Electric fields and collective oscillations in superconductors

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Effects associated with the penetration of an electric field E into a superconductor with deviations from thermodynamic equilibrium are considered. The penetration of a static field E incident to the passage of a current across the boundary between the superconductor and the normal metal (S-N boundary) is analyzed. At temperatures close to the critical temperature the penetration depth l_E of a field E into the S region may be much greater than the correlation length or the London depth and may reach macroscopic dimensions in sufficiently pure specimens. In isotropic superconductors the magnitude of l_E is determined by the branch imbalance relaxation processes. The change in the gap width at the S-N boundary leads to an additional branch imbalance relaxation mechanism which, in pure specimens, is due to Andreev reflection of quasiparticles. The resistance of a superconductor in the intermediate state is calculated. Weakly damped collective oscillations with an acoustic spectrum, which exist in superconductors near the critical temperature, are considered. This collective mode is characterized by oscillations of both the field E and the branch imbalance. The propagation velocity of the oscillations is somewhat lower than the Fermi velocity. Effects associated with the penetration of the field E to great depths in Josephson bridges are analyzed. The theory of the phenomena considered is presented, using the kinetic equation and the equations for the Green's functions. Experiments are described for measuring effects associated with the penetration of a static field E into a superconductor and for detecting the collective oscillations.

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CONTENTS

1. INTRODUCTION

It is well known that a steady current j can flow in a superconductor without energy dissipation in the absence of an electric field E. If the current varies in time there will appear in the superconductor an alternating transverse electric field whose strength falls off within a distance from the boundary of the superconductor of the order of the London depth λ_L [or the skin depth $\lambda_{sk}(\omega)<\lambda_L$, if the frequency ω is high enough]. In this case energy dissipation occurs at temperatures differing from zero. For a long time it was widely believed that an electric field **E** (and especially a static one) cannot exist within a superconductor at distances from the boundary exceeding the characteristic lengths of the superconductor: the magnetic field penetration depth λ_L or the correlation length $\xi(T)$. This belief was apparently based on an examination of the equation of motion of the condensate:

$$\frac{\partial \mathbf{p}_s}{\partial t}=e\mathbf{E}+\nabla\mu, \tag{1.1}$$

where $p_s=(1/2)\nabla\chi-(e/c)\mathbf{A}$ is the momentum of the condensate, $\mu=(1/2)(\partial\chi/\partial t)+e\Phi$ is the gauge invariant potential, χ is the phase of the order parameter, and Φ is the electrostatic potential. Equation (1.1) can be regarded not only as the equation of motion of the condensate, but also as the definition of **E** in terms of the gauge invariant quantities $\mathbf{p}_s$ and μ. In fact, on substituting the expressions for $\mathbf{p}_s$ and μ, we obtain the definition of **E** in terms of the vector and scalar potentials **A** and Φ. Moreover, Eq. (1.1) follows directly from the London equation $(4\pi\lambda_L^2/c)\,\mathrm{rot}\,\mathbf{j}_s+\mathbf{H}=0$, the Maxwell equation $\mathrm{rot}E=-c^{-1}\partial\mathbf{H}/\partial t$, and the expression $\mathbf{j}_s=c^2(4\pi\lambda_L^2 e)^{-1}\mathbf{p}_s$ for the superconduction current.

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If we could neglect the second term on the right in Eq. (1.1) (as will be shown below, it is not, in general, legitimate to do this) the presence of the field E would mean that the condensate was being continually accelerated. This is what led to the conclusion that no electric field can exist within a superconductor far from its boundary. Studies conducted in recent years, however, have shown that under certain conditions an electric field E can penetrate into a superconductor to a depth greatly exceeding the characteristic lengths λ_L and $\xi(T)$ of the superconductor. It is important to note that only the longitudinal part of E, which does not give rise to a magnetic field, penetrates into the superconductor.

Some of the earliest experiments to stimulate interest in this problem were those of Landau[1] and Pippard *et al.*,[2] who measured the electrical resistance ρ^* of a superconductor in the intermediate state produced by the application of a magnetic field H. The resistance ρ^* of the superconductor was measured by passing a weak current I through it in the direction perpendicular to the alternating S and N layers. The experimental dependence of ρ^* on H is shown in Fig. 1. If only the normal phase N contributed to the resistance ρ^*, the H dependence of ρ^* would be represented by a straight line, since the concentration C_N of the normal phase is proportional to H. As is evident from Fig. 1, however, the reistance ρ^* exceeds the reistance ρ of the normal phase. This excess $\rho_s = \rho^* - \rho$ tends to zero at low temperatures and increases as the temperature T approaches the critical temperature T_c. To account for this phenomenon, Pippard *et al.*[2] suggested that a discontinuity in the potential Φ arises at the boundary between the S and N phases, i.e. that a charge double layer is formed. These authors associated the magnitude of the discontinuity with the relaxation time τ_Q for the branch imbalance Q, i.e. the difference between the populations of the electronlike ($\xi = v(p - p_F) > 0$) and holelike ($\xi < 0$) branches of the quasiparticle spectrum $\varepsilon(p)$ (here $v = p_F/m$ is the Fermi velocity). Subsequent theoretical studies, however, showed that Φ is continuous at the S–N boundary, i.e., that the electric field E penetrates into the superconductor. The penetration depth of the field E, however, and therefore also the magnitude of the potential at the S–N boundary, are actually determined by the relaxation time τ_Q. In the following Division we shall derive an equation for the spatial variation of E in a superconductor.

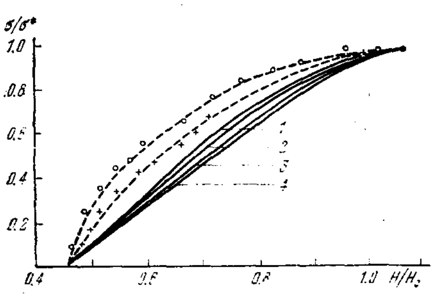

FIG. 1. The effective resistance $\rho^*\sigma = \sigma/\sigma^*$ of superconducting indium in the intermediate state (σ is the conductivity when $H \geqslant H_c$) vs the magnetic field strength H for the following temperatures[1] (°K): 3.37 (circles), 3.3 (crosses), 3.247 (curve 1), 3.164 (curve 2), and 3.09 (curve 3); curve 4 is based on measurements at 2.8, 2.73, 2.31, and 2.12 °K.

2. STATIC ELECTRIC FIELDS IN SUPERCONDUCTORS

a) Generalized Ginzburg-Landau equations for a gapless superconductor

The penetration of an electric field into a superconductor incident to the passage of a current through the S–N boundary was first discussed by Rieger, Scalapino, and Mercereau[3] on the basis of a generalization of the Ginzburg-Landau equations to the nonequilibrium nonstationary case. The authors generalized the equations on the basis of purely phenomenological considerations. Earlier, however, Gor'kov and Éliashberg[4] had shown on the basis of a microscopic theory that generalized Ginzburg-Landau equations could only be obtained, generally speaking, for the special case of a gapless superconductor having a high concentration of paramagnetic impurities ($\tau_s T \ll 1$, where τ_s is the time for spin-flip scattering of an electron from a paramagnetic impurity). In this case the equation for the order parameter $\hat{\Delta}$ has the form[1)]

$$-12\tau_0\left(\frac{\partial}{\partial t} + 2ie\Phi\right)\hat{\Delta} + \xi^2(T)\nabla^2\hat{\Delta} + \hat{\Delta}\left(1 - \frac{|\hat{\Delta}|^2}{\Delta_0^2}\right) = 0, \qquad (2.1)$$

where $\tau_0 = (2\tau_s\Delta_0^2)^{-1}$ and $\Delta_0^2 = 2\pi^2(T_c^2 - T^2)$. Let the superconductor (the normal metal) occupy the region $x > 0$ ($x < 0$). In the stationary case of interest to us the equation for the modulus $\Delta \equiv |\hat{\Delta}|$ of $\hat{\Delta}$ reduces to the Ginzburg-Landau equation:

$$\xi^2(T)\frac{\partial^2\Delta(x)}{\partial x^2} + \Delta(x)\left(1 - \frac{\Delta^2(x)}{\Delta_0^2}\right) = 0. \qquad (2.2)$$

The solution $\Delta(x)$ to this equation that satisfies the boundary condition $\Delta(0) = 0$ is

$$\Delta(x) = \Delta\,\mathrm{th}\frac{x}{\sqrt{2}\,\xi(T)}. \qquad (2.3)$$

If we take the imaginary part of Eq. (2.1) we obtain the equation of continuity for the quasiparticle current, which, in the $\chi = \mathrm{const.}$ gauge, has the form

$$12\sigma\Phi\frac{\Delta^2(x)}{\Delta_0^2} = \xi^2(T)\frac{\partial j_s}{\partial x} = -\xi^2(T)\frac{\partial j_n}{\partial x}. \qquad (2.4)$$

Here we have used the equation of continuity for the total current

$$j = j_n + j_s, \qquad (2.5)$$

and the following expression for j_s:

$$j_s = \frac{\sigma}{4ie\tau_0\Delta_0^2}\left(\hat{\Delta}^*\frac{\partial}{\partial x}\hat{\Delta} - \mathrm{c.c.}\right). \qquad (2.6)$$

Using the expression

$$j_n = \sigma E = -\sigma\frac{\partial\Phi}{\partial x} \qquad (2.7)$$

for j_n and the x dependence of Δ [Eq. (2.3)], we obtain the equation for the x dependence of the potential Φ, and therefore also of the field E, in the S region:

$$12\,\mathrm{th}^2\left(\frac{x}{\sqrt{2}\,\xi(T)}\right)\Phi = \xi^2(T)\frac{\partial^2\Phi}{\partial x^2}. \qquad (2.8)$$

The solution of Eq. (2.8) can be expressed in terms of the hypergeometric function.[5] From the very form of Eq. (2.8), however, it follows that in this case the potential Φ and the field E fall to zero in a distance equal

[1)]We use units in which $\hbar = k_B = 1$ (k_B is Boltzmann's constant).

to the correlation length $\xi(T)$. The x dependences of E and Δ are shown schematically in Fig. 2. Thus, in the case of a gapless superconductor now being considered, there is a peculiar proximity effect for E and Δ: E differs from zero only in the region in which Δ varies with x. A more interesting result is obtained for the case of a superconductor with a gap.

b) The equation for the electric field in an ordinary superconductor with a low impurity concentration (the phonon relaxation mechanism)

In the case of an ordinary superconductor with a gap, the equation for the potential Φ is most easily obtained by starting with the kinetic equation for the quasiparticle distribution function $n(\mathbf{p}, \mathbf{r}, t)$.[6] The kinetic equation is valid provided the reciprocal lengths and frequencies characteristic of the variations of $n(\mathbf{p})$ are small compared with $\xi^{-1}(T)$ and ε_x ($\varepsilon_x \sim \min\{\Delta, T\}$ is the characteristic quasiparticle energy and Δ is the width of the energy gap). The impurity concentration should also be fairly small ($\tau\Delta \gg 1$, where τ is the momentum relaxation time). We shall examine the most interesting case of temperature close to the critical temperature T_c (i.e. when $\Delta \ll T$). At low temperatures ($T \ll \Delta$) the field strength is exponentially low in the S region. In the presence of an electric field and a superconduction current, the kinetic equation has the form[7]

$$\frac{\partial n}{\partial t} + \frac{\partial \tilde{\varepsilon}}{\partial \mathbf{p}} \frac{\partial n}{\partial \mathbf{r}} - \frac{\partial \tilde{\varepsilon}}{\partial \mathbf{r}} \frac{\partial n}{\partial \mathbf{p}} = I_{\mathrm{im}} + I_{\mathrm{ph}}, \tag{2.9}$$

where $\tilde{\varepsilon} = \sqrt{\tilde{\xi} + \Delta^2} + \mathbf{p}_s\mathbf{v}$ is the excitation energy, $\tilde{\xi} = \xi + \mu + (p_s^2/2m)$, $\xi = v(p - p_F)$, and I_{im} and I_{ph} are the collision integrals for collisions with impurities and phonons, respectively. Equation (2.9) is to be used to find the linear response of the system to a weak static electric field $\mathbf{E}$. Suppose that the momentum relaxation is due to scattering from impurities and that $\tau \ll \tau_\varepsilon$, where τ_ε is the energy relaxation time ($\tau_\varepsilon \sim \theta_D^2/T^3$, θ_D being the Debye frequency). Then we may seek the solution to (2.9) in the form

$$\delta n = n - n_F(\tilde{\varepsilon}) = n_0 + \mathbf{n}_1 \frac{\mathbf{v}}{v}, \tag{2.10}$$

in which $n_F(\tilde{\varepsilon}) = (1/2)(1 - \mathrm{th}(\tilde{\varepsilon}/2T))$ is the Fermi distribution function. Only the first two terms remain in the expansion of δn in Legendre polynomials since the other terms are small in terms of the parameter τ/τ_ε. We substitute (2.10) into (2.9) and linearize the resulting equation:

$$\frac{\xi}{\varepsilon} \mathbf{v}\nabla\delta n = -\frac{1}{\tau} \frac{|\xi|}{\varepsilon} \mathbf{n}_1 \frac{\mathbf{v}}{v} + I_{\mathrm{ph}}(n_0), \tag{2.11}$$

where

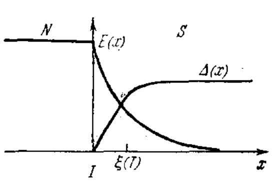

FIG. 2. Electric field strength E in a gapless superconductor vs the coordinate x when current flows across the boundary between the superconductor (S) and the normal metal (N). The x dependence of the order parameter $\Delta(x)$ is also shown.

