L. I. Mandel'shtam and research in radiointerferometry

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An account is given of the work of L. I. Mandel'shtam and of his school in the field of interference at radiofrequencies between coherent radiations with commensurable frequencies, using direct measurements of the phase difference between the interfering oscillations. The various radiointerferometers used under the general direction of L. I. Mandel'shtam in studies of the phase structure and the velocity of radio waves over the earth's surface are described. Possible future developments in radiointerferometry and its applications in various scientific and practical areas are outlined.

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1. INTRODUCTION

Leonid Isaakovich Mandel'shtam's work in radio-interferometry is among his most outstanding and original contributions to science. It clearly illustrates the unique character of L. I. Mandel'shtam's scientific creativity and his gift for broad generalization and the development of new concepts and descriptions created under different conditions and applied to new phenomena.

Radio waves occupy the range from 3×10^{11} to small multiples of 10^3 Hz and are a particular form of electromagnetic radiation, basically of the same kind as infrared, visible, ultraviolet, and x-ray radiation. However, the scale in space and in time of processes occurring in the radio-wave region is very different from the scale characteristic for optical phenomena. Although the theory of the electromagnetic field used in the study of emission and propagation of radio waves is basically the same as that in classical wave optics, the above difference between the scale of the phenomena forces us to take into account many factors that can justifiably be ignored in optics. The basic space scale is the wavelength, which, in the radio-wave band, is often of the same order as or greater than the distances and linear sizes under investigation. The use of sources with linear dimensions comparable with a wavelength, the possibility of maintaining undamped oscillations of elementary vibrators, and many other properties characteristic of the radio band result in different approaches to the theoretical study of radio waves as compared with optics, so that new relationships that would be physically meaningless in optics are established. This refers to the theory of emission and propagation of radio waves, the spatial distribution of radiation intensity, the reception, conversion, and indication of radio waves, and to many other questions.

The differences are very great, but at the same time an effort must be made not to forget the basically common nature of optical and radio waves. Any phenomenon involving one of the types of electromagnetic wave should also occur at other frequencies or wavelengths except that it may appear in a different form and have different features because of the difference of scale. The phenomenon of interference, which has long been known and used in optics, is also found to occur in the radio band. However, prior to L. I. Mandel'shtam's work, the full possibilities of interference phenomena in the radio band were not properly understood. It was only after the extensive studies undertaken by him in collaboration with N. D. Papaleksi, and carried out under their direction by large scientific teams in the 1930's, that fundamental problems connected with the development of different radiointerferometers were solved.

L. I. Mandel'shtam's work in this field began even before 1930, when he and N. D. Papaleksi obtained the first patent for a radiointerference device ("A method for determining the separation between two points with the aid of electromagnetic waves,"[1]), but publications relating to this series of results began to appear only after 1937.

In optics, coherence between interfering rays is achieved by deriving them from the same source of radiation, i.e., by using beam splitters, so that the coherent rays arrive along different paths. In the radio band, on the other hand, it is possible to achieve coherence between different sources (including sources with different but commensurable frequencies) by suitably controlling the devices responsible for the generation of these waves. The observation and detection of interference phenomena in the radio band is much easier than in optics because not only amplitude (power) detection can be performed, but also direct measurements of phase shifts between interfering waves can be carried out.

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These fundamental ideas lie at the basis of different radiointerferometric devices and are now so obvious that no one stops to think about them. However, in the 1930's, when these ideas were first formulated and began to be used in particular radiointerferometric systems, in studies of radio wave propagation, and in many practical applications, these techniques were quite new and well outside the framework of conventional ways of using and studying the possibilities of radio waves. Here again, the depth and scope of the scientific creativity of L. I. Mandel'shtam became quite clear and his ideas were fundamental for many branches of modern physics and technology.

