Nonlinear effects in the propagation of high-frequency sound in normal conductors

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The basic results of the theory of nonlinear acoustic phenomena in metals and semiconductors are reviewed. Short-wavelength sound is assumed: the ultrasonic wavelength is taken to be far shorter than the mean free path of the conduction electrons. The case in which the interaction of the electrons with the sound wave can be described by classical mechanics is studied. The basic purpose of the review is to discuss the theory for nonlinear absorption of sound, which gives the absorption coefficient as a function of the sound intensity. The nonlinearity results from an effect of the field of the sound wave on the motion of "resonant" electrons, i.e., particles which are moving in phase with the sound and which determine its absorption. This effect leads to a substantial distortion of the (quasi-) momentum distribution of this relatively small group of particles. For this reason, this nonlinearity is called the "momentum" nonlinearity. The particular parameters which are the measure of this nonlinearity in different situations are identified. The particular features of the momentum nonlinearity in the presence of a magnetic field are discussed. An extremely unusual variation of the absorption coefficient with the magnetic field which occurs in very weak fields (of the order of a fraction of an oersted) is described. Weak fields at this level do not affect the linear absorption. This unusual variation is due to a suppression of the nonlinearity by the magnetic field, which provides an additional mechanism for removal of particles from the resonant group.

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CONTENTS

1. INTRODUCTION

In this review we are concerned with the theory of the non-linear acoustic effects which occur in metals and semiconductors as short-wavelength sound propagates through them. For our purposes the wavelength $\lambda^{(s)}$ is "short" if it is much smaller than the mean free path of the conduction electrons, l:

$$\lambda^{(s)} \ll l. \tag{1.1}$$

The particular frequency range over which this condition holds varies with the type of conductor, of course, and it also varies with the impurity concentration, the concentration of lattice defects, and the temperature. In practice, this condition can be satisfied only at low temperatures (liquid-nitrogen temperature and below) and in relatively pure materials.

We are interested in only those nonlinear phenomena which are of electronic origin. At low temperatures, the conduction electrons play a governing role in acoustic effects (or at any rate, the effect of these electrons can be reliably identified). Furthermore, the nonlinearities due to the interaction of the sound wave with the electrons are generally manifested at acoustic intensities well below those required for observation of an elastic nonlinearity.

Nonlinear acoustic effects in the case $\lambda^{(s)} \gg l$ have been studied thoroughly, both theoretically and experimentally, and are covered in the reviews in Refs. 1–3, among others. In this particular case the interaction of the condition electrons with the sound can usually be described in macroscopic terms. In the short-wavelength case, (1.1), on the other hand, in which the kinetic properties of the electron system have their greatest consequences, research is in an earlier stage: there has been intense theoretical work in recent years, and the first experiments have been reported. It seems worthwhile at this point to outline what has been learned, to identify the most important theoretical problems awaiting solution, and to point out the most promising directions for future experiments. These are the goals of the present review.

As a sound wave propagates through a medium, the flux density of mechanical energy S transported by the wave is attenuated and dissipated as heat. If the sound wave is excited at a boundary of a crystal, the attenuation of the wave as it penetrates into the crystal is described by

$$\frac{dS}{dx} = -\Gamma S, \tag{1.2}$$

where the sound is propagating along the x axis. The

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quantity Γ is the absorption coefficient for the sound.

If S is sufficiently small, the absorption coefficient is independent of S, and the linear theory applies. This theory yields

$$S(x) = S(0)\, e^{-\Gamma x}. \tag{1.3}$$

As S increases, Γ begins to vary with S, and we have a nonlinear absorption of the sound; the variation of S with x is more complicated than that described by a simple exponential law.

Another nonlinear effect is the acoustoelectric effect, which is the appearance of a direct current (or a static field, if the circuit is open) because of the entrainment of conduction electrons by the traveling sound wave. The acoustoelectric effect was predicted theoretically by Parmenter[4] and subsequently studied theoretically and experimentally by Weinreich *et al.*[5] in n-type Ge. At low sound intensities, the acoustoelectric current is proportional to the intensity, while at higher intensities this current becomes a more complicated function of the intensity.

These nonlinear effects are due to the capture of conduction electrons by the periodic field of the sound wave. In principle, there are two possibilities here: a classical situation (in which there are many levels in the acoustic potential wells) and a quantum situation (the number of levels is of the order of unity). To save space here, we must restrict this review to one of these situations, and we choose the first. To analyze this situation we will use the apparatus of the classical kinetic equation.

In studying the nonlinear effects in the magnetic field, we will correspondingly assume that the field is a nonquantizing field, i.e., is a weak field. We are thus excluding from this review a wide range of effects, which involve, for example, the nonlinear theory of giant quantum oscillations in sound absorption,[6] nonlinear sound absorption under conditions of magnetic breakdown,[7] etc.

Again to save space, we will restrict the discussion to nonlinear acoustic effects in normal conductors. Many of the results found for normal conductors can be extended immediately to superconductors, but the nonlinear acoustoelectric effects which occur in superconductors are extremely unusual, reflecting the unique features of superconductivity, and a special review would be required for a detailed analysis of this case.

Now that we have listed the topics which are not included in this review, we can briefly formulate the topics which are included. Our subject is the classical theory of nonlinear acoustic effects in semiconductors and normal metals, which may be in external fields (electric and magnetic).

Before we begin the review proper, we would like to point out that the basic ideas and methods used to analyze nonlinear sound absorption in conductors can be used in other problems. For example, Vugal'ter and Demikhovskiĭ[8] have shown that an essentially analogous situation arises in the nonlinear absorption of electromagnetic waves in metals. Kagan[9] has shown that the same methods can be used to analyze nonlinear sound absorption in dielectrics.

This review is primarily a review of theoretical results. Since only a few experimental papers have been published at the time of this writing, we decided not to devote a separate section of the review to them. At the end of the fourth and fifth sections there are brief descriptions of experimental observations of nonlinear acoustic effects for the conditions assumed here for semiconductors and metals, respectively.

2. INTERACTION OF ELECTRONS WITH SOUND IN CONDUCTORS

Although the physical mechanism for the classical nonlinearity discussed below is in general features the same for metals and semiconductors, there are some differences in the nature of the interaction of the electrons with the sound. For definiteness we will consider metals first, and then we will point out the differences which arise in the case of semiconductors.

Let us consider the motion of an electron in the self-consistent periodic field of the crystal lattice and of the other electrons, distorted by the sound wave. We describe the electrons of the metal before deformation by the dispersion law $\varepsilon_0(\mathbf{p})$, where $\mathbf{p}$ is the electron quasimomentum.

To describe the change in the electron spectrum upon the deformation, we must specify the coordinate system in which the change is given. This question will be discussed in detail here, since a somewhat hazy understanding of this point has led to confusion in several places.

Let us consider some point in the deformed continuous medium. Before the deformation, the coordinate of this point was $\mathbf{r}$; after the deformation, the coordinate is $\mathbf{r}' = \mathbf{r} + \mathbf{u}$, where $\mathbf{u}$ is the displacement vector. Each point in the continuous medium can be described by either the Lagrange coordinates $\mathbf{r}, t$ or the Euler coordinates $\mathbf{r}', t$. The Lagrange coordinate system accompanies the deformed lattice; the interaction of the conduction electrons with the strain can be described most simply in this coordinate system. Correspondingly, we will write the kinetic equation for the electrons in the varying strain field and also the elastic-theory equations in this coordinate system. This is, of course, a noninertial and curvilinear coordinate system; both circumstances can be taken into account by appropriate correction terms in the equation for the electron energy.

We will describe these additional terms by the method of a generalized effective mass. According to this method, the motion of an electron in the self-consistent periodic field, with respect to pertubrations which vary slowly in time and space, can be thought of as the motion of a quasiparticle with an energy operator $\varepsilon_0(-i\hbar\delta/\delta\mathbf{r})$. The perturbation caused by the elastic deformation of the crystal as the sound wave propa-

gates through it is always of this type. Furthermore, for those problems (classical in nature) which we will be discussing here, the motion of an electron under the influence of an acoustic perturbation can be thought of as motion in an external field of a classical particle with a Hamiltonian $\varepsilon(\mathbf{p}, \mathbf{r})$ ($\mathbf{p}$ and $\mathbf{r}$ are canonically conjugate variables).

An elastic deformation always satisfies the condition

$$\left|\frac{\partial u_l}{\partial r_k}\right| \ll 1. \tag{2.1}$$

Using (2.1), we can write the Hamiltonian as[10,11]

$$\varepsilon(\mathbf{p}, \mathbf{r}) = \varepsilon_0(\mathbf{p}) + \lambda_{ik}(\mathbf{p})\, u_{ik}(\mathbf{r}) - m_0 \mathbf{v}\dot{\mathbf{u}}; \tag{2.2}$$

where $\lambda_{ik}(\mathbf{p})$ is the strain energy tensor, which depends on the quasimomentum[1] $\mathbf{p}$,

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_l}{\partial r_k} + \frac{\partial u_k}{\partial r_l}\right)$$

is the strain tensor, m_0 is the mass of the free electron, and $\mathbf{v} = \partial\varepsilon_0/\partial\mathbf{p}$ is the electron velocity. The second term in (2.2) describes the interaction of the electrons with the strain field. It can be seen that the strain energy is a classical field acting on the electrons. The third term in (2.2) reflects the circumstance that the Lagrange coordinate system is noninertial, and an additional force, proportional to the acceleration, arises in this system and acts on the conduction electrons. This force is due to the Tolman-Stewart effect. Specifically, Eq. (2.2) corresponds to the Hamiltonian of an electron which is moving in an electric field with the vector potential

$$\mathbf{A}^S = \frac{c}{e} m_0 \dot{\mathbf{u}}, \tag{2.3}$$

where c is the speed of light and e is the electron charge. Then in the comoving coordinate system the electron behaves as if it were moving in an external Tolman-Stewart electric field

$$\mathbf{E}^{ST} = -\frac{1}{c}\frac{\partial \mathbf{A}^{ST}}{\partial t} = -\frac{m_0}{e}\ddot{\mathbf{u}}. \tag{2.4}$$

To calculate the absorption we need the nonequilibrium electron distribution function $F(\mathbf{p}, \mathbf{r})$. This function satisfies the kinetic equation

$$\frac{\partial F}{\partial t} + \frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{r}} - \frac{\partial \varepsilon}{\partial \mathbf{r}}\frac{\partial F}{\partial \mathbf{p}} + \hat{I}\{F\} = 0. \tag{2.5}$$

This equation, which corresponds to the classical picture of the interaction of electrons with sound, is applicable if the sound frequency ω is not too high, so that the following condition holds:

$$\hbar q \ll \bar{p}. \tag{2.6}$$

Here $\bar{p}$ is some characteristic value of the quasimomentum of the electron which is interacting with the wave. This value is governed by the particular features of the problem, so that the specific forms of the criterion corresponding to condition (2.6) are different for the linear and nonlinear theories. We will give the corresponding estimates below.

