Leonid Isaakovich Mandel'shtam[1]

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Keynote paper at the May 30, 1979 joint scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the USSR Academy of Sciences, which was devoted to the 100TH anniversary of the birth of Academician L. I. Mandel'shtam. The paper discusses the three main periods in Mandel'shtam's life and work: the Strasbourg period (1899–1914), the time from his education in Russia to his final move to Moscow (1914–1925), and the last, Moscow period (1925–1944), which was the most productive from the standpoint of both scientific and teaching activity. Mandel'shtam was (together with Academician A. F. Ioffe) the founder of Soviet physics, the creator of a physical school exceptional for breadth of its interests and the depth of its research, several of whose representatives went on to form widely known schools of their own, both theoretical (I. E. Tamm, A. A. Andronov, M. A. Leontovich) and experimental (G. S. Landsberg). His coverage of physics, the organic fusion of theoretical and experimental and mathematical and technical physics in Mandel'shtam's creativity, place him among the classics of science. The most important of Mandel'shtam's outstanding scientific and technical achievements are indicated, including the discovery of combination [Raman] and selective scattering of light, establishment of the role of fluctuations in scattering and prediction of the fine structure of Rayleigh scattering, his advancement of the theory of oscillations to the level of an independent scientific discipline and the creation of a nonlinear theory of oscillations, the invention of radiointerferometry, the discovery of the tunnel effect in quantum mechanics, etc. Also outlined are a number of general guiding concepts that Mandel'shtam developed, such as "nonlinear physical thinking," classification of phenomena on the basis of commonality (isomorphism) of the describing relationships, and application of "oscillatory mutual aid" between various fields of physics and engineering. These general ideas unified and directed all of his creative work and have by now thoroughly permeated modern physics.

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I consider it a great honor to submit a paper on L. I. Mandel'shtam at the present session, which is dedicated to the 100th anniversary of the birth of this remarkable man and outstanding Soviet physicist. I am also gratified by the fact that both of the physical divisions are observing this date.

Needless to say, the requirement that the material be fitted into 30–40 minutes does not make what is no easy task to begin with any easier. I shall be obliged to skip over a great deal–any kind of biographical detail, detailed analysis of Mandel'shtam's scientific and teaching legacy, personal memories of the man, including my own, which you would have every right to expect from me as one of his still-living students. I have attempted only to profile Mandel'shtam to some degree as a *scientist* and as a *man of exceptional personal qualities*, spiritual nobility, and charm. I hope that the major and inevitable omissions that I have been obliged to make will be filled by the recently published Anniversary Collection,[2] which contains almost everything that persons who knew Mandel'shtam more intimately and longer than I did have written about him, by the papers in the August number of this journal, and by A. V. Gaponov-Grekhov and M. A. Isakovich, who are scheduled to speak a bit later.

Still, I find it necessary to present a brief biographical sketch of Mandel'shtam for those who not only never saw Mandel'shtam, but know nothing about him.

He passed his childhood and youth at Odessa. He studied at home until he was 12 years of age, and did not enter class III of the gymnasium until 1891, completing the course in 1897. Immediately afterward, he entered the physico-mathematical faculty of the Novo-rossiisk University at Odessa. He had shown no particular interest in physics at the time. Syllogisms, mathematics, and billiards were more to his liking. Two years later, he was expelled from the University in the aftermath of student agitation, and, on the advice of his parents, went to Strasbourg (which was then in Germany) in the same year, 1899, enrolling in the physico-mathematical faculty of Strasbourg University.

From that point on, his life may be divided into three periods: the Strasbourg period up to 1914, i.e., up to the outbreak of the First World War, the period from 1914 to 1925, a time of war, two revolutions, civil war, and subsequent disorders; and, finally, the last, Moscow period, which was the busiest and most productive time of his life and which brought forth his principal achievements—the prime of his scientific and creative activity.

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[1] Paper at joint scientific session of the Division of General Physics and Astronomy and the Division of Nuclear Physics of the USSR Academy of Sciences, May 30, 1979.