L. I. Mandel'shtam[2,3] noted the possibility of very precise measurement of the frequency of waves generated and emitted in the radio band by means of different types of radiointerferometer. This opened up areas that were inaccessible to optical interferometry, so that by studying radio wave propagation solutions to many practical problems could be obtained. In most cases encountered in optics, one is concerned with expressing the length of a standard in terms of the wavelength of optical radiation or, in other words, with the measurement of wavelength. In radiointerferometry, on the other hand, accurate measurement of frequency and the possibility of phase measurement enables us to investigate the phase structure of radio waves, to measure their velocity, and, once these quantities are known, to determine distances and displacements.

Even during the lifetime of Leonid Isaakovich Mandel'shtam, the development of different radiointerferometers and their application to scientific and practical problems led to outstanding scientific and practical results. Radiointerferometry was subsequently very fruitful in studies of the ionosphere and magnetosphere, and in space research. Radiointerferometric methods are at the basis of many of the systems used to determine and control the orbital parameters of artificial earth satellites, and have led to the development of global radionavigation systems and to the solution of many problems in local radionavigation and geodesy.

It therefore seems appropriate to begin with a brief outline of the principles underlying the operation of the most typical radiointerferometric devices, the most important scientific results, and practical applications of radiointerferometry, starting from the time when L. I. Mandel'shtam himself was engaged in research in this field.

In addition, it will be interesting to consider possible further developments in radiointerferometry and some special applications of radiointerferometer systems.

2. EARLY WORK

The interferometric radio rangemeter[4] proposed by L. I. Mandel'shtam and N. D. Papaleksi was based on the use of the variation with frequency of the measured phase difference between oscillations of commensurable frequency, one of which is emitted by the so-called "master" station and the other by the "reflecting" station located at the other end of the distance being measured, for a matched variation in frequency in a precisely defined interval.

In this version of a radiointerferometer, the additional phase acquired by oscillations of frequency ω over a measured distance d is given by

$$\Phi_1 = \omega \int_0^d \frac{ds}{v_1}.$$

After these oscillations are received at the "reflecting" station and their frequency is transformed in the ratio of m/n, where m and n are small integers, they are reemitted and returned by the "reflecting" station to the "master" station so that their phase at the new frequency $(m/n)\omega$ changes by

$$\Phi_2 = \frac{m}{n}\,\omega \int_d^0 \frac{ds}{v_2}.$$

The phase difference recorded at the master station at the original frequency ω is

$$\Psi = \Phi_1(\omega) + \frac{n}{m}\Phi_2\left(\frac{m}{n}\,\omega\right) + \delta_1 + \delta_2,$$

where δ_1 represents the phase shift in the receiving-measuring device at station 1, and δ_2 the phase shift produced in the instrumentation at station 2 (see Fig. 1).

If the frequency ω is varied smoothly by an amount $\Delta\omega$, and if we assume the instrumental phase shift to be constant, the phase change is given by

$$\Delta\Psi = [\Phi_1(\omega + \Delta\omega) - \Phi_1(\omega)] + \frac{n}{m}\left[\Phi_2\left(\frac{m}{n}(\omega + \Delta\omega)\right) - \Phi_2\left(\frac{m}{n}\,\omega\right)\right].$$

In the simplest case of no dispersion or diffraction over a path length d, we have $v_1 = v_2 = v$, so that

$$\Phi_1(\omega) = \frac{\omega d}{v}, \quad \Phi_2\left(\frac{m}{n}\,\omega\right) = \frac{m}{n}\frac{\omega d}{v}, \quad \Delta\Psi = \Delta\omega\cdot\frac{2d}{v},$$

and hence

$$d = \frac{v}{2}\frac{\Delta\Psi}{\Delta\omega}.$$

Thus, when the velocity of the radio waves is known, the separation between two points can be determined by measuring the change in the observed phase difference between radiated waves of frequency ω and coherent received waves of frequency $(m/n)\omega$. Numerous measurements performed with such interference radio rangemeters in the 1930's[3-6] have shown that is is often possible to use such simplifying ideas. They have, in fact, become the basis for very successful applications of this type of radiointerferometer in geodesy and navigational science, using modern versions of the required equipment.[7-11] It is precisely the radiointerferometer method that can be used in high-precision determinations of the phase velocity of radio waves, v, whenever radiointerference rangemeters are effective.

