Einstein and optics

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This is an expanded text of papers presented in Berlin on March 2, 1979 at a session of the Academy of Sciences and Physical Society of the DDR celebrating the 100th anniversary of Einstein's birth, and also on March 28, 1979 in Moscow at a scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the Academy of Sciences of the USSR. Some points of the special theory of relativity and the quantum theory of radiation are treated. Primarily two problems are treated: the problem of a velocity exceeding the velocity of light in a vacuum, and the Doppler effect in a medium in its classical and quantum interpretations.

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INTRODUCTION

It is hardly necessary to say that Einstein exerted a vast influence on the physicsts of our generation. In discussing this, I have turned to the four-volume collection of Einstein's works published in Russian in the series "Classics of Science."[1–4] Reading this brought a feeling of joy and admiration, and withal, understanding of the fact that many of the results familiar to me can serve to illustrate Einstein's ground-breaking studies, or are useful in discussing them. For discussion I have mainly selected two problems: the problem of a velocity exceeding the velocity of light, and the Doppler effect in a refractive medium and its classical and quantum interpretations. Both of these problems are organically connected to the famous studies of Einstein in 1905: the study[5] in which the theory of relativity[1)] was formulated and the study[6] that founded the quantum theory of radiation.[2)] Einstein's development of his ideas in the following 5–10 years is also essential to the discussion. Of course, Einstein's contribution to the development of the problems of optics is considerably broader than the content of this article.

[1)]"Zur Elektrodynamik bewegter Kőrper" (1905) (see Ref. 1, pp. 7–35).

[2)]"Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt" (1905)[6] (see Ref. 3, pp. 92–107).

1. EARLY PUBLICATIONS OF EINSTEIN'S WORKS IN THE USSR

Einstein's worldwide fame spread especially broadly among physicists of the whole world after the end of the first world war. Einstein himself wrote in 1919 of his joy at the restoration of scientific communications with other countries "after the sad period when active contact between scientists broke down. . . ."[3)7] It seems to me that this was especially important for science in the young Soviet government. Before the October revolution in 1917, Einstein's name was known among us only to a narrow circle of scientists who were specialists in the field of theoretical physics.[4)] The situation sharply changes with the beginning of the twenties. Einstein's famous lecture "Geometry and Experiment,"[10] which was read before the Academy of Sciences in Berlin in January 1921[5)] was already published in Russian in the

[3)]Article in the *Times*, Nov. 28, 1919.[7] (See Ref. 1, pp. 677–681).

[4)]In the catalog of the library of the Institute of Physics of the Academy of Sciences of the USSR, I found only two publications of Einstein's studies in Russian before 1917. Of course, it was not fortuitous that they were such famous articles as "On the Development of Our Views on the Essence and Structure of Radiation" (1909)[8] and "The Principle of Relativity and Its Consequences in Modern Physics" (1910).[9] These articles were included in the collected volumes "New Ideas in Physics", which played a significant role in the development of Russian physics.

[5)]See Ref. 2, pp. 83–94.

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next year 1922. The book "On the Physical Nature of Space"[11] was published in the same year, containing both this lecture and the article "The Ether and the Theory of Relativity.[6)12] Einstein wrote a preface for this edition.

This was followed by a whole series of Einstein's books in Russian. I have counted seven books for the period 1921–1923 alone.[7)]

Einstein's popularity in our country was already then exceptional, while the widespread propagation of books on the theory of relativity evidenced the avid interest of the young Soviet science in everything new and progressive happening in the world.

I remember well this time, though then yet a student. The point is that my father, a mathematician, was a brilliant popularizer of the theory of relativity. I heard some of his elegant lectures, and this began my acquaintance with Einstein's work. However, Soviet science is not characterized simply by acquisition of knowledge. In 1922, a now-famous article appeard by the Leningrad scientist Aleksandr Friedmann[33] "On the Curvature of Space."[8)]

As we know well, in 1917 Einstein founded a new science: relativistic cosmology.[16] Friedmann here took the next step, all of whose exceptional importance was discovered only later.

2. EINSTEIN'S STUDY "ON THE ELECTRODYNAMICS OF MOVING MEDIA" AND SOME OF ITS CONSEQUENCES

In this discussion I shall repeatedly turn to Einstein's famous study of 1905, "On the Electrodynamics of Moving Media."[5] A comparison of the results contained in it with those derived by Lorentz and Poincaré lies outside my topic. For me it suffices just to grasp the fact that the theory of relativity is formulated here in almost completed form. The clarity of the postulates on which the study is based and the novelty and significance of a number of their consequences speak for themselves.

Einstein formulates two of his now widely known postulates as: "... all coordinate systems in which the equations of mechanics are valid obey the very same electrodynamic and optical laws ... ." "This hypothesis (whose content will be termed the "relativity principle" below) we intend to convert into a premise, and moreover, to make an extra assumption that only seemingly contradicts the former, namely, that light in a vacuum propagates with certain velocity c independent of the state of motion of the emitting object" (see Ref. 1, p. 7).[9)] Perhaps the most striking point in this study is how Einstein eliminates this apparent contradiction. It took Einstein's genius to note that the seemingly evident concept of simultaneity actually requires definition. In order to synchronize clocks at the mutually stationary points A and B, he proposes to use an exchange between them of light signals. The necessary time of passage of the light signal increases with the distance between the points A and B: time can serve as a measure of distance and vice versa.[10)]

We should note that nowhere in his first study does Einstein state that the velocity c of light in a vacuum is a limiting velocity that cannot be exceeded. He discusses this problem later in 1907[17] by employing the law of addition of velocities. If in any system of coordinates one can receive a signal propagating at a velocity V exceeding the velocity c of light, then, upon observing this signal in a system of coordinates moving away from it at a velocity u smaller than c, one can obtain a negative time of passage of the signal (as the theorem of addition of velocities implies). "This result shows that we are compelled to consider possible a mechanism of transfer of a signal such that an attainable effect precedes the cause. Although this result, as I see it, contains no contradictions from the purely logical standpoint, it still contradicts so much the character of all our experience that the impossibility of $V > c$ seems to be well enough proved" (see Ref. 1, p. 76).

If now, seventy years after he said this, we ask the question of whether a velocity exceeding that of light is possible, the answer is usually: a velocity greater than the velocity c of light in a vacuum is impossible, but a velocity is quite possible that exceeds the velocity of light in a refractive medium in the optical frequency region. As we know, precisely this case is realized in the Vavilov-Cherenkov effect.

This answer is at least incomplete. Einstein's statement actually differs: he states only that a signal cannot propagate at a velocity greater than the velocity of light in a vacuum. As for a velocity that doesn't involve transfer of a signal, the theory of relativity imposes no limitations here. Moreover, we continually encounter such velocities. It is worth taking up this problem in greater detail.

3. AN EXAMPLE OF SUPERLUMINAL VELOCITY

As we have noted above, examples are well known of the optics of superluminal velocities whenever a radia-

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6)See Ref. 1, pp. 682–689.

