Polarization phenomena in electronic and atomic collisions

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A systematic review is given of the phenomenon of spin polarization in electronic and atomic collisions, which has recently been the subject of experimental investigation. These experiments are topical and important because polarization phenomena are associated with interference and thus constitute a very precise and sensitive means of investigating the structure and properties of matter and of analyzing physicochemical processes. A unified theory of polarization phenomena in electronic and atomic collisions, which includes an account of the most recent known experiments, is presented. The unified description is achieved with the aid of the scattering amplitude matrix and the spin density matrix formalism. Particular attention is devoted to processes in two-particle systems consisting of particles with spins 1/2 and 1. Processes involving a change in the spin of the target, which occurs as a result of exchange interactions, are characteristic for electron-atomic collisions. Exchange excitation of atoms and Penning ionization processes are examples of such collisions. The theory of polarization produced in such processes is reviewed and whenever possible, the results are compared with experimental data. Possible future applications of polarized electrons in the physics of electronic and atomic collisions are indicated.

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CONTENTS

1. INTRODUCTION

One of the simplest examples of a polarization effect is the reflection of a plane electromagnetic wave from the separation boundary between two media. The electric field of the wave is then characterized not only by the magnitude of the field vector but also by its direction. The reflection process, which is described by the well-known Fresnel formulas, is therefore much more complicated than, for example, the reflection of longitudinal sound waves in liquids.

Similarly, the scattering of an electron by a force center is much more complicated than the scattering of a spinless particle. The scattering of spinless particles by a force center is completely characterized by an amplitude that is a function of the angle of scattering and the particle energy. In contrast to this, the scattering of particles with spin is characterized not only by a dependence on angle and energy, but also by a dependence on the particle spin component along some special direction before and after scattering, i. e., it is described not by one but by several amplitudes which together form a matrix of the spin variables. We shall

[1)] Spins will be given in units of $\hbar$ throughout this paper.

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refer to this matrix as the amplitude matrix M. For example, when an electron is scattered by a central force, the number of different combinations of electron spin components before and after scattering is four. Consequently, four amplitudes are required for a complete description of the scattering situation. In the case of a collision between two spin 1/2 particles,[1)] the number of different combinations of spin components of the projectile and the target before and after scattering is 16, so that 16 amplitudes are necessary. They form a 4×4 amplitude matrix. It is clear that, in general, the amplitude matrix for the collision of particles with spins s_1 and s_2 has the dimensionality $(2s_1+1)^2(2s_2+1)^2$.

However, not all of these amplitudes are independent, and the actual number of independent amplitudes is less than that indicated above. It will be seen below that the number of such independent amplitudes in the amplitude matrix can be established from relatively simple theoretical considerations based on the invariance of this matrix.

A detailed study of the scattering of particles with spin cannot be based simply on the differential cross section givning the number of particles entering the detector without reference to the spin components. We therefore have to consider the question of the "complete experiment", i. e., the number and nature of measurements necessary to determine the complete set of parameters characterizing the scattering process. Theoretical analysis of possible experiments on the scattering of particles with spin is complicated by the fact that it is practically impossible to achieve accurate spin discrimination for all particles leaving the source, i. e., it is impossible to produce a particular spin state of the beam as a whole in the quantum-mechanical sense of this phrase. All that can be done is to produce a degree of polarization of the particle spins in the beam, which is the analog of partial polarization of a beam of light in optics.

The polarization of a beam of particles is usually characterized by the average spin $\mathbf{P}=\langle\hat{\mathbf{s}}\rangle/s$, where s is the maximum spin component. The factor $1/s$ is introduced to ensure that the maximum value of $|\mathbf{P}|$ is unity. It is clear that $\mathbf{P}$ is pseudovector. For spin 1/2 particles, this is the only possible polarization characteristic. For particles with higher spins, there are also tensorial quantities formed by rotation of the coordinate frame. For example, alignment of spin-one particles can be described by the mean value of the components of the symmetric tensor of rank 2 $s_{ij}=(\frac{1}{2})(s_is_j+s_js_i)-(\frac{2}{3})\delta_{ij}$. A partially polarized beam cannot be described by a single spin wave function for all the particles. This type of beam must therefore be described with the aid of the density matrix[1] ρ.

Three levels of detail can be identified in the theoretical analysis of the scattering of polarized particle beams:

a) Phenomenological analysis of experiments, i. e., determination of (1) the general form of the dependence of the cross section on the polarization characteristics of the beams, and (2) the general connection between the polarization characteristics before and after scattering. The general form of these relationships is established by examining all the possible invariants, made up of the vector and tensor characteristics of the polarization under coordinate transformations. For example, in the nonrelativisitic approximation, the elastic differential cross section for spin 1/2 particles is

$$\sigma=\sigma_0+\sigma_1(\mathbf{P}_1\cdot\mathbf{P}_2), \tag{1.1}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarization vectors of the particle beams and σ_0, σ_1 are functions of the angle of scattering and of energy. Other invariants, containing the momenta of the colliding particles, appear when relativistic effects are taken into account. The general expression for the polarization of one of the beams after the collision, for example, $\mathbf{P}_1'$, can be established in a comparably simple way. Since three pseudovectors can be made up of $\mathbf{P}_1$ and $\mathbf{P}_2$, namely, $\mathbf{P}_1$, $\mathbf{P}_2$, and $\mathbf{P}_1\times\mathbf{P}_2$, we have

$$\mathbf{P}_1'=\alpha\mathbf{P}_1+\beta\mathbf{P}_2+\gamma[\mathbf{P}_1\times\mathbf{P}_2], \tag{1.2}$$

where α,β,γ are functions of angle and of energy. This type of simple and general consideration is sufficient for the phenomenologic analysis of experiments.

b) Determination of the explicit form of the parameters characterizing the cross section and polarization i. e., quantities such as $\sigma_0,\sigma_1,\alpha,\beta,\gamma$ in (1.1) and (1.2) in terms of the elements of the density matrix ρ and the amplitude matrix M. The first of these specifies the initial spin states of the colliding particles and the second describes the collision dynamics. The analysis is then based on formulas for the cross section $\sigma=\mathrm{Sp}(M\rho M^*)$, averaged over the spins, and the average of the spin operator $\langle L\rangle=\mathrm{Sp}(\hat{L}M\rho M^*)/\sigma$ after scattering. These expressions are derived in Sec. b) of Chap. 3.

At this level of detail, the theory is more complicated but, here again, it is often sufficient to use the algebraic structure of the amplitude matrix which is established from general and obvious invariance properties. Examples are given below.

c) Determination of the connection between the elements of the amplitude matrix and the form of the potential energy of the interacting particles. General invariance considerations are now no longer sufficient and the dynamics of the collision process must be examined. At this level, the theory becomes still more complicated and involves the formulation of an equation for the wave functions of the colliding particles and, after the corresponding partial wave analysis, the determination of the scattering amplitude matrix.

Of course, this type of systematic approach to cross sections and polarizations automatically yields expressions that are of the same type as those determined from invariance considerations, except that the coefficients in these expressions are now expressed in terms of variables characterizing the interaction between the particles. Examples of such expressions will be given below.

It is important to note that, when the elastic scattering of an electron by an atom is examined, the polari-

zation characteristics depend only on the resultant spin of the atom. The fact that the resultant spin of the atom consists of the spins of the individual electrons has no effect on the polarization effects and, in this sense, the internal structure of the atom is not relevant. The symmetry properties of the wave function of all the electrons participating in the process, both incident and atomic, are taken into account through the dependence of the scattering amplitude on the resultant spin of the atom-plus-electron system. However, when inelastic collisions, in which there is a change in the spin of the atom, are examined, the atomic structure must be explicitly taken into account. This is achieved by expressing the amplitude matrix in terms of the operators acting on the spin variables of all the electrons participating in the transition.

The fundamentals of the theory of polarization phenomena accompanying nucleon scattering were formulated in the nineteen fifties in connection with nuclear studies (see, for example, the review paper by Bilen'kiĭ *et al.*[2]). However, electron-atom collisions have certain specific features that require further development of the theory.

Even the early experimental and theoretical studies of polarized-electron scattering have shown that experiments with polarized beams are both interesting and promising. Some of them are listed below.

Thus, measurements of the asymmetry of the cross section describing the scattering of polarized electrons by atoms can be used to investigate very interesting and fine relativistic effects that are very elusive in experiments without spin selection. Measurement of the polarization of electrons can, under certain conditions, serve as an indicator of the type of coupling between the electronic angular momenta in atoms and molecules.

Polarized electron beams can be used to produce selective population of states with particular angular-momentum components during the excitation of atoms and the diagnostics of metastable states. Polarization phenomena can be used to investigate the structure of large molecules, surfaces, and so on.

2. METHODS OF PRODUCING POLARIZED ELECTRON BEAMS

A. Scattering of electrons by spinless targets (mott scattering)

Real progress in the physics of polarized electrons began quite recently—less than ten years ago—when sufficiently intense beams of polarized electrons became available. The physical conditions under which free electrons could be polarized were first pointed out by Mott.[3,4] Mott used the Dirac equation to show that electrons scattered elastically by spinless nuclei became polarized as a result of the spin-orbit interaction in the continuous spectrum. Mott estimated the degree of polarization produced by this scattering process and showed that, if the electron velocity was much less than the velocity of light, the expected polarization should be extremely small. This conclusion stimulated experiments with fast electrons but, for a long time, the results were inconclusive. On the other hand, the low-energy electron scattering techniques developed in diffraction experiments, and in studies of the Ramsauer effect, would have been sufficient for the production of high polarizations of relatively slow electrons (up to about 100 eV) immediately after the prediction of the polarization effect, had the right experiments been formulated. In fact, the first experiments (from 1949 onward[7]) that confirmed the presence of appreciable polarization effects were performed only after the calculations by Massey and Mohr,[5,6] who showed that the polarization should already be quite substantial at relatively low electron velocities. At present, the scattering of unpolarized electrons by spinless targets is used as a method of producing oriented electron beams. The amount of data based on Mott scattering is vast, so that we shall confine our attention to the most typical results[8] (Fig. 1). It is clear from Fig. 1 that the polarization is a complicated function of the scattering angle. The energy dependence is also complicated, and this indicates that the theoretical calculation of polarization for real systems in not at all simple. We note that maximum polarization corresponds to minimum differential cross section, which restricts the possibility of further utilization of electron beams produced in this way. However, judicious choice of the scattering angle and collision energy does result in polarized beams suitable for further experiments. If we take the parameter $i = P^2 I$, where P is the degree of polarization and i the current of scattered electrons, as the criterion for the optimum situation, we find that $i \approx 10^{-9}$ A corresponds to moderate effectiveness. More complete data on Mott scattering can be found in the literature.[8,9]