The collisional term $\hat{I}$ in Eq. (2.5) has the standard form[2]

$$\hat{I}(F) = \frac{2\pi}{\hbar}\sum_{\mathbf{p}_1} |V_{\mathbf{p}\mathbf{p}_1}|^2 [F(\mathbf{p}, \mathbf{r}) - F(\mathbf{p}_1, \mathbf{r})]\, \delta[\varepsilon(\mathbf{p}, \mathbf{r}) - \varepsilon(\mathbf{p}_1, \mathbf{r})], \tag{2.7}$$

i.e., vanishes when the Fermi equilibrium function $F_0[\varepsilon(\mathbf{p}, \mathbf{r})]$ is used. Here $V_{\mathbf{p}\mathbf{p}_1}$ is a quantity which is proportional to the amplitude for electron scattering by impurities.

Below we will also need an equation for the electric current density caused by the sound wave. In the comoving coordinates system, this equation is[3] (Ref. 10)

$$j = e\int d\tau_{\mathbf{p}} v F(p, r), \quad d\tau_{\mathbf{p}} = 2\frac{d^3p}{(2\pi\hbar)^3}. \tag{2.8}$$

We note that we can transform from the classical canonically conjugate variables $\mathbf{p}, \mathbf{r}$, in terms of which Eq. (2.5) is written, to the variables $\mathbf{p}', \mathbf{r}'$, which correspond to the laboratory coordinate system, by using a canonical transformation[4] with the generating function

$$\Phi(\mathbf{r}, \mathbf{p}', t) = \mathbf{p}'(\mathbf{r} + \mathbf{u}). \tag{2.9}$$

This transformation is

$$p_i = \frac{\partial \Phi}{\partial r_i} = p_i' + p_k' \frac{\partial u_k}{\partial r_i}, \qquad r_i' = \frac{\partial \Phi}{\partial p_i'} = r_i + u_i, \tag{2.10}$$

$$\varepsilon'(\mathbf{p}', \mathbf{r}') = \varepsilon(\mathbf{p}, \mathbf{r}) + \frac{\partial \Phi}{\partial t} = \varepsilon_0(\mathbf{p}') + (\mathbf{p}' - m_0\mathbf{v}')\dot{\mathbf{u}} + (\lambda_{ik} + p_i' v_k')\frac{\partial u_k}{\partial r'}. \tag{2.11}$$

For free electrons,[10] we have $\lambda_{ik} = -p_i' v_k'$, so that the second and third terms in (2.11) vanish, as they should. In the laboratory coordinate system, the kinetic equation for the function $F'(\mathbf{p}', \mathbf{r}') = F(\mathbf{p}, \mathbf{r})$ takes on a form similar to (2.5). Significantly, the argument of the δ-function in the collisional term of this equation contains the energy $\varepsilon(\mathbf{p}, \mathbf{r})$, expressed in terms of the variables $\mathbf{p}'$ and $\mathbf{r}'$ i.e., the combination

$$\varepsilon'(\mathbf{p}', \mathbf{r}') - \mathbf{p}'\dot{\mathbf{u}}. \tag{2.12}$$

The reason is that the impurity atoms are "frozen" in the deformed lattice and move along with it. The collision operator should thus vanish when a function which is the equilibrium function in the comoving coordinate system is used.

The kinetic equation in the laboratory coordinate system was derived by Kontorovich.[11]

Let us see how the basic equations of the problem are altered if there are macroscopic fields (electric and magnetic) in the conductor.

To incorporate these fields we need to add to the Hamiltonian $\varepsilon'(\mathbf{p}', \mathbf{r}')$ (in the laboratory system) a term $e\varphi(\mathbf{r}')$, where φ is the scalar potential, and we need

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[1] The strain energy was introduced in its simplest from by Titeica[12]; a more general form was introduced by Akhiezer.[13]

[2] We are assuming that the primary mechanism for electron scattering is scattering by impurities. This is usually the case at low temperatures.

[3] This is the usual expression for the current density of Bloch electrons: in the comoving coordinate system, the electron wave functions are Bloch functions.

[4] The idea of this transformation is to be credited to L. D. Landau.

to make the replacement

$$\mathbf{p}' \to \mathbf{P}' - \frac{e}{c}\mathbf{A}(\mathbf{r}'), \tag{2.13}$$

where $\mathbf{P}$ is the canonical momentum, and A is the vector potential ($\mathbf{H} = \mathrm{rot}\mathbf{A}$, $\mathbf{E} = \nabla\varphi - c^{-1}\dot{\mathbf{A}}$). In the comoving system, this replacement corresponds to the replacement[10]

$$\mathbf{p} \to \mathbf{p} - \frac{e}{c}\mathbf{A}(\mathbf{r}) + \frac{e}{c}[\mathbf{uH}]. \tag{2.14}$$

The third term on the right in (2.14) can be thought of as the increment $\Delta\mathbf{A}$ in the vector potential in the new coordinate system. This increment describes the induced field which appears in the corresponding system:

$$\mathbf{E}^{(i)} = -\frac{1}{c}\frac{\partial \Delta\mathbf{A}}{\partial t} = \frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]. \tag{2.15}$$

In the presence of external fields, the left side of the kinetic equation thus acquires an additional term which contains the Lorentz force, and the kinetic equation becomes

$$\frac{\partial F}{\partial t} + \mathbf{v}\frac{\partial F}{\partial \mathbf{r}} + \left(e\mathbf{E} + e\mathbf{E}^{(i)} + \frac{e}{c}[\mathbf{vH}] - \frac{\partial \varepsilon}{\partial \mathbf{r}}\right)\frac{\partial F}{\partial \mathbf{p}} + \hat{I}(F) = 0. \tag{2.16}$$

Here we are retaining the notation $\mathbf{p}$ for the kinematic momentum of the electron; this kinematic momentum is the same as the quasimomentum but is not the same as the canonical momentum $\mathbf{P}$ in the presence of fields.

The complete system of equations of the problem should include the Maxwell equations. Furthermore, for both dynamic deformation (e.g., that caused by a sound wave) and static deformation, the requirement of electrical neutrality serves as yet another equation. For the static deformation, neutrality gives us the following conditions, which must be satisfied identically with respect to the elements of the strain energy tensor[13]:

$$\int d\tau_{p'} \frac{\partial F_0}{\partial \varepsilon'} \lambda_{ik}(\mathbf{p}') = 0. \tag{2.17}$$

In the case of a dynamic deformation, a weak longtudinal electric field arises. Analysis shows[10] that the contribution of this field to the absorption is small, involving at least a factor

$$\frac{w}{v_F} \ll 1 \tag{2.18}$$

(w is the sound velocity and v_F is the Fermi velocity), so that we will ignore this longitudinal field. Choosing a gauge in which $\mathrm{div}A = 0$, we can thus get rid of the term with scalar potential φ.

To take into account the effect of the electrons on sound propagation, we should add to the right side of the elastic-theory equations

$$\rho\frac{\partial u_l}{\partial t} = C_{iklm}\frac{\partial u_{lm}}{\partial r_k} \tag{2.19}$$

a force f, which is exerted on the lattice by the electrons (here ρ is the crystal density and C_{iklm} is the elastic modulus tensor). An equation for this force can be found by varying the energy of the system of electrons with respect to a lattice displacement. Using condition (2.1), kinetic equation (2.5), the Maxwell equations, and the equation for the current density (2.8), we can write the following equation for the force acting on the lattice:

$$f_i = \frac{\partial}{\partial r_k}\int d\tau_p \lambda_{ik}(\mathbf{p}) F(\mathbf{p}, \mathbf{r}) + \frac{1}{c}[\mathbf{jH}]_i + \frac{m_0}{e}\dot{j}_i. \tag{2.20}$$

Here the first term is due to the interaction of the electrons with the strain field, the second is a force of electromagnetic origin, and the third is due to the Tolman–Stewart effect.

The power absorbed per unit volume can be written

$$P = \langle \int d\tau_p \dot{\varepsilon}(\mathbf{p}, \mathbf{r}) F(\mathbf{p}, \mathbf{r}) \rangle, \tag{2.21}$$

where the angle brackets denote averaging over time (over the period of the sound wave) or over space (over the wavelength). The sound absorption coefficient is thus[5] [see (1.2)]

$$\Gamma = \frac{P}{S} = \frac{1}{S}\left\langle \int d\tau_p \dot{\varepsilon} F \right\rangle. \tag{2.22}$$

The general scheme for analyzing sound propagation in a metal thus reduces to an analysis of the equations of elastic theory, the electrodynamic equations for the electromagnetic fields which arise, and the kinetic equation for the electrons in the field of the sound wave and in the electromagnetic field.

