Gluons

Ya. I. Azimov, Yu. L. Dokshitser, and V. A. Khoze

B. P. Konstantinov Leningrad Institute of Nuclear Physics, Academy of Sciences of the USSR
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Contemporary ideas about the gluon—the carrier of the strong interaction—are reviewed. Its manifestations in various reactions and the prospects for studying its properties and interactions are considered. The results of the first experiments on direct observation of hadron jets from the dissociation of the gluon, and also future experiments with such jets, are discussed.

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CONTENTS

INTRODUCTION

The 1970's have been marked by a revolution in elementary-particle physics. Experiments of recent years have convincingly demonstrated that the strongly interacting particles—the hadrons—are by no means elementary. The fundamental objects now appear to be quarks and leptons, the physics of which has emerged as an independent field of knowledge, just as nuclear theory previously emerged from atomic theory, and elementary-particle physcis later emerged from nuclear theory.

What do we know about interactions at the quark–lepton level or about the nature of the fields—the carriers of these interactions?

The electromagnetic field and its quantum—the photon—are well known and are described by good old quantum electrodynamics (QED); the weak interaction of quarks and leptons has been adequately studied (the theory of Glashow, Weinberg, and Salam), and the detection of its carriers—the Z^0 and $W^\pm$ bosons with masses of order 80–90 GeV—is planned for 1981–1983.

Quantum chromodynamics (QCD) is now the only real candidate for the esteemed role of a theory of the strong interaction. Last year, 1979, will evidently go down in the history of physics as the year of the discovery of the gluon—a fundamental object of QCD and the carrier of the strong interaction between quarks. The first successes in the hunt for the gluon in colliding e^+e^- beams were reported in the summer of 1979 at international conferences at Geneva and Batavia, and also in the popular press.

The present review is devoted to the problems of detecting the gluon and studying its properties and interactions.

2. GLUONS AND QUANTUM CHROMODYNAMICS

a) The gluon and the quarks

As is well known, the concept of a quark was developed in three basic stages.

1. Quarks as constituent elements of hadrons: a quark and an antiquark in a meson, and three quarks in a baryon.[1] These are valence quarks, which determine the systematics of the hadrons and many of their low-energy properties.

2. Quarks as point objects—partons, which manifest themselves in so-called hard processes at high energies, for example, in deep inelastic scattering of e, μ, or ν ($\bar{\nu}$) by the nucleon.[2]

3. Quarks as sources of hadron jets. Hadron jets show up most clearly in e^+e^- annihilation.[3-5]

These same stages have occurred in the development of ideas about gluons.

1. The quarks in a hadron must be "glued together" by some field, whose quantum has therefore been designated the gluon.

2. The first experiments on deep inelastic scattering already showed that the parton content of a nucleon is not exhausted by the quarks and antiquarks; there must be partons of another type, these being sterile with respect to the electromagnetic and weak interactions. The simplest possibility was to identify them with the same gluons.

3. If the strong interaction actually has a field character, the quarks should emit gluons in various processes, just as electrons emit photons.

It is natural to expect that these gluons, like quarks,

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produce jets of hadrons.

Chromodynamics in all these roles—the gluon as "glue," the gluon as a parton, and as the universal carrier of the strong interaction—nominates itself as a candidate, namely, a set of eight massless vector fields which interact with the quarks and with each other in a definite way.

b) Colored gluons

We shall briefly discuss the structure of the strong interaction of quarks and gluons in the framework of QCD (a detailed description of this theory can be found in the reviews of Refs. 6–8).

Five types (or, as we say, "flavors") of quarks q are now known: the three "old" light quarks u, d, and s with electric charges $e_u = 2/3$ and $e_{d,s} = -1/3$, and the two new heavy quarks c and b ($e = 2/3$ and $e_b = -1/3$) with masses of order 1.5 and 5 GeV, respectively. (We hope that the reader is already accustomed to the fractional quark charges.) The success of the unified theory of electromagnetic and weak interactions (the theory of Glashow, Weinberg, and Salam)[9] enables theoreticians to predict the discovery of at least one more quark—the t quark with $e_t = 2/3$ (as regards the mass of the t quark, it is now known that it is not less than 18 GeV).

For the construction of a quantum field theory of the gluon interaction, it is important that the quark of each flavor is characterized by a new quantum number, i.e., it can occur in three states, which are conventionally designated by colors, for example, yellow, blue, and red. (For further details about the history of color and related questions, see Refs. 7, 8, and 10.) This arrangement of the quark states has been reliably established experimentally. For example, one of the most striking "miracles" of color is the agreement with experiment of the expected value of the cross section for the process $e^+e^- \to$ hadrons (for colorless fractionally charged quarks, the expected cross section is smaller by a factor of three).

QCD treats quark color as a dynamical degree of freedom and is constructed in close analogy with QED. An attractive hypothesis is the assumption that the group SU_3' of color transformations has an exact symmetry, i.e., the physical properties of a quark in different color states are identical with respect to arbitrary interactions.

Just as the arbitrariness in the choice of the phase of the wave function of a charged particle is intimately related to the electromagnetic field, the requirement of local color symmetry of the theory gives rise to an octet of vector gluon fields and uniquely specifies the entire structure of the gluon–quark interaction. One finds a gauge field theory of Yang–Mills type[11] with the group SU_3'. This is QCD.

A color interaction of Yang–Mills type was first introduced in 1966 by Nambu[12] in the framework of a model of integrally charged quarks. In this model, the SU_3' symmetry was approximate, since it was already broken by electrodynamics.

QCD in its modern form was formulated at the beginning of the 1970's by Gell-Mann and others.[13] It incorporates the hypothesis that the only possible real physical states are states with hidden color, i.e., colorless (white) hadrons formed from three differently colored quarks or $q\bar{q}$ pairs. This hypothesis, which has made it possible to combine the successes of the quark model of hadrons with the experimental absence of free quarks, remains unproved in QCD. However, it looks as if gluon dynamics does not admit the existence of isolated colored objects. This is indicated by the catastrophic growth of the effective coupling constant g_s of the color interaction with increasing distance between the quarks (the "infrared pole" in perturbation theory).

So far, QCD continues to resist attempts to solve the problem of decoloration (or, as we say, quark containment or confinement), but this circumstance even introduces a definite optimism. It offers hope that a theory whose field Lagrangian can be written on a single line is actually sufficiently fundamental for a future description of the entire diversity of hadron physics.

c) Asymptotic freedom

Fortunately, it is not only the prospects of a future explanation of the phenomenon of decoloration that feed the optimism of QCD enthusiasts at the present time. The reverse side of the infrared problem is the unique property of asymptotic freedom,[14] i.e., a weakening of the color interaction at small distances. This property makes it possible to use ordinary perturbation theory as a basis for calculations of a wide range of phenomena in which a decisive role is played by processes taking place at small space-time intervals.

According to QCD, to lowest order in the color constant g_s, the fundamental interactions of the color fields which form the basis of the strong interactions are represented by the Feynman diagrams of Fig. 1. These are the quark–gluon vertex and the trilinear self-interaction of gluons (a four-gluon vertex also occurs in the next order in g_s^2).

The effects of the higher corrections of perturbation theory, for example, in quark–quark scattering (Fig. 2), lead to a dependence of the effective coupling constant $g_s(r)$ on the distance r between the quarks. When $r \to 0$, we have $g_s^2(r) \sim 1/\ln r^{-1} \to 0$.

This is a manifestation of asymptotic freedom, which, in particular, may be the reason for the success of the early variants of the quark model based on the notion

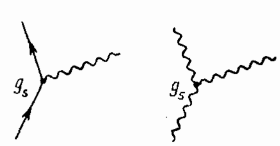

FIG. 1. Fundamental interactions of quarks and gluons in QCD. In the next order in the coupling constant of the color interaction, $\sim g_s^2$, there is also a four-gluon vertex.

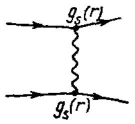

FIG. 2. Effective coupling constant $g_s(r)$ in quark–quark scattering.

that the quarks inside a hadron are practically free.

In connection with the above-mentioned problem of "large distances" and confinement, the construction of quark–gluon dynamics as a rigorous quantum field theory is incomplete. Nevertheless, QCD is already attacking hadron physics on a broad front, using as a bridgehead the asymptotic freedom of the color interaction at small distances. The number of experimental tests of QCD is multiplying.

In recent years, an intensive chromodynamical analysis has been made of various lepton–hadron interactions, such as the above-mentioned deep inelastic scattering, where asymptotic freedom makes it possible to explain the well-known phenomenon of Bjorken scaling on which parton models of hadron structure are based.

QCD provides a description of "strong" (gluon) corrections to the weak interaction; in particular, it gives a qualitative explanation of the famous $\Delta T=1/2$ rule.[15]

The general ideas about the field character of the interaction between quarks at small distances have numerous applications in the theoretical analysis of hadron form factors,[16] in relativistic nuclear physics,[17] and so forth. A psychologically strong argument in favor of a field theory was the observation of the long-awaited transition of the cross section for the production of particles with large transverse momenta in hadronic collisions to the regime $d\sigma/dp_\perp^2\sim p_\perp^4$.[18]

Finally, an excellent proving ground for testing QCD is spectroscopy and the physics of the decays of the new heavy mesons, in particular, the so-called "quarkonia" formed from heavy quarks. (Of these, "charmonium" $c\bar{c}$ has been best studied.[19,20]) Thus, the fact that the lifetimes of the particles of the families ψ ($c\bar{c}$ mesons, with masses $M\sim 3$–4 GeV) and Υ ($b\bar{b}$, with $M\sim 10$–11 GeV) are unusually long for massive hadrons has a natural explanation in terms of the asymptotic freedom of the colored quark–gluon interaction. In all the phenomena enumerated above, the important distances are determined by the small Compton wavelengths of the heavy quarks or are specified by the large energy-momentum transfers in hard processes.

Thus, at distances less than the characteristic scale of the strong interactions, $R_{\rm had}\sim 1\ {\rm GeV}^{-1}$, the "strong interaction" is in essence "weak."

d) "Naive" QCD

But our inability to understand how gluon dynamics ensures the containment of quarks in a hadron does not pass unscathed, even in the analysis of hard processes. We must close our eyes to the problem of decoloration and replace its solution by more or less plausible hypotheses about the stage of transformation of the quarks and gluons into hadrons. This leads to the approach known as "naive" QCD.

