Hadron clusters and half-dressed particles in quantum field theory

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Usp. Fiz. Nauk **132**, 255–291 (October 1980)

Accelerator experiments show that multiple production of hadrons in high-energy collisions of particles involves the formation of unstable intermediate entities, which subsequently decay into the final hadrons. These entities are apparently not only the comparatively light resonances with which we are already familiar but also heavy nonresonant clusters (with a mass above 2–5 GeV). The cluster concept was introduced previously in cosmic-ray physics, under the name "fireballs." To determine what these clusters are from the standpoint of quantum field theory, a detailed and thorough analysis is made of some analogous processes in quantum electrodynamics which are amenable to calculation. The QED analogs of the nonresonant clusters are "half-dressed" electrons and heavy photons. The half-dressed electrons decay into photons and electrons and are completely observable entities, whose interaction properties distinguish them from dressed electrons. In other words, the nonresonant particles are generally off-shell particles (the excursion from the mass shell is in the timelike direction). The assumption that hadron clusters are only resonances would be equivalent to a very specialized assumption regarding the nature of the spectral function of the hadron propagator; it would be different from that in electrodynamics, where the spectral function can be calculated. Nonresonant hadron clusters thus fit naturally into hadron field theory and are nonequilibrium hadrons far from the mass shell in the timelike direction. (In certain cases, their structural distortion is of the same nature as that of a half-dressed electron, so that this term can be conventionally applied to them as well.

PACS numbers: 12.20.Ds

CONTENTS

1. INTRODUCTION

The title of this paper would seem to link two quite different topics. The first topic, that of clusters, deals with a specific problem related to the multiple production of hadrons in collisions of particles at extremely high energies (above $E_{c.m.} \equiv \sqrt{s} \sim 10$ GeV in the center-of-mass frame), at which the multiplicity n is typically large, $\langle n \rangle \gtrsim 10$. More than two decades ago, a picture emerged from cosmic-ray experiments which had this multiple production occurring in two steps: First, heavy blobs of nuclear matter ("fireballs") appear, and then these fireballs decay into the final hadrons.[1] Interestingly, theoreticians had suggested much earlier (although in several different ways) that this is what should happen.[2] At the time, however, the prevailing opinion rejected this idea. Only in recent years, as accelerators have reached the pertinent energy range, have new and more-detailed experiments forced us to return to the fireball picture—although these strange entities are now cautiously called by another name, "clusters,"[3] and they are not necessarily as heavy as was assumed previously.

This picture is not yet accepted universally (although it does appear as an experimental conclusion in one semi-official document[4]). A realistic semiphenomenological model[5] which has been supported by all the experimental data available[6] can be summarized as follows: Among the intermediate entities—"clusters" in the broad sense of the term—we should include both the comparatively light resonances ($\rho, \omega, \Delta^{++}, \ldots$) which are produced directly and which then decay into the final stable hadrons, and the heavy blobs of nuclear matter (with masses $\mathfrak{M} \gtrsim 2-5$ GeV) which are the entities which are properly called "fireballs." The nature of these entities is not yet established *unambiguously*. It may be the uncertainty about just what these fireballs are from the standpoint of quantum field theory which is holding up general recognition of their reality. This is the topic which we will take up first.

The other topic belongs to the fundamental problems of the physics of elementary particles and is related to the concepts of "bare" (or "seed") particles and particles which are "only half-dressed." All the quantum field theories which actually work—quantum electrodynamics (QED), quantum mesodynamics (QMD), and quantum chromodynamics (QCD)—have generally been constructed in accordance with a common principle: A pair of fields, a fermion field and a boson field, is considered (in QED, these are electrons and photons; in QMD they are baryons and mesons; and in QCD they are quarks and gluons). First the bare masses and interaction constants ("charges") of noninteracting particles are introduced in the equations of the theory, and then

 0038-5670/80/100629-22$01.10

they are eliminated from the theory in one manner or another and expressed in terms of the masses and charges of "physical" particles which are actually observable (under certain stipulations). Each such physical particle is a composite of both fields: fermion and boson.

Clearly, this bare particle is an abstraction itself: If it doesn't interact with *anything*, it is unobservable in principle. The only physical meaning which can be attached to the mass of such a particle is that it is an auxiliary parameter which incorporates in some integral manner the effects of other fields and interactions which do not appear explicitly in the given theory. But then another question arises: Is it possible to observe a physical particle with a nonequilibrium structure, san an electron, only partially devoid of the boson (electromagnetic) field which is supposed to be associated with it, i.e. is it possible to observe a "half-dressed" particle? Furthermore, would it be possible to follow the time evolution of the reestablishment of an equilibrium structure—the "dressing" procedure? As it turns out, the cluster question can be associated with these fundamental questions.

We will be working from the basic concepts of quantum field theory. After surviving a period of skepticism, which lasted perhaps 15 years, it has regained acceptance as a result of a series of successes over the past decade (the appearance of non-Abelian gauge theories, and the derivation of a unified theory of weak and electromagnetic interactions). Although quantum field theory is getting more detailed and more complicated, everything basic has survived intact (for example, in the systematic form set forth many years ago by Bogolyubov and Shirkov[7]).

In particular, the right to a space—time description of processes has been preserved, and the principle of the renormalization of the bare quantities originally introduced in the theory has also been preserved.

Since these three field theories are similar, we can begin by examining the specific features of particles with a nonequilibrium structure—their dress in the QED approach, where the necessary calculations can be carried out systematically by perturbation theory. Then we will extend the results to the case of hadron clusters, in which we are interested.

We will see that these clusters can be understood as nonequilibrium or even half-dressed particles, and their decay into final hadrons can be understood as the reestablishment of a normal equilibrium structure.

We will be interested in only the main part of the products produced in the interactions. The momentum transfer for these products is small, so we will not need QCD. Recently, QCD has been used in this field only for analyzing the initial stage in the development of superheavy clusters.[8]

2. RENORMALIZATION AND THE OBSERVABILITY OF INTERMEDIATE STATES

The concepts of bare particles and physical particles and the determination of the relationship between them essentially date back to Lorentz. In his nonrelativistic (and, of course, nonquantum) theory of the electron—a small sphere of radius r_0 and mass m_0, with a charge e_0—Lorentz found that in the equation of motion an awkward self-force $(4e_0^2/3\pi r_0)\ddot{r}$ appears.[1)] In contrast with the other terms, this self-force becomes infinite for a point electron ($r_0 \to 0$), and in addition it generally depends on the structure of the electron. Lorentz noted, however, that this term was of the same form as the inertial force $m_0\ddot{r}$, and if one introduced the notation $\delta m = 4e_0^2/3\pi r_0$ this quantity could be interpreted as an electromagnetic mass which, when combined with the "mechanical" mass m_0, would form the total observable mass (observable even in the limit $r_0/\lambda \to 0$, where λ is the wavelength of the radiation):

$$m = m_0 + \delta m. \tag{1}$$

We would now call the mechanical mass the "bare" mass.

Lorentz was the first to do what we would now call renormalizing the mass which is finite at $r_0 \neq 0$ and infinite in the limit $r_0 \to 0$. The responsibility for the stability of the electron was thus frankly assigned to other forces and fields, beyond those which appear explicitly in electrodynamics itself. These forces and fields can thus represented in an integral manner by the single unknown constant m_0. This was a happy circumstance which allowed the theoreticians to study an immense range of electrodynamic problems which did not require dealing with the structure of the electron. Clearly, however, if one did wish to transcend this restriction it would be naive to expect such a simple method to succeed in taking the nonelectromagnetic forces into account. It is for this reason that we need a unified field theory.

We have discussed these well-known facts in such detail because they correspond completely to the solution of the problem of the charge and mass renormalization in quantum field theory. For example, in a system of two interacting fields—boson and fermion—the Lagrangian is written as the sum of three terms: the Lagrangians of the noninteracting boson field and the noninteracting fermion field, L_b^0 and L_f^0, respectively, by themselves, and the Lagrangian of their interaction:

$$L = L_b^0 + L_f^0 + L_{\text{int}}^0. \tag{2}$$

Initially, these Lagrangians contain only the bare masses of the (bare) particles, m_{0b} and m_{0f}, and the bare interaction constant g_0 (in QED, $g_0 \equiv e_0$). Then they are expressed in terms of the physical masses m_b and m_f and the physical charges g in some manner. This can be done, for example, by writing a perturbation-theory series in g_0 and then expressing all the m_0's and g_0's in this series in terms of m and g, precisely in accordance with the Lorentz example, with $g = g_0 + \delta g$ and $m = m_0 + \delta m$; the fact that δg and δm are infinite is ignored. Actually, it is more convenient to introduce in L some "counterterms" δL, which themselves lead to infinities in the calculations but when summed with L

[1)]We are setting the speed of light, c, and Planck's constant, $\hbar$, equal to unity everywhere.

[see (2)], and with a transformation to m and g, lead to finite results.

One departure from the Lorentz approach was maintained for a long time: In the Lorentz approach the responsibility for the finite value of the overall result was placed on certain other, nonelectromagnetic, forces, which offset the Coulomb repulsion of the parts of the electron, but in quantum field theory until construction of unified theories began, nearly all attempts to construct a systematic theory directly, without divergences, were reduced to modifications within the same system of only two fields (the introduction of nonlinearities, nonlocality, etc.). With hindsight, it seems puzzling that so much effort was devoted to attempts of this sort.

In general, *a renormalization can be carried out formally in the original equations themselves.* All we need to do is examine the Green's functions of these equations for the operator functions of the bosons, $\varphi_0(x)$, and the fermions, $\psi_0(x)$.[2] For a bare boson (a pion, for example), this function (the pion propagator) is, in the momentum representation,

$$D_0 = \frac{1}{k^2 - m_{0\pi}^2}, \tag{3}$$

where $m_{0\pi}$ is the bare mass of the (bare) pion. If the propagator of the physical (dressed) pion is required to have a pole, not at $k^2 = m_{0\pi}^2$, but at the physical value of the mass, $k^2 = m_\pi^2$ ($m_\pi \approx 140$ MeV), with the same residue, unity; and if a similar requirement is imposed on the nucleon propagator G (which has a slightly different form), then it turns out that the bare masses and charges can be eliminated completely from the equations with the help of three constants, Z_1, Z_2, and Z_3, by introducing the "physical," renormalized values[9]

$$g = g_0 Z_1^{-1} Z_2 Z_3^{1/2}, \quad \varphi(x) = \varphi_0(x) Z_3^{-1/2}, \quad \psi(x) = \psi_0(x) Z_2^{-1/2}. \tag{4}$$

The Green's functions, however, do not retain the simple form in (3), with only $m_{0\pi}$ replaced by m_π.

Unfortunately, the calculation method which we have available for finding the Z_i—perturbation theory—again leads to infinities. We will not go into other renormalization methods here [the regularization and renormalization-group method (the results are equivalent)], and we will not discuss a completely different and remarkable method for transforming from zero bare quantities to nonzero quantities which can be used in a certain class of essentially nonlinear field theories with "spontaneous symmetry breaking" (there is a detailed discussion of this method in Ref. 12); after these steps are taken, it is still necessary to renormalize these quantities in order to obtain physical masses.

The successful expulsion of the bare quantities from quantum field theory, at the time when conventional quantum field theory was regarded extremely skeptically, strengthened for decades the conviction that only those particles which are completely dressed should be considered "physical." One result was the derivation of theories such as the axiomatic S-matrix theory. Here the study is confined to the probabilities of transitions between initial ($t = -\infty$) and final ($t \to +\infty$) states of a system, in which all the particles are separated from each other, and they have their own physical, renormalized masses. And it is not allowed to study the time evolution of the interactions among these particles or to study changes in their structure. In the spirit of this conviction, the collision of two nucleons accompanied by the production of a ρ meson, which then decays into two pions (for example), must be treated as a transition from $t = -\infty$ at which there are two spatially separated, "completely dressed" nucleons, with 4-momenta $k_1^{(0)}$ and $k_2^{(0)}$, where $k_1^{(0)2} = k_2^{(0)2} = m_N^2$ (m_N is the renormalized nucleon mass), to $t \to +\infty$, at which there are, in addition to the two nucleons k_1 and k_2, with $k_1^2 = k_2^2 = m_N^2$, two pions, with 4-momenta $\varkappa_1$ and $\varkappa_2$, where $\varkappa_1^2 = \varkappa_2^2 = m_\pi^2$. And only manifestation of the ρ meson in the intermediate state is the fact that the transition amplitude, treated as a function of $\varkappa_1$ and $\varkappa_2$, should have a corresponding singularity at $(\varkappa_1 + \varkappa_2)^2 = m_\rho^2$ where m_ρ is the mass of the ρ meson. The instability of the ρ meson is seen in the circumstance that m_ρ is complex, and the lifetime of this particle, $\tau_\rho^0 \approx (140\ \text{MeV})^{-1}$, is determined by the imaginary part of m_ρ (this time τ_ρ^0 is of the order of the time required for a signal to propagate through the volume of a hadron).

We would hardly expect to remain content with such a "nihilistic" approach (although it is formally allowed), as became particularly clear at high energies, at which the relativistic time dilatation increases the observable lifetime; that of our ρ meson, for example, increases to $\tau_\rho = \tau_\rho^0 E/|m_\rho|$, where E is its energy. At an energy E of the order of 10 GeV, τ_ρ^0 is already so large that the ρ meson becomes a separately observable entity at finite times t.

Analogously, the existence in QED of positronium, which annihilates into γ quanta, would also have to be understood as simply a manifestation of a corresponding singularity in the amplitude for a transition from some initial state, for example, one in which even the positron does not *yet* exist, to a final state in which the positron and the electron *no longer* exist. Positronium is again an entity with which we can experiment separately.

Then, in any case, this ban on considering intermediate states in particle-conversion processes cannot be an absolute ban. We have seen this for resonant intermediate states, even when the lifetime of the intermediate system agrees in order of magnitude with the time required for a signal to propagate across this system—across a hadron, as in the case of the ρ meson. A question which naturally arises is whether there is some quantitative limit on this ban. Could we not, for example, follow the process by which the particles are dressed and undressed if they are not resonant?

It turns out that the answer is affirmative, at least if the energies are so high that the relativistic time dilatation stretches these processes out over a long time—over "macroscopically long" times in some cases.

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[2] Here x is the 4-coordinate, $x = (t, \mathbf{x})$, and $x^2 = t^2 - \mathbf{x}^2$. Analogously, the 4-momentum is $k = (k_0, \mathbf{k})$, $k^2 = k_0^2 - \mathbf{k}^2$.

We will consider this question first in QED, where the calculations can be made sufficiently completely and rigorously. We will see in Sections 3 and 4 that the reality of the half-dressed electron and the various aspects of its behavior are seen in processes which have been studied long ago; at this point we are simply confirming that a particular physical picture which has seen almost no previous use is possible and that as a result it is possible to follow the dressing process. In other words, the reality and observability of the half-dressed electron have already been confirmed experimentally. We will thus be in a position, in Section 5, to extend (in a semiquantitative fashion) this conclusion of the reality of half-dressed, or generally nonequilibrium, particles to hadrons, for which the calculations cannot be carried out as thoroughly. Then we will see (in Section 6) how all this is reflected in certain general equations of quantum field theory, and we will link the reality of nonequilibrium particles to the nature of hadron clusters. The results will be summarized in Section 7.

3. THE DRESSING OF PARTICLES IN CLASSICAL AND QUANTUM ELECTRODYNAMICS (THE "HALF-DRESSED" ELECTRON)

Quantum electrodynamics is a sufficiently complete theory, and all the processes which occur in the electromagnetic surroundings of the electron are of course incorporated automatically. Our problem is simply to distinguish in the theory of certain particular processes those steps in which the dressing process can be followed explicitly (and can be observed in specific experiments). We are interested in situations in which the electron is not playing the role of an equilibrium physical entity with renormalized mass and charge m_e and e but is instead a particle which is largely devoid of its "normal" electromagnetic field. That this should be possible will become clear from the following discussion.

An electron at rest is surrounded by a Coulomb field, with which it interacts, and it is also surrounded by a polarized electron vacuum, in which electrons are partially removed to infinity because of the repulsion, and the excess positive charge is attracted to the electron and to some extent screens the electron charge, reducing $|e_0|$ to $|e|=|e_0|+\delta e$, where $\delta e<0$. If, however, another electron, moving very fast (let us assume that it is ultrarelativistic, for simplicity), is incident on our first electron, which is at rest, and if this fast electron transfers a very large 4-momentum k to it, with $k_0 \sim |\mathbf{k}| \sim \sqrt{k^2}$, then over the collision time

$$T^{\mathrm{coll}} \sim \frac{1}{k_0} \tag{5}$$

nothing will have time to change at large distances $l \gg T^{\mathrm{coll}}$ from the electron: A signal will simply not manage to reach such a point. The main concentration of mass and charge—the bare electron itself along with the nearest part of the field and the vacuum polarization—will have time to move outside a region with dimensions of order T^{coll}. In other words, the Fourier components of the field and polarization for which $|\mathbf{k}| \sim (T^{\mathrm{coll}})^{-1}$ will not be entrained by the ejected complex—the half-dressed electron. We are interested in the fate and properties of this half-dressed electron, on the one hand, on the other, in those of the peripheral part of the field, from which the core has been removed.

