Achieving high resolution in Earth-based optical astronomy

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Progress in astronomy is now a function of unexpected discoveries that have resulted from improvement of its instruments, and specifically of methods for observation of fainter objects and observation with higher angular resolution. The resolution of ordinary astrophotographs obtained with long exposure times is the same for all objects brighter than the limit and is determined only by the atmospheric turbulence generated at existing observatories by orographic atmospheric circulation disturbances. A serious effort to find localities where these disturbances are minor and where the atmosphere is closer to free would make it possible to increase the amount of clear-sky time with images that are very good by present standards (0.5–1″ in a long exposure) to several times those that prevail at existing observatories. Nonclassical, interference methods deliver much better (by factors up to 10^3) resolution at the cost of very insignificant (factors up to 10^3–10^4) deterioration of the ability to register faint objects. The Michelson interferometer at its modern technical level and speckle interferometry are most promising among these methods. Their sensitivity limits are determined by the quantum nature of light and depend strongly, $\sim\beta^{-2}$, on long-exposure image quality. The intensity interferometer gives even better (now down to $10^{-3''}$) resolution, but its low sensitivity permits the use of this method only for bright stars. Adaptive optics does not appear to be widely useful in the astronomy of faint objects. A generalized relation is given for sensitivity as a function of resolution in the various techniques for resolution improvement, and prospects for their future development are discussed.

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1. INTRODUCTION

The attainment of high spatial resolution in the optical band determines the progress of contemporary astronomy. Improvement of the resolution obtained with telescopes makes it possible to detect new structural details of celestial objects and to improve the signal/noise ratio in observations of faint point objects in classical photography and spectrography, where the noise is determined by the skyglow background.[1] It would therefore appear appropriate to discuss improvement of spatial resolution in earth-based optical observations of celestial objects. There is no doubt that the number of ground telescopes capable of producing good images more frequently than do those currently extant will increase in the immediate future. An orbiting optical telescope with a diameter of 2 meters is to be built early in the 1980's. The nonclassical methods of improving resolution that have appeared during recent decades (in chronological order, the intensity interferometer, speckle interferometry, and adaptive optics) are coming into wider use; the periscopic Michelson interferometer may be reinvented at a higher technical level.

In this paper we attempt to appraise the progress of classical methods, to compare them with new methods, and to predict their future usefulness in solving astronomical problems. We shall consider not only the resolution of new methods (which is much higher than that of existing ones), but also their ability to register rather faint objects, a respect in which they are still far behind. Several reviews[2-6,56,57] have been devoted to methods of obtaining high resolution.

2. RESOLUTION OF CLASSICAL LARGE TELESCOPES

We shall first discuss the resolving power of large earth-based telescopes working in the classical photographic or spectrographic regime that is second nature to every astronomer and delivering resolutions of the order of 1″–0″.1. There is obviously no question of substituting something else for wide-angle high-resolution sky photographs; this material forms the basis for the development of modern astronomy, which examines many objects in order to select a few for detailed investigation on the basis of certain criteria. Optical astronomy also finds such objects by itself (peculiar stars and galaxies, interacting galaxies, supernova remnants) or uses reports from observers working in adjacent zones (quasars, pulsars, x-ray sources). A quiet atmosphere and good optics are the foundation of both classical and nonclassical resolution-improvement techniques; the angular diameter of the image has a very strong influence on the stellar-magnitude limits of the

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new methods.

The resolution of modern large telescopes is now determined exclusively by the properties of the atmosphere over the observatories in which they are housed. The optical systems of large modern reflecting telescopes are highly sophisticated, some of them approaching diffraction-limit quality (Fig. 1); practically all of them use the Ritchey-Cretien aplanatic wide-angle scheme with a primary hyperbolic or retouched hyperbolic mirror made from a material with a small $[(1-5)\times 10^{-7}]$ temperature coefficient of expansion. This makes for an optical system that is much more stable than those of previous telescopes and produces a field of good images whose area is 10–20 times larger than those of parabolic, for example 5- and 6-meter, instruments together with better light grasp (to $\sim 24^m$ at a signal/noise ratio of ~5).

The mirrors of modern reflectors are analyzed thoroughly both during the mirrormaking process and in the instrument (Fig. 2); their mechanical systems hold the optics in alignment and make it possible to track the observed object with high precision ($\sim 0''.1$) in its diurnal motion across the sky. Thus, the image quality delivered by large modern instruments is determined solely by atmospheric turbulence.

Although no true observer is ever satisfied with the images that he sees at the focus of his telescope, the world community of astronomers has not done all that it could to site its instruments in the best possible atmospheric conditions. Astroclimate research is no doubt the Achilles' heel of modern practical astrophysics. Writing on this subject, P. Fellgett, who introduced the detection quantum yield concept and Fourier spectroscopy into the astronomy of his day, noted that "Although significant improvements [in methods of investigating astroclimate] were made in the past, they

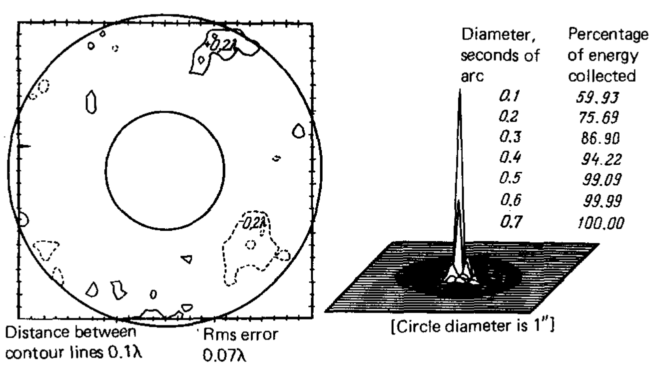

FIG. 1. "Geometric" distribution of light in image produced by the hyperbolic primary mirror of the 4-meter Ritchey-Chretien telescope at Cerro Tololo observatory. The directions of ~1300 normals to the surface of this mirror were determined and a computer was used to find the surface figure that conforms best to these normals. The figure shows the computed mathematical distribution of light in a star image constructed by a parabolic mirror with the same surface errors under ideal atmospheric conditions. (As we know, the hyperbolic mirror does not form a good image; for star observations it must be combined either with a correcting lens or with a second convex hyperbolic mirror in Cassegrain configuration.)

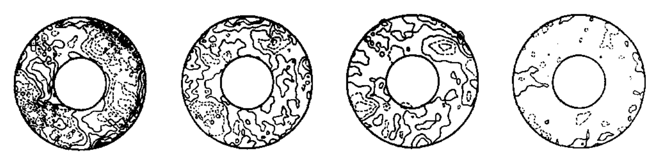

FIG. 2. It takes 2–3 years to make a large astronomical mirror. Quartz and Cer-Vit are used exclusively as materials for modern mirrors. In the final stage, most of the time is spent on tests performed by optical methods. To this day, larger mirrors are generally made in the optics laboratories of astronomical observatories without use of automation; the difficulty of making a mirror increases in proportion to D^4; it is considered reasonable to increase the diameter of the next mirror to no more than 1.5 – 2 times that of the preceding (successful) mirror. The figure shows the evolution of the surface of the 4-meter mirror of Fig. 1 in the course of polishing; the contour lines indicate the deviations of the surface from formula. The amplitudes of these deviations were about equal to λ (0.5μm) at the start of polishing.

have not produced a physical and quantitative description of image quality.

"At the present time, astronomers cannot even characterize, in statistical terms, the rms phase fluctuation caused by the atmosphere over any of their observatories, although they should know the distribution of the inhomogeneities along the line of sight, the temporal and spatial scales of the inhomogeneities, and the displacement rate of each significant component of the total phase perturbation.

"In the sense of cost and effectiveness, the productivity of a telescope costing several million monetary units may be doubled or cut in half, depending on the astroclimate characteristics of the site chosen for it. A program on which a few hundred thousand monetary units are spent and which consists of developing instruments appropriate for objective and absolute measurement of astroclimate parameters (and training of the corresponding specialists.—Author's note) might therefore produce a 100% profit if it were implemented for even one large telescope."

Let us consider certain properties of the distortions of a wave-front propagating through the turbulent atmosphere that are important to the astronomer.