$$I_{\mathrm{ph}}(n_0) = \alpha_{\mathrm{ph}}\theta_D^{-2}\left\{-n_0 \int d\xi' \left[F(\varepsilon, \varepsilon')\left(1 - \frac{\Delta^2}{\varepsilon\varepsilon'}\right) + F(\varepsilon, -\varepsilon')\left(1 + \frac{\Delta^2}{\varepsilon\varepsilon'}\right)\right] + \frac{\xi}{\varepsilon}\int d\xi' \frac{\xi'}{\varepsilon'} n_0(\xi')\left[F(\varepsilon', \varepsilon) + F(-\varepsilon', \varepsilon)\right]\right\},$$
$$F(\varepsilon, \varepsilon') = (\varepsilon' - \varepsilon)|\varepsilon' - \varepsilon|\,\mathrm{ch}\left(\frac{\varepsilon}{2T}\right)\left[\mathrm{sh}\left(\frac{\varepsilon' - \varepsilon}{2T}\right)\mathrm{ch}\left(\frac{\varepsilon'}{2T}\right)\right]^{-1}, \tag{2.11'}$$

and α_{ph} is the coupling constant for the interaction with phonons ($\alpha_{\mathrm{ph}} \sim 1$). In obtaining the expression for I_{ph} we took into account the fact that n_0 is an odd function of ξ (see below). The scattering from impurities is assumed to be isotropic. The equations

$$\mathbf{n}_1 = -l\,\mathrm{sgn}\,\xi \cdot \nabla n_0, \tag{2.12}$$

$$\frac{1}{3}\frac{\xi}{\varepsilon} v \nabla \mathbf{n}_1 = I_{\mathrm{ph}}(n_0), \tag{2.13}$$

for n_1 and n_0, where $l = v\tau$ is the mean free path, follow from (2.11).

The function n_1 determines the quasiparticle current[7]:

$$\mathbf{j}_n = eNp_F^{-1}\int d\xi\,\mathbf{n}_1, \tag{2.14}$$

and the function n_0 is related to the potential Φ (or to the potential μ in the more general nonstationary case). To find the relation between n_0 and Φ we calculate the change in the total number of particles in the superconductor:

$$\delta N = \delta \int d\mathbf{p}\cdot(2\pi)^{-3}\left[u_{\mathbf{p}}^2 n + v_{\mathbf{p}}^2(1 - n)\right] = p_F m\pi^{-2}\left(-e\Phi + \int d\xi \frac{\xi}{\varepsilon} n_0\right),$$

where $u_{\mathbf{p}}^2 = 1 - v_{\mathbf{p}}^2 = (1/2)(1 + (\tilde{\xi}/\varepsilon))$. Now we substitute δN into Poisson's equation

$$\nabla^2\Phi = k_{TF}^2 e\delta N, \tag{2.15}$$

where $k_{TF}^{-1} = (6\pi e^2 N/\varepsilon_F)^{-1/2}$ is the Thomas-Fermi screening length. Since the field penetration depth l_E is much greater than k_{TF}^{-1}, we can neglect the left-hand side of (2.15) and use the quasineutrality condition to obtain the relation between Φ and n_0:

$$e\Phi = \int d\xi \frac{\xi}{\varepsilon} n_0. \tag{2.16}$$

Thus, Φ is related to the angle-independent ξ-odd part of the increment δn of the distribution function $n_F(\tilde{\varepsilon})$. For the normal metal, the integral in (2.16) is the difference between the numbers of electrons and holes;[2)] for the superconductor close to T_c, it determines the difference between the populations $n_>$ of the electronlike ($\xi > 0$) and $n_<$ of the holelike ($\xi < 0$) branches of the quasiparticle spectrum (Fig. 3), i.e. the branch imbalance Q:

$$Q = n_> - n_< = p_F m\pi^{-2}\int d\xi n_0\,\mathrm{sgn}\,\xi. \tag{2.17}$$

Near T_c we have $\varepsilon \sim T$ for the characteristic energy variations of n_0; hence $\xi/\varepsilon \approx \mathrm{sgn}\xi$, and the integrals in (2.16) and (2.17) agree with one another to the first approximation in Δ/T.

To obtain the desired equation for $\Phi(x)$ we multiply (2.13) by ξ/ε and integrate over all ξ near T_c.[3)] In in-

[2)]It should be recalled that the Fermi function $n_F(\tilde{\varepsilon})$ in which the potential $e\Phi$ was added to the chemical potential was used at the outset. The deviation δn from the equilibrium function $n_F(\varepsilon)$, averaged over the angles, vanishes in the normal metal but not in the superconductor.

[3)]A more rigorous analysis based on the expansion of n_0 in pow-

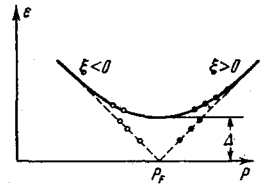

FIG. 3. Single-particle excitation spectrum $\varepsilon(p)$ for a normal metal (dashed lines) and a superconductor (smooth curve). The black circles represent electronlike excitations ($\xi > 0$), and the open circles, holelike ones ($\xi < 0$).

tegrating the left-hand side we may assume that $(\xi/\varepsilon)^2 \approx 1$. Then, using (2.14), we obtain

$$eD\sigma^{-1}\nabla \mathbf{j}_n = \int d\xi \frac{\xi}{\varepsilon} I_{ph}(n_0) = -\int d\xi n_0 \frac{\xi}{\varepsilon} \nu_Q(\varepsilon) \equiv -\nu_Q \int d\xi \frac{\xi}{\varepsilon} n_0, \tag{2.18}$$

where $D = vl/3$ is the diffusion constant, and the frequency ν_Q is defined by the last of Eqs. (2.18):

$$\nu_Q(\varepsilon) = 4\alpha_{ph}\Delta^2\theta_D^{-2} \int d\varepsilon' F(\varepsilon, \varepsilon')(\varepsilon - \varepsilon')(\varepsilon\varepsilon')^{-1}(\varepsilon'^2 - \Delta^2)^{-1/2}\theta(|\varepsilon'| - \Delta).$$

Calculating the frequency $\nu_Q(\varepsilon)$ for $\varepsilon \sim T$, we find

$$\nu_Q(\varepsilon) = \frac{4\pi\alpha_{ph}\Delta\varepsilon^2}{\theta_D^2} \operatorname{cth} \frac{\varepsilon}{2T}. \tag{2.19}$$

The integral on the right in Eq. (2.17) vanishes for the normal metal. This follows directly from (2.19) ($\nu_Q = 0$ when $\Delta = 0$) and is an expression of the law of conservation of particles, since the right-hand part of (2.18) is the collision integral for collisions with phonons summed over all momenta, and this conserves the number of particles (we note that $n_0 = f_0$ when $\xi > 0$ and $n_0 = -f_0$ when $\xi < 0$, where f_0 is the electron distribution function). The frequency ν_Q introduced in (2.18) is accordingly also zero. It is easy to clarify the physical meaning of the frequencies $\nu_Q(\varepsilon)$ and ν_Q on the basis of their definition in (2.18). If we consider the time variation of the branch imbalance Q [Eq. (2.17)] in a spatially uniform system (e.g., in one of the electrodes of a tunnel junction) then in place of (2.18) we obtain

$$\frac{\partial Q}{\partial t} = -\nu_Q Q.$$

Thus, $\tau_Q = \nu_Q^{-1}$ is the branch imbalance relaxation time.[10,11] In the normal metal $\nu_Q = 0$. In other words, if, disregarding the violation of neutrality (assuming, for example, that the electron charge is zero), we produce a population difference Q, then in the normal metal this population difference will not relax as a result of scattering of quasiparticles from phonons, since in scattering, an electron (hole) either passes over into an electron (hole) or recombines with a hole, so that the difference between the number $n_>$ of electrons and the number $n_<$ of holes remains unchanged: $Q = \text{const}$. Actually, of course, in the normal metal the number of electrons is always equal to the number of holes because of the Coulomb interaction (we may neglect the slight deviation from neutrality due to the perturbations under consideration since the perturbations are smooth compared with the Thomas-Fermi screening length). In the superconductor $\nu_Q \neq 0$, since there it is possible for a quasielectron ($\xi > 0$) to undergo a transition into a quasihole ($\xi < 0$) as a result of inelastic scattering from phonons.[11] The magnitude of ν_Q was estimated and measured by Tinkham and Clarke.[10,11]

Now let us calculate the right-hand part of Eq. (2.18). In the case of temperatures close to the critical temperature, which is the one we are considering, n_0 is the same in the S and N regions to the zeroth approximation in Δ/T (we are neglecting Andreev reflection of quasiparticles at the S–N boundary). In the N region,

$$n_0 = -e\Phi \frac{\partial n_F}{\partial \xi} = \frac{e\Phi}{4T} \operatorname{ch}^{-2}\left(\frac{\varepsilon}{2T}\right) \operatorname{sgn} \xi. \tag{2.20}$$

On substituting (2.20) and (2.19) into (2.18), we obtain the desired equation

$$\frac{D}{\sigma} \nabla \mathbf{j}_n = -\tau_Q^{-1}\Phi, \tag{2.21}$$

where

$$\tau_Q = \tau_\varepsilon \frac{4T}{\pi\Delta}, \quad \tau_\varepsilon^{-1} = 14\alpha_{ph} T^3 \theta_D^{-2} \zeta(3), \tag{2.22}$$

and τ_ε is the energy relaxation time. From (2.14) and (2.20) we obtain an expression for the quasiparticle current $\mathbf{j}_n = \sigma\mathbf{E}$ that agrees with (2.7) (here $\sigma = e^2N\tau/m$ is the conductivity in the normal state). By making use of the expression for j_n, we can rewrite Eq. (2.21) in the form

$$l_E^2 \nabla^2 \Phi = \Phi; \tag{2.23}$$

Here

$$l_E = \sqrt{D\tau_Q} = \sqrt{D\tau_\varepsilon \cdot \frac{4T}{\pi\Delta}} \tag{2.24}$$

is the penetration depth of the field into the superconductor.

Thus, when a current flows across the boundary between a superconductor and a normal metal, the field $\mathbf{E}$ penetrates into the S region to a depth l_E that exceeds the energy relaxation length $l_\varepsilon = \sqrt{D\tau_\varepsilon}$, and the latter, in turn, may be much greater than $\xi(T)$ or λ_L. In aluminum, for example, we have $\tau_\varepsilon \sim 10^{-8}$ sec,[12] so in pure aluminum we may have $l_\varepsilon \approx 1$ mm. Since $l_E^{-1} \sim \sqrt{\Delta}$, there is a continuous change in resistivity from the superconducting state to the normal state: at T approaches T_c the field $\mathbf{E}$ penetrates deeper and deeper into the S region from the N region and the resistivity induced in the superconductor increases, approaching the resistivity of the normal metal. Formulas for the resistivity induced in the superconducting region are given in Sections e) and f).

We also note that since we assumed the distribution function to be continuous at the S–N boundary, the field E and the quasiparticle current j_n will be continuous (Fig. 4). The current $\mathbf{j}_s = N_s\mathbf{v}_s$ is found from the condition that the total current j be continuous. In the case of massive S and N regions in contact with one another (e.g., a superconductor in the intermediate state), we have $\mathbf{j}_s = -\mathbf{j}_n$, since the total current vanishes deep in the S region. The total current is carried by pairs and, unlike the quasiparticle current, flows along the S–N boundary in the Meissner layer, and its distribution is determined by the equation

ers of (Δ/T) shows that this integration leads essentially to the condition that Eq. (2.13) for the first correction to n_F have a solution.[8,9]

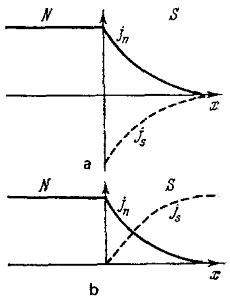

FIG. 4. Quasiparticle (j_n) and superconduction (j_s) currents vs the coordinate x when current flows across the boundary in the case of a massive specimen (a) and a thin film (b).

$$\text{rot rot } \mathbf{p}_s = -\frac{4\pi e}{c^2}\mathbf{j}.$$

If the system under consideration is a thin narrow film of which part is in the normal state and part in the superconducting state, the current j flowing across the S–N boundary will be independent of the coordinate. Hence $j_s = j - j_n$. Thus, the quasiparticle current is converted into superconduction current in the length l_C. Figure 4 shows the distributions of the currents j_n and j_s for the cases of a massive specimen and of a thin film.