3. ABSORPTION OF LONGITUDINAL SOUND IN METALS IN THE ABSENCE OF EXTERNAL MAGNETIC FIELDS

(a) Linear theory[14,15]

We begin with the simplest case, that of longitudinal sound. Since the electron energy is an even function of the electron quasimomentum, the part of the strain energy which is odd in v_x must also be odd in $\mathbf{v}_\perp[\mathbf{v}_\perp = (v_y, v_x)]$. The strain correction to the electron energy can thus be written

$$\lambda_{ik}(\mathbf{p})u_{ik} = \lambda^+_{ik}(\mathbf{v})u_{ik} + m_0 D(\mathbf{v})\left(v_x\frac{\partial}{\partial x}\right)(\mathbf{v}_\perp \mathbf{u}), \tag{3.1}$$

where $\lambda^+_{ik}(\mathbf{v})$ and $D(\mathbf{v})$ are even functions of their argument, λ^* is of the order of the atomic energy, and the dimensionless quantity D is of the order of unity (as before, we are assuming that the OX axis is parallel to the sound propagation direction).

In the case of longitudinal sound ($\mathbf{u}\|\mathbf{q}$), only the first term in (3.1) is nonzero:

$$U \equiv \lambda^+_{ik}(\mathbf{v})u_{ik}(\mathbf{r}). \tag{3.2}$$

It can be shown directly[15] that no macroscopic electric currents arise in this case.

Linearizing the kinetic equation (2.5), we set

$$F(\mathbf{p}, \mathbf{r}) = F_0(\varepsilon(\mathbf{p}, \mathbf{r})) + f(\mathbf{p}, \mathbf{r}), \tag{3.3}$$

where f is a small increment in the equilibrium electron distribution which depends on the total energy[6] $\varepsilon(\mathbf{p}, \mathbf{r}) = \varepsilon_0(\mathbf{p}) + U$.

We assume that the displacement in the sound wave varies in space and time in accordance with

$$\mathbf{u} = \mathbf{u}_0 \exp[i(qx - \omega t)]. \tag{3.4}$$

[5] This equation can be derived in a different way—from the dispersion relation for a sound wave which follows from the equations of the theory of elasticity (2.19), to which the force in (2.20) has been added.

[6] It can be seen that the Tolman-Stewart fields do not affect the absorption of short-wavelength longitudinal sound if small terms w/v_F are ignored.[10]

Then the linearized kinetic equation can be written

$$\left[i(qv_x-\omega)+\frac{1}{\tau}\right]f=\left(-\frac{\partial F_0}{\partial\varepsilon}\right)\dot{U}. \tag{3.5}$$

Here we are using the relaxation-time approximation for the collision operator I, and this approximation will be justified below.

Substituting (3.3) and (3.5) into the equation for the absorbed power (2.21), and averaging, we find

$$P=\frac{\omega^2}{2}\int d\tau_p\left(-\frac{\partial F_0}{\partial\varepsilon}\right)\frac{\tau^{-1}U_0^2}{(qv_x-\omega)^2+\tau^{-2}}, \tag{3.6}$$

where U_0 is the amplitude of U.

Below we will be interested in the case $ql\gg 1$. Restricting the discussion to the lowest order in the parameter $(ql)^{-1}$, we should let τ approach infinity. As a result, the factor $(\tau^{-1}[(qv_x-\omega)^2+\tau^{-2}]^{-1}$ becomes $\pi\delta(qv_x-\omega)$. Making this replacement, and dividing by the acoustic energy flux density, we find the following equation for the absorption coefficient:

$$\Gamma=\frac{\pi\omega}{\rho w^2}\int d\tau_p\left(-\frac{\partial F_0}{\partial\varepsilon}\right)\delta(v_x-w)\,[\lambda^+_{xx}(p)]^2. \tag{3.7}$$

Let us analyze this result, which was first derived in this form by Akhiezer *et al.*[15] As we will see below, the factor $(-\partial F_0/\partial\varepsilon)$ is also present in the equation for the absorption given by the nonlinear theory, and it means that only those electrons contribute to sound absorption which are in the region in which the Fermi distribution is smeared out, i.e. within an energy interval of width of the order of T near the Fermi level. This conclusion is not peculiar to the acoustic case; it is a characteristic result for all kinetic phenomena in metals, except high-frequency phenomena. Then in Eq. (3.7) we can transform from an integration over a volume in p space to an integration over the Fermi surface. Denoting the area element of this surface by ds_F, we write

$$\int\frac{2d^3p}{(2\pi\hbar)^3}\left(-\frac{\partial F_0}{\partial\varepsilon}\right)\to\frac{2}{(2\pi\hbar)^3}\int\frac{ds_F}{v}. \tag{3.8}$$

The factor $\delta(v_x-w)$ is more interesting. This factor means that among the electrons in the region in which the Fermi distribution is smeared out the only ones which participate in the absorption are those for which the x component of the velocity is equal to the sound velocity w. Since the absolute value of the electron velocity is the Fermi velocity, it is clear that the absorption of sound is due to those electrons whose velocities are nearly perpendicular to the propagation direction of the sound: a "belt" on the Fermi surface.

The physical reason for this selectivity in terms of electron velocity is as follows: those electrons for which v_x is quite different from w "perceive" the rapidly oscillating field of the sound wave as they move; on the average over many periods, this rapidly oscillating field has very little effect on them. As a result, the only electrons which interest strongly with the wave are those velocities satisfy the *resonance condition* $v_x=w$. Analyzing (3.6), in which we have not yet taken the limit $\tau\to\infty$, we see that the velocity integral contributing to absorption is finite, given in order of magnitude by

$$\Delta v_x\sim\frac{1}{q\tau}. \tag{3.9}$$

As τ increases, this resonant velocity interval becomes narrower, and the resonance itself becomes sharper; ultimately, Eq. (3.7) becomes independent of τ. For this reason, the sound absorption in this situation is called "collisionless." This absorption is analogous to the collisionless damping of plasma waves studied by Landau.[16] This analogy will be discussed in detail below, but at this point we are interested in some order-of-magnitude estimates of the linear collisionless damping Γ_0. Assuming $\lambda^+\sim\varepsilon_F$ for typical metals, we have

$$\Gamma_0\sim\frac{\omega\varepsilon_F n_0}{\rho w^2 v_F}, \tag{3.10}$$

where n_0 is the electron density.[7)] The quantity ρw^2, with the dimensions of an energy density, is of the order of $\varepsilon_F n_0$, while for the dimensionless ratio Γ_0/q we obtain

$$\frac{\Gamma_0}{q}\sim\frac{w}{v_F}\ll 1. \tag{3.11}$$

This number is of the order of a few thousandths. In other words, the absorption length Γ_0^{-1}, over which the sound intensity is reduced by a factor of e, is of the order of hundreds of acoustic wavelengths.

We also note that the smallness parameter in the estimate (3.11) is none other than $\sqrt{m/M}$, where m is the electron mass, and M is the mass of the ions making up the crystal lattice. The appearance of this parameter is not accidental. In the adiabatic approximation, the electrons "track" the ions, and there is no sound absorption. This parameter is also a measure of the deviation from adiabatic behavior, which determines the absorption.

In typical current experiments absorption lengths of the order of a fraction of a centimeter are being measured. This value is governed by the characteristic lengths of the samples used in the experiments. The mean free path $l=v_F\tau$ and the absorption length Γ^{-1} are thus usually related by

$$\frac{1}{\Gamma}\gg l. \tag{3.12}$$

This condition will be extremely important in the derivation of the nonlinear theory below.

Is the relaxation-time approximation legitimate for calculating the absorption? It is not difficult to see[8)] from Eqs. (3.5) and (3.6) that the increment f in the equilibrium distibution function is confined to a narrow resonant interval of velocities, with a width $\sim 1/q\tau$. Accordingly, it can be seen from the equation for the collision integral (2.7) that in the resonant region in which we are interested the effect of the "incoming" term on f corresponds to processes by which electrons enter a state with a given velocity v from a velocity region occupying a small volume in phase space. On the other hand, the "outgoing" term has an effect on f which incorporates the transition from the state with velocity v to *all* other states. The incoming term in (3.5) can thus be ignored on the basis of the parameter $v_F/q\tau = 1/ql$.

[7)] This equation can also be written in the form $[\Gamma\sim\omega\hbar^{-3}\rho^{-1}w^{-2}(\lambda^+)^2]$, from which it is clear that the absorption does not depend on the electron density (at densities for which $v_F\gg w$).

[8)] These arguments are analogous to those used in Ref. 17.

(b) Nonlinear theory[18,19]

Let us take a qualitative look at the events which occur in a semiconductor as an intense sound wave propagates through it. The sound wave is accompanied by a longitudinal wave that acts on the electrons with an effective field characterized by the potential U. As mentioned above, for the case of short-wave-length sound ($ql \gg 1$) this wave has a significant effect on only those electrons which are from the "resonant" region, which occupies a small volume in phase space. Let us determine the structure and characteristic dimension of this region.

The resonant electrons are divided in turn into "trapped" and "untrapped" electrons. The trapped electrons oscillate in the periodically situated potential wells formed by the sound wave. In the coordinate system moving with the traveling wave, their motion is finite. The depth of each potential well is of the order of the potential amplitude U_0; the typical oscillation velocity is

$$\tilde{v} \sim \sqrt{\frac{U_0}{m}}; \tag{3.13}$$

and the typical oscillation frequency is

$$\omega_0 \sim \tilde{qv} = q\sqrt{\frac{U_0}{m}}. \tag{3.14}$$

Each scattering event significantly changes the direction of the electron velocity and thereby removes this electron from the resonant group. Accordingly, if we wish to assert, for example, that the trapped electrons are executing oscillations in the potential wells of the wave, we must satisfy the inequality

$$\omega_0 \tau \gg 1. \tag{3.15}$$

This is the condition for the formation of a group of trapped electrons. At the same time, the distribution function of the untrapped electrons is greatly distorted in the resonant velocity region near the velocity w, with a width on the order of $\tilde{v}$. As a result of all these factors, there are nonlinear effects in the sound absorption.

If $w_0\tau \ll 1$, then no group of trapped electrons forms, and the width of the resonant region is governed by collisons; it is given in order of magnitude by $1/q\tau$, as mentioned above. In this case the linear theory for sound attenuation is valid.