Knowledge of the distortions to which the wavefront is subject in its passage through the atmosphere makes it possible to compute all of the pertinent characteristics of the star image formed by the telescope. Wavefront distortions can be characterized by a variety of parameters (Fig. 3). Among these, α is the average angle of deviation of the normal to the front, which is also known as the arrival-angle fluctuations. (It will be recalled that inclination of the front shifts the image at the focus of the telescope.) The shaking of images in a small telescope (Fig. 4) or the size of an image in a large telescope is approximately equal to α (we deliberately leave aside any rigorous quantitative analysis, referring the reader to the literature). The path difference structure function $W_x(D)$ is a more complete characteristic of wavefront distortion. By definition, $W_x(D)$ is the mean square of the fluctuations of the path difference x between two points of the wave-

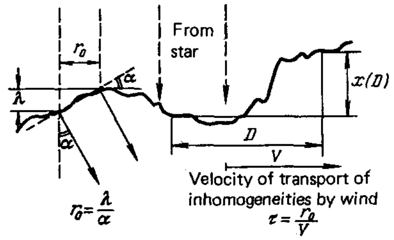

FIG. 3. The atmosphere has its main distorting effect on the phases of the light waves, i.e., it curves the wavefront. The figure shows a section of a wavefront arriving from a point source (star) after distortion by the atmosphere. The physical content of the principal distortion characteristics is indicated: they are the inclination angle α, the coherence radius $r_0 = \lambda/\alpha$, the structure function $W_x(D) \approx \langle x^2(D) \rangle$, and the characteristic time $\tau = r_0/U$ (see text).

front separated from one another by a distance D. Tatarskii[54] showed that, given satisfaction of the Kolmogorov-Obukhov law, the phase structure function $W_\varphi(D)$ [which is obtained by multiplying W_x by $k^2 = (2\pi/\lambda)^2$] has a quite definite form

$$W_\varphi(D) = 6.88 \left(\frac{D}{r_0}\right)^{5/3} \qquad (1)$$

The parameter r_0 in relation (1) was first introduced by Fried[7] and is a measure of wavefront distortion. It is the coherence radius of the wavefront, i.e., the base on which the path-difference fluctuations reach one wavelength in the mean. Since the phase fluctuations are caused primarily by inclinations of the front (see Fig. 3), $r_0 = \lambda/a$. On the other hand, according to

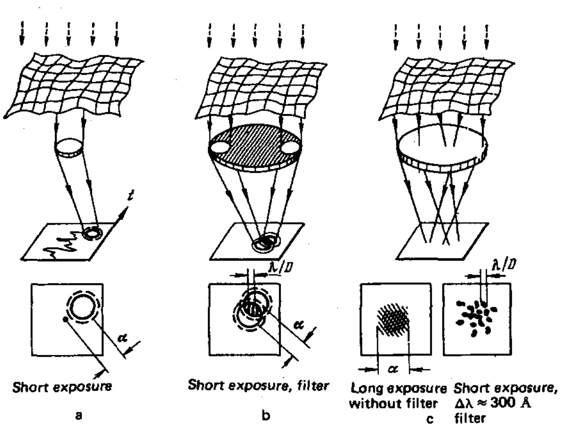

FIG. 4. Telescope imaging process. The atmosphere-distorted wavefront from the star arrives at the entrance pupil. a) small telescope ($r \leq r_0$). A diffraction image is seen; it trembles without distortion and is deflected from its mean position through an angle α. b) Michelson interferometer: a diaphragm with two openings of diameter $d \leq r_0$, separated by a distance D, is placed on the objective of a large telescope. Accordingly, two diffraction disks, which tremble independently, are seen in the focal plane. Where the disks overlap, we see interference bands with a period λ/D if the image is observed with a short exposure in quasimonochromatic light. c) Large telescope with diameter $D \gg r_0$. Rays that have been deflected differently by the atmosphere are collected at the focus. An interference grain pattern is seen in short exposures and with an optical filter; the grain size is on the order of λ/D. In the long-exposure case, the many individual instantaneous images are averaged to produce a diffuse spot of diameter α.

Fried[7], r_0 is the diameter of the aberration-free objective lens whose theoretical (diffraction) resolution is precisely equal to the resolution of a large telescope working through an atmosphere with a given degree of turbulence. Finally, if it it is assumed that the front distortions move uniformly in front of the telescope's pupil at the wind velocity V (the so-called frozen-turbulence hypothesis), it is easy to determine the characteristic time τ during which the image can change significantly: $\tau = r_0/V$. If the exposure time is no longer than τ, it is said to be "short".[1)] A "long" exposure is one that is much longer than τ, so that significant averaging of the image occurs during it.

It is interesting to relate the parameter r_0 to the much more common notion of the frequency-contrast characteristic (FCC) of the telescope or to the distribution of luminous intensity in the image of a star (the latter and the FCC are mutually convertible and hence equivalent). If $M(f)$ is the FCC of a large (diameter $>2r_0$) telescope working with long exposures through an atmosphere with a given r_0, we have

$$M(f) = \exp\left[-3.44 \left(\frac{\lambda f}{r_0}\right)^{5/3}\right], \qquad (2)$$

where f is expressed in rad^{-1}. Among other things, relation (2) states that the FCC is 3% when $\lambda f = r_0$. The diameter β of the circle into which 80% of the starlight collected by the large telescope in a long exposure falls is equal to $18/r_0$ (r_0 expressed in centimeters, $\lambda = 5000$ Å).

If we observe a star through the turbulent atmosphere with telescopes of various diameters r, we can make the following observation (Fig. 5). When $r < r_0$, we see a diffraction image that trembles. The amplitude of trembling is of the order of $\alpha \approx \lambda/r_0$ and decreases slowly ($\propto r^{-1/6}$) with increasing r. When we cross to $r > r_0$, the diffraction image begins to disappear, to "spread out." And this is easily understood: the wavefront can still be considered plane when $r < r_0$, but when $r > r_0$ the objective gathers light from several of its portions of dimension r_0, which deflect the light differently. If the observations are made with a short exposure, trembling will not be detrimental to the resolution. The same effect can be obtained by tracking down random displacements of the images as a whole. Theoretical analysis[7] indicates that for a given r_0 (i.e., for a given amount of atmospheric turbulence) there is a telescope of a certain optimum diameter that will give the best resolution in a short exposure.

We see from Fig. 5 that the resolving power of an objective working through a turbulent medium with astroclimate r_0 increases with increasing telescope diameter, approaching its asymptotic value quite closely even at $r = (2-3)r_0$. Short-exposure resolution does not behave in this way. At $r = 3.8r_0$ (such a telescope might be called atmosphere-matched), it is nearly

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1)For large telescopes, the image is softened slightly, but it trembles more slowly, with a characteristic time of the order of r/V. Tracking out of the trembling improves image quality.[7] An exposure time of the order of τ is necessary in speckle interferometry on any telescope.

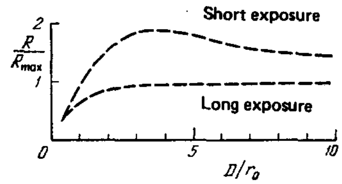

FIG. 5. When atmospheric turbulence permits long-lived flat wavefront areas of size r_0, the best resolution can be obtained on a telescope with a diameter of $\sim 4r_0$ if the random displacements of the object in the field are tracked out manually or automatically; the resolution will be that of an ideal telescope of diameter $2r_0$.

double R_∞—the resolution attainable with a very large objective in a long exposure. Then it decreases gradually, remaining sensibly better than R_∞ even at $r = 10r_0$. An atmosphere with a better r_0 makes it possible to work on larger-diameter telescopes while retaining matching.

The theoretical estimates described above are helpful in that they permit quantitative characterization of the resolution gain that can be obtained by careful guiding during the exposure, which eliminates the displacements of the image as a whole that are caused by the atmosphere. It is known that good observers have always done this (using the Common-Ritchey Cassette) and have also eliminated periods in which the images deteriorate by temporarily closing the shutter in front of the plate. Visual observers obviously do this involuntarily by tracking out, rejecting poor images, and perhaps going over into a speckle interferometry regime from time to time. This last property of the eye merits special notice. This is because even if the telescope does not construct a diffraction image at a given atmospheric turbulence level, it is possible, by shortening the exposure and narrowing the spectral range, to observe bright grains or "speckles" of diffraction size in the spot that it produces. They appear as a result of interference of the light collected at the focus from various areas of the objective (see below).

In the image of a double star or planet, the grains are doubled or join to form a disk, and the trained eye can attain a resolution near the diffraction limit for the particular telescope in observations through the turbulent atmosphere. In confirmation of this statement, we quote one experienced visual observer[34]:

"The skilled, experienced visual observer is able to seize on instantaneous manifestations of the image, an ability which seems to be inherent rather than acquired and which is often described by the phrase 'double-star' eye,: but which is as much mental as physiological....I recall many occasions when W.H. van den Bos and I discussed a phenomenon that puzzled us more than a little: the fact that we had no difficulty in measuring very close *bright* pairs in even the poorest seeing when instead of just one image we would see perhaps dozens in rapid motion, but each recognizable and measurable as a very close double star and resolved *as with the full aperture of the telescope*. We found this last fact difficult to reconcile with atmospheric cells considerably smaller than the telescope aperture. But now, of course, I know; like Monsieus Jourdain who spoke in prose without knowing it, we exploited the speckle technique without knowing it."