The above equation (2.23) for Φ was derived from a kinetic equation that is valid for sufficiently pure superconductors ($\Delta\tau \gg 1$). It turns out, however, that neither Eq. (2.23) nor the length l_E changes for arbitrary impurity concentrations provided only that $l_E > \xi(T)$.[8,9,13]

The results obtained above provide a qualitative explanation for the experimental data on the resistance of superconductors in the intermediate state.[1,2] A quantitative comparison of l_E with the experimentally determined field penetration depth was made in Refs. 14 and 15. In Ref. 15 a thin narrow (~1 μm) film of tin or indium was used to measure l_E. A notch was made in the film so that the width of the film at the notch was reduced by about one-half. A current was passed through the film. An electric field appeared at the narrow spot when the current exceeded a critical value. The length over which the field fell off was determined with microprobes placed close together (~2 μm) near the notch. It was found, in agreement with theory, that l_E increased weakly according to the law $l_E \sim \Delta^{-1/2} \sim (1-(T/T_c))^{-1/4}$ as the temperature T approached T_c. Tinkham and Clarke,[10] and Clarke and Paterson[16] measured τ_Q. The experiment was conducted with an N–I–S tunnel junction. The superconductor was either tin or lead. When a voltage was applied to the junction nonequilibrium quasiparticles were injected into the S electrode and a branch imbalance Q arose (in this case conversion of the quasiparticle current to superconduction current also takes place: quasiparticles flow into the S electrode from the N electrode and a pair current j_s flows out). A potential difference Φ developed between the region of injection into the S electrode and the distant part of the S region (Fig. 5), which was proportional to the relaxation time τ_Q.[11,13,17-18] The relaxation time τ_Q was determined from measurements of Φ. It turned out that $\tau_Q = 2.10^{-10}(1-(T/T_c))^{-1/2}$ sec for tin.

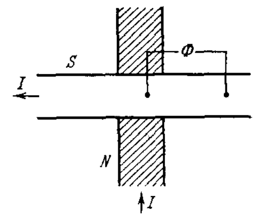

FIG. 5. The S–I–N tunnel system used by Tinkham and Clarke[10] to measure the potential Φ due to the injection of nonequilibrium quasiparticles into the superconductor.

It follows from Eq. (2.21) that a potential Φ arises when the divergence of $\mathbf{j}_n$ differs from zero. This can take place not only in the case of passage of current through the boundary: a divergence arises, for example, when sound or light is absorbed (the acoustoelectric and photoelectric effects), and this leads to the appearance of a quasiparticle current $j_n(x)$ flowing in the direction of propagation (along the x axis) of the sound or light waves.[19] In addition, a quasiparticle current $\mathbf{j}_T = -\beta\nabla T$ arises in the presence of a temperature gradient ∇T, while the total current in the open specimen is zero. At the S–N or S–S' boundary there appears a divergence $\nabla\mathbf{j}_T$ (the coefficients β and β' for the superconductors S and S' must be different) and along with it, a thermoelectric field E_T.[6] A detailed discussion of thermal effects in superconductors will be found in the review by Ginzburg and Zharkov.[20]

c) Andreev reflection at the boundary between a superconductor and a normal metal in the presence of a current across the boundary

In the preceding sections attention was mainly concentrated on the mechanism for the relaxation of the branch imbalance Q and of the quasiparticle current $\mathbf{j}_n$ in a superconductor as a result of inelastic scattering of quasiparticles from phonons. There are also other branch imbalance relaxation mechanisms, which affect the spatial dependence of the field $\mathbf{E}$ in the superconductor. One such mechanism is associated with the coordinate dependence of the gap width Δ. It is well known that Δ varies near the S–N boundary because of the proximity effect. At temperatures close to T_c, the x dependence of Δ is given by formula (2.3). Allowance for the x dependence of Δ leads to different results in the cases of clean and dirty superconductors. In the first case allowing for the x dependence of Δ reduces to allowing for Andreev scattering of quasiparticles at the S–N boundary. As is well known, an electron ($\xi > 0$) moving from the N region with energy $\varepsilon < \Delta$ and velocity $v_{gx} = \partial\varepsilon/\partial p_x = v_x\xi/\varepsilon > 0$, transforms on reflection into a hole with the same energy ε but moving in the opposite direction with $v_{gx} < 0$ ($\xi < 0$).[21] Here the quasiparticle momentum remains unchanged but the group velocity $\partial\varepsilon/\partial p_x = v_x\xi/\varepsilon$ changes sign. Physically, this process amounts to the following: the moving electron undergoes a transition to the condensate, forming a Cooper pair with a second electron having the opposite momentum, and the loss of this second electron results in the appearance of a hole in the N region. In this process the electric current does not vanish in the S and N regions (the current in the S regions is carried by pairs) while the quasiparticle flux and energy vanish.

If the electron energy ε is greater than the gap width Δ, there is a finite probability, which tends to unity for $\varepsilon \gg \Delta$, for the electron to pass into the S region and to move there as an electronlike excitation ($\xi > 0$) with the same energy. From consideration of such processes one can obtain the conditions for matching the distribution functions at the boundary and can then find the potential $\Phi(x)$ by using these conditions and the kinetic equation.[22] We, however, intending also to treat the case of a dirty superconductor, shall obtain the matching conditions and determine $\Phi(x)$ by using the microscopic equations.

d) Microscopic equations

A convenient and at the same time powerful method for investigating nonequilibrium processes in superconductors, and the effects we are considering are among these, is the method of Green's functions integrated over the variable $\xi = v(p - p_F)$. For example, on integration the retarded Green's function becomes

$$g^R = \frac{i}{\pi} \int d\xi G^R(\mathbf{p}, \mathbf{r}, \varepsilon, t). \tag{2.25}$$

Such functions and the equations for them were introduced by Eilenberger,[23] and also by Larkin and Ovchinnikov.[24] These equations were later generalized by Éliashberg[25] to the nonequilibrium case. The technique for investigating nonequilibrium processes on the basis of these functions and equations was developed in Refs. 26 and 27 and was used to analyze the effects of penetration of an electric field into a superconductor in Refs. 13, 8, and 9. Another method for investigating such effects was developed by Galaĭko.[30]

To describe nonequilibrium processes we introduce not only the functions $G^{R(A)}$, but also the function $G = i\langle\psi^+(1')\psi(1) - \psi(1)\psi^+(1')\rangle$.[28] For a superconductor, each of these functions ($G^{R(A)}$ and G) is a matrix. For example,

$$\hat{g} = \begin{pmatrix} g & f \\ -f^+ & \bar{g} \end{pmatrix}. \tag{2.26}$$

The components of $\hat{g}$ are related to one another:

$$\begin{aligned} \bar{g}_{\mathbf{p}}(\varepsilon, \mathbf{r}, t) &= g_{-\mathbf{p}}(-\varepsilon, \mathbf{r}, t) = g^*_{-\mathbf{p}}(-\varepsilon, \mathbf{r}, t), \\ f^*_{\mathbf{p}}(\varepsilon, \mathbf{r}, t) &= f^*_{-\mathbf{p}}(-\varepsilon, \mathbf{r}, t) = f^+_{\mathbf{p}}(\varepsilon, \mathbf{r}, t). \end{aligned} \tag{2.27}$$

The function $\hat{g}$ may be represented as the sum of a regular part and an anomalous part[25-27]:

$$\hat{g}_{\varepsilon_+\varepsilon_-} = \hat{g}^R_{\varepsilon_+\varepsilon_-} \operatorname{th}\frac{\varepsilon_-}{2T} - \hat{g}^A_{\varepsilon_+\varepsilon_-} \operatorname{th}\frac{\varepsilon_+}{2T} + \hat{g}^{(a)}, \tag{2.28}$$

in which $\varepsilon_\pm = \varepsilon \pm (\omega/2)$. The anomalous part $\hat{g}^{(a)}$ differs from zero only when there are deviations from equilibrium. The functions $g^{R(A)}$ and $g^{(a)}$ satisfy the equations[25-27]

$$\begin{aligned} &i(\mathbf{v}\nabla)\hat{g}^{R(A)} + [\hat{\Lambda}_\omega, \hat{g}^{R(A)}] + \frac{i}{2\tau}(\hat{g}_0^{R(A)}\hat{g}^{R(A)} - \hat{g}^{R(A)}\hat{g}_0^{R(A)}) = \hat{I}^{R(A)}_{\mathrm{ph}}, \\ &i(\mathbf{v}\nabla)\hat{g}^{(a)} + [\hat{\Lambda}_\omega, \hat{g}^{(a)}] + \frac{i}{2\tau}(\hat{g}_0^R\hat{g}^{(a)} - \hat{g}^{(a)}\hat{g}_0^A - \hat{g}^R g_0^{(a)} + \hat{g}_0^{(a)}\hat{g}^A) \\ &\quad = \int \frac{d\omega}{2\pi}\Big\{\hat{g}^R_{\varepsilon, \varepsilon'+\omega}[\hat{\Lambda}_\omega - 2\pi\varepsilon'\delta(\omega)\hat{\sigma}_z]\Big(\operatorname{th}\frac{\varepsilon'+\omega}{2T} - \operatorname{th}\frac{\varepsilon'}{2T}\Big) \\ &\qquad - [\hat{\Lambda}_\omega - 2\pi\varepsilon\delta(\omega)\hat{\sigma}_z]\Big(\operatorname{th}\frac{\varepsilon}{2T} - \operatorname{th}\frac{\varepsilon-\omega}{2T}\Big)\hat{g}^A_{\varepsilon-\omega\varepsilon'}\Big\} + \hat{I}_{\mathrm{ph}}, \end{aligned} \tag{2.29}$$

where

$$\hat{\Lambda}_\omega = [2\pi\varepsilon\delta(\omega) - \mathbf{v}\mathbf{p}_s(\omega)]\hat{\sigma}_z - e\mu(\omega)\hat{1} + i\Delta(\omega)\hat{\sigma}_y,$$

$$\hat{g}_0^{R(A)} = \int \frac{d\Omega}{4\pi}\hat{g}^{R(A)},$$

and σ_y and σ_z are the Pauli matrices. The notation $[\hat{\Lambda}_\omega, \hat{g}] = \hat{\Lambda}_\omega\hat{g} - \hat{g}\hat{\Lambda}_\omega$ is intended to include an integration over the internal frequency:

$$\hat{\Lambda}_\omega\hat{g} = \int \frac{d\omega}{2\pi}\hat{\Lambda}_\omega\hat{g}_{\varepsilon-\omega\varepsilon'}.$$

In addition, the functions $\hat{g}^{R(A)}$ satisfy the normalization and orthogonality conditions[27]

$$(\hat{g}^{R(A)})^2 = \hat{1}, \tag{2.30}$$

and

$$\hat{g}^R\hat{g}^{(a)} + \hat{g}^{(a)}\hat{g}^A = 0. \tag{2.31}$$

With the aid of $\hat{g}$ we can find the potential and the current density:

$$\mu = -\frac{1}{8}\int d\varepsilon g_\mu, \quad g_\mu \equiv \operatorname{Sp}\int \frac{d\Omega}{4\pi}\hat{g}, \tag{2.32}$$

and

$$\mathbf{j} = -\frac{e p_F}{8\pi^2}\int d\varepsilon \int \frac{d\Omega}{4\pi}\mathbf{p}\operatorname{Sp}(\hat{\sigma}_z\hat{g}). \tag{2.33}$$

Now let us find the relation between $g^{(a)}$ and the quasiparticle distribution function n (we recall that n is obtained[7] by integrating G over ε). In the case of a pure superconductor and a weak quasiclassical perturbation, the part of $\hat{g}$ averaged over the angles is related to n as follows[7]:

$$g_0 = 2\int d\xi [1 - 2n_0(\xi)][u_{\mathbf{p}}^2\delta(\varepsilon - \varepsilon_{\mathbf{p}}) - v_{\mathbf{p}}^2\delta(\varepsilon + \varepsilon_{\mathbf{p}})].$$

For the deviations from the equilibrium values of $g^{(a)}$ and δn we have

$$\begin{aligned} \theta(|\varepsilon| - \Delta)\operatorname{Sp}(\hat{\sigma}_z\hat{g}_0^{(a)}) &= (g_0^{(a)} - \bar{g}_0^{(a)})\theta(|\varepsilon| - \Delta) = -\frac{8\delta n_+\varepsilon}{\sqrt{\varepsilon^2 - \Delta^2}}, \\ \theta(|\varepsilon| - \Delta)\operatorname{Sp}\hat{g}_0^{(a)} &= (g_0^{(a)} + \bar{g}_0^{(a)})\theta(|\varepsilon| - \Delta) = -8\delta n_-, \end{aligned} \tag{2.34}$$

where the $\delta n_\pm$ are the ξ-even and ξ-odd parts of δn. Thus, the ξ-even part of δn_0 is related to the ξ-odd part of $g_0^{(a)}$, and conversely [see (2.27)].