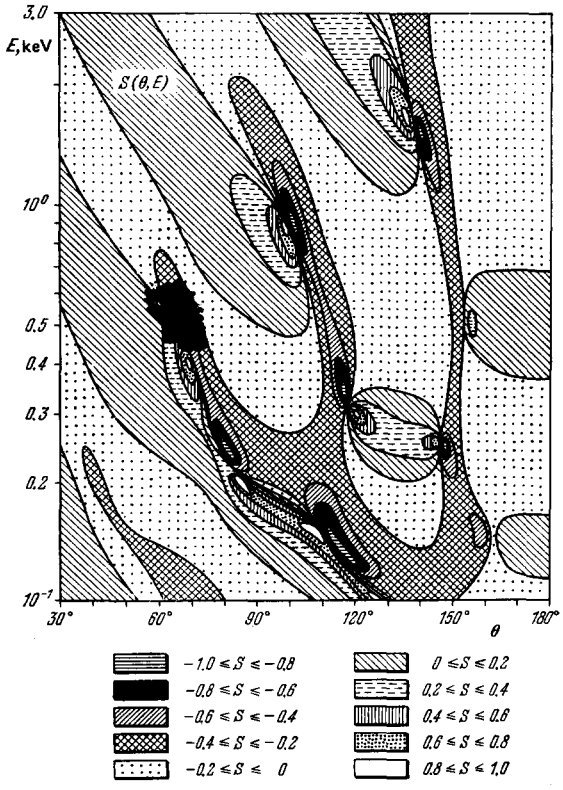

FIG. 1. Polarization profiles $P = S(\theta, E)$ for electrons scattered by mercury atoms.[8]

B. Photoionization of polarized atoms of alkali metals

One of the first ideas relating to the production of polarized electrons was put forward by Fuess and Hellmann,[10] who suggested that oriented electrons could be produced by the photoionization of polarized alkali metal atoms.

The basic idea is to produce a polarized atomic beam by passing it through an inhomogeneous magnetic field, and then expose the atomic beam to ultraviolet radiation in the presence of a magnetic field.

The magnetic field in the photoionization chamber must be low enough (approximately 400 G) to ensure that the electron and nuclear spins can be regarded as noninteracting. If this is not so, electron polarization is substantially reduced by the interaction with the nuclear spin. The best result has been reported for lithium atoms.[11] A pulsed source of light was used, and an electron beam with maximum intensity per pulse amounting to 2×10^9 and degree of polarization of 0.78 was produced. The result reported for potassium atoms[12] is $P\sim0.55$. When continuous light beams are used, the current falls to 10^7 electrons/second.

Despite the basic simplicity of the method, it is found to be more difficult than the Mott method in practice.

C. Photoionization of atoms by polarized light (Fano effect)

Fano is responsible for the original idea of producing polarized electrons without preliminary orientation of the target atoms.[13] Fano used the cesium atom as an example to show that photoionization of unpolarized atoms in the $^2S_{1/2}$ state by circularly polarized light should result in the production of polarized electrons because of the presence of the spin-orbit interaction in the continuous spectrum. In accordance with the selection rules, photoelectrons are produced in the $\varepsilon^2P_{1/2}$ and $\varepsilon^2P_{3/2}$ states in the continuum. The wave functions of these states differ from one another because the spin orbit interaction has different signs for angular momenta $j=1/2$ and $j=3/2$. Since the photoionization cross sections Q_j corresponding to different j are not equal, the electron beams become polarized. It is clear that complete polarization is achieved for light frequencies at which one of the two cross sections $Q_{1/2}$ or $Q_{3/2}$ is zero. This is illustrated in Fig. 2. Figure 2a shows the cross sections corresponding to different directions of the photoelectron spin, and Fig. 2b shows the experimental data for cesium[14] together with the calculations.[13] Apart from its intrinsic interest, the Fano effect is also useful as a means of producing polarized electrons. A pulsed laser has been used[15] to produce 3×10^9 photoelectrons per pulse with average polarization of 0.9.

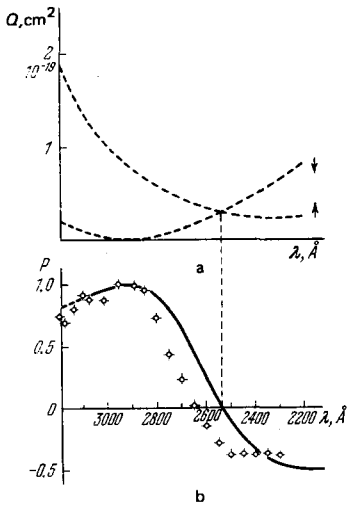

FIG. 2. Cross sections $Q_{1/2}$, $Q_{3/2}$ for the photoinization of cesium (a), and polarization P of emitted electrons (b).[14]

Fano's idea, and other similar ideas, have been extensively used[16-18] in searches for other systems (other than cesium) for which the polarization of the photoelectrons could be high.

D. Collisional detachment of polarized atomic electron

A very promising method of producing polarized electrons is based on the use of oriented atoms in metastable states. As a rule, there is a high probability of detachment of electrons from such states by collision with other atoms or by exposure to radiation.

The orientation of atoms by optical pumping in helium plasma is the most highly developed experimental technique.[19-21] Some of the helium atoms in the discharge are in the metastable 2^3S_1 state, so that exposure to circularly polarized light corresponding to the 1.08-μ resonance line produces the excitation of magnetic sublevels with $m_j=0,1,2$ of the 2^3P_j state (in accordance with the selction rule $\Delta m_j=1$). During the preexcitation of this state as a result of the reverse radiative transition ($\Delta m_j=0,\pm1$), the $m_j=1$ sublevel will, of course, turn out to be overpopulated in comparison with the $m_j=0,-1$ sublevels, so that the helium atoms in the 2^3S_1 state become partially oriented. Detachment of the electron by collision with other atoms in the gas results in the appearance of free polarized electrons. Penning ionization ($A^*+B-A+B^++e$) is among the most effective electron detachment processes.

The electron detachment cross section in the case of Penning ionization is of the order of 10^{-14} cm^2 for optically allowed transitions and remains of the order of the gas-kinetic cross section if the excited atom is in the metastable state. By suitably choosing the collision partners, it is possible to achieve complete transfer of the prepared atomic polarization to the free electron. This method of producing polarized electrons was used in the experiments by Shearer,[22] in which the He(2^3S_1) atoms oriented by optical pumping produce the ionization of cadmium or zinc atoms through the process

$$\mathrm{He}\,(2^3S_1)+\mathrm{Cd}\rightarrow\mathrm{He}\,(1^1S_0)+\mathrm{Cd}^++e.$$

Drukarev *et al.*[23] have shown that the initial polarization of the atom is completely transferred to the liberated electron. There are undoubtedly many other processes in which electron detachment takes place without

depolarization. One is the following[14]

$$\mathrm{He}\,(2^1S_0)+\mathrm{Cs}\,(^2S_{1/2})\rightarrow \mathrm{He}\,(1^1S_0)+\mathrm{Cs}^+ + e.$$

The oriented atom is in the ground state in this process, so that it is possible to increase very substantially the intensity of the polarized electrons as compared with the usual experimental arrangement in which orientation of excited atoms must first be produced. In connection with the above idea based on the Penning ionization of oriented ground-state atoms, we specially mention the process capable of yielding highly polarized electrons of nearly zero energy:[24]

$$\mathrm{O}\,(^1S_0)+\mathrm{Cs}\,(^2S_{1/2})\rightarrow \mathrm{O}\,(^3P_0)+\mathrm{Cs}^+ + e.$$

The resonance defect is $\Delta E = 2169\ \mathrm{cm}^{-1}$.

It is also interesting to consider the reaction

$$\mathrm{O}\,(^1S_0)+\mathrm{Rb}\,(^2S_{1/2})\rightarrow \mathrm{O}\,(^3P_0)+\mathrm{Rb}^+ + e,$$

in which almost complete resonance is achieved ($\Delta E = -115\ \mathrm{cm}^{-1}$), but electron detachment occurs as a result of level crossing.

E. Exchange spin polarization

Burke and Shey[25] were the first to draw attention to the fact that polarized electrons could be produced by elastic scattering of initially unpolarized electrons by oriented one-electron atoms. Clearly, in this case, the mechanism responsible for the polarization is exchange scattering of the electron by the atom in which the atomic polarization is transferred to the free electron. It is equally clear that, when a polarized electron is scattered by the atom, exchange scattering will partially polarize the target and the scattered electrons will become depolarized. The magnitude of exchange spin polarization, even in the case of the simplest two-electron systems, does not explicitly depend on the interaction in the particular atomic system, so that one cannot predict with any degree of certainty the region in which substantial polarization will be produced. Numerical calculations of polarization produced as a result of scattering of electrons by alkali metal atoms have therefore been performed in recent years.[26,27] Typical results are shown in Figs. 3 and 4.

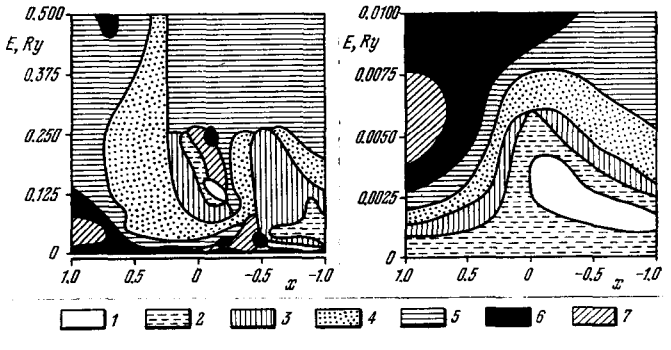

FIG. 3. Electron-spin polarization profiles for electrons scattered by rubidium atoms.[27] 1—$0.8 \leqslant P \leqslant 1.0$, 2—$0.6 \leqslant P \leqslant 0.8$, 3—$0.4 \leqslant P \leqslant 0.6$, 4—$0.2 \leqslant P \leqslant 0.4$, 5—$0 \leqslant P \leqslant 0.2$, 6—$0.1 \leqslant P \leqslant 0$, 7—$0.3 \leqslant P \leqslant -0.1$.

