Experimental verification of the general theory of relativity[1)]

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A review is given of experimental verifications of the general theory of relativity (GTR) in which basic assumptions and consequences of the theory are compared with experiment. It may now be considered that the principle of equivalence and some of the other propositions forming the foundations of GTR, and also the effects predicted by it for weak fields ($|\varphi|/c^2 \ll 1$, where φ is the Newtonian gravitational potential) have, on the whole, been reliably confirmed. However, in the case of strong fields, the theory has not as yet been directly confirmed. In particular, the existence of black holes has not yet been demonstrated, to say nothing of a quantitative agreement between GTR formulas and measurements of the metric near a black hole. Several suggestions are put forward in relation to further verifications of GTR.

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"Experiment never responds with a "yes" to theory. At best, it says "maybe" and, most frequently, simply "no." When it agrees with theory, this means "maybe" and, if it does not, the verdict is "no."

A. Einstein[1]

1. All physical theories must be monitored and verified by observation and experiment. This proposition is now so deeply ingrained and is so much a part of the physicist's daily routine that it may well appear to be simpler than it really is. The point is that verifications of the theory of some particular effect and, in general, discussions of special cases may well involve particular difficulties but, as a rule, do not touch upon fundamental principles. The situation is quite different in the case of experimental verifications of fundamental theories such as the special and general theories of relativity and nonrelativistic quantum mechanics. They involve further questions such as range of validity, degree of completeness, and so on.[2)] This impinges upon the very foundations of physics, so that the substance and purpose of such verifications must be examined with particular care.

In order not to get bogged down in extensive discussions of a general kind, we confine our attention to one characteristic feature that emerges very clearly when fundamental theories are compared with experiment, namely, the profound asymmetry between refutation and confirmation of a theory. It is precisely this point that Einstein puts forward in the epigraph quoted above. The fact is that any discrepancy between conclusions drawn from a fundamental theory and experiment will immediately refute the theory or, at any rate, indicate its restricted range of validity, etc. (we are, of course, concerned with rigorous and definite predictions and sufficiently reliable experiments). On the other hand, agreement between a particular prediction and experiment can in no way be regarded as demonstrating that the particular theory is valid. Firstly, there is the question of the precision of observations and experiments and, secondly (and this is a much less trivial point), the same result (effect or similar) may follow from different theories. The classical example of this is the formula for the energy levels of the hydrogen atom, obtained without taking into account either spin or relativistic corrections. This formula was found by Niels Bohr with the aid of his quantization rule a decade before the creation of quantum mechanics, and is identical with the quantum-mechanical formula even though the Bohr theory is in no way identical with quantum mechanics.

Thus, to maintain that a fundamental theory has been verified and, still more so, verified experimentally is, from the rigorous point of view, an exceedingly difficult matter. In practice, physics develops without the expectation of any rigorous verification and this approach, this strategy, is both natural and justified. Whenever some problem or obscurity left behind is not "resolved" in the course of time, and is found to persist, it is always possible to return to it. Moreover, there is a class of physicists (of course, we speak of physicists only for the sake of being specific) that is particularly interested in these methodological and

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[1)]This paper will also appear in the forthcoming book "O teorii otnositel'nosti" ["The Theory of Relativity" (Collection of Papers, Nauka, M., 1979)] to be published to celebrate the Einstein Jubilee.

[2)]By completeness of the theory, we mean the ability to answer any correctly formulated question relating to its range of validity. Thus, there is the well-known discussion about the completeness of nonrelativistic quantum mechanics in relation to the possible (or impossible) departure from the probabilistic interpretation. According to the generally accepted point of view (exceptions are increasingly rare, so that we feel justified in describing the present situation in this way), nonrelativistic quantum mechanics is complete in the sense that it is capable, within its own range, of answering any question consistent with physical reality. The fact that the theory cannot predict where a particular electron will "fall" in a diffraction experiment is due not to lack of completeness but to the nonclassical nature of microobjects (electrons). This question is outside the scope of the present review (the reader is referred to Ref. 2 for the author's views in this field).

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logical questions, so that continuous progress is being maintained in the analysis of the foundations of physical theories, their connection with experiment, and so on, even in the absence of dramatic events such as "world-shaking" new experiments.

This is precisely the overall situation in relation to the experimental verification of the special theory of relativity and nonrelativistic quantum theory.

These theories lie at the very foundation of physics and have an exceedingly large number of consequences of great variety, but have resulted in no known contradictions. The experimental verification of these theories is, therefore, regarded as essentially complete or, at any rate, it is not looked upon as an urgent problem. Practical experience is the best test: in physics literature (which, like the number of physicists, is now enormous), questions involving the experimental verification and justification (analysis of fundamentals of the special theory of relativity and quantum mechanics are treated very infrequently, especially in relation to any doubts as to the validity of fundamentals and similar questions.[3)]

A different situation has arisen in the case of the general theory of relativity (GTR) which is older by a decade than nonrelativistic quantum mechanics. Experimental verification of GTR is still a topical problem in modern physics and astronomy. For proof of this assertion (how else can this be done?) one can point to the content of physics and astronomy journals and books, including, in particular, recent reviews in the present journal.[4,5] In this paper, we shall try to illuminate the present state of experimental verifications of GTR, not with a view to providing a detailed account of existing data and planned new experiments,[4,5] but in order to exhibit the overall situation obtaining at present.

2. An account of the fundamentals of GTR, including derivations and discussions of formulas for many of the observed effects, can be found in a larger number of readily accessible textbooks and monographs (see, in particular, Refs. 6–9). Nevertheless, we shall have to reproduce and briefly discusss a number of expressions.

GTR is a theory of the gravitational field in which this field is completely described by the metric tensor $g_{ik}(x^l)$, defining the square of the interval

$$ds^2 = g_{ik}\, dx^i dx^k, \tag{1}$$

where x^1, x^2, x^3 are space coordinates that are arbitrary within broad limits, and $x^0 = ct$ is the time coordinate. In the inertial (Galilean) frame (if it exists), and if we use Cartesian coordinates, we have

$$g_{00} = 1,\ g_{11} = g_{22} = g_{33} = -1,\ g_{ik} = 0 \text{ for } i \neq k \tag{2}$$

(other signs are occasionally chosen for g_{00} and $g_{\alpha\beta}$).

The field $g_{ik}(x^l)$ satisfies the Einstein equation

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}, \tag{3}$$

where R^i_{klm} is the curvature tensor (Riemann tensor), $R_{ik} = R^l_{ilk}$ is the Ricci tensor, $R = g^{ik}R_{ik}$ is the scalar curvature, T_{ik} is the energy-momentum tensor of matter (including all fields other than the gravitational field), and $G = 6.670 \times 10^{-8}\,\text{cm}^3 \cdot \text{g}^{-1} \cdot \text{sec}^{-2}$ is the gravitational constant. The tensor R^i_{klm} and hence the tensor R_{ik} and the scalar R can be expressed in terms of g_{ik}, $\partial g_{ik}/\partial x^l$, and $\partial^2 g_{ik}/\partial x^l \partial x^m$, and the corresponding expressions are linear in the second order derivatives of g_{ik}.

Since the four coordinates x^i can be subjected to a transformation corresponding to a different choice of the coordinate frame, four out of the ten components of g_{ik} can be regarded as arbitrary. It follows that six components of the tensor g_{ik} are independent, and Einstein's equations can be used to determine them. In 1917, two years after the equations in (3) were established, Einstein generalized these equations[10] by adding the so-called Λ-term, equal to Λg_{ik}, to the right-hand sides. The introduction of this term (and only this term) is not in conflict with the general requirements leading to (3), but results in a certain generalization of the Newtonian theory of gravitation in the nonrelativistic limit. Moreover, if the cosmological constant Λ is small enough, and this, in fact, follows from cosmological considerations,[11] it should play no significant role not only in weak fields (in particular, within the limits of the solar system), but also in the theory of black holes. On the other hand, the Λ-term may be quite important in cosmology.[4)] However, this will not concern us here and we shall therefore set $\Lambda = 0$.

In a weak gravitational field [this means that the components of g_{ik} may be regarded as being close to the Galilean values (2)], we can set $g_{00} = 1 + (2\varphi/c^2)$, $T^0_0 = \rho c^2$ (ρ is the mass density), and the gravitational potential φ introduced in this way can be seen from (3) to satisfy the equation of the Newtonian theory of gravitation:

$$\Delta\varphi = 4\pi G\rho. \tag{4}$$

It is clear from the foregoing that the condition for the validity of this (Newtonian) approximation and the condition that the field is weak is

$$\frac{|\varphi|}{c^2} \ll 1. \tag{5}$$

[3)] It is not our intention to have these remarks interpreted as a negative attitude to studies involving the history, analysis of measurements, logical foundations, and experimental verification of the special theory of relativity and quantum mechanics. On the contrary, it is surprising how often discussions of "fundamentals" of these theories encounter obscurities and, especially, the absence of an understanding and interest among many physicists. It seems, however, that discussions of the fundamentals of the special theory of relativity and quantum mechanics have become analogous to discussions of the fundamentals of classical mechanics in that they have become part of the methodology and history of physics. An interesting example of an analysis of experimental verification of the special theory of relativity can be found in Ref. 3.