For example, it is natural to expect that the total cross section for e^+e^- annihilation into hadrons can be calculated as the cross section for production of "real" quarks and gluons, assuming that at large energy W their subsequent transformation into hadrons does not alter the result:

$$R_{e^+e^-\to{\rm hadrons}}(W)\equiv\frac{\sigma(e^+e^-\to{\rm hadrons})}{\sigma(e^+e^-\to\mu^+\mu^-)}=3\sum_q e_q^2\left(1+\frac{\alpha_s(W^2)}{\pi}+O\left(\frac{\alpha_s}{\pi}\right)^2\right) \quad (1)$$

(the summation is taken over the flavors of the pair of quarks produced from the virtual photon, and $\alpha_s=g_s^2/4\pi$).

The same hypothesis is also usually exploited in the study of the structure functions in deep inelastic scattering of leptons, for the total hadronic decay widths of quarkonia, and so forth.

Particularization of the properties of the final hadronic state requires the introduction of new hypotheses. In particular, dispensing with the ideas of the parton model, it is assumed that a quark emitted from small distances exists in the form of a jet of hadrons with limited transverse momenta $p_\perp$ with respect to the direction of the parent quark and with a characteristic distribution in the longitudinal momenta (see, for example, Ref. 21). The current experimental data are consistent with the notion of jets. They indicate universality of the hadronic fragmentation of a quark of given flavor in various hard processes.

It is to be hoped that the gluons emitted from small distances will also be realized in the form of hadronic jets, whose properties (composition, multiplicity, and hadron distributions) might distinguish them from the jets produced by quarks (see, for example, Refs. 22 and 23).

We shall discuss (by means of simple examples) the situations in which it is possible to use the perturbation-theory calculations of QCD.

Consider first the emission by a virtual quark with energy E_q of a "bremsstrahlung" gluon with energy zE_q and transverse momentum $k_\perp\ll E_q$ with respect to the quark momentum (Fig. 3a). The spectrum of such emission differs from the ordinary photon spectrum only by a factor 4/3 and, of course, by the substitution $\alpha\to\alpha_s=g_s^2/4\pi$:

$$dw^{q\to qg}=\frac{4}{3}\frac{\alpha_s}{\pi}\frac{dk_\perp^2}{k_\perp^2}dz\,\frac{1+(1-z)^2}{2z}. \quad (2)$$

Similarly, we can consider gluon emission by a gluon, which is specifically characteristic of QCD (see Fig. 3b):

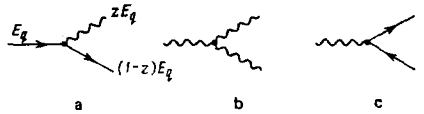

FIG. 3. Elementary "decays" of quarks and gluons.

$$dw^{g\to gg} = 3\frac{\alpha_s}{\pi}\frac{dk_\perp^2}{k_\perp^2}dz\left[z(1-z)+\frac{z}{1-z}+\frac{1-z}{z}\right] \quad (3)$$

and the conversion of a gluon into a quark pair, $g\to q\bar{q}$ (see Fig. 3c):

$$dw^{g\to q\bar{q}} = \frac{1}{2}\frac{\alpha_s}{\pi}\frac{dk_\perp^2}{k_\perp^2}dz\frac{z^2+(1-z)^2}{2}. \quad (4)$$

It is these elementary decays that form the basis of calculations of hard processes in naive QCD.[8]

Allowance for the higher-order corrections of perturbation theory leads to the replacement of the fixed coupling constant α_s in (2)–(4) by $\alpha_s(k_\perp^2)$—the "running" coupling constant[24] (see also Ref. 8). Asymptotic freedom has the consequence that $\alpha_s(k_\perp^2)$ is small at large $k_\perp^2$ and grows with decreasing $k_\perp^2$. Therefore the obvious condition $\alpha_s(k_\perp^2)<1$ for the applicability of perturbation theory gives

$$k_\perp^2 < R_{\text{had}}^{-2}, \quad (5)$$

where R_{had} is some characteristic radius of the strong interaction. For $k_\perp^2 \lesssim R_{\text{had}}^{-2}$, instead of the elementary processes of Fig. 3, there occurs "nonperturbative" production of quarks and gluons, leading to the production of colorless hadrons.

This picture has a perspicuous interpretation in the language of the temporal sequence of events. For example, the time for emission of a gluon by a quark (the quark "lifetime") is determined by its virtuality M_q^2, i.e.,

$$t_{\text{emiss}} \approx \frac{E_q}{M_q^2} = \frac{E_q}{k_\perp^2}z(1-z). \quad (6)$$

It is natural to assume that the transition of quarks and gluons into hadrons is determined by large distances or, more precisely, by the long-wave gluon fields with $\lambda \sim R_{\text{had}}$, which produce an interaction with a large coupling constant $\alpha_s(R_{\text{had}}^{-2})\sim 1$. Then the time for formation of a hadronic jet by a fast quark or gluon (the hadronization time) is approximately equal to the time for formation of long-wave emission, i.e.,

$$t_{\text{had}} \approx ER_{\text{had}}^2 \quad (7)$$

(the linear energy dependence of t_{had} is confirmed by solvable two-dimensional models with "confinement"[25]). By requiring that the elementary decay of Fig. 3 is completed before the hadronization of the decay products, we obtain precisely the condition (5) from the expressions (6) and (7).

It is now easy to understand what is meant by a hard process. For example, in e^+e^- annihilation the characteristic time for production of a $q\bar{q}$ pair falls off with energy:

$$t_{\text{prod}} \approx \frac{1}{E_q}. \quad (8)$$

Therefore, with increasing energy there is an increasingly large gap between the time for production of a pair and the time for its hadronization, i.e., there is a growth of the time interval during which a colored quark propagates as a normal particle and can emit gluons in the same way that an electron emits photons.

Thus, as was hoped at the inception of elementary-particle physics, even in the theory of strong interactions one finds a region—and, incidentally, a very important region—in which the picture of the processes is simple and perspicuous and in which one can make use of clearly formulated computational rules. Therefore it is not surprising that in recent years there has appeared a stream of papers on perturbative QCD, which is continuing to grow. A modern review of these papers can be found in Ref. 26.

e) The value of α_s

For calculations according to QCD, it is obviously necessary to know the "running" coupling constant α_s. By summing the leading logarithmic corrections, this constant is usually parametrized by the approximate formula

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)} \quad (9)$$

where n_f is the number of "unfrozen" quark flavors, i.e., the number of species of quarks whose masses are smaller than the characteristic momentum transfers Q in the process under consideration. Thus, there remains a single parameter Λ. In principle, this parameter can be determined by observing chromodynamical effects in hard processes. However, at currently accessible energy-momentum transfers Q^2, there are important corrections to the formulas of naive QCD, which reflect the physics of large distances (i.e., confinement) and which fall off according to a power law as $Q^2\to\infty$.

At the present time, an approach to the description of a large class of phenomena of charmonium physics, e^+e^- annihilation, the low-lying hadronic states, etc., is being developed on the basis of consistent phenomenological allowance for the power corrections to perturbation theory.[27] This leads to the value $\Lambda\approx 100$ MeV, corresponding to $\alpha_s(m_\psi^2)\approx\alpha_s(10\ \text{GeV}^2)\approx 0.2$.

Respecting tradition, mention should be made of attempts to determine Λ from the so-called violation of scaling in deep inelastic scattering. By analyzing the current data in terms of the explicit formulas of naive QCD (without power corrections), optimists have extracted the parameter value $\Lambda\sim 0.3$–0.7 GeV (for example, Refs. 26, 28, and 29). With $n_f=3$, this gives $\alpha_s\approx 0.3$–0.45 for $Q^2=10\ \text{GeV}^2$.

The apparent discrepancy between the values of Λ is due to the fact that there is currently no consistent method of writing down the power corrections in deep inelastic scattering. In real fits, the introduction of these corrections reduces the value of Λ (even the extremist point of view that $\Lambda=0$ is not ruled out, and all the deviations from scaling that have so far been observed are due to the power terms).[26,30]

To conclude this section, we note that until recently the status of QCD was similar to the status of Yukawa's meson theory of nuclear forces before the discovery of the pion. In spite of all the apparent achievements of gluon dynamics, it is clear that it is the direct observation of the gluon and the subsequent experimental verification of its quantum numbers and interactions that would constitute the decisive step towards the confirmation of QCD as the microscopic theory of the strong interaction.

3. DETECTION OF THE GLUON IN e^+e^- ANNIHILATION

So far, the most favorable conditions for direct observation of the gluon have occurred in e^+e^- annihilation. At the installation PETRA (Hamburg), a total colliding-beam energy $W=36$ GeV has already been achieved; experiments at $W=38$ GeV using this installation are planned for the current year. Also neglected is the installation PEP (Stanford), designed for energies up to $W=36$ GeV.

The production of hadrons in e^+e^- annihilation proceeds via the production of a pair of quarks,

$$e^+e^- \to q\bar{q},$$

which permits optimum utilization of the expended energy. Indeed, in pp scattering, for example, where an energy $W_{pp}^{\max}=62$ GeV has been achieved, the pair energy of the colliding quarks amounts to only $\sim W_{pp}/3$ on the average. Therefore it is e^+e^- annihilation which at the present time makes it possible to obtain quarks with the maximum outgoing energy, and this enhances the conditions of formation of bremsstrahlung gluons.

Moreover, this process offers the best prospects for visualization of the jets produced by quarks and gluons and for the study of their properties.

a) Two jets in e^+e^- annihilation

Thus, the process $e^+e^-\to q\bar{q}$ should lead to two-jet events in the reaction $e^+e^-\to$ hadrons. Experience in the application of the parton model[2] teaches us that the hadrons in a jet produced by a quark have limited characteristic values of $p_\perp$, a plateau in the distribution with respect to the rapidities y, a multiplicity which rises logarithmically with energy, and so forth. However, the jets can obviously be distinguished only in the case when $\langle p_\parallel\rangle \gg \langle p_\perp\rangle$ in a jet.

At low energy, the jets overlap strongly and are unobservable. Their effects become appreciable only at energy $W\gtrsim 7$ GeV.[4] Figure 4 shows three projections of a typical event of e^+e^- annihilation into two jets of hadrons at $W=27$ GeV.[31]

To analyze hadronic jets, it is necessary to know the directions of the outgoing quark and antiquark—the parents. This is determined in each individual event from the momenta of the detected hadrons. Different groups of experimentalists use somewhat different procedures for this purpose. But it is important that with respect to the jet direction found in some particular way the longitudinal momenta of the hadrons grow linearly with energy, with practically constant average transverse momenta (Fig. 5, from Ref. 32). Therefore the two-jet picture improves with increasing W. The measured angular distribution of the jets with respect to the initial e^+e^- beams is well described by the expression $1+\cos^2\theta$, as was to be expected for quarks with spin $\frac{1}{2}$.