7)In 1921, also with a preface by Einstein, a translation was published of the book "On the Special and the General Theory of Relativity" (popular presentation).[13] (Characteristically, a fraction of the first publications for the Russian reader was published in Berlin.) The subsequent publications were mainly of translations from various German publications of this same book on the theory of relativity, which was first published in German in 1917 and was repeatedly reprinted in Germany[13] (see Ref. 1, pp. 530–600).

8)As we know, Einstein initially faced this study with doubt, having published a note[14] in which he said that "the results seem suspicious to me" (see Ref. 2, p. 118). However, upon receiving the author's explanations, he immediately acknowledged his error by publishing[15]: "My criticism, as I have become convinced by Friedmann's letter sent to me by Krutkov, was based on an error in calculations. I view Friedmann's study as being correct and casting new light" (see Ref. 2, p. 119).

9)In this study, instead of the symbol c now generally adopted for the velocity of light, Einstein denotes it by the letter V.

10)This time τ is obviously equal to $\tau = 2AB/c$.

tion source moves in a medium with a velocity exceeding the velocity of light in this medium (for any frequency region). Its velocity of motion V is always smaller than c, and the source traverses its path (e.g., from point A to point B) more slowly than light in a vacuum. Here it turns out that, even in a medium, the source cannot completely overtake all the spectral components of the light that it emits.[24,25] Nevertheless, in the frequency region in which the velocity V surpasses the phase velocity c/n of light, one observes the characteristic feature expected for superluminal velocities. Thus, a uniformly and rectilinearly moving charged particle emits light by expending its kinetic energy.

Instead of a particle moving from A to B, we can treat a light pulse running over a plane along the straight line AB. We can easily convince ourselves that certain restrictions on the velocity drop out here. In fact, let us assume that a brief light pulse is emitted in a vacuum from some point C remote from AB. In the case shown in Fig. 1, the pulse first reaches point A (position 1 of the pulse) and then runs from A to B with the velocity $V = c/\cos\varphi$ (positions *2* and *3*). Thus its velocity of propagation is greater then c, and all the more so as φ approaches $\pi/2$.

When $\varphi = \pi/2$, i.e., with normal incidence of the light, the velocity V becomes infinitely large—the light signal reaches A and B simultaneously. Hence we can use it to synchronize clocks simultaneously at both of these points. Here the light pulse running over the plane from A to B is quite real. We can easily convince ourselves of this by placing, e.g., light scatters on the path from A to B. The incidence of the light pulse on them will be accompanied by emission of flashes of light from these points. In this case there is essentially no difference from a particle moving from A to B and emitting light. Does the fact contradict Einstein here that the velocity V is greater than the velocity c of light? Does it contradict Einstein that clocks at A and B, regardless of the distance between them, can be synchronized as rapidly as we wish, and even instantaneously? First of all, we can easily convince ourselves that there is no contradiction here in the theory of relativity, since it does not forbid a velocity greater than c, but only a velocity of propagation of a signal exceeding the velocity of light in a vacuum. Yet in the given case the pulse arriving at B bears no information about point A, and it cannot be considered a signal proceeding from A. The light pulse arrives at B even when point A does not exist. As for the possible synchronization of clocks, it also does not contradict Einstein. Actually, in the studied cases the light signal is transmitted from point C to points A and B, and it cannot be employed simultaneously for synchronizing a clock at C with one at A nor at B.[11)]

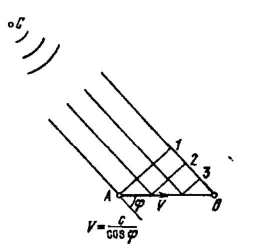

FIG. 1. A brief light pulse emitted from the remote point C reaches point A and then runs along the line AB with a velocity $V > c$.

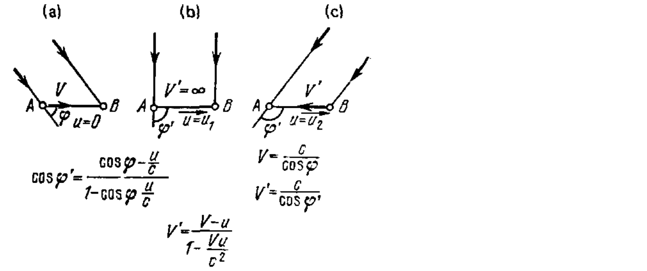

FIG. 2. In contrast to Fig. 1, the segment AB moves at the the velocity u (Figs. b and c). Then aberration of light converts the angle φ into φ'; see in the diagram the relativistic formula relating $\cos\varphi$ to $\cos\varphi'$. The angle φ' corresponds to the velocity of propagation of the pulse along the line AB, which is V'. When $u = u_1$, we have $V' = \infty$ (the angle $\varphi' = \pi/2$), but when $u = u_2$, V' is negative. The sequence of events is altered in this case—the pulse reaches B earlier than A. We can easily convince ourselves (see diagram) that V' is related to V and u by the relativistic formula of addition of velocities.

However, the problem in question is not so simple as it seems at first glance. In fact, V indeed has a real meaning. In order to convince ourselves of this, let us assume, e.g., that the segment AB moves at the velocity u along its length, while moving away from the light source (Fig. 2). Then, owing to aberration of light, the angle φ between the direction of the rays and the line AB will be altered and converted into φ', and V into V'. An acute angle φ' can become a right angle when $u = u_1$ (Fig. 2b) and then the velocity V' becomes infinite. Upon further increase of u to $u = u_2$, φ' is converted into an obtuse angle, and the velocity V' changes sign, so that the light pulse runs from B to A (Fig. 2c). Here we can easily convince ourselves that V' is transformed in exact agreement with the relativistic law of addition of the velocities V and u. Of course, this is not fortuitous (see Appendix 1). In line with what we have said, we obtain the same consequences as Einstein derived for V greater than c. If we could treat the incidence of the pulses at A and B as a signal traveling from A (the cause) and arriving at B as the effect, then the change of sign of V' would imply that the effect precedes the cause: the pulse at B arises before that at A. However, in the given case a velocity V greater than c, though real, yet does not contradict the law of causality.

Let us study another example that also shows the velocity V under consideration to have a close analogy with the velocity of a particle. We shall consider AB to be the boundary between two media whose plane is

[11)]In Ref. 5, Einstein postulates: "If a clock at C runs synchronously with a clock at A and one at B, then the clocks at B and C also run synchronously with respect to one another." This is precisely what happens in the given case (see Ref. 1, p. 10; we have altered the letter notation from Einstein's text).