In summary, the quantity $w_0\tau$ is a parameter which determines the role played by nonlinear effects at large values of ql. Since

$$\omega_0\tau \sim ql\sqrt{\frac{U_0}{\varepsilon_F}}, \tag{3.16}$$

there is a broad range of sound intensities in which the ratio U_0/ε_F is low, while nonlinear effects are already pronounced. A group of trapped electrons does form in this case, but it has only relatively few electrons. The nonlinear effects in the absorption, on the other hand, are already large, since the distribution function is greatly distorted just in this resonant region, and it is this resonant region which causes the absorption.

The nonlinearity parameter of the problem can also be estimated in a different way, from the balance equation for the (quasi-) momentum of the resonant electrons. As the sound is attenuated, these electrons acquire a momentum of the order of

$$\frac{\Gamma S}{w}. \tag{3.17}$$

If the nonlinear effects are to be weak, the momentum transfer by the sound wave should not significantly change the average x momentum of the resonant electrons, which is of the order of δnmw, where δn is the characteristic density of resonant electrons. We thus find the following condition for a slight nonlinearity:

$$\frac{\Gamma S\tau}{w} \ll \delta nmw. \tag{3.18}$$

We have seen that in the linear theory the width of the resonant region is of the order of $(ql)^{-1}$ with respect to the thickness U_0 of the layer at the Fermi surface. We thus have $(\delta n \sim n_0(U_0/\varepsilon_F)(ql)^{-1}$. Substituting this estimate into (3.18), we again find the inequality $w_0\tau \ll 1$ as the condition for the applicability of the linear theory, as expected.

Using similar arguments, we can estimate the asymptotic behavior of $\Gamma(S)$ for the case of a prounounced nonlinearity, in which case condition (3.15) holds. This condition means that the characteristic width of the interaction region in velocity space is on the order of $\tilde{v}$, so that the relative width is of the order of $\tilde{v}/v_F$. Thus $(\delta n \sim n_0(\tilde{v}/v_F)U_0/\varepsilon_e)$. If we also require that the rate at which momentum is transferred to the resonant particles be equal to the rate at which it is dissipated, we find

$$\frac{\Gamma(S)}{\Gamma_0} \sim \frac{1}{\omega_0\tau} \sim S^{-1/4}. \tag{3.19}$$

To get a better understanding of the nature of this nonlinearity, let us compare it with the well-understood case of nonlinear absorption of plasma waves in a collisionless plasma. (As we have already pointed out, the *linear* collisionless attenuation of sound is analogous to the Landau damping of plasma waves.)

In both cases the wave effectively interacts with only a small group of resonant particles, which occupies a small volume in phase space.[9)] There is an important distinction, however: In a collisionless plasma, the mean free path of the particles is much larger than all the other characteristic dimensions of the problem, in particular, the linear damping length [cf. (3.12)]. Thus there actually is no coupling of the resonant particles with the other particles (or with the heat reservoir), and in solving the problem of plasma-wave propagation it is sufficient to consider only the energy balance in the system consisting of the wave and the resonant particles, and this is a *conservative* system.

If the original wave amplitude is low, a wave which has been completely damped, i.e., which has transferred all its energy to resonant particles, does not significantly distort their motion. This situation cor-

[9)] Quantitatively, the groups of resonant particles are selected differently for sound and plasma waves, because of the different ratios of the wave phase velocity and the characteristic velocity of the particles.

responds to linear Landau damping.[16]

If, on the other hand, the plasma wave has a sufficiently high amplitude, the limited capacity of the resonant-particle "reservoir" will make itself apparent, and the wave will significantly distort the distribution of these particles. An equilibrium will ultimately be established in the system consisting of the wave and the resonant particles: the wave amplitude will reach a certain steady-state value, and a plateau will form on the distribution of the resonant particles. The process of establishing this equilibrium is accompanied by oscillations in the wave amplitude.

The problem of nonlinear Landau damping of plasma waves is thus definitely not a steady-state problem. The characteristic nonlinearity parameter in this case is the ratio of the "reservoir-saturation" time (a time of the order of the oscillation period of the trapped particles) to the shorter of two times, the linear damping time and the observation time. The problem of the nonlinear Landau damping of plasma waves has been solved quantitatively by O'Neil[20] (see also the reviews of Refs. 21 and 22).

In the case of a sound wave, collisions are important because of the condition $\Gamma l \ll 1$. As the collisions remove an electron from the resonant group, they cause an exchange of particles between the resonant group and the rest of the electron system. As a result of this particle exchange, the energy acquired from the sound wave by the resonant electrons over a time of the order of the elastic relaxation time τ_p is distributed over the entire electron system and then transferred through inelastic collisions to the heat reservoir. We are thus dealing with a *nonconservative system* consisting of the wave, the resonant electrons, all the electrons, and the heat reservoir. The presence of elastic and inelastic scattering allows us to treat the problem as the steady-state problem of absorption of the sound wave, without going into the processes involved in the establishment of equilibrium, of the type mentioned above.

By virtue of the condition $ql \gg 1$, however, the collisions are still rather infrequent; the removal of energy from the resonant group of electrons is hindered; and this step acts as a "bottleneck." Accordingly, the distribution of resonant particles begins to depend on the wave amplitude when this amplitude is high. This is the basic distinction between the nonlinear and linear cases. A quantitative measure of the difference is the ratio of the saturation time of the resonant group (which is of the order of ω_0^{-1}, as for plasma waves) to the time for removal of energy from the resonant group (the width of the "bottleneck"). As we have seen, this latter time is on the order of τ_p.

Before taking up the quantitative theory for nonlinear absorption of sound, we wish to draw attention to an important circumstance.

The nonlinear terms due to the electron response serve as driving forces in the elastic-theory equations (2.19) and (2.20). These forces drive higher-order acoustic harmonics. If the frequency and wave vector of the driving force are coupled by the dispersion law for free waves, a wave resonance occurs. If, on the other hand, there is a deviation from resonance because of the dispersion of the sound velocity, then the amplitude of the steady-state oscillation will be proportional to the ratio of the amplitude of the driving force to the deviation from resonance or, more briefly, the ratio of the nonlinearity to the dispersion.

In the linear approximation in u_{ik}, the force f exerted on the lattice by the electrons has the spatial and temporal periodicity of the sound wave, but in general its phase is not the same as that of the elastic force, which is governed by the right side of Eq. (2.19). If we resolve this force into two components, a reactive component which varies in phase with the elastic force, and an active component, which is $\pi/2$ out of phase, then the reactive part is due to nonresonant electrons. Its role is one of causing an adiabatic renormalization of the elastic moduli. The amplitude of the active part is lower than the amplitude of the reactive part by a factor of at least w/v_F. The active part of the force causes the absorption of sound. Under nonlinear conditions, a contribution f_N which is nonlinear in u_{ik} appears in addition to the part of the force which is linear in u_{ik}, f_L. It can be shown[19] that the amplitude of this nonlinear contribution is always much less than the amplitude of f_L. The ratio of these amplitudes is governed by the parameters $U_0/\varepsilon_F \sim u_{ik} \ll 1$ for nonresonant electrons and $(\omega_0\tau)^{-1}\omega/v_F \ll 1$ for renonant electrons ($\omega_0\tau \gg 1$). Accordingly, in the case $\omega_0\tau \gg 1$, we have

$$\frac{f_N}{f_L} \sim \max\left(\frac{U_0}{\varepsilon_F}, \frac{1}{\omega_0\tau}\frac{w}{v_F}\right). \tag{3.20}$$

The contribution of the resonant particles to f_N at the different harmonics are generally comparable in magnitude,[10)] while the contributions of the nonresonant particles are proportional to $(u_{ik})^n$ and fall off with increasing harmonic index.

The dispersion mentioned above is characterized by the difference $\omega(n\omega) - \omega(a)$ [$\omega(\omega)$ is the velocity of a free sound wave of frequency ω]. In typical metals under the conditions assumed here, the electron contribution to the dispersion law for the sound does not lead to a frequency dependence of the sound velocity w. Then in the absence of nonelectronic mechanisms which could lead to a dispersion of w, higher-harmonic waves appear in the metal at amplitudes which increase in proportion to the distance from the boundary within a short distance from the boundary (short in comparison with Γ^{-1}). The ratio of the amplitude of the n-th harmonic to the amplitude of the fundamental component in this case is given in order of magnitude by

$$qx\frac{f_N^{(n)}}{f_L}, \tag{3.21}$$

where $f_N^{(n)}$ is the amplitude of the n-th harmonic of the nonlinear contribution to the force. This growth continues to a length of the order of the damping length Γ^{-1}, beyond which the harmonic amplitudes fall of along with that of the fundamental wave. Under typical con-

[10)]Within numerical coefficients associated with the expansion in a Fourier series and falling off with increasing harmonic index.

ditions for typical metals we have $(U_0/\varepsilon_p \ll (1/\omega_0\tau)w/v_F)$; in other words, the resonant particles are primarily responsible for the formation of the higher harmonics. Taking this circumstance into account, and also using (3.20) for the ratio f_N/f_L and (3.19) for the nonlinear absorption coefficient Γ, we easily see that the ratio in (3.21) can reach unity. Under nonlinear conditions, the shape of the sound wave in a metal at a distance from the boundary of the order of the nonlinear damping length can be quite different from sinusoidal.[11)]

It is important to note, however, that the change in shape occurs over a distance of the order of $\Gamma^{-1} \gg l$ [we are assuming condition (3.12)]. Then in treating the problem of the electron distribution in the wave field, we can assume that the shape of the wave is known, even if it is not sinusoidal. Under this assumption, we can calculate the electron distribution and then use this distribution to calculate the force. Vugal'ter and Demikhovskiĭ[23] and Gal'perin and Kozub[24] have analyzed the change in the shape of a wave in a metal. We will not reproduce the results of that analysis here; instead we will focus on a calculation of the absorption coefficient and its variation with the sound intensity.