Equations (2.28) and (2.29) enable us to investigate the effect of coordinate variations of the gap width on the field distribution in the superconductor. Now let us turn to the case of a fairly pure superconductor ($\Delta^{-1} < \tau < \tau_\varepsilon$).

e) Resistance of a superconductor with a low impurity concentration

In this case the kinetic equations (2.9) and Eqs. (2.12), (2.13), and (2.23) are valid when $x \gg \xi(T)$. They can also be obtained from Eqs. (2.28) and (2.29), and relations (2.30) and (2.31). For this we must express $\hat{g}^{(a)}$ in the form

$$\hat{g}^{(a)} = \hat{g}_0 + \hat{\mathbf{g}}_1\frac{\mathbf{v}}{v}, \tag{2.35}$$

and make use of the expression for $g_0^{R(A)}$:

$$\begin{aligned} \hat{g}_0^{R(A)} &= \hat{\sigma}_z g^{R(A)} + i\hat{\sigma}_y f^{R(A)}; \\ g^{R(A)} &= \frac{\varepsilon}{\Delta} f^{R(A)} = \frac{\varepsilon}{\xi^{R(A)}}, \end{aligned} \tag{2.36}$$

$$\xi^{R(A)} = \pm\sqrt{\varepsilon^2 - \Delta^2}\operatorname{sgn}\varepsilon\,\theta(|\varepsilon| - \Delta) + i\sqrt{\Delta^2 - \varepsilon^2}\,\theta(\Delta - |\varepsilon|).$$

We obtain an equation for g_μ of the type of (2.13),

$$\frac{D}{g^R}\frac{\partial^2}{\partial x^2}g_\mu = I_{\rm ph}(g_\mu), \tag{2.37}$$

and a relation analogous to (2.12) between the trace (Spur) of $\hat{\sigma}_z\hat{g}_1$ and g_μ:

$$\mathrm{Sp}\,(\hat{\sigma}_z\hat{g}_1) = -g^R l \frac{\partial}{\partial x} g_\mu. \tag{2.38}$$

When $x \lesssim \xi(T)$ the above relations do not hold and we must take the coordinate dependences of $g^{R(A)}$ and Δ into account. We shall obtain conditions for matching the functions g_μ and g_1 at the S–N boundary that are equivalent to the conditions for matching the quasiparticle distribution functions calculated with allowance for the Andreev reflection conditions.[22]

Let us consider the region $0 < x \ll 1$. Then the collision integrals for collisions with impurities and phonons may be dropped from Eqs. (2.28) and (2.29). In the stationary case we can easily find from (2.29) that

$$iv\frac{\partial}{\partial x}\,\mathrm{Sp}\,\hat{g}^{(a)} = 0, \quad \text{i.e.} \quad [g_\mu] = 0, \tag{2.39}$$

when $\varepsilon > \Delta$; here $[g_\mu] \equiv g_\mu^N - g_\mu^S \equiv g_\mu(0) - g_\mu(x_0)$ and $\xi(T) \ll x_0 \ll 1$. According to (2.32) this means that the potential Φ is continuous at the boundary. When $|\varepsilon| < \Delta$, we have $g_\mu^N = 0$. Let us multiply Eq. (2.29) by $\hat{\sigma}_z$ and Eq. (2.31) by $\hat{\sigma}_y$, and calculate the trace:

$$\begin{gathered} i\cos\theta v \frac{\partial}{\partial x}(g^{(a)} - \bar{g}^{(a)}) = -2\Delta\,\mathrm{Sp}\,(\hat{\sigma}_x\hat{g}^{(a)}), \\ \mathrm{Sp}\,(\hat{\sigma}_x\hat{g}^{(a)}) = -\frac{i}{2}\frac{\mathrm{Im}\,(f^R + f^A)}{\mathrm{Re}\,g^R}\,\mathrm{Sp}\,\hat{g}^{(a)}, \end{gathered} \tag{2.40}$$

where $\cos\theta = v_x/v$. On calculating the corresponding component of Eq. (2.28), we find

$$\left(2\varepsilon + i\cos\theta v \frac{\partial}{\partial x}\right) f^{R(A)} = 2\Delta g^{R(A)}. \tag{2.41}$$

We require energies ε that are much larger than Δ, for which $g^{R(A)} = \pm 1$. At such energies the sum of the Fourier components $f^{R(A)}$ is

$$f^R + f^A = -4\pi i \Delta(q)\,\delta(2\varepsilon - qv\cos\theta). \tag{2.42}$$

On substituting this equation into (2.40) and integrating the first of Eqs. (2.40) over the region in which $\Delta(x)$ varies, we obtain

$$[g^{(a)} - \bar{g}^{(a)}]\cos\theta = 2\left.\frac{|\Delta(q)|^2}{v^2|\cos\theta|}\right|_{q=\frac{2\varepsilon}{v\cos\theta}} (g^{(a)} + \bar{g}^{(a)}). \tag{2.43}$$

The coefficient on the right in (2.43) is small when $\varepsilon \gg \Delta$, so the coefficients of the Legendre polynomials of higher order than the first will also be small. We substitute expansion (2.35) into (2.43), calculate the Fourier component of $\Delta(x)$ [using Eq. (2.3)], and average over the angle; this yields

$$[\mathrm{Sp}\,(\hat{\sigma}_z\hat{g}_1)] = \frac{9}{32}\left(\frac{\Delta}{\varepsilon}\right)^4 g_\mu \equiv \gamma g_\mu. \tag{2.44}$$

The matching conditions (2.39) and (2.44), together with Eqs. (2.37) and (2.38), enable us to find the discontinuity of the field E [more accurately, the change in the field over the distance $\xi(T) \ll l_E$] at the S–N boundary. From (2.32), (2.38), and (2.44) we obtain the following expression for the discontinuity in E:

$$e[E] \equiv e(E^N - E^S) = -\frac{1}{4l}\int_0^\Delta d\varepsilon\,\mathrm{Sp}\,(\sigma_z g_{1x}) - \frac{1}{4l}\int_\Delta^\infty d\varepsilon\,\gamma g_\mu. \tag{2.45}$$

It follows from relations (2.31) that $g_\mu \approx 0$ when $\varepsilon < \Delta$. Near T_c, the main contribution comes from the second term on the right in (2.45). To find $g_\mu(0)$ we must solve Eq. (2.37) in the S and N regions and match the solutions at $x = 0$. We can simplify Eq. (2.37) considerably if we note that at high energies $\varepsilon \sim T$, the leading approximation to g_μ in Δ/T is known: it is equal to the value of g_μ in the N region far from the boundary:

$$g_\mu^N(-\infty) = -\frac{2\Phi}{T}\,\mathrm{ch}^{-2}\frac{\varepsilon}{2T}. \tag{2.46}$$

On the other hand, the term corresponding to the arrival of particles in the collision integral $I_{\rm ph}$(2.11′) for collisions with phonons [also see the connection (2.34) between n_0 and g_μ] is due to the integration over large energies $\varepsilon \sim T$. On substituting (2.45) into this part of the collision integral, Eq. (2.37) becomes

$$D\frac{\partial^2 g_\mu}{\partial x^2} = \tau_\varepsilon^{-1}\left(g_\mu + \frac{2\Phi}{T}\,\mathrm{ch}^{-2}\frac{\varepsilon}{2T}\right), \quad \Delta \ll \varepsilon \ll T. \tag{2.47}$$

Thus, in the investigated energy interval Eq. (2.47) has the same form in the N and S regions, the only difference being that $\Phi^N(x) = \Phi(0) - E^N x$, while $\Phi^S(x) = E^S l_E \exp(-x/l_E)$. On solving Eq. (2.46) with the boundary conditions (2.39) and (2.44), we obtain

$$g_\mu(0) = -\frac{l}{T}\frac{[E]\,l_\varepsilon + 2\Phi(0)}{(\gamma l_\varepsilon/2) + l}\,\mathrm{ch}^{-2}\frac{\varepsilon}{2T}, \tag{2.48}$$

where $l_\varepsilon = \sqrt{D\tau_\varepsilon}$. The characteristic energy ε_x below which (2.48) differs from (2.46) is $\varepsilon_x = \sqrt{3/8}\,\Delta(l_\varepsilon/l)^{1/4}$. Using (2.48), we determine the field discontinuity from (2.45):

$$\begin{gathered} [E] = \frac{(3\pi)^{1/2}}{4}\left(\frac{\Delta}{T}\right)^{1/2}\left(\frac{l_\varepsilon}{l}\right)^{1/4} E^S, \\ 1 \ll \left(\frac{l_\varepsilon}{l}\right)^{1/4} \ll \frac{T}{\Delta}. \end{gathered} \tag{2.49}$$

Here we have rewritten the condition for the applicability of (2.47), using the value of ε_x. The contribution to the resistance (per unit area) of the S–N system introduced by the superconductor is[8,9]

$$\rho^S = \frac{l_E}{\sigma}\left[1 + \frac{(3\pi)^{1/2}}{4}\left(\frac{\Delta}{T}\right)^{1/2}\left(\frac{l_\varepsilon}{l}\right)^{1/4}\right]^{-1}. \tag{2.50}$$

Thus, Andreev reflection of quasiparticles leads to perturbation of the distribution functions in the S and N regions over a distance of l_ε and to a discontinuity of the field determined both by the x dependence of Δ and by the energy relaxation mechanism. The quantity l_ε may be considerably greater than the mean free path l ($(l_\varepsilon/l)^{1/4} = (\tau_\varepsilon/3\tau)^{1/8}$). Nevertheless, because of the weak dependence of $[E]$ on τ_ε/τ the discontinuity in E near T_c is not large and a field E with amplitude E^S close to E^N penetrates into the S region to the depth l_E (Fig. 6).

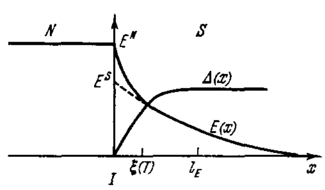

FIG. 6. Electric field strength E vs the coordinate x when a current I flows across the S-N boundary in the case of a superconductor having a gap Δ. The $\Delta(x)$ dependence is also shown.

f) Resistance of a superconductor with a high impurity concentration

Let the impurity concentration be such that the condition $\tau T \ll 1$ is satisfied.[4)] Then all the functions can again be expressed as sums of two Legendre polynomials (2.35). The main difference between this case and the preceding one is that now formulas (2.36) remain valid for the isotropic parts of $g^{R(A)}$ for all values of x, since the gradient terms in the Eilenberger equations (2.28) turn out to be small. The following relation holds[25-27] for $g^{R(A)}$:

$$\hat{g}_1^{R(A)} = -l\hat{g}_0^{R(A)}\nabla\hat{g}_0^{R(A)}. \tag{2.51}$$

Using the known form (2.36) for $\hat{g}_0^{R(A)}$ and (2.51) for $\hat{g}^{R(A)}$, we can obtain the desired equation for g_μ, which is valid for all x [including the region in which $\Delta(x)$ varies].[8,9,13] On substituting $\hat{g}^{(a)}$ into Eq. (2.29), which is considerably simpler in the stationary case now under consideration, we obtain the following equations, which are similar in structure to Eqs. (2.12) and (2.13):

$$\frac{l}{3} v\nabla g_1 + \xi^R [\hat{g}_0^R, \hat{g}_0] = \hat{I}_{\text{ph}}(\hat{g}_0), \tag{2.52}$$

$$iv\nabla\hat{g}_0 + \frac{i}{2\tau}(\hat{g}_0^R\hat{g}_1 - \hat{g}_1\hat{g}_0^A - \hat{g}_1^R\hat{g}_0 + \hat{g}_0\hat{g}_1^A) = 0. \tag{2.53}$$

Using relations (2.30) and (2.31), we can easily obtain the following equation from (2.53):

$$\hat{g}_1 = -l(\hat{g}_0^R\nabla\hat{g}_0 + \hat{g}_0\nabla\hat{g}_0^A). \tag{2.54}$$

This equation is analogous to (2.12). The second term differs from zero in the region in which $\Delta(x)$ varies. Now let us substitute $\hat{g}_1$ from (2.54) into Eq. (2.52), multiply the equation by $\hat{g}_0^R$, and calculate the trace for $|\varepsilon| > \Delta$ (as was already noted, $g_\mu = 0$ when $|\varepsilon| < 0$). Taking Eqs. (2.30), (2.31), and (2.36) into account, we obtain

$$-iD\{\nabla^2 g_\mu - \text{Sp}[(\nabla\hat{g}_0^R)^2 g_\mu]\} = \text{Sp}(\hat{g}_0^R\hat{I}_{\text{ph}}).$$

On substituting the explicit form of $\hat{g}_0^R$ from (2.36) we obtain

$$D\left[\frac{\xi}{i|\varepsilon|}\nabla^2 g_\mu - \frac{\varepsilon(\nabla\Delta)^2}{\xi^3} g_\mu\right] = I_{\text{ph}} = iI_z - \frac{\Delta}{\varepsilon} I_y, \tag{2.55}$$

where $I_{y,z} = \text{Sp}(\hat{\sigma}_{y,z}\hat{I}_{\text{ph}})$. The collision integral for collisions with phonons is again determined by expression (2.11′) and agrees with the right-hand side of (2.47) when $\Delta \ll \varepsilon \ll T$.