[4)] There has been a longstanding dispute as to whether the Λ-term should be included even in cosmology. The present author was among those who never saw any reason for neglecting this term, and this view is now generally accepted (see Refs. 11–13).

On the surface of the sun

$$\frac{|\varphi_\odot|}{c^2}=\frac{GM_\odot}{r_\odot c^2}=\frac{r_{g,\odot}}{2r_\odot}=2.12\cdot 10^{-6},$$
$$r_{g,\odot}=\frac{2GM_\odot}{c^2}=2.94\cdot 10^5\,\mathrm{cm}, \tag{6}$$

since the mass of the sun is $M_\odot=1.99\times 10^{33}$ and the radius of the solar photosphere is $r_\odot=6.96\times 10^{10}$ cm. Equation (6) contains the parameter

$$r_g=\frac{2GM}{c^2}\approx 3\cdot 10^5\frac{M}{M_\odot}\,\mathrm{cm} \tag{7}$$

which is called the gravitational radius.

For the earth (on its surface),

$$\frac{|\varphi_\oplus|}{c^2}=\frac{GM_\oplus}{r_\oplus c^2}=7\cdot 10^{-10},\quad M_\oplus=5.98\cdot 10^{27}\,g,$$
$$r_\oplus=6.37\cdot 10^8\,\mathrm{cm},\quad r_{g,\oplus}=0.86\,\mathrm{cm}. \tag{8}$$

For a circular planetary orbit $|\varphi|/c^2=v^2/c^2$, where v is the velocity of the planet. For the earth's orbit, $|\varphi|/c^2\approx 10^{-8}$, since $v=3\times 10^6$ cm/sec.

As the mass of the body increases and its radius decreases, $|\varphi|$ is found to increase and the field may become strong. For a spherically symmetric (and nonrotating) mass, the metric outside it (Schwarzschild solution, 1916) is

$$ds^2=\left(1-\frac{r_g}{r}\right)c^2dt^2-\frac{dr^2}{1-(r_g/r)}-r^2(\sin^2\theta\, d\varphi^2+d\theta^2). \tag{9}$$

Here, we use the spherical polar coordinates r,θ,φ (in terms of which the length of a circle centered on the center of mass is $2\pi r$). Other coordinate systems are also used, including the "isotropic" spherical coordinates ρ,θ,φ, where $r=\rho[1+(r_g/4\rho)]^2$ and

$$ds^2=\left[\frac{1-(r_g/4\rho)}{1+(r_g/4\rho)}\right]^2c^2dt^2-\left(1+\frac{r_g}{4\rho}\right)^4[d\rho^2+\rho^2(\sin^2\theta\, d\varphi^2+d\theta^2)]. \tag{10}$$

In the weak-field approximation (expansion in $r_g/r\ll 1$ or $r_g/4\rho\ll 1$) we have

$$ds^2=\left(1-\frac{r_g}{r}\right)c^2dt^2-\left(1+\frac{r_g}{r}+\ldots\right)dr^2-r^2(\sin^2\theta\, d\theta^2+d\theta^2),$$
$$ds^2=\left(1-\frac{r_g}{\rho}+\frac{r_g^2}{2\rho^2}+\ldots\right)c^2dt^2 \tag{11}$$
$$-\left(1+\frac{r_g}{\rho}+\ldots\right)[d\rho^2+\rho^2(\sin^2\theta\, d\varphi^2+d\theta^2)].$$

The following device is widely used in verifications of GTR in the case of weak fields. The expressions given by (11) are employed but with arbitrary dimensionless coefficients $\alpha,\beta,\gamma,\ldots$, i.e.,

$$ds^2=\left(1-\frac{\alpha r_g}{r}+\frac{\beta-\alpha\gamma}{2}\frac{r_g^2}{r^2}+\ldots\right)c^2dt^2$$
$$-\left(1+\gamma\frac{r_g}{r}+\ldots\right)dr^2-r^2(\sin^2\theta\, d\varphi^2+d\theta^2),$$
$$ds^2=\left(1-\frac{\alpha r_g}{\rho}+\frac{\beta r_g^2}{2\rho^2}+\ldots\right)c^2dt^2 \tag{12}$$
$$-\left(1+\gamma\frac{r_g}{\rho}+\ldots\right)[d\rho^2+\rho^2(\sin^2\theta\, d\varphi^2+d\theta^2)].$$

In GTR,

$$\alpha=\beta=\gamma=1. \tag{13}$$

In other theories of the gravitational field that are distinct from GTR, the values of α,β,γ and of other coefficients that may be introduced may be different from the values predicted by GTR (in the post-Newtonian approximation for an arbitrary and not merely spherically symmetric gravitational field, there are about ten such coefficients, most of which can be conveniently chosen so that they are zero in GTR; see, particularly, Ref. 9).[5)] Moreover, the mass M of the body is determined from its Newtonian potential $\varphi=-GM/r$ or acceleration $\mathbf{g}=-\nabla\varphi=(GM/r^3)\mathbf{r}$ of a test body. If $\alpha\neq 1$, the potential becomes $\varphi=-\alpha GM/r$ [see (7) and (12)]. Since the mass M cannot evidently be determined in any way other than through measurements of the acceleration $\mathbf{g}$, we must immediately set $\alpha=1$ (and we shall do so).

3. Experimental verifications of GTR are performed in two, not completely distinct, ways. The first method involves the fundamentals of the theory, i.e., its assumptions. The other involves an examination of the consequences of GTR and, in particular, in formal language, the verification of the validity of (13). The asymmetry noted above and characterizing the different ways in which physical theories can be verified suggests that, at any rate, the first method is no less significant than the second. Specifically, this refers, above all, to the principle of equivalence upon which GTR is based.[6)] This principle states that, in a sufficiently small space-time region (an "elevator"), the effect of the gravitational field is indistinguishable from acceleration of the frame of reference. If GTR is accepted, the principle of equivalence follows in a very direct way and is equivalent to saying that, in a sufficiently small neighborhood of a point on a curve or surface, the curve (or surface) can be replaced by the tangent (or the tangent plane). Hence, it is clear that any departure from the principle of equivalence would indicate a breakdown of GTR to the extent that this would happen, i.e., at any rate, it would indicate the limits of validity of GTR. The converse is, of course, untrue because the principle of equivalence can be (especially approximately) satisfied even in non-Einsteinian theories of the gravitational field.

The attention devoted to the verification of equality between inertial and gravitational masses, m_i and m_g, which follows from the principle of equivalence,[7)] is generally understandable and justified. By definition,

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[5)]It is clear from (13) that the second form in (12) is, to some extent, to be preferred because, if GTR is valid, then $\beta-\alpha\gamma=0$ and the term such as $(r_g/r)^2$ in (12) is also zero.

[6)]"According to my understanding, my theory rests exclusively on this principle"—this was the view of Einstein, who also considered that the verification of the principle of equivalence was more important than the verification of the consequences of GTR (here, we have in mind the weak-field GTR effects considered below). To simplify our account, we shall not give references to particular papers by Einstein, especially since they are readily accessible [see Albert Einstein, Collected Scientific Papers (translated into Russian), Nauka, M., 1965–1967] and are also extensively quoted in the literature. Insofar as experimental verifications of GTR are concerned, we shall base our discussion on Ref. 5, which cites a large number of recent publications. References to earlier work can be found in Refs. 14 and 15, where verifications of GTR are reviewed as at 1955 and 1966.

[7)]Historically, the situation was, of course, different, i.e., the principle of equivalence was formulated by Einstein (in 1907) as a generalization of the equality $m_i=m_g$, which had been known to be valid with considerable precision (from the Eötvös experiments).

the masses m_i and m_g appear in the Newtonian law of dynamics $m_i\ddot{\mathbf{r}} = -m_g\nabla\varphi$ for a "mass point" in a gravitational field (potential φ). Consequently, if $m_i = m_g$, all bodies should move ("fall") with identical acceleration $\ddot{\mathbf{r}} \equiv \mathbf{g} = -\nabla\varphi$ in the gravitational field. Since Galileo, this fact has been verified with constantly increasing precision. The experiments of Eötvös and his collaborators, in which the equality $m_i = m_g$ was verified to a precision of 10^{-9}, were published in the early 1920's. Specifically, it was concluded that the ratio m_i/m_g for wood and platinum differed by less than 10^{-9}. A precision of 10^{-11} was achieved by 1964, and the best known measurements,[17] published in 1971, showed that the ratio m_i/m_g for platinum was equal to that for aluminum to a precision better than 10^{-12}. For these very different materials, the contributions of strong and electromagnetic interactions to the proper energy (and hence to the inertial mass) were large enough to affect the experiment if there was a departure from the equation $m_i = m_g$. Weak interactions affect m_i (but not m_g) and would produce an effect of the order of 2×10^{-10} in the experiments reported in Ref. 17. These experiments thus show that the equivalence principle is valid for weak interactions as well to within 0.5% (see also Refs. 5 and 57).