Thus is appears that the visual observer unconsciously uses three methods of improving resolution—switching to short exposures, image selection, and measurements by speckle interferometry.

Therefore visual atmospheric-turbulence estimates made with medium-sized telescopes on single and double stars and satellites of planets are usually highly optimistic, and the star-image diameter determined in this way often has nothing in common with what results in photography on large reflectors.

In investigating astroclimates, therefore, it is more expedient to determine the most pessimistic characteristic of atmospheric turbulence, r_0, i.e., the size of the wavefront area that remains undistorted for several seconds, and then to consider what can be gained by going over to a given observing method. At the present time, r_0 is determined with photoelectric instruments that measure atmospheric trembling[8,9] and with coherence interferometers.[25,10]

Problems requiring high resolution are on the leading edge of contemporary astronomy; it is their solution that determines its progress. It can be shown that the efficiency of a telescope. i.e., the faintness of the faintest object that it can detect, is proportional to Dr_0 in the classical photography and spectroscopy of faint objects; the sensitivity of nonclassical resolution-improving techniques is proportional to r_0 of a power higher than the first. Thus, the potential of an astrophysical observatory is determined by the parameter r_0 of its atmosphere.

The dependence of $\langle r_0 \rangle$ *on the observer's altitude in* the free atmosphere, i.e., the atmosphere unspoiled by local turbulence, was determined by A. Righini *et al.* with microtemperature radiosondes for the Western Atlantic region (see Ref. 11).

It was found that

$$\lg \langle r_0 \rangle_{(\mathrm{cm})} = 0.3 \lg h_{(\mathrm{m})} + 0.3,$$

and that the distribution of r_0 at a given height is near log-normal (Fig. 6). For existing observatories, $\langle r_0 \rangle$ is considerably poorer than in the case of the free atmosphere; the deterioration is due to local turbulence that is caused by the aerodynamic imperfection of the mountain peaks among which these observatories are built, turbulence that sometimes rises to heights of the hundreds of meters (Fig. 7). The topography of local turbulence is now being studied with microthermometers carried aboard aircraft; these measurements, made simultaneously with optical determinations of r_0, can evidently be used to find observatory sites that are closer to the free atmosphere than present ones (Fig. 6). It is definitely possible to find locations having high percentages of images with $r_0 > 20$ cm (Ref. 11; see Fig. 6).

The numerous reports of $r_0 \sim 70-100$ cm always come from visual observers; for the long-exposure case, however, there are practically no such images even in

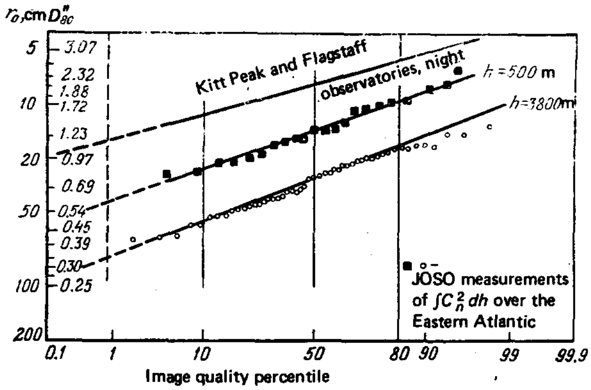

FIG. 6. Temperature fluctuations measured with radiosonde microthermometers can be used to plot the $C_n^2(h)$ distribution and find (given a large enough number of soundings) how often a given value of r_0 can be expected at a given locality. We see that the r_0 for observatories at which they have been measured suffer severely from local turbulence. The search for sites with weaker local disturbances must be continued; construction of observatories at such places would greatly promote the progress of earth-based astronomy.

the free atmosphere. Nevertheless, visual observations, which, as we have seen, can be made in the highly advantageous short-exposure regime with poor-image rejection, are, of course, highly valuable from the standpoint of the astronomical results that they can produce. Thus, for example, the noted double-star investigator Paul Couteau systematically observes them with the 50- and 76-cm refractors of the Nice observatory (the 50-cm instrument happens to be precisely matched to the value $r_0 = 10$ cm that has been registered at this observatory). Stars down to 10^m with equally bright components are resolved down to a distance of 0″.16, and brighter stars to 0″.12. Using the 2-meter McDonald Observatory reflector, G. Kuiper has succeeded in measuring the diameters of Pluto and several statellites of Saturn visually. The resolution here was 0″.05. In these observations, the measured disk of the planet was compared visually with a disk of the same brightness and known diameter.

Thus, the resolving power of earth-based telescopes will be improved as a result of:

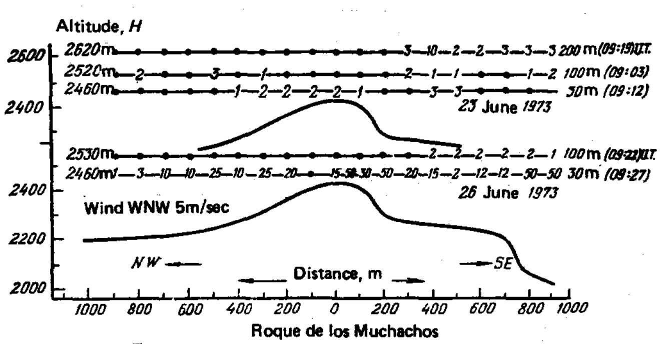

FIG. 7. Aircraft-microthermometer measurements of local turbulence topography near the peak of Roque de los Muchachos in the Canary Islands. At higher wind velocities, conditions for for astronomical observations from the peak of Roque de los Muchachos become much more favorable (this may not be the case for different terrain or a different night).[58]

a) siting them in less turbulent atmospheres;

b) going to short exposures;

c) image selection.

3. THE MICHELSON INTERFEROMETER

Another method of obtaining high resolution in the observation of astronomical objects from the surface of the Earth consists of use of the periscopic Michelson interferometer. The concept was advanced by Fizeau in 1968, but A. Michelson is credited with actually building-building the instrument. From the close contact that he maintained with observing astronomers throughout nearly all of his scientific career, Michelson apparently understood very well how important it was to improve methods for measuring the diameters of celestial objects. Indeed, the astronomer making measurements with the ocular micrometer must sometimes estimate a hardly noticeable broadening of the diffraction-pattern maximum due to the fact that the observed object is not a point object. Interferometric observations consist in the registration of a qualitative effect—the *disappearance* of the interference pattern at certain distances between the entrance apertures of the interferometer. It could be expected that these measurements would be more accurate. Michelson measured the diameters of the Jovian satellites (~1″.5) at the 11-inch Lick Observatory refractor in 1891. The telescope lens was fitted with a cover with two slits the distance between which could be varied by the observer. The measurements were found to be more accurate than those made with the ocular micrometer. In 1903, Michelson formulated the idea of the periscopic interferometer and posed the problem of measuring the angular diameters of stars,[12] but the effort was then suspended for more than fifteen years.

In the fall of 1919, having inserted a diaphragm with two apertures near the focus of the Lick 40-inch telescope, Michelson satisfied himself that atmospheric trembling does not blur interference bands on such a baselength; a month and a half later, the same experiment was repeated on the 60- and 100-inch Mount Wilson reflectors. The low sensitivity of the interferometer to atmospheric interference convinced Michelson and the observatory astronomers that a periscopic version of the device would work; its construction was begun. Working with Michelson at the 100-inch reflector, Anderson observed Capella in 1920; Merrill found that interference bands from the star Betelgeuse (α Ori) did not vanish completely as the baselength was increased to 2.5 meters. Work on the measurement of star diameters was begun in the fall of 1920.[13]

The Mount Wilson astronomer F. Pease continued work on the interferometers after Michelson's death. A horizontal instrument with a baselength of 15 meters was built and used to measure the angular diameters of several stars.[14] But the difficulties of working with this device were exceptionally great, and Pease's death and loss of enthusiasm by the Mount Wilson directors brought an end to the observations.[15] This was again followed by a very long period in which this method attracted no interest; only recently did a visual interferometer appear with a baselength that can be increased to 20 meters—a first repetition, so to speak, of the Michelson-Pease experiments[16] in modern form. It was used to measure the component diameters in a close binary system (Capella), which are comparable to the distance between them and equal to 0″.0057 and 0″.004.

Interest in double-beam interferometry began to revive during the 1950's perhaps as a result of the appearance of new light detectors. At about this time, Finsen designed his visual oscular interferometer, which is used in systematic observations of double stars.[15,17] It can split pairs no fainter than 7^m (the unaided eye sees 6^m stars) separated by no less than 0″.08. A 6-meter visual interferometer designed by V. P. Linnik has been built at the Pulkovo Observatory, but has not yet been used in any serious work. However, the Pulkovo astronomer E.S. Kulagin, who works with the device, has suggested an optical system with superposed beams and localization of the interference bands on the system's entrance pupil, which would be convenient for photo-electric registration.[18] It is interesting to note that back in 1915, S. Pokrowsky proposed what is essentially a similar optical system,[19] although his observing technique cannot be used in the presence of an atmosphere.

