Atoms and hadrons (classification problems)

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A group approach is discussed for classifying two types of objects: hadrons and chemical elements. In this approach, both the hadrons and the atoms are treated as structureless particles. The groups underlying the classifications are different in the two cases: the unitary SU(3) in the hadron case and the orthogonal O(4) group in the atomic case. The classification principles, on the other hand, are identical, and in this sense there is an analogy between atoms and hadrons. Certain aspects of this analogy are discussed.

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1. INTRODUCTION

It had been shown by the middle of the 19th century that the material surrounding us consists of chemical elements of various species. The first step toward a theory of the structure of matter was the classification of a large number of elements. A phenomenological classification was developed by Mendeleev (1869). In constructing his periodic system of elements, Mendeleev drew on a huge chemical literature. A microscopic explanation of this classification was developed by Niels Bohr and Pauli after the structure of the atom had been studied in the early 20th century.

A classification problem arose in the physics of elementary particles when it was learned that the matter in nature does not consist simply of protons, neutrons (atomic nuclei), and electrons (atomic shells). Beginning in the 1940s, progress in experimental apparatus revealed more and more new hadrons. By the 1960s, the number of hadrons had become comparable to the number of chemical elements known to Mendeleev, but not enough information was available to construct a system of the Mendeleev type for hadrons.

Gell-Mann[1] and Ne'eman[2] approached the hadron-classification problem from a completely different direction, making use of the methods of group theory. These methods, which had already proved their efficiency in crystallography (in the work by Fedorov), were rapidly applied in physics after the advent of quantum mechanics. The role of group theory in quantum mechanics was originally only a subsidiary one, consisting essentially of the study of the symmetry properties of the Schrödinger equation. Later, however, group-theory methods became an independent branch of modern theoretical physics. These methods furnish information on the systems described by the differential equations of mathematical physics before these equations have been solved. Group-theory methods are also extremely effective in cases in which the differential equations describing the system are not known at all.

The use of group theory to classify natural objects such as crystals (Fedorov), hadrons (Gell-Mann and Ne'eman), or chemical elements is based on the concept that the classification must be based on some symmetry group appropriate to the objects.

The first classification of physical objects based on group theory was Fedorov's classification of crystals (1895). On the basis of a study of the space groups, Fedorov showed that there exists a finite number of types of crystal lattices, namely, 230. These types are in turn distributed among 32 classes.

There is an important distinction to be made between the group classifications offered by Fedorov, on the one hand, and Gell-Mann and Ne'eman on the other. Fedorov's classification is geometric in nature and is associated with the three-dimensional nature of physical space, while the hadron classification is linked to the internal symmetry and reflects the properties of the intrahadron dynamics. Only now, it appears, are we beginning to see the structure of hadron dynamics. In 1961, when Gell-Mann and Ne'eman proposed the unitary classification, almost nothing was known about the internal structure of hadrons, and these workers made no assumptions regarding the structure of elementary particles in their arguments.

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In this sense, the classification of chemical elements described in the present note is similar to the classification of hadrons: This classification of elements ignores any knowledge of the internal dynamics of atoms, and it is based on a postulated symmetry group. We emphasize that a clear understanding of the group classification of the chemical elements rests on a clear recognition of the fact that this classification is analogous to the phenomenological classification offered by Gell-Mann and Ne'eman, not the Bohr-Pauli microscopic classification of atoms. In this short note, we will attempt to give a clear and brief outline of the basic concepts of this group classification, omitting the mathematical details, which can be found in the original papers.[3-6]

2. UNITARY SYMMETRY OF HADRONS

The clearest example of a classification based on group methods is the classification of hadrons (Gell-Mann[1] and Ne'eman[2]). The SU(3) group[1] on which this classification is based is a group of linear transformations which leave invariant the Hermitian form

$$\bar{Z}_1 Z_1 + \bar{Z}_2 Z_2 + \bar{Z}_3 Z_3,$$

where Z_1, Z_2, and Z_3 are complex quantities, and the superior bar denotes the complex conjugate. According to this group approach, the objects to be classified are combined into certain families or "multiplets."