The same radio rangemeter can be used at a fixed frequency and continuously variable separation between

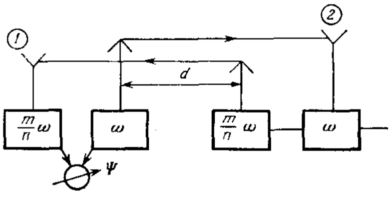

FIG. 1. Block diagram of interference rangemeter.

points 1 and 2 to determine the separation as a function of the phase difference, measured by a phasemeter, between oscillations of frequency ω and $(m/n)\,\omega$.

In this case, the measured phase difference is given by

$$\Psi=\omega\left[\int_1^2 \frac{ds}{v_1}+\int_2^1 \frac{ds}{v_2}\right]+\theta,$$

where θ is the instrumental phase shift, and v_1, v_2 are the phase velocities of the waves of frequency ω and $(m/n)\,\omega$. The change in Ψ due to the displacement of reception point 2 to a point 3 is given by

$$\Delta\Psi=\omega\left[\int_2^3 \frac{ds}{v_1(\omega,\ s)}+\int_3^2 \frac{ds}{v_2(m\omega/n,\ s)}\right].$$

In the simplest case,

$$\Delta\Psi=2\frac{\omega}{v}\,\Delta d.$$

When the radiointerferometer is used with variable separation between the two points, one can use the analogy with the optical interferometer to determine the change in the separation as a function of wavelength:

$$\Delta d=\frac{1}{4\pi}\,\lambda^*\Delta\Psi;$$

where

$$\lambda^*=2\pi\frac{\bar{v}^*}{\omega},$$

and ω is the frequency of the waves radiated by the first station and $\bar{v}^*$ is the velocity of the radio waves averaged in a particular way over the frequencies ω and $(m/n)\,\omega$ and over the interval Δd.

A major series of experiments using this type of variable-separation interferometer was carried out under the direction of L. I. Mandel'shtam and N. D. Papaleksi between 1935 and 1941 (see Refs. 2, 3, 6, 12–14) and resulted in extensive data on the phase structure and velocity of radio waves propagating over the earth's surface, covering the range between short distances (on the wavelength scale) from the radiating antennas up to very considerable distances. The data were obtained both for regular propagation over a level surface and for situations where there were irregularities producing diffraction-type distortion. The fundamental result of this research was the demonstration of the fact that the velocity of the radio waves was close to c, independently of the electrical properties of the soil, and this was also the case for propagation over the earth's surface with low conductivity and over the sea. There are only local distortions of the phase structure of the radio-wave field, accompanied by corresponding changes in the phase velocity. These arise in the transition zone near the radiating antenna and in the neighborhood of perturbing inhomogeneities. Perturbations can be produced both by irregularities in the geometry of the earth-air separation boundary and by changes in the electrical properties of the earth's surface (for example, transitions from dry land to sea, or from open landscape to woodland). These results led to an exhaustive interpretation of the so-called "coastline effect" and confirmed the validity of theoretical calculations performed by Sommerfeld on the propagation of radio waves over the earth's surface. They also demonstrated that the concepts of Zenneck's surface waves were inapplicable to these problems (see the references cited in Recent Studies of Radio Wave Propagation Over the Earth's Surface, Gostekhizdat, M., 1945).

