied the stabilization of an ionization wave which results from a decay involving the formation of two ionization waves with lower frequencies and wave numbers. Although these results are quite interesting, they have apparently not yet been confirmed experimentally.

Several nonlinear problems were solved in Refs. 133 and 135 by the asymptotic method of Ref. 157, which is applicable slightly above the excitation threshold. This method can be used to study the steady-state excitation of striations and the asynchronous-suppression effect.

Nonlinear terms must be taken into account in Eqs. (7) and (8) in order to carry out calculations for the steady-state excitation of moving striations. Corresponding calculations have been carried out for high currents, at which the density of metastable atoms manages to keep up with the changes in the electron density. In this case system (7) can be written as follows, where we are using the dimensionless variables introduced above:

$$\begin{aligned} &\frac{\partial N}{\partial \tau}-\eta_T U=\frac{\partial^2 N}{\partial \xi^2}+\eta_n N+f_1,\\ &\gamma\frac{\partial^2 U}{\partial \xi^2}+\frac{\partial N}{\partial \xi}=\alpha\frac{\partial U}{\partial \xi}+2(N-\tilde{j})+h_T+f_2. \end{aligned} \tag{15}$$

Here f_1 and f_2 are nonlinear functions of $N, U, \tilde{j}$, and their spatial derivatives. Conservative linear terms have been put on the left side of (15), while the nonlinear and nonconservative terms are on the right. The left side of system (15) agrees in the linear approximation with the generating system in Ref. 153. At small wave amplitudes, at which the excitation conditions are approximately single-mode conditions, the right sides of (15) are small in comparison with the left sides. Then a solution can be sought in the form of the sum of a traveling wave and an in-phase component (which includes the reflected wave) with slowly varying amplitudes:

$$N = A(\xi, \tau)\, e^{i(\omega\tilde{\tau}+\varkappa\xi)} + B(\xi, \tau)\, e^{i\omega\tau} + \text{c.c.}$$

Setting $\tilde{j} = C(\tau)e^{i\omega\tau}$ and using the asymptotic method, we can derive truncated equations for the amplitudes A and B (Ref. 133). The relationships among the amplitudes A, B, and C are determined by Ohm's law and the boundary conditions. From the solution of the truncated equations we can find the steady-state striation amplitude and the nonlinear correction to the wave frequency. Under the boundary conditions $N(0) = N(l) = 0$ the striation amplitude near the anode is given by the following expression for conditions near the excitation threshold:

$$|A(l)| = \sqrt{\frac{2\psi_0 l\,(\psi-\psi_0)}{\mu(\psi_0)}};$$

where $\mu(\psi_0)$, the effective saturation coefficient for the forward wave, is a function of the threshold value of the striation growth rate, ψ_0, and the nonlinearity parameters. Figure 23 shows the dependence of $\mu(\psi_0)$ on ψ_0 for certain fixed values of the nonlinearity parameters. We see that $\mu(\psi_0)$ increases with increasing ψ_0 and thus with increasing R_1. At small values of R_1 the quantity $\mu(\psi_0)$ may be negative; this situation corresponds to "hard" striation excitation. Experiments show that for a helium–neon mixture the excitation of striations is hard on both sides of the striation-free region. This can be seen from the hysteresis loop in Fig. 24. As the ballast resistance (R_1) increases, the hysteresis loop becomes narrower, implying that soft excitation conditions are being approached.

The truncated equations for the wave amplitudes have been used to solve the problem of the asynchronous suppression of striations through a modulation of the discharge current. At small values of the difference $\psi - \psi_0$ (slightly above the threshold) the minimum modulation index of the discharge current is given by[133,135]

$$C_t^2 = (\psi-\psi_0)\,\frac{1+(a\omega_1^2/\omega^2)}{b+(c\omega_1^2/\omega^2)};$$

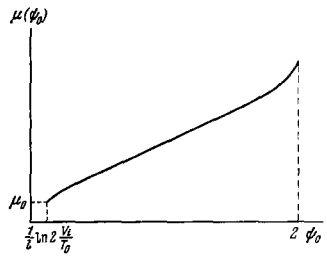

FIG. 23. Dependence of the effective striation saturation coefficent $\mu(\psi_0)$ on the threshold spatial growth rate ψ_0 (Ref. 133).