The construction of Michelson photoelectric interferometers did not begin until the 1970's. Most of them register interference bands by scanning with a black-and-white grating of the same frequency; the modulation of the light that results is measured.[20,21] Currie[22] proposed a different approach: the interference bands are localized in the telescope's exit pupil, which permits easy variation of the baselength, and use of a beam-splitter prism gives two channels with phase-opposed modulation, as in Kulagin's scheme (Fig. 8).

Let us attempt to estimate the sensitivity of the Michelson interferometer without stating the scheme of the instrument in specific terms and working instead from the most general considerations. Our estimates will be valid to within an order of magnitude. Sensitivity is determined by the quantum of light and the associated shot noise, and hence by the amount of interfering light, which depends decisively, as we shall see, on the properties of the atmosphere.

How large should the entrance apertures of the interferometer be made? It is obvious that their maximum size cannot be increased at will, but is related to the wavefront distortions introduced by the atmosphere. For the interference modulation of the light to be strong it is necessary to ensure that the light waves are in phase over the entire aperture area. If its diameter is d and the angle of wavefront inclination caused by atmospheric turbulence is α, this condition written in the form

$$d \leqslant \frac{\lambda}{\alpha}, \quad \text{or} \quad d \leqslant r_0, \tag{3}$$

since r_0 is the "coherence" region of the light that has passed through the atmosphere. This condition means that the two star images formed by the entrance apertures of the interferometer have an angular diameter λ/d larger than their random (atmosphere-caused) displacements α and, consequently, they are almost always superimposed on one another, producing interference bands in the region where they overlap.

Nevertheless, would it not be possible to increase the entrance apertures of the interferometer, i.e., to make $d \gg r_0$ (see Fig. 8)? After all, only N times more light would be collected $[N=(d/r_0)^2]$, and the relative signal fluctuations due to quantum noise would decrease by a factor $\sqrt{N}$, which should, it would appear, increase the sensitivity of the instrument. But then the light at the entrance aperture of the interferometer would no longer be in phase, since the aperture is larger than the coherence region r_0. Therefore specific areas of the aperture (with sizes $\sim r_0$) would produce independent interference signals (bands whose phases differ by an average of π behave as independent bands). The addition of N independent random variables would lower the relative fluctuations of the sum by a factor $\sqrt{N}$, i.e., the interference signal would be weakened by a factor $\sqrt{N}$ without any change in the signal/noise ratio, and the sensitivity would be no higher than before. This fact is well known in other areas of practical optics, i.e., in Fourier spectroscopy (so-called multiplexing). However, it must be remembered that multiplexing increases sensitivity when the noise is due not to the quantum nature of light, but to the radiation receiver (for example, in the infrared). Therefore the largest possible entrance apertures are advantageous for IR interferometry.

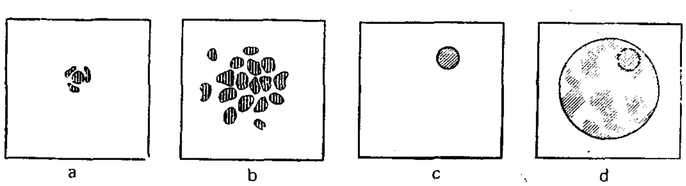

FIG. 8. Appearance of interference bands in various interferometers. a, b) Classical Michelson interferometer when the bands are localized on the image. If the diameter d of the interferometer aperture does not exceed r_0 (a), two diffraction disks appear with bands where they overlap. If $d=r_0$ (b), the images formed by the entrance apertures break up into individual grains, each with its own band system. Observing directly wavefronts that are superimposed on one another (c,d), we can say that the interference bands are localized at the entrance pupil. This situation obtains in the Kulagin[18] and Currie[22] schemes. The atmospheric front distortions now appear as a pattern of light and dark spots with sizes of the order of r_0. If $d \leqslant r_0$ (c), the light at the entrance aperture is coherent and interference modulates the luminous flux fully. When $d \gg r_0$ (d), $N=(d/r_0)^2$ spots fit into the aperture. Then, however, the modulation percentage is lowered by a factor $\sqrt{N}$ (see text).

A large interferometer aperture can be divided into N elementary areas of size r_0, the contrast of the bands measured in each of them and the results averaged. Accuracy (and sensitivity) will increase by a factor $\sqrt{N}$, but it will be necessary to measure the light at N points and this would require a multielement light detector. The theory for this case is given in Roddier's paper.[23] Currie's scheme[22] admits of very easy generalization to a "multielement" variant, and both he himself[24] and other investigators[16] are working in this direction.

The choice of spectral bandwidth $\Delta\lambda$ is also determined by the atmosphere: the fluctuations of the path difference x that it introduces may not exceed the coherence length, so that

$$\Delta\lambda \leqslant \frac{\lambda^2}{x}. \tag{4}$$

Here again, of course, we can attempt independent processing of the interference patterns produced by different segments of the spectrum. But the modulations at different wavelengths will not be independent (since they are determined only by x); this thought has been put to use in the achromatic interferometer of Wickes and Dicke.[21] "Achromatization" of a long interferometer with a 10–100 meter baseline is possible in principle only for bright objects, and not of great interest. The quantity x itself depends, of course, on the interferometer's baselength; it is determined by the phase structure function of the atmospheric disturbances. In the optical band it has thus far been measured only on comparatively short baselengths,[25] and the data are clearly inadequate. The available results indicate that Kolmogorov's law can be used as a first approximation[25] by extrapolating it to the appropriate distance. It must be remembered that the fluctuations will be low-frequency on long bases, so that Labeyrie's estimate[26] ($x = 0.15$ μm in the 0.5–10 Hz band on a 12-meter base) does not appear paradoxically small.

Finally, the characteristic time τ in which the interference phase changes by π is an important parameter. It determines the maximum possible signal-averaging time (not to be confused with averaging of the squared signal fluctuations, which can, in principle, be carried out as long as we please). In the experiments described, $\tau \sim 0.02$ sec. The time τ does not depend on the interferometer base because it is determined by "fast" phase shifts and, consequently, by small temperature inhomogeneities of the atmosphere. The lower the wind velocity and the larger r_0, the larger is τ.

Thus, all of the factors discussed above place limits on the light flux that can be accepted by the Michelson interferometer. Let n photons/cm^2 · sec · Å be the spectral density of the flux from the star (for a 0^m star in the green region of the spectrum, $n \approx 10^3$ cm^2 · sec^1 × Å^{-1}). Then δ photons in the spectral band λ^2/x enter an aperture of size r_0 during a time τ:

$$\delta = \frac{n\lambda^2 r_0^2 \tau}{x} \approx \frac{n\lambda^4\tau}{\alpha^2 x}, \tag{5}$$

where α is the size of the star image in a large telescope.

It is obvious that δ is the number of photons with which interference is observed. If $\delta \gg 1$, it is easy to measure the signal and find the prominence of the bands; if $\delta \ll 1$, the signal is lost in the quantum fluctuations. Thus far, we have been discussing a "simple" interferometer with an aperture of the order of r_0 and a single light detector, but, knowing δ, it is easy to calculate the sensitivity of a multi-element instrument, which will obviously be higher by a factor $\sqrt{N}$. In estimating δ, we do not take account of the quantum yield of the light detector, since the sensitivity will always be 1–1.15 orders below our estimate in practice. The significance of δ can also be explained differently: it is the ratio of the mean-square signal fluctuations due to interference to the mean square of the quantum noise. Knowing δ, we can find the signal/noise ratio for a given accumulation time. Let $\alpha = 1''$ ($r_0 = 10$ cm), $\lambda = 5000$ Å, $x = 5\lambda$, and $\tau = 0.02$ sec. Then $\delta = 1$ for a 15^m star.

We see that a Michelson interferometer can have very high sensitivity. Suppose, for example, that we accumulate the signal for a time $T = 1$ hr and wish to have a signal/noise ratio $K = 10$ for an interferometer with apertures of $d = 1$ m and a multielement detector. Then $N = (d/r_0)^2 = 10^2$ and

$$K = \delta \sqrt{\frac{T}{\tau}} \sqrt{N},$$

from which $\delta_{min} = 2.5\ 10^{-3}$, which corresponds to a $21^m.5$ object.

The signal processing described here ultimately results in averaging of the squared fluctuations. Is this advantageous? Use of a threshold device that accumulates information only at times when bands appear has been proposed, but detection of bands of a faint ($\delta \sim 10^{-2}$) object is an operational impossibility, and this "nonlinear" processing method is applicable only to brighter sources.

