M. A. Kumakhov. *Spontaneous and induced emission by relativistic particles in a crystal and possibilities for utilization of this effect in physics.* The interaction of a channeled particle with the lattice can be described in terms of the averaged potential of an atomic plane or a chain in the cases of planar and axial channeling, respectively. The steady-state motion of the particle in such a field is characterized by transverse-energy eigenvalues. There is a certain probability of transition between these levels with accompanying electromagnetic emission.[1-2]

What is fundamental here is that although the distance between the levels is usually a few electron volts, the frequency of the emission is shifted into the x-ray and γ-ray bands as a result of the relativistic longitudinal motion. In turn, the intensity of the emission rises in proportion to the squared energy of the particle.[1-2]

At high particle energies ($\gamma \gg 1$, γ is the Lorentz factor), it is possible to use classical concepts to calculate the emission. For a positron moving in a planar channel, the potential can be assumed to be close to harmonic, $V = v_0 x^2$ (x is perpendicular to the channel plane); $V_0 = 0.35(4\pi N z_1 z_2 e^2 l b) e^{-lb}$, l is the half-width of the channel, $b = 0.3/a$, a is the screening parameter, and N is the target-atom density. Then the path of the particle is a sine curve with an average radius of curvature R, where $R^2 = 2c^4/\overline{\omega}^4 x_m^2$, $\overline{\omega}^2 = (2V_0/m_0)\sqrt{1-(v_\parallel^2/c^2)}$; and x_m is the amplitude of the oscillation.

Then the intensity of the emission is[1-2]

$$I = \frac{x_m^2 e^2 \overline{\omega}^4 \gamma^4}{3c^3} \sim \gamma^2. \tag{1}$$

The spectral distribution of the emission

$$\frac{dI}{d\omega} = \frac{3I}{\omega_m}\left[1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m}\right)^2\right] \tag{2}$$

has a maximum at $\omega = \omega_m$, where $\omega_m = 2\overline{\omega}\gamma^2 \sim \gamma^{3/2}$ is the maximum frequency of the emission.

The differential distribution of the emission probability W is as follows:

$$\frac{dW}{d\omega} = \frac{3I}{\hbar\omega_m^2}\left[1 - 2\frac{\omega}{\omega_m} + 2\left(\frac{\omega}{\omega_m}\right)^2\right]. \tag{3}$$

This distribution has the form of a parabola symmetric about the minimum at the point $\omega \approx \omega_m/2$. The number of photons emitted along a length Δx is

$$N = 2 \cdot \frac{\Delta x}{c}\frac{I}{\hbar\omega_m}. \tag{4}$$

For motion of an electron with $E \approx 1$ GeV and $\Delta x \approx 10^{-2}$ cm, we have $N \gtrsim 10$. The number of photons emitted into a unit angle interval per second equals

$$\frac{dN}{d\Omega} = \frac{e^2 x_m^2 \overline{\omega}^3}{8\pi\hbar c^3}\,\frac{(1-\beta_\parallel\cos\theta)^2 - (1-\beta_\parallel^2)\sin^2\theta\cos^2}{(1-\beta_\parallel\cos\theta)^4}. \tag{5}$$

The spectral density of the radiation (for example, at $E \sim 1$ GeV) is two or three orders of magnitude higher than the bremsstrahlung density in the range $\omega \approx \omega_m$.[1-2] The radiation is polarized and monochromatic to a high degree.

Axial electron channeling can be represented in first approximation as helical motion with a certain radius $\bar{r}$ around the atom chain. Then the intensity of the emission is

$$I = \frac{2}{3}\frac{e^2 C}{R^2}\gamma^4. \tag{6}$$

Using the chain potential $V(r) = -(\overline{Z}e^2/Z) + C(\overline{Z}$ and C are parameters; for Si, for example, $\overline{Z} \approx 0\ 95)$.

$$R^2 = \frac{m_0\gamma^2 C^2 \bar{r}^2}{\overline{Z}e^2}, \qquad \bar{r} \approx 0.1 - 0.7\ \text{Å}.$$

Radiation from electrons is harder and more intense than radiation from positrons. However, electrons have a broader emission spectrum. In addition, the probability of electron capture into the channeling regime is lower and the dechanneling length shorter than in the case of a positron. A quantum theory of the emission was developed in Refs. 2–4.