The quantitative analysis of the nonlinear absorption is based on the classical kinetic equation (2.5): As mentioned above, the classical description is valid if the momentum transfer $\hbar q$ is lower than the typical value of the electron momentum for the problem, $\bar{p}$. For the present problem, this typical momentum is the larger of $m/q\tau$ and $\sqrt{mU_0}$. The classical description is thus valid under one of the following two inequalities:

$$\frac{\hbar^2q^2}{m} \ll U_0, \tag{3.22}$$

$$\frac{\hbar^2q^2}{m} \ll \frac{\hbar}{\tau}. \tag{3.23}$$

These inequalities have an extremely clear physical meaning. The first says that there are many quantum levels in the potential wells formed by the sound. The second says that the uncertainty in the electron energy due to the collisions is so great that it would be inconsistent to take the term $\hbar^2q^2/m$ into account.

In the opposite limit,

$$\frac{\hbar^2q^2}{m} \gg U_0 \gtrsim \frac{\hbar}{\tau}, \tag{3.24}$$

there are very few levels in the potential wells produced by the sound. The possibility of tunneling between different wells causes these levels to expand into the "acoustic bands" which were first studied by Keldysh.[25] The nonlinear absorption in the case (3.24) was analyzed by Zil'berman[26] and, for the case in which acoustic bands form, by Laĭkhtman and Pogorel'skiĭ.[27]

The nonlinear part of the problem thus reduces to the solution of the kinetic equation in a given field $U(\mathbf{r}, t)$.

[11)] In semiconductors, as we shall see, the electron contribution to the dispersion law for the sound velocity w begins to depend on ω, and in most cases the distortion of the wave shape is slight as long as $U_0 \ll \bar{\varepsilon}$, where $\bar{\varepsilon}$ is the characteristic electron energy.

Since the attenuation length Γ^{-1} is much longer than both the acoustic wavelength and the electron mean free path [by virtue of inequality (3.12)], it can be assumed that the potential U and the solution which we are seeking for the kinetic equation depend only on the difference $x - wt$. We can thus find a local electron distribution and a local sound attenuation coefficient Γ as functions of the sound intensity S. The intensity distribution along the coordinate x can then be found by solving Eq. (1.2), with $S(0)$ specified as a boundary condition.

As in the linear theory, we write the distribution function in the form

$$F_0[\varepsilon_0(\mathbf{p}) + U] + f(\mathbf{p}, x - wt). \tag{3.25}$$

For the function $f(x)$ we find the following kinetic equation:

$$(v_x - w)\frac{\partial f}{\partial x} - \frac{\partial U}{\partial x}\frac{\partial f}{\partial p_x} + \hat{I}f = w\frac{\partial U}{\partial x}\frac{\partial F_0}{\partial \varepsilon}. \tag{3.26}$$

As in the linear theory, we will use the relaxation-time approximation in calculating the absorption, setting $\hat{I}f_p = (1/\tau)f_p$. This approximation is justified in detail for the nonlinear case in Ref. 19. We take into account the fact that electrons with low longitudinal velocities v_x interact with the sound effectively. It can thus be assumed that the quantity τ^{-1} depends only on the transverse components of the electron velocity. In contrast, the distribution function f is a strong function of v_x in the resonant region. It can thus be assumed that

$$\frac{\partial f}{\partial p_x} = m_{xx}^{-1}\frac{\partial f}{\partial v_x}, \tag{3.27}$$

in other words, we can ignore the derivatives $\partial f/\partial v_y$ and $\partial f/\partial v_z$ in comparison with $\partial f/\partial v_x$ (it can be seen that the relative error introduced by this simplification is of the order of $v/v_F \sim \sqrt{U_0/\varepsilon_F} \ll 1$). Accordingly, Eq. (3.26) is one-dimensional. It can be solved by the method of characteristics. The system of equations for these characteristics is

$$\frac{dx}{v_x - w} = \frac{dv_x}{m_{xx}^{-1}\partial U/\partial x} = -\frac{df}{(1/\tau) - w(\partial U/\partial x)\partial F_0/\partial \varepsilon}. \tag{3.28}$$

The characteristics in (3.28) for Eq. (3.26) describe the particle trajectories. The first integral of system (3.28) is an energy integral in the coordinate system moving with the wave:

$$\frac{m_{xx}(v_x - w)^2}{2} + U(x) = E = \text{const}. \tag{3.29}$$

The general solution of Eq. (3.26) is

$$f = w\int_C^x \frac{dx'}{v_x - w}\frac{\partial U}{\partial x'}\frac{\partial F_0}{\partial \varepsilon}\exp\left(-\frac{1}{\tau}\int_{x'}^x \frac{dx''}{v_x - w}\right), \tag{3.30}$$

where

$$v_x - w = \pm\sqrt{2m_{xx}^{-1}(E - U(x))}. \tag{3.31}$$

The classification of electrons as either trapped or untrapped, discussed above, arises here in a graphic manner: for the untrapped particles, we have $E > U_{max}$, while for the trapped particles we have $U_{min} < E < U_{max}$. The equation for the distribution function in (3.30) should be supplemented with boundary conditions which determine the constant C. These conditions are differ-

ent for the trapped and untrapped electrons. For the untrapped electrons we have periodic boundary conditions,

$$f_{\pm}(x) = f_{\pm}(x + \lambda) \tag{3.32a}$$

(here the subscripts $\pm$ correspond to the sign of the difference $v_x - w$).

The trajectories of the trapped electrons have two turning points, which correspond to the roots of the equation $U(x) = E$. We denote the left-hand and right-hand turning points nearest the point x by x_1 and x_2. Since the electrons are reflected at these points from the walls of the potential well created by the wave, we require the following conditions for the trapped particles:

$$f_{\pm}(x_1) = f_{\mp}(x_1),\ f_{\pm}(x_2) = f_{\mp}(x_2). \tag{3.32b}$$

Equation (3.30), with the boundary conditions in (3.32a) and (3.32b), constitutes the complete solution of the problem. It can be seen from (3.30) and (3.31) that the nonlinearity is characterized, as expected, by the parameter

$$\omega_0\tau \equiv q\tilde{v}\tau,$$

where $\tilde{v} = \sqrt{m_{xx}^{-1}U_0}$. The characteristic values of x are of the order of q^{-1}, while those of the difference $v_x - w$ are of the order of $\sqrt{m_{xx}^{-1}U_0}$. In the argument of the exponential function in Eq. (3.30) we thus have a quantity of the order of $(\omega_0\tau)^{-1}$. It can be seen that in the case $\omega_0\tau \ll 1$ Eqs. (3.30) and (3.32a) yield the results of the linear theory and the corrections of order $\omega_0\tau$ to the linear theory which were derived in Ref. 18. In the opposite case, that of a pronounced nonlinearity, $\omega_0\tau \gg 1$, the solution can be expanded in powers of $(\omega_0\tau)^{-1}$, with only the first term being retained. This procedure leads to[19]

$$\frac{\Gamma(S)}{\Gamma_0} = \alpha_1 \frac{\langle (\lambda_{xx}^{+})^2 (\omega_0\tau)^{-1} \rangle_{\perp}}{\langle (\lambda_{xx}^{+})^2 \rangle}, \tag{3.33}$$

where the angle brackets denote the average over the "belt" of the Fermi surface $v_x = w$, and α_1 is a number (of the order of unity) which depends only on the shape of the potential. This latter dependence is weak, and in analyzing the propagation of sound we can treat α_1 as a constant. For a wave of sinusoidal shape its value is[19] $\alpha_1 = 1.1$.

This quantitative calculation thus confirms the order-of-magnitude estimate above which lead to (3.19). We see that in the nonlinear case there is an "acoustic brightening" of the conductor: the nonlinear absorption coefficient Γ is smaller than the linear coefficient Γ_0 by a factor $\omega_0\tau \gg 1$. Here $\Gamma \sim S^{-1/4}$; for a given intensity, the nonlinear absorption coefficient is independent of the sound frequency.

Nonlinear effects of this nature were first observed experimentally by Ivanov *et al.*[28] in a piezoelectric n-type InSb semiconductor. We will discuss the experimental work of Refs. 28 and 29 in the next section, which deals with the particular features of nonlinear effects in semiconductors.

Since the mechanism for the nonlinearity with which we are concerned here involves a distortion of the momentum distribution of the resonant electrons, this mechanism has been called the "momentum nonlinearity."[29] The momentum nonlinearity has also been observed by Fil' *et al.*[30] in ultrapure Ga, with and without a magnetic field.

It can be seen from (3.33) that the ratio $\Gamma(S)/\Gamma_0$ depends on the "outgoing" relaxation time τ_p, which can, in general, be different from the transport time τ_{tr}, which appears, for example, in the equation for the electrical conductivity. Analyzing the asymptotic behavior of the ratio $\Gamma(S)/\Gamma_0$, we can thus obtain interesting information on the mechanisms for the scattering of the current carriers in a conductor, since the ratio τ_p/τ_{tr} can take on quite different values for different scattering mechanisms.

The important point for this analysis of nonlinear absorption of longitudinal sound was the fact that the effective field acting on the electrons is a potential field, and not the particular nature of the polarization of the sound. Since the field is a potential field, it is possible to work simply from the condition of electrical neutrality, avoiding the use of Maxwell equations.

In the case of sound with other than longitudinal polarization, the first term in (3.1), which does not cause the appearance of eddy currents, corresponds to longitudinal fields, so that its contribution to the absorption is described by the theory above, regardless of the polarization.

The second term in (3.1), however, which appears when the sound has other than the longitudinal polarization, causes eddy currents and, by virtue of Maxwell equations, vortical electromagnetic fields. For sound of this type the question arises of the role played by these currents and fields in nonlinear absorption.[12)]

From the linear theory[31,32,11] we know that for transverse sound with a wavelength greater than the skin depth in the case of the anomalous skin effect the electromagnetic absorption (due to the Joule loss during the flow of these eddy currents) can, in general, be of the same order of magnitude as the purely deformational absorption [due to the first term in (3.1)], so that this electromagnetic absorption can be important.

How does electromagnetic absorption change the physical picture of the nonlinearity presented above?

