Acoustic radiating parametric antenna

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The article is a review of the basic papers on parametric antennas published in 1963–1977. Parametric antennas that make it possible to obtain low-frequency radiation of very high directivity with a small aperture of primary source have recently become a subject of active research. Progress in the theory now makes it possible to obtain all the basic characteristics of an antenna. In the review the main attention is given to the axial distribution of sound pressure and to the width of the directivity characteristic at low frequencies. One of the basic shortcomings of such an antenna is low efficiency of parametric transformation. It is noted that regimes close to saturation are as yet insufficiently studied. Optimization of antenna parameters is necessary for improvement of its efficiency. An analysis of the basic experimental results shows that laboratory models of antennas with the primary frequency range of 0.5–1.5 MHz are not optimal. This, probably, is responsible for their efficiency being 10 dB lower than the theoretical value. In the review there are noted important possibilities of applications of such antennas in the field of underwater research.

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Nonlinear interaction of acoustic waves makes it possible, with small dimensions of radiators, to obtain in a medium a distributed radiating or receiving antenna of quite large dimensions. The medium filled with the primary radiation is used as an antenna.

A parametric radiating antenna in the simplest case consists of two primary pumping waves of close frequencies ω_1 and ω_2 radiated by the source 1 (Fig. 1) which generate as a result of nonlinear interaction a wave of low frequency $\Omega = |\omega_1 - \omega_2|$ in addition to high frequency combination waves. This generation is effective only in region 2 where the amplitude of the primary waves is sufficiently large. The length of this region L_a is limited by damping and diffraction of primary waves. For parametric radiation there is formed in the medium an analogue of the running wave antenna, and therefore the directivity characteristic of the low frequency wave is determined not so much by the aperture a of the source of primary waves, as by the effective length of the antenna L_a. For a sufficiently large length L_a, without taking into account the diffraction of the pumping radiation, the width of the directivity characteristic is $\theta \approx N^{-1/2}$, $N = L_a/\Lambda$, where Λ– is the wave length of the low frequency wave. Therefore, a parametric antenna allows one to produce highly-directional low frequency radiation with a small aperture a of the pumping source (often $a < \Lambda$). Besides this substantial advantage in comparison with linear antennas, it has become clear that the high directivity is preserved over a rather wide range of frequencies Ω. Such an antenna, in addition, "corrects" well the defects of the directivity characteristics of the primary radiation, and this allows the elimination of side lobes even in the case where the primary characteristics have a well developed lobe structure. The deficiency of this antenna is the small efficiency of transformation—the part of the energy of primary radiation transformed into the energy of low frequency radiation.[1)] According to Manley-Rowe's relationship, transformation with a substantial decrease of frequency in the interaction of three plane waves allows one to achieve the maximum efficiency of transformation in energy $\eta^2 = \Omega^2/2\omega^2$. The estimates made taking into account diffraction of primary and secondary fields under the best conditions give the same results with an accuracy within a factor of the order of unity. In many applications it would be possible to tolerate low power efficiency if it were possible to obtain sufficiently powerful radiation of low frequency. Here, however, difficulties arise. Not to mention the technical difficulties connected with the possibility of cavitation in liquids, the fatigue of the radiator material, etc., the intensity of the primary radiation of an antenna has a fundamental limitation in the fact that in the absence of dispersion formation of periodic weak ruptures in the primary waves leads to a sharp increase in their damping. Then an antenna is saturated. A further increase in intensity diminishes the effective length L_a of the antenna and gradually it loses its main advantage—the highly directional characteristic.

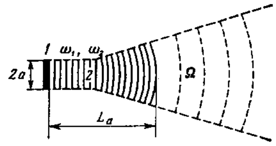

FIG. 1. Scheme of a parametric antenna.

These quite contradictory properties of an acoustic radiating parametric antenna have attracted considerable attention. At present more than a hundred articles have been published in our country and abroad, the more important of which are reviewed briefly in this article. It is necessary to say at the outset that in some applications the advantages, noted above, of such an antenna overshadow its main deficiency—small efficiency and a comparatively low level of radiation.

The theory of parametric antennas is complicated because solutions of problems of diffraction of nonlinear waves are rather difficult to obtain at present. Nevertheless there exist some approximate solutions allowing one not only to choose antenna parameters correctly but also to perform calculations that agree quite well with

[1)]The efficiency of transformation may be defined differently depending on the aim to be achieved because parametric generation and radiation are distributed in space. Examples of different definitions of the efficiency will be given later.