Another version of the radiointerferometer, i. e., the so-called dispersion radiointerferometer, turned out to be a very effective way of studying radio-wave propagation. Here, two coherent waves of frequency ω and $(m/n)\,\omega$ are emitted from the same point with a given phase difference between them. The phase difference is then measured at the point of reception and may differ from the initial phase difference. It may also vary with distance between source and receiver if the phase velocities are functions of frequency, i. e., dispersion occurs. Moreover, in the transition region near the transmitters, and in regions where there are objects producing diffraction due to the difference between wavelengths and the corresponding scale of variation in the field phase structure, there also may be observable variations in the phase difference. The dispersion interferometer has therefore been extensively used and has produced data that have substantially extended the results obtained by other radiointerferometer methods on the emission and propagation of radio waves. The dispersion radiointerferometer is illustrated in Fig. 2.

The measured phase difference on the ω frequency scale is given by

$$\Psi=\omega\left[\int_1^2 \frac{ds}{v_1}-\int_1^2 \frac{ds}{v_2}\right]+\Psi_0,$$

where v_1 and v_2 are the phase velocities of the radio waves with frequencies ω and $(m/n)\omega$, and Ψ_0 is the initial phase difference between the radiated waves.

In the simplest case,

$$\Psi=\omega r\left(\frac{1}{v_1}-\frac{1}{v_2}\right)+\Psi_0.$$

If r varies, i. e., the point of reception is moved relative to the radiator, the observed phase difference is

$$\Delta\Psi=\omega\left(\frac{1}{v_1}-\frac{1}{v_2}\right)\Delta r.$$

The phase of the oscillations at the point of reception can be written in the form

$$\Phi(r)=\frac{\omega}{v}r+\varphi_0(r)+\varphi^*(r),$$

where $\varphi_0(r)$ is the phase shift due to a diffracting object and $\varphi^*(r)$ is the additional phase connected with the process of establishing the radiation pattern. The phase difference observed in the dispersion interferometer is therefore given by

$$\Psi=\Phi_1-\frac{n}{m}\Phi_2=\omega r\left(\frac{1}{v_1}-\frac{1}{v_2}\right)+\left[\varphi_1^*(r)-\frac{n}{m}\varphi_2^*(r)\right]+\left[\varphi_{10}(r)-\frac{n}{m}\varphi_{20}(r)\right].$$

It is thus clear that, in experiments performed with the dispersion interferometer, the change in the observed phase difference accompanying the displacement of the receiving system is determined by three components. The first term, which corresponds to the presence of real dispersion ($v_1 \neq v_2$), is proportional to the range, the second becomes practically constant when the distance from the radiator exceeds 50–60 wavelengths,

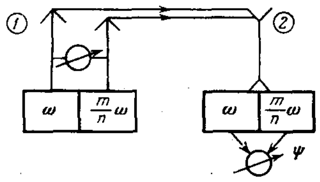

FIG. 2. Block diagram of dispersion radiointerferometer.

and the third contains the functions $\varphi_{10}(r)$ and $\varphi_{20}(r)$ and represents diffraction perturbations of surfaces of equal phase, which are different for the two frequencies because of the difference between the wavelengths λ. The last term varies with distance, but tends to zero as the distance from the diffracting object increases. Numerous investigations, performed in the 1930's using the dispersion interferometer under different conditions and in combination with other radiointerference techniques, have resulted in more detailed information on the process of establishment of phase accompanying emission, on the phase of the emitted radiation. They have also led to a better understanding of the influence of diffraction on the phase structure of the radio-wave field, and confirmed that regular dispersion is practically absent during the propagation of radio waves over the earth's surface.[3,12-15;1]