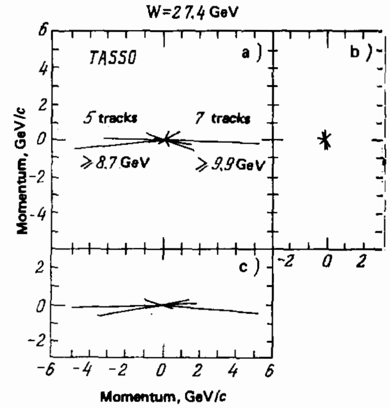

FIG. 4. Three projections of the form of a typical two-jet event[31].

The ratio $\langle p_\perp\rangle/\langle p_\parallel\rangle$ characterizes the average opening angle of a jet, which, as can be seen from Fig. 5, falls off with increasing energy. However, at sufficiently large energies, owing to gluon emission, the regime of limited $\langle p_\perp\rangle$, corresponding to a rapid shrinkage of the angular aperture of a jet as its energy increases, must be replaced by a regime of practically constant [although numerically small, $\sim\alpha_s(W^2)$] emission angles of the hadrons in the jet. This general conclusion can be reached in many ways by studying the various characteristics of the angular distributions of the hadrons in a jet, which reflect the large $\langle k_\perp\rangle$ of the bremsstrahlung gluons emitted by a fast quark.

b) How a bremsstrahlung gluon should show up

As we have already said, it is assumed that the quarks and gluons which have been emitted in the initial hard stage of the process give rise to jets of hadrons. Here the so-called softness hypothesis has become folklore; according to this hypothesis, each quark or gluon in the soft stage dissociates practically independently into a narrow hadronic jet. Of course, the independence of the formation of the jets cannot be exact, since the quarks and gluons are colored, while the final hadrons are colorless. In fact, it is assumed that the mutual compensation of the color of the parents does not lead to a significant redistribution of energy and momentum between the jets (in particular, the fast particles in each jet "remember" the quantum numbers of their parents).

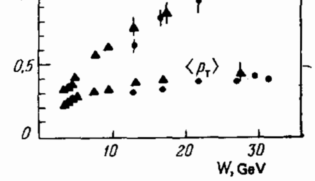

FIG. 5. Energy dependence of $\langle p_\parallel\rangle$ and $\langle p_\perp\rangle$.[32]

According to this hypothesis, sufficiently energetic quarks and the gluons radiated by them, when emitted at large angles from the annihilation region, should give rise to the corresponding number of jets, and this leads to the appearance in the final state of hadrons with large transverse momenta with respect to one another (of the order of their average momentum). However, the cross section for production of many jets with large emission angles is small, since each additional hard decay (see Fig. 3) involves the small factor $\sim\alpha_s(W^2)/\pi$. Therefore it is sufficient to consider only the process of emission by a quark of a single hard gluon as the source of hadrons with large $p_\perp$ in e^+e^- annihilation (Fig. 6).[33] The fraction of events corresponding to this process should be of order $\alpha_s/\pi\sim 1/10-1/15$ at contemporary energies.

It has traditionally been emphasized that the experimental observation of a linear growth of $\langle p_\perp\rangle$ with increasing energy would hardly be the most important stage in testing QCD. The experimentally observed growth of $\langle p_\perp\rangle$ for 100–150 MeV in going over from small jet energies to $E_q\sim 15$ GeV (see Fig. 5) agrees with the expected rise, but this relatively small growth can hardly be regarded as a convincing argument in favor of QCD. Fortunately, a characteristic of the events as general (and hence insensitive to the phenomenon of gluon emission in which we are interested) as $\langle p_\perp\rangle$ is not the only one. In the framework of our picture for the process $e^+e^-\to q\bar{q}g\to$ hadrons (see Fig. 6), we can easily enumerate a number of distinctive features in the distribution of final hadrons, related to its "three-particle" character.

A gluon (as is appropriate for bremsstrahlung) prefers to be emitted at comparatively small angles with respect to the quark: $\pi/2\gg\theta\gg(R_{\text{had}}E)^{-1}$. Therefore, in the bulk of the events, gluon emission does not show up in the form of a clearly distinguished third jet, but looks more like a "swelling" of one of the two jets.[1)]

Let us describe the characteristic features of this swelling.

1. In each event, one of the two jets is "fatter" than the other, since emission of two hard gluons by different quarks is a rare process:

$$\langle p_\perp\rangle_{\text{fat}}>\langle p_\perp\rangle_{\text{thin}},$$

where the average is taken over the hadrons in each jet individually.

2. With increasing energy, $\langle p_\perp\rangle_{\text{fat}}$ rises, while $\langle p_\perp\rangle_{\text{thin}}$ remains practically constant:

$$\langle p_\perp\rangle_{\text{fat}}\sim W,\quad \langle p_\perp\rangle_{\text{thin}}\approx\text{const}=\langle p_\perp\rangle_0.$$

3. The faster particles in the fat jet carry larger $p_\perp$ with respect to the overall axis of the event (Fig. 7a).

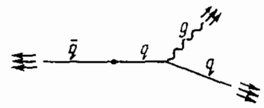

FIG. 6. Emission of a gluon accompanying the outgoing quarks in e^+e^- annihilation. The gluon, like the quarks, produces a jet of hadrons.

Therefore the dependence of $\langle p_\perp\rangle$ in the two jets on the normalized hadron momentum $z_h=2p_h/W$ should have the characteristic form of the so-called seagull effect (Fig. 7b).

4. The jets do not have azimuthal symmetry: in each event, there is a distinguished plane associated with $q\bar{q}g$ emission.

5. The projections $(p_\perp)_{\text{in}}$ of the transverse momenta of the hadrons onto the plane of an event rise with energy, whereas the components $(p_\perp)_{\text{out}}$ orthogonal to the plane remain practically constant:

$$\langle p_\perp\rangle_{\text{in}}\sim W,$$
$$\langle p_\perp\rangle_{\text{out}}\approx\text{const}\sim\langle p_\perp\rangle_0.$$

Thus, on the average the events must become more and more planar with increasing energy.

6. In its center-of-mass system, the fat jet should look as much like two jets (the jets of the quark and gluon) as in e^+e^- annihilation at the corresponding energy (the jets of two quarks).

7. The distributions with respect to the energy E_g and emission angle θ_g of the gluon jet should reflect the specific character of the bremsstrahlung spectrum [see (2)]:

$$d\sigma(E_g,\theta_g)\sim\frac{d\theta_g}{\theta_g}\frac{dE_g}{E_g}. \tag{10}$$

8. If the energy and emission angle of the gluon are sufficiently large, the event should have a pronounced three-jet character. According to the logic of naive QCD, the transverse momenta of the hadrons in each of the three jets should be limited: $\langle p_\perp\rangle_{\text{jet}}\sim\langle p_\perp\rangle_0$ (these jets will also swell with increasing W, owing to the emission of additional gluons).

It should be borne in mind that some of the foregoing properties can manifest themselves to some extent even without gluon emission, in particular, because of the production and decay of resonances in the quark

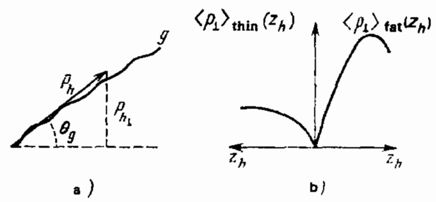

FIG. 7. Scheme of the "seagull" effect. a) Kinematics of the dissociation of a gluon emitted at a large angle θ_g to the quark; the secondary hadron h lies practically in the same direction as the parent gluon; b) the expected dependence of $\langle p_\perp\rangle$ (z_h); the falloff at large values of z_h is due to kinematic effects.

[1)]At first sight, a hadronic jet of a gluon emitted at any small but finite angle to a quark should be distinguished from the jet of a quark at sufficiently high energy. However, this is hindered by the growing emission of additional gluons at small angles as the energy increases. As a result, only jets with emission angles $\theta>\theta_{\text{char}}\sim\alpha_s(W^2)$ can be distinguished.

jets. However, an analysis shows that these mechanisms do not in general reproduce the scale of the phenomena enumerated above and, what is most important, they cannot give the sharp energy dependence of such quantities as $\langle p_\perp\rangle_{fat}$, $\langle p_\perp\rangle_{fat}(z_h)$, and $\langle p_\perp\rangle_{in}$, which is characteristic of gluon emission.

c) What is seen experimentally (gluons as counted in Hamburg)

Three-jet events of the type shown in Fig. 8 were first observed[31,32,34-36] in the colliding e^+e^- beams of the installation PETRA, where in the summer of 1979 four experimental groups at once (TASSO, PLUTO, MARK-J, and JADE) undertook a hunt for gluons at energies up to $W=2\times 16$ GeV. With the present statistics, however, the number of such distinctive events is very small; therefore, to detect the gluon in e^+e^- annihilation, it is crucial to carry out a quantitative analysis of the swelling of jets at different energies. We shall consider briefly the procedure for analyzing the experiment.

First of all, in each individual event one determines a principal axis, i.e., a line chosen to characterize the direction in which the quark and antiquark are emitted. For this purpose, one usually uses the method of sphericity S or "thrust" T: the axis of an event is chosen so as to minimize the transverse components of the hadronic momenta (for S) or maximize their longitudinal components (in the case of T) (see the Appendix).

After isolating the axis, the hadrons are naturally divided into two groups (emitted into the left and right hemispheres), and these are called the two jets. Having determined $\langle p_\perp\rangle$ for the hadrons in each of the two jets individually, one finds the thin and fat jet in a given event. When the characteristics of the fat and thin jets are averaged over the events, one obtains the results shown in Fig. 9.[34]

By analogy with the determination of a principal axis, one can isolate a preferred plane in each event, for example, by minimizing the quantity $\langle p_\perp^2\rangle_{out}$. The distributions of events with respect to the values of $\langle p_\perp^2\rangle_{in}$ and

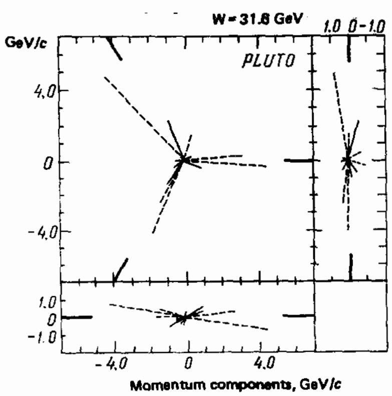

FIG. 8. Three projections of the form of a three-jet event. The dashed lines correspond to neurtal particles, and the solid lines to charged particles; the heavy lines indicate the directions of the total momenta of the jets.

