Modern interpretation of J.I. Frenkel's classical spin theory

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The basic propositions of the classical spin theory are considered. The classical equations of motion of a point particle with half-integral spin are developed. An analysis of the spin equations is given.

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1. INTRODUCTION

It is well known that the hypothesis of the rotating electron was advanced by G.E. Uhlenbeck and S.A. Goudsmit[1] in 1925,[1)] as a convenient classical model of the fourth quantum number introduced by W. Pauli[4] to explain the duality in the properties of an optical electron.[2)] According to the Uhlenbeck-Goudsmit hypothesis, in addition to an intrinsic mechanical angular momentum or spin, the electron must also have an intrinsic magnetic moment. This made it possible to explain a number of experimental facts about the spectra of alkali metals and the anomalous Zeeman effect.

The first attempt to describe the motion of the spin in an electromagnetic field (a Coulomb field + an external magnetic field), by use of the methods of special relativity theory, is due to L. H. Thomas.[5] In fact, however, the foundations of the relativistic theory of spin in classical electrodynamics were laid by J.I. Frenkel[6,7] in 1926. Without, at this point, going into the details of Frenkel's theory, which has also been expounded fully in his well known book on electrodynamics,[8] we point out only that in this theory the intrinsic magnetic moment of the electron in its rest system is set exactly equal to the Bohr magneton.

After the appearance of Frenkel's fundamental work,[7] from time to time over many years papers on the classical theory of spin were published, mainly by foregoing authors (H. Kramers, H. Hönl, and A. Papapetrou, H.J. Bhabha and H.C. Corben, G. Weisshoff and A. Raabe, etc.; see the recent review article, Ref. 36), essentially based on the fundamental elements of Frenkel's theory. Nevertheless these papers did not receive general recognition, owing to the imperfections of the physical bases of the theory, and also to the rapid development of the more consistent quantum theory of spin, first proposed by W. Pauli,[9] and then in a more elegant relativistically covariant form by P.A.M. Dirac.[10]

Only since the appearance in 1959 of the Bargmann-Michel-Telegdi equation,[11] which described the motion of the spin in constant and uniform fields, was there renewed interest in the classical theory of spin. The new classical theory took into account the anomalous magnetic moment, which had been discovered in the meantime[12] and which provided an intuitive concept of the precession of the spin in external fields and a great improvement in precise experiments to measure the g-factors of the electron[13] and of other light particles[14,15] (see also the review papers, Refs. 16, 17). The Bargmann-Michel-Telegdi equation, written in a special coordinate system (the spin is described in the rest system, and the external field and the radiation field, in the laboratory system), led to theoretical confirmation of the effect of radiative self-polarization of relativistic electrons, which was first predicted by the methods of quantum electrodynamics[18-20] and subsequently observed experimentally.[21,22] All this can be regarded as a great success of the classical theory of spin.

It is well established[24-27] (see also Refs. 28 and 29) that the Bargmann-Michel-Telegdi equation follows in the quasiclassical limit from the Dirac equation with Pauli's vacuum interaction.[30]

Finally, comparatively recently R.H. Good, Jr.,[31] P. Nyborg,[32] and a number of other authors[33-36] have derived spin equations which generalize the Bargmann-Michel-Telegdi equations to the case of nonuniform external fields. The correspondence between these equations and quantum theory has been discussed in papers by Plathe.[38]

There is still active interest in the classical theory of spin, owing to its convenience in the explanation of polarization effects for relativistic particles (see Refs. 39–41, etc.). We also note that intuitive classical models have great heuristic significance in the description of the spin properties of microscopic particles.[42]

Unfortunately, the foundation-laying work of J.I. Frenkel[7] is now almost forgotten. The opinion has become accepted that Frenkel's theory "served as the basis for dealing with all questions connected with the dynamics of the spinning electron, up to the time when Dirac created the new quantum-mechanical relativistic theory of the electron."[43]

The purpose of the present paper is to show that Frenkel's theory is not merely of historical interest.

[1)] We note that the idea of a proper rotation of the electron had arisen even earlier (cf., e. g., Refs. 2, 3) but received concrete content only in the hypothesis of Uhlenbeck and Goudsmit.

[2)] Cf. also the comment of N. Bohr on the paper of G. E. Uhlenbeck and S. A. Goudsmit in Nature, in which he calls attention to the importance of the hypothesis of electron spin in the light of the correspondence between classical and quantum mechanics.