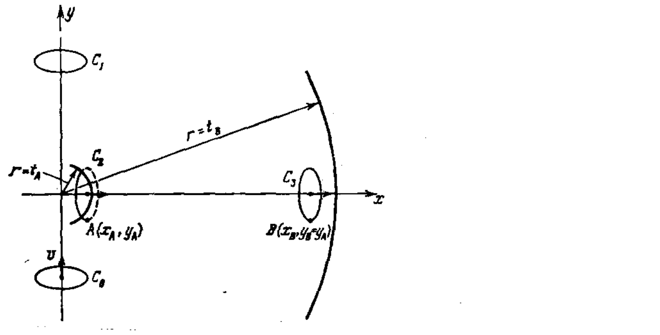

FIG. 1. Scattering of a charge through an angle of 90°, followed by the decay of a "half-dressed" charge into a "dressed" charge and an electromagnetic wave.

Dirac discussed some similar possibilities already in 1933. Using terminology from the theory of holes, he wrote that polarization would cause all the charges introduced into the sea of electrons in negative energy levels to be reduced by 1/137th of their value. He thought (but did not claim to be able to prove, since the corresponding calculations had not yet been carried out) that this would not be the case for electrons moving at a very high velocity, since in this case the "polarization" would simply not have time to be established.[13]

Consequently, the effect is primarily not of a quantum nature and stems from the fact that the propagation velocity of the interaction is bounded. We will therefore first consider everything in classical electrodynamics.[14]

Let us assume that a small charged sphere (the "electron") is moving at a relativistic velocity v $(1-v \ll 1)$ along the y axis, coming from minus infinity, $y=-\infty$ $(y=vt)$. The electron is surrounded by its relativistically contracted electromagnetic field. For example some line of constant field intensity would have the elliptical shape of curve C_0 in Fig. 1, [actually, the ellipse is compressed by a factor of $(1-v^2)^{-1/2}$, far more than in Fig. 1 if $1-v\ll 1$]. We assume that at $t=0$ the sphere is scattered at the point $x=y=0$ through a large angle θ, for example, $\theta=90°$ (actually, it is sufficient to assume $\theta \gg \sqrt{1-v^2}$; see the discussion at the end of Section 4), so that at $t>0$ the sphere is moving along the x axis, and we have $x=vt$. Clearly, at least after a sufficiently long time, $t\to+\infty$, the sphere will again be surrounded by the same normal, relativistically compressed field, but oriented in a different way (C_3 in Fig. 1). Clearly, this change in the field configuration cannot occur instantaneously. Let us say that a signal announcing the scattering reaches the point $A(x_A, y_A)$ not earlier than after a time $t=r_A=\sqrt{x_A^2+y_A^2}$, at which the electron is at the point $x_A=vt$, so that $y_A=x_A\sqrt{1-v^2}/v \approx x_A\cdot m_e/E$, where E is the energy of the electron. At $y>y_A$ at this time, the electron is still lacking its normal field, and this is true in general outside the sphere $r=t$. This missing part of the field is shown by the dashed curve (Fig. 1).

We could describe this situation differently. When the electron begins to move along the x axis, the field, observed in the rest frame of the electron, does not reach the point with ordinate y_1 (*in this frame*) before the time $t_1 = |y_1|$. In the laboratory frame the transverse component retains its value, $y = y_1$, while the time changes, $t = t_1 \cdot E/m$. Accordingly, there is a field at points with the given $x_A = vt$ only at $|y| = |y_1| < t_1 = t \cdot m_e/E \approx x_A \cdot m_e/E$.

This situation is of course reflected by the complete equations for the field. For example, according to the Liénard-Wiechert equations, the electric field at a point at a distance r from $x = y = 0$ has the following value at the time t if there was no charge there at $t < 0$ and if the charge was accelerated instantaneously to a velocity v at $t = 0$:

$$\mathbf{E}(r, t) = e\frac{(1-v^2)(\mathbf{r}(t') - \mathbf{v}r(t'))}{(r(t') - \mathbf{r}(t')\mathbf{v})^3} + e\frac{[\mathbf{r}(t), [(\mathbf{r}(t') - \mathbf{v}r(t')), \dot{\mathbf{v}}]]}{(r(t') - \mathbf{r}(t')\mathbf{v})^3}. \tag{6}$$

Here e is the charge, and t' is the time which satisfies the equation $t' + r(t') = t$ [the field produced by the charge moving along the y axis at $t < 0$ has already left the points $(x > 0, y < 0)$, $|y| \ll vt$; this field is shown by curve C_1 in Fig. 1 and forms one of the two bremsstrahlung cones[3)]]. If there is no charge at $t < 0$, the condition $t' + r(t') = t$ can be satisfied only at $t \geqslant r$. It also follows that at the point with ordinate y there is no field until

$$t \geqslant |y| \frac{E}{m_e}, \tag{7}$$

at which times the electron will be at points with abscissa $x = vt \approx |y| E/m_e$, i.e., $x \gg |y|$ (actually, $t = r = \sqrt{x_A^2 + y_A^2}$ means $t = \sqrt{v^2t^2 + y_A^2}$, i.e. $t = |y_A|/\sqrt{1-v^2}$).

The term with $\dot{v}$ in (6) describes a spherical wave which is emerging from the point $x = y = 0$, while the first term describes the normal, relativistically compressed field of a charge in uniform motion. This field, however, exists only inside the sphere $r = t$. As a result, the field has the structure shown by solid curve C_2. The electron is partially devoid of its own field. As time elapses, however, the front of the spherical wave (moving at a velocity $c = 1$) moves progressively farther from the electron (which is moving at a velocity $v < 1$); at, for example, point $B(x_B, y_B = y_A)$, at the same distance as A from the axis of the motion, the normal field of the electron is restored, and the front of the light wave will move ahead.

An electron which was initially half-dressed thus gets its normal field back and is gradually dressed, but this dressing process is very peculiar: In the limit $t \to \infty$, the half-dressed electron converts into a normally dressed electron and moves ahead of the light wave. We wish to emphasize that we are not considering the interior of the classical electron (the sphere); we are not analyzing the process by which the electron is scattered; we are not examining how elastic waves propagate within the sphere; etc. In other words, we are not considering those Fourier components of the field which have very large $(k_0, \mathbf{k})$, with $|\mathbf{k}| \gtrsim r_0^{-1}$, where r_0 is the radius of the sphere. In particular, we do not know just how the energy is redistributed in the course of the collision among the energy of elastic forces, the kinetic energy of the sphere, and the energy of the electromagnetic field when this field is concentrated within the sphere at small $t > 0$ and then moves outward and forms the field of the dressed electron and also the radiation field.

According to (7), for an electron at an energy $E = 10^3$ GeV $\approx 2 \cdot 10^6 m_e$, for example, the field is restored at a distance equal to an atomic radius, $y \approx 10^{-8}$ cm, only after the electron has traveled a distance $x \approx 0.2$ mm. In cosmic-ray extensive air showers, electrons are found with energies E up to 10^{11} GeV $\approx 2 \cdot 10^{14} m_e$. For such electrons, at an even shorter distance, no greater than the Compton wavelength $m_e^{-1} \approx 4 \cdot 10^{-11}$ cm, the field is restored only after the electron has traveled a distance of 80 m.

Turning to the Fourier representation of the electron field, we see that the absence of a field at distances greater than some $|y|$ means that the only Fourier components present are those for which $\mathbf{k}$ has a transverse component, $k_\perp \lesssim |y|^{-1}$ (along the y axis). Accordingly, inequality (7) can also be read in the following way:[16] The time required for the regeneration of the components of the field of an electron with a given $\mathbf{k}$ (the "regeneration time") is

$$T^{\text{regen}}(\mathbf{k}) \geqslant \frac{1}{k_\perp} \frac{E}{m_e}. \tag{8}$$

This classical result should undoubtedly be correct, if only at "nonquantum" frequencies and distances, $y \gg m_e^{-1}$, $k_\perp \ll m_e$. Clearly, it would not be difficult in principle to design an experiment to verify this point. In this half-dressed state, with $t < T^{\text{regen}}(\mathbf{k})$, the electron should have unusual properties, as we will see below.

Before we go into those effects, however, we would like to examine the same process from the standpoint of QED.[16] We will see that the conclusions remain correct in principle in the case $y \ll m_e^{-1}$ also, although the energy dependence of the regeneration time is slightly different.

In QED, the state of a real charged particle, with an electromagnetic field and a polarizing electron vacuum, can be described by a functional Φ written as a Fock column. Each row of the column describes the effect of a state with a certain number of, say, bare electrons (and positrons), which are described in the momentum representation by the functions $e^-(\mathbf{k})$ [and $e^+(\mathbf{k})$], and also photons, $\gamma(\mathbf{k})$. In the interaction representation, for example, a real electron with momentum $\mathbf{k}$ is described by the functional[4)]

$$\Phi(\mathbf{k}) = \left| \begin{array}{l} a_0 e^-(\mathbf{k}) \\ \int a_1(\varkappa, \mathbf{k})\, e^-(\mathbf{k} - \varkappa)\, \gamma(\varkappa)\, d^3\varkappa \\ \int\int \{a_2^{(1)}(\varkappa_1, \varkappa_2; \mathbf{k})\, e^-(\mathbf{k} - \varkappa_1 - \varkappa_2)\, \gamma(\varkappa_1)\, \gamma(\varkappa_2) \\ + a_2^{(2)}(\varkappa_1, \varkappa_2; \mathbf{k})\, e^-(k - \varkappa_1 - \varkappa_2)\, e^+(\varkappa_1)\, e^-(\varkappa_2)\}\, d^3\varkappa_1\, d^3\varkappa_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \end{array} \right|. \tag{9}$$

[3)]The case of a charge *at rest* at the point $x = y = 0$ at $t < 0$ and then accelerated instantaneously to a velocity v was recently studied in detail.[15]

[4)]We will pass over the circumstance (of no importance here) that there is always an infinite number of photons present with an infinitesimally low frequency.

Here the first row corresponds to a state with a single bare electron; the second row corresponds to a superposition of states [the distribution function is $a_1(\varkappa;\mathbf{k})$] with a single bare electron of momentum $\mathbf{k}-\varkappa$ and one photon of momentum $\varkappa$; the third row corresponds to a superposition of states with one electron and two photons and also with one electron and one electron pair; etc. If we seek Φ from the Schrödinger equation by perturbation theory, we see that the probabilities corresponding to the last rows differ by factors $|a_{i+1}/a_i|^2 \sim 1/137$. Renormalization comes into play beginning in the third row, but we will restrict the present discussion to the lowest approximation in the charge e, and we will consider only two rows:

$$\Phi(k) \approx \begin{pmatrix} a_0 e^-(\mathbf{k}) \\ \int a_1(\varkappa;\mathbf{k})\, e^-(\mathbf{k}-\varkappa)\, \gamma(\varkappa)\, d^3\varkappa \end{pmatrix}. \tag{10}$$

We are thus ignoring the vacuum polarization ($\sim a_2^{(2)}$), which is an effect of higher order in e.

The dressing process can be treated as follows: We assume that at $t=-\infty$ we have only the bare electron; in other words, we are specifying the initial condition

$$\Phi_{t\to-\infty} = \begin{pmatrix} e^-(\mathbf{k}) \\ 0 \end{pmatrix}. \tag{11}$$

We now solve the Schrödinger equation, which takes the form $\partial\Phi/\partial t = -iH_{\text{int}}\Phi$ in the interaction representation, where H_{int} is the Hamiltonian representing the interaction of the particle and the field. We assign H_{int} a factor $\exp(-\xi|t|)$, where ξ is an infinitesimal positive quantity. In doing so, we are turning on the interaction of the bare electron with the degrees of freedom describing the electromagnetic field adiabatically slowly at $t=-\infty$, while at $t\to+\infty$ we are turning it off. Then for the finite values of t in which we are interested, we can, retaining this factor, assume $\xi|t|\ll 1$. After we have completed all the calculations for some particular process (which may last for a long, but still finite time), we must set $\xi=0$. As a result, the quantity $a_1(\varkappa;\mathbf{k})$ assumes the following value after a finite time:[16]

$$a_1(\varkappa;\mathbf{k}) = a_1^{(0)}(\varkappa;\mathbf{k}) = \frac{eM\exp[i(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\varkappa}-\varkappa)t]}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\varkappa}-\varkappa}, \tag{12}$$

$$\varepsilon_{\mathbf{k}} = \sqrt{\mathbf{k}^2+m_e^2}, \quad \varepsilon_{\mathbf{k}-\varkappa} = \sqrt{(\mathbf{k}-\varkappa)^2+m_e^2}, \tag{12a}$$

where M is some factor which depends on the spinor amplitudes of the electron, etc. The distribution of field components in (12) also describes the field ("entrained" field) of an electron which is moving with a momentum $\mathbf{k}$.

In a similar manner, we can now find Φ in the more complicated case in which the electron is scattered (again, at a finite t) by, for example, a Coulomb scattering center (Fig. 1). If we wish to follow the time evolution of the process, we must, of course, from the outset choose the initial state to be a wave packet consisting of functionals as in (10). It turns out that the temporal length of the packet, Δt, and the spatial length $L=v\Delta t\approx\Delta t$, can be taken to be comparatively small if $|\mathbf{k}|$, $\varkappa$, and the scattering angles are sufficiently large (the only requirement is that Δt be greater than the reciprocals of all the momenta, including the momentum transferred during the scattering of the electron: $\Delta t \gg |\mathbf{k}|^{-1}, |\mathbf{k}-\varkappa|^{-1}, |\mathbf{k}_1|^{-1}, |\varkappa|^{-1}$). Then this temporal length will not affect the results in which we are interested. We again solve the Schrödinger equation by perturbation theory (now with respect to the relative scattering potential V).

We set

$$\Phi = \Phi_0 + \Phi_1, \tag{13}$$

where Φ_0 is the solution which is not perturbed by the scattering potential [the wave packet consisting of the states of dressed particles, i.e., consisting of solutions of the type (10) and (12)], while Φ_1 is the change in this solution which results from the scattering. For the packet Φ_1 we obtain a superposition of states, generally of the same type as (10). If we pick out from the packet a term which describes the motion of the electron along the x axis with momentum $\mathbf{k}_1=(\mathbf{k}_1,0,0)$ [generally speaking, of course, this involves an integral over all $\mathbf{k}_1$, $\Phi_1=\int\Phi_1(\mathbf{k}_1;\mathbf{k})d^3k_1$]; then at $t>0$ (more precisely at $t\gg\Delta t$) we find

$$\Phi_1(\mathbf{k}_1;\mathbf{k}) = \begin{pmatrix} a_0 e^-(\mathbf{k}_1)\, V_{\mathbf{k},\mathbf{k}_1} \\ \int \{a_1^{(0)}(\varkappa;\mathbf{k}_1)\,(e^{-it/T_\varkappa(\mathbf{k}_1)}-1) + a_1^{(0)}(\varkappa;\mathbf{k})\, e^{-it/T_\varkappa(\mathbf{k}_1)}\} \\ \times e^-(\mathbf{k}_1)\,\gamma(\varkappa)\, V_{\mathbf{k},\mathbf{k}_1-\varkappa}\, d^3\varkappa \end{pmatrix}. \tag{13a}$$

Here $V_{\mathbf{k},\mathbf{k}'}$ is the amplitude for the scattering (by the potential) of an electron from state $\mathbf{k}$ to state $\mathbf{k}'$. Furthermore,

$$T_\varkappa(\mathbf{k}) \equiv T_\varkappa^{\text{regen}}(\mathbf{k}) = -\frac{1}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}-\varkappa}-\varkappa} > 0. \tag{14}$$

If we denote the transverse (with respect to $\mathbf{k}_1$) component of $\varkappa$ by $\varkappa_\perp$, then we have $\varkappa_\perp \ll \varkappa \le |\mathbf{k}|$ in the relativistically compressed field of the electron and in the emitted radiation. Then from (14) we find $T_\varkappa^{\text{regen}}(\mathbf{k}) \sim k^2\varkappa/(m_e^2\varkappa^2+\mathbf{k}^2\varkappa_\perp^2)$. For the main part of the spectrum, moreover, we have $\varkappa_\perp \sim \sqrt{1-v^2}\,\varkappa \sim \varkappa|\mathbf{k}|/m_e$. Therefore,

$$T_\varkappa^{\text{regen}}(\mathbf{k}) \sim \frac{\mathbf{k}^2\varkappa}{m_e^2\varkappa^2+\mathbf{k}^2\varkappa_\perp^2} \xrightarrow[\varkappa_\perp\sim\sqrt{1-v^2}\,\varkappa]{} \frac{1}{\varkappa_\perp}\frac{|\mathbf{k}|}{m_e}. \tag{14a}$$

We have a reason for calling this quantity the "regeneration time," as in (8). Let us examine in more detail a field structure (the second row) in Φ_1 in (13a) which is more complicated than that in (10) and (12).