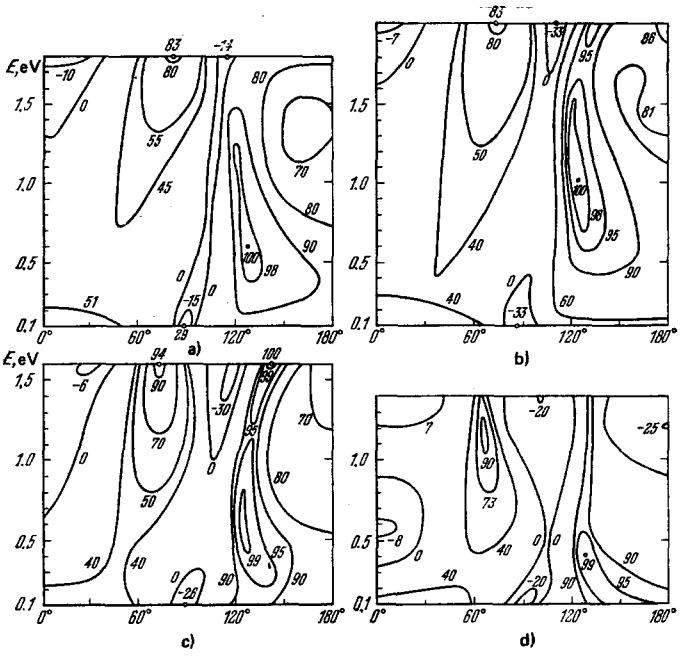

FIG. 4. Exchange polarization of electrons scattered by lithium (a), sodium (b), potassium (c), and cesium (d) as a function of electron energy and scattering angle.[26]

It is important to note that, in general, maximum polarization of scattered electrons corresponds to collisions for which the differential cross section is close to its minimum.[28] This means that the intensity is quite low. For example, in the experiment reported by Campbell *at al.*,[29] the polarization was $P \sim 0.5$ for an average current $I \sim 10^{-13}\ A$. Typical values of i in the range $10^{-14} - 5 \times 10^{-12}\ A$ refer to low effectiveness of exchange scattering as a means of producing polarized electrons.

F. Low-energy diffraction of electrons by surfaces

Spin polarization of electrons reflected from the surface of solids was predicted theoretically by Jennings and Sim[30] and by Feder.[31] In the case of diffraction by surfaces, the phenomenon is determined by the combined effect of several factors that are absent in the case of scattering by free atoms for which the final result is determined by the atomic field alone. These factors include, above all, the periodicity of the crystal lattice, the surface potential barrier, and multiple scattering. The first experimental results appear to confirm the existence of this effect.[32]

G. Multiphoton ionization

Examples of multiphoton ionization of atoms by polarized laser radiation, in which polarized electrons were produced, have been reported.[33-35] Thus, for an atom in the $P_{1/2}$ ground state, it is possible to choose a photon so that first absorption excites the intermediate $S_{1/2}$ state in which m_j is equal to m_s and, by virtue of selection rules, only sublevels with $m_s = 1/2$ are excited. All the excited atoms are then completely polarized and release these electrons after absorption of a

second photon. Experimental studies of this effect were begun by Lambropoulos *et al.*[36] with sodium and Zeman[37] with cesium.

H. Emission of electrons from magnetic materials

The most obvious idea is to produce polarized electrons by detachment from ferromagnets. However, real progress in this field has been achieved only in recent years as a result of improvements in cryogenic vacuum techniques. The experimental specimen (needle) is cooled and subjected to a strong electric field or ultraviolet radiation. Relatively high (of the order of 0.5) polarization has been achieved for relatively low intensities.[38,39]

3. SCATTERING OF SPIN 1/2 PARTICLES BY SPIN 1/2 TARGET

A. Amplitude matrix

We shall consider the scattering of an electron by a spin 1/2 target, for example, an atom with one electron outside a filled shell. For brevity, we shall refer to this as a two-electron system. As noted in the Introduction, there are 16 possible amplitudes with different spin components of the two electrons. However, not all are independent and the determination of the number of independent amplitudes is equivalent to the identification of the algevraic structure of the amplitude matrix. This structure can, in fact, be found from the general requirement of the invariance of the M matrix. This means that, since the amplitude matrix is a scalar, it can include vector characteristics of the system only in combinations that are invariant under shift, rotation, and inversion of the coordinate frame and under time reversal. The only vectors in the two-electron system under investigation are the Pauli spin vectors $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ and the wave vectors $\mathbf{k}$ and $\mathbf{k}'$ of the incident electron before and after scattering. Three mutually perpendicular unit vectors can be constructed from the two wave vectors:

$$\mathbf{n}=\frac{[\mathbf{k}\times\mathbf{k}']}{|[\mathbf{k}\times\mathbf{k}']|}, \quad \mathbf{p}=\frac{\mathbf{k}+\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|}, \quad \mathbf{q}=\frac{\mathbf{k}-\mathbf{k}'}{|\mathbf{k}-\mathbf{k}'|}, \tag{3.1}$$

the first of which is perpendicular to the plane of scattering. We also note that, by virtue of the well-known properties of Pauli matrices, we need not consider invariants containing $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ raised to powers greater than one. We are now in a position to write out all the invariants made up of the above vectors. Since, under space reflection

$$\mathbf{n}\to\mathbf{n},\quad \mathbf{p}\to-\mathbf{p},\quad \mathbf{q}\to-\mathbf{q},\quad \boldsymbol{\sigma}_1\to\boldsymbol{\sigma}_1,\quad \boldsymbol{\sigma}_2\to\boldsymbol{\sigma}_2,$$

and under time reversal

$$\mathbf{n}\to-\mathbf{n},\quad \mathbf{p}\to-\mathbf{p},\quad \mathbf{q}\to\mathbf{q},\quad \boldsymbol{\sigma}_1\to-\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2\to-\boldsymbol{\sigma}_2,$$

we can form only seven invariants, namely,

$$I,\ \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2,\ \boldsymbol{\sigma}_1\cdot\mathbf{n},\ \boldsymbol{\sigma}_2\cdot\mathbf{n},\ (\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n}),\ (\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p}),\ (\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}). \tag{3.2}$$

Next, since

$$\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2=(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})+(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p}_1)+(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}) \tag{3.3}$$

we are left with only six variants. This leads us directly to the required result, namely, the algebraic structure of the amplitude matrix:

$$M=a_1I+a_2(\boldsymbol{\sigma}_1\cdot\mathbf{n})+a_3(\boldsymbol{\sigma}_2\cdot\mathbf{n})+a_4(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})+a_5(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p})+a_6(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}). \tag{3.4}$$

Strictly speaking, instead of $\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2$ we should write $\boldsymbol{\sigma}_1=\boldsymbol{\sigma}_1\otimes I,\boldsymbol{\sigma}_2=I\otimes\boldsymbol{\sigma}_2$ where the symbol $\otimes$ represents the direct product, but we shall not need this more sophisticated notation in the ensuing analysis.

The coefficients $a_1,\ldots,a_6$ in (3.4) play the role of the scattering amplitudes, and their number thus turns out to be six. These amplitudes describe all the possible processes that can take place during the scattering of an electron by a spin 1/2 target. In particular, they include relativistic scattering, which occurs in addition to potential scattering.

It is important to emphasize here that knowledge of the amplitude matrix in the form given by (3.4) is sufficient for many theoretical purposes. For example, it will be shown later that this is sufficient for the analysis of scattering asymmetry. However, it is also clear that determination of the explicit form of the amplitudes $a_1,\ldots,a_6$ can be found in the literature.[40] Here, we shall confine our attention to examination of the physical meaning of these amplitudes.

To begin with, it is convenient to rewrite (3.4) in the form

$$M=a_1I+a_2'(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}+a_2''(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}+a_4(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})+a_5(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\mathbf{p})+a_6(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q}). \tag{3.5}$$

We can then use the properties of the operators $(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}$ and $(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}$ to show that the amplitude a_2' corresponds to spin-orbit interaction in the system, which conserves total spin, whereas a_2'' represents all the interactions that do not conserve the total spin of the two electrons. The amplitudes a_4, a_5, and a_6 describe scattering accompanied by a change in the component of the total spin due to relativistic interactions in the system. In the absence of such interactions, the amplitudes a_2', a_2'' are both zero and the amplitudes a_4, a_5, and a_6 retain only the part describing scattering with the conservation of the particle spin component. As a result, the M matrix reduces to

$$M=aI+b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2, \tag{3.6}$$

where the amplitudes a,b can be related to the usual direct and exchange scattering amplitudes [see (3.4)]. Next, if the relativistic interaction can be represented by the spin orbit interaction $(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{L}$, the M matrix can be written in the form

$$M=aI+b\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+h(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{n}. \tag{3.7}$$

We note that the operator form of the expression for M can, if necessary, be replaced by the matrix form. However, the latter depends on the choice of the quantization axis, so that the operator form is more convenient for the purposes of general analysis. Ideas similar to those used above will be encountered in the en-

suing sections in which the amplitude matrices for other systems will be constructed.

B. Density matrix for the final state

We shall now derive the basic formula used in calculations of the cross section and polarization characteristics for collisions between oriented particles.

Suppose that a particle of spin s_1 collides with a target of spin s_2. The scattering process can be described by the amplitude matrix whose elements will be denoted by $\langle m_1' m_2' | M | m_1 m_2 \rangle$ where m_1, m_2 are the spin components of the colliding particles along some given direction in the initial state, m_1' and m_2' are the spin components in the final state, and M is the operator acting on the spin variables and depending on the scattering angle and collision energy. The differential cross section summed over the spin components in the final state and averaged over the spins in the initial state is given by

$$\sigma = \sum_{m_1', m_2'} \overline{\langle m_1' m_2' | M | m_1 m_2 \rangle^* \langle m_1' m_2' | M | m_1 m_2 \rangle} \tag{3.8}$$

where the bar represents averaging. Since M is Hermitian, we have, by definition,

$$\langle m_1' m_2' | M | m_1 m_2 \rangle^* = \langle m_1 m_2 | M^+ | m_1' m_2' \rangle.$$

If we substitute this in (3.8), we can evaluate the sum over $m_1' m_2'$ (using the matrix multiplication rule). The result is

$$\sigma = \overline{\langle m_1 m_2 | M^+ M | m_1 m_2 \rangle}. \tag{3.9}$$

To perform averaging over m_1, m_2, we must know the probabilities of the different values of spin components in the initial state.