When sources of coherent waves of frequency ω and $(m/n)\omega$ are at a distance d from one another, surfaces corresponding to constant phase difference under idealized wave-propagation laws will be hyperboloids with foci at the points of emission. This means that a radio beacon can be produced because a constant phase difference at the point of reception means that the motion of the point of reception takes place over a preselected hyperboloid (or the corresponding hyperbola when a two-dimensional system is used). A system incorporating three radiators can also be employed. One of these can be the master source and the other two subordinate sources at which waves received (with constant retarded phase) are transformed in frequency into $(m_1/n_1)\omega$ and $(m_2/n_2)\omega$, respectively, and are then radiated with a predetermined phase shift. By monitoring the variation in the phase differences between the received waves with frequencies ω and $(m_1/n_1)\omega$ and ω and $(m_2/n_2)\omega$ one can then determine the displacement of the receiver over the corresponding hyperbolic grid consisting of two sets of hyperboloids with foci at the point of emission. Additional devices that can easily be developed can then be used not only to check that the observed phase differences are constant (or to establish any changes), but also to find the true values of the differences themselves and thus determine the position of the receiving point relative to the corresponding hyperbolic coordinate system. The development of these radio beacons is important from the practical point of view. It became possible only after L. I. Mandel'shtam, working with N. D. Papaleksi, introduced radio interference between coherent sources of commensurable frequency and the phase method of detection, and after sufficient information became available on the laws governing the propagation of radio waves over the earth's surface.

All the researches into radiointerferometry performed during the lifetime of L. I. Mandel'shtam have the common feature that they have led to new practical applications of radio waves. The methods for achieving radio-wave interference, developed as a result of this work, opened up a new era in radiophysics and laid the foundations for applications of radio-wave interference in navigation and geodesy. They led to exhaustive studies of the phase structure and velocity of radio waves propagating over the earth's surface and to the determination of the special features of the effect of the properties of the earth's surface on the processes of establishment of the phase structure of radiowaves emitted by an antenna located on the air-ground separation boundary. This, in turn, led to estimates of the possible precision and future possibilities of different radio-beacon type rangemeters and navigational systems when the influence of the ionosphere could be neglected. Moreover, the pioneering work in this series of investigations, in which applications of radio interference were studied in relation to the properties and perturbations of the ionosphere, provided the foundation for many subsequent investigations that are successfully developing at the present time.[2]

3. SUBSEQUENT WORK IN RADIOINTERFEROMETRY

We shall now try to elucidate the subsequent fate of radiointerference methods, their development, and their application in modern technology and physics research.

We must first note the widely used radionavigational phase systems in which phase measurements are used to locate objects in space by means of interference between two or more coherent radiations with commensurable frequencies. Such radionavigational systems operate in different wavelength bands, depending on their purpose. Long-wave radiation is used for global systems, whereas short and ultrashort radio waves can be used for navigational purposes and are very precise when the distances involved are short. All this is, of course, accomplished with the aid of automatic phase-

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[1] Here again, we shall not reproduce the very numerous data obtained as a result of these investigations with the dispersion interferometer and will limit ourselves to referring the reader to references where such data can be found together with the corresponding generalizations ensuing from these results that have realized the original ideas of Leonid Isaakovich Mandel'shtam. It is important to remember, however, that all these conclusions are valid only for separations for which the earth's surface can be regarded as flat and diffraction effects due to curvature of the earth's surface can be neglected. Subsequent theoretical and experimental studies performed at Leningrad University under the general direction of G. I. Makarov have shown that, for large distances, the phase velocity is a function of frequency and of the electrical properties of the soil,[37] i.e., definite dispersion is found to occur.

[2] Some general conclusions and generalizations of possible ways of realization and utilization of radio-wave interference on the basis of these studies were examined in a previous paper,[15] published in 1947 in this journal.

meters, often based on counting the number of complete cycles of variation in the phase difference between oscillations reduced to a single frequency, using phase detectors.

The possibility of frequency conversion with controlled phase distortion now allows us to produce by means of frequency multiplication an effective amplification of phase shifts and thus to increase substantially the sensitivity and precision of measurements of small phase differences.

Modern interferometers with long baselines, which are used in radio astronomy and in tracking systems for satellites and other space vehicles, and which ensure high angular precision in locating these objects, take the form of radiointerferometers with a single source of radiation (see Ref. 15, p. 372) and belong to the general family of radiointerferometric devices. The development of modern versions of these systems was directly stimulated by researches into radiointerferometry performed under the direction of L. I. Mandel'shtam and N. D. Papaleksi.