Here the second term, $a_1^{(0)}(\varkappa;\mathbf{k}_1)e^-(\mathbf{k}_1)\gamma(\varkappa)\exp[-it/T_\varkappa^{\text{regen}}(\mathbf{k})]$, describes an electron which is moving in a new direction with a momentum $\mathbf{k}_1$, according to Eqs. (10) and (12), but in combination with the field $\gamma(\varkappa)$; which is distributed over $\varkappa$ in accordance with $a_1^{(0)}(\varkappa,\mathbf{k})$. Consequently, this field is distributed as it would be for an electron moving with the initial momentum, $\mathbf{k}$. This term thus describes the bare electron after scattering (with momentum $\mathbf{k}_1$), combined with the previous field, which is again moving along the y axis. This is the field which surrounded the original electron and which had been stripped from it (contour C_1 in Fig. 1). This field forms one of the two bremsstrahlung cones (the "forward cone") (we recall that we are dealing with the ultrarelativistic case, $1-v\ll 1$, in which the field of the electron is almost purely transverse and may become a radiation field).

We are particularly interested in the first term in braces in the second row in (13a), which has the factor $\exp(-it/T_{\varkappa}^{regen}(\mathbf{k}_1)-1)$, which is of fundamental importance. At $t \ll T_{\varkappa}^{regen}(\mathbf{k}_1)$, this term disappears completely. This disappearance means that as the electron moves along the new direction it initially does not have its field. In complete accordance with Fig. 1, it is still half-dressed; *it is bare with respect to those components of the field* $\varkappa$ *for which the following inequality holds*:

$$\varkappa_\perp \leqslant \frac{1}{t}\frac{|\mathbf{k}|}{m_e}. \tag{15}$$

The smaller t is, the larger is the part of the spectrum which is missing.[5)]

At $t \gtrsim T^{regen}(\mathbf{k}_1)$, however, the term $a_1^{(0)}(\varkappa;\mathbf{k}_1)[\exp(-it/T_{\varkappa}^{regen}(\mathbf{k}_1))-1]$ splits in two. The first of the new terms has according to (12), the normal field of an electron with a new momentum $\mathbf{k}_1$, while the second new term corresponds to the same radiation field as that which has been stripped off, but now it is oriented along the $\mathbf{k}_1$ axis. This is the second bremsstrahlung cone. Accordingly, as in classical electrodynamics, *the regeneration of the field of the scattered electron occurs through decay into a normally dressed electron and a radiation field.*

There is another useful way to interpret this term [we are still talking about the first term in braces in (13a)]. We can say that superimposed on the normal field of an electron with a spectrum $a_1^{(0)}(\varkappa;\,\mathbf{k}_1)$ there is a radiation field which has precisely the same spectrum but the opposite phase (a minus sign inside the braces), so that these fields cancel each other out at $t \ll T^{regen}$. Only later do these fields become spatially separated because of their different velocities: The normal field of the electron is restored, while the radiation field moves off in the forward direction (to see this, it is necessary to consider the entire packet).

But the estimate of the regeneration time in (14a), which agrees with the classical value in (8), was derived in (14a) only for those photon wave vectors $\varkappa$ (and, correspondingly, for $y \sim \varkappa_\perp^{-1}$) which describe the general features of the field structure, $\varkappa_\perp \sim m_e\varkappa/|\mathbf{k}| \lesssim m_e$, i.e., for $y \gtrsim m_e^{-1}$. For smaller distances from the center of the electron $y \ll |\mathbf{k}|(\varkappa m)$, we find from the central term in (14a)

$$T_{\varkappa}^{regen}(\mathbf{k}) \gg \frac{\varkappa}{\varkappa_\perp^2} \sim \frac{1}{\varkappa_\perp}\frac{|\mathbf{k}|}{\varkappa_\perp} \qquad \left(|\mathbf{k}| \gg \varkappa_\perp \gg \frac{m_e\varkappa}{|\mathbf{k}|}\right). \tag{14b}$$

This quantity is still much larger than the distance $y \sim \varkappa_\perp^{-1}$ from the axis of the motion; i.e., the relativistic dilatation of the regeneration time occurs even in the case $y \ll m_e^{-1}$ (however the dependence on k and $\varkappa$ turns out to be somewhat different). The dilatation effect disappears only when we go to the limit allowed by the electron energy, $\varkappa_\perp \sim |\mathbf{k}|$, in which case we find $T^{regen} \sim \varkappa_\perp^{-1} \sim y$. Since the times (including T^{regen}) must exceed $L, L \gg |\mathbf{k}|^{-1}$, we are justified in considering the region $y \gg |\mathbf{k}|^{-1}$ alone. At ultrarelativistic energies, of course, we would still be dealing with a physically observable particle, although it would be devoid of its field in a very large region of space. According to (14b), for example, an electron of energy $|\mathbf{k}| \sim 10$ GeV $\sim 10^4 m_e$ can travel the distance between the regular sites in a crystal lattice, $d \sim 10^{-8}$ cm $\sim 100 m_e^{-1}$, and its normal field would be restored only at very short distances. According to (14b), the field is restored (i.e., we have $T_{\varkappa}^{regen} \sim d$) only at distances $y \sim \varkappa_\perp^{-1}$ such that $d \gtrsim |\mathbf{k}|y^2$ or

$$y \leqslant \sqrt{d\,|\mathbf{k}|} \sim 10^{-1}\, m_e^{-1} \sim 10^{-12}\,\text{cm}.$$

Although this is a nearly bare electron, it is still a physical object, fundamentally different from a bare electron.

To summarize: 1) *Elementary nonquantum arguments regarding the regeneration time which are based on the signal propagation velocity are valid even inside such a complicated relativistic quantum system as an electron with its own field* (at least for $y \gtrsim m_e^{-1}$). 2) In classical mechanics, the normal structure of the particle is restored as the result of a gradual increase in the dimensions of the region in which the field becomes normal, with a gradual detachment of the light wave which forms the forward front of the expanding system (Fig. 1), but in the quantum case the half-dressed particle decays into a normally dressed particle and a photon (the *probability* of observing the given state increases gradually).

If we had taken into account the following rows in Φ in (9), we also would have found events involving a decay into an electron plus several photons, into an electron plus a pair, etc. Due to the smallness of the number $e^2 \approx 1/137$, however, the corresponding probabilities are reduced by a factor of $(e^2)^{\nu-2}$, where ν is the number of the row.

We will conclude this section with a comment regarding the history of this question. That an electron at rest should, upon instantaneous acceleration, give rise both to an "entrained" transverse field and to radiation and that the energies of the two fields should be equal was first pointed out a very long time ago in a QED study of nonrelativistic motion by Ginzburg.[17,18]

4. OBSERVABLE PROCESSES FOR A HALF-DRESSED ELECTRON

We will now show that certain aspects of the behavior of a half-dressed electron have in fact been observed in phenomena which have been under study for a long time. In order to observe such an electron we must first produce it (for example, as a result of scattering of a normal electron, as in Fig. 1), and then we must watch it as it interacts with some other object. This, however, is a process of higher order than simple scattering,

[5)]It might appear that in the limit $t \to 0$ we always obtain from this a completely bare initial electron. However, the condition $t \gg \Delta t \approx L$ must hold, where L is the length of the packet, and $L \gg |\mathbf{k}_1|^{-1}$. Accordingly, a field remains in some part of the volume, if only a small part. Since the electromagnetic energy of the electron depends only logarithmically on the size of the region occupied by the field, and the electromagnetic mass is formed from the contributions of the deepest regions, the energy of the missing part of the normal field is very small in comparison with m_e, even at $t \sim \Delta t$. This is a physically real entity, even though it is "nearly bare."

and in the framework of QED we can study it as a single process, without concerning ourselves with what the electron looks like in the intermediate state. The answer will of course be correct. However, if the intermediate state exists long enough, then it is not only meaningful to segregate this state from the general result and analyze it independently; it is in fact necessary to do this, for a purely utilitarian goal: By utilizing the processes for which calculations are carried out in QED we can construct models for and understand corresponding processes in hadron physics, where reliable calculations can be carried out only partially.

We note first of all that Coulomb scattering (or some other deflection in an external field) remains the same for the *half*-dressed electron in our approximation, (10), as in the case of a dressed electron, and a renormalization of neither the mass nor the charge does yet have any effect. Accordingly, the effect in which we are interested should be sought only in processes in which there is a redistribution of the peripheral field, i.e., in radiation processes. The most suitable of these processes are those in which a role is played by the "radiation formation zone" or "coherence length," which is important in the theory of electromagnetic processes at very high energies.

The beginnings of this concept can already be seen in classical physics. In a 1942 paper,[19] Frank studied the radiation field of an electron which was initially at rest, which was accelerated instantaneously, which traveled a distance L in uniform motion and which was then stopped instantaneously (see also the paper by Tamm[49]). It turned out that the resulting radiation, for example, at the wavelength λ, observed at an angle ϑ from the axis of motion, was missing if the distance traveled was equal to the length of the Fresnel zones, $\lambda/(1-v\cos\vartheta)$, multiplied by some integer. The radiation reached a maximum when this number was a half-integer. From this we already see that the radiation can be thought of as being formed over the entire distance L. However, it may also be thought of as being the result of a superposition of two waves, emitted from the initial and final points, where the acceleration has opposite values. This conclusion also follows from the general theorem of Rubinowicz.[20]

The formation-zone concept appeared in mature form in a paper by Ter-Mikaelyan[21,22] (see also Ref. 23) for the general case (QED, with recoil, etc.). The effect can be summarized as follows.

If an electron of momentum $\mathbf{k}_0$, scattered by a "third body" (an atom or a crystal), emits a photon with momentum $\varkappa$ and goes to a final state with momentum $\mathbf{k}$, then the entire process occurs (at ultrarelativistic energies) in a highly elongated spatial region, whose longitudinal dimensions (parallel to $\mathbf{k}_0$) increase with increasing $|\mathbf{k}_0|$. This conclusion follows immediately from the fact that the matrix element for this process includes an integral of the type

$$M \sim \int e^{i(\mathbf{k}_0-\mathbf{k}-\varkappa)\mathbf{r}}\, W(\mathbf{r})\, d^3r, \tag{16}$$

where $W(\mathbf{r})$ carries information on the interaction with the scatterer. The exponential factor itself shows that a region of space with a longitudinal dimension $\sim l_f$ is important:

$$l_f = \frac{1}{|k_0 - k_\parallel - \varkappa_\parallel|}, \tag{17}$$

where $k_\parallel$ and $\varkappa_\parallel$ are the longitudinal components of the vectors $\mathbf{k}$ and $\varkappa$. At high energies, the scattering and emission angles are small, $\vartheta_\varkappa, \vartheta_\mathbf{k} \sim m/|\mathbf{k}|$, and this expression becomes $l_f \approx |\mathbf{k}_0|\,|\mathbf{k}|/\varkappa m_e^2$, so that in the principal bremsstrahlung region, $k_0 \sim k$, we have

$$l_f \approx \frac{1}{\varkappa_\perp}\frac{|\mathbf{k}_0|}{m_e}. \tag{18}$$

If, for example, an electron is incident on a crystal along its axis, and the condition $l_f \gg d$ holds (d is the distance between adjacent sites), then all $N \sim l_f/d$ sites within the zone act coherently on the electron; the amplitudes resulting from the scattering of different atoms are summed; and W is higher than the value corresponding to a single atom by a factor of N. On the other hand, if there is no more than a single lattice site on this path then the emission occurs independently at each site, and it is the probabilities rather than the amplitudes which are summed.[21,22]

This result was originally perceived as paradoxical, since the crystalline structure of the scatterer was important—not at *low* energies (and at large wavelengths, $|\mathbf{k}_0|^{-1}$ and $|\mathbf{k}|^{-1}$ comparable to d), as would be expected—but at *high* energies, where the wavelengths are much smaller than d, but l_f which is proportional to $|\mathbf{k}_0|$, is greater than d. When this result was finally understood, it became the basis for theories for many other radiation processes in media.[23] It was only a year later, for example, that Landau and Pomeranchuk[24] noted that if an emitting electron underwent strong multiple scattering in a medium [or if there was some other distortion of the plane waves in the integral in (16)] within the formation zone then the integration range would effectively be cut off, and the bremsstrahlung intensity would be reduced (Ter-Mekaelyan[25] showed that a similar role would be played by a refractive (index of the medium, that differs from unity, etc.). Because of this scattering, the vectors $\mathbf{k}$ and $\varkappa$ fluctuate within the integration range, and over a certain distance l_s the mean square scattering angle may exceed the typical emission angle $\vartheta \sim m_e/|k_0|$, which determines the effective value of l_f in (17). In the case $l_s \ll l_f$, the value of M is reduced by a factor $\sim l_f/l_s$.

We approach this question from a somewhat different standpoint. We will show that the reason for these features of the radiation process is that here the electron is half-dressed in a certain stage (within l_f). This conclusion is indicated by the agreement of two expressions—that for the path travelled by the scattered electron in the half-dressed state (with respect to the component of the field $\varkappa$), $l^{\text{regen}} = vT_\varkappa^{\text{regen}} \approx T^{\text{regen}}$ in (8), and that for the coherence length or the length of the photon-formation zone, l_f in (18). However, we can get a better picture from the following example.

We assume that, after being scattered through a large angle (Fig. 1), the electron is moving without external perturbations for a time $t \gg T_\varkappa^{\text{regen}}$. It ultimately decays into a dressed electron and a photon. This means that

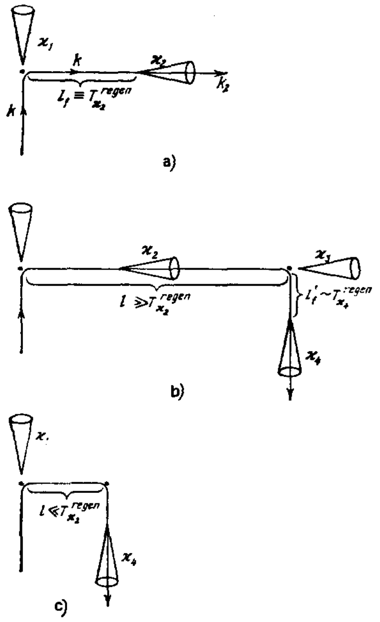

FIG. 2. Bremsstrahlung in single scattering (a) and in double scattering (b and c) for equal ratios of the distance (l) between the scatterers to the distance over which the electron cloud is regenerated ($T^{\text{regen}}_{\varkappa_2}$).

the overall result of the scattering at the point $x = y = 0$ is normal bremsstrahlung confined to two cones,[6)] for $\varkappa_1$ and $\varkappa_2$ (Fig. 2a). If the electron (now dressed) is scattered a second time at a distance $l \gg l_f \equiv T^{\text{regen}}$, this event will be independent of the preceding event. Accordingly, there will again be two ordinary bremsstrahlung cones $\varkappa_3$ and $\varkappa_4$ (Fig. 2b). If, however, the second scatterer is at a *short* distance, $l \ll T^{\text{regen}}$, then the electron (still half-dressed; the necessary field component does not exist within the cloud) does not have time to decay into a dressed electron plus a photon. Accordingly, there will be no second cone $\varkappa_2$ as in Fig. 2a, but there will the cone $\varkappa_3$, which arises in the second scattering event (Fig. 2b), since the half-dressed electron has nothing to shed. The general picture is as shown in Fig. 2c; i.e., the overall result of the double scattering is two, not four, bremsstrahlung cones. We can say that the bremsstrahlung cross section (or the cross section for inelastic scattering, σ_{inel}) is half that for two independent scatterers. Calculations of the sequential steps in QED lead to the same picture, of course.[16] This is, in fact, the Landau–Pomeranchuk effect.[24]

We can thus say that *a half-dressed electron is an observable entity, seen in well-studied phenomena, and the procedure by which the electron becomes dressed can be followed by quantum field theory* (in this case, QED).[7)]

However, the interactions of a nonequilibrium particle (for example, a half-dressed electron) with different scattering centers, i.e., the interactions of this particle during its regeneration time, can by no means always be reliably distinguished from each other. Strictly speaking, these interactions constitute a single process, which should be treated at the amplitude level, rather than at the probability level. In Fig. 2c, for example, rescattering *through a large angle* suppresses the emission of the photon $\varkappa_2$. Here there is a simple shielding. We can say in this case that the nonequilibrium particle is an entity which can be characterized by, for example, its cross section for scattering, for the bremsstrahlung emission of a photon $\varkappa_4$ (but there is no $\varkappa_3$), absorption, etc. If, on the other hand, the second scattering is through a *small* angle, $\vartheta_e \lesssim m_e/E$ or $\vartheta \lesssim \varkappa_1/\varkappa$, then interference will be very important. An example of this case is the bremsstrahlung in a crystal, in which the basic effect (the emission of the photon $\varkappa$) is *intensified* as a result of rescattering by subsequent lattice sites within the distance l_f (the Ter-Mikaelyan effect). A sort of *"antishielding"* is occurring. We cannot speak in terms of the bremsstrahlung cross section of the nonequilibrium particle as an independent entity. We will see in Section 5 that a similar distinction between situations can occur in hadron physics.