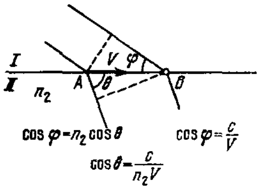

FIG. 3. The segment AB lies in the plane of the phase boundary between the vacuum and the medium II having the refractive index n_2. A light pulse reaching the phase boundary yields a refracted wave at the angle θ to the surface. It is easily established that $\cos\theta$ is related to the velocity V by the same relationship as for the characteristic angle in Vavilov-Cherenkov radiation, namely, $\cos\theta = c/Vn_2$.

perpendicular to the plane of the drawing. Let the refractive medium II with refractive index n_2 fill the half-space below the boundary AB. Then a light pulse from C running along the phase boundary will give rise to a refracted wave in the medium II that proceeds at the angle θ to the phase boundary (Fig. 3). If we employ the law of refraction of light ($\cos\varphi/\cos\theta = n_2$) and pay attention to the fact that $\cos\varphi$ is associated with the velocity V of propagation of the wave along the phase boundary, then we find that $\cos\theta = c/Vn_2$. That is, it obeys the same relationship as does the characteristic angle of the Vavilov-Cherenkov effect for a particle of velocity V (see Fig. 3). This analogy can be extended. For example, we can easily convince ourselves that the threshold velocity in the Vavilov-Cherenkov effect corresponds to the V, less than V_0, at which total internal reflection from the medium II begins, i.e., the refracted wave vanishes (see Appendix 1). However, while in the Vavilov-Cherenkov effect the velocity of the particle is always less than c, here it can have any value.[12)] We see from the above how essential Einstein's statement is that only the velocity of propagation of a signal is limited.

4. EINSTEIN'S APPLICATION OF THE LAWS OF CONSERVATION OF ENERGY AND MOMENTUM: THE LAW $E = mc^2$. EINSTEIN'S TRANSITION PROBABILITY COEFFICIENTS

The next section of this article will discuss the problem of the Doppler effect. In this connection, one of Einstein's results that was presented in his first study on the theory of relativity merits attention.[5] Einstein treats the case in which light is emitted during a given short time interval τ. Then the emitted light flux propagating from the source is contained at each instant of time t inside a sphere whose radius increases as ct.

In a moving coordinate system, this sphere is transformed into an ellipsoid, and here the amount of energy propagating in each given direction depends on the angle of observation. It turned out that, in a moving coordinate system, the energy of the light complex transforms in just the same way as the frequency. Qualitatively, we would now say this: N quanta are emitted in a given direction, whose energy is $N\hbar\omega_0$. In a moving coordinate system, owing to the Doppler effect, the frequency would change from ω_0 to ω, and their total energy to $N\hbar\omega$. Thus the energy transforms like the frequency (see Appendix 2). Did Einstein note this feature? Certainly he noted it, and in presenting this result, he writes[5]: "It is remarkable that the energy and the frequency of the light complex vary according to the same law as the state of motion of the observer changes" (see Ref. 1, p. 28). Neither here nor in this article as a whole did he say a word about quanta. Nevertheless, several months before this, in a no less famous article, presenting the fundamentals of the theory of quanta,[6] he wrote: "According to this assumption made here, the energy of a light beam leaving some point is not distributed continuously throughout the growing volume, but is composed of a finite number of indivisible quanta of energy localized in space, which are absorbed or emitted only as a whole" (see Ref. 3, p. 93). Of course, Einstein had not forgotten these words of his that he had said in the same year 1905, and we can suppose that this is just why he noted as a remarkable fact the identity of the transformation of energy and frequency. However, in a study concerned with the macroscopic properties of light, he deemed it superfluous to speak of quanta.

In an article in 1907,[18] in discussing the problem of the necessity of quantum representations, while at the same time noting their incompleteness, he writes on the basis of very cogent arguments: "Nevertheless, as long as we do not have a picture of the world at our disposal that corresponds to the stated requirements, we shall naturally employ, without fear of falling into error, the existing theory in all problems that do not deal with transformations of elementarily small quantities of energy, and also which do not pertain to relationships in which the entropy figures" (see Ref. 1, p. 54).

While the result on the transformation of energy was not employed in connection with quantum representations, yet it served as the basis for deriving another very important result of 1905 that essentially completed the special theory of relativity, namely, the establishment of the law of proportionality of energy and mass[19]—the famous relationship

$$E = mc^2.$$

To do this, Einstein treats a very simple thought experiment in which he applies for the first time the laws of conservation of energy and momentum to radiation processes: an emitter at rest emits two equal portions of energy in opposite directions. Obviously the light bears away zero momentum, and the emitter remains at rest. In a moving coordinate system the emitter will have the same velocity as it would have had if it had not emitted light. Upon treating the transformation of the energy of the light in transforming to the moving coordinate system and applying the law of con-

[12)]Initially it was precisely the analogy with the Vavilov-Cherenkov effect that attracted attention to the case treated here,[26] and then the problem was noted and discussed of velocities greater than the velocity of light in a vacuum, including also this case.[27] A separate chapter in V. L. Ginzburg's book[28] is devoted to this problem: "On Superluminal Radiation Sources".

servation of energy, Einstein showed that the kinetic energy of a light source that has emitted energy must differ from that of one that has not emitted. When the velocity remains the same, the kinetic energy can differ only if the masses are different. This unequivocally implies that the energy E of the light bore away a mass m such that $E=mc^2$.

Undoubtedly, Einstein attributes a very great significance to this result, which he derived in 1905. In 1906 and 1907 he again devotes articles[18,20] to it in order to substantiate the hypothesis that the relationship $E=mc^2$ is universal. Perhaps we should note that Einstein as yet had not even a hint of the very widespread erroneous formulation of the allegedly possible transformation of mass into energy. Even the first article in 1905[19] ends with the words: "If the theory corresponds to the facts, then emission transports inertia between the emitting and absorbing bodies" (see Ref. 1, p. 38). Thus, energy possesses mass, which it transports. The article of 1906 has the title "The Law of Conservation of Motion of the Center of Gravity and the Inertia of Energy"[20] and the 1907 article "On the Inertia of Energy as Required by the Principle of Relativity."[18] Thus every energy possesses a mass, and every mass contains an equivalent amount of energy.

We should note that Einstein considered the laws of conservation of energy and momentum to be so fundamental that the relationship $E=mc^2$ inferred from them was indubitable to him, though it did not seem possible at that time to test it experimentally. One could not expect that, in less than three decades, practical application of this law would become necessary for nuclear physics. This became evident only after the possibility had arisen of studying processes that occur in individual particles of matter. In particular, electromagnetic processes were studied in which the entire mass of particles arises at the expense of the mass of a photon. Figure 4 shows the case of the creation of an electron-positron pair arising in krypton in the disappearance of a particle of light (a photon) of energy $E=2.6$ MeV.

An essential point in the treatment below will be the fact that Einstein applied the laws of conservation of energy and momentum in his quantum theory of radiation. In contrast to the case in which one can apply classical representations, where an emitted wave usually propagates in all directions, he showed that the emission of a quantum of light is always directional.

FIG. 4. Photograph of an electron-positron pair produced by γ-rays of photon energy 2.6 MeV. (Picture taken in a Wilson chamber by L. V. Groshev and I. M. Frank.)