We first note that again in the case of electromagnetic absorption there is a distinct resonant group of electrons, which make the major contribution. We have already seen that in the case of longitudinal sound the effect of the effective longitudinal field on the motion of these electrons leads to nonlinear absorption. The transverse electromagnetic fields, however, also affect the electron motion, by virtue of the Lorentz force. Since the resonant electrons are moving nearly perpen-

[12)] This question is pertinent only for typical metals. In semiconductors, the carriers occupy a small volume in p space, so it turns out that the ratio of the second term in (3.1) to the first is always small.

dicular to the sound wave vector q, the Lorentz force acting on these electrons is essentially parallel to q. The sign of this force depends on both the direction of the electron velocity and the wave phase. In summary, the effect of the wave on the motion of the resonant electrons corresponds to the one-dimensional picture. The force exerted on a particle by the vortical field behaves in this sense in the same way as the force due to the potential field in the case of longitudinal sound. We can thus expect that the physical picture of the nonlinearity (and, in particular, the value of the nonlinearity parameter) presented above also holds in this situation. The only change we have to make is to replace the potential force $-qU$ by the Lorentz force in the equation for ω_0.

The equation for the absorption coefficient, however, is quite different from the corresponding equation in the case of the purely deformational absorption. The electromagnetic absorption is a quadratic function of the components of the vortical electric field, whose coefficients are the elements of the conductivity tensor. The distribution of resonant frequencies directly affects the effective conductivity. Furthermore, this distribution affects the magnitude of the vortical electric fields, which determine the Joule loss. These fields are in turn governed by the degree of shielding of the "seed" eddy currents, and this shielding itself depends on the effective conductivity.[13)] On the one hand, the nonlinear decrease in the effective conductivity (by a factor of $\omega_0\tau_p$ in the case $\omega_0\tau_p \gg 1$) for a given field level leads to a decrease in the absorption. On the other hand, a decrease in this quantity also weakens the shielding and causes a corresponding increase in the magnitude of the vortical field. The behavior of $\Gamma(S)$ is thus more complicated than in the case of deformational absorption; in particular, it can be nonmonotonic.

A nonlinear theory for electromagnetic absorption was derived in Ref. 33. Here we will give the result for only the most interesting case, that in which the wavelength is much larger than the skin depth in the case of the anomalous skin effect, $\delta(q\delta \gg 1)$. In this case, in the linear region, the electromagnetic contribution to the absorption is generally of the same order of magnitude as the deformational contribution. In the nonlinear region, on the other hand, the electromagnetic contribution becomes definitely predominant, and it determines the total absorption. The behavior of $\Gamma(S)$ is nonmonotonic: at $S \ll S_0$ we have $\Gamma \sim \omega^3 S^{1/2}$ (i.e., Γ increases with increasing intensity S), while at $S \gg S_0$ we have $\Gamma \sim \omega^3 S^{-1/4}$. Here S_0 is a characteristic value of the sound intensity, given by

$$S_0 = \rho\omega^3 (ql)^{-4} (q\delta)^{-6}. \tag{3.34}$$

In the first region, the leading role is played by the nonlinear decrease in the shielding of the vortical fields, while in the second region the leading role is played by the nonlinear behavior of the effective conductivity, which appears directly in the equation for the Joule loss.

For $q = 2 \cdot 10^4$ cm^{-1}, $l = 10^{-1}$ cm, and typical metallic properties, S_0 is 0.2 W/cm^2. The nonmonotonic variation of $\Gamma(S)$ can thus be observed with the experimental apparatus presently available.

4. PARTICULAR FEATURES OF NONLINEAR ABSORPTION AND AMPLIFICATION OF SOUND IN SEMICONDUCTORS

The current carriers in semiconductors, in contrast with metals, occupy regions of small volume in p space near the extrema of the energy bands. Correspondingly, the strain energy tensor in semiconductors with a simple band can be assumed to be independent of p. On the other hand, since the number of carriers in a semiconductor is small in comparison with that in a metal, electrical neutrality generally does not hold, and there can be a local redistribution of space charge in the field of the sound wave. For this reason, the interaction of the electrons with the sound wave, $\lambda_{ik}u_{ik}$, which is independent of the quasimomentum p, is not completely shielded, as it would be in metals. In semiconductors the shielding is only partial. The actual degree of shielding is governed by the value of the parameter qR_D, where R_D is the Debye-Hückel radius in the case of a nondegenerate semiconductor or the Thomas-Fermi radius in the case of a degenerate semiconductor. The problem of sound absorption in a semiconductor thus differs from that in a metal in that the Poisson equation instead of the electrical neutrality condition must be analyzed to determine the effective potential acting on the carriers.[14)]

The most important point is that the interaction through the strain potential is not the only interaction or even the most important interaction in many semiconductors. Semiconducting properties are exhibited by many piezoelectric crystals, in which electric fields proportional to the strain arise when the crystals are deformed. The piezoelectric interaction is very anisotropic; i.e., it is very sensitive to the polarization and propagation direction of the sound. In the piezoelectrically active directions, the piezoelectric interaction can be much stronger than the deformation interaction. In these directions the electron absorption of the sound in piezoelectric semiconductors can be many times greater than the lattice absorption.

Semiconductors may exhibit a new type of acoustic effect, which is not possible in metals. In contrast with a metal, it is possible to apply a strong electric field E which causes a carrier drift at the drift velocity

$$v = \mu E,$$

where μ is the carrier mobility (in the general case, a tensor). In strong electric fields the drift velocity v

[13)] We recall that in the case of longitudinal sound the shielding is governed by the response of the entire electron system, against whose background the contribution of the resonant electrons is small.

[14)] This assertion is not correct for so-called multivalley semiconductors such as n-type Ge and Si. To save space here, we will not examine the nonlinear effects which are peculiar to these materials and which lead, for example, to the formation of acoustic shock waves.[34]

can exceed a certain critical value, of the order of the sound velocity, at which the absorption coefficient for the sound will change sign; i.e., absorption will give way to amplification.[15] This amplification of sound by current-carrier drift is most interesting in piezoelectric semiconductors, because of the strong interaction of the electrons with the sound.

The amplification can raise the sound intensity by several orders of magnitude. The question of nonlinear acoustic effects thus originally arose in connection with the amplification of sound. It was necessary to examine these effects in order to answer an important question: up to what limit can the sound be amplified?

In the case of the piezoelectric interaction, the piezoelectric force

$$f_i^p = -\beta_{i,kl}\frac{\partial^2\varphi}{\partial r_k\,\partial r_l}, \tag{4.1}$$

where $\beta_{i,kl}$ is the piezoelectric-constant tensor, appears in the equations of the theory of elasticity. Furthermore, there is a piezoelectric increment in the electric displacement D,

$$D_i = \varepsilon_{ik}E_k + 4\pi\beta_{i,kl}u_{kl}, \tag{4.2}$$

which leads to a corresponding change in the Poisson equation. As a result, in first order in the wave amplitude we find the following equation for the increment in the electron energy:

$$U^e = U^b\frac{(qR_D)^2}{1+(qR_D)^2}, \tag{4.3}$$

where $U^b = 4\pi\beta_{x,kx}u_k$ for the piezoelectric interaction, and $U^b = \lambda_{xx}^* u_{xx}$ for the deformation interaction. This result is derived by ignoring terms of the order of $w/\bar{v}$ (where $\bar{v}$ is the characteristic electron velocity), which result from the time variation of the wave. In metals, $\bar{v}$ is the Fermi velocity; in semiconductors, depending on the carrier statistics, it is either the Fermi velocity or the average thermal velocity, $\sqrt{T/m}$. In either case, the ratio $w/\bar{v}$ is small, and we will adopt it as the small parameter of the theory.

At a low sound intensity, i.e., in the linear region, we find for the absorption coefficient a result which we already have, Eq. (3.8), except that $\lambda_{xx}^* u_{xx}$ must be replaced by U^e. As a result, for the piezoelectric interaction we have $\Gamma_0 \sim \omega^3$ for $q \ll R_D^{-1}$ and $\Gamma_0 \sim \omega^{-1}$ for $q \gg R_D^{-1}$ (Ref. 39b); in the case of the deformation interaction, the corresponding expressions are[16] ω^5 and ω.

For the analysis of the momentum nonlinearity in semiconductors, we assume

$$\frac{U^e}{\bar{\varepsilon}} \ll 1, \tag{4.4}$$

where $\bar{\varepsilon}$ is the characteristic electron energy. In metals, $\bar{\varepsilon} = \varepsilon_F$ is of the order of an electron volt, and condition (4.4) is always satisfied. In semiconductors, condition (4.4) becomes a limit on the sound intensity.[17]

Under condition (4.4), the change in the electron density in the wave field which is nonlinear in u_{ik} is small in comparison with the linear change. Then the density change can be assumed to be linear in u_{ik} in the Poisson equation. As a result, Eq. (4.3) for the effective potential retains its form.

On the other hand, the shielding of the interaction at $q \lesssim R_D^{-1}$ has the consequence that the force exerted on the lattice by the electrons is not proportional to the frequency ω, as it is in metals. The result is a dispersion of the sound velocity, which suppresses the higher harmonics when the inequality (4.4) holds. The entire analysis is thus precisely the same as in Section 3, except for the one simplification that the wave can be assumed to be sinusoidal here. As in the case of metals, Eq. (3.33) holds for the ratio $\Gamma(S)/\Gamma_0$. This equation is applicable even when Γ_0 is negative, i.e., even when the sound is amplified rather than absorbed.

It should be kept in mind that the amplitude of the electron potential energy U_0 is now governed by (4.3) and, in particular, may depend on the frequency, in contrast with the result for metals.