A tensor $(A, B = 1, 2, 3)$ is associated with each SU(3) multiplet. This tensor has a certain type of symmetry in the three-dimensional complex space. In this classification, a hadron is associated with each independent tensor element $T^{A1\cdots An}_{B1\cdots Bm}$, so that the number of hadrons in an SU(3) multiplet is equal to the number of independent elements of the corresponding tensor. For example, the tensor T_A corresponds to a multiplet which consists of three particles (quarks). The tensor T^A_B (where $\sum^3_{A=1} T^A_A = 0$) describes an octet of hadrons. The hadrons which have been studied most thoroughly comprise the meson octets, the octet of baryons, and the decuplet of baryons, the latter being described by the tensor $T^{(ABC)}$, which is symmetric with respect to all its indices (there are then elements).

The hadrons in each SU(3) multiplet are also grouped into multiplets, whose structures are governed by the smaller group SU(2).

A convenient geometric display of the tensors $T^{A1\cdots An}_{B1\cdots Bm}$ (multiplets) consists of tables in which each box corresponds to a certain element of these tensors (and thus to a certain hadron).

As an example, we show here the classification of the hadrons making up the octets and the decuplet (Table I).

TABLE I.

a) Mesons

η	548.8	ω	782.6
K^- $\bar{K}^0$	493.7 497.7	K^{*-} $\bar{K}^{*0}$	892.2 896.3
K^0 K^+	497.7 493.7	K^{*0} K^{*+}	896.3 892.2
π^- π^0 π^+	139.6 135 139.6	ρ^- ρ^0 ρ^+	765.9 770.2 765.9

b) Baryons

Λ	1115.6	Ω^-	1672.2
Ξ^- Ξ^0	1321.3 1314.9	Ξ^{*-} Ξ^{*0}	1535.0 1531.8
N P	939.6 938.3	Σ^{*-} Σ^{*0} Σ^{*+}	1387.5 1382.0 1382.3
Σ^- Σ^0 Σ^+	1197.4 1192.5 1189.4	Δ^- Δ^0 Δ^+ Δ^{++}	1232 1232 1232 1232

Each box in Table I shows the designation of the hadron (meson or baryon) and its mass, in MeV.

The greatest success of this classification was its prediction of the decuplet particle Ω^- (1672.2), which was discovered experimentally soon after the prediction of its existence.

The partitioning of the SU(3) multiplets into isotopic multiplets involves a breaking of the SU(3) symmetry, by a mechanism which is not yet understood. The hadrons in each SU(3) multiplet are similar: They have similar masses and similar properties. In the limit in which this symmetry is exact, the hadrons in the SU(3) multiplets behave identically. In addition to the similarity of the hadrons in a column, we observe a similarity of the hadrons in a row. In the latter case, the hadrons in a common row in the tables of identical dimensionality have identical charges and identical hypercharges. In this sense, these hadrons are similar to the chemical analogs found in Mendeleev's table.

3. ORTHOGONAL SYMMETRY OF THE CHEMICAL ELEMENTS

As we mentioned earlier, Mendeleev constructed the table of chemical elements by analyzing and systematically classifying a huge amount of chemical information. The hadron classification proposed by Gell-Mann and Ne'eman used far less phenomenological information, but was based from the very beginning on a postulated symmetry [SU(3), etc.].

A question arises here: Are the properties of the chemical elements controlled by some symmetry group in the same manner as the properties of hadrons are controlled by the SU(3) group?

