V. A. Bazylev and N. K. Zhevago. *Electromagnetic radiation emitted by particles channeled in a crystal.* In the effect in which a relativistic particle is channeled in a crystal, it travels macroscopic distances along planes or chains of the crystal's atoms without being scattered through relatively large angles.[1] The particle's transverse motion is limited approximately to interatomic distances and is relatively slow (nonrelativistic).

In this case, the total energy of the particle can be represented as the sum of the energy of longitudinal motion $E_{\parallel}$ and the energy of transverse motion $E_{\perp} \ll E_{\parallel}$. The latter takes on discrete values by virtue of the finiteness of the transverse motion. It is found that the "transverse" energy levels are determined by a Schrödinger equation in which the longitudinal energy takes the part of the mass of the particle. Therefore the transverse-energy levels depend parametrically on the longitudinal energy.

Electromagnetic emission occurs on spontaneous transfer of the particle from an initial state i of the transverse motion to a final state f. The energy of the emitted photon does not, generally speaking, agree with the difference $\mathscr{E}_i - \mathscr{E}_f$ between the energy levels.

The longitudinal motion makes the energy of the photon dependent on the direction of observation. For this reason, as Kumakhov[2] has observed, the spectral energy density maximum of the emission is shifted to increasingly hard frequencies and increases with increasing longitudinal energy of the particle.

We should like to draw attention to a number of interesting features of emission of radiation by channeled particles. The results given below were a consequence of a further development of Kumakhov's original classical calculations.[2] These results overlap to some degree with the results of his later papers (see Refs. 3 and 4).

Formally, the channeled particle can be treated as an atom that is "one-dimensional" in planar channeling and "two-dimensional" in axial channeling. Here the transverse energy plays the role of the internal energy of this atom. On the other hand, the crystal structure also influences the electromagnetic field of the emitted radiation. Frequency and spatial dispersion of the field occur in the crystal. The influence of dispersion on the emission spectrum of the relativistic channeled particle is found to be significant even in the x-ray frequency band, where the dielectric permittivity of the crystal differs only slightly from unity. This result follows from simple reasoning based on energy and momentum conservation in emission[5] which is expressed by

$$E_i'' - E_f'' + \omega_{if} = \omega, \quad \mathbf{p}_i - \mathbf{p}_f = \mathbf{k} - \mathbf{K}_h;$$

here E_i'', $\mathbf{p}_i$; E_f'', $\mathbf{p}_f$ are the energy of the longitudinal motion and the momentum of the particle before and after emission of a photon with energy ω and momentum $\mathbf{k}$ and $\omega_{if} = \mathscr{E}_i - \mathscr{E}_f$ is the difference between the transverse-motion energy levels. When the frequency dispersion is taken into account, $k = \omega\sqrt{\varepsilon'(\omega)}$ where ε' is the dielectric permittivity. The spatial dispersion is manifested in the fact that a momentum equal to one of the reciprocal-lattice vectors $\mathbf{K}_h$ can be transferred to the crystal as a whole during the emission process.

In the case of weak frequency and spatial dispersions ($|\varepsilon'(\omega) - 1| \ll 1, K_h \ll \omega$ ultrarelativistic energies ($E'' \gg 1$), small angles ($\theta \ll 1$), and relatively soft ($\omega \ll E$) emission frequencies, we obtain the following relation between θ and ω:

$$\theta = \sqrt{\frac{2(K_h^{(v)} + \omega_{if})}{\omega} - \left(E^{-2} + \frac{\omega_p^2}{\omega^2}\right)};$$

here $K_h^{(v)}$ is the projection of $\mathbf{K}_h$ onto the direction of the longitudinal particle velocity and $\omega_p^2 = 4\pi Ne^2$ is the plasma frequency. Emission is possible only if the radicand is positive. Leaving the spatial dispersion out of account ($K_h^{(v)} = 0$), this is possible in the frequency band

$$E^2\left(\omega_{if} - \sqrt{\omega_{if}^2 + \omega_p^2 E^{-2}}\right) = \omega_{\min}^{(j)} \leqslant \omega \leqslant \omega_{\max}^{(j)} = E^2\left(\omega_{if} + \sqrt{\omega_{if}^2 + \omega_p^2 E^{-2}}\right).$$

From this it follows, in particular, that there is a minimum particle energy (of the order of a few tens of MeV for electrons and positrons) at which the particle can still emit x-ray photons.

When spatial dispersion is taken into account ($K_h^{(v)} > 0$), emission of an x-ray photon with transfer of the particle to a higher transverse-motion level ($\omega_{if} < 0$) is found to be possible. This process is also possible at $K_h^{(v)} = 0$ if the susceptibility is positive ($\varepsilon' - 1 > 0$). A detailed calculation of the spectral and angular characteristics of the emission in these cases will be found in our paper (Ref. 6).

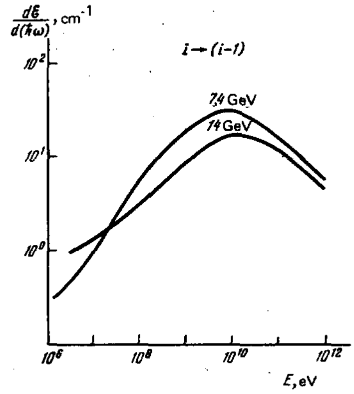

FIG. 1. Maximum value of spectral energy density of emitted radiation as a function of the energy of an electron channeled by the (100) plane of silicon. Curve 1 corresponds to the parabolic-well approximation of the potential of the plane and curve 2 to the rectangular-well approximation.

At medium particle energies ($\lesssim 1 - 10$ GeV for electrons and positrons), the emission is of dipolar nature. This case has been analyzed in detail.[2-6]

The emission pattern changes significantly as the energy of the particles rises. First of all, the multipole expansion of the emission field becomes generally inapplicable. In this sense, the emission from the channeled particle differs from the emission from atoms. Secondly, the distance between adjacent energy levels i and $f = i - 1$ begins to decrease more rapidly than $E_{\parallel}^{-1/2}$. Finally, the upper limit of the emission spectrum is generally determined by the expression $\omega_{\max} = 2E^2\omega_{if}/(1 + 2E\omega_{if})$, which takes into account the recoil on emission of a sufficiently hard photon by the channeled particle.

One consequence of the nondipolar nature of the emission is that radiative transitions can proceed to many levels even in a parabolic well, and a set of bands (partially overlapping) appears in the spectrum, mov-

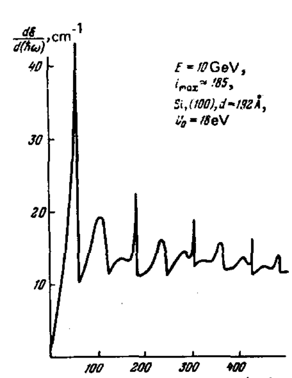

FIG. 2. Spectral energy density of emission per unit path of a 10-GeV electron channeled by the (100) plane of silicon as a function of emitted frequency.

ing toward increasingly hard frequencies, up to $\omega \sim E_i''$, as $i-f$ increases. The spectral energy density maximum of the emission begins to decline as the energy of the particle rises (Fig. 1). In contrast to dipolar emission, the positions of the maxima in the spectral density do not generally coincide with the bounding frequencies $\omega_{max}^{(f)}$ (Fig. 2)

The influence of the shape of the interplanar potential on the emission spectrum was investigated in Ref. 6. Figure 1 illustrates these results.

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