It must be remembered that a polarized beam of particles is not, in general, described by some definite wave function, but by a more general quantity, namely, the density matrix ρ. A detailed account of how such states can be described with the aid of the density matrix is given in the well-known paper by Fano.[1]

The required probabilities are the diagonal elements of the density matrix $\langle m_1 m_2 | \rho | m_1 m_2 \rangle$, which can be used to show that the average cross section is given by

$$\sigma = \sum_{m_1, m_2} \langle m_1 m_2 | \rho | m_1 m_2 \rangle \langle m_1 m_2 | M^+ M | m_1 m_2 \rangle, \tag{3.10}$$

i. e., it reduces to the sum of the diagonal elements of the product of ρ with M^+M:

$$\sigma = \mathrm{Sp}\,(\rho M M^+) = \mathrm{Sp}\,(M \rho M^+). \tag{3.11}$$

We note that the expression $\rho' = M \rho M^+$ may be looked upon as the density matrix after the collision. If we use this, we can find the average value of any quantity L after the collision from the formula

$$\langle L \rangle = \frac{\mathrm{Sp}\,(\hat{L}\rho')}{\sigma}. \tag{3.12}$$

We conclude this section with a summary of the formulas for the scattering cross section and polarization of the scattered electron, which will be useful later. If we use (3.11) and (3.12) with the matrix M given by (3.4) and the density matrix in the form

$$\rho = \frac{1}{4}\left(I + \sum_{\alpha} P_{1\alpha}\sigma_{1\alpha} + \sum_{\alpha} P_{2\alpha}\sigma_{2\alpha} + \sum_{\alpha, \alpha'} Q_{\alpha\alpha'}\sigma_{1\alpha}\sigma_{2\alpha'}\right), \tag{3.13}$$

where $P_{1\alpha}$ and $P_{2\alpha}$ are the initial electron polarizations and the elements $Q_{\alpha\alpha'}$ form a matrix of electron correlations in the directions α and α', respectively, we find that the differential cross section is given by

$$\sigma = \sigma_0 [1 + \mathbf{P}_1^0 (\mathbf{P}_1 \cdot \mathbf{n}) + \mathbf{P}_2^0 (\mathbf{P}_2 \cdot \mathbf{n}) + \sum_{\alpha\alpha'} P_{1\alpha} P_{2\alpha'} C_{\alpha\alpha'}], \tag{3.14}$$

where σ_0 is the cross section for the scattering of unpolarized electrons by polarized atoms, P_1^0, P_2^0 are the polarizations of the electrons and atoms produced during the scattering of unpolarized particles, respectively, and $C_{\alpha\alpha'}$ is a tensor made up of the amplitudes $a_1, \ldots, a_6$. Finally, the components of the polarization of the electron scattered by the initially unoriented system are given by

$$P_{2\alpha}' = P_2^0 \delta_{\alpha n}. \tag{3.15}$$

In the case of arbitrary initial polarizations, the formula for $P_{1\alpha}'$, $P_{2\alpha}'$ can be found in Ref. 40.

C. Asymmetry of cross section and spin polarization

Farago has used (3.14) and (3.15) as a basis for a simple method of studying relativistic effects by low-energy scattering.[41] The idea is that the scattering of unpolarized electrons by oriented atoms should be accompanied by an azimuthal (left-right) asymmetry in the cross section if there is interference between the potential and the relativistic interactions. Actually, it follows from (3.14) that, if we define the asymmetry by

$$A = \frac{\sigma(\theta) - \sigma(-\theta)}{\sigma(\theta) + \sigma(-\theta)}, \tag{3.16}$$

then, for $P_2 = 0$ and $P_1 \neq 0$, we have

$$A = P_1^0 (\mathbf{P}_1 \cdot \mathbf{n}) \equiv A' (\mathbf{P}_1 \cdot \mathbf{n}), \tag{3.17}$$

or, explicitly in terms of amplitude,

$$A' = \frac{2\,\mathrm{Re}\,(a_1 a_2^* + a_3 a_4^*)}{\sigma_0}. \tag{3.18}$$

The expression for P_2^0 is

$$P_2^0 = \frac{2\,\mathrm{Re}\,(a_1 a_3^* + a_2 a_4^*)}{\sigma_0}. \tag{3.19}$$

The exact connection between the asymmetry in the cross section and the polarization assumed by the electron on scattering by an initially unoriented system as a result of relativistic interactions that do not conserve the resultant spin, or the spin components, can be found in the literature.[42] If we now introduce the amplitudes $a_2' = a_2 + a_3$ and $a_2'' = a_2 - a_3$, we can define the polarization

$$P_c = \frac{2\,\mathrm{Re}\,(a_1 a_3'^* + a_1' a_4^*)}{\sigma_0}, \tag{3.20}$$

due to interactions that conserve the total spin of the system (but do not conserve its component), and the polarization

$$P_n = \frac{2\,\mathrm{Re}\,(a_2'' a_4^* - a_1 a_3''^*)}{\sigma_0}, \tag{3.21}$$

due to processes that do not conserve the total spin of the two-electron system in the external field. According to (3.19), the total polarization is $P_2^0 = P_c + P_n$ and it then follows from (3.18) that the asymmetry is

$$A' = P_c - P_n. \tag{3.22}$$

This leads to the important result that the asymmetry is equal to the polarization in all interactions conserving the total spin. Measurement of the asymmetry in an oriented system is, therefore, the alternative to the detection of the polarization acquired by the electron in the unoriented system. It is clear that, under suitable conditions, the above equality can be used as a basis for the detection of polarization in two-electron systems. The polarization and asymmetry in the electron + cesium atom system has been calculated[43] by solving the Dirac equation with the relativistic Hartree-Fock potential for Cs and the interelectron interaction in the Breit form.

Numerical results obtained for $E = 1.427$ eV show that the asymmetry and polarization are practically equal and that A' assumes the value of -0.587 at the maximum. As the energy increases, the asymmetry tends to vanish, whereas polarization retains an appreciable value. Form the point of view of the above analysis [see Eq. (3.22)], this means that processes conserving spin and those that do not do so assume comparable importance. It may therefore also be said that measurement of the asymmetry and polarization can be used for the analysis of the role of different interactions in collisions. Finally, it is important to note that asymmetry can be measured in a single scattering experiment which can be performed without any practical difficulties.

Farago's idea can be augmented by measurement of asymmetry in the scattering of polarized electrons by unpolarized atoms. According to (3.14), this should give rise to the asymmetry

$$A = P_2^0(\mathbf{P}_2 \cdot \mathbf{n}) \equiv A'(\mathbf{P}_2 \cdot \mathbf{n}), \tag{3.23}$$

which is exactly the same as the polarization of the electron after scattering by the unoriented system. Both types of experiment in which only one of the two electrons is initially polarized provides us, at least in principle, with the possibility of determining the polarization P_n produced as a result of the nonconservation of total spin in the system. It is quite clear that this polarization is specific for many-electron systems and is fundamentally different from the polarization mechanism that was discussed by Mott and is well known in nuclear physics.

D. Scattering in the case of spin-dependent interaction

We now specialize our analysis by assuming that, in addition to the central field, there is also the spin-orbit interaction in the continuous spectrum. The interaction potential can be written in the form

$$v = v_1(r) + \frac{1}{2} v_2(r)(\sigma_1 + \sigma_2)\cdot \mathbf{L} = v_1(r) + v_2 \mathbf{S}\cdot\mathbf{L}, \tag{3.24}$$

where L is the angular momentum operator. This interaction conserves S^2 and L^2 but does not conserve the components of the angular momenta. To find the amplitude matrix corresponding to this type of interaction, we must perform the partial-wave analysis in order to separate the angular and spin variables in the scattering equation. This procedure can be performed in the standard way and yields the following result for the operator M:

$$M = \frac{1}{4}(3F + G) I + \frac{1}{4}(F - G)\sigma_1 \cdot \sigma_2 + H\mathbf{S}\cdot\mathbf{n}; \tag{3.25}$$

here G, F are the scattering amplitudes in the singlet and triplet states, respectively, and H is the amplitude due to the spin-orbit interaction. Comparison with (3.4) will show that (3.25) is obtained from the general expression for $a_2 = a_3$, $a_4 = a_5 = a_6$, but this qualitative result cannot, as noted above, be obtained from the invariance principle; it follows from the partial-wave analysis for the specific interaction. It is also clear that the amplitude a_2'' which describes processes with the nonconservation of total spin will vanish, as expected, because v conserves total spin.

Instead of G and F, which describe scattering in the total angular momentum representation, it is possible to introduce two other amplitudes in a natural way on the basis of the following considerations. If we construct the matrix elements of M for the different spin states, we find that (α and β are spinors with positive and negative components along the quantization axis)

$$\begin{aligned} \langle \alpha_1\beta_2 | M | \alpha_1\beta_2 \rangle &= \tfrac{1}{2}(G + F) \equiv f, \\ \langle \alpha_2\beta_1 | M | \alpha_1\beta_2 \rangle &= \tfrac{1}{2}(G - F) \equiv g. \end{aligned} \tag{3.26}$$

Hence, it follows that the amplitudes f and g can be interpreted as the direct and exchange scattering amplitudes. In a completely similar way, we can establish that the amplitude H describes scattering in the triplet state with a change in the component of the total spin. In fact, if we denote the spin functions for the triplet states by χ_{1M_S}, we can show that

$$\langle \chi_{11} | M | \chi_{10} \rangle = \sqrt{2}\, ie^{i\varphi} H. \tag{3.27}$$

We note, for comparison, that, in the case of scattering of an electron by a spinless target, the amplitude matrix takes the form

$$M = m + h\sigma\cdot\mathbf{n}, \tag{3.28}$$

and that $\langle \beta | M | \alpha \rangle = ie^{i\varphi} h$ enables us to interpret h as the spin rotation amplitude. Thus, the spin-orbit interaction in two-electron systems will rotate the spin, whereas, in one-electron systems, it will turn it over (this is the well-known spin-flip phenomenon).