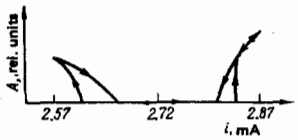

FIG. 24. Dependence of the striation amplitude on the discharge current near the striation-free region for a helium–neon mixture. $\alpha = 10$, $pR = 0.4$ torr·cm, $L = 14$ cm, $R_1 = 277$ kΩ.

where a, b, and c ($a \sim 1, b \sim \varepsilon c$) are coefficients determined by the nonlinearity parameters. Figure 25 illustrates the dependence of C_t on the ratio of the modulation frequency, ω_1, to the natural striation frequency, ω. The dashed part of the curve is the part near the resonance, where this calculation is not applicable (in this region the current modulation leads to stimulated striations, which are not taken into account in the calculations). The ω_1 dependence of C_t is in qualitative agreement with experiment (Fig. 12).

Ohe and Takeda[51] have also attempted to explain the asynchronous suppression of striations, using the Pekarek model and an analogy with the classical problem of the asynchronous suppression of self-excited oscillation in a Van der Pol oscillator.[95] This analogy, however, is strained, if simply because the striations are described by an equation which is of first degree in the time, rather than of second degree as in a Van der Pol oscillator.

d) Stationary striations in molecular and inert gases

While the linear theory gives a rather good explanation of the growth and dispersion properties of moving striations, the situation is much more complicated in the case of stationary striations. The growth rate is not resonant in nature in the region in which the $\omega(k)$ curve intersects the wave number (k) axis. For this reason, those "possible values of the curve of the temporal growth rate for the case of undamped stationary striations" which are shown by Fig. 25 in Pekarek's review[5] are actually not possible. The only way to explain the existence of stationary striations is to appeal to a nonlinear theory. Nevertheless, attempts have been made to extract stationary striations from the dispersion relations. Nedospasov,[120] for example, attempted to explain the existence of stationary striations on the basis of the thermodiffusion term in the ambipolar diffusion equation, since the $\omega(k)$ curve intersects the wave number axis when this term is taken into account.

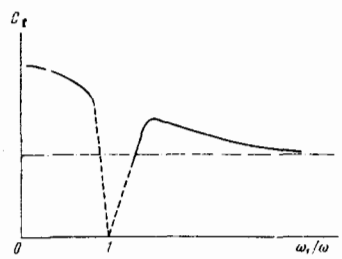

FIG. 25. Dependence of the threshold modulation amplitude C_t on the ratio of the modulation frequency ω, to the natural striation frequency ω (Ref. 133). The dashed part of the curve shows the region near the resonance.

Lee and Garscadden[49,50] derived stationary striations as natural solutions of the Pekarek equations[5] with specified boundary conditions. The spatial damping rate for the stationary striations which they calculated, however, turned out to be much larger than that observed experimentally under corresponding discharge conditions. The apparent reason for the discrepancy lies in the original model, which, like the early papers by Pekarek, ignores the nonlinear dependence of the ionization rate on the electron density.

Chapnik[158] attempted a nonlinear study of stationary striations by using an analog computer to solve a nonlinear equation for the electron temperature, ignoring the spatial variation of the density. This is an extremely crude approximation, which is at odds with experimental results (e.g., those of Refs. 108 and 113), the relative changes in the density are much larger than those in the temperature over the length of a striation. According to Ref. 113, for example, the electron density varies by an order of magnitude, while the temperature varies by a factor of only 1.5.