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experiments. Here we will discuss briefly some results of the theory relating to the axial distribution of low frequency parametric radiation and to its directivity characteristics. These two properties of a parametric antenna play probably the most important role in the majority of applications.

The general form of the solution for the axial distribution of the amplitude of the vibrational velocity v_Ω can be presented as

$$\frac{v_\Omega}{v_0}=\frac{\pi\varepsilon M}{\Lambda}X(x,\ L_d,\ l_d,\ \Phi,\ L_D)\,, \tag{1}$$

where v_0 is the pumping amplitude, and $\varepsilon=(\Gamma+1)/2$ is the nonlinear parameter of the medium ($\Gamma=c_p/c_v$ for gases, Γ is an empirical parameter for liquids; for water, in particular, $\varepsilon\approx 4$), $M=v_0/c_0$, c_0 is the sound velocity. The function representing the axial distribution of the secondary field X depends upon the damping length of pumping $L_d=\alpha_\omega^{-1}$ and upon that of the low frequency radiation $l_d=\alpha_\Omega^{-1}$, where α are the corresponding damping coefficients, and also upon the reduction in frequency $\Phi=\omega/\Omega$ and diffraction length $L_D=\omega a^2/2c_0$. This function has a maximum at a distance x_1; in this region the transfer of energy from the pump wave compensates damping and diffraction losses of the low frequency wave. The form of the X function is determined by the model of the primary field, by the regime of the antenna operation and, in particular, by the relationship between L_d and L_D.

At $L_d \ll L_D$ pumping is already damped before reaching the region where the influence of diffraction becomes substantial. In this case generation of a low frequency wave occurs in the near field of pumping where the wave can be considered quasiplane. The solution of the problem of parametric radiation for the case of a plane wave has been given in Ref. 1. Evidently, it does not allow one to obtain information about the directivity of an antenna; however, it does give estimates, correct in order of magnitude, of the transformation efficiency in different regimes of operation of such an antenna. At comparatively low pumping intensities we have $R=v_0\lambda/\nu \ll 1$, where λ is the wavelength of pumping, and ν is the kinetic viscosity of the medium. At a distance x_1 the efficiency of transformation of the amplitude is $\eta(x_1)=v_\Omega(x_1)/v_0=R/\Phi\ll 1$. At considerable intensities of pumping we have $R\gtrsim 1$, $\eta(x_1)\approx\Phi/2$ which is in accordance with the Manley-Rowe efficiency with an accuracy within a factor of the order of unity. Because the efficiency at $R\gtrsim 1$ is larger it would seem that this operational regime of an antenna might be desirable. However, in this regime, as has been noted already, ruptures are formed in the pumping wave and the antenna passes into the saturation regime. This regime is unattractive for two reasons: due to a sharp increase in damping the effective length of the antenna decreases and, therefore, the directivity characteristic broadens; and also parametric radiation, although having larger amplitude, is located mainly near the source of pumping. More even distribution of it gives the regime of an unsaturated antenna.

In the other limiting case $L_D \ll L_d$ pumping propagates to much longer distances than the extent of its nearest field. In this case the parametric generation occurs mainly in the region of the spherical divergence of the pumping wave. The one-dimensional problem of a parametric antenna with a spherically divergent primary beam without saturation was considered in Ref. 2. At $x > L_d$ the efficiency of transformation reduced to the radius of the radiating sphere x_0 and given by $\eta=-\varepsilon Rz\ln z/\Phi_\gamma$, $z=\gamma x_0/L_d\ll 1$, where γ is Euler's constant, is substantially smaller than in the case of a plane wave. This result is natural, since in a spherically divergent wave parametric generation is considerably less effective. In the region of spherical divergence, however, the antenna is not so sensitive to the transition into the saturation regime, which allows one to increase the power of pumping considerably.