In pursuing this question we must ignore the chemical and spectroscopic data available, and we must ignore the description of atoms as structured objects consisting of more-elementary particles (nuclei and elec-

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[1] It was established later that a larger unitary group—the $SU(4)$ group [or perhaps even the $SU(5)$ group]—was necessary for classifying the hadrons. It is nevertheless clear that the classification principles remain the same for any unitary group.

trons). Rumer and Fet have demonstrated that the SO(4) group can be used for such a classification of chemical elements. The SO(4) group is that group of transformations which leave invariant the quadratic form

$$\xi_1^2+\xi_2^2+\xi_3^2+\xi_4^2.$$

Like the hadrons in the SU(3) classification, the chemical elements are combined into groups: SO(4) supermultiplets. Each SO(4) supermultiplet is described by a tensor

$$T_{nl\nu} \quad (n = 1, 2, \ldots, \infty;\ l = 0, 1, \ldots, n-1;\ \nu = l, l-1, \ldots, -l)$$

in a real four-dimensional space in which each element of this tensor is associated with a certain chemical element. We note that the specification of the number n for the tensor $T_{nl\nu}$ is analogous to the specification of the symmetry type of the tensors $T_{B1\ldots Bm}^{A1\ldots An}$ in the SU(3) group. The tensors $T_{nl\nu}$ can be described clearly by a table of chemical elements.

Table II, which is based on the SO(4) group, contains only half as many boxes as there are elements, so that each box contains two elements, for example, hydrogen and helium, nitrogen and oxygen, manganese and iron, and copper and zinc. While the two chemical elements in the common box in the last two of these examples are "similar" those in the common box in the first two examples are completely dissimilar. A more careful analysis shows that this "doubling" of the number of classified objects in comparison with the number of positions in the table results from the fact that the SO(4) group does not give a complete description of the system of elements. It must be expanded to the Spin (4) group[3] or the SO(2, 4) group,[4] both of which give a complete description of the properties of the table of elements. To describe the Spin (4) or SO(2, 4) group would require more mathematics than would be appropriate for this note, so we will drop this complication here and restrict the discussion below to the SO(4) group.

The numbers $n = 1, 2, 3, \ldots$ in Table II are the designations of the SO(4) supermultiplets, which are arranged vertically. Each supermultiplet consists of a certain number (n) of SO(3) multiplets. These multiplets in turn consist of $2l+1$ boxes [$2(2l+1)$ boxes, when the doubling is taken into account], in which the chemical elements are placed. The "address" of each element is thus specified by the three numbers n, l, ν

TABLE II.

$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$		
H	Li	Na	K	Rb	Cs	Fr	Ia	$l=0$
He	Be	Mg	Ca	Sr	Ba	Ra	IIa	
	B	Al	Ga	In	Tl		IIIb	$l=1$
	C	Si	Ge	Sn	Pb		IVb	
	N	P	As	Sb	Bi		Vb	
	O	S	Se	Te	Po		VIb	
	F	Cl	Br	J	At		VIIb	
	Ne	Ar	Kr	Xe	Rn		VIIIb	
		Sc	Y	Lu	Ac		IIIa	$l=2$
		Ti	Zr	Hf	Krc		IVa	
		V	Nb	Ta			Va	
		Cr	Mo	W			VIa	
		Mn	Tc	Re			VIIa	
		Fe	Ru	Os			VIIIa	
		Co	Rh	Ir			VIIIa	
		Ni	Rd	Pt			VIIIa	
		Cu	Ag	Au			Ib	
		Zn	Cd	Hg			IIb	
			Lanthanides	Actinides				$l=3$
			Ce	Th				
			Pr	Pa				
			Nd	U				
			Pm	Np				
			Sm	Pu				
			Eu	Am				
			Gd	Cm				
			Tb	Bk				
			Dy	Cf				
			Ho	En				
			Er	Fm				
			Tu	Mv				
			Yb					
			La	Lw				

(where $\nu = l, l-1, \ldots, -l$). When doubling is taken into account, we would have two boxes instead of a single box $[(n, l, \nu, +)$ and $(n, l, \nu, -)]$. Each box contains, in addition to the designation of the element, all its excited states, isotopes, and so forth. It can be hoped that the finer classifications based on more complicated groups will make it possible to distinguish these isotopes and excited states.