There are several reasons why it is particularly important to determine the contribution of the gravitational interaction to the masses m_i and m_g (apart from the fact that the gravitational interaction is the weakest of the known interactions, and $m_i = m_g$ in GTR even when gravitation is taken into account). The ratio of the energy of gravitational interaction $E \sim Gm^2/a$ (in the case of a body of radius a and mass m) to the rest energy mc^2 is (we assume that the density of the body is $\rho \sim 5$)

$$\Delta \sim \frac{Gm}{c^2 a} \sim \frac{r_g}{a} \sim \frac{4\pi G\rho a^2}{3c^2} \sim 10^{-27}a^2. \tag{14}$$

Under laboratory conditions, the ratio Δ is of course entirely negligible but, for the earth as a whole, $\Delta \sim 3\times10^{-10}$ because $a \sim 6\times10^{8}$ cm. For Jupiter, $\Delta \sim 10^{-8}$ and, for the moon, $\Delta \sim 2\times10^{-11}$. If inclusion of the gravitational interaction were to violate the equation $m_i = m_g$ in the case of the earth, and thus contradict the principle of equivalence, the lunar orbit would oscillate in a particular way.[5,18] Two independent groups[19] published in 1976 the results of the corresponding observations performed by laser ranging techniques. The conclusion arising from these observational data was that the gravitational energy of the earth provided the same contribution to m_i and m_g to within 2–3%. In other words, the value of m_i/m_g for the earth is equal to that for the moon to a precision of the order of 10^{-11}.

Thus, the equation $m_i = m_g$ has now been verified to a precision that is several orders higher than was available at the time when GTR was developed. Moreover, it has been established, with the precision indicated above, that gravitational energy (in the Newtonian sense of this idea) provides equal contributions to m_i and m_g. In GTR, as in Newtonian gravitational theory, the gravitational constant G is, in fact, a constant, i.e., it is time-independent (and independent of spatial coordinates). The question as to whether G and certain other "constants" are, in fact, functions of time has been a matter for long-standing discussion. Specifically, it has been hypothesized that G decreases with time and was much greater some billion years ago (this point is important for geophysics and cosmology). This reduction in G would, among other things, produce an increase in the size of the orbits of planets and satellites, including, for example, an increase in the earth-moon separation (by, say, a few centimeters per annum). There are no reliable data at present that would support such changes in G. At any rate, we may conclude that

$$|dG/dt|/G < 4\cdot10^{-10}\ \mathrm{yr}^{-1}.$$

Since GTR is also based on the special theory of relativity, we could also consider the validity of the latter. However, we shall not do this and will confine our attention to one further remark. The mass m of a body is a scalar quantity both in classical mechanics and in special relativity. This means that, in particular, the mass is the same for all directions of acceleration or applied force. However, what if the mass is, in fact, anisotropic and, specifically, is different in the case of acceleration toward the center of the Galaxy and, say, in the direction of the galactic pole? This possibility cannot be rejected *a priori*, and the corresponding experiments have been formulated (see Ref. 20, Chap. 6). The most accurate of these experiments is based on the use of nuclear magnetic resonance and has led to the conclusion that the relative mass anisotropy is $\Delta m/m < 5\times10^{-23}$.

Summarizing, we may conclude that none of the experiments that we have somewhat arbitrarily associated with the verification of GTR provides any indication as to the limitation of its range of validity.

4. In the course of his development of the general theory of relativity, Einstein pointed out three consequences of the theory, which are referred to in the literature as the critical, classical, standard, or celebrated effects. These are the red (gravitational) shift of the frequency of spectral lines, the deflection of a beam of light passing near the sun, and the precession of the perihelion of Mercury.

According to GTR, the proper (true) time τ at any fixed point is related to the coordinate time $t = x^0/c$ by $\tau = (1/c)\int\sqrt{g_{00}}\,dx^0$. In a constant (time-independent or static) gravitational field, the frequency of light measured in coordinate (world) time is constant along a light beam, so that the measured frequency $\nu = 1/\tau_0$ (τ_0 is the period of the oscillations measured in proper time τ) is different at different points. The ratio of the frequencies ν_2 and ν_1 at points 2 and 1 is

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{g_{00}(1)}{g_{00}(2)}}. \tag{15}$$

In a weak field, $g_{00} = 1 + (2\varphi/c^2)$ and, to within terms of order φ/c^2, we have

$$\frac{\nu_2 - \nu_1}{\nu_1} \equiv \frac{\delta\nu}{\nu_1} \approx \frac{\delta\nu}{\nu} = \frac{\varphi_1 - \varphi_2}{c^2}. \tag{16}$$

Hence, it follows, for example, that the spectral lines of radiation emitted by the solar photosphere and recorded on the earth's surface are shifted toward the red in such a way that [see (6)]

$$\frac{\delta\nu}{\nu} \approx -\frac{GM_\odot}{c^2 r_\odot} = -2.12\cdot 10^{-6}. \quad (17)$$

Unfortunately, the gravitational red shift of lines in the solar spectrum are masked by other effects, mainly due to motion of the gas in the photosphere. The red shift of lines in the solar spectrum was therefore a matter of some dispute for a long time and led to some contradictory conclusions (see Ref. 14 and the references cited therein). However, it is now clear that data on the solar spectrum are in agreement with (17), but verifications of this formula hold only to within one or even a few percent. Somewhat earlier (in 1960 and 1965), more accurate measurements of the gravitational shift of the frequency were performed on the earth using the Mössbauer effect for gamma rays, and the formula given by (16) was confirmed to within 1%. Further improved measurements were recently performed, using an aeroplane and a rocket. The expression given by (16) was verified to within 0.04% (see Ref. 5 in which the original papers are cited). It is important to note that the gravitational frequency shift pointed out by Einstein in his very first paper on GTR in 1907 follows from the principle of equivalence and the special theory of relativity. This effect can therefore be regarded as being not only one of the consequences of GTR, but also associated with its foundations (see above); in particular, measurements of the gravitational frequency shift provide no information on the magnitude of β and γ in (12).[8] Although Eötvös-type experiments verify the principle of equivalence to a high precision, red-shift experiments involve different objects (macroscopic masses and photons), and the two sets of experiments merely complement one another (we shall not pause to analyze this question beyond the limits of GTR; see Ref. 21).

In 1907, and in greater detail in 1911, prior to the derivation of the basic GTR equation given by (3), Einstein pointed out that light rays passing near the sun would be deflected. However, only the change in the component $g_{00} \approx 1 + (2\varphi/c^2)$, due to the presence of the sun, was taken into account. The expression obtained for this deflection was, therefore, smaller by a factor of two as compared with the expression given below [see (19)], i.e.,

$$\alpha' = \frac{2GM_\odot}{c^2 R}. \quad (18)$$

It is interesting to note that it became clear later that this formula was obtained as far back as 1801 (!) by Soldner on the basis of the corpuscular theory of light (see Ref. 14 for references and the derivation of this formula). After Eq. (3) was established in 1915, Einstein used it in the same year to consider the deflection of light rays and the precession of the perihelion of Mercury. The expression given by (18) for the angle of deflection of a beam of light was replaced by the following result which ensues from GTR:[9]

$$\alpha = \frac{4GM_\odot}{c^2 R} = \frac{2r_{g,\odot}}{R} = 1''.745\,\frac{r_\odot}{R}, \quad (19)$$

where R is the impact parameter or, for practical purposes, the distance of closest approach between the beam and the center of the sun. In other words, rays passing in the immediate neighborhood of the solar limb (disk) are deflected by 1.75 seconds of arc. An attempt to measure the deflection was undertaken as far back as 1914 when a German expedition was dispatched to Russia but was interned when war broke out. Had this expedition succeeded in carrying out its measurements, it would have concluded that Einstein's theory was refuted because the result given by (18) was considered correct! The first successful observations of the deflection of light rays were performed in 1919 at two different points and yielded the values (for $r_\odot/R = 1$) $\alpha = 1.98 \pm 0.18$ and $\alpha = 1.69 \pm 0.45$. Thus, not only was the existence of the effect demonstrated but (19) and not (18) was confirmed. It is precisely after these observations that GTR attracted the attention of the general public and became famous. It is now sixty years since these original observations of the deflection of light, but progress in optical measurements of the effect over this long period of time has been depressingly slow. Specifically, the formula given by (19) or, more precisely, the value 1″.75 for the maximum deflection (but not the variation of α with R) has been tested (and confirmed) as a result of observations of a number of eclipses but only to within about 10–20% (see, for example, Refs. 8, 9, and 14). Judging by the reports in the literature, proposed non-eclipse optical measurements of the angle α (both from the earth and from satellites) have turned out to be difficult to carry out and have not as yet been performed. In terms of the parameters β, γ, and so on, measurement of the angle α will directly determine the parameter γ because

$$\alpha = \frac{2r_{g,\odot}}{R}\left(\frac{1+\gamma}{2}\right).$$

The optical measurements mentioned above show that γ lies somewhere between 0.9 and 1.3, which is not, of course, satisfactory. Measurements of the angle α in the radio band (quasar radio emission received by interferometers) have been performed in the course of the last decade. Recent data[5] yield the GTR value of 1 for the parameter γ to within about 2%.

An effect of the same nature as the deviation of rays of light but involving quite different measurement techniques[22] began to be considered quite recently (in 1964). It involves the relativistic delay of an electromagnetic

[8]As an example of one of the possible derivations of (16), we note the following, given in quantum-mechanical language (Planck's constant h does not appear in the result and an analogous derivation can be carried out by replacing the photon energy $h\nu$ with the energy E of a wave train and using the adiabatic invariance of the ratio E/ν). The mass of the photon is $m_i = m_g = h\nu/c^2$ and the change in this energy $h\delta\nu$ as it crosses a gravitational potential difference $\varphi_2 - \varphi_1$ is $h\delta\nu = -(h\nu/c^2)/\varphi_2 - \varphi_1)$. The expression given by (16) follows directly from this.