In the remarkable paper of 1916 "On the Quantum Theory of Radiation,"[21] the following phrase is italicized: *"It turns out that in a non-contradictory theory we encounter only the case in which all elementary processes are considered fully directional"* (see Ref. 3, p. 394). This implies that: In an elementary process of spontaneous emission, the molecule receives a recoil momentum of the amount $h\nu/c$, while according to the current state of the theory the direction is determined only by "chance" (see Ref. 3, p. 406). If we know all this and employ the laws of conservation of energy and momentum, we can, in particular, derive Doppler's law from an elementary quantum treatment also. Insofar as I know, this was first done by Schrödinger in 1922.[29] Both in the above-mentioned study by Einstein in 1916[21] and in the study[22] that had preceded it in the same year, the transition probability coefficients were first introduced and employed to derive Planck's formula:

1) the coefficient A_m^n for spontaneous transition from the state Z_m to the energetically lower Z_n, with emission of the quantum $\hbar\omega$ (Fig. 5a);

2) the coefficient for induced emission, which "can cause with equal success either a decrease or increase in energy"[22] (see Ref. 3, p. 390).

In the former case, an induced photon is emitted, and the transition probability is proportional to B_m^n. That is, the incident photon generates another photon $\hbar\omega_0$, and hence is converted into two photons (Fig. 5b′). In the latter case the photon is absorbed. The probability is proportional to B_n^m (see Fig. 5b).

Einstein considers all three processes to be essentially quantum processes, just because the transition occurs between two discrete Bohr states. However, he does not distinguish the process of induced emission of a photon as being a pure "quantum" process, in contrast to what people usually do now. For him B_m^n and B_n^m have equal rights. On this topic he writes[21]: "If a Planck resonator lies in a radiation field, then the energy of the resonator changes because the electromagnetic field performs work on the resonator; this work can

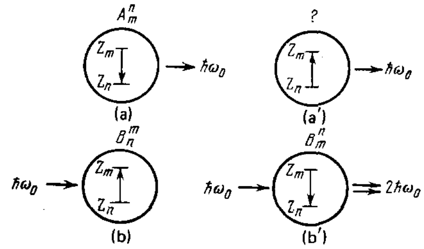

FIG. 5. Illustration of the Einstein transition probability coefficients A and B. The circles are marked with the corresponding quantum transitions. The coefficient A_n^m is lacking (question mark over the circle in Fig. a′).

be positive or negative, depending on the relationship of phases of the resonator and the oscillating field" (see Ref. 3, p. 396). Unfortunately, people often forget this classical analogy of absorption and induced emission.[13)]

Attention is called to the fact that, whereas the transitions $Z_m \to Z_n$ and $Z_n \to Z_m$ exist for an induced process, the spontaneous process has only $Z_m \to Z_n$. Correspondingly, only the coefficient A_m^n exists for the spontaneous process, but not the coefficient A_n^m. The difference seems obvious—the spontaneous process can only release energy, since there is no external agent here that could supply the energy. However, this ceases to be obvious if we imagine the radiation source moving in a medium with kinetic energy much greater than the energy of an emitted photon. The law of conservation of energy will not be violated in this case, since the kinetic energy suffices both for emission of a photon and for the transition $Z_n \to Z_m$. Then why is it always impossible for a spontaneous process to have a coefficient A_n^m along with the coefficient A_m^n? The answer lies in the fact that the law of conservation of momentum must be satisfied as well as the law of conservation of energy. It is not fortuitous that Einstein repeatedly stresses the importance of both these laws for radiation processes. He especially focuses attention on this in the article "On the Quantum Theory of Radiation."[17] He writes in the conclusions of the article: "Almost all theories of thermal radiation are based on treating the interaction between the radiation and the molecules. However, they are generally limited to treating the exchange of *energy* without accounting for the exchange of *momentum*" (see Ref. 3, p. 406). This happens, says Einstein, because the amount of change of momentum is usually small. "But in a theoretical treatment, one must consider such small actions equally important alongside the obvious transport of energy by radiation" (see Ref. 3, p. 406). Several lines before this, in speaking of the importance of the results he has derived, he writes: "... The most important conclusion is the one on the momentum that is imparted to the molecule in spontaneous and induced emissions" (see Ref. 3, p. 405).

By applying these two laws jointly, we can easily convince ourselves that they cannot simultaneously be satisfied in the uniform and rectilinear motion of a charge in a vacuum at a velocity less than that of light.[14)] In accord with the laws of electrodynamics, an electric charge in such a motion actually does not emit light. A different situation obtains with a charge moving in a medium. The velocity of the emitter here can be greater than either the phase or the group velocity of light for any frequency region. The process of spontaneous emission becomes possible, and in fact, the well-known Vavilov-Cherenkov radiation arises. Yet if the emitter has no charge but possesses a characteristic frequency, then the so-called anomalous Doppler effect arises.

5. THE DOPPLER EFFECT IN A REFRACTIVE MEDIUM

The set of fundamental problems treated in the first paper on relativity[5] also includes the Doppler effect. Einstein deemed this problem fundamental. Much later, in 1916 in the article "On the Quantum Theory of Radiation"[21] he writes "... Whatever form the theory of electromagnetic processes takes, the Doppler principle and the law of aberration is in any case conserved..." (see Ref. 3, p. 409).

Einstein treated Doppler's formula repeatedly, while citing it also for the case in which the light source is at rest while the medium moves (see, e.g., Ref. 5), and conversely, for motion of the source and a medium at rest (see, e.g., Ref. 17). Of course, both of these formulas are interconnected in elementary fashion.

If a medium at rest is filled with matter having the refractive index $n(\omega)$ differing from unity, then one can easily derive Doppler's law by analogy with the case for a vacuum by simple wave considerations. For nonrelativistic velocities, the Doppler shift in a refractive medium has been known for a long time, and was treated, in particular, by Einstein as early as the paper of 1905[5] in connection with the problem of the entrainment of light by a refractive medium. He treated it in general form for the same problem in 1907.[17]

For a relativistic velocity, Doppler's law has the form represented by the formula in Fig. 6a. It corresponds to the case in which a light source having the characteristic frequency ω_0 is moving in a medium, while the medium is at rest. Here θ is the angle between the light ray and the velocity of the particle as measured in the stationary system of coordinates associated with the medium. The difference between this formula and the case of a vacuum consists only in the fact that the cosine in the denominator has been multiplied further by the refractive index. This is quite natural—instead of the ratio of the velocity v of the particle to the velocity of light in a vacuum, i.e., β, the ratio figures here of the velocity of the particle to the velocity of propagation of the wave c/n in the medium, which is smaller than in vacuum by a factor of $n(\omega)$, i.e., $\beta n(\omega)$.

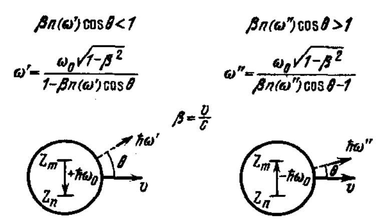

FIG. 6. Doppler formulas for a medium (light source moving while the medium is at rest). The normal Doppler effect is on the left and the anomalous effect on the right. The quantum transitions are shown below that correspond to the appearance of the normal and anomalous frequencies.

[13)]As N. G. Basov noted in discussing my report, this probably happens because actually the analogy is not very elementary.

[14)]In order to convince ourselves of this, it suffices to transform to a system of coordinates in which the charge is at rest. Evidently such a system is equally valid with respect to the original one.

