Free-electron lasers

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Recent studies devoted to the problem of constructing a free-electron laser are reviewed. Possible approaches to description of the physical processes on which free-electron lasers are based are discussed briefly, and means of increasing the efficiency of laser arrangements are considered.

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CONTENTS

1. INTRODUCTION

In the optical region, as a rule, transitions between levels of the discrete energy spectrum of quantum systems are used for amplification and generation of coherent radiation. Nevertheless, there are also possibilities of constructing devices of this type with the continuum. Here monochromaticity and coherence of the radiation can be achieved by giving the process an induced character. In particular, the free-elctron laser is such a device. The term electron laser (e-laser) is usually applied to devices based on the radiation of a relativistic electron flux moving in external electromagnetic fields. These may be the spatially varying field of a wiggler, intense laser radiation (Compton laser), the field of the periodic structure of crystals (Čerenkov laser), etc. If an electron radiates while moving in the coherent field of an external signal, then the new photon, in accordance with Bose statistics, is emitted with a high probability at the frequency and phase of the external stimulating field.

The mechanism described for amplification and generation of electromagnetic radiation has been achieved experimentally relatively recently.[1,2] In the first of these studies the authors report their investigations on the interaction of a relativistic electron flux with the transverse magnetic field of a helical undulator. The experimental arrangement, which is shown in Fig. 1, consisted of a superconducting magnet with a spatial period 3.2 cm and a field intensity 10 kG. The qualitative form of the dependence of the radiated power generated by the system at wavelength $\lambda = 10.6$ μm on the electron energy is shown in Fig. 2a. Analysis of this radiation, however, showed that it is not coherent. The authors then stimulated this process by illuminating the interaction region by the beam of a CO_2 laser ($\lambda = 10.6$ μm). Here the apparatus began to amplify or attenuate the laser radiation efficiently, depending on the energy of the electrons. The dependence of the gain G of the field of the CO_2 laser on the energy of the electron flux is shown in Fig. 2b. Its maximum value at a current of 70 mA reached 7%.

In a subsequent publication of the same authors they reported construction of the first free-electron laser. It was based on the scheme described above, but the parameters of the apparatus were somewhat changed—the magnetic field strength in the undulator space was decreased and the energy of the electron flux was increased. It now reached 43 MeV. The maximum power of the radiation generated at wavelength $\lambda = 3.4$ μm reached 7 kW, with an average value 0.36 W.

The Stanford experiment confirmed the theoretical ideas and served as a good stimulus for further developments in this region. An illustration of the interest which has arisen in the problem of electron lasers is the Tenth International Conference on Quantum Electronics held in 1978 at Atlanta, where problems related to the creation of electron lasers were collected for discussion in a special section.

We note some obvious advantages of electron lasers. First, the problem of creating a population inversion of energy states is solved relatively simply in such systems. In fact, any monoenergetic electron beam is a system with a population inversion with respect to the entire energy spectrum with lower energy. In addition to this, it is possible to control the frequency of radiation of electron lasers easily be changing the energy of the electron flux. For example, for a Compton laser the ratio of the frequency of radiation to the

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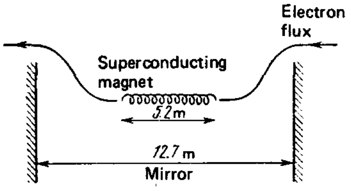

FIG. 1.

pumping is $4\mathscr{E}^2/(mc^2)^2$, where $\mathscr{E}$ is the electron energy and m is its rest mass. By using relativistic electron beams it is possible to obtain coherent radiation in the ultraviolet and x-ray regions. It is improtant also that in free-electron lasers there are no fundamental limitations on the power of the generated signal.

Of the wide range of problems associated with construction of a free-electron laser, we shall dwell on the problems of development of a Compton laser of a free-electron laser using the undulator effect. The principle of action of the latter is based on the radiation of an electron flux moving in a spatially varying magnetic field. According to the Weizsäcker-Williams approximation[3,36] the behavior of an electron flux in the field of an undulator is equivalent to its motion in the field of an electromagnetic wave. In view of this the mechanism of operation of the two types of free electron lasers can be described from a unified point of view.