[9]It is clear, for example, from (11) that, to this order in φ/c^2, both g_{00} and $g_{rr} = -1 + (2\varphi/c^2)$ will change. This produces a doubling of the result given by (18) and the final expression is given by (19). The physical point is that the deviation of the light rays is also connected with the curvature of space and not merely a consequence of the principle of equivalence by virtue of which $g_{00} = 1 + (2\varphi/c^2)$.

signal during its propagation in a nonuniform gravitational field. In practice, the experiment involves the reflection of radio waves from Mercury or Venus. When the GTR effect is ignored, the signal will reach the planet and will return to earth in a time $t_0 = 2r/c$, where r is the line of sight separation between the earth and the target planet (the influence of the interplanetary medium is neglected since it can be quite accurately monitored or eliminated by using different carrier frequencies). However, when the curvature of space-time is taken into account [and, in the present case, this is based on (12)], the signal should be delayed by a further amount δt, which depends on the mutual disposition of the planets and the sun. The delay is a maximum when the signal passes near the solar limb, i.e., the target planet is in superior conjunction. Under such and similar conditions (see Refs. 7–9 for further details) we have

$$\delta t \approx \frac{4GM_\odot}{c^3}\left[1+\left(\frac{1+\gamma}{2}\right)\ln\frac{4r_1r_2}{R^2}\right], \tag{20}$$

where R is the "impact parameter" [see (19)], r_1 is the earth-sun separation, and r_2 is the separation between the sun and the target planet (Mercurcy or Venus) which, by assumption, lies on the other side of the sun. Of course, the value $(\delta t)_{\max}$ is reached for $R = r_\odot$. In the case of Mercury $(\delta t)_{\max} = 2.4\times10^{-4}$ sec (for $\gamma = 1$, this corresponds to GTR), whereas $t_0 \approx 23$ min [consequently, $(\delta t)_{\max}/t_0 \sim 2\cdot10^{-7} \sim 0.1|\varphi\ |/c^2$; see (6)]. A very important point is that the delay δt varies during the motion of the earth and the target planet, so that it is possible to perform differential measurements and, in particular, examine the logarithmic term in (20). The precision with which the time δt can be determined and thus GTR verified by this method is determined by the precision with which we know the planet's position (and some of its parameters) or the position of the space probes (artificial earth or planetary satellites). Preliminary data available to us, obtained by the Viking satellite of Mars, indicate that $|1-\gamma| \leqslant 1\%$ (see Ref. 5). Generally speaking, the parameter γ may now be considered as departing from unity by not more than 1–2%. In the scalar tensor theory of gravitation (Brans-Dicke theory), $\gamma = (\omega+1)/(\omega+2)$, where ω is a free parameter introduced into this theory. For example, if $\gamma = 0.99$, then $\omega = 98$, whereas GTR corresponds to the limit as $\omega \to \infty$. Until quite recently (when one could not be sure that γ differed from unity by less than 10%), the parameter ω could have been regarded as being not too large (it was usually assumed that $\omega \sim 5$–6). If, however, $\omega \sim 100$, the validity of a scalar field becomes particularly unlikely.

The last and, in a sense, the most important of the "critical" tests is the advance of the perihelion of Mercury or, more generally, the rotation (precession) of the orbits of planets and their satellites. It is well known that, if all perturbations are neglected in Newtonian mechanics, the orbit of a planet is an ellipse, one of whose foci contains the sun (or, more precisely, the center of gravity of the sun-planet system). Actually, planetary orbits rotate slowly in their planes, so that the perihelion, i.e., the apex of the ellipse nearest to the sun, is found to move under the influence of perturbations mainly due to the other planets.[10] As a result, the perihelion of Mercury, for example, is found to precess at a rate of 532 seconds per century. As far back as the middle of the nineteenth century, it was established that perturbations due to the other known planets could not completely explain this effect: there was an unexplained residual precession of the perihelion of Mercury of about 40 seconds per century. Attempts were made to associate this precession with some as yet unknown planet, with departures from Newton's laws of motion, and so on. The first triumph of GTR in the form of an application to a particular phenomenon or effect was the resolution of this particular problem.

According to GTR, the angular displacement (in radians) of the perihelion of a planet per revolution is

$$\Psi = \frac{6\pi GM_\odot}{c^2a(1-e^2)} = \frac{24\pi^3a^2}{c^2T^2(1-e^2)}, \tag{21}$$

where a is the semimajor axis of the ellipse (orbit), $e = \sqrt{(a^2-b^2)/a}$ is the eccentricity of the orbit (b is the semiminor axis), and T is the orbital period of the planet around the sun [the second expression in (21) is obtained by using Kepler's law $a^3 = (GM\ /4\pi^2)T^2$]. We note that, when gravitational theories other than GTR are used, the expression given by (12) yields (21) with the additional factor $(2-\beta+2\gamma)/3$. The precession of the perihelion is thus found to depend not only on γ but also on β, and is thus found to depend on terms of the order of $(r_g/r)^2$ in (12). There is, however, no reason to consider that the precession of the perihelion of planets is an effect of the order of $(\varphi/c^2)^2 \sim (GM/rc^2)^2$ because the value of Ψ is clearly of the order of $|\varphi|/c^2$. On the other hand, the dependence of Ψ on the term $\beta r_g^2/2\rho^2$ in (12) is due to the fact that the motion of the planet depends on φ (or, more precisely, on $\nabla\varphi$) even in the Newtonian approximation, so that the relativistic correction appears in a combination of the form $\varphi(\varphi/c^2) \sim (1/c^2)\times(GM\ /r)^2$. Nevertheless, measurements of the advance of planetary perihelions are, of course, particularly valuable because they yield additional information as compared with measurements of the deflection of rays or the delay of signals (see above).

According to (21), for Mercury, we have

$$\Psi = 43''.03 \text{ per century}. \tag{22}$$

Einstein's paper, mentioned above, in which (21) was derived, quotes the value of $45'' \pm 5''$ as the residual unexplained observed value of Ψ. Twenty to thirty years ago, the value of Ψ, adopted as a result of analysis of observational data, was $42''.56 \pm 0.96$ and, later, $43''.11 \pm 0.45$ (see the bibliography given in Refs. 14 and 15). The currently quoted result is:[5] $(2-\beta+2\gamma)/3 = 1 \pm 0.01$, i.e., the precession of the perihelion of Mercury is in agreement with the GTR predictions expressed by (21) and (22) to within about 1%. If we suppose that $\gamma = 1$ to

[10]In addition to this "secular" perturbation, which is cumulative in time, there are also periodic perturbations of the orbital elements. In addition to the precession of the perihelion, some of these perturbations produce a secular displacement of the orbital nodes.

within 2%, then $\beta=1$ to within 7%.

As we can see, there was essentially no progress in the determination of the angle Ψ during the last two or three decades although the new results are, of course, more reliable than the old. However, this period saw the rather dramatic suggestion that the sun was an oblate body and, probably, had a substantial gravitational quadrupole moment, so that the agreement between observations and (22) was fortuitous (see, for example, Refs. 8, 9, and 15 for references to this work). The suggested oblateness of the sun appears to have been reliably refuted[4,5] in recent years, and the agreement between the GTR predictions and measurements of the precession of the perihelion of Mercury can again be treated with confidence.

5. New developments in physics and technology, including the launching of artificial satellites and space probes, have been accompanied by a very considerable expansion of experimental possibilities, and this has influenced verifications of GTR. Thus, measurements of the deflection of radio waves near the sun and of the delay of radar signals were quite impossible not only when GTR was created but even thirty years ago. Naturally, many projects have been put forward (and, in many cases, observations have already been performed and analyzed), aimed at verifying the equation $m_i=m_g$ and GTR effects generally with constantly increasing precision. Space studies[4,5,14,15] occupy a conspicuous position among them. Thus, although the use of artificial satellites of the earth, the sun, and the planets was proposed well before the launching of the first satellite (on October 4, 1957),[11)] the last two decades have not seen an extensive deployment of satellites in gravitational experiments requiring the compensation of satellite "drift," the use of delicate equipment, and so on. There are reasons to suppose, however, that we are now on the threshold of such experiments and that they will be performed in the near future (including the launching of a solar probe, a satellite carrying a precision gyroscope, and so on). We shall not examine these projects in detail here since there is very little we can say in addition to the review given in Ref. 5, to which we refer the reader (see also Ref. 4). It is hoped that these experiments will result in determinations of γ to within 0.01% and of β to within 0.1%. Both space and laboratory (terrestrial) experiments[25] should take verifications of the fundamentals and consequences of GTR in weak fields to "new heights" by increasing the precision by two or three orders of magnitude, or even more, as compared with that achieved so far.