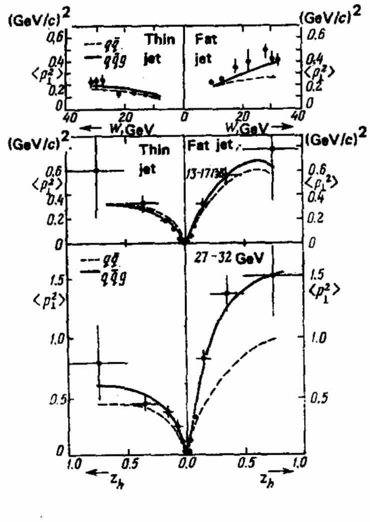

FIG. 9. Characteristics of the fat and thin jets at various energies (PLUTO group).[34]

$\langle p_\perp^2\rangle_{out}$ at various energies are presented in Fig. 10.[34] In Fig. 11,[35] use has been made of another method of demonstrating the azimuthal asymmetry of the events—the distribution with respect to the oblateness O (see the Appendix).

For comparison, we give the calculated curves in Figs. 9–11 for two models: 1) the model of production of a $q\bar{q}$ pair, which dissociates into hadrons with a Gaussian distribution with respect to the transverse momenta in the jet[21] [$\sim\exp(-p_\perp^2/2\sigma_q^2)$]; 2) a model which also takes into account $q\bar{q}g$ production[37]; the properties of a gluon jet were assumed to be similar to the properties of a quark jet. The events generated by a Monte Carlo method on the basis of either model, and with allowance for the effect of the detector, were subjected to the same analysis as the actual events. It must be borne in mind that even without gluon emission there is a difference between the properties of the thin and fat

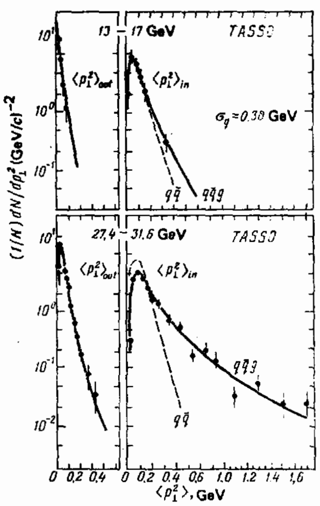

FIG. 10. Distributions of events with respect to $\langle p_\perp^2\rangle_{in}$ and $\langle p_\perp^2\rangle_{out}$ at various energies[32] and their comparison with a model QCD calculation[39] for $\Lambda = 0.6$ GeV and $\sigma_q = 0.3$ GeV (solid curves) and a two-jet calculation (dashed curves).

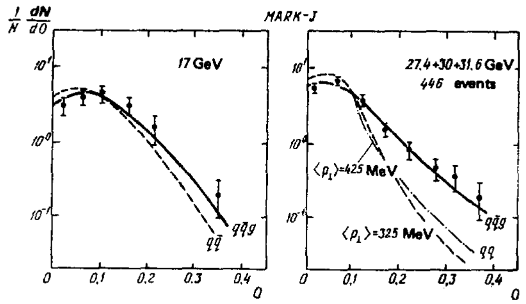

FIG. 11. Distributions of events with respect to the oblateness O (Ref. 35) and their comparison with calculations according to QCD and in the model of two jets with limited transverse momentum $\langle p_\perp \rangle$ of the hadrons.

jet, and also between the quantities $\langle p_\perp^2 \rangle_{\rm in}$ and $\langle p_\perp^2 \rangle_{\rm out}$; this is a result of statistical fluctuations, whose importance is greatly increased by the procedures of separating the jets or isolating the plane of an event.

It can be seen from the figures presented here that at energies $W \lesssim 17$ GeV gluon emission does not have any major consequences for the distributions of hadrons in the process of e^+e^- annihilation. At the same time, at higher energies the "old" two-jet models are inadequate: the three-jet effects enumerated above show up conspicuously against the "background" of the two-jet events.

There have also appeared data on the structure of the fat jet for $W \sim 27-30$ GeV (see items 6 and 7 above).[38] Thus, the degree of the two-jet character in its center-of-mass system (the distribution with respect to T and $\langle k_\perp \rangle$ of the hadrons in each jet) was found to be the same as in e^+e^- annihilation at an energy $W = 9.4$ GeV, which is close to the mean invariant mass of the fat jet. The data on the energy and angular spectra of the jet with lowest energy (of the three jets)—the candidate for gluons—comes close to the expectations of QCD, in agreement with (10).

In addition to the general analysis of the phenomenon of jet swelling, all four experimental groups used various criteria to select "planar" or "three-jet" events with the aim of enhancing the effects of the gluon. In each case, the experimental number of selected events (several tens from among 300–500 for $W = 27.4-31.6$ GeV) was appreciably greater than the number expected in the model of two quark jets with limited $p_\perp$ of the hadrons, and at the same time it was in impressive agreement with the prediction of QCD.

Attempts to extract the value of α_s from the data of various groups lead to values near $(\alpha_s)_{\rm eff} \approx 0.2$.[38,39] To determine the parameter Λ [see (9)], this value is usually assigned to the quantity $\alpha_s(W^2)$, which would correspond to $\Lambda \approx 500$ MeV. However, this prescription evidently leads to an overestimate of Λ, since in reality the argument of α_s is some effective value of the transverse momentum of a bremsstrahlung gluon, $(k_\perp^2)_{\rm eff} < W^2$. It must also be borne in mind that the concrete procedures for describing hadronization (in particular, of heavy quarks and a gluon) used to extract α_s are as yet not free from criticism. This remark also applies to attempts to take into account the higher-order corrections of QCD.[39]

This is the experimental status of the bremsstrahlung gluon in e^+e^- annihilation at the beginning of 1980. Although there still remains definite room for reasonable skepticism,[40] the balance of the evidence clearly favors the discovery of the gluon in 1979.

The accumulated experience in the analysis of experimental information is extremely important, and this is what led to the problem of explicit separation and detailed experimental investigation of the properties of the gluon jet.

4. PROPERTIES OF GLUON JETS AND THEIR EXPERIMENTAL FUTURE

a) Generators of gluon jets

We shall briefly consider in what phenomena, other than gluon bremsstrahlung by quarks, it is possible to observe gluon jets and study their properties.

1. A good way of isolating the hadronic jet produced by a gluon is the process of production of photons with large $p_\perp$ (or massive lepton pairs) in $\pi^- p$ or $\bar{p}p$ collisions. For values of $x_\perp = 2p_\perp/\sqrt{s}$ which are not especially small, this process is determined mainly by the annihilation of a pair of valence u and $\bar{u}$ quarks, as shown in Fig. 12a. The transverse momentum of the photon is then compensated by the gluon jet. Comparison with the analogous process in pp collisions, where a mechanism similar to the Compton effect (Fig. 12b) dominates, makes it possible to study the specific features of gluon hadronization.

2. Great interest attaches to the study of the production of particles with $x_\perp \ll 1$ in hadronic collisions at high energy. Here a major role is played by the interaction of two gluons from the "clouds" of the colliding hadrons (Fig. 13). The gluons rescatter (Fig. 13a) much more frequently than they are transformed into a $q\bar{q}$ pair (Fig. 13b), this being determined mainly by the color combinatorics. This process is of special interest in that it makes it possible to study not only the hadronization of gluons, but also their self-interaction.

3. The decay of "quarkonia" provide a "clean" source of gluons. States with $J^{PC} = 1^{--}$, which are observed directly in e^+e^- annihilation, can decay into three gluons (Fig. 14), whereas P-wave states wtih $J^{PC} = 2^{++}$ and 0^{++}

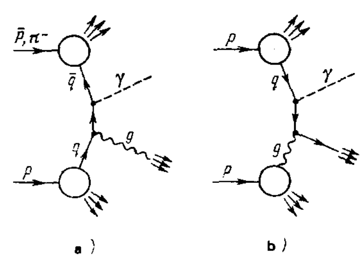

FIG. 12. Jets accompanying the production of a photon with large $p_\perp$ ($x_\perp = 2p_\perp/\sqrt{s} \sim 1$). a) Gluon jet in a $\bar{p}p$ or $\pi^- p$ collision; b) quark jet in a pp collision.

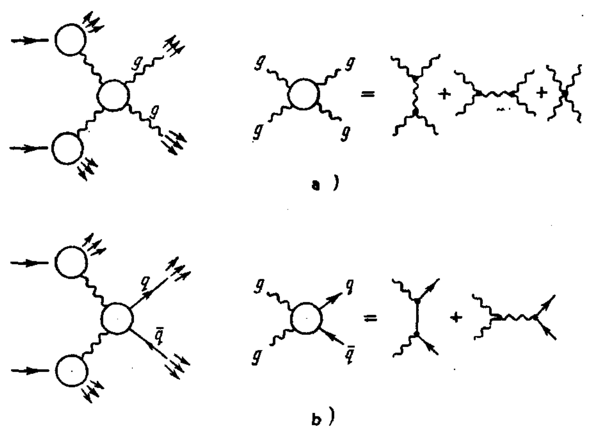

FIG. 13. Jets with large $p_\perp$ for $x_\parallel = 2p_\perp/\sqrt{s} \ll 1$. Gluon rescattering (a) dominates over $q\bar{q}$ pair production (b).

can also be transformed into two gluons (Fig. 15).

If we recall that hadronic jets from quarks begin to show up only at an energy above 3–3.5 GeV per jet, it is clear that in decays of the J/ψ with $M = 3.1$ GeV it is simply not possible to observe jets from three gluons. Even in the decays of Υ and Υ' ($M \approx 10$ GeV), the jets still overlap very strongly, although there is already hope of detecting their existence.

It is possible that the situation will improve with the study of still heavier quarkonium consisting of hitherto undetected t quarks. Since the $(t\bar{t})$ mass must exceed 30 GeV,[32,36] we can hope that the overlap of the jets will be much weaker in its direct decays.

The first experiment to isolate gluon jets in quarkonium decay was a study of three-jet structure in the direct hadronic decays of the Υ.[41,42] Historically, these experiments provided the first evidence for direct effects of gluons.

The events inside and outside the Υ peak were analyzed by the method of "triplicity" T_3 (see the Appendix), which provides an optimum separation of the final hadrons into three "jets." These groups of particles cannot be literally identified with the actual jets from the dissociation of three gluons, which for the mass of the Υ meson strongly overlap. Therefore, as in the search for $q\bar{q}g$, these data were compared with model Monte Carlo calculations taking into account the overlap of the jets. Figure 16 (Ref. 41) shows the distributions with respect to T and T_3, with respect to the "jet" energies ($x^J_i = 2E^J_i/W$, $x^J_1 > x^J_2 > x^J_3$), and with respect to the angles θ^J_i between them. The curves show the results of the model calculations, and the points show the experimental data for the direct decays of Υ and for the background events. Three models were used: a) the two-jet

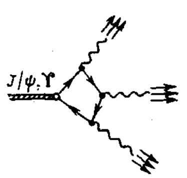

FIG. 14. Decay of vector quarkonia into three gluons.