We note particularly the application of the dispersion interferometer to measurements of electron density in plasma near the earth's surface and in cosmic plasma, using rockets, artificial earth satellites, and other space vehicles. The development of the dispersion radiointerferometer for the meter, decimeter, and centimeter wavelength bands, in which the transmitter of coherent radiations with commensurable frequencies is carried by a rocket or some other space vehicle, has led to measurements of electron density along the path between the rocket or space vehicle and the phase-sensitive receiving system. Such transmitting devices (code name "Mayak") are now routinely incorporated in the equipment carried by research rockets, artificial earth satellites, and other space vehicles. The 1954–1958 experiments, which were performed with geophysical rockets, formed the beginning of this type of investigation in the Soviet Union. The mounting of a source of coherent frequencies on a rocket with nearly vertical trajectory meant that measurements of the phase difference between waves of commensurable frequency after passage through the ionosphere could be used to determine the electron density as a function of height.

The measured change in the phase difference $\Delta\Psi$ is related to the dispersion which, in turn, is due to the fact that the effective refractive index of the ionized atmosphere is a function of frequency because of the presence of free electrons:

$$n_\omega = 1 - \frac{2\pi N e^2}{m\omega^2},$$

where N is the effective electron density, e and m are the charge and mass of the electron, and ω is the frequency of the radio waves. When the trajectory of the rocket is vertical, the phase difference between frequencies ω and $p\omega$ is

$$\Delta\Psi = \frac{2\pi N_{\mathrm{cp}} e^2}{cm\omega} \frac{p^2-1}{p} \Delta H.$$

Hence the average electron density over an interval ΔH is

$$N_{\mathrm{av}} = \frac{cm\omega}{2\pi e^2} \frac{p}{p^2-1} \frac{\Delta\Psi}{\Delta H}.$$

When the sources of coherent frequencies are carried by an artificial earth satellite or some other space vehicle whose trajectory is not vertical, the determination of the local electron density is possible only under certain definite assumptions and not always. The interpretation of the data is complicated by the presence of the geomagnetic field and by dynamic processes in the plasma near the earth which lead to different inhomogeneities.

If the sources of coherent waves with commensurable frequencies ("Mayak" transmitters) are carried by a geostationary satellite, measurements of the phase difference between these oscillations at a point on the earth will contain information on the integrated electron density along the entire path between the earth and satellite and on the variations in it.[17,18]

The great majority of articles on the applications of the dispersion interferometer to ionospheric research was based on the classical two-frequency version of this device although three-frequency[19] and four-frequency[20] interferometers have also been used in some cases.

General analysis of the possibilities of dispersion interferometers in studies of cosmic plasma from space vehicles led to the development of the theory of phasometric systems with a generalization of the dispersion interferometer method to the case of an arbitrary number of coherent signals and a derivation of an expression for the measured quantity as a linear combination of the total phases of all the signals used. Systems were proposed and developed based on the use of a coherent interrogation signal.[21-23] They were designed both for radionavigational purposes and for the study of the properties of the space plasma traversed by the coherent signals.

Questions relating to the study of interplanetary plasma, the plasma near the earth, and the ionospheres of other planets, with the aid of measurements of transmission through them of coherent-frequency signals radiated by satellites or other space vehicles, have been extensively discussed in scientific literature. Major reviews include those of Evans[24] and Al'pert,[25] and there are also papers by Kolosov *et al.*,[26,27] and Vasil'ev *et al.*[28,29] Without going into detail, we note that experiments performed with the dispersion interferometer (which is referred to in foreign literature as the Doppler-difference method) have yielded the integrated and local electron density at different heights under different conditions of the earth's atmosphere as well as estimated values of many of the parameters of atmospheric inhomogeneities, the electron density of interplanetary plasma, and many of the properties of the ionospheres of Venus and Mars.