But what are the astronomical prospects for the Michelson interferometer in its modern form? At a glance, we perceive the familiar analogy with radiointerferometers, whose baselengths have now grown to be of the order of the earth's diameter and which have enriched contemporary astrophysics with priceless data on the fine structure of cosmic radio sources. However, these instruments differ in that the path-difference fluctuations introduced by the atmosphere in the radio band are small ($< \lambda$), while the phase shift can be measured easily and accurately. It is difficult to measure phase shifts in optics, and their atmospheric fluctuations are wide; therefore star-disk images cannot be expected from long-baseline optical interferometers. In principle, according to theory, optical phase shifts can be measured even under the conditions of the atmosphere. Perhaps future interferometers will be capable of constructing genuine images of celestial objects, as their radioastronomical brothers do now. Information on the exact position of the object with respect to the interferometer base vector is also present in the band phase shifts (radiointerferometers are now being used successfully in positional astronomy); this makes it difficult to use the Michelson optical interferometer to measure star coordinates, although the idea of using it for this purpose was suggested some time ago by

V. P. Linnik. Improving accuracy in such a highly developed area as optical positional astronomy is an exceptionally difficult matter; progress is more likely to appear when position measurements begin to be made at localities with good r_0 on instruments that have been adjusted to the optimum thermal regime, since the various methods of registering star transits across the axis of the instrument all deliver practically the same accuracy.

However, the large Michelson optical interferometer is unquestionably a promising tool. Resolution better than that of speckle interferometers, fair light grasp, and feasibility even with present-day electronic and optical components make the construction of such a device highly attractive. As for the problem areas with which it will deal, it may, like any other good instrument, prove "smarter" than its creators and find its own optimum objects of study. At the moment, apart from the boundless field of double-star activity (there are about 10^7 stars brighter than 15^m in the sky, and no fewer than 40% of them are doubles) and size estimates on peculiar objects, it is difficult to say anything more definite.

If the atmosphere distorts the wavefront, should it not be possible to "straighten it out" again artificially in an optical instrument ? The desirability of compensating phase fluctuations introduced by the atmosphere in real time was stated in very general terms in 1953 by H. Babcock,[27] who indicated a specific actuating mechanism (an oil film bombarded with electrons), but, of course, could tell us almost nothing of the phase-fluctuation sensor. This idea has recently been developed under the name of adaptive (or active) optics, and we shall now consider prospects for its use in astronomy (Fig. 9).

4. ADAPTIVE OPTICS

Dyson demonstrated the feasibility of this method in principle in 1975. However, even his strongly idealized estimates[28] indicated that phase tracking is possible only at $\delta > 1$, i.e., for stars no fainter than 15^m, and δ must be much larger than unity in practical work, Dicke[29] proposed an original optical scheme for adaptive compensation of wavefront distortions. The first successful experiments in this area have already been reported,[30] but the experimentors state that the 14^m limit can be reached only under exceptionally good atmospheric conditions. All experimental adaptive-optics systems that work through the atmosphere still have one property in common: they stop tracking irregularly and spontaneously, and resume it just as spontaneously. These tracking interruptions make for difficulty in adaptive-optics use of interferometers that cannot distinguish different bands from one another. The reader is directed to Hardy's review,[31] which has its own bibliography, if he wishes to gain a general acquaintance with the theory and practice of adaptive optics.

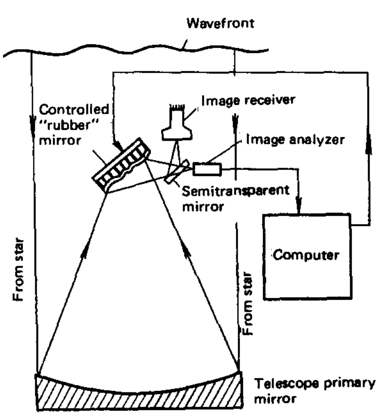

FIG. 9. Adaptive optics: one of the possible systems. The wavefront distortions are measured with a multielement image receiver; then the voltages that must be applied to the piezoelements to bend the "rubber mirror" and thus compensate for atmospheric turbulence are computed. When this is done, the telescope will construct an ideal (diffraction) image of the object.

The region of the field of view in which adaptive optics compensates distortions (the region of isoplanatism) is determined by the distribution of the optical disturbances along the line of sight and evidently cannot be more than a few seconds of arc. To compensate distortions in observing an extended object (for example, part of a planet), it is necessary to register a much larger number of photons than in the case of a star and to use complex computing algorithms. Finally, the number of mobile mirror elements is $(D/r_0)^2$, i.e., very large for large telescopes. All of these factors suggest that adaptive optics may not find a good "point of application" in astronomy. In principle, it would be possible to use it to improve the images of comparatively bright stars on the Coude-spectrograph slits of large telescopes, thus substantially improving the efficiency of these instruments, but much the same thing could also be accomplished with straightforward optical attachments. The possibility of observing the shapes of bright objects in real time with the aid of adaptive optical systems may, however, prove valuable in certain other areas of practical optics.

Although adaptive optics cannot compete with passive methods in sensitivity, we note that there is still one important particular case in which application helps the astronomer. This is the "matched" telescope of which we spoke above with fast automatic guiding to compensate wavefront inclinations. On a good night ($r_0 = 20$ cm), a one-meter telescope will produce images of only half diffraction quality on photographs made with trembling tracked out as a whole. Tracking-out itself is possible on rather faint stars (~13^m) using a very simple single-channel system. Such a system would be very useful in photography, say, of galaxies and star clusters.

5. SPECKLE INTERFEROMETRY

Speckle interferometry, a method proposed by Labeyrie in 1970,[32] also increases the resolution of telescopes working through a turbulent atmosphere. It is named for the fact that any more or less monochromatic image is of granular form; to verify this, it is sufficient to look attentively at objects illuminated by a laser beam. Why does this happen? Because laser light is coherent and it is not the intensities, but the amplitudes of the individual waves that are summed at the detector (for example, on the retina of the eye). The graininess is the result of the random interference that occurs when many individual independent amplitudes are summed. The sum amplitude will, naturally, be normally distributed, and the intensity I of the light (the square of the amplitude) will be distributed in accordance with the law $p(I)=(1/I_0)e^{-I/I_0}$. Simply put, the amplitudes cancel one another in some spots and intensify one another in others.

It is remarkable that the distribution of intensity in a speckle image does not depend at all on the nature of the scatter. The atmosphere is the scatterer in the case of star images. Light arriving from various areas of the wavefront, which have random phases, is collected and interferes at the telescope focus. The chaotic-interference phenomenon is also well known in radar: the level of the return signal is subject to rapid irregular variations caused by the same thing: different areas on the target reflect the radio waves with different phases, so that they either cancel or intensify one another.

The grains of the speckle pattern are sometimes identified with rays of light that have passed through different inhomogeneities and have therefore been focused at different points of the field. This is incorrect: as a result of diffraction, a slender bundle of rays always forms a large diffuse spot, rather than a "speckle," at the telescope focus. Only interference of several such bundles can produce fine details in the image, i.e., grains.

The characteristic size of the grains is determined by the entrance aperture of the telescope. As in the double-beam interferometer, the band period here is equal to λ/d. The baselength d is approximately equal to the diameter D of the telescope objective (more precisely, the average distance between interfering elements is of the order of $D/2$), and the grain size averages λ/D.

Astronomers have long been aware that star images are grainy, and even make use of it in visual measurements on double stars (see p. 650). But extensive use of this effect became possible only after Labeyrie proposed a convenient practical scheme for analysis of the resulting image. The observing methods and results of French astronomers who investigated double stars were no doubt familiar to him, and the appearance of lasers made it possible to identify the causes of image graininess. The emergence of speckle interferometry can also be traced to the rapid development of the theory and practice of image processing, which was also stimulated by the development of laser techniques and in praticular by the results obtained by Francon and May in speckle-pattern analysis.

The wavefront changes rapidly; therefore the image must be photographed with a short (of the order of τ) exposure in order to obtain a speckle pattern. Since the graininess is an interference phenomenon, it is necessary to ensure coherence of the light over the entire aperture of the telescope by limiting the spectral band by condition (4) or by the condition

$$\Delta\lambda < \frac{\lambda^2}{\alpha D}, \tag{7}$$

which is derived from (4) with $x=\alpha D$ (the path difference is governed by the wavefront inclinations); this is indeed nearly the case for Kolmogorov turbulence. The practical choice is $\Delta\lambda \sim 300$ Å.