These experiments are, however, very labor-consuming so that, for this and other reasons, we have to face the question as to whether GTR verifications should continue and, if so, in which directions, with which particular aims, and so on. It is, of course, possible to maintain that, in general, any improvement in experiments and observations, especially whenever one is dealing with verifications of fundamental theories, is useful and justified. However, this type of answer avoids the real question. The means and resources at the disposal of physicists and astronomers are very considerable but, undoubtedly, limited. This becomes abundantly clear when we recall the enormous number of problems facing science. Attempts to do everything are both impossible and irrational. On the contrary, the history of science and technology clearly demonstrates the efficacy of concentration of resources on the most important lines of research although concentration on "hot spot" problems to the exclusion of anything else would be equally unreasonable. The present author has already had the opportunity of examining such questions elsewhere.[26] Here, it will be sufficient to emphasize that the choice of strategy and tactics in GTR verifications is not merely a matter for idle speculation.

The question is: when is the verification of some particular fundamental theory a particularly pressing problem? The answer is, firstly, that this is so when the theory had just appeared and is being used as a foundation for some particular predictions. For example, it became quite clear soon after the creation of GTR that the detection of the deflection of light rays by the solar field and measurements of the gravitational frequency shift were essential. Secondly, verifications of a theory become essential whenever contradictions, alternative proposals, and so on are put forward. This was the situation when it was suggested that the sun had a large quadrupole moment although the importance of this experiment is not comparable with the last two. Thirdly,

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[11)] Here we would like to introduce a historical digression. As far back as 1918, Lense and Thirring[23] pointed out that the rotation of a central body (the sun, earth, Jupiter, and so on) should produce a "rotation effect" (in the sense that it was due to the rotation of the central body in addition to the precession of the perihelion and the displacement of the nodes of planetary orbits, and a corresponding change in the orbits of planetary satellites. In 1956, the present author published a note (see Ref. 24 and Refs. 14 and 15) in which he discussed this question in relation to the artificial earth satellites. It turns out that the relativistic "rotation effect" for close satellites may amount up to 60″ per century in the precession of the perihelion of a satellite, and up to 20″ per century in the precession of the orbital nodes. In the same time, the perihelion of the lunar orbit advances by only 3×10^{-4} seconds of arc per century as a result of the rotation of the earth. In practice, various incomparably greater perturbations due to the nonspherical nature of the earth, and so on, ensure that the "rotation effect" has not so far been detected for the earth satellites, and it is hardly likely that it will be detected in the foreseeable future. We have considered this question mainly for another reason. When he became familiar with Ref. 24, Thirring recalled an interesting episode (reported in a letter published in a duplicated bulletin of an Austrian Society, sent to the present author and now, unfortunately, lost). On a May evening in 1918, Thirring mentioned to Einstein his paper published in collaboration with Lense and the fact that the "rotation effect" was exceptionally small in the case of the moon. Looking at the night sky, Einstein exclaimed, "Wie Schade dass wir nicht einen Erdmond haben, der gerade nur ausserhalb der Erdatmosphere umlauft! (What a shame that we don't have a moon rotating just outside the limits of the earth's atmosphere!)". We now have artificial moons moving near the earth but, for the reason indicated above, the "rotation effect" is still not measurable.

verifications of a theory attract the attention of researchers whenever the possibility of fundamentally new experiments, extending well beyond the limits of existing experiments and observations, is seen to emerge.

What is the situation in relation to GTR verifications in weak fields if we accept the above guidelines? There are, at present, no known reliable deviations, even the smallest, from results that are in agreement with GTR. In the light of what we have said at the beginning of this paper, the importance of this statement should not be underestimated. Moreover, all the GTR predictions relating to the region of weak gravitational fields that have been verified have turned out to be correct. There is, therefore, no reason to doubt the validity of GTR in weak fields [condition (5) or, more precisely, restriction to effects of the order of φ/c^2]. Beyond this, further conclusions cannot avoid being colored by subjective considerations. Our view is that, if we use the above criteria, further experimental verifications of GTR in weak fields cannot be regarded as an attractive and pressing problem.[12)] However, this conclusion should not be interpreted as an objection to current and planned experiments designed to verify GTR. The point is that, in real life, one doesn't always have the complete freedom of choice. For example, if a radioastronomer finds that he is in a position to determine the deflection of radio waves by the solar field, he is fully justified in using his radiointerferometer, which may have been designed and mainly used for completely different purposes. Similarly, it is probable that new gravitational experiments that have become possible as a result of progress in measurement technology will serve many other purposes as well. Here, we mention particularly experiments involved in the detection of gravitational waves. The reception of such waves can be used within certain limits (which, in practice, are very modest) to verify GTR.[13)] On the other hand, the main effect of the detection of gravitational waves will be the creation of a new branch of astronomy, namely, gravitational wave astronomy. It follows that the application of resources to the development of gravitational wave detectors (see Ref. 5 and the literature cited therein) is completely justified irrespective of the connection with the problem of verification of GTR.

In view of the foregoing, there is no doubt that verifications of GTR in weak fields (e.g., the solar field) will continue. Unless something unforeseen occurs (this is always possible—and adds fuel to the fire), we shall see, in the course of the next five to ten years, verification of weak-field GTR with precision much higher than is now available. What will this indicate and what will it give us in relation to the verification of GTR as a whole?

[12)]We have expressed this view before (see, for example, Ref. 27). It is quite widely held. Only for a limited period weak-field verifications of GTR have reentered the arena of physics in connection with the argument mentioned above about the possible oblateness of the sun.

[13)]Variations in the orbit of the double pulsar PSR 1913+ 16 have led to the conclustion[56] that it emits gravitational radiation at a level in agreement with calculations based on GTR.

6. Strictly speaking, it is only now that we are in a position to consider the genuinely topical problem, namely, the verification of GTR in a strong gravitational field. Let us suppose, for the sake of argument, that GTR has been reliably verified not only to within terms of order $|\varphi|/c^2$, but also to within terms of order $(\varphi/c^2)^2$, which, within the limits of the solar system, are smaller by at least six orders of magnitude [see (6)]. Can we conclude from this that GTR is valid in strong fields in which $|\varphi|/c^2\sim 1$ or, more precisely, when the deviations of the components of g_{ik} in (9) or (10) from the Galilean values are of the order of unity? If we approach this question in a purely formal fashion, it is obvious that the answer to this question is a definite 'no.' For example, the component

$$-g_{rr}=\frac{1}{1-(r_g/r)}=1+\frac{r_g}{r}+\left(\frac{r_g}{r}\right)^2+\left(\frac{r_g}{r}\right)^3+\ldots,$$

in (9) and knowledge of the first three terms in this series [which is evidently precisely equivalent to knowing the terms of the order of φ/c^2 and $(\varphi/c^2)^2$] will in no way guarantee that the sum of all the terms is equal to $[1-(r_g/r)]^{-1}$. On the other hand, if we know only the first two terms in the series, which is the case in reality (for $\gamma=\beta=1$, which corresponds to GTR), we are still less justified in saying anything about the entire sum of terms. In practice, the situation is less gloomy. GTR is not at all equivalent, if we can use this language, to the Schwarzschild solution (9). It is based on a series of profound principles, not all of which are obviously necessary but which, when taken together, are very difficult to modify (here, we also use the assumption that GTR is valid in weak fields; the foundations of GTR are discussed in Refs. 6–9 and 28). An "empirical" indication of this is that, despite numerous attempts extending over many years, no-one has yet succeeded in constructing a theory of the gravitational field that differs from GTR but is identical with it in weak fields and gives rise to no objections (for greater detail see below).

In one way or another, GTR will have to be verified for strong fields. Within the limits of the solar system, one would hope to be able to measure terms of the order of $(\varphi/c^2)^2\sim 10^{-12}$, in the first instance, with the aid of a solar probe, i.e., an interplanetary station capable of close approach to the sun. For white dwarfs, the parameter $|\varphi|/c^2=GM/r_0c^2$ is greater by two orders of magnitude as compared with the sun, but an independent determination of the radius r_0 of the photosphere (other than from the red shift of spectral lines) is very difficult. For neutron stars, $|\varphi|/c^2=GM/c^2r_0$ reaches 0.1–0.3 (for example, when $M=M_\odot$ and $r_0=10^6$, we have $|\varphi|/c^2\approx 0.15$), and measurements of the relative red shift of different lines in the x-ray and gamma-ray bands can be used[29] to verify the principle of equivalence in a relatively strong gravitational field. There is one other possibility, namely, studies of the motion of a pulsar (magnetized neutron star) in a sufficiently tight binary system. The first such pulsar (PSR1913 +16) was discovered in 1974 and the associated possibilities are discussed in Refs. 5 and 56. We note that the relativistic precession of the orbit in the case of a binary system with component masses $M_{1,2} \sim M_{\odot}$ and observed orbital period $T \approx 7.7$ h is about 4 degrees of arc per annum, which is greater by four orders of magnitude as compared with Mercury. This comparison is, however, somewhat misleading. The physical parameter is not the precession of the orbit per some particular period of time, say, per annum, but the precession per revolution. This is the significance of the angle Ψ given by (21). For Mercury, $\Psi \approx 5 \times 10^{-7}$ radians per revolution, whereas, for the above pulsar in the binary system, $\Psi \sim 5 \times 10^{-5}$, i.e., greater by only two orders (and not four). Generally, even for a pulsar in a binary system, the parameter $|\varphi|/c^2$ is much less than unity, and we are still involved with measurements of only the first-order effects in this parameter. A further point is that, although the value of $|\varphi|/c^2$ for the pulsar is much greater than for Mercury, this is largely offset by the fact that the orbital elements of the pulsar are not accurately known. It is therefore better to use GTR to analyze the behavior of the pulsar and hence of its emission[56] rather than verify GTR in the still relatively weak field.