We also note that the operator M can be written in another way in terms of the singlet and triplet projection operators:

$$\hat{\Pi}_0 = \frac{1}{4}(I - \sigma_1\sigma_2), \quad \hat{\Pi}_1 = \frac{1}{4}(3I + \sigma_1\sigma_2). \tag{3.29}$$

We thus obtain

$$M = G\hat{\Pi}_0 + F\hat{\Pi}_1 + H\mathbf{S}\mathbf{n}. \tag{3.30}$$

In the absence of the spin-orbit interaction ($H = 0$), the structure of this expression gives us a recipe for constructing the operator M for colliding particles with arbitrary spins, namely, $M = \Sigma G_S \hat{\Pi}_S$ where $\hat{\Pi}_S$ are the projection operators onto states with total spin S, and G_S are the scattering amplitudes in these spin states. We conclude this section by evaluating the cross section symmetry due to the spin-dependent interaction. If one

of the particles is polarized prior to collision, the scattering cross section is given by

$$\sigma=\frac{1}{4}(3F\bar{F}+G\bar{G})+2H\bar{H}+(\bar{F}H+F\bar{H})\,\mathbf{P}_1\mathbf{n}\equiv\sigma_0(\theta)+S(\theta)\,\mathbf{P}_1\mathbf{n}. \quad (3.31)$$

Hence the asymmetry is given by

$$A=\frac{S(\theta)}{\sigma_0(\theta)}\,\mathbf{P}_1\mathbf{n}\equiv A'\mathbf{P}_1\mathbf{n}. \quad (3.32)$$

It is clear from this that nonzero asymmetry arises as a result of interference between potential scattering in the triplet state and scattering due to the spin-orbit interaction. Measurement of this asymmetry is, therefore, a direct means of detecting the presence of interactions differing from the central interaction.

If we evaluate the polarization of the electron after scattering by the unoriented system, we find that

$$\mathbf{P}_1=\frac{nS(\theta)}{\sigma_0(\theta)}. \quad (3.33)$$

Comparison of the last two results will show that $A'=P'$, which is in agreement with the general relation given by (3.22) for interactions that conserve total spin ($P_n=0$).

E. Exchange polarization

At low energies, when the spin-orbit interaction can be neglected, the exchange in the polarization of the initially oriented system occurs as a result of exchange scattering. In fact, exchange scattering in the two-electron system leads to the process "atomic electron ⇄ free electron." An unpolarized electron beam thus becomes partially polarized as a result of scattering by an oriented atom, and the atom itself is depolarized. Similarly, the atom can become polarized as a result of a collision with a polarized electron beam. Exchange redistribution of polarization will, in general, take place when both electrons are initially oriented. This type of polarization is referred to as exchange polarization. This also emphasizes the fact that polarization effects are consequences of the Pauli principle and are unrelated to the spin dependence of the interaction.

The quantitative theory of exchange polarization is based on the amplitude matrix

$$M=G\hat{\Pi}_0+F\hat{\Pi}_1, \quad (3.34)$$

and the spin density matrix given by (3.13). Calculations first reported by Burke and Shey[25] lead to the following expressions for the scattering cross section and electron polarizations:

$$\left.\begin{aligned}\sigma P_e'&=n(\theta)\,P_e+p(\theta)\,P_a+iq(\theta)\,[\mathbf{P}_e\times\mathbf{P}_a],\\ \sigma P_a'&=n(\theta)\,P_a+p(\theta)\,P_e-iq(\theta)\,[\mathbf{P}_e\times\mathbf{P}_a],\\ \sigma&=\sigma_0(\theta)+m(\theta)\,(\mathbf{P}_e\cdot\mathbf{P}_a),\end{aligned}\right\} \quad (3.35)$$

where

$$\left.\begin{aligned}n(\theta)&=\tfrac{1}{4}(2F\bar{F}+\bar{F}G+F\bar{G}),\\ p(\theta)&=\tfrac{1}{4}(2F\bar{F}-\bar{F}G-F\bar{G}),\\ q(\theta)&=\tfrac{1}{4}(\bar{F}G-F\bar{G}),\\ \sigma_0(\theta)&=\tfrac{1}{4}(3F\bar{F}+G\bar{G}),\\ m(\theta)&=\tfrac{1}{4}(F\bar{F}-G\bar{G}).\end{aligned}\right\} \quad (3.36)$$

It is clear that the exchange interaction does not produce polarization in an oriented system ($P_e=P_a=0$).

The new feature encountered in the physics of atomic collisions, which is connected with the use of polarized electrons, is that such experiments can be used to separate direct from exchange scattering, and to determine $|f|$, $|g|$, and the phase difference between these amplitudes. All that needs to be done is to determine the scattering cross section for the unoriented system

$$\sigma_0(\theta)=\tfrac{1}{2}(f\bar{f}+g\bar{g})+\tfrac{1}{2}|f-g|^2, \quad (3.37)$$

from the experimental data, and to measure $\sigma_d=f\bar{f}$ and $\sigma_{ex}=g\bar{g}$.

Experiments designed to determine σ_0 are conventional, whereas the determination of σ_d and σ_{ex} involves measurement of the depolarization of electrons scattered by an unoriented target and the resulting polarization P_a' of the target.

Let us verify this. When $P_a=0$, (3.35) yields

$$\left.\begin{aligned}\frac{\sigma_0(\theta)P_e'}{P_e}&=n(\theta)=|f|^2-\tfrac{1}{2}(\bar{f}g+f\bar{g}),\\ \frac{\sigma_0(\theta)P_a'}{P_e}&=p(\theta)=|g|^2-\tfrac{1}{2}(\bar{f}g+f\bar{g}).\end{aligned}\right\} \quad (3.38)$$

From these expressions, we obtain

$$\left.\begin{aligned}|g|^2&=\sigma_0\left(1-\frac{P_e'}{P_e}\right)=\sigma_{ex},\\ |f|^2&=\sigma_0\left(1-\frac{P_a'}{P_e}\right)=\sigma_d,\end{aligned}\right\} \quad (3.39)$$

which establishes the above propositions. Measurement of the three cross sections as indicated above will thus yield $|f|^2$, $|g|^2$, and $|f-g|^2$, which, in turn, means that three of the four parameters in the complex quantities $f=|f|e^{i\varphi}$ and $g=|g|e^{i\psi}$, have been determined, namely,

$$|f|,\ |g|,\ \cos(\varphi-\psi)=\frac{|f|^2+|g|^2-|f-g|^2}{2|f||g|}. \quad (3.40)$$

Systematic calculations of the exchange polarization $P_e'=p(\theta)/\sigma_0(\theta)$ produced by scattering by Li, Na, K, and Cs atoms in the strong-coupling approximation have been performed by Karule.[26] The results are shown in Fig. 4.

One further interesting result follows from these calculations, namely, that the energy E_c at which complete polarization $P_e'=P_a$ is achieved is proportional to the atomic number: $E_c\approx\text{const}\times Z$.

A general proof of this result has not yet been given.

4. SCATTERING OF PARTICLES IN TRIPLET STATES BY SPINLESS PARTICLES

The initial spin state of spin 1 particles is characterized by the vector and tensor polarizations

$$\left.\begin{aligned}P_\alpha&=\tfrac{1}{2}\,\mathrm{Sp}\,(\rho,\ \sigma_{1\alpha}+\sigma_{2\alpha}),\\ Q_{\alpha\alpha'}&=\mathrm{Sp}\,(\sigma,\ \sigma_{1\alpha}\sigma_{2\alpha'}).\end{aligned}\right\} \quad (4.1)$$

The density matrix for the triplet state is

$$\rho=\tfrac{1}{4}\Big[I+\sum_\alpha P_\alpha(\sigma_{1\alpha}+\sigma_{2\alpha})+\sum_{\alpha,\alpha'}Q_{\alpha\alpha'}\sigma_{1\alpha}\sigma_{2\alpha'}\Big], \quad (4.2)$$

where

$$\sum_{\alpha} Q_{\alpha\alpha} = \langle \sigma_1 \sigma_2 \rangle = 1. \tag{4.3}$$

The amplitude matrix for the scattering of this particle by a spinless target can be constructed from the two amplitudes F and H if the interaction is described by (3.24) and can be written as follows in operator form:

$$M = F\hat{\Pi}_1 + H\mathbf{S}\cdot\mathbf{n}. \tag{4.4}$$

It is clear from the general form of the operator M that scattering will polarize metastables. Calculation of this polarization in accordance with the scheme described above yields

$$\mathbf{P}' = \frac{n\mathbf{S}(\theta)}{\sigma_0(\theta)}, \tag{4.5}$$

where $\sigma_0 = FF + (8/3)H\bar{H}$. It is also a relatively simple matter to verify that, in the case of scattering of an oriented metastable by a spinless target, the cross section is given by

$$\sigma = \sigma_0 + 2S(\theta)\,\mathbf{P}\cdot\mathbf{n}, \tag{4.6}$$

and, consequently, we have the asymmetry

$$A' = \frac{2S(\theta)}{\sigma_0(\theta)}, \tag{4.7}$$

which is related to the polarization as follows:

$$A' = 2P'. \tag{4.8}$$

This result is a consequence of the fact that triplet-singlet transitions are forbidden in the above interaction. It can also be verified that, for the same reason, the change in the tensor polarization of the atom occurs so that $\Sigma Q_{\alpha\alpha} = \text{const}$, i.e., this quantity is conserved in the collision.

Polarization studies involve an interesting aspect of collision theory, namely, the use of optical potentials and the determination of the parameters of these potentials. In fact, for energies corresponding to the maximum value of the polarization, inelastic channels are energetically open and their influence on elastic scattering can be taken into account within the framework of the optical model. In particular, if we suppose that

$$v = v_1(1 + i\zeta) + \eta\frac{1}{r}\frac{\partial v_1}{\partial r}(\sigma_1 + \sigma_2)\cdot\mathbf{L}, \tag{4.9}$$

we can show[44] that scattering produces vector polarization $P' = P(\zeta, k, \theta)$, whose dependence on ζ is as follows:

$$\mathbf{P}' = \mathbf{n}\cdot\frac{8\zeta\eta k^2\sin^2\theta}{3(1+\zeta^2+8k^4\eta^2\sin\theta)}. \tag{4.10}$$

This, by determination of the polarization, establishes the parameter ζ in the optical potential, which can then be used for various problems in the physics of atomic collisions. Measurement of polarization, used in conjunction with (4.8), can in turn replace measurement of asymmetry.