Table II differs from Mendeleev's table in that the symmetry group which controls the properties of the system of chemical elements is taken into account explicitly. In the original version of Mendeleev's table, all the actinides and lanthanides were in a common box, outside the table; in Table II, the actinides and lanthanides completely fill two multiplets. Elements in the same horizontal row are chemical analogs; in particular, the actinides are chemical analogs of the lanthanides. This property of the chemical elements is analogous to the property of the hadrons mentioned above. Similarity of the elements within the multiplets is observed for the lanthanides and actinides ($l = 3$). This similarity is not as apparent for the multiplets with $l = 2$, and for $l = 0$ and $l = 1$ this property is not exhibited at all. For hadrons, this property is exhibited within each multiplet. We wish to emphasize that when we talk about a similarity within multiplets we mean a similarity in the energy (or mass) characteristics in the case of hadrons, while in the case of the chemical elements we mean a similarity in the chemical properties (in particular, the valence or even the chemical activity). If this distinction is missed, one might reach the false conclusion that this classification of chemical elements in SO(4) and SO(3) multiplets is based on a similarity of the elements in terms of mass (although such a similarity in terms of mass is in fact observed in Table II). In both cases, the sets of allowed multiplets are infinite. At present, only a few such multiplets are filled. The vacant boxes (and multiplets) apparently correspond to very unstable hadrons and chemical elements.

In this short note we can do no more than briefly outline the basic features of the group classification of the chemical elements. The reader is referred to Refs. 3–6 for a detailed analysis of the corresponding groups and quantum numbers, the rules for arranging the chemical elements among the boxes, and a detailed discussion of the physical and chemical properties of the elements.

4. CONCLUSION

We see that the classification method based on group theory can be applied to two different types of objects: hadrons and chemical elements. While the mathematical apparatus for the classification is identical in the two cases, the properties of the objects on which the classification is based are very different: In the case of the hadrons, the properties are physical (the mass), while in the case of the elements the properties are chemical. We believe that this is a profound analogy between the systems of hadrons and chemical elements and that there are several (very different) ways to pursue this analogy.

Let us examine the hadrons and chemical elements from this standpoint. To both systems we can apply a group classification method which corresponds to the phenomenological knowledge level, but only in the case of the chemical elements do we understand the internal dynamics (from quantum mechanics). The internal dynamics of the hadrons is only now beginning to be understood (from quantum chromodynamics). This example of the system of chemical elements shows that there is a possible relationship between the phenomenological description in group-theory terms and the exact description (quantum mechanics), and it helps us understand some general behavioral aspects which are independent of the particular nature of the objects. In principle, it would be possible to determine the dynamical origin of the quantum numbers and to determine what information on the internal dynamics is embodied in the group with which these objects can be classified. It would be easy to add to this list of questions.

By working from an analysis of the system of chemical elements, which can be pursued to the finest detail, we can attempt to identify some general features of the internal dynamics of another system of objects: hadrons.

In summary, the classification method based on symmetry groups raises new opportunities for reaching an understanding of the properties of physical objects.

[1] M. Gell-Mann, Report CTSL-20, California Inst. of Technology, 1961.
[2] Y. Ne'eman, Nucl. Phys. 26, 222 (1961).
[3] Yu. B. Rumer and A. I. Fet, Teor. Mat. Fiz. 9, 203.
[4] B. G. Konopel'chenko, Preprint 40–72, Institute of Nuclear Physics, Novosibirsk, 1972.
[5] A. I. Fet, Teor. Mat. Fiz. 22, 323 (1975).
[6] V. M. Byakov, V. I. Kulakov, Y. B. Rumer, and A. I. Fet, "Group-theoretical classification of chemical elements" (Preprint ITEP No. 26-I); "Physical foundations" (No. 90-II); "Description of applied groups" (1976) (No. 7-III); "Comparison with the properties of elements" (1977).

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