The solution to Eq. (2.55) is to be sought in the same way as in the case of a superconductor with a low impurity concentration. If $l = \sqrt{D\tau_\varepsilon} \gg \xi(T)$, the second term in brackets in (2.55) can be replaced by

$$\delta(x)\frac{\varepsilon}{\xi^3} g_\mu \int_0^\infty dx \left(\frac{\partial\Delta}{\partial x}\right)^2 = \frac{\varepsilon}{\xi^3}\frac{\sqrt{2}}{3}\frac{\Delta^2}{\xi(T)}\delta(x). \tag{2.56}$$

It is further necessary to solve Eq. (2.55) to the left and right of the boundary, dropping the second term in brackets [in this case Eq. (2.55) coincides with (2.47)], and then to match the solutions, using the continuity of g_μ and the matching condition for $[\partial g_\mu/\partial x]$ given by the function (2.56). When $\Delta \ll \varepsilon \ll T$ and $x = 0$, the function $g_\mu(x)$ takes the form

$$g_\mu(0) = -\frac{2}{T}\frac{[E]\,l_\varepsilon + 2\Phi(0)}{2 + (\varepsilon_0/\varepsilon)^2}\,\text{ch}^{-2}\frac{\varepsilon}{2T}, \tag{2.57}$$

where

$$\varepsilon_0^2 = \frac{\sqrt{2}}{3}\frac{\Delta^2 l_\varepsilon}{\xi(T)}.$$

The field discontinuity, calculated with (2.45), is

$$[E] = \sqrt{\frac{\pi\Delta l_\varepsilon}{3\sqrt{2}\,T\xi(T)}}\,E^S, \quad 1 \ll \sqrt{\frac{l_\varepsilon}{\xi(T)}} \ll \frac{T}{\Delta}. \tag{2.58}$$

In this case the contribution from the superconducting region to the resistance of the S–N system is[8,9]

$$\rho^S = \frac{l_E}{\sigma}\left(1 + \sqrt{\frac{\pi}{3\sqrt{2}}}\sqrt{\frac{\Delta l_\varepsilon}{T\xi(T)}}\right)^{-1}. \tag{2.59}$$

The T dependence of ρ^S obtained in (2.59) differs from the results of Ref. 13 obtained with the aid of a computer and from those of a more recent paper[55] in which the case of arbitrary temperatures was analyzed.

g) Other quasiparticle-current relaxation mechanisms (paramagnetic impurities, condensate flow, anisotropy)

There are also other effective mechanisms for relaxation of the branch imbalance Q that determine the field penetration depth. These include scattering from paramagnetic impurities,[8,9,13,29] relaxation of Q in a superconductor in which a sufficiently strong condensate current j_s flows (so that the term p_s^2 must be taken into account),[30] and also anisotropy of the superconductor.[9,11]

As regards the first mechanism, it was partially analyzed in Section a) of Division 2, where it was assumed that the concentration of paramagnetic impurities was high ($\tau_s T \ll 1$). In that case, however, the basic characteristics of the superconductor—the density of states, the critical temperature, etc.—are strongly altered. It is interesting to examine the case of low impurity concentration ($\tau_s\Delta \gg 1$), in which the effect of paramagnetic impurities on the thermodynamic characteristics may be neglected. In this case the equation for g_μ has the form of Eq. (2.55), but differs from it in having the additional term $2\Delta^2(\varepsilon^2\tau_s)^{-1}g_\mu$ on the right, which describes the relaxation of g_μ due to scattering by paramagnetic impurities.[8,9] If scattering by paramagnetic impurities dominates scattering by phonons ($\tau_s \ll \tau_\varepsilon(\Delta/T)^2$), the solution to this equation in the S region (neglecting the second term on the left, i.e. neglecting the field discontinuity at the S–N boundary) is

$$g_\mu(x) = (2E^N l_\varepsilon + \Phi(0))\,T^{-1}\left(1 + \frac{\sqrt{2}\,\Delta l_\varepsilon}{\varepsilon l_s}\right)^{-1}\exp\left(-\frac{\sqrt{2}\,\Delta x}{\varepsilon l_s}\right)\text{ch}^{-2}\frac{\varepsilon}{2T},$$

where $l_s = (D\tau_s)^{1/2}$. Calculating the potential $\Phi(x)$ shows that the field E falls off in the S region in the length $l_E \approx l_s T/\Delta$. If $\tau_s \gg (\Delta/T)^2\tau_\varepsilon$, however, two cases are possible: a) $\tau_s \gg \tau_\varepsilon$; then l_E is independent of τ_s and is determined by formula (2.24). b) $\tau_s \ll \tau_\varepsilon$; then $l_E = l_\varepsilon(4T/\pi\Delta)^{1/2} \times (\tau_s/2\tau_\varepsilon)^{1/4}$.[8,9,29]

Taking the finite velocity of the condensate in the S region into account leads to the appearance of a term of the form $D\Delta^2 p_s^2\varepsilon^{-2}$ in Eq. (2.55).[26,30] Since this additional term has the same structure as the term describing scattering by paramagnetic impurities, the analysis given above remains entirely valid provided l_s is replaced by $\sqrt{2}p_s^{-1}$.

In anisotropic superconductors, relaxation of the

[4)]This case was first treated by Schmid and Schön.[13]

branch imbalance Q takes place as a result of elastic scattering by ordinary impurities.[11] In this case, for sufficiently pure superconductors l_E is again equal to $\sqrt{D\tau_Q}$, while $\tau_Q = \tau T/\langle a^2\rangle\overline{\Delta}(1+(T^*/\overline{\Delta}))$, where $\overline{\Delta}$ is the gap width averaged over all directions and $\langle a^2\rangle$ is the anisotropy factor. For example, $\langle a^2\rangle = 0.02$ for tin.[11] As the impurity concentration increases (with $\Delta\tau < 1$) the effect of anisotropy decreases.

h) Resistance of a superconductor in the intermediate state

A good system for investigating field penetration effects is a superconductor in the intermediate state. By altering the magnetic field one can vary the relation between the thicknesses of the superconducting (L_S) and normal (L_N) layers (for simplicity we consider the case of alternating plane parallel layers), on the one hand, and the characteristic lengths (l, l_ε, l_E) that determine the penetration of the field into the S region, on the other hand. From measurements of the effective resistance of the system,

$$\rho^* = \frac{\overline{E}}{j}, \quad \overline{E} = (L_S + L_N)^{-1} \int_0^{L_S+L_N} E(x)\,dx \tag{2.60}$$

one can extract information on the mean free path, the energy relaxation mechanism, etc. The formulas for the field discontinuity $[E]$ at the S–N boundary and for the resistance ρ^* are not directly applicable in the present case since the solutions to the diffusion equation

$$l_\varepsilon^2 \frac{\partial^2 n_0}{\partial x^2} = n_0 - \frac{\Phi}{4T}\,\mathrm{ch}^{-2}\frac{\varepsilon}{2T}, \qquad \Delta \ll \varepsilon \ll T, \tag{2.61}$$

were chosen as functions that fall off exponentially from the boundary. However, it is not difficult to generalize the results obtained.[22]

Let us consider the most interesting case of a fairly pure superconductor of the first kind ($\Delta\tau \gg 1$). Two limiting cases are possible:

1) $l \ll l_\varepsilon, L_{S,N}$. The function n_1 that determines the current is given in the S region as before by Eq. (2.12), but in the N region by

$$n_1 = -l\,\mathrm{sgn}\,\xi\left(\frac{\partial n_0}{\partial x} + \frac{\partial n_F}{\partial \varepsilon}\frac{\partial p_s}{\partial t}\right). \tag{2.62}$$

The second term takes account of the presence of a phase difference at the N layer that increases with time. It follows from (2.62) and (2.1) that

$$j_n = \sigma E^N. \tag{2.63}$$

In this case the matching conditions (2.39) and (2.44) take the form

$$[n_0] = 0, \quad \left[\frac{\partial n_0}{\partial x}\right] + \frac{\partial n_F}{\partial \varepsilon}\frac{\partial p_s}{\partial t} = -\frac{\gamma}{l}\,n_0. \tag{2.64}$$

The solution to (2.61) is

$$n_0^{S,N} = \frac{\partial n_F}{\partial \varepsilon}\left\{-\Phi + C^{S,N}\,\mathrm{sh}\left[\left(x \mp \frac{L_{S,N}}{2}\right)\frac{1}{l_\varepsilon}\right]\right\}. \tag{2.65}$$

The constants $C^{S,N}$ are found from conditions (2.64), and the field discontinuity, from (2.45):

$$[E] = \frac{\alpha\sqrt{3\pi}}{4}\left(\frac{\Delta}{T}\right)^{1/2}\left(\frac{l_\varepsilon}{\alpha l}\right)^{1/4}\mathrm{th}\left(\frac{L_S}{2l_E}\right)E^S \equiv \eta E^S, \tag{2.66}$$

where $\alpha = (1/2)(\mathrm{cth}(L_S/2l_\varepsilon) + \mathrm{cth}(L_N/2l_\varepsilon))$. Here the field E is independent of x in the N regions, but in the S regions we have $E(x) = E^S\,\mathrm{ch}((x - L_S/2)l_E)/\mathrm{ch}(L_S/2l_E)$ (Fig. 7). The effective resistance (2.60) is given by

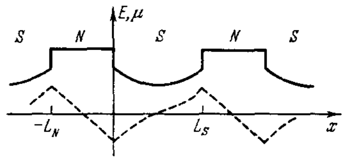

FIG. 7. Electric field strength E and potential μ (dashed curve) vs the coordinate x when a current flows through successive layers of S and N. It is assumed that $\xi(T) \ll L_{S,N}$, so the change in E over the correlation length $\xi(T)$ is represented as a discontinuity.

$$\rho^* = \frac{1}{\sigma}\frac{L_N + 2l_E(1+\eta)^{-1}\,\mathrm{th}(L_S/2l_E)}{L_N + L_S}.$$

2) $l \gg L_{N,S}$.[5)] In this case it is Andreev reflection that exerts the dominant influence on the motion of an electron (especially at low temperatures). The collision integral for collisions with impurities and phonons may be neglected in the zeroth approximation. The distribution function n_0 has the form

$$n_0^{S,N} = C^{S,N}\left(x \pm \frac{1}{2}L_{S,N}\right).$$

The matching conditions (2.64) remain in force, but in the second condition one may neglect the right-hand part. The fields $E^{S,N}$ are determined directly from formula (2.16),

$$eE^S = \frac{\partial p_s}{\partial t}\frac{L_N}{L_N + L_S}\left(1 - \mathrm{th}\frac{\Delta}{2T}\right) = \frac{\partial p_s}{\partial t} - eE^N, \tag{2.67}$$

and the electrical conductivity is given by

$$(\rho^*\sigma)^{-1} = \frac{\sigma^*}{\sigma} = 1 + \frac{L_S}{L_N}\,\mathrm{th}\frac{\Delta}{2T}.$$

This formula is also valid at low temperatures. The lower bound on T is determined by the smallness of the right-hand side of the second of the matching conditions (2.64):

$$T > \Delta\,\frac{L_S L_N}{l(L_S + L_N)}.$$

It is evident from (2.67) that at low temperatures ($\Delta \gg T$) E^S is exponentially small as compared with E^N.

3. COLLECTIVE OSCILLATIONS IN SUPERCONDUCTORS

In this Division we shall obtain equations for the linear response of a superconductor to an alternating electric field. In particular, these equations describe weakly damped collective oscillations with an acoustic-type spectrum, which exist in the superconductor at temperatures close to T_c. Such oscillations were experimentally detected by Carlson and Goldman[31] and were obtained theoretically by Schmid and Schön[32] for the limiting case of a dirty superconductor ($T\tau \ll 1$), and by the present authors[33] for the case of a pure superconductor ($\Delta\tau \gg 1$).