Another aspect connected with oriented metastables can be seen in Penning ionization processes. For example, the radiation emitted by Cd^* ions in the process

$$\mathrm{He}(2^3S) + \mathrm{Cd} \to \mathrm{He}(1^1S) + \mathrm{Cd}^+ + e,$$

is characterized by an intensity asymmetry $I_\pm$ between the right and left circular polarizations. In particular, for the $5^2D_{5/2} \to 5^2D_{1/2}$ transition ($\lambda = 4416$ Å) in Cd^*, the radiation asymmetry A is related to the initial polarization of the metastable by $A = (I_+ - I_-)/(I_+ + I_-) = 0.7P$. Measurement of the radiation asymmetry in an optical experiment can thus be used to obtain the initial polarization of the atom.

5. SCATTERING OF AN ELECTRON BY A SPIN 1 TARGET

The scattering of an electron by an atom or molecule in the ground or a metastable state with $s = 1$ has not been extensively investigated. In this section, we shall review some of the basic results of the theory of polarization for such systems.[45-47]

The requirement of invariance demands that, in the nonrelativistic approximation, the amplitude matrix must have the structure

$$M = AI + B\mathbf{s}\cdot\sigma. \tag{5.1}$$

Since $\langle \mathbf{s}\cdot\sigma \rangle = 1$ when the total spin is $j = 3/2$ and $\langle \mathbf{s}\cdot\sigma \rangle = -2$ for $j = 1/2$, we can rewrite (5.1) in the form

$$\begin{aligned} M &= \tfrac{1}{3}(2F+G)I + \tfrac{1}{3}(F-G)\mathbf{s}\sigma \\ &= G\cdot\tfrac{1}{3}(1-\mathbf{s}\cdot\sigma) + F\cdot\tfrac{1}{3}(2I+\mathbf{s}\cdot\sigma) \equiv G\hat{\Pi}_{1/2} + F\hat{\Pi}_{3/2}, \end{aligned} \tag{5.2}$$

where F and G are, respectively, the quadruplet and doublet amplitudes, and $\hat{\Pi}_{1/2}$ and $\hat{\Pi}_{3/2}$ are the projection operators acting on spin functions with total spin j. By analogy with the two-electron system, we can replace F and G with the direct and exchange scattering amplitudes f and g through the relations

$$\begin{aligned} &\left\langle \tfrac{1}{2}1 \middle| M \middle| \tfrac{1}{2}1 \right\rangle = \tfrac{1}{3}F \equiv f, \\ &\left\langle -\tfrac{1}{2}1 \middle| M \middle| \tfrac{1}{2}0 \right\rangle = \tfrac{\sqrt{2}}{3}(F-G) \equiv -\sqrt{2}g. \end{aligned} \tag{5.2'}$$

The second of these expression shows that exchange scattering leads to the appearance of the oppositely directed spin in the scattered beam, i.e., it stimulates the relativistic phenomenon of spin flip. The M matrix in these amplitudes has the form

$$M = (f-g)I - g\mathbf{s}\cdot\sigma. \tag{5.3}$$

We shall write the density matrix in the form of the product $\rho = \rho_1\rho_2$, where $\rho = \frac{1}{2}(I + \mathbf{P}_e\sigma)$ is the spin density matrix of the electron and ρ_1 is the corresponding matrix for the target. Let us take the basis matrices in the expansion for ρ_1 in the form of the matrices s_i corresponding to spin 1 and the components of the second-rank tensor

$$s_{ij} = \frac{1}{2}(s_i s_j + s_j s_i) - \frac{2}{3}\delta_{ij}I, \tag{5.4}$$

subject to $\Sigma s_{ii} = 0$. We then have

$$\rho_1 = \frac{1}{3}\left[I + \frac{3}{2}\sum P_{ai}s_i + 3\sum Q_{ij}s_{ij}\right], \tag{5.5}$$

where $P_{ai} = \langle s_i \rangle$ are the components of the polarization vector describing orientation and $Q_{ij} = \langle s_{ij} \rangle$ are the components of the polarization tensor describing alignment of the target. The spin state of the target with $s = 1$ is thus, in general, characterized by eight real parameters instead of the three used in the case of spin 1/2 particles. If the initial polarization of both

particles is arbitrary, the . sections and polarizations are given by

$$\sigma = |f-g|^2 + 2|g|^2 + (2|g|^2 - \bar{f}g - f\bar{g})(\mathbf{P}_e\cdot\mathbf{P}_a), \quad (5.6)$$

$$\sigma P'_{ai} = \left(|f-g|^2 - \frac{2}{3}|g|^2\right) P_{ei} + (3|g|^2 - \bar{f}g - f\bar{g}) P_{ai} + i(f\bar{g} - \bar{f}g)[\mathbf{P}_a\cdot\mathbf{P}_e]_i + 2|g|^2 \sum_j Q_{ij}P_{ej}, \quad (5.7)$$

$$\sigma P'_{ai} = \frac{i}{3}(3|g|^2 - \bar{f}g - f\bar{g}) P_{ei} + (|f-g|^2 + |g|^2) P_{ai} + \frac{1}{2}(\bar{f}g - f\bar{g})[\mathbf{P}_a\cdot\mathbf{P}_e]_i - (\bar{f}g + f\bar{g}) \sum_j Q_{ij}P_{ej}. \quad (5.8)$$

The results can be used to classify experiment for the determination of $|f|$, $|g|$, and the relative phase ψ of these amplitudes. Measurement of the depolarization of electrons by an unpolarized target can be used to find the cross section for pure exchange scattering:

$$\sigma_{ex} = \frac{9}{8}\sigma_0\left(1 - \frac{P'_e}{P_e}\right) \equiv 3|g|^2. \quad (5.9)$$

The direct cross section $\sigma_d = |f|^2$ is found by measuring the resulting polarization of the target

$$\sigma_d = \sigma_0\left(1 - \frac{3P'_a}{2P_e}\right). \quad (5.10)$$

If, in addition, we measure the cross section for scattering without polarization

$$\sigma_0 = |f|^2 + 3|g|^2 - (\bar{f}g + f\bar{g}), \quad (5.11)$$

we can obtain the quantities $|f|^2$, $|g|^2$, and $\bar{f}g + f\bar{g}$ from these three measurements, and, hence, $|f|$, $|g|$, and cos ψ. It is clear from the foregoing formu- the foregoing formulas that the polarization of the atom after the scattering event does not exceed $(2/3)P_e$, whereas the scattering of an unpolarized beam of electrons by an oriented atom may be accompanied by the complete transfer of polarization to the electron.

Finally, we note the following general points. Exchange scattering by an unoriented system does not produce polarization. An electron beam becomes polarized only if the target has a vector orientation. Tensor polarization is insufficient for the appearance of $P'_e \neq 0$. The scattering of polarized electrons produces only the vector polarization $P_a = (2/3)P_e$ in the unoriented target. It may also be verified that the maximum polarization of the beam is achieved for energies and scattering angles for which the cross section is close to its minimum. Experiments on the scattering of polarized electrons by atoms in triplet states have not as yet been performed. Figure 5 shows the results[48] of calculations of the polarization produced by scattering of electrons by the metastable He(2^3S).

These results can be used to describe the scattering of polarized electrons or molecules. We note a possible application of polarized electrons to the study of electronic states of molecular negative ions. Many of the processes that occur during the scattering of electrons by molecules are resonant in character because of the formation of molecular negative ions during the

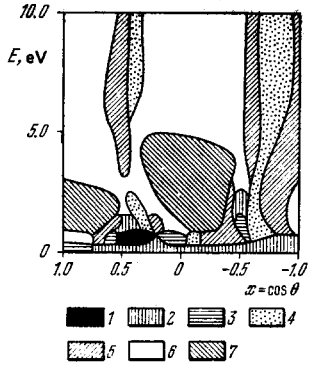

FIG. 5. Exchange polarization of electrons scattered by helium atoms. 1—$0.8 \leqslant P \leqslant 1.0$, 2—$0.6 \leqslant P \leqslant 0.8$, 3—$0.4 \leqslant P \leqslant 0.6$, 4—$0.2 \leqslant P \leqslant 0.4$, 5—$0.0 \leqslant P$ 0.2, 6—$0.2 \leqslant P \leqslant 0$, 7—$0.45 \leqslant P \leqslant -0.2$.

intermediate stage of the reaction. Measurement of the depolarization of electrons scattered by unpolarized spin-1 molecules (for example, O_2, NH, SO, S_2, etc.) can be used to establish the multiplicity of the molecular ion. Thus, in terms of the doublet and quartet amplitudes G and F, the electron depolarization is given by [see (5.6)–(5.7)]

$$D = \frac{5F\bar{F} - 2F\bar{G} - 2\bar{F}G - G\bar{G}}{3(2F\bar{F} + G\bar{G})}. \quad (5.12)$$

Hence, it follows that, when the collision energy corresponds to the energy of the intermediate doublet state, the depolarization should be close to its minimum value of $-1/3$, whereas D approaching 5/6 suggests the presence of scattering through the quartet state of the molecule. The most desirable type of experiment is measurement of depolarization in inelastic processes because the above resonances are than most clearly defined.

To conclude this section, let us consider the influence of target structure on polarization effects. It is noted in the Introduction that polarization characteristics depend only on the total spin of the target and are independent of the fact that this total spin is made up of the spins of the individual electrons. This is valid provided the target spin is conserved during the collision process. However, when exchange or relativistic interactions lead to a change in the target spin (singlet-triplet transitions in the system), the amplitude matrix must include the spin operators of all the electrons. For example, in the three-electron system, the most general invariant expression for the nonrelativistic M matrix is

$$M = aI + b\sigma_1\cdot\sigma_2 + c\sigma_1\cdot\sigma_3 + d\sigma_2\cdot\sigma_3, \quad (5.13)$$

where $a\ldots, d$ are the scattering amplitudes. Direct though tedious calculations show that the three-particle approach based on (5.13) is equivalent to the two-particle approach based on (5.1) when the spin of the target is unaffected by collision. However, in inelastic collisions, the exchange interaction produces triplet-singlet transitions and the only adequate approach to this case is the description based on the three-particle matrix (5.13) which explicity includes the structure

of the target (see Sec. A. of Chap. 8 for further details).