An important antenna characteristic is the distance x_z at which the amplitude of the low frequency parametric radiation becomes equal to the pumping amplitude. It is possible to show that at $R\ll 1$ the condition $x_2\approx -L_d\ln\eta$ holds for plane as well as for spherical waves and, therefore, the "rupture" (at $x>x_2$) of the low frequency radiation occurs the earlier the greater is the damping in the medium and the higher is the efficiency of transformation. This allows one to draw a qualitative conclusion about the usefulness of the parametric regime for the active direction finding. It is useful (naturally if special methods of increasing the signal-to-noise ratio are not applied) with the reflector situated at distances greater than x_z and with the signal level (reflected and arriving at the point of reception) higher than the noise level, which is possible at sufficiently high transformation efficiency and sufficiently high damping of the pumping in the medium. These requirements are, however, contradictory because it is impossible to form a long antenna and to obtain high directivity characteristics in a medium with very high damping. Such antennas have found application (see below) in the case when large damping takes place beyond the region of generation of the low frequency radiation (for example, in studies of bottom sediments, when the antenna is formed in water and radiation propagates into strongly absorbing sediments). It is necessary to note that in all antennas studied in water the distance x_z is sufficiently large; at these distances the magnitude of the parametric signal is comparable to or less than the level of the sea noise.

The first quasi-two-dimensional theory of an antenna was proposed by Westervelt[3]. The theory is based on the fact that one-dimensional equations of nonlinear acoustics in approximations higher than the first are inhomogeneous wave equations with the inhomogeneity known from the solutions of equations of lower order approximation. The equation of the second order approximation is the equation with sources whose distribution is determined by the primary field. The primary field in the theory[3] was supposed to be a narrow collimated beam. The application of the method of retarding potentials has yielded in the far field

$$X(x,\ \theta)=\frac{\pi a^2 L_d}{2x\Lambda}\left[1-4\left(\frac{2\pi L_d}{\Lambda}\right)\sin^4\frac{\theta}{2}\right]^{-1/2}. \tag{2}$$

The half-width of the directivity characteristic θ_W determined from the condition of decreasing v_Ω^2 by a factor of two for a long antenna ($L_d\gg\Lambda$) is

$$\theta_W \approx \sqrt{\frac{\Lambda}{\pi L_d}}. \quad (3)$$

The limitations of Westervelt's theory are considerable:

1. The diffraction of the primary field and the transverse distribution of secondary sources were not considered. This, in fact, is what made possible the use of a one-dimensional system of equations with the artificial introduction of two dimensions in obtaining the second approximation. It is evident that such discussion becomes possible at $L_d/L_D \ll 1$ and $a \ll \Lambda$, the parameter N_D being $N_D = L_d/L_D\Phi \ll 1$. This parameter, as will be seen later, is important for parametric antennas taking diffraction effects into account.

2. The near field of secondary radiation is excluded from consideration. Although, as a rule, characteristics in the far field are of more interest in applications, in studies of antennas and their optimal working conditions, the process of formation of the parametric radiation is of considerable interest.

3. The approximate character of the theory is evident from the fact that the second approximation was used and, therefore, the theory is applicable only for low-power primary beams.

This led to the fact that there was no complete agreement between theory and experiment neither in the level of radiation nor in the width of the directivity characteristic. In particular, under some experimental conditions the directivity characteristic was narrower than what would follow from equation (3). It is necessary to note, however, that Westervelt's theory had considerable influence on the development of antenna concepts. Many papers were concerned with the development of this theory. In the articles of Refs. 4–6, in particular, the transverse distribution of secondary sources was taken into consideration. In many articles models of the primary field more complicated than that of a collimated beam were studied, including the spherically divergent directed wave, 7, 8, the collimated beam in the near field and the cone of spherically divergent waves in the far field. The calculations of the near field of an antenna were performed on the basis of model representations of the primary field.[11,12]

A fundamentally new theory of an antenna taking into account in a natural manner the nonlinear diffraction of three-dimensional waves became possible on the basis of the solution of the Khokhlov-Zabolotskaya equation for nonlinear sound beams.[13] This has allowed the formulation of a theory of an antenna practically free from the limitation's theory. In the first work of this direction[14] the problem of parametric radiation in the near field at $x < L_D$ was considered, that is, the diffraction of the primary radiation was not taken into account. The more general case was studied later[15-17] taking into account the diffraction of both the secondary and primary fields. The integral representation of the formation and radiation of a low frequency wave was attained as a result. This representation is substantially simplified in cases of greatest practical interest for axial distributions of low frequency radiation and can be found in explicit form. Antennas with $N_D \gtrsim 1$ are of practical interest, that is, antennas in which the region of the primary wave interaction is not only larger than the diffraction length of the primary radiation and includes, therefore, the far field of this radiation, but is also larger than the region of the effective parametric generation, whose length is $\sim L_D\Phi$. The case considered by Westervelt ($N_D \ll 1$) evidently is not optimal because the primary radiation is attenuated too quickly. The other limiting case $N_D \gg 1$ also has no special interest because in this case the principal region of generation of parametric radiation is transferred to the region of the spherically divergent pumping where, as has been already noted, the efficiency of transformation is small. For the most interesting case $N_D \gtrsim 1$ at the distances $x < L_d$ the axial distribution obtained in Ref. 17 is