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Starting from general requirements of relativistic covariance, we have established that if one phenomenologically introduces into Frenkel's equations an arbitrary intrinsic magnetic moment which in the rest system is not equal to the Bohr magneton (for instance, it includes the anomalous magnetic moment of the particle), one gets equations that reduce in the case of constant uniform fields to the Bargmann-Michel-Telegdi spin equation and the ordinary classical equation of motion of the charge. Moreover, it follows from an exact analysis of the Frenkel equations that after elimination of nonphysical terms they lead to generalized spin equations that agree, in particular, with the equations of Good, Nyborg, and others.

2. FUNDAMENTAL PROPOSITIONS OF THE CLASSICAL RELATIVISTIC THEORY OF SPIN

Following J.I. Frenkel,[7] we shall start from a point model of a particle with half-integral spin, which possesses a charge and an intrinsic magnetic moment. Unlike Frenkel, however, we take as the magnetic not the Bohr magneton $\mu_0 = e\hbar/2m_0c$ but an arbitrary value

$$\mu = g\mu_0 s. \quad (1)$$

Inclusion of the g factor here takes into account the anomalous magnetic moment of the particle. For the electron we can take[12] $g = 2(1+\alpha/2\pi)$, and the spin $s = \frac{1}{2}$.

For convenience in comparing with the Frenkel theory we write the intrinsic magnetic moment (1) also in the form

$$\mu = \frac{g}{2}\varkappa\hbar s, \quad (2)$$

where $\varkappa = e/m_0c$ is a notation used by Frenkel.

We shall describe the intrinsic magnetic moment of the particle with the antisymmetric tensor $\mu^{\alpha\beta}$ introduced by Frenkel, and the related (dimensionless) spin tensor which we denote by $\Pi^{\alpha\beta} = (\Phi, \Pi)$. According to Eq. (1) we have

$$\mu^{\alpha\beta} = \mu\Pi^{\alpha\beta}. \quad (3)$$

To obtain the correct relativistic equations of motion of a particle with half-integral spin it is necessary to observe the following rules:

I. In the rest system of the particle the spin tensor $\Pi^{\mu\nu}$ must have only purely spatial components

$$\Pi_0^{\mu\nu} = (0, \Pi_0). \quad (4)$$

This requirement is secured by Frenkel's condition[3)]

$$v_\mu \Pi^{\mu\nu} = 0. \quad (5)$$

As a consequence of this, we have the relation

$$\Phi = [\beta\Pi]. \quad (6)$$

II. In the rest system of the particle the spin equation must have the classical form

$$\frac{d\Pi_0}{dt} = \frac{\mu}{s\hbar}[\Pi_0 H_0]. \quad (7)$$

III. Besides the three ordinary degrees of freedom, a point particle with spin must also have a "rotational" degree of freedom, which correspond to the $2|s|+1$ possible orientations of its spin (see Ref. 44). Since the rotational motion of the particle brings in two degrees of freedom (*sic*), we require that the tensor $\Pi^{\mu\nu}$ have only two independent components. We note that so far, with antisymmetry and the condition (6) taken into account, the tensor $\Pi^{\mu\nu}$ has three independent components. We can impose a supplementary condition on the invariant $\Pi_{\mu\nu}\Pi^{\mu\nu}$, fixing its value as

$$\frac{1}{2}\Pi_{\mu\nu}\Pi^{\mu\nu} = \Pi^2 - \Phi^2 = 1. \quad (8)$$

Then according to Eq. (3) we have

$$\frac{1}{2}\mu_{\alpha\beta}\mu^{\alpha\beta} = \mu^2. \quad (9)$$

From the condition (8) we immediately obtain

$$\Pi_{\mu\nu}\frac{d\Pi^{\mu\nu}}{d\tau} = 0, \quad (10)$$

where τ is the proper time.

IV. In accordance with the postulate of special relativity theory we have have[4)]

$$v_\mu v^\mu = -c^2 \quad (11)$$

and consequently

$$v_\mu \dot{v}^\mu = 0, \quad (12)$$

where the dot indicates the derivative with respect to the proper time.

V. In the rest system the force acting on a particle with charge e and intrinsic magnetic moment μ must be determined by the well known expression

$$\mathbf{F}_0 = m_0\frac{d\mathbf{r}}{dt} = e\mathbf{E}_0 + \mu\nabla(\Pi_0 \mathbf{H}_0). \quad (13)$$

The last term in this equation is due to the potential energy

$$U = -(\mu\mathbf{H}) \quad (14)$$

of the intrinsic magnetic moment in the magnetic field.

With these rules we can formulate unambiguously the spin equations of motion. We shall show how this can be done.