The first studies devoted to the problem of induced scattering—induced Compton effect—are apparently those of Schrödinger.[4] The model considered by him is actually identical to the scheme of the Compton laser. We must also recall here the work of Kapitsa and Dirac,[5] who discussed the scattering of electrons by a standing electromagnetic wave.

The publications mentioned appeared long before the creation of the first lasers. Therefore we can consider as the pioneer works in the field those of Milburn,[6] Arutyunyan and Tamanyan,[7] and Pantell and co-workers,[8] who discussed means of producing sources of coherent radiation in the short-wavelength region using energy of an electron flux.

The further development of studies in this field occurred along two directions. The first was based on the discussion of single scattering of a photon by a free electron in the presence of an external electromagnetic wave. This is a quantum approach to the problem. The second direction considers induced effects from the point of view of the laws of collective behavior of particles in regular external fields. These are studies which make use of the ideas of classical physics.

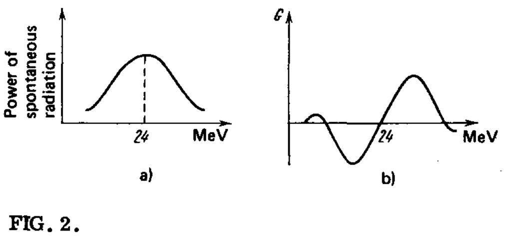

FIG. 2.

2. CLASSICAL THEORY OF FREE-ELECTRON LASERS

The scattering becomes an induced process in the presence of a large number of coherent photons moving in the direction of the scattered radiation. This circumstance is the condition for applicability of classical methods. Thus, it can be stated that the generating laser is a classical device and only the language of its description permits some arbitrariness in selection.

The mechanism of induced scattering can be represented pictorially in terms of ideas of classical physics. The electric field of the pumping wave builds up oscillations of the electrons in a direction perpendicular to the motion of the electron flux. Then the magnetic field of the scattered wave picks them up—so-called Miller-Gaponov forces arise.[9] The total action of the two waves (signal and pumping) on the electron flux leads to appearance of an electron density wave with frequency $\Omega = \omega_s - \omega_p$ and wave vector $\mathbf{K} = \mathbf{K}_s - \mathbf{K}_p$. Then the beam is said to be bunched. If the velocity of the electron flux satisfies the synchronism condition $(v \approx v_{\text{ph}} \approx (\omega_s - \omega_p)/(K_s + K_p))$, an intense exchange of energy occurs between the waves and the electron flux. This interaction appears particularly simple in the K' frame where the pumping frequency ω_p' and the signal frequency ω_s' are equal. In this case the signal gain is determined by the reflection of the pumping wave from the periodic structure, which is presented by the modulated electron flux in the K' system.

In discussing studies devoted to the classical consideration of the problem, we can distinguish two directions, as is done in microwave electronics. The first is the hydrodynamical or single-particle description of the electron flux. Studies using this approach discuss the behavior of an electron moving with velocity $v \approx v_{\text{ph}}$ in the field of the two waves.[9-16] In these studies it has been shown that the model considered reduces to the problem of oscillations of a pendulum. This analogy has made it possible to obtain an expression for the laser gain, the saturation effect, and the coherent modulation of the electron flux. Hagenbuch[17] analyzed the motion of an electron in an electromagnetic field with inclusion of radiation damping. In the scattering process, in addition to transfer of energy from one electromagnetic wave to the other,[18] a redistribution of energy occurs between the electron flux and the electromagnetic field. Mayer[19] draws a parallel between this effect and the similar effect leading to Landau damping in a plasma. The single-particle approximation permits rather complete study of the behavior of an electron in electromagnetic fields. However, as in devices in the microwave range, this approach does not provide complete information on the process.