Since the Doppler frequency ω' is an essentially positive quantity, the inequality of Fig. 6a must be satisfied. Here both it and the Doppler equation contain the refractive index for the Doppler frequency ω', rather than for the characteristic frequency ω_0 of the emitter. In a number of cases, this circumstance can lead to essential features. For example, possibly not one, but simultaneously several Doppler frequencies can be emitted in a given direction θ for a given ω_0 and β—this is the so-called complex Doppler effect[15)26] (the equations of Fig. 6 have several solutions).

The spectrum of Doppler frequencies determined by the equation of Fig. 6a is termed normal. This equation always has a solution, i.e., for any ω_0 and arbitrary β, including even a dense medium at small θ.[26] For a relativistic particle with $\beta n(\omega_0) > 1$, the fulfillment of the inequality of Fig. 6a is ensured in this case by the shift of the Doppler frequency into the region of anomalous dispersion, where $n(\omega')$ is close to or less than unity.

The lower part of Fig. 6a shows schematically the mechanism of appearance of the normal Doppler frequencies. As usual, the emitter undergoes spontaneous transition with decrease of internal energy by the amount $\hbar\omega_0$, and here a photon of energy $\hbar\omega'$ arises.

An essential point for the treatment below is that the equation in Fig. 6a is not unique. In a dense medium having $n(\omega)$ greater than unity and for a relativistic particle, the so-called anomalous Doppler effect can occur[26] as defined by the equation in Fig. 6b.

Evidently an inequality must also be satisfied to keep the denominator in Fig. 6b positive, but now it is the inverse of that in Fig. 6a. We can easily convince ourselves that the Doppler frequency defined by this equation behaves in a quite unusual way. In fact, we are accustomed to the idea that a moving light source emits light of greater frequencies forward (blue shift in the spectrum), and lower frequencies backward (red shift).

This relationship differs for the anomalous Doppler effect. Thus, in a region of the spectrum in which the refractive index varies little, the Doppler frequency does not decline with increasing angle θ, but on the contrary increases. We can easily convince ourselves that the anomalous Doppler effect is possible only for acute angles θ, and that there is a threshold velocity necessary for its appearance for every ω_0 for which it is possible. The Doppler effect here is always complex, not only because normal and anomalous frequencies arise together in some frequency region, but also because the anomalous effect itself is always complex. The question arises as to the nature of the emission for which the anomalous Doppler effect arises, and how it differs from the normal form of emission. While taking things out of turn, we should say that spontaneous excitation of the emitter occurs in the anomalous Doppler effect with emission of the photon $\hbar\omega''$ (see Fig. 6b).

We are accustomed to the situation in which, if a light-emitting oscillator exists, then the electromagnetic field that it creates must act on it to damp the oscillations. This is natural because emission of light carries away energy. Hence, according to the law of conservation of energy, the energy of the oscillations must decrease. That is, they are damped. However, it is not *a priori* obvious what the reaction of the field on the emitter will be if the latter moves faster than the light wave. The light source will overtake the wave, and in a system of coordinates associated with the source, the wave will seem to meet it head on—here the vector **k** changes sign. It turns out that this has the effect that the reaction of the field on the emitter also changes, and a force arises that tends to pump it by the wave that it emits. We can easily convince ourselves that no contradiction arises here with the law of conservation of energy. In fact, the radiation field can retard the moving particle, and hence the kinetic energy of the motion of the emitter will be transformed into the energy of vibrations and radiation.[16)] The fact that the radiation affects the translational motion is quite evident if we adopt the quantum viewpoint. The emitted photon bears away momentum, and hence the emitter undergoes recoil. If the recoil is directed oppositely to the velocity, then the motion is retarded, and if it is in the same direction, it is accelerated. Thus, in order to understand the Doppler effect, we must treat the laws of conservation of energy and momentum as applied to a moving light source. This returns us directly to Einstein's remarkable studies on the theory of radiation, and as we have noted, it is an elementary consequence of them. Here Einstein's words are essential that "whatever form" the theory of radiation "takes, the Doppler principle ... is conserved in any case." In fact, if we assume in the quantum treatment that the energy of the emitted photon is small in comparison with the kinetic energy of motion (which corresponds to the assumption that the motion occurs at constant velocity), then Doppler's classical formulas are a necessary consequence. The only problem that is not completely elementary in this treatment is what momentum we should ascribe to the photon in the medium. V. L. Ginzburg first showed in the quantum theory of the Vavilov-Cherenkov effect that this momentum in the

[15)]More detailed treatment shows that not only the value of $n(\omega')$ and its frequency-dependence are essential for the appearance of a complex Doppler effect. It turns out that the group velocity of the light is also essential. This is an interesting problem that falls outside the scope of this report.[24]

[16)]The effect of the threshold velocity at which the velocity of the emitter begins to exceed the phase velocity of the light for any frequency is especially graphically manifested in the case of uniform, rectilinear motion of a charge. As long as the velocity is below the threshold, motion occurs freely—a reaction of the field on the moving charge is absent. However, above the threshold, the components of the field at the frequencies for which the vector **k** changes sign create a force that retards the motion. As we should expect, the work done by this force equals the energy of the Vavilov-Cherenkov radiation.[30] The same would happen also for a charge moving in a vacuum at a velocity greater than c. Sommerfeld found this as early as 1904–1905,[31] while Tamm showed that there is a direct analogy here with the Vavilov-Cherenkov effect.[32]

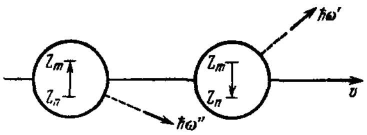

FIG. 7. Spontaneous light emission in the case in which the anomalous Doppler effect is possible. Emission of photons corresponding to the anomalous Doppler effect ($\hbar\omega''$) and to the normal effect ($\hbar\omega'$) alternate.

medium differs by a factor of $n(\omega)$ from that of the photon in a vacuum, where $n(\omega)$ is the refractive index.[33] Thus, in the medium the momentum is[17)] $p=\hbar k=\hbar\omega n(\omega)/c$.

