M. A. Liberman. *A contribution to the theory of ionizing shock waves in magnetic fields.* The development of high-power electro-magnetic shock tubes in which a dense hot plasma is formed behind the front of a strong ionizing shock wave (SW) in a magnetic field, has become a promising trend in the field of controlled thermonuclear fusion.[1] In nature, such waves are formed in the ionosphere and in outer space. In spite of more than twenty years of study of ionizing SW in magnetic fields, aspects of the problem still remain unclear. Moreover, the calculated structures of such waves disagree even qualitatively with observations.[2] Whereas theory[3,4] predicts that compression of the magnetic field should be observed behind the gas-dynamic discontinuity in the structure of the wave, the experimentally observed magnetic-field compression[5] leads the pressure pulse.[6]

1)A more detailed exposition of this problem will be found in the paper, which the authors have submitted to Uspekhi Fizicheskikh Nauk [Soviet Physics-Uspekhi].

 0038-5670/79/030190-05$01.10

Special theoretical interest in the problem is associated with formulation of boundary conditions. Since the three conservation equations (mass flux, momentum, and energy) are insufficient for derivation of the Hugoniot relations for the four variables that characterize the state of the gas (density, velocity, temperature, and magnetic field), an additional equation is needed. A generalized Chapman-Jouguet condition for normal ionizing waves was used as this supplementary condition in Refs. 7 and 8, while in Refs. 3, 4, and 9 the supplementary condition was treated as a corollary of the existence of wave structure.

In solving these problems, it is natural to seek an additional relation as a consequence of the stability requirement, in much the same way as is done in blast theory. Then ionization stability of the gas before the wavefront is necessary in addition to hydrodynamic stability.[10,11] Let us consider the elementary case of a transverse shock wave. If v_1 and H_1, v_2 and H_2 are the velocities and magnetic fields in front of and behind the wave, the magnitude of the induced electric field (as yet undetermined) in the nonmoving gas ahead of the front is $E=(v_1H_1-v_2H_2)/c$. The ionization-stability principle means that not only the temperature, density, and velocity, but also the electron density must be homogeneous in the gas ahead of the ionization-wave front. (The conductivity σ of the gas can be regarded as zero at a given electron concentration if the magnetic Reynolds number is small: $\mathrm{Rm}=4\pi\sigma v_1 L/c^2 \ll 1$.)

Thus, the rate of production of electrons in the induced electric field should be zero ahead of the wavefront, i.e., the induced electric field should be equal to the threshold value of the breakdown field E_b of the gas. Thus the sought additional relation is

$$v_1H_1 - v_2H_2 = cE_b. \qquad (1)$$

The possible magnetic structures of the ionizing wave depend on the relation between the intensity of the wave and the value of E_b for the particular gas. If the largest value of the induced field is $E_{max}=(v_1-v_2)H_1/c > E_b$, the boundary condition is given by formula (1). Then the structure of the wave consists of a region of compressed magnetic field behind which there may be a gasdynamic (viscous) discontinuity if the shock-wave intensity is above critical. (The appearance of a gasdynamic discontinuity is necessary for high-intensity waves because more energy than $H^2/8\pi$ cannot be dissipated on compression of the magnetic field.)[6,10] Figure 1 shows experimental values of v_2H_2/v_1H_1 for an ionizing shock wave in hydrogen and the values calculated from (1).

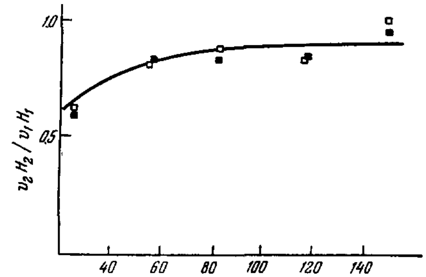

FIG. 1.

At low magnetic fields and high gas densities, when $E_{max} < E_b$, the ionization-stability principle can be satisfied only if the degree of ionization of the gas ahead of the front equals zero everywhere up to the gasdynamic discontinuity. Then if the magnetic field is not compressed in the gasdynamic discontinuity, we have $H_1 = H_2$ as the supplementary boundary condition instead of (1). Estimates show that nontrivial magnetic structure (with magnetic-field compression in the gasdynamic discontinuity) is possible only in a very dense gas in weak magnetic fields:

$$Na\,(\mathrm{cm}^{-3}) > \frac{v_1H_1}{cI\sigma_{ea}} \sim 10^{30}H_1,\quad H_1\,(\mathrm{Oe}) < 10^{-1}\frac{I}{\sigma_{aa}\sqrt{T/m}} \sim 10^{-4}.$$

[1]R. A. Gross, Nucl. Fus., 15, 729 (1975).
[2]B. P. Leonard, J. Plasma Phys., 17, 69 (1977).
[3]C. K. Chu, Phys. Fluids, 7, 1349 (1964).
[4]A. G. Kulikovskiĭ and G. A. Lyubimov, Dokl. Akad. Nauk SSSR 129, 52 (1959) [Sov. Phys. Dokl. 4, 1185 (1960)].
[5]C. E. Stebbins and G. C. Vlases, J. Plasma Phys. 2, 633 (1968).
[6]A. L. Velikovich and M. A. Liberman, Zh. Eksp. Teor. Fiz. 73, 891 (1977) [Sov. Phys. JETP 46, 469 (1977)].
[7]W. B. Kunkel and R. A. Gross, in: Plasma Hydromagnetics. ed. D. Berchader, California, 1962.
[8]R. T. Taussig, Phys. Fluids 9, 121 (1966).
[9]A. G. Kulikovskii, Prikl. Mat. Mekh. 6, 1125 (1968).
[10]M. A. Liberman and A. L. Velikovich, Plasma Phys. 20, 439 (1978).
[11]A. L. Velikovich and M. A. Liberman, Zh. Eksp. Teor. Fiz. 74, 1650 (1978) [Sov. Phys. JETP 47, 86 (1978)].