5. NONEQUILIBRIUM HADRONS

In the quantum field theory for strong interactions, it is not possible to carry out the direct calculations which are possible in QED. Nevertheless, the clear physical meaning of the results for the electron (Sections 3 and 4) allows us to extend these results (in a semiquantitative manner) to hadrons at the model level; by using the experience gained in QED we can work from the rigorous general relations of quantum field theory to obtain some auxiliary conclusions. A more detailed study (although still at the model level) has already been begun of the interaction of nonequilibrium particles at the field-theory level.

The main point for this section is the basic conclusion of the preceding discussion: *Structural changes in the quantum particle can be treated in coordinate space*;

[6)]The formation zone is sometimes identified with the interaction region. Actually, the interaction with the *scatterer* may occur at a point or in a very small region, as in Fig. 1, while the formation zone is determined by the change in the field structure (by the spatial separation of the radiation field and the field of the electron, which are interfering with each other); this change occurs over a much longer distance during the subsequent free motion of the nonequilibrium system. If, however, there are many new interaction regions in the overall formation zone, then these concepts may mean the same thing (as in the case of a crystal lattice[21,22]). The role played by the formation zone in the case of transition radiation has been analyzed in detail.[26]

[7)]Since it decays into a dressed electron and a photon, its mass, m_e^*, is larger than m_e. It may seem strange that an electron's mass would increase when it lost part of its cloud, i.e., when it lost part of the positive energy of its field, $\int [(E^2+H^2)/8\pi]\, d^3r$. However, as was emphasized earlier (Section 2), we are completely ignoring those field components (with very large $\varkappa_\perp$) which, along with other ("mechanical") forces, keep the particle stable. In the course of the scattering, there is a redistribution of the energy (in the classical Lorentz model, elastic waves, etc., should propagate in the electron). It is this energy redistribution which ultimately leads to a $k^2 \equiv m_e^{*2}$ which is larger than m_e. The mass increases, of course, because of the decrease in kinetic energy, which we are ignoring.

they occur in a time no shorter than the time required for a signal to propagate across the volume of the particle. Because of the short-range nature of the nuclear forces, the concept of a volume has a more definite meaning for a hadron than for an electron, so this conclusion is even simpler and clearer in the case of hadrons.

Furthermore, as we will see, *the regeneration time is independent of e^2, i.e., is independent of the interaction force between fermions and bosons.*[16] At first glance, this is a surprising result: We would expect that as the interaction constant became larger (in QED, this interaction constant is $e^2 = 1/137$) the electron would radiate its field off and polarize the vacuum more rapidly. However, as e^2 is increased the equilibrium field which must be produced itself becomes more intense. The two effects operate in opposite directions and cancel each other out, so that an explanation based solely on the signal-propagation time is not contradictory. The calculated value of T^{regen} is thus definitely correct as long as perturbation theory is valid, e.g., if e^2 is raised to a value of the order of $1/10$. There is no reason to believe that there will be any important changes in the limit $e^2 \to 1$, for example, in quantum mesodynamics.

Let us consider the collision of two nucleons of mass m_N and energy $E \gg m_N$ in their center-of-mass system, in which their longitudinal dimensions, $r_0 \sim m_\pi^{-1}$, are reduced by a factor of E/m_N. Hadron collisions are known[3] to be peripheral collisions (in reactions involving a "normal" small transverse momentum transfer): The hadrons release some of their energy to a certain subsystem, while they themselves move forward with an energy E^* of the same order as E. In general, their internal state is distorted, and their mass satisfies[8)] $m_N^* \geq m_N$, so they may decay into stable particles. This is the "fragmentation" part of the reaction products. We can assume that the duration of the collision is $T^{\text{coll}} \sim r_0 m_N/E$. Working from experience in electrodynamics, we can make several assertions.

a) The restoration of a normal structure requires a time no shorter than the time required for the signal to propagate across the volume. In the rest frame of the particle, this time is $\gtrsim r_0$, so that in the c.m. system we have

$$T^{\text{regen}} \gtrsim r_0 \frac{E^*}{m_N^*} \sim r_0 \frac{E}{m_N}. \tag{19}$$

The collision and the regeneration are thus sharply distinguished, as in the case of the electron, which underwent a sharp change in momentum in the first collision (Fig. 1):

$$\frac{T^{\text{regen}}}{T^{\text{coll}}} \gtrsim r_0 \frac{EE^*}{m_N m_N^*} \sim r_0 \frac{E^2}{m_N^2} \gg 1. \tag{20}$$

b) The restoration process involves a decay into equilibrium hadrons of various types. If m_N^* is very large, and $n \gg 1$ particles are produced, the process may be a gradual quasiclassical expansion of the type in Fig. 1.

There are two types of interactions which perturb the internal state of the particle: diffractive dissociation, in which the charge, strangeness, etc., of the incident particle do not change; and an interaction which does change these characteristics. Inequalities (19) and (20) hold in both cases. The uncertainty in the mass m_N^* which results from the instability of the particle is usually unimportant: $\Delta m \sim (T^{\text{regen}})^{-1} \lesssim m_\pi$.

There are similar relations for those particles which arise because of the energy transferred by the primary nucleons, $E - E^* \equiv KE$. If (as is assumed in certain models) a subsystem forms and moves slowly in the c.m. system (a heavy cluster), then it also will decay into stable hadrons. This part of the products is referred to as the "pionization" part (or the "central" part). If this part is at rest as a whole in the c.m. system, then the decay occurs over a time no shorter than the time which follows from the dimensions of that system which should result from the decay into n hadrons, $r_0 \sim m_\pi^{-1}\sqrt[3]{n}$. In the laboratory system, this time is longer by a factor $\sim E^*/m_N^*$. In any system, however, this time will be shorter than that for the fragmentation part by the same factor.

The quantity r_0 is of order m_π^{-1}, but we can specify it more accurately: According to data on electron–proton scattering, the rms radius of the proton is $\sqrt{\overline{r^2}} \approx 0.77$ fm $\approx (2m_\pi)^{-1}$. The mean radius is clearly even smaller, so for a nucleon it is reasonable to assume

$$T^{\text{regen}} \gtrsim \frac{1}{m_{\text{eff}}} \frac{E}{m_N}, \quad m_{\text{eff}} \sim (2-3)\, m_\pi. \tag{21}$$

Here, however, there is an important distinction from electrodynamics.

a) To some extent, we can still think of the nucleon as a particle surrounded by its field: a pion cloud. We can expect that the polarization of the nucleon vacuum is significant only over distances of order $(2m_N)^{-1}$. Consequently, our concept of a half-dressed nucleon devoid of a certain peripheral part of its field is permissible. For mesons, however, this picture can be adopted only with extreme reservations. It is better in this case to speak in terms of a *disruption of the equilibrium structure* of a more complicated nature.

The regeneration process (which of course also occurs through a decay into stable hadrons) also requires a time no shorter than the time required to propagate across the volume of the final system.

We will nevertheless see that in certain models (e.g., the parton model) (see Figs. 4–6 below) this deviation from equilibrium takes the form of a hadron which has spatial dimensions smaller than those of a stable hadron with the same quantum numbers. There is accordingly some justification for the term "half-dressed."

b) The fact that part of the field of the electron is missing has only a slight effect (in terms of higher order in e^2) on the intensity of the subsequent scattering. The hadrons, on the other hand, interact with all the

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[8)] Here m_N is the energy of the lowest-lying stable state (with the given quantum numbers) of the system of fermions plus bosons. Accordingly, any perturbed state of this system will have a higher energy.

elements of their volume, and the scattering of a half-dressed hadron (which is generally a nonequilibrium hadron) may be very different from that for an equilibrium hadron. For example, if a ρ meson results from the first scattering event (this is an example of a *resonant* nonequilibrium structure), and this meson is capable of decaying into pions, we can discuss a model in which there is a superposition of states with different numbers of pions, and for this model we must take into account the scattering of each of these pions in calculations for the subsequent interaction.

c) For hadrons, there are many types of resonances (in electrodynamics, there are only systems of the positronium type). If the collision puts the hadron in or near a resonant state (for example, if N under goes a transition to N^* or to a Δ particle), then the regeneration time may be much longer than the right side of (19). This situation corresponds to the Wigner theorem which states that in an elastic collision of two particles in the resonance region the time which the particles spend within the interaction radius is greater than the transit time of the free particles.[27]

d) For sufficiently large values of m_N^*, a decay into many hadrons may be no less probable than a decay into a dressed nucleon and one "field quantum" (a pion or kaon, etc.), in contrast with QED. The functional for the nucleon and the nucleon-like state, Φ_N, or that for the pion and the pion-like state, Φ_π, which has a meaning analogous to that of (9), has a structure which can be described symbolically as follows (the notation is self-explanatory):

$$\Phi_N = \begin{pmatrix} C_0^N N \\ C_1^N N\pi \\ C_2^N N\pi\pi \\ \cdots \\ C_i^N NK\bar{K} \\ C_{i+1}^N NK\bar{K}\pi \\ \cdots\cdots \\ C_j^N NN\bar{N} \\ \cdots\cdots \end{pmatrix}, \qquad \Phi_\pi = \begin{pmatrix} C_0^\pi \pi \\ C_1^{\pi'}\pi\pi\pi + C_1^{\pi''}\pi\rho \\ C_2^{\pi'}(5\pi) + C_2^{\pi''}\pi\rho\rho \\ \cdots\cdots\cdots \\ C_i^\pi \pi K\bar{K} \\ \cdots\cdots\cdots \\ C_j^\pi \pi N\bar{N} \\ \cdots\cdots\cdots \end{pmatrix}. \tag{22}$$

In each row here there is to be an integration over the relative momentum, as in (9). The expansion can be carried out in any orthogonal system of functions; for example, the particles in the rows can be considered free and bare particles, or they may be considered free and equilibrium, dressed particles. Some definite combination of rows—a certain set of the coefficients C_m^s—describes a stable hadron. For it, all the C_m^s except C_0^s are zero in the expansion in terms of dressed particles. After a collision which leaves the hadron in a nonequilibrium state, the set of coefficients is different from the equilibrium set, and a decay should occur. Since the interaction is strong, the various rows may contribute with different weights; i.e., the decay may be a multiple decay, into $n \gg 1$ final particles. For large values of $\langle n\rangle$, however, the decay will be quasiclassical, as we mentioned earlier, with a gradual expansion in coordinate space, until completely dressed hadrons are formed, each with a radius $r_0 \sim m_\pi^{-1}$. Correspondingly, the total regeneration time (in the rest frame of the entity) can be $\gtrsim m_\pi^{-1}\sqrt[3]{n}$. We see why this process can be described by the quasiclassical Pomeranchuk statistical model or even by the Heisenberg-Landau hydrodynamic model (Ref. 2; see also the reviews in Ref. 28).

With these circumstances in mind, let us examine an interaction of a nonequilibrium hadron, i.e., an interaction which occurs during the regeneration time after a collision which has "damaged" the internal structure of a hadron or after the instant at which the hadron is produced. This question is already the object of a huge and growing literature, since recent hadron experiments at accelerator energies have made it possible to reach large relativistic dilatations of the regeneration time. This effect is seen in hadron–nucleus collisions in experiments of two types: In the first approach, a study is made of the secondary interaction of the incident hadron in the same nucleus in which it has already interacted once. In the second approach, there is a study of the secondary interaction of a hadron produced in this nucleus.[9)]

A problem arose two decades ago, when cosmic-ray experiments ($E_{lab} \gtrsim 100$ GeV) led to a strange result (at the time these experiments were not very reliable): A proton loses, on the average, almost precisely the same fraction $\langle K\rangle$ of its energy in collisions with different nuclei, even if they have extremely different atomic numbers A. An extremely important fact was established for protons passing through the atmosphere: For air, we have $\langle K\rangle \approx 1/3 - 1/2$ over a huge energy range, $E_{lab} \sim 10 - 2000$ GeV or even $10 - 10^5$ GeV (Ref. 29) (this value was soon refined to $\langle K\rangle = 0.45 \pm 0.05$). As it turned out, however, the value of $\langle K\rangle$ was roughly the same in the heavy nuclei of a photographic emulsion, in copper, and in lead.[30] Even if we have $\langle K\rangle = 1/3$ in air, it would seem that in a heavy nucleus, in which the proton travels a distance greater by a factor ~2.5 (this distance is proportional to the nuclear radius, $R = r_0 A^{1/3}$), the proton would undergo a number of collisions greater by the same factor, and in lead it should be $\langle K\rangle \sim 0.65$. Later, when accelerator energies reached these levels, this conclusion was established even more firmly. We now know that in a proton–proton collision we have $\langle K\rangle \approx 0.5$, so that we can expect $\langle K\rangle_{p,Pb} = 0.9 - 1.0$. In a collision of a proton with nuclei, up to the very heaviest, the A dependence is very weak, $\langle K\rangle_{pA} \sim A^{0.06\pm(0.02-0.03)}$ (cosmic rays, up to $E_{lab} \sim 10^3$ GeV; Ref. 30c).

Accordingly, everything points to the conclusion that a nucleon does not undergo a second collision in a nucleus.

There is another fact pointing toward this conclusion: the extremely weak A dependence of the multiplicity $\langle n\rangle$ of the hadrons produced in proton–nucleus collisions. Even if the new hadrons produced in a nucleus

9) These are the particles which are sometimes called "immature" or "young." Drawing from the terminology used in chemistry, with reference to the activity of the atoms of a substance immediately after a chemical reaction in which they appear, we can say that these are particles "in statu nascendi."[14b]

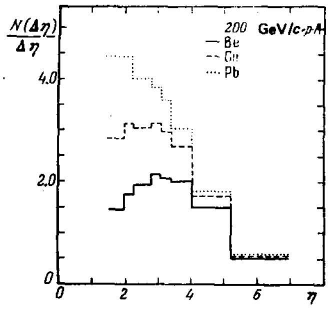

FIG. 3. Distribution with respect to the pseudorapidity $\eta=-\ln \mathrm{tg}\,(\theta_L/2)$($\theta_{\mathrm{lab}}$ is the angle at which the particle is emitted in the laboratory frame) of the particles in the multiple production which occurs in the collision of protons with various nuclei (Be, Cu, Pb) at an energy of 200 GeV (Ref. 32). The distribution was obtained by electronic methods. Similar distributions have been obtained previously by various workers by exposing photographic emulsions in accelerators. The particles which are moving into the forward hemisphere, $\theta_C<90°$, in the c.m. frame for the proton–nucleon collision correspond to $\eta>3$; $\theta_C\leqslant 45°$ corresponds to $\eta>4$.

underwent absolutely no multiplication in the same nucleus, but additional particles were produced solely in subsequent collisions of the primary nucleon, then the value of $\langle n\rangle$ at a high energy would have to be approximately proportional to the average distance traveled by the nucleon in the nucleus, $\langle n\rangle\sim A^{1/3}$ (in this estimate we are ignoring the energy lost by the primary particle in each collision; the multiplicity is a very weak function of the energy). Cosmic ray experiments, on the other hand, definitely showed a far weaker dependence, as was first pointed out in Ref. 31. Accelerator experiments now give us the entire distribution of product particles over the angle θ_{lab} in the laboratory frame. Or, as we would say today, the distribution with respect to the pseudorapidity $\eta=-\ln\tan(\theta_{\mathrm{lab}}/2)$, for collisions with various nuclei. Figure 3 shows that as A increases there is a rapid increase only in the number of product particles which are moving backward in the nucleon–nucleon c.m. frame (corresponding to large emission angles in the laboratory frame): The number of fast particles moving forward in the c.m. frame, remains essentially constant for all values of A. As a result, if we parametrize the dependence as $\langle n\rangle\sim A^{\alpha}$ we find that α is 0.10 – 0.15, depending on the conditions adopted for selecting particles. In fact, a detailed study[32] for p, $\bar{p}$, π, and K collisions with nuclei yielded $\alpha=(0.19-0.23)\pm(0.01-0.02)$. For the fastest protons moving in the forward direction ($\eta\approx 4-6$), however, this value is $\alpha\leqslant 0.1$ (see Fig. 17d in Ref. 32). We see again that the primary leading nucleon undergoes no significant number of repeated interactions.