Bearing this in mind, we can easily write the laws of conservation of energy and momentum for the moving light source. Here, if the light emitter goes from one energy state to another in emitting a photon, then according to Einstein we must consider the change of rest mass that alters the kinetic energy at the given velocity. Starting with this, we actually arrive at Doppler's classical formulas of Fig. 6[25,35] by elementary calculations (see Appendix 2). The results of the treatment here unambiguously imply that the normal Doppler effect (the formula of Fig. 6a) corresponds to the process that is usual for radiation—spontaneous transition from a higher energy state to a lower one that differs in energy by the amount $\hbar\omega_0$. As for the anomalous Doppler effect, the spontaneous emission of the photon $\hbar\omega''$ is actually accompanied by self-excitation, i.e., a transition to a higher energy state separated from the initial one by the amount $\hbar\omega_0$.[35] Thus we see that in spontaneous emission not only the Einstein coefficient A_m^n can be essential, but also the A_n^m. Here, just as Einstein saw it, this not only allows but requires application of the conservation laws. However, this happens only when the velocity of the emitter exceeds the phase velocity of light for the emitted frequency. We should bear in mind the fact that, although the formulas of Fig. 6 for the Doppler effect in a medium do not differ from the classical formulas, their interpretation in the quantum approach differs. In the classical treatment, the moving oscillator emits simultaneously both the normal and anomalous frequencies (if they are possible). The reaction of the field for the normal frequencies tends to damp the vibrations of the oscillator, while radiation at the anomalous frequencies tends to pump them.[18)] From the quantum standpoint these processes are not simultaneous, but occur successively: e.g., the emitter is spontaneously excited with emission of the anomalous frequency, followed by spontaneous transition to the lower state with emission of the normal frequency (Fig. 7), etc. The source of energy is the kinetic energy of motion, which must decline hereby.[30]

To the abovesaid we should add a few words on the Einstein coefficients B_m^n and B_n^m for stimulated processes. Evidently, if the photon $\hbar\omega$ can be emitted at the

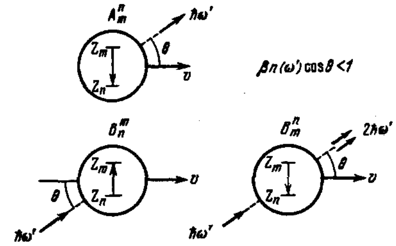

FIG. 8. The role of the Einstein coefficients A and B in the case of the normal Doppler effect. Everything occurs as indicated by Einstein (see Fig. 5).

angle θ, then the same photon directed at the same angle can be absorbed. The absorption process must be the inverse of emission. This implies that we obtain a different result, depending on whether a photon $\hbar\omega'$ corresponding to the normal process, or a photon $\hbar\omega''$ corresponding to the anomalous process, acts on the moving emitter. In the former case, we have $\beta n(\omega') \times \cos\theta<1$ (Fig. 8), and everything occurs in the manner indicated by Einstein. Excitation occurs upon absorption of the photon, while transition to the lower state occurs in induced emission of a photon. In the case of the anomalous process, we have $\beta n(\omega'')\cos\theta>1$. As we saw for spontaneous emission, the emitter is excited (Fig. 9), and hence a transition from the upper to the lower state must occur upon absorption of the photon $\hbar\omega''$. As for the induced emission of a photon, vice versa, it must be accompanied by excitation. Thus the roles of the Einstein coefficients B_m^n and B_n^m are interchanged, as shown in Fig. 9. We recall that no contradiction with the law of conservation of energy arises here, since both the internal energy of the emitter and its kinetic energy figure in the energy balance.

In the case of light scattering by a particle moving in a medium, we can also discover essential features. In the action of light on a particle moving in a vacuum, a scattered photon arises, but induced emission of light cannot occur here, since to do this the light must interact with the already oscillating particle. In a medium the situation differs—a photon of induced emission can arise, not with damping, but conversely, with pumping of the oscillations. For example, if light is propagating in the medium in the same direction in which the particle is moving, with the velocity of the particle greater than the velocity of the light, then the result will be an unexpected one, as shown in Fig. 10. Not only can a scattered photon of frequency higher than the original

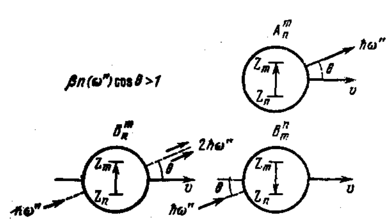

FIG. 9. The role of the Einstein coefficients A and B in the case of the anomalous Doppler effect. In contrast to the usual situation, spontaneous emission is governed by the coefficient A_n^m. The roles of the coefficients B are interchanged: the coefficient B_m^n corresponds to absorption, and B_n^m to stimulated light emission (cf. Fig. 5).

[17)]The necessity of this value of p is discussed in Ref. 34.

[18)]This treatment within the framework of the classical theory is contained in Ref. 36. Damping of the vibrations prevails in an isotropic medium.

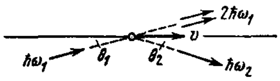

FIG. 10. Anomalous light scattering by a particle moving in a refractive medium. The incident photon $\hbar\omega$ give rise to an induced photon, and also a scattered photon $\hbar\omega_2$ arises. The spontaneous process corresponding to this is simultaneous emission of the photons $\hbar\omega_1$ and $\hbar\omega_2$ (two-photon Vavilov-Cherenkov effect). In both cases we must have $\beta n(\omega_1)\cos\theta_1 > 1$ and $\beta n(\omega_2)\cos\theta_2 > 1$, or conversely.

frequency arise, but the original photon will give rise to an induced photon, i.e., will be converted into two photons (see Appendix 2). The light scattering will cause the intensity of the original beam to increase.[37]

Inasmuch as the induced process can occur, so can the spontaneous process. We can easily convince ourselves that this will be Vavilov-Cherenkov radiation with simultaneous emission of two photons.[37] Such a process has not yet been studied experimentally.

Turning to the problem of spontaneous emission, we can say this: motion of a particle at a velocity greater than the velocity of light in a vacuum is impossible, but motion can occur in a medium at a velocity greater than the phase velocity of the waves. Nevertheless, nature does not completely lift its prohibitation. In fact, we see with the example of the Vavilov-Cherenkov effect and the anomalous Doppler effect that spontaneous emission arises here. Since kinetic energy is spent here on emission, the motion ceases to be free, and it is retarded.

I have examined in this article only a limited set of problems involving the special theory of relativity and the quantum theory of radiation.

The magnificent creation of the general theory of relativity and its amazing consequences for modern physics and astrophysics have remained outside the field of view of this paper. It would be impossible to encompass all this in a single report.

APPENDIX 1
DISCUSSION OF FEATURES THAT ARISE AT SUPERLUMINAL VELOCITY

As we have noted, the velocity V of propagation of a light pulse along the direction AB (see Figs. 1 and 2) actually possesses many properties that are usual for the velocity of a particle. If the segment AB moves at the velocity u in the direction AB, then in the coordinate system associated with AB, the angle between the direction of the rays and AB is altered by the light aberration. It changes from φ to φ' (see Figs. 2b and c), and here the velocity V changes to V' can be found by the relativistic law of addition of the velocities V and u. This is an obvious result, because Einstein derived the law of aberration from the law of addition of velocities. Hence the inverse transformation should also be correct. In fact, we have the following expression for the aberration of light:

$$\cos\varphi' = \frac{\cos - (u/c)}{1 - \cos\varphi\,(u/c)}. \qquad (1.1)$$

It suffices to assume that $V = c/\cos\varphi$, i.e., $\cos\varphi = c/V$, and correspondingly, $\cos\varphi' = c/V'$, so that we get directly from (1.1):

$$V' = \frac{V - u}{1 - (Vu/c^2)}. \qquad (1.2)$$

Now let us assume that the points AB are at rest and lie on the bounding surface of a half-space in which the light source C exists with the medium II having the refractive index n_2 (see Fig. 3). The law of refraction implies that $\cos\varphi = n_2\cos\theta$, or consequently,