Calculations for stationary striations can be carried out from the same hydrodynamic model as was used for moving striations [Eqs. (7)]. Because of the considerable difference in parameters, this calculation should be carried out differently for the cases of molecular and inert gases.[137,138]

1) *Stationary striations in molecular gases.* Substantial electric fields arise in molecular gases because of the large inelastic energy loss of electrons. The parameters η_T and η_n are thus much smaller than in inert gases. For nitrogen with $pR = 0.5$ torr.cm and $i/R = 0.01$ A/cm, for example, we have $\eta_T \approx 0.5$. If we assume $\eta_T \lesssim 10, \eta_n \lesssim 1, h_T \sim 10$, then Eqs. (7) can be written as follows in terms of dimensionless variables:

$$\begin{aligned} &\frac{d^2N}{d\xi^2} + \eta_T U + \eta_n N = f_1, \\ &\frac{dN}{d\xi} - 2N - h_T U = f_2, \end{aligned} \tag{16}$$

where f_1 and f_2 are nonlinear functions, and $\eta_n = \eta_m(q/J + P_n)$, $\eta_T = P_T\eta_m + \bar{\eta}_T$. In deriving (16) we eliminated the density of metastable atoms, M, and we discarded the derivatives with respect to time, since they vanish in the case of steady-state excitation of stationary striations. Furthermore, the variation in the discharge current upon the excitation of striations was ignored. In the linear approximation, the solution of (16) is

$$\begin{aligned} N &= Ae^{-\psi\xi}\sin(\varkappa\xi + \varphi_0), \\ U &= A\sqrt{\frac{\varkappa^2 + (2+\psi)^2}{h_T}}\, e^{-\psi\xi}\sin(\varkappa\xi + \varphi_1), \end{aligned} \tag{17}$$

where

$$\psi = \frac{\eta_T}{2h_T}, \quad \varkappa = \sqrt{\eta_n - \frac{2\eta_T}{h_T}\left(1 + \frac{\eta_T}{8h_T}\right)}, \tag{18}$$

φ_0 is the initial phase, determined from the boundary condition at $\xi = 0$, and

$$\varphi_1 - \varphi_0 = -\operatorname{arctg}\frac{\varkappa}{2+\psi} \tag{19}$$

is the phase shift between the variations in the electron density and temperature.

It follows from (18) that the spatial damping rate of the striations, ψ, is smaller, the smaller the ratio η_T/h_T. Furthermore, stationary striations will be observed if $\varkappa/2\pi$ is at least larger than ψ (damping should occur over a distance larger than the length of a striation). We thus have a condition on η_n:

$$\eta_n > \frac{2\eta_T}{h_T}\left(1+\frac{\pi^2\eta_T}{2h_T}\right). \tag{20}$$

Since the parameter η_n falls off with increasing current at intermediate and high currents, condition (20) can be satisfied only in a limited current range. The ratio of the damping rate of the standing striations to the wave number should increase with increasing current.

If $\eta_T/h_T \ll \eta_n$, then the length of the striations is, in dimensional units, $\lambda = 2\pi T_0/E_0\varkappa = 2\pi R/(\sqrt{Z_n}\beta)$, where $Z_n = \tau_{m\ eff}/\tau_m + P_{n'}$. Since Z_n is smaller than or of the order of unity, the length of slightly damped stationary striations in molecular gases must be larger than or of the order of the tube diameter—in agreement with experiment.

As can be seen from (19), the phase shift between U and N may vary from π at $\varkappa \ll 2+\psi$ to approximately $(3/4)\pi$ at $\varkappa = 2+\psi$, depending on the parameters. This conclusion agrees with experiment (see Subsection b2 of Section 1).