$$X(x) = \frac{2L_D}{\sqrt{4\Phi^2 + \Phi^2 x_H^2}} \sqrt{\operatorname{arctg}^2(2\Phi x_H) + \frac{1}{4}\ln^2\frac{1+4\Phi^2 x_H^2}{(1+x_H^2)^2}}, \quad (4)$$

where $x_H = x/L_D$. It follows from this relation that the parametric radiation attains a maximum at $x_1 = L_D$, and at the maximum we have

$$X_m \approx \frac{\pi L_D}{\sqrt{5}\,\Phi}\left(1 + \frac{1}{\pi^2}\ln^2\Phi^2\right).$$

In the far field for $x \gg L_d$, according to Ref. 15, 16, we have

$$X(x) = \frac{2L_D}{x}\sqrt{\frac{1}{4}\ln^2(1+N_D^2) + \operatorname{arctg}^2 N_D}. \quad (5)$$

It can be seen from this relation that the axial distribution depends weakly on the parameter N_D at far distances in the spherically divergent low frequency wave, with the order of magnitude of the quantity (5) corresponding to (2) at $N_D \ll 1$.

The regimes considered above, in general, referred to an unsaturated antenna, that is to conditions when there are no ruptures in the interaction region. In Refs. 15, 18, 19 an approximate analysis of the saturation regime was made which, however, allows one to draw only qualitative conclusions. The use of a saturated regime for which the rupture is formed in the region $X \leqslant L_D$, as has been noted already, allows high efficiency. However, in this case the region of comparatively high level of the low frequency wave is close to the region of the saw-tooth wave and is shorter than in the case of an unsaturated antenna. At large distances the saturation regime, probably, can not provide a substantial advantage. The weak dependence on the boundary conditions, as is well known, is characteristic of systems satisfying equations similar to the Burgers equation. It is necessary to mention the relation between the width of the directivity characteristics of primary and secondary radiation. The relation (2) was obtained under the condition that $N_D \ll 1$ and the directivity characteristic of the primary radiation is δ-function-like (a plane wave). In this case, naturally, the width of the low frequency characteristic is greater than that of the primary radiation. In the case $N_D \gtrsim 1, L_a \sim L_D\Phi$ and it can be easily seen that $\theta_0 \sim a/\lambda$, that is, the width of the secondary characteristic is of the same order as that of the primary characteristic. The possibility of obtaining a "quadratic" characteristic narrower than the primary one was predicted for $N_D \gg 1$ in those theoretical papers in which the nonuniformity of the primary field was taken into account by means of models. These papers, however, are based on an incorrect account of the interaction in the near and intermediate fields of an antenna because the directivity characteristic of the primary radiation in this region is considered as having been already formed. The correct account of the interaction in all regions will probably not allow one to obtain a low frequency characteristic narrower than the characteristic of the primary radiation. The problem of the directivity characteristic at $N_D \gg 1$, apparently, is of no practical interest, because it is evident from the previous discussion that this regime of antenna operation is inefficient.

The complicated relation between nonlinear diffraction and dissipative effects in a parametric antenna leads to the necessity of optimization of the antenna's parameters in order to obtain the highest level of the radiation while preserving a narrow directivity characteristic.[17,20,21] Optimization, of course, is determined also by acoustic parameters of the medium; for example, an antenna works better in water than in air because of lower attenuation, and this is observed experimentally. Unless some special additional conditions are imposed the optimal frequencies for water are in the range 10^4–10^5 Hz. The optimal diameter of the source increases sharply at lower frequencies. The higher frequency waves have high damping and the length of an antenna becomes too small. In the optimal case the length of the rupture formation region must be of the order of the length of effective parametric generation[17] and the attenuation length must be not less than this length, that is $N_D \gtrsim 1$.