$$\cos\theta = \frac{c}{n_2 V}. \qquad (1.3)$$

That is, θ obeys the same relationship as the angle of emission in the Vavilov-Cherenkov effect. This result is trivial, since it implies only that the phases, both of the incident and of the refracted wave, propagate along the phase boundary with the same velocity V. (In the medium II, this velocity is $V_2 = c/n_2\cos\theta = V$.) Nevertheless, the requirement that the phase of the wave run in the direction of motion of the particle with a velocity equal to its velocity v is the condition for appearance of the Vavilov-Cherenkov effect. This is just why the v at which Vavilov-Cherenkov radiation arises cannot be smaller than the phase velocity c/n. In this case, V is greater than c, and this condition is satisfied for any n_2 greater than 1. Evidently a refracted ray always arises here. However, we can also assume that the point C lies in the medium having the refractive index n_1, we assume that n_1 is greater than n_2. Then the velocity V_1 for the rays incident on the phase boundary will be smaller by a factor of n_1 than in the case of a vacuum: $V_1 = c/n_1\cos\varphi$. Since we have assumed that $n_1 > n_2$, total internal reflection becomes possible. We can easily convince ourselves that the following condition is satisfied for the angle of total interval reflection φ_0:

$$V_0 = \frac{c}{n_2}. \qquad (1.4)$$

Here we have $V_0 = c/n_1\cos\varphi_0$. It is evident from what has been said above that an analogy actually exists here with the threshold velocity in the Vavilov-Cherenkov effect.

A light pulse running along the phase boundary of two media can also be used for explaining the Doppler effect (this has already been done in Ref. 26). In order to do this, let us assume that a diffraction grating is imposed on the surface refracting the light, with fringes perpendicular to the direction AB. The grating, whose period is l, will modulate the intensity of the light pulse with the period

$$T = \frac{l}{V_1} = \frac{2\pi}{\omega_0'}. \qquad (1.5)$$

Here ω_0' is the frequency of modulation.

Let us study the first-order diffraction spectrum in the medium II. A wave of frequency ω directed at the angle θ, propagates in the medium II along the direction AB with the velocity V_2. As we have seen, the latter is equal to $V_2 = c/n_2\cos\theta$. This wave traverses the distance l along the surface in the time

$$T_2 = \frac{l}{V_2} = \frac{l}{c}\,n_2\cos\theta = \frac{2\pi}{\omega_0'}\,\frac{V_1}{c}\,n_2\cos\theta. \qquad (1.6)$$

On the right-hand side of (1.6), the quantity l has been eliminated by using Eq. (1.5). The equation $V_2 = V_1$ that we have been employing thus far corresponds to the zero-order diffraction spectrum, i.e., refraction of light. For the first-order spectrum that we shall study, we have $V_2 \neq V_1$ and the phase of the wave in the medium *II* must precede or lag behind the wave in the medium *I*, upon passing through one lattice period, by the time

$$T_\omega = \frac{2\pi}{\omega}. \tag{1.7}$$

Here ω is the frequency of the wave in the diffraction spectrum. Thus we must have[19)]

$$T_2 - T = \pm T_\omega. \tag{1.8}$$

Upon employing Eqs. (1.5)–(1.7), we get an analog of Doppler's equation directly from (1.8):

$$\omega = \frac{\pm \omega_0'}{(V/c)\, n_2 \cos\theta - 1}. \tag{1.9}$$

The plus sign corresponds to the anomalous Doppler effect $\omega = \omega''$, and the minus sign to the normal $\omega = \omega'$ (see Fig. 6). Here ω_0'' has the meaning of the frequency measured in the stationary coordinate system, i.e.,

$$\omega_0'' = \omega_0 \sqrt{1-\beta^2}.$$

Upon fixing the frequency-dependence of n_2, one can study these solutions.

APPENDIX 2
QUANTUM DERIVATION OF DOPPLER'S FORMULA FOR A MEDIUM AND FREQUENCY CONVERSIONS IN ANOMALOUS LIGHT SCATTERING

Following Einstein, let us apply the laws of conservation of energy and momentum to a moving particle that emits light. The total energy of the particle and its momentum are

$$W = \frac{mc^2}{\sqrt{1-(v^2/c^2)}}, \quad p = \frac{mv}{\sqrt{1-(v^2/c^2)}}. \tag{2.1}$$

We shall assume that the photon carries away an energy and momentum that are small in comparison with the values W and p. Then the quantum treatment must lead to the relationships derived from classical wave considerations.

Upon differentiating (2.1) with respect to v and assuming that the rest mass is invariant, we get

$$\Delta W = v\Delta p. \tag{2.2}$$

Here Δp is the change in p in Eq. (2.1), i.e., the change in its absolute value. The quantity $\Delta W = -\hbar\omega$ must equal the energy of the emitted quantum, while Δp equals the change in the momentum of the particles arising from the recoil received in emitting the photon. As we have noted, V. L. Ginzburg[33] took the momentum of the photon in the medium from quantum considerations to be

$$p(\omega) = \frac{n(\omega)}{c} \hbar\omega. \tag{2.3}$$

Here $n(\omega)$ is the refractive index for the frequency ω. If the photon is emitted at the angle θ_0 to the direction of the velocity of the particle, then when $\Delta p/p \ll 1$, the value of Δp must be

$$\Delta p = -\frac{n(\omega)}{c} \hbar\omega \cos\theta_0. \tag{2.4}$$

Upon substituting this Δp into (2.2) and assuming in it that $\Delta W = -\hbar\omega$, we get

$$\cos\theta_0 = \frac{c}{vn(\omega)}. \tag{2.5}$$

This is actually the well-known relation from classical physics for the angle θ_0 of Vavilov-Cherenkov radiation.

What has been said above implies, first, that the condition (2.3) on the momentum is actually correct. The second fundamental consequence is that the laws of conservation of energy and momentum permit spontaneous emission of light by a uniformly moving particle if (2.5) is satisfied, i.e., if the velocity of the particle exceeds the phase velocity of light

$$v > \frac{c}{n(\omega)}. \tag{2.6}$$

Now let us take the next step and assume that the emission of light involves spontaneous transition from one quantum state to another, and hence, the rest mass is altered by the amount

$$\Delta m = m_2 - m_1 = \frac{\hbar\omega_0}{c^2}. \tag{2.7}$$

Here ω_0 is the characteristic frequency in a coordinate system stationary with respect to the emitter, while a positive Δm corresponds to transition from a smaller mass m_1 to a larger m_2, i.e., excitation of the emitter. In this case the relationship (2.2) for $\Delta m/m \ll 1$ is written as:

$$\Delta W = v\Delta p + c^2 \Delta m \sqrt{1-\frac{v^2}{c^2}}. \tag{2.8}$$

It remains now to assume again that $\Delta W = -\hbar\omega''$ and $\Delta p = -[n(\omega'')/c]\hbar\omega'' \cos\theta$, and replace Δm by the quantity on the right-hand side of (2.7). Then we get

$$-\omega'' = -\omega'' v \frac{n(\omega'')}{c} \cos\theta + \omega_0 \sqrt{1-\beta^2}, \tag{2.9}$$

whence we have

$$\omega'' = \frac{\omega_0 \sqrt{1-\beta^2}}{\beta n(\omega'')\cos\theta - 1}. \tag{2.10}$$