A number of papers[20-22] discuss the possibility of multiple utilization of an electron bunch in the generation process. In fact, a scheme which is very attractive at first glance is a circulating electron flux which gives up part of its energy in individual portions of the orbit and makes it up in other portions, as is done in electron cyclic storage rings. The central question here is the effect of induced radiation on the electron flux, which reduces its useful properties and determines the maximum possible gain and efficiency of the system. It is just the nature and degree of this change which determine the possibility of multiple use of a beam for radiation. The answer to this question can be given only by a kinetic approach to description of the electron flux. A theory of this type is based on the combined solution of the system consisting of the Boltzmann kinetic equation and the Maxwell equation:

$$\frac{\partial f}{\partial t}+\dot{\mathbf{r}}\frac{\partial f}{\partial \mathbf{r}}+\dot{\mathbf{p}}\frac{\partial f}{\partial \mathbf{P}}=0, \tag{1}$$

$$\Box \mathbf{A}=\frac{4\pi}{c}\mathbf{j}. \tag{2}$$

Compton scattering in the classical interpretation has been investigated previously as one of the possibilities for heating a plasma by laser radiation (see for example Ref. 23). In application to a free-electron laser, kinetic methods were first used in Refs. 24–26. Introducing new functional dependences, the authors reduced the initial system of equations to relations similar to the generalized Bloch equations, which are then solved by the method of successive approximations.

The kinetic approach formed the basis for discussion of one of the aspects of interaction of electrons with waves in a short-wavelength free-electron laser.[27] In Ref. 28 an analysis of the operation of an electron laser was carried out in terms of the Klimontovich formalism.

3. QUANTUM TREATMENT OF THE PROBLEM

The convenient representation of the mechanism of induced scattering given by the classical approach to the problem does not, however, permit us to treat the problem purely classically from beginning to end. In fact, a necessary condition for the applicability of these methods is a comparatively high strength of the electromagnetic fields. The latter requirement, however, is not satisfied at the initial moment of a free-electron laser. Before the beginning of induced scattering, there occurs for some time a process which cannot be considered classical, since the number density of photons in the scattered radiation is still small and spontaneous scattering dominates. Thus, induced radiation begins not immediately, but after a time τ, the evaluation of which is extremely important because of the smallness of the total time of passage of the electrons through the interaction region.

Let us consider briefly the quantum approach to the problem. In the papers which develop this direction, single scattering of a photon $(\omega_1\mathbf{K}_1)$ by an electron with momentum $\mathbf{P}$ in the presence of n_2 photons $(\omega_2\mathbf{K}_2)$ is considered. The differential cross section for scattering is determined in this case by the relation

$$d\sigma_i=(1+n_2)\,d\sigma_{sp}, \tag{3}$$

where $d\sigma_{sp}$ is the cross section for the spontaneous scattering process, which can be found in accordance with the Klein-Nishina-Tamm formula.[29] The frequency ω_2 of the scattered radiation can be determined from the formula

$$\omega_2=\omega_1\left[1-\left(\frac{v}{c}\right)\cos\theta_1\right]\left(1-\frac{v}{c}\cos\theta_2+\frac{\hbar\omega_1}{\mathscr{E}}\right)(1-\cos\theta)^{-1}; \tag{4}$$

here θ_1 and θ_2 are the angles between the electron velocity vector and the wave vectors of the initial photon $\mathbf{K}_1$ and the scattered photon $\mathbf{K}_2$, and θ is the angle between $\mathbf{K}_1$ and $\mathbf{K}_2$. It is easy to see that the cross section for spontaneous scattering and the photon frequency will depend on the relative orientation of the vectors $\mathbf{P}$, $\mathbf{K}_1$, and $\mathbf{K}_2$. These dependences are rather complicated and therefore it is not surprising that only processes in which these vectors are collinear have been studied, especially since this choice gave the largest region of interaction of the electron flux with the electromagnetic field. The traditional choice of the one-dimensional model also becomes understandable in this connection.

In the pioneer work of Pantell and co-workers[8] and in the subsequent review[30] the basic energy characteristics of induced inverse Compton were given, such as the expression for the scattered power, gain, etc. The functional form of the gain G was:

$$G\sim\frac{N_eJ}{\omega_1^3\mathscr{E}}\frac{df}{d\mathscr{E}}, \tag{5}$$

where N_e is the electron density, J is the power incident on unit area, and $f(\mathscr{E})$ is the electron energy distribution function. Molchanov[31] made numerical estimates in accordance with this formula. Thus, if a pumping wave with $\lambda=1.06$ μm is characterized by a photon density $N_\nu=1.8\times10^{22}$ cm^{-3}, then for an electron energy of 2 MeV and an electron flux density $N_e=2\times10^{13}$ cm^{-3} the gain at a wavelength $\lambda=166$ Å amounts to $G\approx2.2$ cm^{-1}.