Interesting features are also associated with relativistic interactions in the three-particle system. General considerations show that such interactions produce polarization of the incident electron in the unoriented system. It is possible to distinguish three mechanisms in this phenomenon that lead to the appearance of orientation. The first of these is connected with the change in the component of the spin of the incident electron and is completely analogous to the polarization mechanism examined by Mott. The second is connected with a change in the total spin of the system (doublet-quartet transitions) and is analogous to the mechanism noted above in the case of two-electron systems [see (3.21)]. In this case, therefore, we are concerned with the change in the total spin of the system as the reason for the appearance of polarization. Finally, electron polarization may arise as a result of a change in the spin of the target (singlet-triplet transition) due to relativistic interaction with the incident electron. The resultant polarization which accompanies the change in spins and their components is quantitatively related to the asymmetry in the elastic electron scattering cross section of the vectionially polarized target. Detailed analysis of these questions will be given elsewhere. Here, we merely note the desirability of experimental studies of polarization and asymmetry in the interests of further development of the physics of electron-atom collisions.

6. SCATTERING OF PARTICLES WITH SPIN 1

Collisions in optically pumped helium plasma He(2^3S) are an example of this type of process. The complete description of all possible processes in systems of spin-1 particles is shown by partial wave analysis to involve 19 different scattering amplitudes. In the nonrelativistic approximation, the amplitude matrix contains only three amplitudes and, in operator form, is given by the following invariant expression:

$$M = aI + b\mathbf{s}_1\mathbf{s}_2 + c\,(\mathbf{s}_1\cdot\mathbf{s}_2)^2. \tag{6.1}$$

This expression can be rewritten in terms of the spin projection operators:

$$\left.\begin{aligned}\hat{\Pi}_0 &= -\tfrac{1}{3}[I-(\mathbf{s}_1\mathbf{s}_2)^2],\\ \hat{\Pi}_1 &= \tfrac{1}{2}[2I-\mathbf{s}_1\mathbf{s}_2-(\mathbf{s}_1\mathbf{s}_2)^2],\\ \hat{\Pi}_2 &= \tfrac{1}{6}[2I+3\mathbf{s}_1\mathbf{s}_2+(\mathbf{s}_1\mathbf{s}_2)^2].\end{aligned}\right\} \tag{6.2}$$

If we also introduce the amplitude

$$G_S = \frac{1}{2ik}\sum_L (2L+1)\,T_{SL}P_L(\cos\theta), \tag{6.3}$$

which describes the scattering for total spin $S = 0, 1, 2$, we obtain, instead of (6.1),

$$M = \sum_S G_S\hat{\Pi}_S. \tag{6.4}$$

Taking the density matrix in the form $\rho = \rho_1\rho_2$, where $\rho_{1,2}$ are defined by (5.5) with $Q_{ij} = \delta_{ij}Q_i$, we obtain the following expression for the cross section:

$$\sigma = A + B\mathbf{P}_1\cdot\mathbf{P}_2 + C\sum_i Q_i^{(1)}Q_i^{(2)}. \tag{6.5}$$

The expressions for the coefficients in this result, given in terms of the amplitudes G_S, and the formulas for the polarizations of the particles after scattering, can be found in the literature.[49] These results can now be used to formulate an experiment for the determination of the moduli of the amplitudes and the two relative phases. To carry out the minimum program, i.e., three scattering experiments are sufficient to determine the moduli of the amplitudes. One is the conventional experiment involving measurement of the cross section σ_0 for the scattering of unpolarized particles. In the other two, a determination is made of the cross section σ for the scattering of particles with the same vector polarization P and of the depolarization D of one of the particles after scattering. The choice of states with only the vector polarization is dictated by the fact that such states can be obtained experimentally by optical pumping or by scattering polarized electrons by the spin 1 target,[5] and by the fact that the overall experimental arrangement is then particularly simple. Writing $\sigma_1 = D\sigma$, and noting that $B = (\sigma - \sigma_0)/P^2$, we obtain the following formulas for the moduli of the scattering amplitudes:

$$|G_0|^2 = \tfrac{1}{19}(81\sigma_0 - 40B - 62\sigma_1),$$

$$|G_1|^2 = \tfrac{1}{19}(45\sigma_0 + 20B - 26\sigma_1),$$

$$|G_2|^2 = \tfrac{1}{19}(-9\sigma_0 - 4B + 28\sigma_1).$$

As expected, a further three independent experiments are necessary if the relative phases between the amplitudes are to be determined.

We note in conclusion that analysis of the total 9×9 amplitude matrix enables us to establish certain features connected with the nature of the interaction in a system of spin 1 particles. In particular, it turns out that the spin-orbit interaction changes the initial state in such a way that one can speak of the rotation of the spin of one of the particles. It can be shown that the interaction which reverses the spin of one of the particles generates amplitudes of the form $H = \sum r_L P_H^{(2)}$ and the interaction leading to the reversal of the spins of both particles (double spin flip) generates the amplitudes $R = \sum r_L P_h^{(4)}$.

It can also be verified, following the analysis of Sec. B of Chap. 3, that the spin-orbit interaction produces an asymmetry of the scattering cross section which is proportional to the polarization produced by the scattering of unpolarized particles in the presence of the spin-orbit interaction.

7. TRANSITIONS INVOLVING A CHANGE IN THE SPIN OF THE TARGET

These collisions are characteristic for many-electron systems in which exchange scattering can change the spin of one (or both) partners during the collision. Examples of such collisions are given below.

We shall confine our attention to interactions which conserve the total angular momenta and their components. We can then introduce the operator $\hat{A}_{SM_S}$ which

transforms the initial spin state with the function $\chi^{(i)}_{SM_S}$ (S is the total spin of the system and M_S is its projection) into the final state with the function $\chi^{(i)}_{SM_S}$ in such a way that

$$\hat{A}_{SM_S}\chi^{(i)}_{SM_S}=\chi^{(f)}_{SM_S}. \tag{7.1}$$

If the process is allowed for several values of the total spin, the amplitude matrix becomes

$$M=\sum_S \hat{A}_{SM_S}G_S(\theta), \tag{7.2}$$

where $G_S(\theta)$ are the amplitudes corresponding to the values of the total spin S for which transitions with the redistribution of the spins of the individual particles in the system take place.

A. Singlet-triplet excitation of the target

In the case of the singlet-triplet transition in an atom, the total spin of the system, which includes both the atom and the incident electron, is 1/2. Assuming that the two-electron target is described by the LS-coupling approximation, and constructing the spin functions $\chi^{(i)}_{1/2m}$ and $\chi^{(f)}_{1/2m}$ for the three-electron system, we find from (7.1) that

$$\hat{A}_{1/2}=\frac{1}{2\sqrt{3}}\sigma_1\cdot(\sigma_2-\sigma_3), \tag{7.3}$$

where 2, 3 label the atomic electrons. A new spin state arises after excitation and is described by the density matrix

$$\rho'=\sum_{M_LM_S}\hat{A}_{1/2}\rho\hat{A}^{\dagger}_{1/2}|G_{LM_L}|^2, \tag{7.4}$$

where G_{LM_L} is the amplitude for the transition to the ^3L state and ρ is the initial density matrix:

$$\rho=\frac{1}{8}(I+\mathbf{P}\cdot\sigma_1)(I-\sigma_2\cdot\sigma_3). \tag{7.5}$$

It is readily verified that new spin correlation between the atomic electrons appears after the transition and corresponds to the atomic spin $s=1$. Using (7.4), we find that

$$\langle\sigma_2\cdot\sigma_3\rangle'=\frac{\mathrm{Sp}(\rho'\sigma_2\cdot\sigma_3)}{\mathrm{Sp}\rho'}=1, \tag{7.6}$$

as expected in the triplet state. Evaluating the polarizations after collision, we find that the excited atom assumes the polarization $(2/3)P$ and the inelastically scattered electron is depolarized so that $P'/P=-1/3$.

Hanne and Kessler[51] have reported measurements of the depolarization of electrons due to the $^1S\to{}^3P$ transition in the mercury atom. The depolarization differs from $-1/3$ and turns out to be a complicated function of the collision energy. Hence, it follows that the LS-coupling approximation is insufficient for the description of a heavy atom in the scattering process. It is therefore clear in advance that, with suitable theoretical interpretation, depolarization can serve as a characteristic of the type of atomic coupling.

B. Atomic fine structure

Satisfactory agreement with experimental data on depolarization can be achieved by taking into account the fine structure of the atom and the intermediate type of coupling.

If the spin-orbit interaction inside the atom can be taken into account within the framework of perturbation theory, with the LS-coupling approximation as the zero-order result, the M matrix for the transition becomes

$$M(JM_J)=\sum_{M_L+M_S=M_J}C^{JM_J}_{LSM_LM_S}\hat{A}_{SM_S}G^S_{LM_L}(\theta). \tag{7.7}$$

Calculations with the final state density matrix

$$\rho'(J)=\sum_{M_J}M(JM_J)\rho M^+(JM_J) \tag{7.8}$$

[see (7.5) for ρ] then yield the depolarization of the electrons responsible for the excitation of the individual components of the ^3P_J triplet. Calculations of the depolarization $D(^3P_J)=P'/P$ of electrons scattered in the forward direction after the $^1S\to{}^3P_J$ excitation in mercury[50] give

$$D(^3P_0)=-1,\quad D(^3P_1)=0,\quad D(^3P_2)=-0{,}4. \tag{7.9}$$

Recent experimental data[51] on $D(^3P_J)$ show that they agree with the calculated values to within experimental error.