The "grains" in the image of the star are distributed randomly if it is a single star. But when the star is a double (Fig. 10) the image consists of two identical grain patterns that are slightly shifted with respect to one another, since each component produces it own grainy image. These two images are identical if the components are close to one another and the paths of the light from each of them through the disturbing layers of the atmosphere are the same. Normally, the maximum angular separation of the stars at which this condition is still satisfied is about 1"; this is referred to as the region of isoplanatism. For a double star, therefore, a regular pattern that can be detected statistically exists in the chaos of grains.

Image autocorrelation offers a convenient method for processing speckle images. Let $I(x, y)$ be the intensity distribution in the image and let $C(p, q)$ be the autocorrelation function, which can be defined as follows:

$$C(p, q)=\left\langle \iint I(x, y)\, I(x+p, y+q)\, dx\, dy \right\rangle, \tag{8}$$

where the brackets indicate averaging. For a single star, autocorrelation indicates a central maximum of width λ/D, which reflects the fact that the image has granular structure. In the case of a double star, there are two symmetrically disposed "peaks" in addition to the central maximum, and from their positions and heights we can judge the distance between the components of the star, their position angle (accurate to 180°),

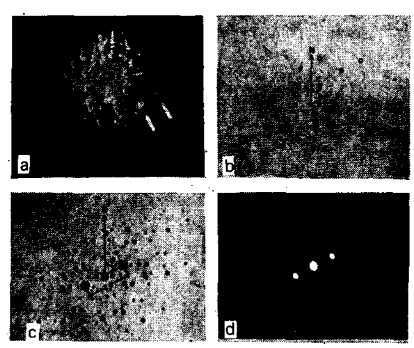

FIG. 10. Illustrating the principle of speckle interferometry.[34] The image of a double star (a) consists, as it were, of two speckle patterns superimposed on one another and then shifted; thus each speckle is repeated twice. The arrows indicate the characteristic grain configuration seen in the images of the first and second components. Figure b) shows a schematic double-star image composed of six pairs of randomly disposed points; the points simulate speckles. Figures c) and d) show the autocorrelation of this "point" image, i.e., the number of coinciding points on self-superposition of the shifted images, as a function of the magnitude and direction of the shift.

and the difference between their luminosities. If a "resolvable" star whose diameter exceeds λ/D is observed, the grains in its image will be coarser than those in the image of a single star, and the maximum in the autocorrelation function will also become broader. Star diameters are measured by speckle interferometry in this way.

Instead of the autocorrelation function of the random process (in this case that of the speckle image), it is also possible to study its power spectrum. This is convenient in that the Fourier transform of the image is easily calculated as an analog quantity by optical-coherence processing (Fraunhofer diffraction is expressed directly in terms of the Fourier transform). This simple and inexpensive processing technique is being used with success,[32,34] in spite of the difficulty of quantitative power-spectrum measurement. But this is not even necessary for double stars; there are bands in their Fourier spectra, and their frequency and orientation can be measured.[34]

The frequency-contrast characteristic (FCC) can also be introduced in the case of speckle interferometry, in which case it refers to the image power spectrum of a single star. The effect of granularity is that the FCC does not vanish at high frequencies, where it reproduces the FCC of an ideal telescope of diameter D in the absence of atmosphere, but reduced by a factor N (here N is the number of grains in the image, $N = (\alpha D/\lambda)^2$). Several studies have been devoted to theoretical analysis of speckle-interferometry FCCS.

Observations in the speckle-interferometry method consist of rapid registration of strongly magnified images in a narrow spectral band and subsequent reduction of the records. For bright objects, the autocorrelation (or power spectrum) is usually calculated by averaging from 20 to 100 frames. For observations of faint stars it is better to use a television image receiver with photon counting; the number of resolution elements (pixels) of conventional television devices is quite adequate for solution of this problem.

A simpler variant is a combination of an electronic image amplifier (EIA) with a motion-picture camera. Stars down to $6-8^m$ are accessible to such systems. Thus, H. McAlister makes systematic double-star observations on the 160-inch Kitt Peak telescope with the device shown in Fig. 11. He has succeeded in attaining rather high accuracy in his measurements (the closest pairs measured are $0''.035$ apart) and good output (more than 100 stars in a night). Study of close double stars is one of the serious applications of speckle interferometry, one that will, in the future, greatly increase the number of reliably measured stellar masses.[34] Good star-mass determinations are comparatively few in number, but are urgently needed for testing of stellar evolution theories.

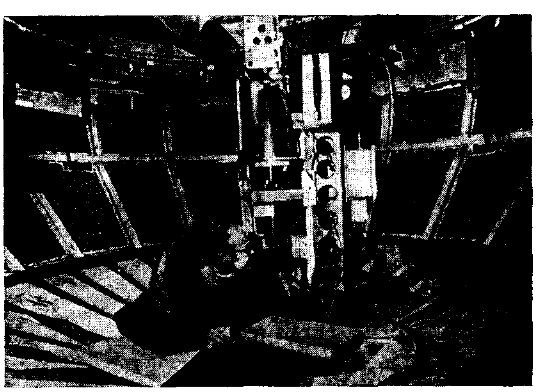

FIG. 11. Speckle interferometer at the Cassegrain focus of the 160-inch Kitt Peak telescope and its builder, C. R. Lynds.[34] A disk image of Betelgeuse (α Ori) has been obtained with this instrument (see Fig. 12), which is being used in systematic observations of double stars.

The image processing described above yields the autocorrelation function of the observed object, which is often enough. However, speckle interferometrists are also developing methods for reconstruction of the actual images,[37] which can be broken down into three groups:

1) Methods based on isolation and analysis of single grains,[37,38] which can work on comparatively "minute" objects. This approach was used in an attempt to obtain a disk image of Betelgeuse (Fig. 12).

2) Phase averaging of the Fourier transform, which has been proposed by Sodin.[39]

3) The Knox method[40] (correlation of Fourier transforms). The first application of this method to real images[41] (sunspot photographs) appeared along with computer illustration. Here the object must not be larger than the isoplanatism region (~1"), since otherwise it will be necessary to reconstruct it by parts and then assemble a "mosaic" image. It must be borne in mind that the more complex the observed object, the lower (other conditions the same) will be the sensitivity of speckle interferometry in registering it.

Let us make a trial estimate of sensitivity in speckle interferometry, which obviously depends on the number δ of photons per speckle. We know the number of speckles N (see above), the spectral bandwidth $\Delta\lambda$ [see (4)], and the exposure time τ. Clearly,

$$\delta = \frac{D^2\tau\Delta\lambda \cdot n}{N} = \frac{\lambda^4\tau}{x\alpha^2} n. \quad (9)$$

It is easily verified that δ is the same for speckle interferometry and the Michelson interferometer.

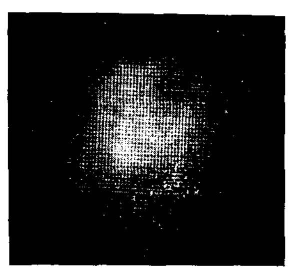

FIG. 12. Disk image of the red giant Betelgeuse reconstructed by C. R. Lynds using speckle interferometry.[38] There is still doubt as to the existence of the coarse details on the surface of the star.[55]

Therefore the sensitivities of the two methods are also equal in principle.

It is interesting to consider the method used by Blazit *et al.*[42] to process the speckle-interferometry pattern of an object that is very faint (14^m) by contemporary standards. For a faint object ($\delta \ll 1$), the image will consist of a small number of photons whose number is smaller than the number of grains N. The autocorrelation of this binary (i.e., having only two intensity levels, zero and one) image is easily calculated. It is sufficient to construct all of the possible vectors joining pairs of points of photon arrival and accumulate their histogram in a computer memory.[43] Suppose observations are being made on a double star with equally bright components. Those vectors whose directions and lengths do not correspond to the components of the double star connect photon arrival points that belong to different grains and are independent of each other. But a vector whose direction and length coincide with the separation and position angle of the star pair will be encountered more often than the others, since, in addition to random photons, it will sometimes connect photons belonging to corresponding grains in the images of the components: as a result, there will be more of these events. Needless to say, there will also be a large number of vectors that are shorter than λ/D and connect photons from the same grain. As a result, the histogram (autocorrelation) will have the form familiar to use, with a central maximum, two "peaks," and a "base" of random coincidences (Fig. 13). Let us break it up into cells of size λ/D. The average number of photons falling in one cell of the image equals δ; the number of coincidences in two arbitrary cells is δ^2, and for the entire image the number of these events corresponding to a given vector is $N\delta^2$. On accumulation of K images, $KN\delta^2$ events will correspond to a given vector; this is what determines the level of the random-coincidence background (for observations spanning $T=10$ hr, $K=T/\tau \approx 10^6$; $N \approx 10^3$). Suppose that 100 events are necessary to bring out "peaks" among the Poisson fluctuations of the "base" (the height of the "peaks" is of the same order as the height of the "base"). Then $KN\delta^2=100$, from which $\delta=3\cdot10^{-4}$, and if $\delta=1$ corresponds to a 15^m star, we have "detected" duality of a 24^m star. When actual experimental conditions are considered, about 20^m seems reliable as an estimated limit for speckle interferometry.[33,36,43]

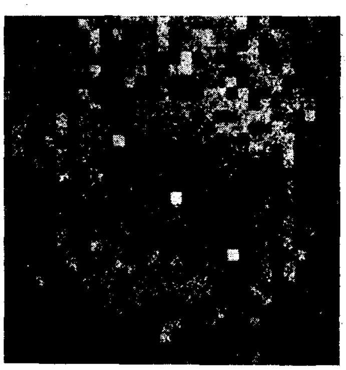

FIG. 13. Two-dimensional autocorrelation function of a large number of speckle images of a double star (computer simulation).[43] The size of each square corresponds to the size of a single grain of the speckle image, and its brightness is proportional to the number of vectors connecting points of photon arrival at the image and having a certain length (the position of each cell corresponds to the tip of one of these vectors; see text). Note the maximum at the center and the two collateral maxima, which suggest duplication.