An important aspect, which substantially limits the gain of a Compton laser, is the finiteness of the region of interaction of the electrons with the electromagnetic field.[32] These same authors, after pointing out the theoretical limit for the gain of a Compton laser with a finite interaction length, turned their attention to possible means of increasing the efficiency of interaction of the radiation with the electron flux.[33] They made use of the fact that the scattering cross section increases in the presence of a constant magnetic field directed along the electron beam axis. The magnetic field strength is chosen so that the cyclotron frequency of an electron is approximately equal to the frequency of the microwave radiation in the electron rest system. This effect was named by the authors magnetic Compton scattering. It can be understood also without complicated quantum-electrodynamics calculations. In fact, according to the ideas of classical physics, the cross section for scattering at a frequency ω in the presence of a magnetic field depends on the factor $\omega^2/(\omega-\omega_c)^2$, where $\omega_c=eH/mc$. If we take into account in the calculation that not all electrons are in resonance and the frequency of radiation can be determined only within a quantity $(\Delta\tau)^{-1}$, where $\Delta\tau\approx L/c\gamma$ is the electron interaction time measured in its rest system, we finally obtain

$$\frac{G_{H\neq0}}{G_{H=0}}=\frac{\omega^2}{(\omega-\omega_c)^2}\sim\left(\frac{L}{\lambda_1}\right)^2. \tag{6}$$

In regard to this work and to magnetic Compton scattering as a whole we note that this effect is a combination of induced Compton scattering and cyclotron radiation which is well known in microwave radiation physics (see for example Ref. 34). We shall give here also estimates for the magnetic field strength. Let a pumping wave with $\omega_1 \sim 10^{15}$ sec^{-1} (optical frequency) be scattered by a flux of 5 MeV electrons. Then resonance radiation at a frequency $\omega_2 \sim 10^{17}$ sec^{-1} (x-ray region) will be achieved at a magnetic field strength $H_0 \sim 10^9$ Oe. This is four orders of magnitude higher than values achieved at the present time.

4. MEANS OF OPTIMIZATION OF A FREE-ELECTRON LASER SCHEME

In a recent review[35] it has been correctly noted that the interaction of electrons with an electromagnetic wave will be efficient when it has a collective nature ($\lambda_D K \approx 1$), i.e., when the frequencies of the interacting signals are close to the plasma frequencies characteristic of the given electron flux. Single interactions such as, for example, induced Compton scattering are extremely low in efficiency. As a result the question of possible means of increasing the gain of laser arrangements is quite urgent. One attempt at this is the study discussed in the previous section on magnetic Compton scattering.

Another theoretical attempt to search for a means of increasing the efficiency of interaction of a wave with an electron flux was undertaken in Refs. 37 and 38. This idea originated from the analogy between free-electron laser and microwave devices of the O-type, in particular, traveling-wave tubes. The authors propose to pass an electron flux through a corrugated waveguide (Fig. 3) in order that the electromagnetic wave be able to interact efficiently with the electron flux through its longitudinal component. The calculations made in this study showed that for a wavelength $\lambda = 10$ μm and a current $I = 1$ mA the gain per pass will amount to 6.8%.

An interesting means of stimulation of the scattering process is developed in Ref. 39–41, which are devoted to the theory of the Compton laser. The basic idea of this method consists in producing an additional slow electromagnetic wave at the difference frequency $\Omega = \omega_s - \omega_p$ and with a wave vector $\mathbf{K} = \mathbf{K}_s - \mathbf{K}_p$. This three-wave interaction is rather efficient (according to the calculations of the authors,[41] for a stimulating-wave intensity 1 mW/cm^2 the gain in the probability of induced scattering may reach a value of 10^7); however, practical utilization for realizing a free-electron laser in the ultraviolet or x-ray region encounters difficulties because of the need for creating a system for slowing down the wave with frequency Ω. Such systems are realized relatively simply in the microwave region, i.e., for a free-electron laser with insignificant conversion of the radiation in frequency.