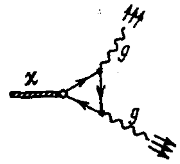

FIG. 15. Decay of a heavy C-even quarkonium into two gluon jets.

model[21]; b) the phase-space model; c) a model involving production of three gluons. It can be seen that, as expected, the Υ decays are in good agreement with the three-gluon model, and the background processes with the quark–antiquark model. From the standpoint of QCD, this is a very gratifying result. However, because of the insufficient jet energy, it cannot by itself serve as a basis for declaring that gluons have been discovered.

4. There seem to be excellent prospects for a study of decays of the type $\Upsilon' \to \gamma\chi_b$, where χ_b is a $b\bar{b}$ state with $J^{PC} = 2^{++}$ or 0^{++}.

It is expected that the decay of the χ_b takes place in the form (see Fig. 15)

$$\chi_b \to 2g,$$

after which the gluons should produce two jets of hadrons. Since the χ_b have mass ~9.5–10 GeV, each jet receives an energy ~5 GeV, and we can hope that they overlap weakly (unlike the three jets in the decays of Υ and Υ'). Therefore it becomes possible to make a careful study of the structure of the hadronic jet produced by a gluon. However, fears have been expressed[43] that for the mass of the χ_b a major part of its hadronic decays will be given by $\chi_b \to 3g$.

As we have already noted, a better solution of the

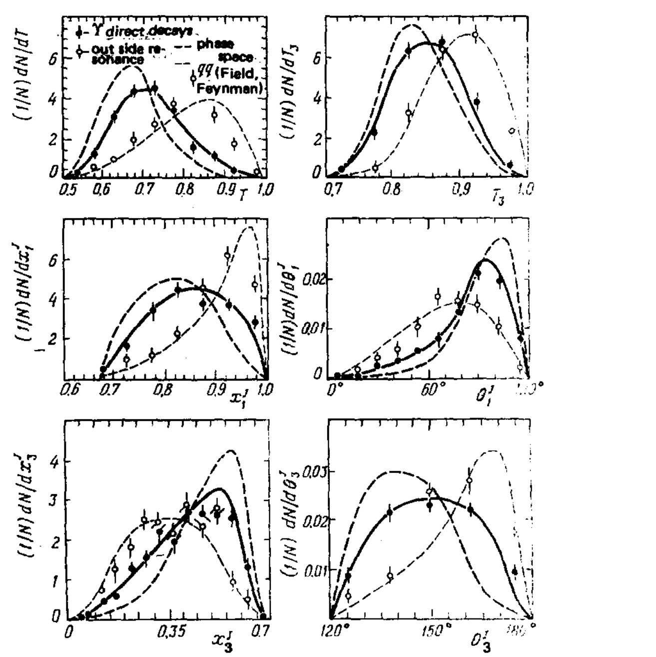

FIG. 16. Comparison of the experimental distributions with respect to the quantities T, T_3, x^J_1, x^J_3, θ^J_1, and θ^J_3 in the Υ resonance and outside it[41] with Monte Carlo calculations in various models.

problem of studying gluon jets will be obtained for t-onia. For a vector toponium, it may also be of interest to study radiative decays of the type

$$(t\bar{t}) \to \gamma gg \to \gamma + \text{hadrons}.$$

Here one also expects two clearly defined jets produced by gluons. Owing to the large charge $e_t = 2/3$, the probability of photon decays may be appreciable:

$$\frac{(t\bar{t}) \to \gamma gg}{(t\bar{t}) \to ggg} \sim 20\%.$$

b) What do we expect of a gluon jet?

Since the order of the day includes experimental generation of gluon jets, it is natural to attempt to imagine their properties.

1. Since the gluon is electrically neutral, the average charge of the fast hadrons in a jet should be zero. In a quark jet, it is expected to be equal to the charge of the parent quark, which agrees with experiment.[44]

2. The gluon interacts in a universal manner with the quarks of all flavors. Therefore, at first sight the various quarks (with sufficient energy) should occur with different probabilities among the products of gluon dissociation. However, even when phase-space effects are neglected, the effective α_s falls off with increasing quark mass. Consequently, in reality there will always be fewer heavy quarks (c, b, t) than light quarks (u, d, s) in a gluon jet (but more than in an ordinary quark jet). There will also be an appreciable violation of the $SU(3)$ singlet character of the jet, whereas the isotopic singlet character should be preserved (since the mass m of the strange quark is much greater than $m_u \sim m_d$).

3. The gluon is a completely new object, and the study of its dissociation into hadrons may completely surprise us. We may find an unusual set of particles in a gluon jet. In particular, we can expect an abundance of mesons with isospin $I = 0$ $(\eta, \eta', \omega, \varphi, \ldots)$ among the fragmentation products of a gluon.

Gluon jets may contain even more interesting objects, for example, bound states of gluons. It has been argued that such hadrons, if they exist, should be rather massive[45]: in particular, the mass of the pseudoscalar gluonium has been estimated as $m_{gg} \approx 2.5$ GeV. It has been remarked by V. N. Gribov that the appearance of a large characteristic mass in the gluon interaction would facilitate the solution of important pressing problems of strong-interaction physics; for example, it would explain the small value of the slope of the vacuum trajectory:

$$\alpha'_P \sim \frac{1}{m_{gg}^2} \ll 1\,\text{GeV}^2.$$

A convenient "laboratory" for the search for gluonia may be the radiative decays of quarkonia, for example, $J/\psi \to \gamma + gg$. The production of gluonium in this process, whose experimental investigation is already in progress,[46] might appear as a peak in the photon spectrum.

4. As we have already said, a gluon emits additional gluons more intensively than a quark. In particular, it follows from a comparison of Eqs. (2) and (3) that the probabilities for emission of a soft gluon ($z \ll 1$) by a gluon and by a quark are in the ratio 9/4.

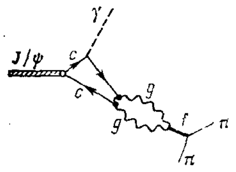

FIG. 17. Picture of the decay $J/\psi \to f\gamma$.

As was noted in Ref. 22, this simple fact has an important consequence: the swelling of a gluon jet with increasing energy begins earlier and takes place more rapidly than in the case of a quark jet; the hadronization of the gluon emission should evidently lead to a more rapid growth with energy of the multiplicity of hadrons in a gluon jet than in quark jets. An indirect manifestation of intensive bremsstrahlung should be a greater violation of scaling in the energy distributions of the hadrons produced by a gluon.

The study of gluon jets is also related to the determination of the quantum numbers of the gluon (its spin, parity, etc.). Various methods of measuring the gluon spin are already available in the literature (see, for example, Refs. 47–50). Several have already been tested experimentally. One such method is the investigation of the decay $J/\psi \to \gamma f \to \gamma\pi\pi$,[47] where $f(1270)$ is the well-known resonance with $J^{PC} = 2^{++}$. If there are two gluons in the intermediate state (Fig. 17) and these can be regarded as free massless particles, the f meson does not acquire all the possible polarization states (a massless vector gluon cannot have helicity 0). Experiment[51] shows that the angular distributions of the pions from the decay of the f meson agree with the predicted distributions. What seems more convincing is the analysis of the angular and energy distributions of gluon jets in the decay $\Upsilon \to ggg$ (see Sec. 4a and Fig. 16), which depend strongly on the spin and parity of the gluon. A comparison of the calculations with the data on the angular distribution of the jets with respect to the electron–positron beams rules out scalar gluons and is consistent with vector gluons.[49]

Perhaps the clearest evidence against spinless gluons comes from the comparison of the hadronic widths of the pseudoscalar and vector quarkonia. In QCD, these decay into hadrons mainly via gg and ggg, respectively, and the widths differ by about two orders of magnitude (for example, Refs. 19 and 20). In the case of spinless gluons, the pseudoscalar quarkonium cannot decay into gg, and the difference between the widths is much smaller.[2)]

Obviously, the quantum numbers of the gluon can also be measured by means of the characteristics of the gluon bremsstrahlung. Thus, a reliable experimental confirmation of the "doubly logarithmic" spectrum (10) would prove that the gluon has spin 1.

[2)] If the spin – parity of the gluon is 0^+, then a pseudoscalar quarkonium decays into at least four gluons, and its width is parametrically smaller ($\sim \alpha_s^4$) than for a vector quarkonium ($\sim \alpha_s^3$).

We note here that for the theoretician there are no problems about either the existence of gluons of their quantum numbers, since QCD as a theory of the strong interactions no longer has any rivals. However, such fundamental problems of hadron physics nevertheless undoubtedly require direct experimental tests. The need for tests is even greater for the rules of the game of naive QCD, which are exploited in deriving the predictions of the theory.

5. MULTIPLE EMISSION OF GLUONS AND THEIR SELF-INTERACTION

In our discussion of the growth with energy of the characteristic transverse momenta of the hadrons in e^+e^- annihilation, we considered only the simple picture of single emission of a gluon by one of the quarks, having noted the infrequency of processes of production of several (N) energetic jets with large emission angles (in addition to the two quark jets):

$$\frac{1}{\sigma}\Delta\sigma^{(N+2)}\sim\left(\frac{\alpha_s}{\pi}\right)^N. \tag{11}$$

At the same time, the integrated probability for production of a gluon or a $q\bar{q}$ pair in the hard stage of the process becomes of order unity with increasing energy. Indeed, as can be seen from (4), in the total probability for the decay $g\to q\bar{q}$ the small factor α_s is compensated because of the broad distribution of the products with respect to $k_\perp$:

$$W^{\mathrm{F}}\sim\frac{\alpha_s}{\pi}\int\limits_{R_{\mathrm{had}}^{-2}}^{E^2}\frac{dk_\perp^2}{k_\perp^2}\approx\frac{\alpha_s}{\pi}\ln(ER_{\mathrm{had}})^2. \tag{12}$$

If we consider gluon emission as in (2) or (3), the total probability rises even more rapidly, since the bremsstrahlung is "logarithmic" both with respect to the emission angle $\theta\approx k_\perp/\omega\ll 1$ and with respect to the gluon energy $z=\omega/E\ll 1$:

$$W^{\mathrm{G}}\sim\frac{\alpha_s}{\pi}\int\limits_{R_{\mathrm{had}}^{-1}}^{E}\frac{d\omega}{\omega}\int\limits_{R_{\mathrm{had}}^{-2}}^{E^2}\frac{dk_\perp^2}{k_\perp^2}\approx\frac{\alpha_s}{\pi}\ln^2(ER_{\mathrm{had}}). \tag{13}$$

In both cases (12) and (13), the characteristic emission angle of the decay products is relatively small:

$$dW^{\mathrm{G,F}}\sim\frac{d\theta}{\theta},\quad 1\gg\theta_{\mathrm{char}}\gg(R\omega)^{-1}\geqslant(RE)^{-1},$$

so that the produced gluons and $q\bar{q}$ pairs (Fig. 18) are emitted in practically the same direction as the parent quark. The quantity (13) is related to the number of "soft" ($z\ll 1$) "collinear" ($\theta\ll 1$) gluons; with sufficient energy [when the parameter of (12) becomes of order unity], these also include energetic gluons (with $z\sim 1$) and additional $q\bar{q}$ pairs with appreciable probability. We recall that the transverse momenta of all these gluons and quarks are assumed, as usual, to be bounded below by the relation

$$k_\perp^2>R_{\mathrm{had}}^{-2},$$

as a result of which the emission can occur before the onset of the process of hadronization [see (6) and (7)]. Nevertheless, such gluons and $q\bar{q}$ pairs are traditionally said to be collinear (with the direction of the parent) in order to distinguish them from the rare energetic gluons emitted at large angles, which provide the three-jet planar events for the experimenters working at the installation PETRA.