The possible existence of weakly damped collective excitations in superconductors has been repeatedly discussed in the literature. In 1958 Bogolyubov (see Ref. 34) and Anderson[35] considered weakly damped oscillations of the order-parameter phase, which have an

[5)] It is assumed that $L_{N,S} \gg \xi(T)$, so the Josephson effect need not be taken into account.

acoustic spectrum in the case of an uncharged Fermi liquid. These oscillations, however, involve disturbance of the electron density, so in real superconductors, owing to the Coulomb interaction, the frequency of such oscillations rises to the plasma frequency. But since the plasma frequency is several orders of magnitude higher than the width of the energy gap in a superconductor, the difference between such oscillations in a superconductor and plasma oscillations in a normal metal may be neglected.

It is well known that several types of weakly damped collective excitations with an acoustic spectrum exist in superfluid He II.[36] Of these, first, second, and fourth sound are bulk oscillations. The question arises whether oscillations of these types can propagate in superconductors. Acoustic oscillations in He II can be described in terms of the two-fluid model within the limitations of the hydrodynamic approximation. Two types of excitations, first and fourth sound, are associated with density oscillations, either of both the normal and superfluid components of the liquid (first sound), or of the superfluid component alone (fourth sound). These modes are accompanied by perturbations of the total density of the Fermi liquid; hence, as in the case of the modes considered by Bogolyubov and Anderson, the Coulomb interaction converts them into plasma oscillations.[37]

Second sound consists of temperature (entropy) oscillations; it has nothing to do with oscillations of the density of the liquid (although it is associated with oscillations of the density of excitations) and in principle it might exist in superconductors. The possibility that second sound might exist in superconductors was considered by Bardeen[38] and Ginzburg,[39] who showed that second sound would be highly damped except under very rigid conditions on the parameters of the superconductor that cannot be met in practice. This is due to the fact that the frequency of the sound must be large compared with the frequency of collisions with impurities and phonons, but small compared with the reciprocal of the time required for establishing local thermodynamic equilibrium in the quasiparticle gas (i.e. $\omega \ll \tau_{ee}^{-1}$, where τ_{ee} is the quasiparticle-quasiparticle collision time). In addition, the wavelength $\lambda = v_2/\omega$ (v_2 is the velocity of second sound) must be large compared with $v\tau_{ee}$, i.e. $\omega \ll \tau_{ee}^{-1} v_2/v$. Thus, we should have $\tau_{ph}^{-1} \ll \omega \ll \tau_{ee}^{-1} v_2/v$, and since $v_2 \ll v$, this condition on the frequency is very difficult to meet.

The collective oscillations that we shall consider in this Division are not analogous to the known acoustic oscillations in liquid He II. Neither are they associated with second sound. In particular, they do not involve the temperature and excitation-density oscillations associated with second sound. Unlike second sound, the collective oscillations under consideration cannot be obtained from the hydrodynamic equations for the superfluid liquid, which were derived under the assumption that the system is in local thermodynamic equilibrium, i.e. that the quasiparticle distribution function has the equilibrium form but with the thermodynamic characteristics varying with position and time. As will be evident from what follows, collective oscillations can exist in a superconductor only at frequencies exceeding $1/\tau_\varepsilon$ and $1/\tau_Q$, the reciprocals of the energy and branch imbalance relaxation times. In these oscillations the distribution of quasiparticles between branches is not an equilibrium distribution, and the form of this distribution is used in an essential way in deriving the equations for the oscillations. The collective modes to be considered consist of oscillations of the branch imbalance Q, accompanied by oscillations of the phase of the order parameter and, correspondingly, of the condensate velocity. Incident to these oscillations, there also arises in the superconductor an alternating longitudinal electric field $\mathbf{E} = -(\nabla\mu/e) + (\partial \mathbf{p}_s/\partial t)$, but the field is weak ($|\mathbf{E}| \ll |\nabla\mu/e|$) in the region in which the oscillations are weakly damped. The normal-excitation and condensate currents arising in the superconductor are directed opposite to one another, so that the total current vanishes at each instant. Since there is no current, and hence no magnetic field, these oscillations can exist in the bulk of the superconductor where the electric field cannot penetrate owing to the Meissner effect.

a) Equation for the electric field in the nonstationary case

Let us generalize Eqs. (2.7) and (2.21) to the nonstationary case. We shall consider a uniform superconductor. We shall impose no limitations on the impurity concentration (i.e., we treat the parameters $T\tau$ and $\Delta\tau$ as arbitrary). We shall use the equations for the anomalous Green's functions $\hat{g}^{(a)}$.

Assuming that the wavelength of the disturbance is large compared with the mean free path $l = v\tau$, we express $\hat{g}^{(a)}$ as in (2.35). From the linearized equation (2.29) for the Fourier components $\hat{g}_0$ and $\hat{g}_1$, we obtain

$$\frac{1}{3} v\mathbf{k}\hat{\mathbf{g}}_1 - \hat{\lambda}_+\hat{g}_0 + \hat{g}_0\hat{\lambda}_- = \alpha\mu_k(\omega)(g_+^R - g_-^A), \tag{3.1}$$

and

$$\mathbf{k}v\hat{g}_0 - \hat{\lambda}_+\hat{\mathbf{g}}_1 + \hat{\mathbf{g}}_1\lambda_- - \frac{i}{2\tau}(\hat{g}_+^R\hat{\mathbf{g}}_1 - \hat{\mathbf{g}}_1\hat{g}_-^A) = \alpha v\mathbf{p}_s(\omega)(\hat{g}_+^R\hat{\sigma}_z - \hat{\sigma}_z\hat{g}_-^A), \tag{3.2}$$

where

$$\alpha = \omega\left(2T\,\mathrm{ch}^2\frac{\varepsilon}{2T}\right)^{-1}, \quad \varepsilon_\pm = \varepsilon \pm \frac{\omega}{2},$$

$$\xi_\pm^{R(A)} = \xi_{\varepsilon\pm\omega/2}^{R(A)}, \quad \hat{\lambda}_\pm = \varepsilon_\pm\hat{\sigma}_z + i\Delta\hat{\sigma}_y, \quad \hat{g}_\pm^{R(A)} = \frac{\hat{\lambda}_\pm}{\xi_\pm^{R(A)}}$$

(the $\hat{g}^{R(A)}$ are the equilibrium retarded and advanced functions). We have not written the collision integral with phonons in Eq. (3.1); it has the same form as in (2.11′). To express $\hat{g}_1$ in terms of $\hat{g}_0$, we use the orthogonality condition (2.30) and condition (2.31),

$$\hat{g}_+^R\hat{g}_0 + \hat{g}_0\hat{g}_-^A = 0, \quad (\hat{g}^R)^2 = \hat{1}.$$

and obtain

$$\hat{\mathbf{g}}_1(\varepsilon, \omega) = il\alpha\beta\mathbf{p}_s(\omega)(\hat{\sigma}_z - \hat{g}_+^R\sigma_z\hat{g}_-^A) - i\mathbf{k}l\beta\hat{g}_+^R\hat{g}_0, \tag{3.3}$$

where $\beta = (1 - i\tau(\xi_+^R + \xi_-^A))^{-1}$. Substituting (3.3) into (3.1), we determine the isotropic part $\hat{g}_0$,

$$\hat{g}_0 = -\alpha\mu(\omega)\varkappa^{-1}(1 - \hat{g}_+^R\hat{g}_-^A) + i\alpha\beta\varkappa^{-1}D\mathbf{k}\mathbf{p}_s(\omega)(\hat{g}_+^R\sigma_z - \hat{\sigma}_z\hat{g}_-^A), \tag{3.4}$$

where $\varkappa = \xi_+^R + \xi_-^A + i\beta Dk^2$. It follows from Eq. (3.4) that the function $g_\Delta = \mathrm{Sp}(\hat{\sigma}_z\hat{g}_0)$, which describes the perturbation of the energy distribution of the quasiparticles and, accordingly, the perturbation of the gap width Δ, vanishes in the linear approximation in μ and $\mathbf{p}_s$. In the kinetic equation method, g_Δ corresponds to δn and is odd in ξ [see (2.34)]. The perturbation of the branch imbalance is described by the function $g_\mu = \mathrm{Sp}\hat{g}_0$, which, according to (3.4), is given by

$$g_\mu = -2\alpha\varkappa^{-1}\mu(\omega)\left[1 - \frac{\varepsilon^2 - \Delta^2 - (\omega^2/4)}{\xi_+^R\xi_-^A}\right] + 2i\alpha\beta\varkappa^{-1}D\mathbf{k}\mathbf{p}_s\left(\frac{\varepsilon_+}{\xi_+^R} - \frac{\varepsilon_-}{\xi_-^A}\right). \tag{3.5}$$

We find the function $g_1 = \mathrm{Sp}(\hat{\sigma}_z\hat{g}_1)/2$, which determines the anisotropic part of the quasiparticle distribution function, by substituting (3.4) into (3.3), multiplying (3.3) by $\hat{\sigma}_z$, and taking the trace:

$$\mathbf{g}_1 = i\alpha\beta\varkappa^{-1}l\left\{\mathbf{p}_s(\omega)\left[1 - \frac{\varepsilon^2 - \Delta^2 - (\omega^2/4)}{\xi_+^R\xi_-^A}\right](\xi_+^R + \xi_-^A) + \mathbf{k}\mu(\omega)\left(\frac{\varepsilon_+}{\xi_+^R} - \frac{\varepsilon_-}{\xi_-^A}\right)\right\}. \tag{3.6}$$

The function g_1 determines the quasiparticle current [see (2.33)].

It will be evident later that the collective oscillations will be weakly damped provided the conditions

$$Dk^2 \ll \omega \ll \Delta \ll T, \quad \omega \ll \tau^{-1} \tag{3.7}$$

are satisfied. Conditions (3.7) simplify the calculations substantially. Let us calculate the current j_n. The main contribution to the integral (2.33) comes from energies $\varepsilon \sim T$, the only exception being the term in (3.6) that contains the product $\xi_+^R\xi_-^A$, for which low energies are also important. Under conditions (3.7) and the further condition $|\xi_\pm| \gg \omega$, the functions $\varkappa$ and β have the form

$$\beta = \begin{cases} (1 - i\omega\tau\varepsilon/\xi^R)^{-1} \approx 1, & |\varepsilon| > \Delta, \\ (1 + 2\tau|\xi|), & |\varepsilon| < \Delta, \end{cases}$$
$$\varkappa = \begin{cases} \varepsilon\omega/\xi^R + iDk^2 \approx \varepsilon\omega/\xi^R, & |\varepsilon| > \Delta, \\ 2i|\xi|, & |\varepsilon| < \Delta. \end{cases} \tag{3.8}$$

Taking (3.8) into account, we can obtain the following expression for the current $\mathbf{j}_n$:

$$\mathbf{j}_n = \sigma\left[-\frac{i\mathbf{k}\mu(\omega)}{e} - i\omega\mathbf{p}_s(\omega)\left(1 + \frac{\Delta}{T}J\right)\right] = \sigma\mathbf{E} - i\omega\sigma\mathbf{p}_s(\omega)\frac{\Delta}{T}J, \tag{3.9}$$

where

$$J = \int_1^\infty \frac{dx}{\sqrt{(x^2-1)\{[x + (\omega/\Delta)]^2 - 1\}}}\{1 + [1 + 4\Delta^2\tau^2(x^2-1)]^{-1}\}. \tag{3.10}$$

To emphasize the manner in which (3.9) differs from the static case [see (2.7)] we have expressed μ in terms of E with the aid of (1.1).

J has the following asymptotic behavior:

$$J = \begin{cases} \ln\frac{\Delta}{\omega}, & \Delta\tau \ll 1, \\ \frac{1}{2}\ln\frac{\Delta}{\omega}, & \Delta\tau \gg 1. \end{cases} \tag{3.11}$$

The kinetic equation (2.9) cannot be used to obtain the correct form of the term with J in (3.9), since the kinetic equation was derived under the assumption that the characteristic values of the energies ε and ξ substantially exceed ω and τ^{-1}, whereas energies $\xi \sim \omega \ll \Delta$ turn out to be important in calculating J.