Examination of a broad class of radiointerference systems with coherent interrogating and responding signals has shown that the interferometer with two interrogating and three responding communication links is an effective version of systems with coherent response. It should ensure determination of the parameters of nonstationary processes occurring in plasma along the path traversed by the radio waves, and of the position of the points at which these processes take place along the communication route. In contrast to systems with single-frequency interrogation and response communication links, i. e., the usual systems of orbital measurement of radial velocity of space vehicles, in which precise compensation of the Doppler frequency shift cannot be achieved for the received signal, the measured quantity in this version should be independent of the kinematic parameters of the space vehicle, which improves sensitivity and precision.

We also note that experiments based on the Faraday rotation of the plane of polarization of one of the components of the radio waves propagating through plasma in a magnetic field are also, in effect, versions of the radiointerference technique using interference, at the same frequency, between two differently polarized components.

In many cases, the experimental conditions themselves force us to measure not the phase differences but the changes in them, i. e., they lead to frequency measurements.

On the other hand, the development of highly stable quantum-mechanical frequency standards has ensured that frequency conversion can be carried out practically without disturbing the initial frequency stability of the original oscillations, and thus leads to new modifications of radiointerference devices.

As an example, consider the radiointerferometer using quasicoherent radiation with frequency conversion and capable of eliminating (or substantially reducing) the effect of the ordinary Doppler frequency shift when the responder is carried on board a satellite. This experiment was carried out by Basov *et al*.[30,31] who tried to estimate the gravitational shift in the frequency of waves emitted by a source on the ground and returned by the satellite after the appropriate frequency conversion. The satellite carried a quantum-mechanical frequency standard with a stability of the order of 10^{-11} Hz. The interrogation signal sent from the earth with frequency ω_0 after having been received by the satellite had the new frequency $\omega_0 + \Delta\omega_0$, where $\Delta\omega_0$ is the shift due to the Doppler effect, the effect of the medium, and other factors. It can be assumed that $\Delta\omega_0 \ll \omega_0$.

The frequency of the wave emitted by the standard oscillator carried by the satellite was chosen to be $2\omega_0$ (as closely as possible). By subtracting the frequency of the received oscillations from the frequency of the local oscillator, one obtains a signal of frequency

$$\omega' \approx \omega_0 - \Delta\omega_0.$$

Back on earth, it is received at a frequency

$$\omega_1 = \omega' + \Delta'\omega',$$

where, as before, $\Delta'\omega'$ is the change in the frequency of the signal received on the earth due to the Doppler effect, the effect of the ionosphere, and so on:

$$\omega_1 \approx \omega_0 - \Delta\omega_1 + \Delta'\omega_0.$$

Comparison of the frequency of this signal with the interrogation frequency ω_0 yields

$$\Delta\omega \approx \Delta\omega_0 - \Delta\omega_0.$$

Since the nonrelativistic Doppler effect and the change in frequency due to the influence of the medium are independent of the direction of propagation, they cancel out in the expression for $\Delta\omega$. On the other hand, the gravitational frequency shift and the relativistic Doppler effect have different signs for the interrogation and response signals, so that they are doubled in the expression for $\Delta\omega$.

The above frequency conversion scheme cannot be realized in practice because reception and emission cannot be carried out at the same frequencies. In practice, the interrogation and response frequencies are slightly different, and the compensation of the Doppler effect, though incomplete, is sufficient to avoid the masking of the expected frequency shift due to the gravitational effect (Fig. 3). Chikhachev *et al*.[32] have reported the results of an experiment of this type, which qualitatively confirm the presence of the gravitational shift. A nonstationary interference pattern was produced as a result of the interaction between oscillations emitted by similar but independent quantum-mechanical sources of stable oscillations, so that experiments of this type are referred to as interference with quasicoherent response. It is clear that the stability of quantum-mechanical frequency standards, which is of the order of 10^{-11}–10^{-12} Hz, can ensure constant scanning of the interference pattern at carrier frequencies of the order of 10^{-7} Hz to not better than 10^{-5} $\sec^{-1}$ when the frequencies of the oscillators on the earth and on the satellite are exactly equal, or it will ensure determination with at least this precision of the rate of variation of the change in phase difference, i.e., of the observed difference frequencies (beat frequencies).