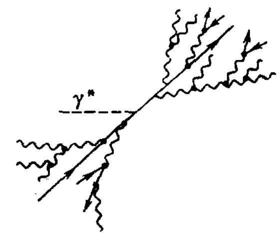

FIG. 18. Shower-like multiplication of soft gluons in e^+e^- annihilation.

a) Gluon cascades

Thus, we have arrived at the conclusion that even for comparatively low annihilation energies neither of the two quarks produced by a photon remains in isolation: on entering the "hadronizer," i.e., at the instant of time $t\approx WR_{\mathrm{had}}^2$, the quark is not bare but is completely surrounded by a perceptible cloud of gluons.

Although the detailed description of the hadronization of such a quark–gluon conglomerate is a problem for a future theory, one important consequence of the picture of the accompanying emission already seems certain: the fast growth with energy of the number of gluons must also entail an increase in the multiplicity of slow hadrons—a growth of the "plateau."

As regards "soft" gluons with $k_\perp^2\ll\omega^2$ and $z\ll 1$, QCD is extremely productive, since the gluon self-interaction participates very effectively in their production. The multiplicity of gluons with $z\ll 1$ produced by shower-like processes of the type shown in Fig. 19 can be estimated as follows. An important part is played by decays into gluons with gradually decreasing fractions of the energy and emission angles; summing the contributions of k-step cascades, we obtain a gluon spectrum which rises rapidly as $z\to 0$:

$$z\frac{dn_g^{(q)}}{dz}\sim\sum_k\left(\frac{3\alpha_s}{\pi}\right)^k\frac{1}{k!}\left(\ln\frac{1}{z}\right)^k\frac{1}{k!}(\ln W^2R_{\mathrm{had}}^2)^k$$
$$\approx\exp\left(2\sqrt{\frac{3\alpha_s}{\pi}\ln W^2R_{\mathrm{had}}^2}\sqrt{\ln\frac{1}{z}}\right). \tag{14}$$

For the gluon spectrum, and also for $q\bar{q}$ pairs with small z, it is possible to obtain more accurate formulas[52,8] taking into account, in particular, the effect of the "running coupling constant" $\alpha_s(Q^2)$.[3)] Thus, for example, the mean multiplicity n_g of soft collinear gluons in the quark jet

$$n_g^{(q)}=\int\limits_{z_{\min}}^{1}dz\frac{dn_g^{(q)}}{dz},\quad z_{\min}=\frac{E_{\min}}{E_q}$$

(E_q is the energy of the parent quark) has the form

$$n_g^{(q)}(E_q)\approx\frac{C_s}{N}(I_0(v)-1)\,e^{-a\xi}, \tag{15}$$

[3)]Equation (14) and other formulas of this type give an unlimited growth of the density of gluons in phase space, which contradicts the unitarity condition. Actually, these formulas are inapplicable for sufficiently high gluon density, when it is necessary to take into account not only gluon multiplication, but also the inverse process of gluon fusion.

FIG. 19. A Gluon cascade. The energy of the gluon and the transverse momentum of the decay products gradually decrease in each decay.

where

$$v=\sqrt{16N\xi\ln\frac{1}{z_{\min}}},\quad C_2=\frac{4}{3},\quad N=3,$$
$$a=\frac{11}{3}N+\frac{2}{3}\frac{n_f}{N^2}=\frac{101}{9}\quad \text{for}\quad n_f=3;$$

$I_0(v)$ is the modified Bessel function,[53] which for large values of the argument v is given by

$$I_0(v)\approx\frac{1}{\sqrt{2\pi v}}e^{v}.$$

The symbol ξ in (15) denotes the well-known perturbation-theory variable[24]

$$d\xi(Q^2)=\frac{\alpha_s(Q^2)}{4\pi}\frac{dQ^2}{Q^2},$$
$$\xi(E_q^2)=\int_{R_{\text{had}}^{-2}}^{E_q^2}d\xi(Q^2)\approx\frac{1}{b}\ln\frac{\alpha_s(R_{\text{had}}^{-2})}{\alpha_s(E_q^2)},\quad b=11-\frac{2}{3}n_f,\tag{16}$$

the value of which characterizes the degree of development of the chromodynamical picture of emission of collinear gluons and pairs in hard processes. From deep inelastic scattering, it can be estimated[8] that $\xi\approx 0.2$ when Q^2 is of the order of several $(\text{GeV})^2$.

The multiplicity of soft collinear gluons in a gluon jet differs from the multiplicity (15) in a quark jet in that $C_2/N=4/9$ is replaced by unity:

$$n_g^{(g)}(W)\approx\frac{9}{4}n_g^{(q)}(W).\tag{17}$$

The cascade multiplication of gluons and quark pairs may be the reason for the fast growth of the plateau observed in e^+e^- annihilation[54,55] and the related enhancement in the energy dependence of the total number of charged hadrons $\langle n_{ch}\rangle$. In Fig. 20 the data on $\langle n_{ch}\rangle$ are compared with the naive formula

$$\langle n_{ch}(W)\rangle=n_0+K_{ch}\cdot 2n_g^{(q)}\left(\frac{W}{2}\right),\tag{18}$$

where we have introduced "by hand" the coefficient K_{ch} to convert the total number of gluons in the two quark jets into charged hadrons. The curve in Fig. 20 corresponds to Eqs. (15) and (18) with $n_0=2$, $K_{ch}=1$, $\xi=0.2$, and $E_{\min}=0.2$ GeV. These parameter values seem reasonable in the framework of a simplified picture of hadronization in which each additional gluon leads on the average to the production of a single meson. Clearly, the curve correctly reproduces the scale of the effect. A detailed description of $\langle n_{ch}(W)\rangle$ would require accurate allowance for the specific features of the dissociation of heavy quarks, especially the c quarks, which are responsible for almost half of the events (for further details, see Ref. 40). Note, however, that equations analogous to (15) and (18) also provide a reasonable description[40] of the observed growth of the plateau,[54] which, as expected, is more directly related to the dissociation of light quarks. It is interesting that a similar phenomenon of growth of the plateau is also observed in pp interactions at high energy.[55]

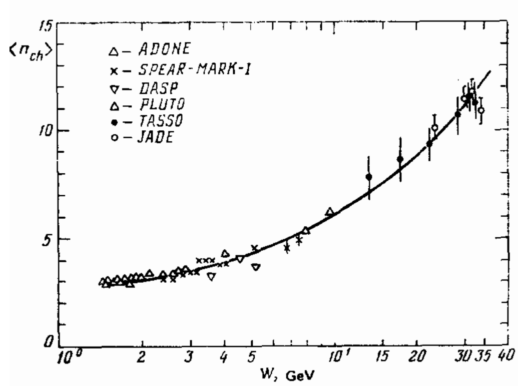

FIG. 20. Dependence of $\langle n_{ch}\rangle$ on the total energy W in e^+e^- annihilation.[5]

If the growth of the hadron multiplicity in e^+e^- annihilation at high energy is really due to the emission of collinear gluons, we can obtain simple qualitative predictions for the direct (three-gluon) decays of quarkonium with very large mass M_O. For example, their multiplicity should be described approximately by the relation

$$\langle n_{ch}^{O}\rangle\approx\frac{3}{2}\cdot\frac{9}{4}\Delta n_{ch}^{e^+e^-}\left(W=\frac{2}{3}M_O\right)+\langle n_{ch}^{J/\psi}\rangle,\tag{19}$$

where $\Delta_{e^+e^-}(W)$ is the increment of the multiplicity in e^+e^- annihilation from low energies (of the order of 2 GeV) to W. The factor 3/2 in (19) is due to the transition from two jets to three jets, and the coefficient 9/4 reflects the larger probability of gluon emission by a gluon than by a quark [see (17)]. Using the naive formula (19) and the data of Fig. 20, we find a reasonable value of $\langle n_{ch}\rangle$ for the Υ meson, and we are compelled to expect that for t-onia the multiplicity in the direct decays will be approximately twice as large as in the background (for $M\approx 45$ GeV, we have $\langle n_{ch}\rangle\sim 30$).

Multiple bremsstrahlung of gluons by gluons also leads to observable effects in hard hadronic collisions. Thus, for example, it is interesting to compare the production of massive lepton pairs l^+l^- and C-even quarkonia χ in pp collisions.[56] The fusion of two gluons into a χ meson (Fig. 21b) is accompanied by more intense gluon emission than in $q\bar{q}$ annihilation (Fig. 21a). As a result, the distribution with respect to the transverse momenta $p_\perp$ is much broader in the case of χ than for a lepton pair with the same mass.[8] For $p_\perp^2\ll M^2$, the spectra are described qualitatively by the following

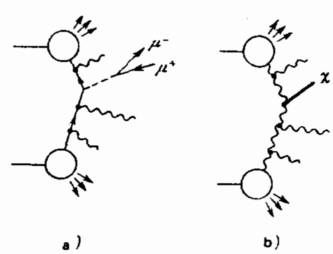

FIG. 21. Gluon bremsstrahlung accompanying the production of a massive lepton pair (a) and the production of a χ meson (b).

formulas, which are specific for QCD:

$$\frac{p_\perp^2}{\sigma}\frac{d\sigma^{(\chi)}}{dp_\perp^2} \sim \exp\left(-\frac{3}{2}\frac{\alpha_s}{\pi}\ln^2\frac{M_\chi^2}{p_\perp^2}\right), \tag{20a}$$

$$\frac{p_\perp^2}{\sigma}\frac{d\sigma^{(l^+l^-)}}{dp_\perp^2} \sim \exp\left(-\frac{2}{3}\frac{\alpha_s}{\pi}\ln^2\frac{M_{l^+l^-}^2}{p_\perp^2}\right). \tag{20b}$$

The arguments of the exponential functions in (20) now differ by the above-mentioned factor 9/4 [see (17)].