In view of the foregoing, the prospects for strong-field GTR verifications seem exceedingly modest. It is, therefore, important to emphasize that, within the framework of GTR, not only strong but even ultrastrong gravitational fields can and even should be encountered. In particular, this is so in cosmology and in the case of black holes.

Nonstationary isotropic and homogeneous cosmological models (Friedmann, 1922 and 1924) are known to have singularities. Open models have one singularity at $t=0$, whereas closed models have two singularities, one at $t=0$ and the other at the end of the contraction phase (we assume that the general properties of such models are known to the reader; see, for example, Refs. 6–9 and 11). The density of matter ρ increases without limit when a singularity is approached in a model of this kind. It seemed at one time that the appearance of singularities in GTR solutions was associated with the high degree of symmetry of the models considered. However, it was established later[6,30,31] that this was not so and that the appearance of the singularities (infinite density or infinite curvature of space even in the absence of matter) were very general properties of the GTR equations. In particular, a singularity appears whenever one considers the gravitational collapse of some mass (for example, sufficiently massive star) that leads to the formation of a black hole. However, in this case, the singularity lying at $r=0$ (at the origin of coordinates) is "invisible" to the observer located at infinite [i.e., for $r \to \infty$; r is the radial coordinate in the Schwarzschild solution (9)].

How can we assess the appearance of a "true" singularity, i.e., infinite density of matter and/or infinite curvature of space (and hence infinitely strong tidal forces; see, for example, Ref. 9, Sec. 31.2)? According to the currently most widely held view (to which the present author has always adhered), the existence of "true" singularities undoubtedly indicates that GTR—a specific classical theory of the gravitational field[14)]—has a limited range of validity. Apart from general considerations, this conclusion reliably ensues from the analysis of GTR itself when the fundamentals of quantum theory are taken into account. In point of fact, the quantum-mechanical point of view necessarily demands the existence of zero-point oscillations in the gravitational field and, indeed, in all other fields. The classical theory can be used only where all these fluctuations are small enough. Specifically, the quantum-mechanical fluctuations δg_{ik} in the metric itself must be small in comparison with the classical values of g_{ik}. This eventually ensures (see Refs. 9, 12, 32, and the references therein) that the limits of validity of GTR are determined by the following parameters:

$$\left.\begin{aligned} &l_g=\sqrt{\frac{\hbar G}{c^3}}=1.6\cdot 10^{-33}\,\text{cm}, \quad t_g=\frac{l_g}{c}=5.4\cdot 10^{-44}\ \text{c}, \\ &\rho_g=\frac{\hbar}{c l_g^4}=\frac{c^5}{\hbar G^2}=5.2\cdot 10^{93}\ \text{g}\cdot\text{cm}^{-3}, \\ &M_g=\sqrt{\frac{\hbar c}{G}}=\rho_g l_g^3=\frac{\hbar}{c l_g}=2.2\cdot 10^{-5}\ \text{g}. \end{aligned}\right\} \qquad (23)$$

This means that the classical theory is invalid if the radius of curvature of space is comparable with the Planck length l_g, or the density ρ is comparable with ρ_g. In other words, the classical theory (GTR) is valid only for $\rho \ll \rho_g$, for "point" bodies with mass $M \gg M_g$, for time intervals $t \gg t_g$, and for lengths $l \gg l_g$. These inequalities can be applied to specific situations (i.e., the precision of the classical approximation can be estimated) only on the basis of the quantum theory of gravitation which cannot as yet be regarded as complete. It is, therefore, clear that, if we accept GTR say, for densities $\rho < 10^{93}-10^{94}$ g/cm^3, we introduce a certain arbitrariness because it may turn out that some numerical factors will ensure that GTR is valid to, say, 1% only for $\rho < 10^{90}$ g/cm^3, and so on. There is, however, a less trivial reservation. We have assumed that GTR is restricted only on the side of the quantum theory, so to speak. In all other respects, it has been assumed that we are at liberty to consider space and time in the same way as in macroscopic physics and in astronomy, down to scales of the order of l_g and t_g. On the other hand, modern physics (in this case, microphysics or, as it is sometimes described, the physics of high energies or elementary-particle physics) has managed to reach down to only $l \sim 10^{-15} - 10^{-16}$ cm. This means that existing physical experiments more or less guarantee the validity of the usual space concepts down to scales of length of this order (we note, for comparison, that the Compton length of a nucleon is $\hbar/mc \sim 10^{-14}$ cm and $\hbar c/E \sim 10^{-16}$ for particle energies $E \sim 1$ erg $\sim 10^{12}$ eV). It was suggested relatively recently that there is a fundamental length $l_f \sim 10^{-17}$ cm such that, for $l \lesssim l_f$, space itself becomes "unusual," i.e., it is somehow quantized, and so on. The development of a unified theory of weak and electromagnetic

[14)]In order to avoid misunderstanding, we use the abbreviation GTR in relation to the classical theory alone. Whenever the quantization of GTR is involved or, more correctly, the quantum-mechanical generalization of GTR, this will be emphasized in the terminology.

interactions has meant that there is now no basis for introducing the length $l_f \sim 10^{-17}$ cm, and the question of the fundamental length has been almost universally ignored. It is, however, very striking that there is a break of almost 17 orders of magnitude (!) between the smallest "investigated" scales $l \sim 10^{-16}$ and the gravitational length $l_g \sim 10^{-33}$. Even if we perform a very extravagant extrapolation and suppose that the fundamental length is, say, $l_f \sim 10^{-20}$ cm, the gap between l_g and l_f is still quite colossal. On the other hand, so far as we can tell, there is no experimental evidence against the existence of a fundamental length $l_f < 10^{-17}$ cm. This does not, of course, mean that this length does, in fact, exist. It is possible that the gravitational length l_g plays, in fact, the role of l_f. However, in view of the foregoing, we cannot conclude that the limits of validity of GTR are defined by the parameters given by (23). On the contrary, if there exists a fundamental length l_f, one might imagine that the limits of GTR are defined by

$$l_f,\ t_f \sim \frac{l_f}{c},\quad \rho_f \sim \frac{\hbar}{c l_f^4} \sim \rho_g \left(\frac{l_g}{l_f}\right)^4. \tag{24}$$

For $l_f \sim 10^{-17}$ cm, we have $t_f \sim 10^{-27}$ sec and the maximum density is $\rho_f \sim 10^{30}$ g/cm$^3 \sim 10^{-64}\rho_g$. However great the difference between l_f, t_f, ρ_f, and the gravitational parameters given by (23) for $l_f \sim 10^{-20}$ cm, it affects only microscales and is important[33] near the classical singularities,[15)] and in the theory of relict black mini-holes. The last topic, namely, the question of black holes of low mass that "evaporate" (emit photons and other particles) when quantum-mechanical effects[34] are taken into account is among the most interesting questions in recent physics and astrophysics. However, this and the associated problems are outside the framework of the present review (see Refs. 35–37).

The fact that the limits of validity of GTR (even if it is exactly valid in the classical region) appear when quantum-mechanical effects are taken into account and, possibly, even when the fundamental length $l_f \gg l_g$ is shown to exist,[16)] is of major fundamental importance. The properties of the space-time region near the singularities that appear in GTR, the development of a quantum theory of gravitation and quantum cosmology, and the connection of all this with microphysics are all of major importance and continue to attract constant attention. This is a reflection of the fact that, after many years of isolation (if not total oblivion), both GTR and the theory of gravitation as a whole have emerged into broad daylight and occupy a conspicuous position in physics. Moreover, it is neither appropriate nor useful to relate the singularity problem to the question of experimental verification of GTR. On the contrary, it is more likely that it will be extremely important to verify the validity of GTR in the classical region before it is generalized to the quantum region. It is indeed this that should dictate the aim of experimental verifications of GTR, namely, the demonstration of its validity in strong gravitational fields but still well away from the "true" singularities.

7. It is exceedingly important that this problem, i.e., the verification of GTR in strong fields but well away from a singularity, is very closely related to the physically, astrophysically, and cosmologically topical questions involving the existence and behavior of black holes.