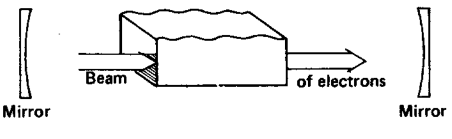

FIG. 3.

Another result obtained in Ref. 40 is of interest. According to the calculations of the authors the bunching due to interaction of noncollinear wave is proportional to the product of the orthogonal components of the electric fields of the two waves. This aspect is important, since in this case the efficiency of interaction is substantially increased (by approximately $c/(c - v_0)$, where v_0 is the velocity of the relativistic electron). This circumstance brings to mind the desirability of achieving the interaction of waves at an angle. However, in the case of a Compton laser this leads to a decrease of the interaction region. Therefore it is interesting to consider the problem of optimization of the angles between the wave vectors of the signal and pumping waves, and also their orientation with respect to the electron flux.

The basic problem of optimization of a free-electron laser consists of accelerating the bunching of the electron flux. This fact is important because during the time of interaction of the electrons with the electromagnetic field—and it is small—there must be accomplished, first, a modulation of the electron flux in density and, second, the modulated electron flux must be able to transfer to the electromagnetic field the maximum possible portion of its energy. The arrangement of a free-electron laser can be optimized if the region of interaction of the electron flux with the external field is separated by an extensive drift space.[42] In this region a kinematic grouping is accomplished, and the size of the region is chosen from the condition of maximum modulation of the electron flux in density. In the papers of Vinokurov and Skrinskiĭ[43,44] where this idea was first expressed, it is proposed to accelerate the kinematic grouping by introducing three narrow magnets into this space (Fig. 4). The time of traversal of this system by the electrons depends rather strongly on their energy. The deflection angle β must be chosen so that the longitudinal bunching at the exit of the drift space is maximal. The electron flux prepared in this way is again subjected to the action of external fields and begins to radiate efficiently. A free-electron laser modified in this way is very similar in its principle of action to a klystron. In fact, the first region where the electron flux undergoes for the first time the action of the external field and where its modulation in energy mainly occurs is similar to the first resonator gap of a transmission klystron. The role of the drift space of the klystron is played by the magnetic system, and the second interaction region is similar to the second resonator gap, since in it the electrons, on the average, transfer energy to the field.

5. CONCLUSION

A number of other studies devoted to this subject discuss particular questions of the mechanism of interac-

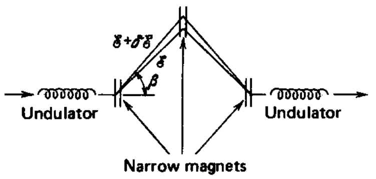

FIG. 4.

tion of laser radiation with free electrons, such as the scattering cross section,[45–46] the frequency of the scattering radiation,[47] possible undulator schemes,[48] and so forth.

Let us dwell further on the paper by Stolyarov.[49] Here the author notes that, in addition to Compton scattering (which is incoherent with respect to the beam), there is the possibility of using reflections (coherent with respect to the beam) from the boundary of a moving electron bunch or plasma. It is shown in the article that a severe restriction on processes of this type is imposed by the spread in the front edge of the beam. An essential point here is that the smearing is important not only with respect to the incident wave, but also for the shorter-wavelength reflected wave. In the authors's opinion, for the efficient conversion of optical radiation $\lambda \sim 10^{-4}$ cm to the hard x-ray region $\lambda \sim 10$ Å a beam with energy 8 MeV and density 10^{14} A/cm^2 is required. In addition the rise time of the beam front must be no greater than 10^{-18} sec, which apparently is beyond present technical possibilities.

We note that this is essentially the only study where the presence of a resonator is not proposed. In all other articles whose results can be transferred to the x-ray and ultraviolet regions, the amplification mode of a free-electron laser has actually been analyzed. The lack of good resonators in this region pointedly raises the problem of the generation threshold of a one-pass laser based on free electrons. The solution of this problem will enable us to judge how close we are to creation of an x-ray free-electron laser.

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