A very weak dependence of $\langle n\rangle$ on A (with $\alpha=0.19$) was predicted in Landau's hydrodynamic model (the same explanation has been adopted by Busza *et al.*[32]). This dependence is attractive in that it automatically incorporates all the subsequent interactions of not only the initial hadron but also all the products. In its original form, however, this model ran into several contradictions in principle and in fact. First, it denies the existence of leading nucleons in the final state (nucleons which lose only half their energy, on the average). This contradiction can be eliminated if we assume that the nucleon undergoes a peripheral collision of the ordinary type, but that the energy which it gives up goes to a hydrodynamic subsystem which is formed. This "peripheral–hydrodynamic" model works better.[33] Nevertheless, if the leading nucleon remains active, the number of hydrodynamic subsystems would have to increase in proportion to $A^{1/3}$, but this is not what happens.

A difference in multiplicities for particles moving forward and backward was also discovered a long time ago (around 1961) in cosmic-ray work at incident-nucleon energies $E_{\mathrm{lab}}\gtrsim 2000$ GeV, and it was explained on the basis of the following model: Two fireballs of mass $\mathfrak{M}_{\mathrm{fb}}\sim 3$–5 GeV are produced. They move respectively forward and backward in the c.m. frame. They have small dimensions, $r_{0,\mathrm{fb}}\sim\mathfrak{M}_{\mathrm{fb}}^{2}$, so that their cross sections for collision with other nucleons of the nucleus are small. The fireball moving forward, however, expands because of the relativistic time dilatation, and it decays only after it has left the nucleus; the fireball moving backward in the c.m. frame, on the other hand, is moving slowly, and it manages to decay while still inside the nucleus. Its products undergo a further multiplication in the nuclear matter.[1]

We see that in either approach the experimental data indicate that repeated collisions of the incident nucleon in the nucleus do not occur (or at best are of minor importance).

The recognition of this fact more than two decades ago, on the basis of no more than the cosmic-ray data, led to a daring hypothesis (although it was in very rough form when first offered)[30a,34]: If the proton is thought of as consisting of a core—the nucleon proper—and a peripheral pion, it can be suggested that the proton loses its pion in the first collision and is left in a passive state until it acquires a pion cloud again. Nothing is said here about the mechanism for the restoration of this cloud, etc., and the description of the structure and of the interaction of the nucleon is primitive, but the basic concept of a "bare" and therefore passive nucleon is very important. As we have seen, this concept soon found support in an analysis of the behavior of the electron in electrodynamics, where theoretical calculations are possible. These calculations showed that the half-dressed electron, at least, was a physical reality. The hadron data, which we have already discussed, with a very crude interpretation, do in fact suggest that something similar also occurs in the case of a nucleon.

All these events focused more attention on the cross section for the interaction of a particle immediately after a collision.

The process most suitable for study here is diffractive dissociation in nuclei of hadrons with a high energy E_{lab} such that the nonequilibrium particle, as it moves within the nucleus, does not have time to undergo a a change in structure: $T^{\mathrm{rec}}\gg R$, where R is the radius of the nucleus (in practice, an energy $E_{\mathrm{lab}}\gtrsim 15$ GeV is sufficient). A large number of experiments, carried out from 1972 on, have customarily been interpreted on the basis of the optical model.[35] It is assumed here that at a certain point in the nucleus the incident hadron undergoes a transformation, acquiring a mass m^* and a transverse momentum $k_\perp = \sqrt{-t}$. Before it reaches the transformation point, it is absorbed in nuclear matter, in accordance with the total hadron–nucleon collision cross section, σ_1, known from experiment; after the transformation, it is absorbed with some cross section σ_2, to be determined. The total product yield for the dissociation of a hadron h by a nucleus A in this model is expressed in terms of the cross section for the same dissociation of h by an isolated nucleon N. For the distribution with respect to the invariant product mass m^* and also with respect to t the following is found:

$$\left(\frac{d^2\sigma}{dm^*\,dt}\right)_{hA} = \left(\frac{d^2\sigma}{dm^*\,dt}\right)_{hN} |F|^2, \tag{23}$$

$$F = 2\pi \int_{-\infty}^{\infty} dz \int_0^{\infty} b\,db\,e^{ik_\parallel z} J_0(|t|b)\,\rho(b, z) \times \exp\left[-(1-i\alpha_1)\frac{\sigma_1}{2}T_1 - (1-i\alpha_2)\frac{\sigma_2}{2}T_2\right], \tag{23a}$$

$$T_1 = \int_{-\infty}^{z} \rho(b, z')\,dz';\ T_2 = \int_z^{\infty} \rho(b, z')\,dz',\ \int \rho(r)\,d^3r \equiv \int \rho(b, z)\,d^2b\,dz = A. \tag{23b}$$

It is assumed that the distribution of the nuclear matter, $\rho(r)$, is known from other experiments (for example, Hofstadter's). Here α_1 and α_2 are the ratios of the real and imaginary parts of the amplitudes for elastic scattering by a nucleon at zero angle; they can be used to relate the absorption of the waves (the initial hadron h and the nonequilibrium hadron h^*) to the total cross section. The quantity α_1, like σ_1, is known from hN scattering, while α_2 (which is small, $\alpha_2 \approx 0.0 - 0.3$) and σ_2 are chosen to get the experimentally observed A dependence of the diffractive—dissociation yield. This method has been used for a detailed study of the reactions $\pi \to n\pi$ with $n = 3$ and 5; $N \to n\pi + \mathrm{N}$, with $n = 1$ and 2; and $K \to n\pi + K$, with $n = 1$ and 2. Naive expectations had σ_2 turning out to be equal to the sum of the known cross sections of the hadrons observed in the final state, provided that m^* did not fall in a resonance (having the same quantum numbers as h); for the reaction $\mathrm{N} \to \mathrm{N} + \pi$, the value $\sigma_2 = \sigma_{\mathrm{NN}} + \sigma_{\pi\mathrm{N}} \approx 60$ mb might have been expected, $\sigma_2 \approx 3\sigma_{\pi\mathrm{N}} \approx 70$ mb might have been expected for $\pi \to 3\pi$, and an even larger value might have been expected for $\pi \to 5\pi$. The experimental results shattered these expectations: All the σ_2 turned out to be approximately equal to σ_1 or even smaller. This was a surprise to the experimentalists, but there is a natural explanation if we think in terms of a finite rate for the restructuring of the hadron. For example, the initial pion cannot instantaneously expand to the volume of three pions. The process goes as $\pi \to \pi^* \to 3\pi$ etc.; under the condition

$$R \gg T^{\mathrm{regen}} \gtrsim \frac{1}{m_\pi}\frac{E^*}{m^*}, \tag{24}$$

the decay $\pi^* \to 3\pi$ occurs long after emission from the nucleus. What is actually being measured, therefore, is the cross section $\sigma_2 = \sigma_{\pi^*\mathrm{N}}$ or, in general, $\sigma_2 = \sigma_{h^*\mathrm{N}}$. According to (24), we can expect $\sigma_2 \lesssim \sigma_{h\mathrm{N}}$ i.e., a value of the order of the cross section of the initial hadron or even smaller. Experiment apparently agrees herein; different intervals of the mass m^* and for slightly different E_{lab} the following results are found: $\sigma_{(K\pi\pi)\mathrm{N}} = (11 - 22) \pm (2 - 2,5)$ mb, $\sigma_{(3\pi)\mathrm{N}} = (16 - 29) \pm (1 - 2)$ mb, $\sigma_{(5\pi)\mathrm{N}} = 17 \pm 5$ mb, $\sigma_{(\mathrm{N}\pi)\mathrm{N}} = (33 - 39) \pm 7$ mb, and $\sigma_{(2\pi+p)\mathrm{N}} = (17 - 36) \pm (1 - 2)$ mb (see, for example, Ref. 36). These results, however, are for the total cross sections. In the reaction $\pi \to 3\pi$, for example, it is apparently possible to distinguish among the effects of the various resonant and nonresonant states. For them, the situation is more complicated. For the 1^+ state which is most important here, $\sigma_2 = 17 \div 24) \pm (1 - 2)$ mb is found, while for the 0^+ state the result found in one range of energies and masses is $\sigma_2 = 30 \pm 5$ mb, while the results found in three other ranges are $\sigma_2 = (52 - 60) \pm (12 - 33)$ mb (Ref. 36). Nevertheless, the experimentalists apparently do agree that σ_2 is small, more precisely, that the value found for σ_2 is usually approximately equal to the cross section of the initial particle or even smaller; at any rate it is much smaller (frequently by a factor of several units) than the sum of the cross sections for the final hadrons observed far from the nucleus. This circumstance, expressed in such general form, confirms that the *nonequilibrium intermediate hadron is a real entity* and that the *restoration of the equilibrium structure requires a time T^{regen} which is at least as long as the time required for the propagation of the interaction* (or signal) across the system. Specific models should make it possible to trace the mechanism by which this equilibrium structure is restored, but they must obey this restriction.

Thus it would seem that everything fits in well with expectations. However, the situation is not quite so simple. As we mentioned at the end of Section 4, for small-angle scattering of an electron, it is by no means always possible to break up the process into independent successive collisions of a nonequilibrium particle, so that an independent cross section σ_2 would be meaningful. Furthermore, subsequent coherent interactions could reinforce each other (for example, in the case of an electron which radiates upon incidence on a crystal). In case (23) it is in fact small angles which are involved. It is not surprising that the very method used to extract σ_2 from the optical model, (23), has recently come under severe criticism.

In the first place, the quantum-field model-based estimate showed (although there were several assumptions involved here) that the first rescattering of the excited hadron h^* could lead to a strong "antiscreening" i.e., to an increase in the yield of reaction products. If this possibility is ignored [as it is in (23)], we would explain the increase in the yield as the result of weak absorption, i.e., of a particularly small cross section σ_2 (Ref. 37). For this reason the value of σ_2 found from the experimental data with the help of (23) gives us some average value over many rescattering events.

In the second place, there is the "theoretical experiment" devised by Miettinen and Pumplin.[38] Calculations are carried out for an h*N collision inside a nucleus in a model of the eikonal type, with parameters chosen for a correct description of diffractive dissociation by the isolated nucleon (more on this below). It turns out that the value of σ_2 calculated directly for this collision is vastly *larger* than that which we find if we apply the optical-model estimate, (23), to the same results.

These two studies thus lead to opposite conclusions regarding the sign of the deviation of the actual value of σ_2 from that yielded by the optical model.

On the other hand, Verebryusov and Ponomarev[39] have shown that a longitudinally polarized ρ meson has an extremely small cross section (for the same transverse momenta $k_\perp$ that are involved in diffractive dissociation), by calculating the cross section for an interaction of a ρ meson with a nucleon through the use of a dispersion relation continued analytically with respect to mass into the region of decay masses of the ρ meson and incorporating the interaction of the nucleon with each of the two pions into which the ρ meson may decay. Consequently, if we assume that π^* is the combination $\pi+\rho$, we find the cross section for the reaction $\pi\to\pi^*\to 3\pi$ to be $\sigma_{\pi^* N}\approx\sigma_{\pi N}$ (in fact, if we make some other assumptions—for which we cannot make a strong case—regarding an admixture of an A_1 resonance and regarding a small cross section for this resonance, we can obtain $\sigma_{\pi^* N}<\sigma_{\pi N}$). This model-based analysis serves an an example to show what specific mechanism can, within the framework of field theory, lead to a small cross section for the nonequilibrium particle during the regeneration time. This result is of course consistent with the circumstance that this small value of the cross section was linked earlier to general principles: the finite rate at which a signal would propagate in the volume of a particle and the relativistic time dilatation (provided that we assume that the concept of a cross section σ_2 independent of earlier interactions is meaningful). This model-based analysis thus favors the optical model, (23). However, rescattering is completely ignored here.

Obviously, the applicability of the optical model and the validity of the values of σ_2 which it yields require further study. At this point we can take this σ_2 to be some overall semiphenomenological characteristic of the result of all subsequent collisions of h* in the nucleus before it escapes.

A possible mechanism for the appearance of a nonequilibrium (half-dressed) state, which interacts with a reduced cross section at times $t\ll T^{\rm regen}$, can be seen in the two following models, which are crude but do explain the essential features of the situation.

1. We begin with diffractive dissociation. We consider the model of Ref. 14, which is based on a nonrelativistic model for the diffractive dissociation of the deuteron (into a proton and a neutron) which was used successfully by Glauber[40] some time ago. Let us assume that a hadron h incident on a nucleus is scattered by some nucleon of the nucleus (or by some group of nucleons) without any transfer of charge, baryon number, strangeness, or any other such characteristic. Furthermore, there is no significant transfer of energy (the nucleus is heavy). Only momentum is transferred. As a result, the hadron is excited, becoming h*, and then decays into stable hadrons. If we have a nucleon

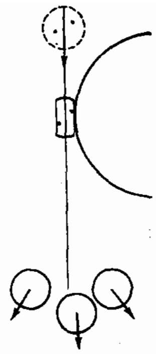

FIG. 4. The diffractive dissociation $\pi\to 3\pi$ in a model of the Glauber type.

in mind, we would have h = N and h* = N*; h and h* would each consist of three valence quarks; and π and π^* would each consist of two valence quarks. An excitation without a change in the state of the heavy target can occur only if the hadron is scattered at a time at which its constituent elements (valence quarks, the gluons around them, and the sea of quark pairs) are anomalously close together (in the transverse plane), because of a fluctuation. Then we *know* that the internal structure of the hadron h* is described immediately after the collision by a functional $\Phi_t\equiv\Phi_{h*}$ which is truncated in coordinate space (Fig. 4). Because of relativistic time dilatation, the internal elements of this system do not have time to undergo displacements during the collision, and only after a time $t\gtrsim T^{\rm regen}$ in (24) does it decay into the final particles. In the expansion (in terms of equilibrium particles) of the functional Φ_t (for definiteness, we assume h $\equiv\pi$),

$$\Phi_t = C_0^\pi\Phi_\pi+\sum C_{3\pi}^\pi\Phi_{3\pi}+\sum C_{\pi\rho}^\pi\Phi_{\pi\rho}+\sum C_{5\pi}^\pi\Phi_{5\pi}+\ldots \quad (25)$$

[here each of the functionals on the right side contains only a single element from the column in (22)], $|C_i^\pi|^2$ gives the probability for finding a given final state in the limit $t\to\infty$. The mass of the intermediate state Φ_t is m^*. It seems physically reasonable that as the pion state is "truncated" more and more the dimensions of the system will become smaller, the perturbation will become greater, and n^* will thus become larger. Before the decay into the final products, the system "does not yet know" just which state it is going to. These aspects of the nonequilibrium system are in fact seen in certain features observed in diffractive dissociation. Specifically, if we believe the data extracted from experiment with the help of the optical model, then σ_2 for π^* (and for N^*) actually turns out to be smaller, the larger is m^* (although the decrease is not great). Furthermore, the distribution with respect to the resultant transverse momentum $k_\perp^*$ of all the hadrons detected finally (i.e., the distribution with respect to the transverse momentum of the particle h*) is described by the function $\exp[-B(m^*)k_\perp^{*2}]$; here $B(m^*)$ is smaller (the decrease is by a factor of several units), the larger m^*. This result can be taken as an indication that as m^* increases the effective impact parameters for the scattering of the system decrease; i.e., the "truncated state is smaller (in the transverse direction), the larger is m^*. Finally, the values of $B(m^*)$ for the various final states, $\pi \to 3\pi$ and $\pi \to 5\pi$ are the same (as is found in dissociation by a nucleon). This situation corresponds to the idea that $B(m^*)$ describes the properties of an intermediate h* system which is the same system for 3π and 5π if their masses are equal (actually, we would also have to make sure that the angular momenta are equal).

This nonequilibrium system, "truncated" in space, and retaining the normal number of valence quarks, could of course be called a "half-dressed particle."[10)] It comes into being due to quantum fluctuations in the transverse dimensions.