The mechanism for the momentum nonlinearity is the same in general features for semiconductors and metals. In semiconductors, however, in contrast with metals, there can be another nonlinearity mechanism, which can, generally speaking, compete with the one described above. This other mechanism is the heating of the electrons by the field of the sound wave. The characteristic electron energy $\bar{\varepsilon}$ in a semiconductor is small in comparison with that in a metal, where it is of the order of several electron volts. Furthermore, in semiconductors with a high mobility the energy relaxation time τ_ε can be long. In n-type InSb, for example, the ratio τ_ε/τ can reach 10^2–10^4 (depending on the temperature and purity of the sample), so even at low sound intensities there could presumably be large deviations of the electron energy distribution from the equilibrium distribution, i.e., a heating of electrons. This heating in turn makes the absorption dependent on the sound intensity.[18]

At what sound intensities should this heating be important? To answer this question, we compare the energy acquired by the resonant electrons with the energy transferred to the lattice through inelastic scattering. In the absorption of sound, the power absorbed per unit volume is ΓS, so that the power absorbed per electron is $\Gamma S/n_0$ (in this estimate we are dividing by the total electron density, since over the time τ_ε the absorbed energy is redistributed between the resonant electrons

[15] Sound amplification by current-carrier drift for the case $gl \ll 1$ has been studied in Refs. 35–37, etc. The case $gl \gg 1$, which is the case of interest here, has been studied in Refs. 38, 39, etc.

[16] The function $\Gamma(\omega)$ for $q \gg R_D^{-1}$ and for the deformation interaction is the same as in metals. This is a natural result, since the shielding is inconsequential if $q \gg R_D^{-1}$.

[17] Kagan[40] has derived a nonlinear theory for the absorption of sound under the conditions $gl \gg 1$ and $U_0/\varepsilon \gg 1$ for the case of electron scattering by phonons. Kagan reached the conclusion that the function $\Gamma(S)$ in this case is the same as in the case $gl \ll 1$ (Refs. 41 and 42).

[18] The heating of electrons by sound in the case $gl \ll 1$ was studied in Refs. 43, 44, etc.

and all the other electrons with the same energy). Clearly, if the electron energy distribution is to deviate only slightly from the equilibrium distribution, the energy absorbed over the time τ_ε must be much smaller than the characteristic energy $\bar{\varepsilon}$. We accordingly find the following condition for slight heating:

$$\frac{\Gamma S \tau_\varepsilon}{n_0 \bar{\varepsilon}} \ll 1. \tag{4.5}$$

If we use the estimate in (3.19) for Γ and express S in terms of U_0 in accordance with (4.3), we can rewrite (4.5) as

$$\left(\frac{U_0}{\bar{\varepsilon}}\right)^{3/2} \frac{\tau_\varepsilon}{\tau} \left(\frac{w}{\bar{v}}\right)^2 \ll 1. \tag{4.6}$$

As a rule, the product $(\tau_\varepsilon/\tau)(w/\bar{v})^2$ does not greatly exceed unity in order of magnitude, so that condition (4.6) is more or less equivalent to (4.4). Under this condition, the heating is unimportant.[19)] The nonlinearity of the absorption or amplification, on the other hand, can be marked: Since

$$\omega_0 \tau = ql \sqrt{\frac{U_0}{\bar{\varepsilon}}},$$

the condition for a pronounced nonlinearity, $\omega_0 \tau \gg 1$, can be satisfied at the same time as condition (4.4), because of the large value of the parameter ql.

Let us discuss some particular features of sound propagation under amplification conditions. As we have seen, the nonlinear amplification coefficient Γ falls off with increasing sound intensity. Then when the intensity reaches a certain S_0 in the course of the amplification, the amplification coefficient can become comparable to the absorption coefficient corresponding to lattice absorption, Γ_l (although $|\Gamma_0| > \Gamma_l$). Beginning at this point, a wave with a stationary amplitude S_0 can propagate in the crystal, and if the condition $U_0 \ll \bar{\varepsilon}$ holds then the wave is essentially sinusoidal (as mentioned above). This is a major distinction from the case $ql \ll 1$, in which the nonlinear stationary wave is definitely nonsinusoidal.[41,42]

To conclude this section we turn to the experimental results of Refs. 28 and 29, where a momentum nonlinearity was first observed in the amplification of sound at frequencies of 1–2 GHz in n-type InSb. In this frequency range the parameter ql is 5–10. The characteristics of the sample were chosen such that conditions (3.22) and (3.23) do not hold at low "input" sound intensities. Under this condition, the classical approach adopted above cannot be used, and quantum theory must be used to analyze the nonlinear absorption. As mentioned earlier, this problem is treated in Refs. 26 and 27. According to those papers, in the case $U_0\tau/\hbar \gg 1$ we have

$$\frac{\Gamma(S)}{\Gamma_0} \approx \frac{\hbar}{U_0 \tau} \sim S^{-1/2}. \tag{4.7}$$

As the sound intensity increases, however, condition (3.23) eventually begins to hold, and there should be a transition from the behavior $\Gamma \sim S^{-1/2}$ (quantum theory)

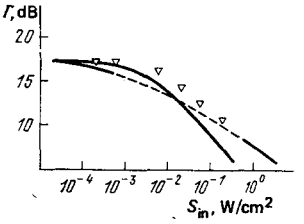

FIG. 1.

to the behavior $\Gamma(S) \sim S^{-1/4}$, predicted by the classical theory, according to the arguments above. Figure 1 shows the electron amplification over the length of the crystal as a function of the intensity of the sound which enters the crystal, taken from Ref. 28. At low intensities, the experimental points are described better by the quantum theory, while at high intensities they are described better by the classical theory. A more detailed comparison of these experiments and the theory has been made by Kozub.[46] We note that a transition to a steady-state sinusoidal wave, as mentioned above, was observed in Ref. 29.

5. NONLINEAR EFFECTS IN THE PROPAGATION OF SOUND IN A METAL IN AN EXTERNAL MAGNETIC FIELD[47,48,45]

The momentum nonlinearity takes an unusual form in the presence of an external magnetic field. Since the absorption of low intensity sound in magnetic fields has been studied in detail both theoretically and experimentally (see, for example, Ref. 49), we will simply outline the corresponding physical picture here.

We assume that a wave is propagating in a conductor in a nonquantizing magnetic field ($\hbar\Omega \ll T$, where Ω is the characteristic frequency of the periodic motion in the magnetic field), and we assume that we can speak in terms of a classical trajectory of the electron in the magnetic field. If the field is classically strong ($\Omega\tau \gg 1$), the electron manages to traverse many periods of the trajectory during the time between collisions. The trajectory can take different shapes, depending on the shape of the Fermi surface and on the experimental geometry. As an example, we show the projections of the trajectories onto a plane passing through the sound wave vector (Fig. 2).

Let us assume that the characteristic dimension of the trajectory, $R \sim v/\Omega$, is much larger than the sound wavelength. Clearly, the electron interacts effectively with the wave only on those parts of the trajectory where the projection of the electron velocity onto the sound wave vector is small (in the present case, these regions are the classical turning points along x). In these regions, the electron "senses" a certain wave phase for a long time interval. The contribution of the electron to the absorption is thus governed by the sum of the contribution of these regions over the part of the

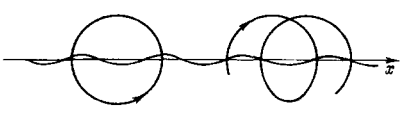

FIG. 2.

[19)] This is the case only in the absence of an external magnetic field. In a magnetic field, the heating can be more important, generally speaking, than the momentum nonlinearity.[45]

trajectory traversed during the time between collisions. The correlation of the wave phases corresponding to different regions of effective interaction is of course extremely important. This correlation, which is governed by the geometry of the trajectory and the strength of the magnetic field, leads to a nonmonotonic, oscillatory variation of the absorption with the field. The particular type of variation depends on the experimental situation: geometric oscillations,[50,31] magnetoacoustic resonance,[51] the tilt effect,[52] etc.

In weak fields ($\Omega\tau \ll 1$ or $l \ll R$), the electron manages to cover only a small part of the trajectory (a distance much shorter than a period) during the time between collisions. Each scattering event changes the direction of the particle's velocity, so the particle switches to another trajectory. The only contribution to the effective interaction is made by the motion of the electron on the parts of the trajectory near the classical turning points. Because of the random nature of the jumps during the scattering, there is no correlation between the different regions of this type. According to the linear theory, the sound absorption here is independent of the magnetic field, equal to its value at $H=0$ [if small corrections $\sim(\Omega\tau)^2$ are ignored].

If the sound intensity is increased, the effect of the sound wave on the electron motion in the regions of effective interaction becomes important, leading to a distortion of the trajectory near the turning points. The magnitude of this distortion can be seen by comparing the force exerted on the electron by the magnetic field, $\sim(e/c)v_F H$, with the force due to the effective field of the sound wave,[20] $\sim qU$. This comparison shows that the distortion is governed by the dimensionless parameter

$$b^{-1}=\frac{U_0}{\varepsilon}qR. \qquad (5.1)$$

These arguments can be illustrated by assuming that the electron velocity is nearly perpendicular to the sound wave vector in the regions of effective interaction, so that the force exerted by the magnetic field acts along the wave vector. Then on these parts of the trajectory the magnetic field behaves as some effective electric field which accelerates the electron along the x axis. As a result, the electron motion near the turning point x_0 can be described as the motion in some effective electric field $\sim U_F H/c$. Correspondingly, we can introduce the energy of the one-dimensional motion along the x axis, $F(x)$:

$$F(x)=\frac{mv_x^2}{2}+U(x)=\frac{ev_F H}{c}(x-x_0)+U(x_0) \qquad (5.2)$$

(Fig. 3). The slope of the lines in Fig. 3 is governed by the parameter b. If $b \lesssim 1$, the wave greatly distorts the trajectory; in particular, the group of trapped particles is singled out, and the untrapped particles can have turning points only at the crests of the wave profile. This distortion of the trajectory of course leads to changes in the contributions of the electrons to the absorption and thus to the nonlinearity.

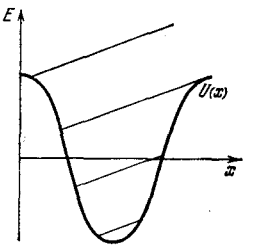

FIG. 3.

We should point out that the parameter b itself depends on the magnetic field: as the field is strengthened, the nonlinearity is weakened, and it is suppressed completely at $b \gtrsim 1$. Correspondingly, there should be an additional variation of the absorption with the magnetic field. Since these discussions have referred to only a small part of the trajectory, they are valid in both strong and weak ($l \ll R$) magnetic fields, so that the suppression of the nonlinearity should occur in weak magnetic fields also. What is the physics of this effect?