3. DERIVATION OF THE EQUATIONS OF MOTION OF A PARTICLE WITH HALF-INTEGRAL SPIN

In accordance with Frenkel's condition, Eq. (5), we shall assume that the tensor $\Pi^{\mu\nu}$ is spacelike [see Eq. (4)]. We write the covariant generalization of Eq. (7) in the form

$$\frac{d\Pi^{\mu\nu}}{d\tau} = \frac{\mu}{\hbar s}(H^\mu{}_\rho\Pi^{\rho\nu} - H^\nu{}_\rho\Pi^{\rho\mu}) + \frac{1}{c^2}(v^\mu X^\nu - v^\nu X^\mu). \quad (15)$$

The second term on the right side of this equation is inserted in order to satisfy the condition (8). By using the spacelike nature of $\Pi^{\mu\nu}$, the condition (12), and also the identity

$$\Pi_{\mu\nu}H^\nu{}_\rho\Pi^{\rho\mu} = 0, \quad (16)$$

we can verify that Eq. (10), and consequently the condition (8), are actually satisfied.

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3)The numbers placed to the left of equations are those given to them in Frenkel's paper, Ref. 7.

4)The metric tensor used in $g^{\mu\nu} = (-1, +1, +1, +1)$.

It remains to determine X^μ. For this purpose we calculate the derivative $d(v_\mu \Pi^{\mu\nu})/d\tau$, which is equal to zero according to Eq. (5). Differentiating, we get

$$\dot{v}_\mu \Pi^{\mu\nu} + \frac{\mu}{\hbar s} v_\mu H^\mu{}_\rho \Pi^{\rho\nu} - \left(\delta^\nu{}_\rho + \frac{1}{c^2} v^\nu v_\rho\right) X^\rho = 0. \tag{17}$$

From this we find

$$X^\nu = \left(\dot{v}_\rho + \frac{\mu}{\hbar s} v_\mu H^\mu{}_\rho\right) \Pi^{\rho\nu} + x^\nu. \tag{18}$$

The arbitrary timelike vector x^ν satisfies the relation

$$x^\nu + \frac{1}{c^2} v^\nu v_\rho x^\rho = 0. \tag{19}$$

Substituting X^ν in Eq. (15), we get Frenkel's spin equation with arbitrary μ:

$$\frac{d\Pi^{\mu\nu}}{d\tau} = \frac{\mu}{\hbar s} [H^\mu{}_\rho \Pi^{\rho\nu} - H^\nu{}_\rho \Pi^{\rho\mu} + (v^\mu \Pi^{\nu\rho} - v^\nu \Pi^{\mu\rho}) a_\rho]. \tag{20}$$

The vector $a\rho$ in this equation is defined by the formula

$$a_\rho = \frac{1}{c^2 (\mu/\hbar s)} \left(\frac{\mu}{\hbar s} H_{\rho\sigma} v^\sigma - \dot{v}_\rho\right). \tag{21}$$

We shall now find the equation for the force acting on the charged particle with spin. The covariant generalization of Eq. (13) can be written in the form

$$M\dot{v}^\mu = \frac{e}{c} H^{\mu\nu} v_\nu + \frac{\mu}{2} \Pi_{\rho\sigma} D^\mu H^{\rho\sigma}. \tag{22}$$

The meaning of the mass coefficient M will be made clear further on in a deeper analysis of the corresponding Frenkel force equation.

To satisfy the condition (12), the derivatives in Eq. (22) have been "extended" with an added term:

$$\partial^\mu \to D^\mu = \partial^\mu + \frac{1}{c^2} v^\mu v_\rho \partial^\rho. \tag{23}$$

If we now substitute Eq. (22) in the spin Eq. (20), we get a self-consistent version of the Frenkel equation

$$\frac{d\Pi^{\mu\nu}}{d\tau} = \frac{\mu}{\hbar s} (H^\mu{}_\rho \Pi^{\rho\nu} - H^\nu{}_\rho \Pi^{\rho\mu}) + \frac{1}{2c^2} (v^\mu \Pi^{\nu\rho} - v^\nu \Pi^{\mu\rho}) \left[\left(\frac{\mu}{\hbar s} - \frac{e}{Mc}\right) H_{\rho\sigma} v^\sigma - \frac{\mu}{M} \Pi_{\alpha\beta} D_\rho H^{\alpha\beta}\right]. \tag{24}$$