Many different radiointerferometers with quasicoherent response have now been described in the literature. They are designed for the observation of nonstationary processes in the plasma near the earth and in space, and for the investigation of discontinuities in the magnetosphere and in interplanetary space. The possibility of observing interference between oscillations that are not stricly coherent, and the corresponding frequency conversion, has considerably extended

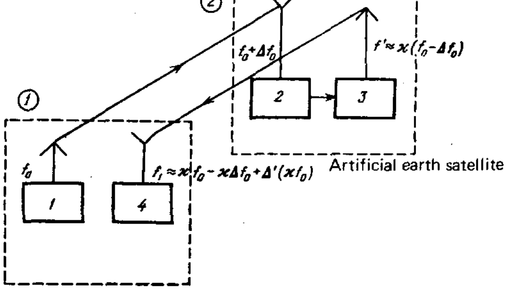

FIG. 3. Block diagram of interferometer with noncoherent response as used in the observation of the gravitational frequency shift ($\varkappa$ is a coefficient close to unity).

the potential of the radiointerference technique (see Refs. 33 and 34).

In conclusion, we must say a few words about the widely used methods by which processes occurring in the ionosphere and in space plasma are examined by studying changes in the frequency of received oscillations due to the Doppler and other similar effects associated with changes in the optical path length. These experiments are discussed in the now extensive specialist literature, and are naturally looked upon as a further extension and improvement of the original radiointerference technique. Methods based on observing the variation in the phase difference between two coherent signals, one of which has been transmitted through the region of space under investigation, were also examined and used by L. I. Mandel'shtam and his school.[3,15,35] However, they are useful only for studying relatively slow and large-scale processes. In the case of fast processes, it is more convenient to record not the phase difference but its derivative, i.e., frequency. Measurements of the variation in the original frequency are, therefore, a natural development of the radiointerference technique in which the second coherent oscillation is replaced by a local but sufficiently stable reference source with which the observed oscillation is compared. This method, now known as the Doppler method, began to be intensively developed in the 1960's in connection with the study of the properties and dynamics of the ionosphere and the properties of space plasma. However, a more detailed account of special features of this technique and of the results obtained with its aid would take us outside the scope of the present paper. We therefore refer the reader to the very exhaustive 1975 review paper by Namazov *et al.*[36]

4. CONCLUSIONS

As reviewed above, the development of radiointerferometry begun in the 1930's demonstrates the exceptional productivity of the foundation-laying ideas of L. I. Mandel'shtam in relation to the realization and application of radio-wave interference. The various radiointerferometers which he proposed and developed together with N. D. Papaleksi, which used interference at commensurable frequencies and phase methods of measurement, have been used to solve a broad range of scientific and practical problems. They have opened up new ways for the practical utilization of radio waves, and led to new methods of investigation radio-wave propagation and the properties of media transmitting these waves.

Both the original work performed during the lifetime of L. I. Mandel'shtam and the subsequent development of the entire subject have demonstrated the exceptional value of radiointerference methods in radionavigation, geodesy, ionispheric studies, and space research. The fact that the fundamental principles underlying the many versions of the radiointerferometer are now regarded as being completely obvious only serves to emphasize the depth of the "general oscillatory" approach of Leonid Isaakovich and his physical intuition in relation to very different physical phenomena, and his talent for finding aspects of these phenomena that escaped the attention of other researchers but which, after elucidation by him, became quite clear. The entire series of researches into radiointerferometry is an excellent example of the scientific creativity of L. I. Mandel'shtam. It is fundamental to modern radiophysics and to many technological applications of radio waves.

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