Let us now consider experimental investigations. It should be noted that of the large number of experimental papers the majority, but by no means all, use a laboratory model of an antenna. In Fig. 2a and 2b the directivity characteristics of primary waves of frequencies (a) 418 and (b) 482 kHz are shown; the directivity characteristic of the secondary wave of frequency 64 kHz is shown in Fig. 2c.[22] These results show that the complex lobe structure of the primary radiation disappears practically completely in the secondary radiation; its level is lower than 40 db. However, such good results were obtained in far from all the investigations. The direct emission of low frequency radiation by the source of primary waves brought about by detection in radio circuits may be the cause of the "pear"-shaped characteristic. In the region of the lowest frequencies

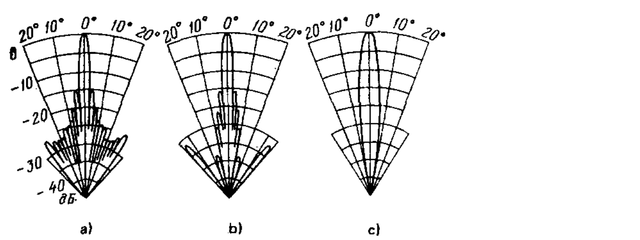

FIG. 2. Directivity characteristics of primary (a, b) and secondary (c) radiations.[22]

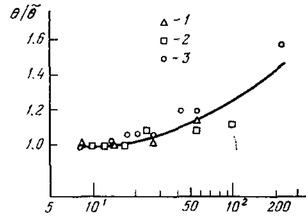

FIG. 3. Dependence of the relative width of the directivity characteristic on the frequency reduction. 1—according to Ref. 23, 2—according to Ref. 24, 3—according to Ref. 10.

when an antenna becomes short ($N \sim 1$) a nonlinear transformation of the directivity characteristic is possible which leads to sufficiently strong side radiation.[37] The width of the directivity characteristic in most of the experimental investigations was somewhat larger than the width of the characteristic of the primary radiation, approaching the latter at $N_D \gg 1$. The broadband properties of the antenna can be seen from Fig. 3 where the relative increase in the width of the directivity characteristic of the secondary radiation ($\tilde{\theta}$—the width with a reduction of $\Phi \leq 10$) with a decrease in the low frequency is shown using the results of several investigations.[23,24,10] The transition into the saturation regime is accompanied by a broadening of the directivity characteristic[25,18,19,26] proportional to $\sqrt{p_0}$, as is shown in Fig. 4, and by generation of harmonics of Ω: 2Ω, 3Ω, etc.[27] As theory[28] shows, the special features of harmonic generation depend upon the type of modulation of the primary wave, and their directivity characteristics substantially differ from the characteristic for Ω.

The results of measurements of the axial distribution of the levels of the primary (I) and secondary (II) radiation in relation to the sound pressure of 1μPa according to data of several authors are shown in Fig. 5 and the corresponding characteristics of antennas are shown in Table I. The level of secondary radiation at distances of 10–20 m, as can be seen from Fig. 5, lies 40–45 dB lower than the level of the primary radiation. The levels must become equal at the distances $x_2 \sim 10$–100 km where they will be of the order of 1 dyn/cm^2. In the $Y_{II} - Y_I = -20 \log 2\Phi$ column of Table I the order of magnitude of the maximum theoretical efficiency is shown to be approximately 10 dB higher than the experimental values. This shows that the antenna characteristics are not optimal. The results shown in Table I refer to laboratory models of an antenna with the primary frequencies in the range of 0.5–1.5 MHz; this region is not optimal for water as has been noted earlier. In the study of Ref. 31 the regime was apparently close to optimal because at $\omega/2\pi = 105$ kHz, $2a = 92$ cm the value obtained

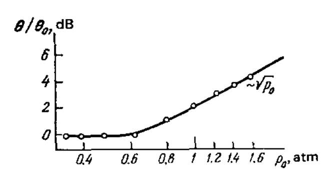

FIG. 4. Dependence of the relative width of the directivity characteristic on the amplitude of primary radiation.[26]