It is very interesting that the concept of a black hole was essentially introduced by Laplace as far back as the end of the eighteenth century.[40] Laplace noted that, according to the corpuscular theory of light and the Newtonian theory of gravitation, a sufficiently massive body would not emit light. In actual fact, a particle of mass m will escape to infinity (will be emitted) only if its initial kinetic energy $\frac{1}{2}mv_0^2$ on the photosphere of a star of mass M and radius r is greater than or equal to GmM/r (it is, of course, assumed that the inertial and gravitational masses of the particles, m, are equal to one another). When the velocity is equal to the velocity of light, $v_0 = c$, light can be emitted (i.e., departs to large distances from the star) only when $r > r_g = 2GM/c^2$. The fact that this method yields the correct expression for the gravitational radius [see (7)] is, of course, fortuitous because the above expressions are not valid in the case of light. However, if we were to adopt the more modern approach and replace $\frac{1}{2}mv_0^2$ not with $\frac{1}{2}mc^2$ but with mc^2, the two expressions would differ by a factor of only two. At the same time, the calculation would correctly represent the essential physics of the situation. Black holes appear to have been mentioned for the first time nearly 250 years later, i.e., in 1939, and then on the basis of GTR.[41] A further 25–30 years elapsed (nearly 50 years after the creation of GTR) before black holes assumed a significant and then a conspicuous position in physics and astronomy. The reason for this can be seen in the fact that the gravitational radius of a star of mass M that is of the order of the solar mass $M_\odot$ is only a few kilometers [see (7)], which is very much less than the radius of normal stars (for the sun, $r\ /r_g \sim 2\times 10^5$, where $r_\odot = 7\times 10^{10}$ cm is the radius of the photosphere). Under such conditions, the Schwarzschild solution (9), which applies only to the vacuum (outside the star), is needed only to provide corrections of the order of $|\varphi|/c^2 \sim r_g/r$, which we have already discussed. This is why, even now, some courses on the general theory of relativity treat gravitational collapse and black holes as relatively exotic features.

At the same time, it is precisely when the validity of GTR may be regarded as established that one would ex-

[15)]The phrase, "classical singularity," indicates that we have in mind a singularity appearing in the classical (nonquantum) theory and, in particular, in GTR. There is no reason to suppose that a particular singularity will persist into the quantum region and, in general, into the "true" theory of gravitation going beyond the framework of GTR. We note that an analysis of the behavior of physical clocks as they approach the classical singularity is given in Ref. 38, where it is concluded that any clock will "give up" and will not measure the proper time when it is close enough to a singularity. The fact that GTR is not valid near the singularity follows from this.[38] It is clear, however, that the corresponding restriction is probably not stronger than the quantum-mechanical restriction $M \gg M_g$, $l \gg l_g$, etc.

[16)]See Ref. 39, where an argument in favor of $l_f \sim l_g$ is given.

pect the appearance of a large number of black holes. The point is that cold (burnt up) stars can become white dwarfs only if their mass is $M \lesssim (1.2-1.4)M_\odot$. In the case of neutron stars, $M \lesssim 3M_\odot$. Although the last result depends on certain (not totally rigorous) assumptions about the equation of state of nuclear matter, it is, nevertheless, reasonably reliable. Cold stars with $M > 3M_\odot$ or, at any rate, with $M > 5M_\odot$ should, according to GTR, collapse without limit and form black holes. This means that, in the reference frame in which the stellar surface is at rest, the surface will cut the Schwarzschild sphere of radius r_g at some particular time. However, for a distant observer, the star will cool down and rapidly approach the size $r = r_g$. The field due to a star that has cooled down in this way is described in GTR by the solution (9) for which $g_{00} \to 0$ and $g_{rr} \to -\infty$ as $r \to r_g$. The gravitational field of a cold star (or, alternatively, of the resulting black hole) is, therefore, very strong and cannot be approximated by a few terms of expansions such as (12). It is, of course, important to emphasize that this so-called Schwarzchild singularity, which occurs in the Schwarzschild solution (9) for $r = r_g$, is not a "true" singularity of the gravitational field in the sense in which this concept was used above. In particular, as $r \to r_g$, the components of the space curvature tensor are finite and the determinant $g \equiv |g_{ik}| = -r^4 \sin^2\theta$ has no singularities. The "observer" falling together with the stellar material will not "note" the crossing of the Schwarzschild sphere so that the singularity on the $r = r_g$ sphere is not a "true" one.[17] However, it cannot be referred to as "fictitious" either because, for an external observer, it is precisely the Schwarzschild sphere that acts as the "horizon of events," i.e., it is the sphere on reaching which the star "cools down," as indicated above. For rotating black holes, the situation is somewhat more complicated, but the horizon of events does, in general, exist (this involves the Kerr solution; the reader is referred to Refs. 9 and 11 for further details).

Many stars have masses $M \gg M_\odot$ so that it would appear that many of them will convert into black holes at the end of the cooling process. The gas accumulated in the central regions of globular clusters and galaxies may also be expected eventually to produce black holes of large mass. This is indeed the reason for the statement above that GTR leads us to expect the appearance of black holes not as exceptional objects but as common byproducts of the evolution of massive bodies.

The fact is, however, that not a single black hole has so far been reliably detected despite the fact that a constant search has been in progress for the last fifteen years, both among stars and in the interiors of galaxies and quasars. This situation calls for some comment.

Despite the fact that GTR undoubtedly predicts that black holes can exist, their appearance is not at all mandatory. In fact, the star can explode during the burning up of the nuclear fuel, and this has undoubtedly been observed in some cases (in the case of novas and supernovas). The star sheds a shell during the explosion, and the residual mass can be less than $3M_\odot$, or even less than $1.2M_\odot$, so that the formation of a black hole can be avoided. Examination of a number of stellar models leads to precisely this result. In the case of galactic cores, collapse can again be avoided, at least in principle, either by invoking nuclear explosion or the redistribution of energy among the stars in the cluster (in the latter case, we assume that the galactic core is a dense stellar cluster).[42] It must then be remembered that stars, their clusters, and gaseous masses are usually rotating, so that contraction at constant angular momentum should produce an increase in the rotational velocity, which impedes collapse.[18] The loss of angular momentum, on the other hand, is a relatively slow process. Briefly speaking, the formation of black holes occurs with considerable difficulty and quite slowly, or it may not actually occur at all over a period of ten to twenty billion years.

Thus, the absence of detectable black holes does not indicate a breakdown in the validity of GTR in a strong field, but it does require special explanation. The existence of stable cold stars of mass $M > 3M_\odot$ (or, for safety, of still greater mass) could be regarded as evidence against GTR. The behavior of a compact (high mass) star conflicting with the predictions of GTR (absence of a horizon of events when, according to GTR, the horizon must exist, as in the case of a nonrotating or slowly rotating star[19]) would also be evidence against GTR. *On the other hand, the detection of black holes would be a very far-reaching confirmation of GTR* although, for a quantitative verification [say, of the validity of (9)], the mere fact of existence of a horizon of events is not enough.

What is the present state of searches for black holes? A black hole can be detected either by its gravitational

[17] We note that, for sufficiently massive black holes (with $M_\odot \gg M$), the density ρ is still not too high as the stellar radius approaches the gravitational radius; for example, it is not greater than the density of nuclear matter, $\rho_n \sim 3\times10^{14}$ g.cm^{-3}. In fact, $\rho_0 = 3M/4\pi r_g^3 = 3c^6/32\pi G^3 M^2$) because $r_g = 2GM/c^2$; hence, for example, for $M = 10M$, the density is $\rho_0 \sim 2\times10^{14}$ g.cm^{-3}.

[18] For a gas cloud with $r_1 \sim 10^{18}$ cm and characteristic gas velocity $v_1 \sim 10^6$ cm/sec, we have $r_1 v_1 \sim 10^{24}$ and the angular momentum of the gas per gram is of the same order. When the cloud contracts down to r_2 of the order of the gravitational radius r_g, the gas velocity is $v_2 \sim c$ and, consequently, $r_2 v_2 \sim c r_g \sim 10^{16} M/M_\odot$ cm^2/sec, where M is the mass of the gas. It is clear that, when $M/M_\odot \sim 10$, we have $r_1 v_1 / r_2 v_2 \sim 10^7$, i.e., the angular momentum of the moving cloud must be reduced by 6–7 orders of magnitude prior to the formation of the black hole. In the magnetoid model,[44] in which the effect of the magnetic field is taken into account, the reduction in the angular momentum is, of course, produced in a relatively short time.[44b] However, fragmentation and nuclear explosions attending this can still impede collapse with the formation of a black hole.

[19] According to the so-called cosmic censorship hypothesis within the framework of the general theory of relativity the *true singularity that appears as a result of collapse is always surrounded by a horizon of events provided only that the metric is Galilean at infinity.* This hypothesis has not been proved—hence the caution exercised in the formulation of the conclusion that the absence of a horizon is in conflict with GTR.

effect on another star in a binary system, or by the specific radiation that appears during the accretion of gas incident on the black hole from the interstellar medium or issuing from the companion star.[11a] Searches for black holes in binary systems have so far provided us with only one possible "candidate," namely, the system Cyg X-1, a relatively bright "x-ray star" which has been under investigation for several years. The mass of the compact component of this source appears to exceed $5M_\odot$, so that it is suspected that one is observing a black hole. Oscillations in the x-ray luminosity of this source and its spectrum are also unusual, namely, they differ from those expected in the case of accretion on a neutron star. Nevertheless, the nature of Cyg X-1 is still not clear. The latest review in this field,[43a] written by three American and two Soviet astrophysicists, begins with the words, "We would be happy if Cyg X-1 were to turn out to be a black hole. However, frankly speaking, we are not completely sure." On the other hand, the same paper ends with the words, "It would appear that Cyg X-1 should be the nearest black hole undergoing accretion," but the entire question is still open (the alternative possible assumption is that Cyg X-1 is not a binary but a ternary system[20)]). The situation is still more indeterminate in the case of galactic cores and quasars (or, more precisely, the centers of these cores and quasars). There are, at present, three competing models of such central cores,[44a] namely, dense stellar clusters, a plasmomagnetic body (magnetoid or spinar) and a massive black hole. The last model is, at any rate, no more probable than the magnetoid model. In addition, there are some arguments[42] against the presence of very massive black holes at the center of the galaxy and, possibly, of other galaxies and globular clusters. Details of all this are outside the scope of the present review but we have the impression that black holes with stellar masses $[M<20M_\odot-50M_\odot]$ are, in any event, very rare. There is no direct evidence for the existence of massive black holes $[M>10^2M_\odot-10^3M_\odot]$ although there are some indications[44c] (for example, in the core of the radiogalaxy Vir$A=M\,87$) leading us to suspect the presence of a black hole (which, however, has not only *not* been verified but has raised some serious objections).