C. Depolarization as a test of the type of atomic coupling

It is interesting that depolarization can be used to test for the type of atomic coupling. Thus, consider the situation in which the $^1P_1$ singlet state is excited in addition to the $^3P_1$ triplet state. As noted above, forward scattering is accompanied by the complete depolarization of electrons that have excited the $^3P_1$ state whereas, obviously, depolarization does not occur during the excitation of the singlet state. Consequently, in the case of the intermediate coupling, when the singlet state forms a definite admixture to the triplet state, depolarization should assume some intermediate value between 0 and 1, depending on the weight of the admixture. If we use the matrix for the transition to the mixed state $\psi(^3P_1)=\alpha\psi(^3P_1)+\beta\psi(^1P_1)$ which is given by

$$M=\alpha M(1,M_J)+\beta g_{1M_L}\hat{\Pi}_0, \tag{7.10}$$

where g_{1M_L} is the singlet amplitude for the excitation of the $^1P_1$ level and $\hat{\Pi}_0$ is the singlet projection operator [$M(1,M_J)$ – see (7.7)], we obtain the following expression for the depolarization of electrons during the excitation of the mixed state:

$$D=\frac{-\alpha^2|G^1_{11}|^2+\beta(|g_{10}|^2+2|g_{11}|^2)}{\alpha^2(|G^1_{10}|^2+2|G^1_{11}|^2)+\beta(|g_{10}|^2+2|g_{11}|^2)}. \tag{7.11}$$

In the case of forward scattering

$$D=\frac{\beta^2|g_{10}|^2}{\alpha^2|G^1_{10}|^2+\beta^2|g_{10}|^2}. \tag{7.12}$$

Hence, it is clear that, as the energy increases and the contribution of exchange scattering $\sim|G^1_{10}|^2$ begins to fall, depolarization tends to 1 from $D=0$ at the threshold, which is in complete qualitative agreement with experiment.[51] Numerical calculations[52] based on (7.12) and on some simple assumptions with regard to the amplitude are in satisfactory agreement with the above experiment (Fig. 6).

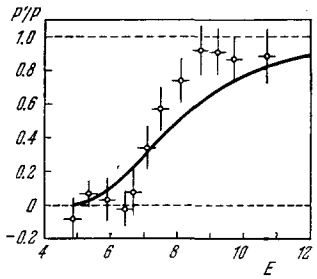

FIG. 6. Ratio of final to initial electron polarization for forward-scattered electrons after excitation of the 6^3P_1 state of the the mercury atom, plotted as a function of electron energy: points—experimental[51], curve—theoretical.[52]

D. Effect of molecular spin-orbit interaction on electron depolarization

In a completely similar way, we can use depolarization of electrons that have excited the singlet-triplet transition in a molecular to elucidate the role of spin-orbit interaction within the molecule. One of the manifestations of this interaction in molecules is the existence of the so-called mixed states which do not have definite values of the total spin. For such states, the transition intensity increases by several orders of magnitude in comparison with transitions between pure states with definite spin for which the transition is forbidden in the electric-dipole approximation. It may be seen that the admixture of a state with a different spin, for example, the admixture of the singlet to the triplet state, will have a clear effect on the depolarization of electrons that have excited this mixed state. Thus, using (7.3) to describe transitions for LS-coupling, we obtain the following depolarization after some simple calculations:

$$D=\frac{-(1/3)\,a^2g\bar{g}+b^2f\bar{f}}{a^2g\bar{g}+b^2f\bar{f}}, \tag{7.13}$$

where a,b are the weights of the pure states and g,f are the amplitudes for the excitation of these states. Since, with increasing electron energy, direct excitation begins to dominate exchange excitation which acts on the pure triplet state, D will vary between $-1/3$ near the threshold and 1. In the absence of the singlet-state admixture, D should remain equal to $-1/3$ at all energies (there is no fine structure, i.e., it is assumed that we are dealing with transitions between Σ states).

E. Spin-orbit interaction in the continuum

Polarization of electrons through scattering by spin 0 targets which assume spin 1 as a result of an inelastic process is a generalization of the well-known Mott problem. For the $^1S\to{}^3S$ transition (atom without fine structure), partial-wave analysis[53] of the three-electron system under investigation has led to the following expression for the inelastically-scattered electron when the spin-orbit interaction was taken into account in the continuous spectrum:

$$\mathbf{P}=\frac{\mathbf{n}\,\mathrm{Im}\,(m\bar{h})}{|m|^2+|h|^2}; \tag{7.14}$$

where m and h are the excitation amplitudes. When the spin-orbit interaction is weak, we have $h\to 0$, and the electrons remain unpolarized.

F. Fine structure and polarization of scattered electrons

The fact that fine structure due to the spin-orbit interaction in the atom should be reflected in the polarization of the scattered electrons was predicted by Hanne.[54] The polarization is perpendicular to the plane of scattering. The distinguishing feature of this polarization is the absence of a small parameter connected with the spin-orbit interaction between the atomic electrons. This effect has not as yet been confirmed experimentally. A possible experimental way of detecting polarization of this kind is to use elastic scattering of slow electrons by atoms with well-defined fine structure (for example, oxygen). Low energies can be used to exclude polarization due to Mott scattering and thus examine the effect in its pure form.

G. Penning ionization in optically pumped helium plasma

Ionization processes in helium, namely,

$$\mathrm{He}\,(2^3S)+\mathrm{He}\,(2^3S)\to\mathrm{He}+\mathrm{He}^+ + e, \tag{7.15}$$

where one of the atoms undergoes a triplet-singlet transition, form another type of process involving a change in the spin of one of the colliding particles. Nonconservation of the spin of the atom due to the fact that the two-electron system can be in either triplet or singlet states means that the problem cannot be reduced to a two-particle problem as in Chap. 6, but must be interpreted as a four-electron problem, i.e., with a maximum degree of detail.

Hence, it follows, in particular, that rigorous description of polarization phenomena involves, 16×16 matrices. Since the process defined by (7.15) is allowed for total spins $S=0$ and 1, it follows that, according to (7.2), the amplitude matrix can be written in the form

$$M=\hat{A}_0G_0+\hat{A}_1G_1. \tag{7.16}$$

For the transition operators $\hat{A}_s$, we find, using the definition given by (7.1) and the explicit form of the spin functions for the system (7.15), that

$$\begin{aligned}\hat{A}_0&=\frac{\sqrt{3}}{4}(\sigma_2-\sigma_1)\cdot(\sigma_3-\sigma_4),\\ \hat{A}_1&=\frac{\sqrt{3}}{4}(\sigma_2-\sigma_1)\cdot(\sigma_3+\sigma_4).\end{aligned} \tag{7.17}$$

Using the two-electron density matrices for each atom, we obtain

$$\left.\begin{aligned}\sigma&=A+B\,(\mathbf{P}_a\cdot\mathbf{P}_b)+C\sum_i Q_i^{(a)}Q_i^{(b)},\\ A&=\tfrac{3}{4}(|G_0|^2+3|G_1|^2),\\ B&=-\tfrac{3}{2}(|G_0|^2+|G_1|^2),\\ c&=\tfrac{3}{4}(|G_0|^2-|G_1|^2).\end{aligned}\right\} \tag{7.18}$$

In particular, it is clear that, for a pure state cor-

responding to initial spin $S = 2$ (in this case, $\mathbf{P}_a\mathbf{P}_b = 1$, $\sum Q_i^{(a)}Q_i^{(b)} = 1$), the cross section for the transition is identically zero. This result is in agreement with Wigner's rule on total spin conservation. The remaining results given below refer to vectorially polarized atoms. Since, for atoms with initial vector polarization $Q_{ij} = (\frac{1}{3})\delta_{ij}$, the cross section becomes

$$\sigma = |G_0|^2 + 2|G_1|^2 + B(\mathbf{P}_a \cdot \mathbf{P}_b) \equiv \sigma_0 + B(\mathbf{P}_a \cdot \mathbf{P}_b). \quad (7.19)$$

it is clear that, for atoms with the same initial polarization $P_a = P_b = P$, the moduli of the amplitudes are related to the cross sections as follows:

$$|G_0|^2 = \frac{4}{3}B - \sigma_0, \quad |G_1|^2 = -\frac{2}{3}B + \sigma_0, \quad B = \frac{\sigma - \sigma_0}{P^2}. \quad (7.20)$$

This means that two experiments, namely, measurement of the cross section σ_0 of unpolarized atoms and the ionization cross section σ of atoms with the same initial polarization are sufficient to determine the moduli of the amplitudes for Penning ionization. The actual polarization assumed by the electron after ionization in oriented helium plasma is of considerable practical interest. This is a topical problem in view of attempts to use Peening ionization in oriented media as source of polarized electrons. It is generally believed that the initial polarization of the atom in (7.15) is completely transferred to the electron. More detailed studies[49] show that, under typical experimental conditions, the electron polarization is $P' \sim (3/4)P$, i.e., it is high. A favorable feature of this process is that the polarization is practically independent of the collision energy when the cross section for the process is large.

In view of the other features of this process, we note that polarization studies can be used to analyze the initial metastable states of atoms and to investigate the effect of orientation and alignment on the various characteristics of plasmas such as, for example, electrical conductivity, luminosity, and so on.

8. CONCLUDING REMARKS

It is clear from the foregoing that analysis of the scattering of polarized electrons by atoms and molecules is a very detailed and sensitive way of studying elementary processes and the properties and structure of targets. In particular, this kind of analysis can be used to investigate new scattering characteristics (for example, scattering asymmetry and the moduli of the amplitudes), the role of relativistic interactions, the properties of metastable states, the type of atomic coupling, the geometric structure of large molecules, the parameters of quasistationary states, and so on. Further progress in the physics of polarized electrons will, in the first instance, depend on the development of new experiments with polarized electrons. The number of such experiments has so far been exceedingly small.

We conclude by listing certain types of experiment which seem to be the most realistic for the near future:

1) Determination of asymmetry in the scattering of electrons by oriented spin 1/2 and spin 1 targets. Investigation of the connection between asymmetry and polarization accompanying scattering in unoriented system [5];

2) Studies of new mechanisms for the polarization of electrons. This range of problems includes mechanisms responsible for polarization connected with non-conservation of the total spin of the system, changes in the spin of the target, and experimental investigation of polarization due to the fine structure of the atom.

3) Polarization studies under the conditions of resonance scattering by atoms and molecules with a view to estimating the parameters of quasistationary states and the classification of these states.

4) Measurement of the depolarization of scattered electrons with a view to establishing the type of coupling, the role of fine structure, and so on.

5) Studies of large molecules, simple crystals, and the structure of magnetic materials with the aid of polarized electrons.

The above list is not intended to be complete. There is no doubt that new and interesting practical applications of polarized phenomena will emerge in the course of research into the properties of polarized electrons.

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