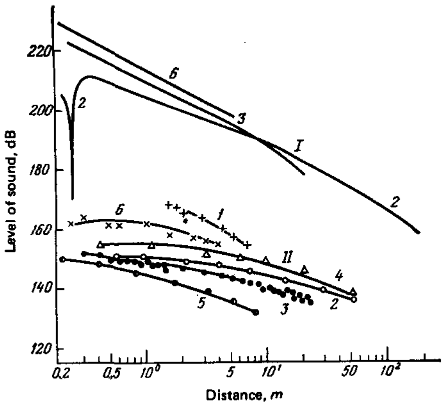

FIG. 5. Levels of the Primary (I) and secondary (II) radiations at the axis as a function of the distance. 1—according to Ref. 27, 2—Ref. 7, 3—Ref. 29, 4—Ref. 24, 5—Ref. 23, 6—Ref. 30.

at the frequency $\Omega/2\pi = 5$ kHz was $\eta = 10^{-2}$, which is close to the theoretical maximum value. The decrease in the level of the parametric radiation with increase in the degree of reduction in frequency is shown in Fig. 6 according to the data of Refs. 31, 29. The continuous line corresponds to the functional dependence $p_\Omega \sim \Omega^{1,7}$; in accordance with the theory of Ref. 3 this dependence must be Ω^2.

One might suppose that for long antennas under real sea conditions fluctuations in the velocity of sound, dispersion, and refraction can exert an influence. As the investigations of Ref. 32 have shown, velocity fluctuations do not significantly affect the operation of an antenna. In the case of dispersion which is theoretically possible because of the strong reduction in frequency, it can be easily shown[33] that parametric radiation will be maximal at the angle

$$\theta = \arccos \frac{k_1 - k_2}{k_d} = \sqrt{\frac{2\Delta c}{c_0}}$$

to the antenna axis. Here k_1, k_2 and k_d are wave numbers of the primary and secondary waves, and $\Delta c = |c_\omega - c_\Omega|$. Under real conditions $2\Delta c/c_0$ is not higher than a fraction of one per cent which gives $\theta \approx 1,7°$. Antennas, as a rule, have somewhat larger widths. However, it can be seen from this estimate that the dispersion widening of the characteristic must be taken into account.

A few words about antenna applications must be said in conclusion.[22] Presently the main application of an antenna is investigating the sea bottom profile with high resolution and studying the structure of sea bottom precipitates.[34,36] The latter is based on the fact that the low frequency radiation is much more weakly absorbed in the precipitates than high frequency radiation. There is a report on the possibility of penetration to 75 m into bottom precipitates at a depth of 1 km. The narrow directivity characteristic of the antenna allows its use in searching for foreign objects in the bottom precipitates in oceanic archeology, in particular.[22] The broad band characteristic of the antenna allows optimal processing of hydrolocation signals and the detection of "weak" targets and also the study of frequency characteristics of sound scattering layers and bottom precipitates. The same property of an antenna was used for coded underwater communication at distances up to 4 km, a high reliability of transmission being noted.[35] The structure of the secondary field free of lobes allows one to work practically in a one-mode regime in a shallow sea and eliminates bottom and surface reverberation.[22] Parametric antennas find applications as broadband sources of low frequency sound for the calibration of hydro-acoustical receivers under pool conditions when reflections from the walls of the pool must be minimal. An attempt was made to use the parametric regime in "ultrasonic x-raying". A hypothesis was advanced that the parametric regime forms the basis of the echo-location apparatus of the dolphin, which is known to have very high angle resolution at low frequency with a small source.[22,36]

TABLE I. Characteristics of antennas.

References	Level of primary radiation, dB	$\omega/2\pi$, kHz	$\frac{\Omega}{2\pi}$, kHz	$2a$, cm	Φ	Ld/L_D	N_D	$Y_{II}-Y_I$, dB
27	237	700	14,2	10	50	22	0.44	−40
7	210	450	64	7.5	7	150	21	−23
29	220	890	40	2	22	270	12	−33
24	220	1064	50	3.5	21	66	3.1	−33
23	214	1435	50	2.0	29	42	2.9	−35
30	230	840	50	1×1	17	1200	71	−30

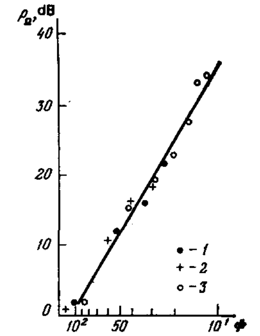

FIG. 6. Dependence of the level of the secondary radiation on the frequency reduction. 1—according to Ref. 31 at frequency of 105 kHz, 2—according to Ref. 31 at frequency of 123 kHz, 3—according to Ref. 29.

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