The substantial difference between the theoretical (20^m) sensitivities is easily explained. Blazit *et al.*[42] indicated that a 13^m star yields two photoelectrons per image (i.e., $\delta \approx 2\cdot10^{-3}$). Therefore the processing procedure is correct, and improvements to the sensitivity of speckle-interferometric techniques is being held up by inadequate development of image-registration techniques.

Even now, speckle interferometry is acquitting itself well in areas other than the investigation of close double stars. Luminous gas has been detected in the vicinity of certain stars,[42] expansion of the shell thrown off by Nova Cygni 1975 has been recorded, and an upper bound has been found for the optical dimensions of a quasar.[42] It has been proposed that speckle interferometry be used on a space telescope to detect planets near stars[45]: This should make it possible to record emission nine orders of magnitude fainter than the light of the star itself.

6. THE INTENSITY INTERFEROMETER

Let us now analyze the capabilities of the intensity interferometer. This method was quite current in the 1950s and 1960s and fascination with it was no doubt detrimental to the improvement classical interferometers.

The intensity interferometer is based on measurements of the correlation between the fluctuations of light received by two detectors spaced a distance D apart. Here we refer to the fluctuations due to the random nature of light. They can be envisaged as beats between different frequencies, a kind of "instantaneous" interference. If optical radiation is observed in a frequency bond $\Delta\nu$, the characteristic beat frequencies will also naturally be of the order of $\Delta\nu$. Coherence of the light incident on the two detectors implies concordance of the vibrations of the light, and therefore the beats between them will also coincide. In other words, the fluctuations of the light are correlated if it is coherent. Accordingly, partially coherent light will give incompletely correlated fluctuations. In an elementary analysis of the case in which a double star emits only two frequencies, ν and $\nu+\Delta\nu$, one can determine the correlation of the signals as a function of the interferometer's base D. Thus, the operating principle of the intensity interferometer is, contrary to prevailing opinion, very simple, admitting of explanation within the framework of classical electrodynamics.

The random nature of the light vibrations makes it possible to describe them as a Gaussian process for which the intensity fluctuations $\langle \Delta I^2 \rangle = I^2$. For two partially coherent light fluxes I_1 and I_2, we have $\langle \Delta I_1 \Delta I_2 \rangle = \gamma I^2$, where γ is the coherence coefficient and is to be measured (I is the average intensity of the light). Knowing γ as a function of the base D, we can determine the dimensions of the observed object as in the Michelson interferometer. If M independent oscillations (modes) of total intensity I are registered, the fluctuations will be less wide:

$$\langle \Delta I_1 \Delta I_2 \rangle = \frac{\gamma I^2}{M}. \qquad (10)$$

Let us now take the quantum nature of the light into account (a more rigorous theory can be found for example, in Perina's book[46]). Let n_1 and n_2 be the number of photons registered by the two detectors during the characteristic period of the fluctuations, $\langle n_1 \rangle = \langle n_2 \rangle = n$. Then

$$\langle \Delta n_1 \Delta n_2 \rangle = n + \frac{\gamma n^2}{M} = n\left(1 + \gamma \frac{n}{M}\right). \qquad (11)$$

Here the first term describes the Poisson fluctuations and the second is fully identical to the classical intensity fluctuations. The quantity $\delta = n/M$ is called the degeneracy parameter: it equals the number of photons incoming per mode of the radiation. We see that δ characterizes the magnitude of the classical fluctuations as compared to the quantum fluctuations. To understand the content of δ, let us attempt to calculate it directly for a star. Let β (rad) be its angular diameter on the sky, and $\Delta\lambda$ its spectral bandwidth. Then the coherence length at the surface of the earth will be λ/β and the coherence time $\lambda^2/c\Delta\lambda$. If the star emits as a black body of temperature T, its brightness

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}. \qquad (12)$$

An energy E falls on the coherence area during the coherence time, and we divide it by the quantum energy hc/λ to obtain δ:

$$\delta = \frac{E}{hc/\lambda} = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \Delta\lambda \frac{\lambda^2}{c\Delta\lambda} \beta^2 \left(\frac{\lambda}{\beta}\right)^2 \frac{\lambda}{hc} = 2\frac{1}{e^{hc/\lambda kT} - 1}. \qquad (13)$$

The multiplier 2 actually disappears after polarization is taken into account, and we arrive at the well-known formula for the Bose-Einstein distribution. For $T = 6 \cdot 10^3$ K and $\lambda = 5000$ Å we obtain $\delta = 2.6 \cdot 10^{-2}$.

Incidentally, it follows from (13) that it is easier to observe hot (blue) stars. In reality, δ is even smaller, since the area D^2 of the telescope mirror is smaller than the coherence area $(\lambda/\beta)^2$; therefore

$$\delta \approx D^2 \Delta\lambda \frac{\lambda^2}{c\Delta\lambda} n = \frac{D^2\lambda^2}{c} n = 3 \cdot 10^{-6} n$$

for $D = 6$ m and $\lambda = 5000$ Å, where, as before, n denotes the flux density of the photons from the star. Thus, δ is extremely low. Therefore, the classical fluctuations are very small compared to the quantum noise, and only bright (and hot!) stars are accessible to the intensity interferometer.

An instrument with baselengths up to 200 meters and mirrors 6.5 meters in diameter (their optical quality is unimportant) is now in operation at Narrabri, Australia[47] and can reach stars no fainter than $2^m.5$. The diameters of 32 stars measured with this instrument have been published.[48]

Is there a way to improve the sensitivity of the method? The diameter D of the mirrors could be increased (although it may not exceed the coherence length λ/β, so that β must be small), but prospects for improved efficiency are not bright here. The frequency band $\Delta\nu$ is determined by the speed of the system's electronic circuits and must obviously be maximized. Light is, of course, received in a section of the spectrum broader than $\Delta\nu$, but this does not increase sensitivity, and the situation here is exactly the same as in all multiplexing instruments. True, it would be possible to take N independent detectors working in parallel in different segments of the spectrum for a gain of $\sqrt{N}$ (this method, incidently, can also be used in the Michelson interferometer). In general, sensitivity could be improved greatly by increasing the number of spectral channels, but this cannot be done at present because of the technical difficulties.[2)] This exhausts the ways to improving the intensity interferometer. Thus, it is possible to build an improved version of the intensity interferometer[5] to reach the limit 6^m, but...the Michelson interferometer is much more competitive, promising higher sensitivity at lower cost. The intensity insensitivity to the atmosphere, which makes it possible to lengthen the base without limit. But the method still remains uncompetitive in terms of the resolution achieved.

We have discussed three interferometric techniques that deliver high resolution. From the standpoint of signal analysis, they all boiled down to the detection of intensity fluctuations against a quantum-noise background, and it was therefore possible to compare their sensitivities by introducing the parameter δ—the number of quanta arriving during the characteristic fluctuation time. The fluctuations themselves are of different natures (they are caused by the atmosphere in the first two cases and by the properties of light in the third). In principle, the same parameter δ was found for the Michelson and speckle interferometers. It would be logical to call it the atmospheric degeneracy parameter. The atmospheric δ can be applied irrespective of the method used to register the light.

Indeed, if there were no atmosphere, a plane wave arriving from a star would have the same amplitude and phase at the entrance aperture and would produce one diffraction image—in other words, the light field would have one degree of freedom. In the presence of atmospheric distortions, which are an unknown factor at any given moment (this is essential, because they could otherwise be compensated), the light arriving at the telescope can be treated as an aggregate of several waves or modes that are independent of one another. The number of these modes is called the number of degrees of freedom of the field; it is equal to the number of light amplitude correlation zones on the pupil $(D/r_0)^2$ or to the number of grains in the image. The received light flux is distributed among the various modes in such a way that there are δ protons for one of them. It is this that is the atmospheric degeneracy parameter.