Setting here [cf. Eq. (2)]

$$\frac{\mu}{\hbar s} = \frac{ge}{2m_0 c}, \qquad M = m_0, \tag{25}$$

we arrive at the equation obtained by P. Nyborg[32] (see also Ref. 35). For constant uniform fields we have the equation investigated by M. Kolsrud.[27,37]

If we use not a tensor, but the vector

$$S^\alpha = \frac{1}{2c} \varepsilon^{\alpha\beta\mu\nu} v_\beta \Pi_{\mu\nu}, \tag{26}$$

which was derived in classical spin theory, in a correct manner, by Yu. M. Shirokov,[45] Eq. (24) goes over into an equation of the form

$$\frac{dS^\alpha}{d\tau} = \frac{\mu}{\hbar s} H^{\alpha\mu} S_\mu + \frac{1}{c^2} \left(\frac{\mu}{\hbar s} - \frac{e}{Mc}\right) v^\alpha v_\beta H^{\beta\mu} S_\mu + \frac{\mu}{Mc^2} v^\alpha S^\beta \partial_\beta E^{\nu\sigma} S_\nu v_\sigma, \tag{27}$$

where $E^{\rho\sigma}$ is the tensor dual to the electromagnetic field tensor H.

For $M = m_0$ Eq. (27) for a particle with half-integral spin is identical with the equation obtained by R. H. Good[31] and several other authors.[32-35] For constant uniform fields the substitution (25) becomes identical with the Bargmann-Michel-Telegdi equation[11]:

$$\frac{dS^\alpha}{d\tau} = \frac{ge}{2m_0 c} H^{\alpha\mu} S_\mu + \frac{(g-2)e}{2m_0 c^3} v^\alpha v_\beta H^{\beta\mu} S_\mu. \tag{27'}$$

4. ANALYSIS OF THE FRENKEL SPIN EQUATIONS

In deriving a system of equations describing the motion of a point charged particle with spin, J. I. Frenkel used Hamilton's variation principle. The supplementary conditions I–IV were satisfied either automatically (Points II and III), or by the introduction of Lagrange multipliers a_ρ (Point I) and λ (Point V).

The force equation (in our notation) was thus found in the form

$$\frac{d}{d\tau}(\lambda v^\alpha - \mu \Pi^{\alpha\beta} a_\beta) = \frac{e}{c} H^{\alpha\beta} v_\beta - \frac{\mu}{2} \Pi_{\rho\sigma} \partial^\alpha H^{\rho\sigma}. \tag{28}$$

The spin equation agreed completely with the Eq. (20), which Frenkel had previously derived by a different method.

At first glance, Eq. (28) differs decidedly from the spin Eq. (22).[5)] It turns out, however, that this is due to the fact that Frenkel did not make use of the further proposition formulated in Point V.

If we repeat the procedure for determining the Lagrange multipliers λ and a^σ, which differs from that used by Frenkel only by the fact that the intrinsic magnetic moment has an anomalous part, which is taken into account with the g factor shown in Eq. (2), we get[6)]

$$\lambda = m_0 - \frac{\mu}{2c^2} H_{\rho\sigma} \Pi^{\rho\sigma}, \tag{29}$$

$$a^\alpha \approx \frac{1}{c^2 (g/2)\varkappa} \left(\frac{g-2}{2} \varkappa H^{\alpha\beta} v_\beta - \frac{\mu}{2m_0} \Pi_{\rho\sigma} \partial^\alpha H^{\rho\sigma}\right). \tag{30}$$

Frenkel succeeded in finding the multiplier a^α only approximately. After substituting λ and a_ρ in Eqs. (20) and (28), we get a system of equations which, in the case of constant uniform fields, is identical with the classical theory of spin constructed on the basis of the Bargmann-Michel-Telegdi equation. In fact, if we use the vector representation of the spin, Eq. (26), then using the Lagrange multipliers (29) and (30) we get the equations

$$m_0 \dot{v}^\mu = \frac{e}{c} H^{\mu\nu} v_\nu + \frac{\mu}{2} S_\rho v_\sigma \partial^\mu E^{\rho\sigma}, \tag{31}$$

$$\frac{dS^\alpha}{d\tau} = \frac{ge}{2m_0 c} H^{\alpha\beta} S_\beta + \frac{e(g-2)}{2m_0 c^3} v^\alpha v_\beta H^{\beta\gamma} S_\gamma + \frac{\mu}{m_0 c^2} v^\alpha S_\beta \partial^\beta E^{\rho\sigma} S_\rho v_\sigma. \tag{32}$$

It is now obvious that our assertion is correct. Moreover. Eq. (32) is identical with Good's equation for a particle with half-integral spin, and for $g = 2$ it becomes the equation found also by I. E. Tamm [see Ref. 47, Eqs. (49) and (49')].