It can be seen from Fig. 3 that a strengthening of the field tends to remove particles from the resonant region. On the other hand, we saw earlier (Section 3) that the nonlinearity is governed by a competition between the supply of energy to the group of resonant electrons and processes tending to transfer this energy to the entire system of electrons. The nonlinearity is pronounced if the latter processes are relatively ineffective and become a "bottleneck." In the absence of a magnetic field, particles are removed from the resonant group as the result of infrequent collisions. When the magnetic field is applied, removal by the magnetic field becomes important: the "bottleneck" expands, and the nonlinearity is suppressed.[21]

At what magnetic fields does the suppression of the nonlinearity become important? As a measure of the effectiveness of the removal of particles by the magnetic field we can estimate the time over which the force exerted by the magnetic field produces an x component of the velocity which is of the order of the characteristic velocity of the trapped particles, $\sim\sqrt{U/m}$: $\tau_1 \sim \omega_0/\Omega qv$. If $\tau_1 < \tau$, the field-induced removal is more important than the removal by collisions. Since the parameter b introduced above is equal to $1/\omega_0\tau_1$, as is easily shown, we conclude that, again in the case of a magnetic field, the role of the nonlinear effects is governed by the relationship between the energy supplied to the resonant particles and the width of the "bottleneck" through which this energy escapes.

As before, the quantitative analysis is based on a solution of the kinetic equation incorporating an external magnetic field.[22] This solution is expressed in terms

[20] For simplicity we are ignoring the electromagnetic contribution to the absorption; this contribution can be important for transverse sound.

[21] In semiconductors, electrons can be removed from the resonant group by applying an external electric field, as shown by Zil'berman and Mishin.[53]

[22] It can be shown[45] that in an external magnetic field the conditions under which the classical description is legitimate are $(\hbar^2 q^2/m \ll \max(\hbar\Omega\sqrt{qR}, U_0, \hbar/\tau)$.

of an integral along the electron trajectory:

$$f = \int_{-\infty}^{t} dt' \left(\dot{U}\frac{\partial F_0}{\partial \varepsilon}\right)\Big|_{t'} \exp\left[-\frac{1}{\tau}(t-t')\right]. \qquad (5.3)$$

Equation (5.3) is similar to (3.30), but the particle trajectory is much more complicated than in the case dealt with in Section 3, since now we must take into account the magnetic field as well as the field of the sound wave. There is a simplifying circumstance here: as mentioned above, the motion of the electron in the regions of the effective interaction (where the field of the sound wave is important) can be assumed to be one-dimensional.

In weak fields, $l \ll R$, because of the rapidly decreasing factor, the integral in (5.3) can be affected by only one effective-interaction region, and it can be shown that if $\tau_1 \ll \tau$ the exponential function can be ignored in this region. Using (5.2), we can write

$$\int dt'\dot{U} = \int \frac{dx\, w\, \partial U/\partial x}{\pm\sqrt{(2/m)\{U(x_0) - U(x) + (evH/c)(x-x_0)\}}}. \qquad (5.4)$$

This equation is the transformation from an integral over time to an integral over the coordinate; the denominator in (5.4) is the electron velocity. We thus find the following estimate of the absorption coefficient for the case $l \ll R$[45]:

$$\Gamma \sim \frac{1}{\omega_0 \min(\tau_1, \tau)}\Gamma_0. \qquad (5.5)$$

Since $\tau_1 \sim \omega_0/\Omega qv$, Γ is proportional to H in the case $\tau_1 < \tau$. We thus reach an important conclusion: in the nonlinear case, the absorption becomes sensitive to the magnetic field at *weak* fields, for which there is no such sensitivity in the linear case. Estimates show that in pure samples the suppression of the nonlinearity may begin in a field of the order of an oersted or even lower.

For classically strong fields ($R \ll l$), the electron manages to traverse many periods of the trajectory during the time between collisions. The contribution of the several turning points is thus important in (5.3); the correlation of the wave phases corresponding to these points leads to oscillatory effects in the absorption in the linear situation. In the nonlinear situation, a further analysis is required, to determine how a sound wave of finite amplitude affects the distribution of turning points, i.e., the shape of the entire trajectory (this additional analysis significantly complicates the calculations).

To see what happens in the nonlinear situation, we return to Fig. 3. Here the trapped electrons are represented by line 3, which can be assumed straight by virtue of the condition $qR \gg 1$. It can be seen from Fig. 3 that in the case $b \ll 1$ (i.e., in the case $\tau_1 \ll \tau$) the magnetic field has only a slight effect on the motion of the trapped particles. Their contribution thus remains the same as in the absence of a magnetic field, and if $\tau_1 \ll \tau$ this contribution can be ignored in comparison with that of the other (untrapped) particles. It can be seen from (5.2) that the field of the sound wave allows the untrapped particles to have turning points only near the crests of the potential profile; the wave "synchronizes" the distribution of turning points, tying them to the crests. The result is an important change in the variation of the absorption with the field.

The resulting pičture is complicated and extremely sensitive to the experimental geometry (and to the geometry of the Fermi surface); accordingly, we will restrict the present discussion to the basic features of this picture.

In the nonlinear situation, generally speaking, there is a nonlinear contribution to the absorption which varies monotonically with the field. This contribution reduces the depth of the modulation of the oscillations in comparison with the depth in the linear situation. The variation of the amplitude of the resonant magnetoacoustic oscillations with the magnetic field is sharper than in the linear situation. The reason for this result is that the trajectory distortion caused by the sound wave makes the turning points more sharply defined. Furthermore, a distortion of the trajectories complicates the oscillation picture: as shown in Ref. 48, additional peaks appear on the curve of $\Gamma(H)$, along with the peaks which are predicted by the linear theory of Kaner *et al.*[51]

The nonlinear absorption coefficient can be written in the form[45]

$$\Gamma = [C_1 b + C_2(\Omega\tau)\, b + C_3(\Omega\tau)\, b^3\gamma(H)]\,\Gamma_0; \qquad (5.6)$$

here γ is a function normalized to unity which describes the oscillations, and C_{1-3} are numerical coefficients of the order of unity. The coefficient C_2 may vanish because of symmetry considerations. In particular, it does vanish if the Fermi surface is isotropic, and it also vanishes if the sound is propagating along a high-symmetry axis.

To conclude this section, we would like to discuss the experiments of Fil' *et al.*,[30] who observed nonlinear acoustic effects in ultrapure Ga with an electron mean free path $l \sim 2$ cm. In the experiments, the absorption coefficient for a weak signal at a frequency of 59 MHz was studied as a function of the amplitude of a strong "pump" signal at 154 MHz. The results are shown in Fig. 4. At the same time, the change in the sound velocity was measured. We see that there is a pronounced change in the absorption, while the change in the sound velocity is negligible. This result agrees with the physical picture drawn above for the momentum nonlinearity: the corrections to the sound velocity are governed by nonresonant electrons, whose distribution is essentially undistorted by the sound wave. The $\Gamma(S)$ curve shown in Fig. 4 can be found only by placing the sample inside a superconducting shield: the geomagnetic field suppresses the nonlinearity. Figure

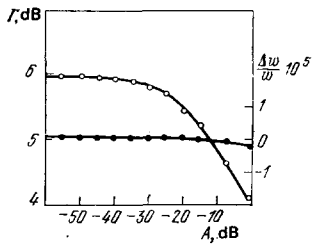

FIG. 4.

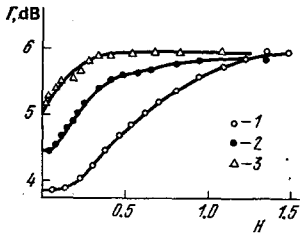

FIG. 5.

5 shows the variation of the absorption coefficient with the magnetic field for weak fields and for various input sound intensities, $S_{pump}=90$, 6, and 1 W/cm² (curves 1, 2, and 3, respectively). We see that the nonlinearity is suppressed by the magnetic field. The typical field required for suppression of course varies with the sound intensity. This value is in order-of-magnitude agreement with the condition $\omega_0\tau_1 \sim 1$, which follows from the theory described above.

Unfortunately, it is not possible to make a quantitative comparison with the theory, for several reasons, of which the most important are as follows: the theory is derived for a single signal, rather than for two; the theory ignores the particular features resulting from the complicated Fermi surface of Ga; and the sound intensity used in the experiments is not high enough to satisfy the inequality $\omega_0\tau \gg 1$ with a wide margin.

6. CONCLUSION

In conclusion we would like to suggest some worthwhile directions for further experimental and theoretical work in the near future.

Actually, there has been no direct experiment to measure ultrasonic absorption in metals under conditions corresponding to a momentum nonlinearity in a single-wave situation. In the experiments of Ref. 30, mentioned above, the absorption of a weak signal was studied in the field of a strong acoustic pump. No theory is available for this case. The special case in which the frequency of one wave is a multiple of the frequency of the other, however, was studied by Vugal'ter and Demikhovskii.[23]

It would be very interesting to observe the momentum nonlinearity in metals by making use of the acoustoelectric effect.[23)] We know that at a low sound intensity the acoustoelectric current density $j(x)$ is related to the absorption coefficient Γ by the Weinreich equation,[54]

$$j^{ac}=\frac{\sigma\Gamma S}{en_0 w}.$$

This order-of-magnitude equation follows from energy and quasimomentum conservation in the interaction of a sound wave with conduction electrons.

The theory of Ref. 19 shows that this equation should also hold under conditions corresponding to the momentum nonlinearity. The theory also gives the numerical coefficient: in a metal, this coefficient is one, while in a semiconductor it depends on the scattering mechanism. A study of the acoustoelectric effect can thus furnish several additional types of information about the momentum nonlinearity.

We have thus examined several specific nonlinear effects which occur as high-frequency sound propagates through pure conductors. It can be hoped that this field of solid state physics will develop rapidly for at least the next few years. Then, in addition to the problems discussed here, some new problems should arise. At this point it would be impossible to predict the appearance of these problems, but their formulation and solution will constitute a major thrust in this field in the near future.

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