The idea that diffractive dissociation results from fluctuations in the internal state of the incident hadron is developed far more extensively in the model for the process based on the "eigenstate" picture.[41] If the actual hadron is a superposition of certain states (e.g., partons), each of which is absorbed in a nucleon with a certain corresponding amplitude t_k (for simplicity, let us say that this amplitude is real), then it can be shown that the cross section for diffractive dissociation, σ^{DD}, is determined by the fluctuation of the components in the initial state, $\sigma^{DD} \sim \langle t^2 \rangle - \langle t \rangle^2$, where, for example, $\langle t^2 \rangle = \langle \Phi_h | \hat{t}\hat{t} | \Phi_h \rangle$. Here $\hat{t} = \hat{S} - 1$, if $\hat{S}$ is the scattering matrix for the scattering of a parton by a nucleon, etc. Miettinen and Pumplin[42] have shown for Feynman's parton model that fluctuations of three types are involved (although their roles are qualitatively and quantitatively different): fluctuations in the number of partons (quark pairs, gluons), fluctuations in the parton distribution in the transverse plane (these are the only fluctuations which are incorporated in the model in Fig. 4) and fluctuations in the longitudinal rapidities of the partons. By choosing three corresponding distribution functions we can find a good description of the dissociation in the reaction $NN \to (N\pi)N$ [it was the use of this method to calculate σ_2 which led to the criticism of the optical model, (23), which we mentioned above[38]].

2. Inelastic collisions of a nondiffractive type are also frequently treated on the basis of Feynman's parton model, under the assumption that only the slowest

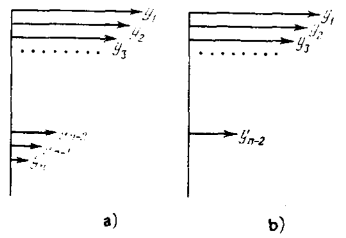

FIG. 5. Change in the structure of a hadron after an interaction in Feynman's parton model.

partons, with rapidities $y \sim 1$ (in the target frame), are capable of interacting. The "ladder" or "comb" diagram in Fig. 5a is particularly popular. Here the length of arrow i corresponds to the rapidity of parton $i, y = \ln(\varepsilon_i/m)$, where ε_i is the energy of the parton and m is the mass; more precisely, it is some characteristic parameter of the order of 1 GeV. The usual approach is to start from an equidistant (or at least approximately equidistant) distribution of partons along the y axis, $\Delta y = y_i - y_{i+1} = \text{const} \sim 1$. It is assumed that a core, stripped of the partons which have interacted, moves off in the forward direction after the interaction (Fig. 5b). Restoration of the normal structure requires a time which is the same as the regeneration time discussed above:

$$\tau \gtrsim \frac{1}{m}\frac{E_L}{m}, \quad \tau \approx T^{\text{regen}}. \tag{26}$$

At $t \ll \tau$, the core has no partons, $y \sim 1$, and is thus passive; it cannot interact with the nucleons in its path, for example, those in the same nucleus in which the first interaction occurred.[43,44]

It is easy to see that this picture is to a large extent equivalent to the model in which a hadron which has been "chopped off" in coordinate space is moving forward after the interaction (in the same manner in which the scattering in Fig. 1 is followed by the emission of a half-dressed electron, or pion in Fig. 4). If the multiperipheral comb model for inelastic collisions is to lead to the correct—Regge—behavior of an elastic collision, the structure of the chain in Fig. 5 must be completely definite in *coordinate* space, as was pointed out a long time ago.[45] In the so-called "target plane," which is oriented perpendicular to the collision axis, n links of the chain must be distributed in accordance with a certain type of Brownian motion (Fig. 6; we wish to emphasize that y is the parton rapidity here, rather than the transverse coordinate, as in Fig. 1). The di-

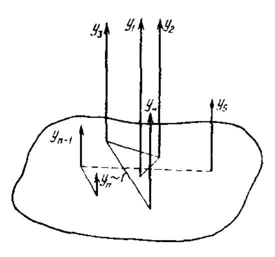

FIG. 6. Coordinate-space distribution of the partons of the model of Fig. 5a (in the target plane). Here y_i are the rapidities of the partons.

[10)]From the standpoint of the model of the present paper, the formulation of the question in Ref. 39 and the result found there can be described as follows. If an expansion in terms of real ("dressed") particles is used, the incident pion is described by the functional in (22), with only the first row not equal to zero, $c_0^\pi = 1$. Experiments show that in a certain range of the mass m^* the reaction $\pi \to 3\pi$ is dominated by the process $\pi \to \rho\pi \to (2\pi)\pi$; i.e., the coefficients C_0^π and $C_1^{\pi''}$ play a leading role in the expansion of the nonequilibrium pion π^*. The other coefficients can be assumed small. Accordingly, the amplitude for the scattering of the π^* by the next nucleon of the nucleus is a coherent sum of the amplitudes for ρN and πN. This amplitude was also calculated in Ref. 39, but in a certain range of the mass m^* there may be decays to both 3π and 5π. For a given angular momentum, the π^*N scattering should be determined by the sum of many amplitudes (e.g., $\pi\rho\rho$, etc., should be added). From the standpoint of the model of a spatially truncated system, the dimensions are even smaller for such an excited π^*, and the summation of all these amplitudes should yield an even smaller cross section for π^*N. This is the case, however, only in a simple model of this sort (see the next paragraph of the text).

mension of the system in the direction transverse to the collision axis is thus of order $\rho \sim \sqrt{n \langle \Delta k_\perp \rangle^{-2}}$, where $\langle \Delta k_\perp \rangle^{-2}$ is the reciprocal of the average square of the transverse momentum transfer in a single link. Furthermore, since $n \sim y_1/\Delta y \sim \ln(E_L/m)$, the dimensions of the particles turn out to have a Regge behavior, $r^2 \sim \ln E_{lab}$.

Here, however, the partons with $y \sim 1$ at the end of the chain have a broader distribution then that of the "inactive" partons. Consequently, a *hadron which is chopped off* (after the first interaction) *in the space of the rapidity y* (Fig. 5b) *is also chopped off to a certain extent in coordinate space.* The restoration of the normal parton structure over the time τ in (26) is the ordinary restoration in coordinate space which we have examined in detail for QED.[11)] We can again speak in terms of a "half-dressed" particle, although the use of this term is quite arbitrary here.

We should emphasize a circumstance which is very frequently ignored when the parton model is applied, for example, to hadron—nucleus collisions: Frequently, it is simply assumed that over a time τ the hadron is passive, and after this time it undergoes "regeneration," transforming into a normal hadron. As we have seen, this regeneration occurs through a decay into $n > 1$ stable particles. If n is small, this is a quantum transition, but if $n \gg 1$, then it is a quasiclassical expansion followed by a decay. The nucleons of the nucleus may of course also be interacting during the expansion. This model was developed by Kalinkin and Shmonin.[46]

We mentioned above yet another possible type of experiment on the interaction of "immature" hadrons. This would be to study the emission from a nucleus of fast (leading) hadrons which have interacted in the nucleus or which have been produced in it and emitted at a large angle from the collision axis. A program of experiments along this line, based on proton—nucleus collisions at $E \sim 20$ GeV, has been undertaken recently[47] and is in a developmental stage at present. The experiments are furnishing some very interesting indications that the emitted nucleon has a cross section smaller than normal, while the difference for π and K mesons is insignificant. This conclusion must be considered only tentative at present, however, since the same difficulties are arising here as in the application of the optical model to diffractive dissociation (discussed above), and not much experimental information has been acquired yet. Furthermore, a calculation in the Glauber approximation, with multiple interactions in the nucleus and a realistic shape of the nucleon density distribution at the nuclear boundary, has led Alaverdyan *et al.*[50] to conclude that the experimental data discussed in Ref. 47 can be understood if a normal collision cross section is preserved for the equilibrium nucleon in each interaction.

Accordingly, although the concept of a hadron which has lost, at least partially, its ability to interact during a regeneration time, (26), is widely used in the parton model for interactions in nuclei, and although it does agree with much experimental evidence, both old and new, the conclusions being reached in more accurate model-based calculations (including quantum field calculations) are turning out to be ambiguous. The theory of individual specific processes which are involved in this concept remains a serious problem. On the other hand, the physical clarity of the basic fact—that the internal state of a particle which has been disrupted in the course of an interaction cannot be "healed" faster than a signal can propagate across the volume of particle (a fact which can be analyzed completely in QED)—makes both experimental and theoretical research on this problem seem extremely urgent.

6. THE PLACE OF NONRESONANT HADRON CLUSTERS QUANTUM FIELD THEORY

Up to this point we have been discussing nonequilibrium particles either in terms of QED, in which calculations of rigor sufficient for our purposes are possible, because perturbation theory can be used, or on the basis of crude models, through the application of our experience in QED to hadrons. In quantum field theory, however, there are certain general relations which go further to justify the linkage of the problem of nonequilibrium particles to the problem of hadron clusters. These are the Källen–Lehmann spectral-relations for the Green's functions.[9]

We mentioned in Section 2 that in quantum field theory the renormalization can be based on the requirement that the propagator (in the momentum, k, representation) for each field has a pole at $k^2 = m_i^2$, where m_i is the mass of the renormalized "physical" particle of species i. Then it follows from the most general positions of quantum field theory that the renormalized propagator of, for example, the pion ($i \equiv \pi$) must be of the form

$$D(k^2) = \frac{1}{k^2 - m_\pi^2} + \int\limits_{(\varkappa_\pi^0)^2}^{\infty} \frac{\rho(\varkappa^2)\, d\varkappa^2}{k^2 - \varkappa^2}; \tag{27}$$

here $\varkappa_\pi^0$ is the threshold for excited states of the system. Furthermore, $\rho(\varkappa^2)$ is the "spectral function" of the propagator: It is determined by the particular dynamics of the two interacting fields. We could calculate this spectral function if we were able to solve the equation of motion for a system of baryon and pion fields. In electrodynamics, analogous spectral functions are calculated by perturbation theory. All we know from general considerations is that ρ is a finite positive quantity: $0 \leq \rho(\varkappa^2) < \infty$. The factor $(k^2 - \varkappa^2)^{-1} = D_0(k^2; \varkappa^2)$ in the integral is, according to (3), the propagator of the entire system with mass $\sqrt{\varkappa^2}$ and internal quantum numbers the same as those of the pion. By no means does this system have to be a stable particle. It may be a unique entity: a nonequilibrium particle in which all the field interactions have already been taken into ac-

[11)]Admittedly, a comparison of the consequences of the parton models of this type with experiment for hadron—nucleus interactions yields the estimate $m \sim 0.5$–1 GeV in (26), instead of $m \sim m_\pi \sim r_0^{-1}$. This estimate is ambiguous, however, since the models on which it is based are very crude; in particular, they ignore the comment in the next paragraph of the text [see also (21)].

count, but perhaps, of course, by means of a set of particles which have not interacted and which are in different types of motion; for example, there may be two such particles, with 4-momenta k_1 and k_2, $(k_1+k_2)^2 = \varkappa^2$. Such systems are taken with a weight $\rho(\varkappa^2)$. For the pion, because it is a pseudoscalar, the excited state corresponds to at least three pions, so that $(\varkappa_\pi^0)^2 = (3m_\pi)^2$.

Furthermore, the renormalization constant Z_3 can also be expressed in terms of $\rho(\varkappa^2)$,

$$Z_3^{-1} = 1 + \int_{(\varkappa_\pi^0)^2}^{\infty} \rho(\varkappa^2)\, d\varkappa^2, \tag{28}$$

so that we have $0 \leq Z_3 < 1$ [we have $Z_3 = 1$ only if $\rho(\varkappa^2) = 0$ i.e., only if there are no excited states of any sort—and this is an unrealistic assumption]. Finally, this function $\rho(\varkappa^2)$ also determines the difference between the renormalized and bare masses.

Analogously, there is a spectral representation for a fermion (here, a nucleon) propagator, with its own spectral function (which also determines the constant z_2 and the relationship between the renormalized and bare masses of the fermion). However, we will not use this representation. For photons (their operator wave function is a vector, not a pseudoscalar) the propagator is a tensor, $D_{\mu\nu} = (\delta_{\mu\nu} - k_\mu k_\nu/k^2) D_\gamma(k^2)$, and relations like (27) and (28) apply to $D_\gamma(\varkappa^2)$.

The function $\rho(\varkappa^2)$ obviously reflects the effect (in the composite system represented by a dressed particle) of the components with the given 4-momentum $\varkappa$. Indeed, we find $|\varphi|^2 = |\varphi_0|^2 Z_3^{-1}$ from (4), and it is the function $\rho(\varkappa^2)$, according to (28), which shows how the states $\varkappa^2$ affect the difference between the renormalized field operator φ and the bare operator φ_0.

Perturbation-theory calculations of $\rho(\varkappa^2)$, particularly in QED, lead to confusing results: They show functions $\rho(\varkappa^2)$ which fall off with increasing $\varkappa^2$ too slowly for convergence of the integrals in the basic relations, (27) and (28). In meson—nucleon theory, for example, we can calculate the effect on ρ of excited states with a single nucleon—antinucleon pair. For both scalar and pseudoscalar neutral mesons, we find $\rho(\varkappa^2) \sim \varkappa^{-2}$ in the limit $\varkappa^2 \to \infty$ in all orders of perturbation theory; in other words, the integral in (28) diverges.[11] In QED for the photon, in the low order corresponding to the incorporation of only excited states with a single e^+e^- pair (more on this below), we have $\varkappa^0 = 2m_e$ and

$$\rho_\gamma(\varkappa^2) \approx \rho_\gamma^{(1)}(\varkappa^2) = \frac{e^2}{12\pi^2}\frac{1}{\varkappa^2}\left(1 + \frac{2m_e^2}{\varkappa^2}\right)\sqrt{1 - \frac{4m_e^2}{\varkappa^2}} \sim \frac{1}{\varkappa^2} \quad \text{as } \varkappa^2 \to \infty. \tag{29}$$

In other words, the integral in (28) again diverges. Since the effect of each excited state is positive, the effect of considering other states could be only to increase ρ.

In summary, many of the two-field theories presently available contradict the spectral relations based on general principles.

We could, however, ignore this contradiction and assume that somewhere as $\varkappa^2 \to \infty$ the function $\rho(\varkappa^2)$ falls off fast enough for convergence of all the integrals. At any rate, ordinary calculations cannot be carried out here. Let us consider an example: In QED there is the bare electron mass m_0, which, as we have already mentioned, reflects in some overall manner the roles played by certain nonelectromagnetic interactions. This simple approach may prove inadequate at large values of α^2. At the same time, we know that such interactions clearly exist. Even before the discovery of strong and weak interactions, for example, it was known that electrons and electromagnetic fields are subject to gravitational forces. These forces were ignored, because they were "too weak." However, as the radius of the electron, r_0, is reduced (in QED, this decrease corresponds to an increase in the 4-momentum transfer $\varkappa$, $\varkappa^2 \sim r_0^{-2}$) the electromagnetic mass diverges very slowly—only logarithmically in quantum field theory (the use of perturbation theory for the calculation presupposes $m_0 \approx m_e$):

$$\delta m_e \sim m_0 \frac{3e^2}{4\pi} \ln \frac{1}{m_0^2 r_0^2} \approx m_e \frac{3e^2}{4\pi} \ln \frac{1}{m_e^2 r_0^2}. \tag{30}$$

It was pointed out a long time ago[48] that this leads to $\delta m_e \sim m_e$ only if r_0 is much smaller than the radius of the Schwarzschild sphere, r_S. At $r \lesssim r_S$, the geometry used in QED, which ignores gravitational forces and the general theory of relativity, is incorrect: where this geometry is valid, at $r \gg r_S$, an electromagnetic mass much smaller than m_e is acquired. Consequently, the very fact that gravitational forces exist keeps us from accepting the $\rho(\varkappa^2)$ behavior which is found in QED by perturbation theory for $\varkappa^2 \to \infty$. In actual fact interactions other than electromagnetic, but more significant than the gravitational interaction, should change $\rho(\varkappa^2)$ already at values of r immeasurably larger than r_S. Accordingly, the divergences in a system of only two fields, with the other forces represented by only integral characteristics (bare masses), still tell us nothing about the actual convergence of the spectral integrals. For this reason, we need not be disturbed by the behavior of $\rho(\varkappa^2)$ in the limit $\varkappa^2 \to \infty$.

With this thought in mind, we will attempt to learn something about the possible form of $\rho(\varkappa^2)$ at *finite* $\varkappa^2$ for hadrons, in which case exhaustive calculations cannot be carried out. We will base our arguments on an analogy with QED and on known experimental facts.