As for the force Eq. (31), it is easy to see that it does not satisfy the condition (12), which is due to the fact that the Lagrange multiplier a^α has been found only approximately.

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5) When a_β is substituted in Eq. (28) the second derivative of the velocity appears, $\ddot{v}_\beta$, which has given rise on occasion to criticism of Frenkel's theory.[43]

6) Considerably later the equations of motion for a classical spinning particle with this value of λ were derived by H. C. Corben,[46] but he gave no reference to Frenkel's work.[7]

We shall now show that if one carries out an exact analysis of Frenkel's spin Eq. (28), with the requirement of Point V taken into account, all the difficulties are eliminated. In fact, after we differentiate with respect to τ on the left side of Eq. (28) it can be put in the form

$$m_0\dot{v}^{\alpha}=\frac{e}{c}H^{\alpha\beta}v_{\beta}+\frac{\mu}{2}\Pi_{\rho\sigma}D^{\alpha}H^{\rho\sigma}+\frac{\mu}{2c^2}H_{\rho\sigma}\Pi^{\rho\sigma}\dot{v}^{\alpha}+R^{\alpha}. \tag{33}$$

We see that here the "extended" derivative D^{α} expected according to Eq. (12) has made its appearance, but additional terms have also appeared, one of them proportional to the acceleration $\dot{v}^{\alpha}$, and the other corresponding to the rather complicated expression

$$R^{\alpha}=\frac{\mu}{c^2}\Big[\frac{1}{2}v^{\alpha}H_{\rho\sigma}\dot{\Pi}^{\rho\sigma}+\dot{\Pi}^{\alpha\beta}\Big(H_{\beta\sigma}v^{\sigma}-\frac{2}{g\varkappa}\dot{v}_{\beta}\Big)+\Pi^{\alpha\beta}\Big(H_{\beta\gamma}\dot{v}^{\gamma}+v_{\rho}\partial^{\rho}H_{\beta\gamma}v^{\gamma}-\frac{2}{g\varkappa}\ddot{v}_{\beta}\Big)\Big]. \tag{34}$$

We can, however, note that these additional terms are spacelike vectors. In the case of the term $\sim\dot{v}$ this is obvious, and for R^{α} our assertion can be proved by means of the relation

$$H_{\rho\sigma}\dot{\Pi}^{\rho\sigma}=\frac{2}{c^2}v_{\alpha}\dot{\Pi}^{\alpha\beta}H_{\beta\gamma}v^{\gamma}, \tag{35}$$

which is a simple consequence of Eq. (20).

However, owing to the condition (12), the acceleration vector is in general defined up to an arbitrary spacelike vector. Accordingly, if we consider the requirement (13), we can drop out the term R^{α} as a vector which has no physical meaning. It is appropriate to include the additional term proportional to the acceleration in the left side of the Eq. (33). We then get

$$\Big(m_0-\frac{\mu}{2c^2}H_{\rho\sigma}\Pi^{\rho\sigma}\Big)\dot{v}^{\alpha}=\frac{e}{c}H^{\alpha\beta}v_{\beta}+\frac{\mu}{2}\Pi_{\rho\sigma}D^{\alpha}H^{\rho\sigma}. \tag{36}$$

Comparing this equation with Eq. (22), we find that the quantity M introduced earlier *a priori* is given by the expression

$$M=m_0-\frac{\mu}{2c^2}H_{\rho\sigma}\Pi^{\rho\sigma}, \tag{37}$$

and has a quite definite physical meaning; it is the effective mass of the particle in the external electromagnetic field. We note that in the quantum theory a similar expression for the effective mass follows from the Dirac and Pauli equations.[30]

It is interesting that in constant and uniform fields the effective mass is an integral of the motion. It is easy to show this if we consider the relation (35) and the identity

$$v_{\mu}H^{\mu\nu}\Pi_{\nu\rho}H^{\rho\sigma}v_{\sigma}=0. \tag{38}$$

It follows from this that in the Bargmann-Michel-Telegdi equation the effective mass can be taken into account by the simple change $m_0=M=\text{const}$.

Summing up, we conclude that if we phenomenologically introduce into the Frenkel equations an arbitrary intrinsic magnetic moment in the rest system, then after removal of nonphysical terms from the force equation we get a system of equations which, along with the inclusion of the anomalous magnetic moment, contains as special cases all the best known equations of the classical theory of spin that have been published since 1926.

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