We note that the form of the spectra [in particular, (20a)] is very sensitive to the value of the parameter Λ (the exact formulas of the leading logarithmic approximation for the distributions discussed above can be found in the review of Ref. 8).

The fact that a significant fraction of the χ mesons should have comparatively large values of $p_\perp$ raises hopes that the states χ ($b\bar{b}\sim 10$ GeV) may even be detected in contemporary experiments.

b) Critical tests of QCD: infrared-stable quantities and experiment

The predictions of naive QCD are derived, as a rule, on the basis of a set of qualitative hypotheses and phenomenological quantitative notions about the dynamics of the transition of colored quarks and gluons into hadrons. At the present time, many of these ideas appear to be prescriptions which have no theoretical basis. To test the claims of QCD, it would therefore be of special interest to carry out a theoretical analysis and experimental investigation of those characteristics of hard processes (in particular, hadronic jets) which may be insensitive to the physics of large distances.

The method of seeking quantities that can be claimed to have such insensitivity is quite simple. The prescription for doing this is to forget about the existence of hadrons and to calculate any particular quantity in terms of quarks and gluons according to the rules of perturbative QCD. The result of a calculation of Feynman diagrams must be finite, i.e., it must not contain infrared divergences, which are typical for a field theory with massless vector particles. In this case, the quantity under consideration is said to be an "infrared-stable" quantity[57] and it is asserted that the value prescribed for it by naive QCD (without hadronization) can differ from the true value only by power corrections of the type

$$\sim\left(\frac{1}{Q^2R_{\rm had}^2}\right)^n,$$

which fall off rapidly with increasing hardness of the process, characterized by large energy-momentum transfers Q^2.

The quantity of this type which is best known and whose special role as a test of QCD was emphasized even before the formulation of the hypothesis of infrared stability is the total cross section (1) for e^+e^- annihilation into hadrons, which we have already discussed above. It is important that the correction to R calculated in second order [$\sim(\alpha_s/\pi)^2$] is numerically small (for a review of the results, see, for example, Ref. 26), and the nonasymptotic power terms are negligibly small for $W\gtrsim 2$ GeV.[27] Therefore, outside the region of the J/ψ and Υ resonances and structures associated with near-threshold production of heavy (charmed, etc.) particles, the cross section for e^+e^- annihilation should be described by the simple QCD formula (1). A measurement of the cross section with accuracy of order 2–3% would make it possible to visualize the phenomenon of asymptotic freedom: the quantity R must fall off slowly with increasing energy, together with $\alpha_s(W^2)$. With this approach (although it is a rather thorny one), one can hope to measure the fundamental parameter Λ of the theory. The accuracy of contemporary measurements of R is within 10–15%. This program may be greatly facilitated by measurements of the total and partial widths of the Z boson, whose peak in e^+e^- annihilation rises sharply (by a factor ~10^?) above the background.

Of the quantities characterizing the topology of hadronic jets, one can also distinguish infrared-stable quantities. Those which are best known are T and S' (spherocity, in contrast to sphericity). The guarantee that such quantities are infrared-stable lies in their linearity with respect to the momenta (energies) of the detected hadrons.[58-60]

The values of $\langle T\rangle, \langle S'\rangle$, etc., averaged over the events at fixed energy W, can be calculated by perturbation theory in the form of a series in α_s:

$$\langle T\rangle = 1-1.05\frac{\alpha_s}{\pi}, \quad \langle S'\rangle = \frac{\alpha_s}{\pi}1.09. \tag{21}$$

By comparing these values with the data of experiments at high energies (with allowance for nonasymptotic effects associated with the production of heavy quarks), one can, as in the case of the total cross section R, determine $\alpha_s(W^2)$.

It is particularly interesting to study the distributions of the numbers of events with respect to T and S' in the region of the values of these variables that single out events with a prominent two-jet character:

$$1-T \ll 1-\langle T\rangle, \quad S' \ll \langle S'\rangle, \text{ etc.}$$

The point is that this choice greatly restricts gluon bremsstrahlung, and this leads to a strong form-factor type of suppression of the spectra when allowance is made for the higher-order corrections of perturbation theory; for example,[61]

$$\frac{d\sigma}{dT} = 2C_q\frac{\alpha_s}{\pi}\sigma_0\frac{\ln(1-T)|}{1-T}\exp\left[-c_q\frac{\alpha_s}{\pi}\ln^2(1-T)\right], \quad C_q = \frac{4}{3}. \tag{22}$$

It is important to note that, whereas the expansions of the average quantities (21) [like the series (1) for R] are typical for any field theory with a dimensionless coupling constant (for example, with scalar gluons), distributions of the type (22) and (20) are a characteristic feature of bremsstrahlung of vector particles and are therefore more sensitive to the structure of the theory.

A comparison of the fractions of strongly collimated hadronic events in quark and gluon jets (for example, in the decay $\Upsilon\to$ hadrons) enables us to satisfy ourselves about the existence of the self-interaction of gluons. Indeed, as we already know, it is because of this that bremsstrahlung of a gluon is much more intense than in the case of a quark, as a result of which the hadrons from a gluon jet are emitted less frequently into a narrow cone (i.e., with small S' and large T): the coefficient C_g in (22) for the jet of a gluon g exceeds C_q by a factor which is already familiar to us:

$$C_g = \frac{9}{4} C_q. \tag{23}$$

The same physical picture of jet swelling associated with gluon bremsstrahlung is the basis of the well-known jet form factor $f(\varepsilon, \delta)$ proposed by Sterman and Weinberg in their pioneering paper of Ref. 57, which initiated the ideology of "infrared stability."

An infrared-stable quantity which is of practical interest is the pair correlation of the energy fluxes of the hadrons in e^+e^- annihilation.[62,8]

We shall briefly describe the formulation of the problem (for further details, see Ref. 8). In each event, one constructs the distribution with respect to the angle θ between two hadrons, weighted with the product of the fractions of energy $z = 2E/W$:

$$K(\theta, W) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{d\cos\theta} \equiv \frac{1}{\sigma_{\text{tot}}} \sum_{a,b} \int\int dz_a\, dz_b\, z_a z_b \frac{d\sigma}{dz_a\, dz_b\, d\cos\theta}. \tag{24}$$

We note at once that from the experimental point of view the measurement of this quantity has definite advantages over other methods of studying the physics of jets [for example, the distribution (22) with respect to T], since, first, it does not require a rather delicate procedure for determining the axis of an event, and second, it ensures rapid collection of statistics: the pair combinations of all the final particles "come into play," i.e., each event provides on the average $\langle n^2\rangle/2$ pair angles θ_{ab}.

On the other hand, the physical information contained in the correlation function (24) is quite rich. First of all, QCD predicts approximate scaling for the quantity K with respect to the angle θ at high energies:

$$K(\theta, W) \approx K(\theta), \quad \frac{\partial}{\partial \ln W} \ln K \sim \frac{\alpha_s(W^2)}{\pi} \ll 1, \tag{25}$$

which is in sharp contrast to the old parton picture with limited $p_\perp$ in the jets, where $K(\theta, W)$ is concentrated in narrow angular ranges that shrink rapidly with increasing energy:

$$\theta \lesssim \frac{\langle p_\perp\rangle}{W}, \quad \theta_a \equiv \pi - \theta \lesssim \frac{\langle p_\perp\rangle}{W}.$$

Further, the correlation function (24) is infrared-stable. This special property of the energy flux, which, like other quantities that are linear in the momenta (T, S', etc.), is cluster-invariant, i.e., it is not changed by collinear decay of a particle. For comparison, the angular correlator, for example, of the squares of the energies of hadrons a and b [weight z_a^2 and z_b^2 in (24)], or, say, of the multiplicity (weight 1), no longer has this property and depends significantly on the details of the process of hadronization of quarks and gluons.

Finally, comparatively simple closed expressions can be obtained for the function K over the entire range of angles $0 \leqslant \theta \leqslant \pi$. Three ranges of values of θ can be distinguished. For large angles $\theta \sim \theta_a \sim 1$, the single-gluon approximation works,[63] whereas for small angles θ or θ_a, the problem of determining the value of $K(\theta, W)$ involves parametrically large contributions from the higher-order corrections of perturbation theory:

$$\frac{\alpha_s}{\pi} \ln \frac{1}{\theta} \sim 1 \quad \text{for} \quad \theta \ll 1,$$

$$\frac{\alpha_s}{\pi} \ln^2 \frac{1}{\theta_a} \sim 1 \quad \text{for} \quad \theta_a \ll 1,$$

which modify the lowest approximation, $K \sim \theta^{-2}, \theta_a^{-2}$. Multiple bremsstrahlung of gluons comes into play,[62,64] and this leads to a singly and doubly logarithmic suppression, respectively, of the number of collinear and anticollinear pairs $\{a, b\}$ described by the form factors

$$\tilde{T} \approx \exp\left(-2.57 \frac{\alpha_s}{\pi} \ln \frac{2}{\theta}\right), \quad \theta \ll 1, \tag{26a}$$

$$T_F \approx \exp\left(-\frac{4}{3} \frac{\alpha_s}{\pi} \ln^2 \frac{2}{\theta_a}\right), \quad \theta_a \ll 1. \tag{26b}$$

The correlation function (24) in these angular ranges has the universal form of a derivative with respect to $\cos\theta$ of $\tilde{T}$ and T_F^2.[62] In particular, for angles $\theta_a \ll 1$, where higher-order gluon effects manifest themselves most strongly, the correlation K is given by the formula

$$K(\theta, W) \approx \frac{1}{\sin^2\theta_a} \frac{\partial}{\partial \ln \operatorname{tg}(\theta_a/2)} T_F^2(k^2, W^2), \tag{27}$$

where

$$k^2 = W^2 \operatorname{tg}^2\left(\frac{\theta_a}{2}\right) \ll W^2.$$

The effective form factor T_F has the obvious meaning of the probability of *nonemission* by a quark of bremsstrahlung gluons with transverse momenta greater than a particular value, and is given by the expression[8,65]

$$T_F(k^2, W^2) \approx \exp\left[-\int_{k^2}^{W^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_s(k_\perp^2)}{\pi} \frac{4}{3} \int_{k_\perp/W}^{1} dz \frac{1+(1-z)^2}{2z}\right], \tag{28}$$

which, in the crude doubly logarithmic approximation, reduces to (26b).