Examining the conditions for the spontaneous excitation of stationary striations, we find that they are not resonant in nature and can be satisfied simultaneously for a broad range of wave numbers. It can be shown,[138,139] however, that all wave numbers for which the excitation conditions hold approach a single value because of the strong dependence of the wave number on the striation amplitude. This value is approximately that given by Eq. (18). Because of this dependence, the boundary conditions at $\xi = l$ are also satisfied. Analytic calculations can be carried out for these nonlinear effects under the condition $\psi \ll \varkappa$, with $\psi \lesssim 0.1$ and $\varkappa \sim 1$. In this case the quantity N is small, and the nonlinear functions f_1 and f_2 can be expanded in series. Retaining the quadratic terms of these series, and eliminating U from (16), we find an equation for N:

$$\frac{d^2N}{d\xi^2} + 2\psi\frac{dN}{d\xi} + \varkappa^2 N - \rho N^2 = 0. \tag{21}$$

Equation (21) is analogous to the oscillation equation of a nonlinear oscillator, and its solution is expressed in terms of slightly damped Jacobi elliptic functions.[159] For the boundary conditions $N(0) = N(l)$ we have

$$N(\xi) = \frac{2\theta^2(\xi)k^2(\xi)}{3\rho} \times \left(\operatorname{sn}^2\frac{\theta(\xi)(\xi-\xi_0)}{2} - \operatorname{sn}^2\frac{\theta(\xi)\xi_0}{2}\right), \tag{22}$$

where $k(\xi)$ is the modulus of the elliptic function, which falls off slightly from the cathode toward the anode, $\theta = \varkappa(1+k^4-k^2)^{-1/4}$, $\operatorname{sn}^2[(\xi_0\theta_0)/2] = (1+k^2-\sqrt{1+k^4-k^2})/3k^2$. Near the anode the modulus k can take on one of the eigenvalues

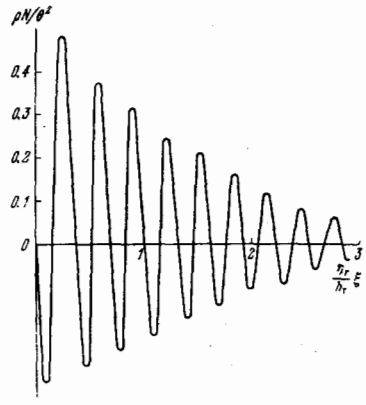

FIG. 26. Shape of the stationary striations calculated from Eq. (22).

$$k_m(l) = \frac{2\sqrt{2}}{15^{1/4}}\left(\frac{\varkappa l}{m\pi} - 1\right),$$

where $m = 1, 2, \ldots, m_0$; $m_0 = \text{entier}\{\varkappa l/\pi\}$ (m_0 is the largest integer in $\varkappa l/\pi$).

We see from (22) and Fig. 26 that the shape of the stationary striations near the cathode may be quite nonsinusoidal; the difference fades toward the anode. The length of the striations should fall off slightly with distance from the cathode. These results agree well with experiment (Fig. 17).

A complete solution of the problem of the existence of stationary striations must await a study of the stability of stability of solution (22). This stability study is a complicated problem, which has yet to be solved. The approach suggested by Yakubovich[160] may prove helpful here.

2) *Stationary striations in inert gases.* The parameters η_T and η_n are large in inert gases, as we mentioned earlier, and in a certain current range we can set $\eta_T \sim \varepsilon^{-3}$, $\eta_n \sim \varepsilon^{-2}$, $h_T \sim \varepsilon^{-1}$. Ignoring the small terms in (15), and setting $\partial N/\partial_T = 0$, we find

$$\frac{d^2N}{d\xi^2} + \eta_T U + \eta_n N = f_1,$$
$$\frac{dN}{d\xi} + \gamma\frac{d^2U}{d\xi^2} = f_2.$$

In the linear approximation,

$$N(\xi) = Ae^{-\psi\xi}\sin(\varkappa\xi + \varphi_0) + B + Ce^{2\psi\xi}, \tag{23}$$

where

$$\psi = \frac{\varepsilon_1 - \varepsilon_2}{2}, \quad \varkappa = \frac{\sqrt{3}}{2}(\varepsilon_1 + \varepsilon_2),$$
$$\varepsilon_{1,2} = \left[\left(\frac{\eta_T}{4\gamma^2} + \frac{\eta_n^3}{27}\right)^{1/2} \pm \frac{\eta_T}{2\gamma}\right]^{1/3}.$$