If black holes are not going to be detected, the question of verification of GTR in strong fields will probably remain open for an indefinite time. On the other hand, as already noted, the detection of black holes would provide qualitative confirmation of GTR, and studies of such holes would be an effective way of verifying the validity of GTR in strong fields.

In this situation, there is, of course, considerable interest in possible theories of gravitation in which black holes cannot appear. Such theoretical schemes are known.[45-48] For example, there has been an attempt[47] to construct a new (and different from GTR) theory of gravitation, motivated precisely by doubts as to the existence of black holes. Such doubts are widely held (for the most recent known case of this, see, for example, Ref. 28). We must, therefore, emphasize once again that black holes seem to us to be reasonable and noncontradictory consequences of GTR that do not at all require its modification (in contrast to the situation in the case of "true" singularities). However, whenever the question of verification of GTR is raised, we cannot assume in advance that black holes can, in fact, exist.

All the papers mentioned above[45-48] are based on the introduction of an "*a priori* geometry"[9] in the sense that, in addition to the metric tensor g_{ik}, they also discuss a further metric tensor γ_{ik} (the theory of Ref. 47 is therefore referred to as the bimetric theory and this terminology can be justifiably applied to other similar schemes). The tensor γ_{ik} is specified in advance and corresponds to flat space-time [i.e., it can be taken in the form given by (2) with $\gamma_{00}=1$, etc.]. The description of the gravitational field by only one metric tensor g_{ik} is based on the principle of equivalence. The introduction of the second tensor, γ_{ik}, is, therefore, a departure from the principle of equivalence but, of course, particular bimetric theories are constructed so that they do not conflict with experimental facts. As an example, we note that the set of equations chosen in Ref. 47 is such that the spherically symmetric vacuum solution for g_{ik} is given by[21)]

$$ds^2=e^{-r_g/\rho}c^2\,dt^2-e^{r_g/\rho}[d\rho^2+\rho^2(\sin^2\theta\,d\varphi^2+d\theta^2)]$$
$$=\left[1-\frac{r_g}{\rho}+\frac{1}{2}\left(\frac{r_g}{\rho}\right)^2+\ldots\right]c^2\,dt^2 \qquad (25)$$
$$-\left[1+\frac{r_g}{\rho}+\frac{1}{2}\left(\frac{r_g}{\rho}\right)^2+\ldots\right][d\rho^2+\rho^2(\sin^2\theta\,d\varphi^2+d\theta^2)].$$

From this and from (10) and (11), it follows that GTR and the bimetric theory predict the same weak-field expressions for ds^2, and this ensures that the predictions of effects observed in the solar system are also the same. On the other hand, the solution given by (25) has no singularities at the gravitational radius r_g and, specifically, the component g_{00} does not vanish at $r=4\rho=r_g$, in contrast to the Schwarzschild solution (9) and (10). According to (25), therefore, black holes do not exist. The theory reported in Ref. 47 has been used[49] to solve the problem of the maximum possible mass, $M_{\max}$, of a neutron star. Under certain assumptions with regard to the equation of state of the material, which in GTR predict that $M_{\max}=1.46M$, it was found[49] that $M_{\max}=8.1M$ (although the question of stability of the solution was not investigated). If $M>M_{\max}$, the bimetric theory[49] predicts a collapse, but a horizon of events (black hole) is not produced, i.e., a distant "observer" will be able to see the incidence of

[20)] It has also been suggested[43b] that, when a very strong magnetic field is present, the mass of a cold star that has not as yet collapsed can be substantially greater than $3M_\odot$. A further suggestion is[43c] that the Cyg X-1 radiation is connected not with accretion of gas on a compact star but with magnetic effects in the binary stellar system.

[21)] The notation used here has been modified as compared with Ref. 47. The main change is that the radial variable ρ is assumed to be the same as in "isotropic" spherical coordinates [see (10)]. We emphasize this because it is well known that the choice and meaning of coordinates in comparisons between observations and theory is not a trivial problem in gravitational theories.

matter on the center of the star. This would appear to be the "true" singularity, but the question as to its character or even as to its very existence has not, as far as we know, been investigated in the necessary detail.

These remarks essentially bring us to the end of our review. Bimetric theories are generally more complicated than GTR and have only just begun to be examined. Moreover, even GTR, which has been under investigation for many decades, has frequently presented us with surprises and can hardly be regarded as simple and clear in all its mathematical and physical aspects. Bimetric theories have been shown[50,51] to encounter certain specific difficulties, but it is still not clear whether a noncontradictory bimetric theory can be constructed.[22)] There is also no reason why we should restrict our attention to bimetric theories, since other proposals deserving attention have also been put forward.[28,52,53] For example, there is the theory[53] with a skew-symmetric tensor g_{ik} that was considered in Einstein's last paper.[55] The attempt[28] to construct a tetrad theory of gravitation, satisfying all the basic principles of GTR but being distinct from it, seems to us to be particularly interesting, at least for macrophysics. Unfortunately, analysis of this version of the theory is still far from completion and its fate is still not clear.

The general theory of relativity began its life more than seventy years ago and has been under investigation in its modern form for more than six decades. This period has seen the emergence of several generations of physicists and astronomers and many discoveries of lasting importance. For many years (especially between the end of the 1920's and, roughly, the beginning of the 1950's), GTR seemed to languish on the sidelines of the main thrust and development of science. However, at present, it occupies (one might say *again* occupies) the center of attention. In the somewhat more restricted area of experimental verification of GTR, there have also been several ups and downs connected with the emergence of each particular vagueness or doubt (for example, we recall once again the difficulties encountered with the observation of the red shift of spectral lines in the solar spectrum and the oblateness of the sun). In fact, GTR is now in a better situation than it has ever been in relation to its agreement with experiment and observations. There is not the least evidence for any departure from GTR in weak gravitational fields, and all the predictions that could be checked have been verified. Experimental studies in weak fields are continuing and the corresponding data will, after a few years, become much more extensive than it is now. At the same time, the "center of gravity" in research on GTR and on its applications and verifications has clearly shifted toward strong gravitational fields. If GTR as a classical theory of gravitation is exact, the main problem is its quantum-mechanical generalization and its features near classical singularities. This is where cosmology and GTR as a whole come into contact with microphysics. In the classical (nonquantum-mechanical) domain, the black-hole problem, i.e., the problem of their detection and properties, is central for GTR and its verifications. Until black holes are found it is difficult, if not impossible, to be quite certain about the validity of GTR in strong fields. The detection of black holes would be a further genuine triumph for GTR, and their subsequent investigation would be a means of quantitative verification of GTR in strong fields.

When will this situation be clarified? It is very difficult to answer this question but, although the detection of a black hole could occur at anytime, quantitative verification of GTR involving "the use of" black holes will, clearly, require a considerable amount of time. Fortunately, as has been already emphasized, science does not develop in a strictly consecutive fashion whereby each step follows the demonstration of complete validity of the preceding step. GTR has contributed important results even before it could be verified to the extent that it now is. Progress will of course be maintained, and we must view only as a warning signal the fact that black holes have not yet been reliably detected and that the general theory of relativity as applied to strong gravitational fields has not as yet been directly experimentally verified.

On the other hand, we must also emphasize here that, in addition to logical arguments, experimental data, and theoretical analysis, each physicist is also guided by his own intuition and has faith in one or another future course of research. Of course, caution frequently restricts prediction and, indeed, the probability of error in estimates relating to fundamental questions is quite high. Nevertheless, whilst concluding this review with a factual statement, the author would not wish to conceal his own intuitive view. The great simplicity and elegance of the general theory of relativity and all its past history lead us to think (or, if you prefer, believe) that this theory is strictly valid even in strong fields right up to some space-time region in the neighborhood of true singularities where quantum-mechanical and, possibly, some other microphysical effects must be taken into account. From this point of view, black holes can undoubtedly exist and, if they are not detected, this will have to be related to the conditions under which they are produced. These conditions may well turn out to be relatively unfavorable in the particular cosmic situation. At the same time, we repeat once again that it is quite possible that the discovery of black holes is imminent.

[22)]According to Ref. 51, existing bimetric theories predict the emission of gravitational waves with negative energy, so that the radiating system (for example, a binary star) may increase its energy by emission. This is clearly an unacceptable situation, indicating that the theory is not valid. It is still not clear whether it is possible to construct a bimetric theory free from this defect, but it is apparent that this is a profound difficulty. The appearance of solutions with negative energy also prevents us from generalizing GTR by introducing terms with higher-order derivatives[54] for which there appears to be good justification.[12]

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