For a long time, the two-step nature of the hadron-production process was thought of as one of the most interesting properties of multiple production in the physics of strong interactions at high and ultrahigh energy. At a sufficiently high energy, this process involves an intermediate state, in which comparatively heavy, unstable formations are present, and the stable final hadrons are produced only as a result of the subsequent decay of these unstable hadron formations.[3] However, bremsstrahlung in electrodynamics also goes through an intermediate stage involving the formation of a nonequilibrium particle, as we saw in Sections 3 and 4 (see Fig. 2). The occurrence of two stages is thus by no means peculiear to strong interactions, and it is of no essential importance in quantum field theory.

For a long time, the accelerator experiments kept physicists believing that these unstable formations were simply the known resonances: the meson resonances ($\rho, \omega, \ldots$, etc.) and the baryon resonances (Δ, etc.). As the collision energies increased to $E_{\rm lab} \sim 10^2 - 10^3$ GeV, however, the concept of fireballs, or of clusters, as we would now say, became increasingly attractive, as we saw in Section 1. Analysis of the accelerator data under the assumption that the collision of nucleons involves the production of several identical, independent, "average" clusters, which subsequently decay into pions (this is the "independent cluster emission model" or ICEM) led to the conclusion that one of these "average" clusters would decay into three or four pions and would have a mass of about 1.5 GeV. However, the more realistic model which allows the production of both light resonances (ρ) and heavy nonresonant clusters,[5] which we mentioned earlier, shows, when taken along with the accelerator data, that this "average" is actually obtained by summing light resonances and far heavier clusters.[6] If, for example, we select only high-multiplicity events, with $n_s \geq 6$ (n_s is the number of relativistic charged particles), as was done in cosmic-ray work at one time,[1] we would be emphasizing the effects of heavy clusters. Then the average cluster mass is ~3–5 GeV, and the number of decay products is ~6–10. These conclusions agree with the characteristics assigned to fireballs.

Furthermore, as we mentioned earlier, an unstable, heavy, intermediate hadron formation may have a reduced cross section for interactions with nucleons of the same nucleus in which this formation appeared (Section 5). In the cosmic-ray work, also, as we mentioned in Section 5, it was found that the fireball emerges freely from the nucleus in which it was produced, having small dimensions before the decay.[1]

There are two points of view regarding the nature and mass spectrum of the heavy clusters. According to one, these are resonances which have not yet emerged distinctly and perhaps overlap—similar to the familiar entities at $m^* \lesssim 1.5$ GeV and perhaps lying further along the same Regge trajectories. The other view has these heavy clusters being nonresonant blobs of hadron matter. This rather fuzzy definition requires some interpretation.

The question of the nature of a heavy cluster is frequently confused with the question of the nature of its decay (which is usually treated thermodynamically, but which also permits a sequential emission of particles, for example, in accordance with a dual-resonance model). But the point of interest to us is what such a cluster is from the standpoint of quantum field theory, if there is in fact a place for it in this theory. We will see that this cluster does in fact find a natural place in quantum field theory.

To choose between resonant and nonresonant natures (or some compromise) for the cluster is equivalent to choosing the form of the spectral function $\rho(\varkappa^2)$. Let us consider the Feynman diagram M (Fig. 7a) for the collision of two hadrons in which an intermediate state is formed with 4-momentum k, a large mass $m^* = \sqrt{k^2}$, and certain quantum numbers. This intermediate formation subsequently decays. We will not concern ourselves with the other elements of the diagram, shown by the dots in Fig. 7a. The probability or cross section σ for the process is determined by the square of the modulus of the amplitude M, i.e., by the diagram in Fig. 7b. Since the final particles have definite "physical" masses (as shown by the cross bars on the corresponding lines), the cross section σ is determined by the imaginary part of the diagrams in Fig. 7b:

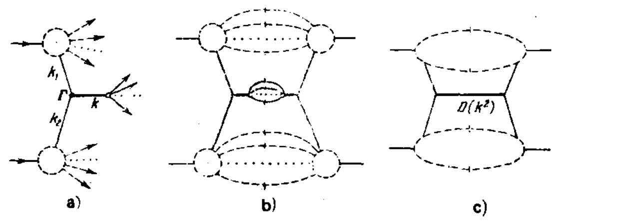

FIG. 7. a) Feynman diagram of multiple production which occurs through the decay of an unstable intermediate particle; b) the corresponding diagram describing the cross section for the process; c) the same, expressed in terms of the cluster propagator.

$$\sigma \sim \mathrm{Im}\, M. \tag{31}$$

We could also find this value, however, by ignoring the decay into final particles and instead replacing the diagram in Fig. 7b by that in Fig. 7c, but understanding $D(k^2)$ to be the propagator of the renormalized particle, which incorporates the virtual decays of this particle into other particles. Since the cross section in which we are interested also contains the Γ vertex functions, then

$$\sigma \sim |\Gamma|^2\, \mathrm{Im}\, D(k^2). \tag{32}$$

However, the imaginary part of the propagator D, taken in the spectral representation in (27), is[9]

$$\mathrm{Im}\, D(k^2) = -\pi\rho(k^2). \tag{33}$$

To some extent, therefore, $\rho(\varkappa^2)$ determines the probability for the formation of a state of mass $m^* = \sqrt{k^2}$ with given quantum numbers.

If we assume that the clusters are simply resonances, we would thereby be assuming that $\rho(\varkappa^2)$ reduces to a set of resonant maxima, that there is no nonresonant background, and that the amplitude M can be written as the sum of resonant terms of the Breit–Wigner type. This approach, however, is not realistic.

Let us consider QED first. On the one hand, we know (Section 3) that an electron e^* with a nonequilibrium field (a half-dressed electron) may appear in the course of the interaction and then decay—in the first approximation in e^2—into a stable electron and a γ-quantum (Fig. 8a; in higher-order approximations, there may

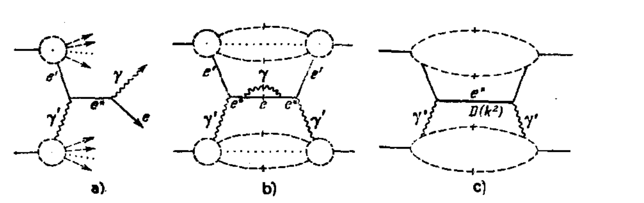

FIG. 8. Electromagnetic analog: Bremsstrahlung of an electron as the decay of a nonresonant "cluster" e*.

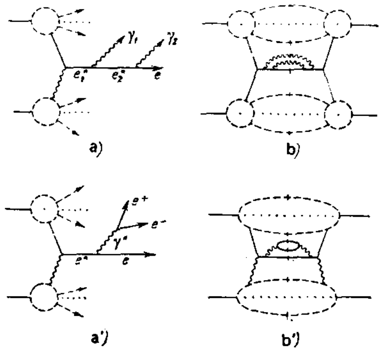

FIG. 9. Multiparticle decay of a "cluster" e* in the next order of perturbation theory.

be more photons and electron pairs, as in Fig. 9a′). The cross section for this process (Fig. 8b) can be described by the diagram in Fig. 8c, with the corresponding propagator for e^*. Evaluating the propagator in Fig. 8b with help of an equation analogous to Eq. (30) for the γ-quantum, we find the spectral function for this electron propagator (in the first approximation in e^2). The higher-order approximations in e^2 for this spectral function arise from processes of the type of those in Fig. 9, etc.

A better approach, however, is to examine in detail the spectral function of the photon propagator, rather than the electron propagator. In the higher orders in e^2, this propagator has contributions from $D_\gamma^{(1)}, D_\gamma^{(2)}, D_\gamma^{(3)}, \ldots$—corresponding to the processes in Figs. 10a, 11a, 12a, … . Calculating the average elements in diagrams b, we determine the propagators and consequently the contributions to the spectral function of the propagator for the γ-quantum:

$$\rho_\gamma(k^2) = \rho_\gamma^{(1)}(k^2) + \rho_\gamma^{(2)}(k^2) + \rho_\gamma^{(3)}(k^2) + \ldots. \tag{34}$$

The first term, $\rho_\gamma^{(1)}$, was given earlier, in (29). We will not calculate the lengthy terms which follow, but we will discuss their general behavior. First we wish to call attention to the fact that we have ignored the possible formation of a resonance state: positronium (e^+e^-), with a mass $m^{(p)} = 2m - \varepsilon^{(p)}$, where $\varepsilon^{(p)}$ is the binding energy of the electron and the positron. This event would lead to processes of the type shown in Fig. 13. Taking all these contributions into account, we can offer a qualitative description of the behavior of $\rho_\gamma(\varkappa^2)$ for the photon—as shown in Fig. 14. We emphasize that these curves are not drawn to scale, particularly with regard to the relative heights and widths of the resonance peaks and the smooth curves. Furthermore, we have not distinguished the contributions $\rho_\gamma^{(1)}$ and $\rho_\gamma^{(2)}$,

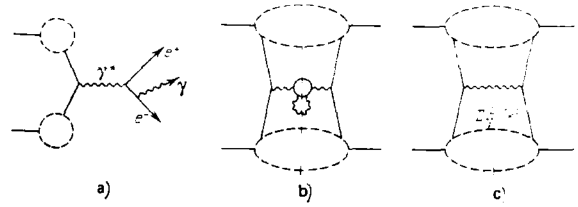

FIG. 11. The same as in Fig. 10, in the next order of perturbation theory.

since the term $\rho_\gamma^{(2)}$ (smaller than $\rho_\gamma^{(1)}$ by a factor ~137) is added over the entire $\varkappa^2$ range, because of the zero mass of the photon. The general conclusion which we need, however, is clear: $\rho(\varkappa^2)$ consists of smooth curves, which rise at each threshold for the production of an additional electron pair and then fall. Superimposed on these curves are peaks corresponding to a positronium resonance.

A point of fundamental importance is that these processes go through an intermediate stage in which *one* nonequilibrium particle is produced (e^* in Figs. 8 and 9 or a heavy photon γ^* in Figs. 10–12). This particle is an off-shell particle with a timelike $k, k^2 > 0$, i.e., with a mass $m^* = \sqrt{k^2}$, so that it is capable of decaying into a certain number of stable final particles.

We turn now to the hadron interactions.

Since the general structure of meson–baryon theory is the same as the QED structure, we can assert that the spectral function of the pion propagator, $\rho_\pi(\varkappa^2)$, also has slope changes at the thresholds at which new hadrons can appear, in the number allowed by the quantum numbers of the particles. Specifically, the diagrams in Fig. 15a with $\varkappa^2 > (3\pi)^2$, Fig. 15b with $\varkappa^2 > (5\pi)^2$, etc., may be possible. The interaction of two final-state pions, however, may also lead to ρ and π mesons (Fig. 15c), which will cause a smeared resonant peak (or step) at $\varkappa^2 = (m_\rho + m_\pi)^2$, where m_ρ is the mass of the ρ meson. Other resonances are also possible of course. At even larger values of $\varkappa^2$ there are contributions from nucleon pairs to the final state, etc. (Fig. 16).

The primary distinction from the QED case is that in the region in which the various diagrams are energetically possible they have the same order. For example, in the region $(3\pi)^2 \leqslant \varkappa^2 \leqslant (7\pi)^2$ the $\rho^{(a)}$, $\rho^{(b)}$, and $\rho^{(c)}$ contributions (Figs. 15a, 15b, 15c) are generally of the same order of magnitude. However, of course, in order for the integrals in (27) and (28) to be meaningful, the sum of the effects of all the diagrams should begin

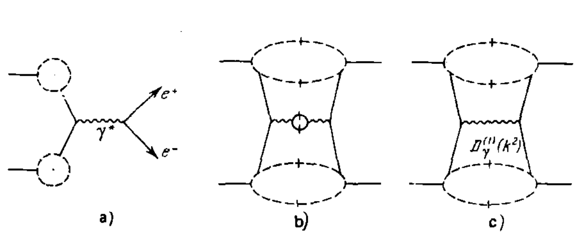

FIG. 10. Heavy photon as an electromagnetic nonresonant "cluster."

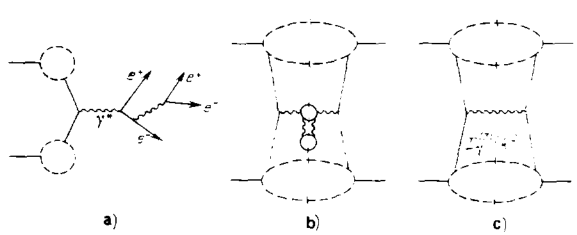

FIG. 12. The same as in Fig. 11, in the next order of perturbation theory.

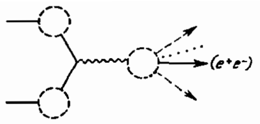

FIG. 13. A resonance (positronium) appearing in the decay of an electrodynamic "cluster."

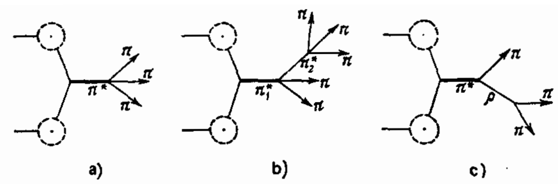

FIG. 15. Pion cluster π^* (different decay modes).

to fall off at certain values of $\varkappa^2$ for reasons which will be made clear to us in some future theory.

7. CONCLUSIONS

We can draw several conclusions from this comparison of the hadron and electromagnetic spectral functions for the propagators, from our knowledge that a half-dressed electron is observable, and from our understanding of the relativistic dilatation of the time required for this half-dressed electron to decay into stable electrons and photons (in other words, our understanding of the slowing of the bremsstrahlung process), which is manifested, in particular, in the concept of a formation zone (Sections 3 and 4).

1. *The two-step nature of the multiple production of hadrons is a property which is by no means peculiar to strong interactions.* In QED also, in the production of new particles (electrons and γ-quanta) the first step is the formation of an unstable, nonequilibrium (but actually observable) entity, for example, a half-dressed electron e* (Fig. 8), which then decays, after a time T^{regen} [Eqs. (14) and (14b); this time is "macroscopically long" if the energy is high enough], into two stable particles (in the lowest approximation in e^2) or more stable particles (in higher-order approximations): $e+\gamma, e+e^++e^-, e+2\gamma, \ldots$ (Figs. 8 and 9). In a completely similar way, an unstable, nonequilibrium (and nonresonant) heavy photon γ^* (Figs. 10–12) decays into stable final electron pairs and photons. The relativistic dilatation of the e* decay time is of the same nature as that for the nonequilibrium hadrons in the quantum field theory of strong interactions. The only unique feature of strong interactions is that the amplitudes for processes with different numbers of final particles (the numbers allowed by the rather large mass of the intermediate formation, $m^* = \sqrt{k^2}$) are of the same order of magnitude. Correspondingly, there is another unique feature: Since the terms of different orders are identical in hadron physics, the diagrams with rescattering of the final particles, in Fig. 17b, may be equally as important as the diagrams without rescattering, in Fig. 17a; this situation is to be contrasted with that in electrodynamics, where the rescattering diagrams are suppressed by factors $(1/137)^{2v}$, where v is the number of rescattering events. Then the effect of the production of groups of equilibrium particles *which have not interacted with each other*, with a total mass $\sqrt{k^2}$, on the nonresonant background will be suppressed here, and the heavy nonresonant clusters will be more important than in QED.

2. *In principle, the nature of the intermediate formation in QED is the same as that in hadron physics*: For given quantum numbers, this is a corresponding particle *far from the mass shell* (the departure from the mass shell is in the timelike direction, $k^2 = k_0^2 - \mathbf{k}^2 > 0$).

3. *The disruption of the equilibrium structure in the intermediate formations may take different forms.* This "virtual" particle (which is actually a physically observable particle) may be, for example, missing some outer parts of its structure (while the internal quantum numbers are conserved) down to some depth (the depth increases with increasing mass), in which case it may be called half-dressed (the half-dressed electron, in Fig. 1; or, in the parton model, the hadron devoid of gluons with rapidities $y \sim 1$, in Figs. 5 and 6; or the hadron in the diffractive-dissociation process, according to the model of Fig. 4). The structure of this particle may be distorted in a different way, however, if, say, it becomes an ordinary resonance (i.e., $\rho, \omega, \ldots$), etc. From this point of view, *a resonance is an extremely special case* of a disruption of the structure of a normal stable particle.