For very small θ_a, when $T_F^2(\theta_a) \sim \theta_a$, the doubly logarithmic approximation is inadequate. Here $K(\theta)$ ceases to depend on the angle.

The characteristic form of the quantity

$$\sin^2\theta \cdot K(\theta, W) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{d \ln \operatorname{ctg}(\theta/2)}$$

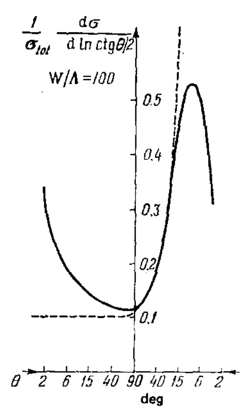

FIG. 22. Characteristic form of the pair correlation of the energy fluxes in e^+e^- annihilation. The dashed curve corresponds to the lowest order of perturbation theory, and the solid curve includes the effects of multiple gluon emission (the horizontal axis to the right indicates θ_a).

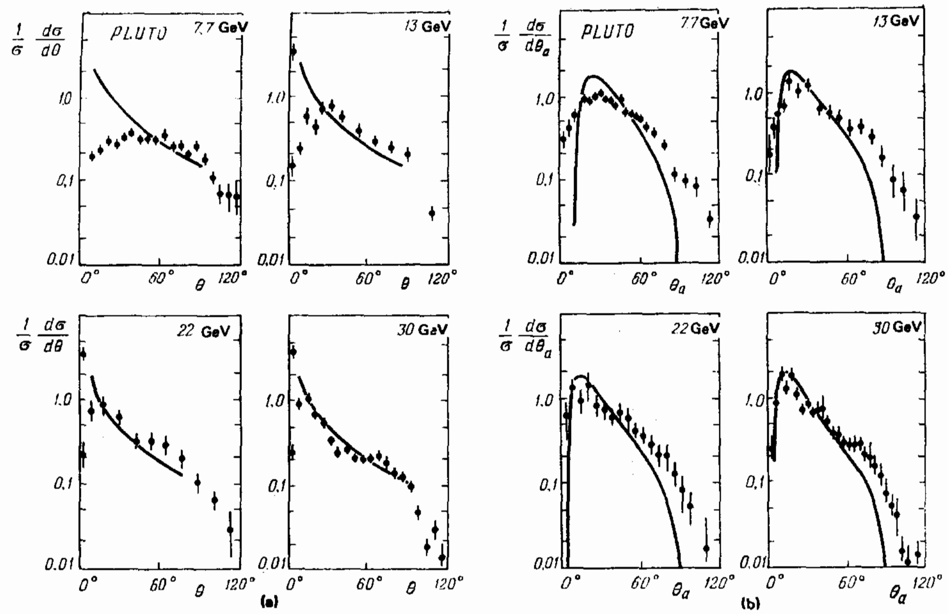

FIG. 23. Angular correlation of the energy fluxes of hadrons[66] in one jet (a) and from opposite jets (b); $\theta_a = \pi - \theta$. The curves correspond to $\Lambda = 500$ MeV.

over the entire range of angles is illustrated in Fig. 22, where the dashed curve corresponds to the lowest order of perturbation theory.[63] The rise at small θ is related to the growth of $\alpha_s(k^2)$ with decreasing virtuality $k^2 \sim W^2\theta^2$, which is a specific feature of QCD. The bend in the curve for $\theta_a \to 0$ is due to the doubly logarithmic effects of multiple gluon emission.

The first experiment to measure the correlation of the energy fluxes was recently performed by the PLUTO group[66] at energies W from 7.7 to 31.6 GeV. The results of the experiment at four energies for the correlation function in a somewhat different representation $(1/\sigma)d\sigma/d\theta = \sin\theta K(\theta, W)$ are presented in Fig. 23 separately for the pairs of particles from one jet and from the opposite jets. The theoretical curves[4)] are constructed in accordance with the formulas of Ref. 62; the parameter Λ is chosen to have the value 500 MeV.

At low energies, the correlation function has a broader angular distribution than that predicted by the theory. With increasing W, this function rapidly approaches the theoretical function. It can be seen from Fig. 23 that the data already agree with the theory in the region $W \sim 30$ GeV (note that the curves in Fig. 23 are not intended to describe the region of large angles θ and θ_a, where it would be necessary to use the exact calculation in the lowest order of perturbation theory).

The observed behavior of the correlation function for small θ_a may provide a strong argument in favor of multiple gluon emission in e^+e^- annihilation, since it is the higher-order effects of gluon emission that suppress the correlation at these angles (see Fig. 22).

[4)]We note that the expression for T_F given in Ref. 62 is erroneous. The correct expression in the doubly logarithmic approximation was obtained from simple physical considerations by Parisi and Petronzio.[65] The error in the derivation of the form factor was rectified by the orginal authors in the review of Ref. 8. The authors of Ref. 67 also participated in the establishment of the correct result. The numerical difference between the expression for T_F used by the experimenters (the curves in Fig. 23b) and the expression (28) is slight.

6. CONCLUSIONS

The fundamental objects of quantum chromodynamics—the only candidate for the role of a theory of the strong interactions—are colored quarks and gluons. There is now no doubt about the existence of quarks, especially since the discovery of charmed particles and the detection of hadronic jets in e^+e^- annihilation. The observation of gluons and the study of their properties and interactions have become problems of the highest priority.

The greatest successes in this direction have now been achieved in e^+e^- annihilation, which for several years has already been a center of attention in elementary-particle physics. Here, with increasing beam energy, conspicuous deviations from the "old" two-jet picture of hadron production have been detected. One finds a class of events which can be interpreted most naturally as a manifestation of bremsstrahlung of a hard gluon by a quark. An analysis of the decay of the Υ meson also indicates the observation of gluons with spin-parity 1^-, exactly as expected from QCD.

More detailed information about the gluon will undoubtedly become available in the near future, as the statistics of the events improve and the energies of the colliding beams continue to increase. Valuable information will also come from the study of gluon jets in hadronic processes. The physics of heavy quarkonia also seems very important; here it is possible to study the characteristics of gluon dissociation into hadrons, to elucidate the self-interaction of gluons, which is a specific feature of QCD, and, possibly, to find exotic quarkless mesons—gluonia.

On the whole, we can say that elementary-particle physics has entered a new stage of its development: we are beginning to see a detailed study of elementary processes involving quarks and gluons, which form the basis of the strong interactions. It looks as if we are here returning to a comparatively simple and perspicuous picture of field theory, which in many respects is reminiscent of good old quantum electrodynamics. But the agonizing problem of color "confinement" stands in the way of the application of this picture to the description of the entire diversity of phenomena of hadron physics. If QCD is to become the same working tool as QED, we must not only find a qualitative explanation of the phenomenon of decoloration of quarks and gluons, but also learn to describe it quantitatively. The solution of this problem, in particular, will obviously be facilitated by the experimental study of the physics of gluons.

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APPENDIX. DESCRIPTION OF THE FORM OF THE EVENTS OF $e^+e^- \to$ HADRONS

A rather large number of quantities have been proposed in the literature to characterize the two-jet or three-jet character of the system of final hadrons. These quantities are associated with procedures for finding a preferred axis and a preferred plane in each individual event. We shall give here only those quantities which have been used in this review (for further details, see, for example, Ref. 40).

1. Two-jet character

The most frequently used quantities are the sphericity[68,4]

$$S=\frac{3}{2}\frac{\min\sum_i p_{i\perp}^2}{\sum_i p_i^2} \tag{A.1}$$

and the thrust[69,58,59]

$$T=\frac{\max\sum_i |p_{i\parallel}|}{\sum_i |p_i|} \tag{A.2}$$

Use is also sometimes made of the spherocity[59,60]

$$S'=\left(\frac{4}{\pi}\right)^2\min\left(\frac{\sum_i |p_{i\perp}|}{\sum_i |p_i|}\right)^2. \tag{A.3}$$

The summation is taken in each event over all the detected hadrons, and the minimization (maximization) is with respect to the direction relative to which the transverse (longitudinal) components are measured. The direction found in this way is taken as the axis of the event.

In place of the momenta of the individual hadrons one can use the energy flux (the Poynting vector) in finding the axis of an event.[35]

Theoretically, the quantities T and S' are preferred, since they are infrared-stable.

TABLE I. Properties of events with characteristic configurations.

	Cigar	Three-prong star	Disk	Sphere
S	0	3/4	3/4	1
S'	0	0.54	0.66	1
T	1	2/3	0.64	1/2
T_3	1	1	0.83	0.65
O	0	0.57	0.64	0

2. Planarity

We shall describe one of the prescriptions for constructing the plane of an event. We project the momenta of all the hadrons onto the plane perpendicular to the axis of the event. In this plane, we then seek the axis that maximizes the sum of the momentum projections onto this axis (or minimizes the projections transverse to it), by analogy with what was done in (A.1)–(A.3). Taken together with the axis of the event, the second axis found in this way determines the plane of the event.

Use is made of didferent quantities which indicate the extent to which an exent lies in (or does not lie in) a plane. Here we shall consider only the oblateness[35]

$$O=\frac{|E_{\perp\,\mathrm{in}}|-|E_{\perp\,\mathrm{out}}|}{E_{\mathrm{vis}}} \tag{A.4}$$

where E_{vis} is the total visible energy in an event, and $E_{\perp\mathrm{in}}$ ($E_{\perp\mathrm{out}}$) is the component of the Poynting vector transverse to the axis of the event and lying in the plane of the event (or outside this plane). Obviously, $O=0$ for azimuthally symmetric events. It is interesting that the distribution with respect to O produced by statistical fluctuations in the purely two-quark mechanism is insensitive to the properties of the quark fragmentation function.

3. Three-jet character

One of the methods of distinguishing three jets in an individual event is associated with the triplicity[70]

$$T_3=\frac{\max\left(\left|\sum_{C_1} p_i\right|+\left|\sum_{C_2} p_i\right|+\left|\sum_{C_3} p_i\right|\right)}{\sum_i |p_i|}. \tag{A.5}$$

The maximization is taken over all possible partitions of the particles into three groups C_1, C_2, and C_3. The grouping which gives the maximum determines the three hadronic jets. The plane of the event is simultaneously constructed in accordance with the momenta of these jets. For any event in the form of three ideal thin jets, $T_3=1$.

We note that T can be defined in the same form as T_3, but with the partition of the particles into two groups. Therefore $T_3 \geq T$ in all cases.

To help the reader to understand the quantities described here, we present in Table I their values for events having four characteristic configurations: a) a cigar, i.e., two thin jets on a single axis; b) a 3-prong star, i.e., three thin jets at angles 120° to one another; c) a disk; d) a sphere.

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