This is the Doppler equation in which the emitter is spontaneously excited (Δm positive), which is allowed only when the denominator of (2.10) is positive, i.e.,

$$\beta n(\omega'') \cos\theta > 1. \tag{2.11}$$

Now let us assume that Δm is negative, and hence a transition occurs from an excited state of mass m_2 to a state of mass m_1. That is, in (2.7) we have $\Delta m = -\hbar\omega_0/c^2$. Then instead of (2.9) we get

$$-\omega' = -\omega' v \frac{n(\omega')}{c} \cos\theta - \omega_0 \sqrt{1-\beta^2}. \tag{2.12}$$

This leads to the formula for the normal Doppler effect:

$$\omega' = \frac{\omega_0 \sqrt{1-\beta^2}}{1-\beta n(\omega')\cos\theta}. \tag{2.13}$$

This requires that

$$\beta n(\omega') \cos\theta < 1. \tag{2.14}$$

Equations (2.10) and (2.13) for the Doppler frequency do not contain $\hbar$, and they coincide with the equations

[19)]Here we are already treating the case of a light pulse running over the surface so as to have an analogy with a moving particle. However, this is not essential either for refraction or diffraction, and the same result is obtained for a continuous flux of white light. We can easily convince ourselves of this by treating it as an infinite sequence of light pulses.

derived by classical wave considerations. In this regard the question arises: is the method applied here for deriving them specifically a quantum method? Let us treat this problem in greater detail. We can assume that an emitter moving in a vacuum emits in the direction θ a portion of energy that is equal to ΔE_0 in the coordinate system associated with the source. We shall not consider this quantity to be quantized. Neither is there anything of a specific quantum nature in the emission of light in the given direction. We can always assume that we employ some optical system for this, e.g., a parabolic mirror.

In line with the consequences of classical electrodynamics, the existence of light pressure gives rise to a momentum that is taken up by the system emitting the light. The projection of this momentum on the direction of the velocity is evidently $\Delta p = (\Delta E'/c)\cos\theta$ (here $\Delta E'$ is the energy corresponding to ΔE_0 in a coordinate system with respect to which the light source is moving).

Thus, in Eq. (2.8) we can set

$$\Delta W = -\Delta E', \quad \Delta p = -\frac{\Delta E'}{c}\cos\theta, \quad \Delta m = -\frac{\Delta E_0}{c^2}.$$

Hence we have

$$\Delta E' = \frac{\Delta E_0 \sqrt{1-\beta^2}}{1-\beta\cos\theta}. \tag{2.15}$$

Consequently we obtain the result of which Einstein said: "It is remarkable that the energy and frequency of the light complex vary according to the same law as the state of motion of the observer changes."[5]

To go from this to the Doppler relationship, it suffices to assume that ΔE_0 and $\Delta E'$ consist of an identical number of light quanta. This is the only specifically quantum assumption contained in the derivation applied here, which employs the laws of conservation of energy and momentum. In essence this assumption is already contained in classical physics, and is even implied unambiguously by it if we compare Eq. (2.15) with the Doppler formula stemming from simple wave considerations. Essentially only the quantity $\hbar$ remains arbitrary here.

Now let us treat the problem of absorption of light and its induced emission by a moving emitter. If the photon $\hbar\omega$ is not emitted, but absorbed, then in Eqs. (2.9) and (2.12) we must change the sign of both terms containing ω. In order to obtain (2.10) and (2.11) again for ω'', and Eq. (2.13) and (2.14) for ω', we must also change the sign of ω_0. This is obvious: in absorption and emission of light, the sign of Δm must reverse. Hence, in the case of the anomalous process of (2.11), absorption of light is accompanied by transition from an excited state to a lower state. Conversely, when (2.14) is satisfied, as in the usual case, absorption of light increases the mass of the emitter, i.e., excites it. It is also easy to treat the problem of induced emission of light. Evidently, the incident photon is absorbed and emitted afresh in this case, and hence it changes neither in momentum nor energy. As for the second, induced photon, it is emitted, and hence again we arrive at Eqs. (2.9) or (2.12). They imply that the induced photon in the case of (2.14) causes a transition from the upper to the lower state, and vice versa in the case of (2.11). Thus, the Einstein coefficients B are interchanged in the case of (2.11) in induced processes in absorption and emission of a photon (see Figs. 8 and 9).

It remains only to treat the problem of light scattering, as was mentioned at the end of the article.

To do this, let us turn again to Eq. (2.2) and bear in mind the fact that the primary photon $\hbar\omega_1$ is absorbed, while the secondary photon $\hbar\omega_2$ is emitted. Hence we get the following from (2.2), with (2.4) taken into account:

$$\hbar\omega_1 - \hbar\omega_2 = \frac{vn(\omega_1)}{c}\hbar\omega_1\cos\theta_1 - \frac{vn(\omega_2)}{c}\hbar\omega_2\cos\theta_2. \tag{2.16}$$

This implies for the scattered photon ω_2 that

$$\omega_2 = \frac{\omega_1[1-\beta n(\omega_1)\cos\theta_1]}{1-\beta n(\omega_2)\cos\theta_2}. \tag{2.17}$$

This is the so-called equation of normal light scattering. It requires that both ω_1 and ω_2 obey (2.11) or also that (2.14) is satisfied for both frequencies. Now let us assume that the light scattering is accompanied by induced light emission. Then, as we have seen, we need not allow for the primary photon in the conservation laws, but only for the photon or induced emission. That is, we reserve the sign in (2.16) of both terms containing ω_1 (both the photon $\hbar\omega_1$ and the photon $\hbar\omega_2$ are emitted). Then we get[37]

$$\omega_2 = \frac{\omega_1[\beta n(\omega_1)\cos\theta_1 - 1]}{1-\beta n(\omega_2)\cos\theta_2}. \tag{2.18}$$

Satisfaction of Eq. (2.18) requires that, whenever ω_1 obeys (2.11), ω_2 must necessarily obey the inequality (2.14), or vice versa. Here the induced photon can to an equal extent obey either the one or the other inequality, provided that the second photon obeys the opposite relationship.

Several words in closing concern the Vavilov-Cherenkov effect, Eq. (2.16) with the signs changed in the terms containing ω_1 is a consequence of the law of conservation of energy and momentum for the case of spontaneous emission of two photons. Thus it is the condition for the two-photon Vavilov-Cherenkov effect, which was mentioned in the report.

Finally, some last remarks will be useful. As we have seen, when (2.6) is satisfied, Eq. (2.2) allows spontaneous emission. Nautrally one can invert the problem, and then Eq. (2.5) is the condition for absorption of a photon by the moving charge. This same condition can be treated also as the condition for stimulated emission of a photon. Returning to what Einstein said about the coefficients B, we should note that the problem of whether absorption or stimulated emission of a photon will occur in a given case is determined by the phase of the wave acting on the moving particle.

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