Let us obtain the equation of continuity for the quasiparticle current, using formula (2.32) and expression (3.5). Here it must be borne in mind that the integral of the first term in (3.5) over the energy region $\varepsilon \sim T$ gives μ_ω and therefore cancels with the left-hand part. Hence we cannot neglect Dk^2 compared with ω in the expression for $\varkappa$, and in integrating this term we must take the contribution from the energy region $\varepsilon \sim \Delta$ into account. As a result, we obtain

$$i\omega\frac{\pi}{4}\frac{\Delta}{T}\mu = Dk^2\mu + D\omega\mathbf{k}\mathbf{p}_s. \tag{3.12}$$

Now let us perform an inverse Fourier transformation and, as was done in the static case, take the phonon collision integral into account:

$$\frac{\pi}{4}\frac{\Delta}{T}\left(\frac{\partial}{\partial t} + \tau_\varepsilon^{-1}\right)\mu = -D\nabla\mathbf{E}. \tag{3.13}$$

We also write the expression for the superconduction current, which is determined by the regular part of $\hat{g}$ [see Section d) of Division 2]. When conditions (3.7) are satisfied, $\mathbf{j}_s$ has the form

$$\mathbf{j}_s = \frac{eN_s}{m}\mathbf{p}_s, \qquad N_s = \frac{m^2p_Fe\Delta^2}{2\pi T}Dy(T\tau), \tag{3.14}$$

where

$$y(T\tau) = \frac{8}{\pi^2}\sum_1^\infty\{(2n+1)^2[(2n+1)\cdot 2\pi T\tau + 1]\}^{-1} = \begin{cases} 1, & \tau T \ll 1, \\ \frac{7\zeta(3)}{2\pi^3 T\tau}, & \tau T \gg 1. \end{cases}$$

When conditions (3.7) are satisfied, Eqs. (3.9), (3.13), and (3.14), together with Maxwell's equations, determine the response of a superconductor to a longitudinal field $\mathbf{E}$.

In particular, it follows from these equations that the law according to which an alternating field E of frequency ω falls off within the superconductor is determined by the wave number

$$k_\omega^{-1} = il_E\sqrt{(1 - i\omega\tau_\varepsilon)\left[1 - \frac{2i\omega T}{\pi\Delta^2 y(T\tau)}\right]}\Big/\sqrt{1 - \frac{2i\omega J}{\pi\Delta y(\tau T)}}. \tag{3.15}$$

In the static case ($\omega = 0$), Eq. (3.15) reduces to the well known result $k_0^{-1} = il_E$.

b) Spectrum of the collective oscillations

To calculate the spectrum of the collective excitations of a superconductor one must find the range of values of ω and k for which Eq. (3.13) has a nontrivial solution with $j = j_n + j_s = 0$. The corresponding dispersion equation has the form

$$\omega^2 = D\frac{N_s}{\sigma m}\frac{4T}{\pi\Delta}(k^2 + l_E^{-2}) - i\omega\left(\frac{N_s}{\sigma m} + \frac{1}{\tau_\varepsilon}\right) - \frac{4}{\pi}i\omega Dk^2. \tag{3.16}$$

The oscillations described by (3.16) will be weakly damped provided the conditions[6)]

$$\tau_\varepsilon^{-1}, \quad \frac{N_se^2}{\sigma m} \ll \omega \ll \frac{T}{\Delta}\frac{N_se^2}{\sigma m}\frac{1}{J}. \tag{3.17}$$

are satisfied. They have an acoustic spectrum

$$\omega = k\sqrt{2D\Delta y(T\tau)}\left[1 - \frac{i\pi\Delta^2}{4\omega T}y(\tau T) - \frac{i\omega J}{\pi\Delta y(\tau T)}\right] \tag{3.18}$$

and their propagation velocity $V = \mathrm{Re}(\omega/k)$ is

$$V = v\begin{cases} \sqrt{\frac{7\xi(3)}{3\pi^3}\frac{\Delta}{T}}, & T\tau \gg 1, \\ \sqrt{\frac{2\tau\Delta}{3}}, & T\tau \ll 1. \end{cases} \tag{3.18'}$$

In the same limiting cases, conditions (3.17) for weak damping can be rewritten as follows:

$$\max\left\{\tau_\varepsilon^{-1}, \tau^{-1}\left(\frac{\Delta}{T}\right)^2\right\} \ll \omega \ll \tau^{-1}\frac{\Delta}{T}, \quad T\tau \gg 1,$$
$$\max\left\{\tau_\varepsilon^{-1}, \frac{\Delta^2}{T}\right\} \ll \omega \ll \Delta, \quad T\tau \ll 1. \tag{3.19}$$

---

6) It can be shown that in pure superconductors ($\omega \gg 1/\tau$) the oscillations die out because of Landau damping.

Thus, the propagation velocity of the oscillations differs somewhat from the Fermi velocity v by a factor $\approx 0.5(1-(T/T_c))^{1/4}$ when $\tau T \gg 1$ and by a factor $\approx 1.5((T_c - T)\tau^2 T)^{1/4}$ when $\tau T \ll 1$. It is evident from conditions (3.17) and (3.19) that the oscillations will be weakly damped in a temperature interval that is close to T_c and is bounded both above and below. The conditions for weak damping obtained in the earliest studies[32,33] in which the spectrum of the oscillations was calculated differed somewhat from those given above. Thus, in Ref. 32 the damping that limits the frequency from above was not taken into account, and in Ref. 33, in which the kinetic equation was used, the factor $\ln(\Delta/\omega)$ was not present. Ovchinnikov[40] found the exact conditions for damping in the case of arbitrary impurity concentration.[7)] He also explained how the presence of a steady superconduction current increases the damping of the oscillations. This phenomenon is in agreement with the conclusion drawn in Section g) of Division 2 that the branch imbalance relaxation rate is increased in the presence of p_s. The presence of paramagnetic impurities also leads to the same effect. The spectrum of the oscillations for the case of low concentration of magnetic impurities ($\tau_s\Delta \gg 1$) was obtained by Galaĭko *et al.*[41] The corresponding oscillations in superconductors having a high concentration of magnetic impurities was investigated in Refs. 42 and 43. In those studies, however, the charge of the electron and the presence of the field E were not taken into account.[44] Taking them into account led to additional damping of the oscillations, which turned out to be small only in the practically unachievable temperature interval $1-(T/T_c) < 4\cdot 10^{-6}$.

c) Experimental observation of the collective modes

Weakly damped collective excitations in superconductors were detected experimentally by Carlson and Goldman,[31] who measured the current-voltage characteristic of an Al-*I*-Pb Josephson tunnel junction and determined the contribution δI to the single-particle current due to order-parameter phase fluctuations $\delta\chi$. The contribution δI from the fluctuations could be determined by applying a magnetic field that was strong enough to suppress the Josephson supercurrent and thus allow the single-particle current to be measured. As Ferrell[45] and Scalapino[46] showed, the presence of intrinsic oscillation modes $\mathbf{p}_s = (i/2)\mathbf{k}\delta\chi$ in the superconductor leads to the appearance of peaks in the dynamic structure factor $S(\mathbf{k},\omega) = \langle \Delta^*(\mathbf{r},t)\Delta(0,0)\rangle_{\mathbf{k},\omega}$ at the corresponding values of ω and $\mathbf{k}$. On the other hand, $\delta I \sim S(\mathbf{k},\omega)$, where the frequency ω is related to the potential at the Josephson junction by the formula $2eU = \hbar\omega$, and the wave vector $\mathbf{k}$ is related to a weak applied magnetic field H: $k = 2eH(\lambda + (1/2)d)(\hbar c)^{-1}$,[47] where d is the thickness of the aluminum film and λ_L is the London penetration depth into the lead ($d_{Pb} > \lambda_L$). The measurements were made at a temperature T close to T_{cAl} so the fluctuations $\delta\chi$ in the lead film could be neglected. The mean free path in the aluminum film was very short, so that the condition $\tau T \ll 1$ for a dirty superconductor was satisfied. From measurements of δI as a function of U and H, the relation between U and H corresponding the maximum of δI was determined. It was found that $U \sim H$, which corresponds to a linear relation between ω and k. Figures 8 and 9 show the experimental and theoretical dependences of S on ω for fixed k, and the dispersion law for the collective oscillations. It will be seen that the theory is in good agreement with experiment.

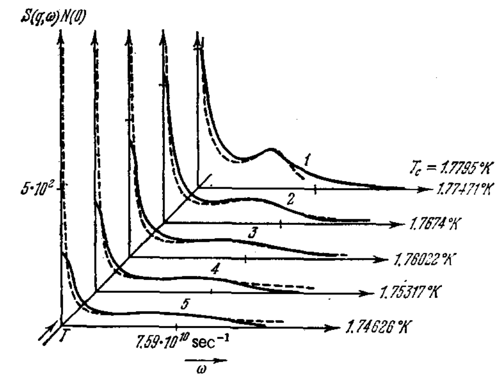

FIG. 8. Dynamical structure factor $S(q,\omega)$ for various values of q: $Dq^2/T = 0.02$ (1), 0.04 (2), 0.06 (3), 0.08 (4), and 0.1 (5). The dashed curves represent theory,[53] and the full curves, experiment.

Thus, weakly damped oscillations of acoustic type can propagate, under certain conditions, in the electron Fermi liquid of a superconductor, just as they can in other superfluid liquids. Refined experimental technique was required to detect the oscillations, since in the Carlson-Goldman experiment[31] the amplitude of the oscillations was determined by thermal fluctuations and was therefore very low. The problem of observing collective modes in superconductors would evidently be simplified if the oscillations could be excited by external action. As was noted above, what oscillates in the modes under consideration is the branch imbalance Q, which is brought about by divergence of the quasiparticle or Cooper-pair current. Such oscillations can therefore be excited in a nonuniform system. In the following Division we shall show that collective oscillations analogous to those considered above can actually

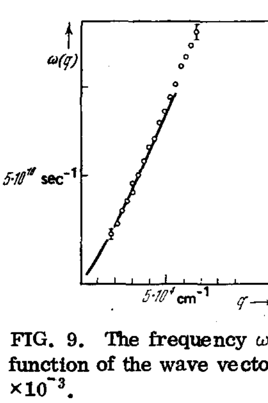

FIG. 9. The frequency ω at which $S(q,\omega)$ is maximum as a function of the wave vector q (Ref. 53). $t = (T_c - T)/T = 6.3 \times 10^{-3}$.

[7)]The spectrum of the collective oscillations has also been calculated in a recent study[56] for arbitrary frequency and impurity concentration. In that study, however, there was no damping of the collective oscillations that would limit the frequency from above.

be excited in a system consisting of a number of Josephson bridges lying close together and connected in series.

4. THE JOSEPHSON EFFECT AND LONGITUDINAL ELECTRIC FIELDS

The large penetration depth of an electric field into a superconductor in the presence of collective oscillations leads to interesting features in the behavior of weakly coupled superconductors.[48] This is especially the case for those types of weak coupling in which there is no concentration of the total current.

That is the situation, for example, in Mercereau-Notarys bridges,[49] in which the gap depends on a single coordinate and is locally depressed by the proximity effect (Fig. 10). The resistance of such Josephson bridges is determined by the field penetration depth.

Let us consider a simple model of such a junction.[48] Let the critical temperature T_c of a thin strip depend on the coordinate x measured along the strip and let it be lower in a certain small section of the strip than elsewhere: $T_c(x)=T_c^*$ when $|x|<d$, and $T_c(x)=T_c>T_c^*$ when $|x|>d$ [d is of the order of the correlation length $\xi(T)$]. Let us assume that the temperature is close to T_c and that the parameter $v_0^2=(T-T_c^*)/(T_c-T)$ is much greater than unity. In addition, we shall assume that the mean free path is short ($\tau T \ll 1$). Then the critical current j_c of the bridge will be exponentially small compared with the Ginzburg-Landau critical current j_{GL} for a uniform strip: $j_c=(3\sqrt{3}/2)j_{GL}(2v_0\,\mathrm{sh}[2v_0 d/\xi(T)])^{-1}$. In the weak coupling region ($|x|<d$), under this condition, the anomalous terms, which describe the deviations of the distribution functions from their equilibrium values, make only a small contribution, and the equation for the complex order parameter $\hat{\Delta}$ reduces to the Ginzburg-Landau equation, from which the cubic term may be dropped because it is small:

$$\xi^2(T)\frac{\partial^2\hat{\Delta}}{\partial x^2}-v_0^2\hat{\Delta}=0. \tag{4.1}$$