[2)]The 5000–5500 Å band accomodates $2 \cdot 10^5$ channels, each 200 MHz wide, the combined utilization of which would increase sensitivity by a factor $\sqrt{2 \cdot 10^5}$, i.e., by 7^m. However, $2 \cdot 10^5$ light detectors and correlators would be necessary.

A more rigorous treatment of this problem based on analysis of eigenfunctions and eigenvalues is also available.[49]

Interferometric methods can deliver high resolution in observations through the atmosphere. It is sometimes stated that these methods totally eliminate all of the problems associated with astroclimate. As we have seen, this is not the case. Although it is not detrimental to resolution, the atmosphere is still decisive in determining the sensitivity of high-resolution systems: the latter is proportional to α^{-2} for a single-element interferometer and to α^{-1} for a multielement instrument.

Another property of the obtained sensitivity estimates is their strong dependence on the wavelength of the light (as λ^4): interferometry is much easier at longer wavelengths. Sensitivity ceases to depend on the atmosphere altogether in the radio band.

As for heterodyne interferometers that work in the infrared, their sensitivity is severely limited by the narrow spectral bandwidths that can be obtained in these devices by electronic methods ($\Delta\nu/\nu \sim 10^{-3}$ at present).

The question arises: if sensitivity is determined by the atmosphere, would it not be better to move the instrument out into space? Knowing the hypothetical parameters of the space vehicle (primarily the precision and stability of its tracking of the object), we can find δ in the same way as for the atmosphere. Given all the complexity of the space experiment, it would make sense only if it were possible to raise δ by at least one or two orders above the values of earth-based systems.

We may therefore conclude that progress in the observation of faint objects will very likely come with improvement of earth-based classical interferometers. Speckle-interferomety resolution is limited by the diameters of the largest optical telescopes, and its sensitivity will increase with improvement of multielement light detectors.

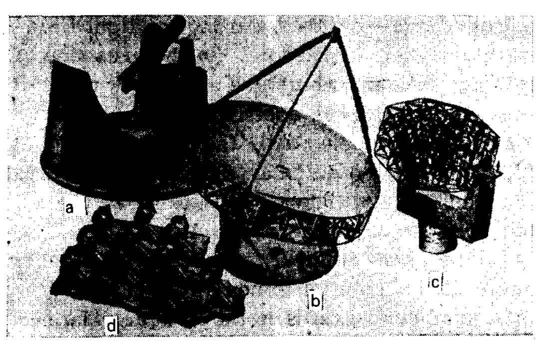

FIG. 14. Four possible design variants of the 25-meter telescope.[50] The giant mirror will consist of several hundred individual elements. It can rotate only around a vertical axis (so-called rotary tower, Fig. a) or in two coordinated (b). Light can be collected from a large number of individual telescopes, which are either mounted on a common frame (multimirror telescope, Fig. c) or carried on individual equatorial mounts (d).

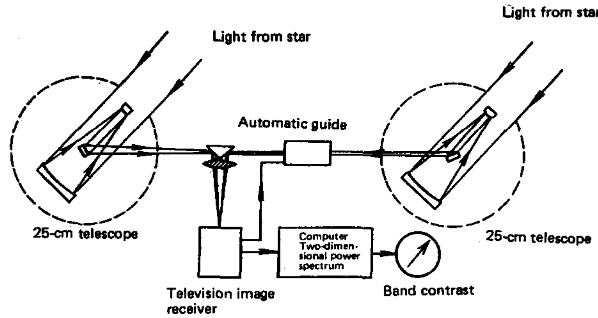

FIG. 15. Scheme of "long" Michelson interferometer (after Labeyrie). The two telescopes send their light to a common focus, where interference occurs. A television receiver and computer are used to detect the bands and measure their contrast. The number of telescopes could be increased later on.

If, of course, a 25-meter telescope[50, 57] were to be built—and the feasibility of building one is now being discussed in the United States and other countries (Fig. 14), the resolution attainable in speckle interferometry would increase sharply. Sensitivity, however, might prove to be slightly lower as a result of dephasing of the individual blocks of which the giant mirror would consist [these errors have the effect of x in formula (4)]. An image of the disk of Betelgeuse with a resolution of 20×20 elements could be obtained on the 25-meter aperture. Possibilities in the observation of lunar occultations would also be broadened (a magnitude limit of $12^m.5$ at a resolution of $0''.005$ is possible). Further increases of resolution would require the creation of longer-base Michelson interferometers (Fig. 15).

7. COMPARISON OF VARIOUS INTERFERENCE METHODS OF INCREASING RESOLUTION

Figure 16 gives a schematic representation of angular resolution and sensitivity (in stellar magnitudes on the left and in the fluxes from the object in relative units on the right) for the various observing methods used in astronomy. However surprising it may seem, all of the most "advanced" methods have fallen along the same line, which therefore reflects the sensitivity levels attained today as a function of resolution. The dashed line represents the progress that can be expected in the not too distant future. It depends on improvement of image detectors in the case of speckle interferometry and on construction of the 25-meter telescope for lunar occulations. The parameters of an improved version of the intensity interferometer[5] and a "long" Michelson interferometer are also indicated. The inclined straight lines represent the magnitudes and diameters of stars wtih effective temperatures of 2000 and 20 000 K at various distances from us. The figure shows that the sensitivity of the Michelson interferometer will make it possible to observe objects of low surface brightness. At the same time, very bright, compact objects (for example, optical pulsars) are still inaccessible to interferometry, which will evidently have double-star observations as its main program.

There is Miller's "long interferometer" project,[51-53]

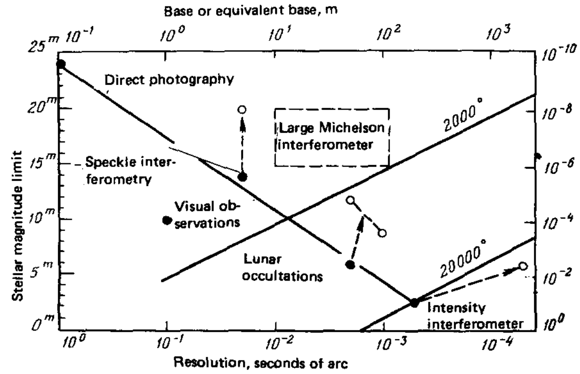

FIG. 16. Sensitivity of various nonclassical methods of measuring angular dimensions as a function of their resolution. The ordinate is the stellar magnitude limit (the corresponding luminous fluxes are indicated on the right in relative units), and the abscissa the angular resolution in seconds of arc. The line drawn from the upper left corner represents the sensitivity level that has now been attained as a function of resolution; the dashed line indicates the progress that may be expected in the not too distant future. The two inclined lines indicate the angular sizes and magnitudes of stars with effective photosphere temperatures of 2000 K and 20 000 K at various distances from us. (The stars emit almost as black bodies in the visible region of spectrum.) The overwhelming majority of stars fall between these lines on the diagram, and as we move to the right and upward along these lines (i.e., with increasing distance from the observer), the number of stars naturally increases sharply, since we are studying a steadily increasing volume of space. This diagram can be used to judge the degree to which a given method is useful for measurement of star diameters. We see that the intensity interferometer measures basically hot stars, while the Michelson interferometer "likes" red giants.

which is already a bit obsolete, although its ideas respecting the detection of interference bands have been developed further. Building a vacuum tunnel has been suggested as a way of reducing distortion in horizontal propagation of the beam along the interferometer base, but this would hardly be worthwhile. It would be easier simply to reduce the cross-section of the beam, as was done by Labeyrie,[6] or to raise the path above the surface of the earth. Measurements of temperature fluctuations in the surface air layers have shown that a path 50 meters long running at a height of 10 meters above the ground would produce the same disturbances as the entire remainder of the atmosphere, and that the reduction of beam cross section would make its contribution neglibibly small. However, development of a large interferometer presupposes the availability of the following hardware in highly efficient form:

1) High-precision optical and especially mechanical systems. Equalizing the arms of the interferometer would evidently give rise to the most serious difficulties. The mechanical system would have to be computer-controlled with the computer receiving information from laser distance-measuring instruments.

2) A multielement light detector capable of registering single photoelectrons. Such detectors exist but have not yet been perfected.

3) An appropriate information-processing system in the form of a special-purpose[42] or appropriately adapted computer. It is interesting to note that the fainter the object being observed, the smaller is the volume of computations required. The signalprocessing techniques and apparatus can be taken directly from speckle interferometry.[23,33]

There is no doubt that present-day technology can meet these requirements. Thus, opportunities for significant sensitivity and resolution improvements in observation of faint objects are at hand even now. In addition to answering the questions already posed, these observations will certainly produce much that is unexpected.

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