At energies of more than a few GeV, the condition for dipolar emission is not satisfied. This problem was discussed by Bazylev and Zhevago,[5] who showed that the emission spectrum is then modified. They also showed that there is a lower limit in the hard-emission spectrum due to polarization of the medium. With increasing energy, the right end of the spectrum rises rapidly[1-2] and the left end rises somewhat more slowly,[5] so that the emitted frequency band becomes broader.

Terhune and Pantell[6] studied the spontaneous emission of electrons in MgO and confirmed that its intensity is much higher than the bremsstrahlung intensity.

A paper by Akhiezer's group[7] discussed the transition from coherent to spontaneous emission at angles of the order of the critical Lindhard angle. The classical calculation[7] confirms the results of Refs. 1-2 under channeling conditions.

Beloshitsky[8] derived a general formula for the spontaneous-emission intensity, including the optical region.

Podgoretskiĭ[9] considered the difference between electron and positron emissions.

Induced emission occurs along with spontaneous emission. The effect of amplification of induced emission and the possibility of building a continuously tunable laser on this basis were discussed in Refs. 2, 4, and 10.

The amplification coefficient is

$$C=\frac{3}{\pi}\lambda\lambda_{\min}\frac{\Delta N}{\Delta\omega\cdot\tau}, \quad (7)$$

where λ is the wavelength to be amplified, $\lambda_{\min}=\pi c/\gamma^2\omega_{ab}$, ω_{ab} is the transition frequency, ΔN is the population inversion of the transverse-energy levels, τ is the lifetime of the level, and $\Delta\omega$ is its width. The level width is governed by the nonmonochromaticity of the beam, multiple scattering, and band broadening due to the periodicity of the lattice. Values of $\Delta\omega\cdot\tau\sim 1$ may be reached at $E\sim 1$ GeV; at E of a few MeV, $\Delta\omega\cdot\tau\sim 10^2-10^3$. Amplification can be obtained at wavelengths of a few thousand angstroms at beam densities $\sim 10^3$ A/cm^2 (which are already available in several laboratories). Amplification in the x-ray band requires densities $\sim 10^6$ A/cm^2. The beam must scan the crystal to prevent damage to it.[10] One advantage of this laser over the magnetic-undulator laser is that a given gain could be obtained at beam energies smaller by two or three orders than those in the undulator. In addition, development of an x-ray laser might be attempted at reasonable energies.

We do not yet have sufficiently powerful γ sources in the hard range ($\hbar\omega\approx 0.1-100$ MeV). Synchrotron emission decreases exponentially in this range, and bremsstrahlung has two disadvantages: its spectral distribution is very diffuse and is also dispersed over the angles after leaving a thick target as a result of multiple electron scattering. In a thin target, on the other hand, it is necessary to produce high photon flux densities. Spontaneous emission suffers from neither of these shortcomings. The crystal itself directs the channeled particle, i.e., the direction of emission is fixed; further, the spectral density of the emission may be much higher than the bremsstrahlung density. It is therefore possible to obtain strong γ fluxes in a predetermined range of frequencies and angles by using the spontaneous-emission effect.

Since the frequency range covers the entire nuclear range, it is possible to manipulate nuclear transitions selectively and with high efficiency (nuclear "pumping").

At electron energies $E\gtrsim 1$ GeV, not only the differential (at $\omega\sim\omega_m$), but also the integral intensity of the emission is already an order of magnitude higher than the bremsstrahlung density; the difference grows to two orders at $E>10$ GeV. At high energies, therefore, we have extremely rapid pumping of electron kinetic energy into photon energy, i.e., ultra-high-power emission occurs.

Because of its high power and strong energy dependence, the spontaneous-emission effect can be used for energy and mass detection of particles at high energies, when Cherenkov counters become ineffective.[11]

Note should also be taken of the possibility of transforming certain elements into others by means of (γ,p), $(\gamma,2p)$ and other reactions, and of the possibility of producing neutrons in (γ,n) reactions. Estimates indicate that record-high neutron fluxes could then be obtained in heavy-current accelerators (current ~1 mA) at a beam energy of a few hundred MeV.

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