4. *The time required for the regeneration of the normal structure cannot be shorter than the time required for a signal to propagate across the normal volume of the particle.* In the rest frame of the particle the regeneration time for a hadron is $T^{\text{regen}} \gtrsim m_\pi^{-1}$; for motion in the laboratory frame, for example, the regeneration undergoes a relativistic dilatation, and we have $T^{\text{regen}} \gtrsim (1/m_\pi)E_L/m^*$. The regeneration time may be much longer than this minimum time, for two reasons: a) if the intermediate formation is a resonance, so that the time spent by the final particles in the region in which they interact is greater than the transit time across

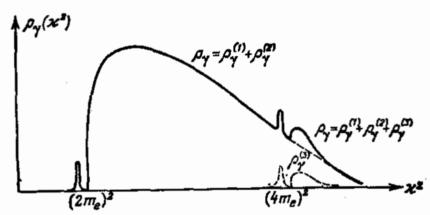

FIG. 14. Spectral function of the photon propagator (not drawn to scale).

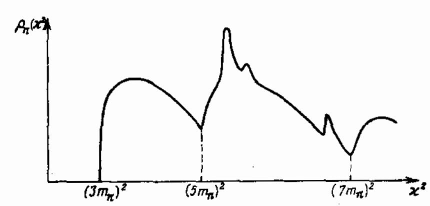

FIG. 16. Qualitative behavior of the spectral function of the pion propagator.

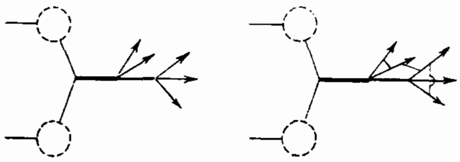

FIG. 17. a) Decay of a hadron cluster; b) with rescattering in the final state.

this region, in accordance with the Wigner theorem;[27] b) if the mass m^* is very large, if the total number of decay products is large, and if there is a corresponding increase in the volume of the final system in which the equilibrium is to be established.

5. *A nonresonant cluster fits naturally into quantum field theory.* This nonresonant cluster is also a nonequilibrium particle—a particle far from the mass shell, in the timelike direction, with $k^2>0$, provided that we are not in the vicinity of a resonance. Whether such clusters are actually formed depends on the value of the vertex Γ. We do not, however, see why Γ in (32) should be anomalously small for all $k^2>(\varkappa^{(0)})^2$. The experience in QED, where nonequilibrium e^* and γ^* are emitted and then decay into stable particles, shows that the emission of a group of stable independent particles (not clusters) at the very least does not exhaust the nonresonant part of $\rho(\varkappa^2)$.

It is assumed in the quark models that the first step in reactions of the type $e^+e^- \to h+h+\ldots$ is the production of a quark and an antiquark, $q+\bar{q}$, which then pick up one or several pairs of quarks from the vacuum. This step compensates for the color and the fractional quantum numbers and leads to hadrons (Fig. 18). There is no reason to expect, however, that the stable hadrons would be produced immediately in this pickup step. It is the subsequent decay of nonequilibrium hadrons which leads to the jet structure in the final state.

6. The dynamics of the cluster decay is a separate question. In the dual-resonance model,[51] where the clusters are assumed to be simply heavy resonances, the decay is analyzed by means of tree diagrams, which show the sequential emission of hadrons and a progressive conversion of the cluster into progressively lighter resonances (Fig. 19). The result is a hadron energy spectrum which is similar to the thermodynamic spectrum, $\exp(-\varepsilon/T_{\rm eff})$, where $T_{\rm eff}$ (not to be confused with the regeneration time!) is a parameter determined by the slope of the Regge trajectory of the initial resonance, $\alpha'(0)\approx 1\ {\rm GeV}^{-2}$, and by the dimensionality D of the equivalent oscillators in the formal apparatus of the model. For reasonable values of D, ~4–7, we find $T_{\rm eff}=\sqrt{3/2\pi^2\alpha'(0)D}\sim m_\pi$, i.e., a value close to that observed experimentally ($T_{\rm eff}$ does not, of course, have the physical meaning of a temperature of some sort).

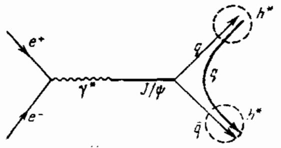

FIG. 18. Formation of a decaying hadron cluster in electron–positron annihilation which results in hadrons.

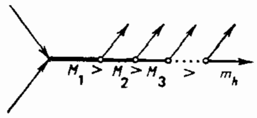

FIG. 19. Sequential (treelike) decay of resonances in the dual-resonance model.

There are decay models of other types: thermodynamic and even hydrodynamic.[2,3,28] Here we wish to emphasize that these models also essentially assume, or at least allow, a sequential decay. For example, in Landau's hydrodynamic model the expanding blob of nuclear matter converts into the final hadrons when the temperature drops to $T\sim m_\pi$ in some volume element of the blob (in the proper frame of the element). In the c.m. frame of the entire blob, this event will be no means occur simultaneously for all volume elements.

In conclusion, I wish to thank several of my colleagues, especially I. V. Andreev, V. L Ginzburg, I. M. Dremin, D A. Kirzhnits, and V. Ya. Faĭnberg, for extremely useful discussions and criticism.

[1] See, e.g., the review by M. Miesowitcz in: Progress of Elementary Particles and Cosmic Ray Physics (ed. J. G. Wilson and S. A. Wouthyusen), Vol. X, Amsterdam, 1971, p. 165. See also E. L. Feinberg, Phys. Rept. **C5**, 237 (1972).

[2] W. Heisenberg, Z. Phys. **101**, 533 (1936); **113**, 61 (1939); **126**, 569 (1949); **133**, 65 (1952); E. Fermi, Progr. Theo. Phys. **5**, 570 (1950); I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **78**, 889 (1951); L. D. Landau, Izv. Akad. Nauk SSSR. Ser. Fiz. **17**, 51 (1953).

[3] C. De Tar, in: Eighteenth Intern. Conf. on High Energy Physics, Tbilisi, 1976, JINR, Dubna, 1977, Vol. 1, p. A3–4; I. V. Andreev and I. M. Dremin, Usp. Fiz. Nauk **122**, 37 (1977) [Sov. Phys. Usp. **20**, 381 (1977)]; I. M. Dremin and C. Quigg, Usp. Fiz. Nauk **124**, 537 (1978) [Sov. Phys. Usp. **21**, 265 (1978)].

[4] W. K. Jentschke, CERN director's report for 1971–1975, CERN Courier **16**, No. 1, 4 (1976).

[5] I. M. Dremin, I. I. Roĭzen, and D. S. Chernavskiĭ, Usp. Fiz. Nauk **101**, 385 (1970) [Sov. Phys. Usp. **13**, 438 (1971)]; I. M. Dremin and A. M. Dunajevskii, Phys. Reprt. **C18**, 159 (1975).

[6] E. I. Volkov, I. M. Dremin, A. M. Dunaevskiĭ, I. I. Roĭzen, and D. S. Chernavskiĭ, Yad. Fiz. **20**, 149 (1974) [Sov. J. Nucl. Phys. **20**, 78 (1975)]; I. M. Dremin, A. M. Orlov, and E. I. Volkov, Lebedev Inst. Preprint No. 120, Moscow, 1978; I. M. Dremin and E. L. Feĭnberg, Fiz. Elem. Chastits At. Yadra **10**, 996 (1979) [Sov. J. Part. Nucl. **10**, 394 (1979)].

[7] N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh poleĭ (Introduction to Quantum Field Theory), Gostekhizdat, Moscow, 1957. (Engl. Transl., Interscience).

[8] E. V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [Sov. Phys. JETP **47**, 212 (1978)].

[9] See any text on quantum field theory, e.g., E. M. Lifshits and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), Part 2, Nauka, Moscow, 1971; see also Refs. 7, 10, and 11.

[10] S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Row, Peterson and Co., Evanston, Illinois, 1961 (Russ. Transl. IL, Moscow, 1968).

[11] G. Barton, Introduction to Dispersion Techniques in Field Theory, Benjamin, New York, 1965 (Russ. Transl. IL, M., 1968).

[12]D. A. Kirzhnits, Usp. Fiz. Nauk 125, 169 (1978) [Sov. Phys. Usp. 21, 470 (1978)].
[13]P. A. M. Dirac, in: The Atomic Nucleus. Abstracts, First All-Union Nuclear Conference, 24–30 September 1933, Leningrad [Russian translation], GTTI, Moscow, 1934, p. 142.
[14]a) E. L. Feĭnberg, in: Problemy teoreticheskoĭ fiziki, Sb. stateĭ pamyati I. E. Tamma (Problems of Theoretical Physics; Collection Dedicated to I. E. Tamm), Nauka, Moscow, 1972; b) E. L. Feinberg, Lebedev Inst. Preprint No. 166, Moscow, 1972.
[15]B. M. Bolotovskiĭ, V. A. Davydov, and V. E. Rok, Usp. Fiz. Nauk 126, 311 (1978) [Sov. Phys. Usp. 21, 865 (1978)].
[16]E. L. Feinberg, in: Proc. Tenth Intern. Cosmic Ray Conf., Vol. II, London, 1965; E. L. Feĭnberg, Zh. Eksp. Teor. Fiz. 50, 202 (1966) [Sov. Phys. JETP 23, 132 (1966)].
[17]V. L. Ginzburg, Dokl. Akad. Nauk SSSR 23, 775, 889 (1939); 24, 181.
[18]V. L. Ginzburg, Teoreticheskaya fizika i astrofizika (Theoretical Physics and Astrophysics), Nauka, Moscow, 1975, Chapter I.
[19]I. M. Frank, Izv. Akad. Nauk SSSR. Ser. Fiz. 6, 10 (1942); see also I. M. Frank, cited in Ref. 14a.
[20]A. Rubinowicz, Ann. Phys. (Leipzig) 53, 257 (1917); 73, 339 (1924).
[21]M. L. Ter-Mikaelyan, Interferentsionnoe izluchenie sverkhbystrogo élektrona v kristalle (Interference Radiation of an Extremely Fast Electron in a Crystal), Dissertation, Lebedev Physics Institute, Moscow, 1952.
[22]Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. 25, 289, 296 (1953).
[23]M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokikh énergiyakh (Effect of a Medium on High-Energy Electromagnetic Processes), Izd. Akad. Nauk Arm. SSR, Erevan, 1969; High-Energy Electromagnetic Phenomena in Medium, Wiley, New York, 1972; I. M. Ryazanov, Usp. Fiz. Nauk 114, 393 (1974) [Sov. Phys. Usp. 17, 815 (1975)].
[24]L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 92, 535, 735 (1953); see the discussion in Ref. 23.
[25]M. L. Ter-Mikaelyan, Dokl. Akad. Nauk SSSR 94, 1033 (1954).
[26]V. L. Ginzburg and V. N. Tsytovich, Usp. Fiz. Nauk 126, 553 (1978) [Physics Reports 49 (1), 1–89 (January 1979)].
[27]M. L. Goldberger and K. M. Watson, Collision Theory, Wiley, New York, 1964 (Russ. Transl. Mir, Moscow, 1967), Chapter 8, §5.
[28]P. Carruthers, Ann. N. Y. Acad. Sci. 229, 9 (1974); I. L. Rozental', Usp. Fiz. Nauk 116, 271 (1975) [Sov. Phys. Usp. 18, 430 (1975)]; E. V. Shuryak, cited in Ref. 3, p. A3–1; E. I. Daibog, I. L. Rosental, and Ju. A. Tarasov, Fortschr. Phys. 27, 313 (1979).
[29]G. T. Zatsepin, Zh. Eksp. Teor. Fiz. 19, 1104 (1949); G. T. Zatsepin and L. I. Sarycheva, Dokl. Akad. Nauk SSSR 99, 951 (1954); N. L. Grigorov, Usp. Fiz. Nauk 58, 599 (1956).
[30]a) B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. 3, 237 (1958); b) E. Lohrman, M. W. Teucher, and M. Schein, Phys. Rev. 122, 672 (1961); S. A. Asimov, A. M. Abdulaev, V. M. Myalkovsky, and T. S. Yuldashbaev, in: Proc. Intern. Conf. on Cosmic Rays, Jaipur, 1963, Vol. 5, p. 69; L. Lal, R. Raghavan, T. N. Rangaswamy, A. Subramanian, S. C. Tonwar, and R. H. Vatcha, in: Proc. Intern. Conf. on Cosmic Rays, Jaipur, 1963, p. 377; c) K. G. Gulamov, G. M. Chernov, and B. S. Yuldashev, in: Mezhdunarodnyĭ seminar po mnozhestvennoĭ generatsii chastits (International Seminar on Multiple Production), Dubna, 1975; A. I. Dem'yanov, V. S. Murzin, and L. I. Sarycheva, Yadernokaskadnyĭ protsess v plotnom veshchestve (Nuclear Cascading in Dense Matter), Nauka, Moscow, 1977, p. 131.
[31]I. L. Rozental' and D. S. Chernavskiĭ, Usp. Fiz. Nauk 52, 185 (1954).
[32]W. Busza, J. E. Elias, C. Halliwell, D. Luckey, P. Swartz, L. Votta, and C. Young, Preprint Fermilab-Pub 79/47-Exp. 7185.178 (submitted to Phys. Rev.), June 1979.
[33]É. V. Shuryak, Yad. Fiz. 24, 630 (1976) [Sov. J. Nucl. Phys. 24, 330 (1976)].
[34]G. T. Zatsepin, in: Proc. Intern. Conf. Cosmic Rays, Kyoto, 1961, Vol. III, 1962, p. 495.
[35]K. S. Kolbig and B. Margolis, Nucl. Phys. B6, 85 (1968).
[36]G. Bellini, in: Multiparticle Production on Nuclei at Very High Energies: Proc. of the Topical Conference, Trieste, 1976, Trieste, 1977, p. 505; J. Rosen, in: Multiparticle Production on Nuclei at Very High Energies: Proc. of the Topical Conference, Trieste, 1976, Trieste, 1977, p. 305.
[37]A. B. Zamolodchikov, B. Z. Koleliovich, L. I. Lapidus, and S. V. Mukhin, Zh. Eksp. Teor. Fiz. 77, 451 (1979) [Sov. Phys. JETP 50, 229 (1979)].
[38]H. I. Miettinin and J. Pumplin, Phys. Rev. Lett. 42, 204 (1979).
[39]V. S. Verebryusov and L. A. Ponomarev, Preprint ITÉF-114, Moscow, 1979.
[40]R. Glauber, Phys. Rev. 99, 1515 (1955).
[41]L. Caneschi, P. Grassberger, H. Miettinin, and F. Nenyey, Phys. Lett. 56B, 359 (1975); K. Fialkowski and H. Miettinin, Nucl. Phys. 103, 247 (1967).
[42]H. Miettinen and J. Pumplin, Phys. Rev. D18, 169 (1978).
[43]O. V. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. 18, 465 (1973) [JETP Lett. 18, 274 (1973)]; N. N. Nikolaev, cited in Ref. 30; V. I. Zakharov and N. N. Nikolaev, Yad. Fiz. 21, 434 (1975) [Sov. J. Nucl. Phys. 21, 227 (1975)].
[44]Yu. P. Nikitin, I. L. Rozental', and F. M. Sergeev, Usp. Fiz. Nauk 121, 3 (1977) [Sov. Phys. Usp. 20, 1 (1977)] (Section 2, Subsection 9); N. N. Nikolaev, Multiparticle Production on Nuclei at Very High Energies, Topical Meeting, Trieste, 1976, ICTP, Trieste, 1977.
[45]E. L. Feĭnberg and D. S. Chernavskiĭ, Usp. Fiz. Nauk 82, 3 (1964) [Sov. Phys. Usp. 7, 1 (1964)] (see the end of Subsection III.4); V. N. Gribov, in: Élementarnye chastitsy, shkola ITÉF (Elementary Particles; School of the Institute of Theoretical and Experimental Physics), Atomizdat, Moscow, 1973, No. 1, p. 65.
[46]B. N. Kalinkin and V. L. Shmonin, Yad. Fiz. 21, 628 (1975) [Sov. J. Nucl. Phys. 21, 325 (1975)].
[47]A. I. Demianov, V. S. Murzin, and L. I. Sarycheva, in: Proc. Intern. Conf. on Cosmic Rays, Kyoto, 1979, Vol. 6, p. 227.
[48]M. A. Markov, Zh. Eksp. Teor. Fiz. 17, 846 (1947).
[49]I. E. Tamm, Sobranie nauchnykh trudov (Collected Works), Vol. 1, Nauka, Moscow, 1975, p. 94.
[50]G. B. Alaverdyan *et al.*, Yad. Fiz. 31, 776 (1980) [Sov. J. Nucl. Phys. 31, 402 (1980)].
[51]V. I. Makarov *et al.*, Phys. Lett. B43, 72 (1973); Lett. Nuovo Cimento 8, 151 (1973).

Translated by Dave Parsons