The first term in (23) describes the striations proper; the second and third determine the aperiodic change in the electron density over the length of the column. The quantity C must be negligibly small in comparison with A and B in order to satisfy the boundary conditions at the anode. From the expressions for $\varkappa$ and ψ we see that $\psi < \varkappa$; the ratio $\psi/\varkappa$ is smaller, the larger η_n. At $\eta_n \ll \eta_T^{2/3}$ ($\eta_n \lesssim \varepsilon^{-1}$) the ratio $\psi/\varkappa$ tends toward $1/\sqrt{3}$. As we mentioned earlier, the striations can be observed if $\psi/\varkappa < 1/2\pi$. This condition imposes a restriction on η_n. Since η_n decreases with increasing current at interme-

diate and high currents, the ratio $\psi/\varkappa$ should increase with increasing current, and the stationary striations should become unobservable at a certain current. These results agree with the experiments of Zaĭtsev and Savchenko (Fig. 16).

The length of the striations in dimensional units is

$$\lambda=\frac{2\pi T}{\varkappa E}\sim\begin{cases}\dfrac{2\pi R}{\beta\sqrt{Z_n}} & \text{for } \eta_n\gg\eta_T^{2/3},\\ \dfrac{2\pi R}{\sqrt{3}}\left[\dfrac{\varepsilon}{R}(P_T+Z_T-1)\right]^{-1/3} & \text{for } \eta_n\ll\eta_T^{2/3}.\end{cases}$$

We see that in the first case, in which the striations are damped relatively slightly (the damping occurs over several wavelengths), their length must be greater than or of the order of the tube diameter. In the second case, on the other hand, with strong damping, the striation length may be several times the diameter at small ε. The addition of an impurity to an inert gas increases the electric field, so that the parameter ε increases, and the length of the striations should decrease.[114,161]

The amplitude and shape of stationary striations in both inert and molecular gases are found from a solution of the nonlinear equation for the density and the boundary conditions at $\xi=l$. For inert gases, however, this equation is more complicated than it is for molecular gases.

CONCLUSIONS

The ionization waves which have been the subject of this review are simply one example of a stratification phenomenon which is quite common in nature: stratification of homogeneous media. The stratification may result from quite different factors, and even in gas discharges the stratification effects may be quite different in physical nature. Superficial similarities have led to the use of the term "striation" to refer to a variety of effects in gas discharges: ionization waves, caused by the ionization of gas atoms; electrothermic waves, which arise in crossed electric and magnetic fields[4,162-181]; waves in electronegative gases, caused by a dependence of the attachment frequency on the electron temperature[182-190]; etc. In order to describe these different types of waves it is necessary to consider different physical processes in the plasma, so that different original models are used. From the standpoint of oscillation theory, on the other hand, all these phenomena are self-excited oscillations in distributed systems, so that they can all be studied by a common approach.

As has been mentioned repeatedly in the literature, ionization waves constitute a special class of waves, with a peculiar dispersion law. The pecularity of the dispersion law leads in turn to peculiar laws for the reflection and refraction of these waves at interfaces; specifically, the phase velocity of the reflected wave is in the same direction as the incident wave, while the energy is moving in the opposite direction.

Ionization waves (striations) may be either moving or standing waves. It follows from the results of this review that the waves are of the same physical nature in the two cases and are caused by the same physical processes.

It can be concluded from this discussion that the hydrodynamic model gives a qualitatively, and frequently quantitatively, correct description of both the linear and nonlinear properties of moving and stationary striations at intermediate and high currents. It is apparently necessary to use the kinetic theory only at low currents and well above the excitation threshold, at least for a qualitative study of striations.

We have not touched on the excitation of striations under more complicated discharge conditions, e.g., in a contracted column, an rf discharge,[116,123,191] or a magnetic field.[161,192-204] Under these conditions there are several specific effects which deserve special study.

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