When $|x|>d$, the modulus of $\hat{\Delta}$ is the same as in the equilibrium case, since in this region the effect of the superconduction current on $|\hat{\Delta}|$ is small: $\hat{\Delta}(x)=\Delta\,\mathrm{th}((x+x_0)/\sqrt{2}\,\xi(T))\exp(i\chi(x))$. Matching this function to the solution of (4.1) at $|x|=d$ and using expressions (3.9) and (3.14) for the currents j_n and j_s, we obtain

$$j=\sigma E(t)+j_c\sin\varphi, \tag{4.2}$$

where $\varphi=2\chi(d)$ is the phase difference in the case of weak coupling and $E(t)$ is the field in the region $|x|<d$, where it is independent of the coordinates. To close Eq. (4.2) we must find the relation between $E(t)$ and $\varphi(t)$. To find this relation we express the Fourier component of the field $E(t)$ in terms of $\nabla\mu$, using the equation of continuity for the total current $j=j_n+j_s$ and Eq. (1.1):

$$eE_\omega(x)=-\frac{\partial\mu_\omega}{\partial x}\left(1-i\Omega\frac{\Delta^2}{\Delta^2(x)}\right)^{-1}, \tag{4.3}$$

where $\Omega=2T\omega/\pi\nabla^2$. Substituting this expression into (3.13), we obtain the equation for $\mu_\omega(x)$:

$$\frac{\partial}{\partial x}\left[\frac{\partial\mu_\omega}{\partial x}\left(1-\Omega i\frac{\Delta^2}{\Delta^2(x)}\right)^{-1}\right]=\frac{\pi\Delta}{4TD}(-i\omega+\tau_\varepsilon^{-1})\mu_\omega.$$

We shall solve this equation under the assumption that the penetration depth $|k_\omega^{-1}|$ of μ into the S region exceeds $\xi(T)$. Finding $\mu_\omega(x)$ and taking account of the relation

$$[\mu_\omega(d)-\mu_\omega(-d)]=\frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)_\omega-2eE_\omega d,$$

we determine the desired relation between the Fourier component E_ω of the field in the bridge region ($|x|\leqslant d$) and $(\partial\varphi/\partial t)_\omega$:

$$eE_\omega=\frac{1}{4}\left(\frac{\partial\varphi}{\partial t}\right)_\omega[d+ik_\omega^{-1}(1-i\Omega)]^{-1}.$$

We have assumed for simplicity that $|k_\omega|\xi(T)v_0^2\ll 1$. This allows us to neglect the dependence of Δ on x in (4.3) and in the equation for $\mu_\omega(x)$. Expression (3.15) for k_ω simplifies in the case of a dirty superconductor at frequencies $\omega\ll\Delta$: $k_\omega^2=(k'+ik'')^2=-\pi(-i\omega+\tau_\varepsilon^{-1})\times(1-i\Omega)(\Delta/4TD)$. The junction potention $U_\omega=2E_\omega(d+ik_\omega^{-1})$ is the sum of the potential drop $2E_\omega d$ in the case of weak coupling and the quantity $2ik_\omega^{-1}E_\omega$, which is due to the penetration of the field E into the superconductor at the critical temperature T_c. From the expression for U_ω and Eq. (4.3) for E_ω it follows that the Josephson relation between U_ω and $(\partial\varphi/\partial t)_\omega$ is not satisfied when $\omega\gtrsim\Delta^2/T,\tau_\varepsilon^{-1}$. However, this relation is satisfied for the time averages $(\bar{U}\sim U_{\omega=0})$ as it should be: $2e\bar{U}=\overline{\partial\varphi/\partial t}$.

The relation between the average potential $\bar{U}$ and $\partial\varphi/\partial t$ will differ from the Josephson relation if the normal-metal electrodes used to measure the potential $\bar{U}$ are close enough to the weak coupling point.[3] In this case the field and the potential will not vanish near the measuring electrodes, and the potential drop between symmetrically disposed electrodes will be $\bar{U}=2\bar{E}(d+l_E(1-\exp(L/l_E))$, where $2(d+L)$ is the distance between the electrodes. The difference from the Josephson relation in the case of a tunnel junction arises in just the same way when the electrodes used to measure U are close enough to the quasiparticle injection point, where the potential μ differs from zero.[3,50]

The collective phenomena considered in the preceding Division manifest themselves especially clearly in a system of proximity-effect Josephson bridges connected in series. In practice, the number N of junctions may reach 2000, the distance between neighboring junctions may be of the order of a micron, and the spread of the values of the critical currents may amount to ~10%.[51] It has been found that the Josephson oscillations in the junctions take place synchronously, i.e. with the same frequency and phase, over a fairly wide range of cur-

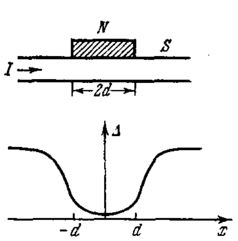

FIG. 10. Schematic diagram of a proximity-effect Josephson junction: S—superconducting strip, N—normal strip. The dependence of the gap width Δ on the coordinate x measured along the strip is also shown.

rents. Let us obtain an equation for the interaction of the junctions. We shall consider N junctions (Fig. 11). Assuming again that $|k_\omega|\xi(T)v_0^2 \ll 1$, we use the equation for $\mu_\omega(x)$ to the left (−) and to the right (+) of the n-th junction to obtain

$$\mu_n^\pm = A_n^\pm \cos k_\omega x + B_n^\pm \sin k_\omega x. \tag{4.4}$$

Proceeding as in the case of a single junction, we express $\mu_n^+(d) - \mu_n^-(-d)$ in terms of the phase difference φ_n and the field E_n. Using Eq. (4.3) we express the fields $E_{n\pm1}(\omega)$ in terms of the $\mu_n^\pm(\pm L)$ and add them:

$$\frac{1}{2}(E_{n+1}+E_{n-1}) = \frac{1}{4}\left(\frac{\partial\varphi_n}{\partial t}\right)_\omega k_\omega (1-i\Omega)^{-1}\sin k_\omega L + E_n[\cos k_\omega L - k_\omega d(1-i\Omega)^{-1}\sin k_\omega L]. \tag{4.5}$$

This equation, together with Eq. (4.2) (written for each junction) describes the interaction between the junctions and makes it possible to determine, for example, the current-voltage characteristic of the system. The characteristic interaction length $(k_\omega'')^{-1}$ may be substantially greater than the correlation length $\xi(T)$, This was just the situation in the experiment of Palmer and Mercereau[51] (the correlation length in the niobium bridges used in this experiment was ~100–200 Å). Synchronism is lost at high enough frequencies $[(k_\omega'')^{-1}$ decreases with increasing $\omega]$. An equation of the type (4.5) for the interaction of the junctions will be valid not only in the case of one-dimensional bridges, but also in the case of pinched bridges. This is associated with the fact that the total quasiparticle current remains unchanged on passing from one bridge to the next, provided $Lk_\omega'' < 1$. Consequently, the field E_n will affect the fields $E_{n\pm1}$.

The spectrum of the collective oscillations in a system of Josephson junctions is distorted, and bands of forbidden and allowed frequencies appear; in addition, weakly damped oscillations are possible at frequencies at which oscillations in a uniform superconductor would be damped out. To find the form of the spectrum we use Eq. (4.2) to express the field E_n for $j=0$ as $E_n = -(j_c/\sigma)\varphi_n$ ($\varphi_n \ll 1$) and transform to the collective variable $E_n = \sum_q E_q \exp(inqL)$ in (4.5). If the conditions

$$\tau_\varepsilon^{-1} \ll \omega, \quad \frac{\Delta^2}{T} \ll \omega \ll \Delta, \tag{4.6}$$

are satisfied the dispersion equation will have the form

$$\lambda\frac{\omega}{\omega_0}\sin\frac{\omega}{\omega_0} = \cos\frac{\omega}{\omega_0} - \cos qL,$$

where $\lambda = \omega_0^2/\omega_J^2$, $\omega_J^2 = (16DTj_c/\pi L\sigma\Delta)$, and $\omega_0 = \sqrt{2D\Delta}/L$. We shall consider two limiting cases: a) $\lambda \ll 1$. In this case when $|\omega/\omega_0 - \pi n| \gg \lambda$ we obtain the spectrum of the oscillations of a uniform superconductor, $\omega = q\sqrt{2D\Delta}$. When $|\omega/\omega_0 - \pi n| \lesssim \lambda$ there is a splitting of the branches due to the interaction of the junctions, and bands of allowed and forbidden frequencies appear. The separation between the bands is $\delta\omega_n = 2\pi n\lambda\omega_0$. b) $\lambda \gg 1$. Bands also appear in this case. In the first band the spectrum has the form of the spectrum of acoustic phonons in a crystal: $\omega = \omega_J \sin(qL/2)$. It is interesting that the oscillations in the first band are weakly damped even when the second of conditions (4.6) is not satisfied, i.e. when they would be damped in a uniform superconductor. The oscillations in the other bands are weakly damped under the conditions (4.6). Their spectrum is determined by the formula $\omega/\omega_0 = \pi n + (\lambda\pi n)^{-1}(1-(-1)^n \times \cos(qL))$, $n = 2, 3, 4, \ldots$. The spectrum of the oscillations is shown in Fig. 12. The presence of weakly damped collective modes leads to peculiarities of the impedance and of the current-voltage characteristic of the system. The impedance of the system has the form

$$Z(\omega) = \frac{U(\omega)}{I(\omega)} = -\frac{i\Omega}{1-i\Omega}\left[1 - \frac{\mathrm{tg}(k_\omega L/2)}{i\Omega k_\omega L\lambda - (1-i\Omega)\,\mathrm{tg}(k_\omega L/2)}\right].$$

When conditions (4.6) are satisfied, impedance peaks appear at frequencies that satisfy the equation $\mathrm{tg}(\omega/2\omega_0) = -\lambda(\omega/\omega_0)$. The peculiarities of the current-voltage characteristic appear in the case of potentials $U = \pi\omega_0 n/e$ $(n = 1, 2, \ldots)$. No experimental study of the collective modes of a system of Josephson junctions has yet been conducted.

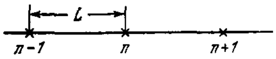

FIG. 11. Schematic representation of a system of Josephson bridges connected in series (each bridge is indicated by a cross).

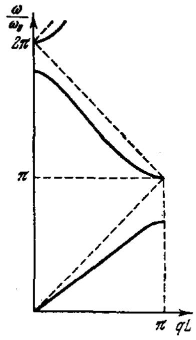

FIG. 12. Spectrum $\omega(q)$ of the collective oscillations of a system of Josephson bridges connected in series. The dashed lines show the spectrum in a uniform superconductor.

5. CONCLUSION

The effects of the penetration of an electric field **E** into a superconductor discussed above are nonequilibrium phenomena, which may be elicited, for example, by injecting quasiparticles into a superconductor. Other nonequilibrium phenomena in superconductors include effects that arise, for example, when electromagnetic radiation acts on a superconductor (depression of Δ under the action of laser light or, conversely, increase of Δ near T_c under the action of uhf waves) (see the review articles of Refs. 52, 55, and 57). The difference between these phenomena is that in the former case the perturbed part δn of the distribution function is asymmetric in ξ_p (the appearance of electron-hole asymmetry), while in the latter case δn is symmetric in ξ_p, i.e. the number of electronlike excitations remains equal to the number of holelike ones. The branch imbalance arises when the divergence of the quasiparticle (or Cooper-pair) current does not vanish. This means that nonequilibrium phenomena of the first type (in which the perturbed part of the distribution function is asymmetric in ξ_p), unlike the nonequilibrium phenomena of the second type, can appear only in nonuniform systems (or in uniform systems under the action of nonuniform perturbations). In addition, in this case the deviation from equilibrium is linear in the perturbation. This makes it possible in some cases to obtain accurate results for observed quantities directly from the microscopic equations, or from simpler equations (the kinetic equation or the generalized Ginzburg-Landau equations) derived on the basis of the microscopic theory.

The principal result of the investigations examined in this review is the establishment of the fact that a longitudinal field **E** can penetrate into a superconductor to a depth l_E much greater than the lengths $\xi(T)$ and λ_L characteristic of the superconductor. A magnetic field, however, penetrates no farther into the body of the superconductor (when $\Delta \neq 0$) than to the London depth λ_L. We may therefore say that the expulsion of a magnetic field is a more fundamental property of a superconductor than the lack of electrical resistance to a steady current.[54]

Another important result is the detection near T_c of weakly damped collective oscillations of **E** and $\mathbf{p}_s$, the search for which began as soon as the microscopic theory of superconductivity was constructed. Both these facts—the penetration of a static field **E** and weakly damped oscillations of **E**— have been fairly well verified by experiment. These phenomena, however, will doubtless continue to be investigated. In particular, it would be very interesting to measure l_E for pure superconductors, where it might be comparable with the dimensions of the specimen. As was already noted, the study of the penetration of a field E into a superconductor near T_c is of the greatest interest, since at low temperatures the discontinuity in the field at the S–N boundary is large while the field strength at depths greater than $\xi(T)$ in the S region is exponentially small. Even at low temperatures, however, the penetration of a field E into the S region may be accompanied by interesting phenomena— for example, by a logarithmic growth of $E(x)$ in the superconducting and normal regions at distances from the S–N boundary of the order of the mean free path (see Ref. 22 and papers cited in Ref. 9). In addition, collective excitations have not been investigated experimentally either in superconductors with a low impurity concentration or in a system of Josephson junctions.

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