[2] The collection "Academician L. I. Mandel'shtam: On the 100th Anniversary of his Birth," Nauka, Moscow, 1979.

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At Strasbourg, Mandel'shtam advanced from student to University *280* Professor (1913)—a rare honor for a foreigner. While there, he studied mathematics and the works of the classical physicists thoroughly, including those of Rayleigh, who had a special influence on him. Rayleigh's writings struck a highly responsive chord: they were, as N. D. Papaleksi put it, congenial with him. "And this was no accident," writes Papaleksi. "The pathways of their scientific creativity often paralleled each other and crossed more than once." The "atmosphere" of electromagnetic oscillations and waves into which Mandel'shtam was immediately plunged by his first teacher-Ferdinand Braun (Braun himself had transferred to radio measurements a year before Mandel'shtam arrived), together with the study of Rayleigh, marked the beginning of Mandel'shtam's *oscillatory approach* to *all* of physics.

"Wireless telegraphy" drew Mandel'shtam to Papaleksi, forming the basis for many years of collaboration and friendship.

A series of outstanding studies made at Strasbourg won Mandel'shtam European recognition, not only as a radio specialist, but also as a broad-profile physicist. His work in the areas of radio measurements and directional antennas drew Mandel'shtam's attention to *phase* relations in oscillations and waves, and this was no doubt a factor later on, when he created radiointerferometry. The extension of *oscillatory* "ideology" to *optics* also took place here, Mandel'shtam began work on light scattering, and clarification of the role of fluctuations in scattering followed at the hands of Smoluchowskiĭ, Einstein, and Mandel'shtam himself. The papers on dispersion theory and the discovery of Planck's error in the problem of attenuation of light due to scattering also date from this period (1904). The 25-year-old lecturer managed to start an argument with a figure as towering as Planck and win it.

Also while at Strasbourg, Mandel'shtam wrote his outstanding paper on the scattering of light at the interface between two liquids-as a result of thermal fluctuations of this boundary. We might note that as recognition, he received a postcard from Einstein (1913) in which the latter reported that he had read this excellent paper at his own colloquium.

His intensifying interest in optics and the problems of resolving power produced a paper on a rigorous theory of image formation in the microscope, in which the Abbe theory was improved and the integral-equation formalism, which was new at the time, was first applied to a physical problem. Again an argument arose, this time with Lummer, and Mandel'shtam again emerged the winner.

Finally, mention should be made of one more study quite typical of Mandel'shtam: he became interested in what would be produced by an optical source in wavelength-scale proximity to an interface, i.e., under the same conditions as in radio. He built such a source very cleverly, using total reflection at a boundary between glass and a fluorescent liquid, and studied the entire problem in both experiment and theory.

"All of these studies," writes Papaleksi, "which include the brilliant and highly instructive polemics with Planck, Fleming, and Lummer, in which Mandel'shtam pointed out the errors of their perceptions, won him recognition and wide repute."

To complete our story of the Strasbourg period, we must also mention the fact that it was in his student years that Mandel'shtam suddenly became deeply interested in literature, music, and the depictive arts. He began to absorb artistic literature in various languages, to visit galleries and attend concerts.

The war interrupted his work at Strasbourg. At the end of July, 1914, just having completed his courses, he hurriedly returned to his native land, arriving at Odessa on the day on which war was declared. From then until 1925, Mandel'shtam moved from city to city and job to job in a search for scientific-research opportunities and a complete education. I shall mention only the three longest and, in certain respects, most important stages of this period.

For two years, Mandel'shtam worked as a scientific consultant at the Siemens and Halske plant at Petrograd (now the Kozitskiĭ plant). E. Ya. Shchegolev, for many years a colleague of Mandel'shtam and Papaleksi and at the time a student in training at this plant, writes of a number of Mandel'shtam's noteworthy inventions and instruments. He writes that everyone was astonished at "how beautifully and sometimes ingeniously Mandel'shtam found simple solutions to very unsimple technical problems—solutions so simple that each of us would find himself asking "Why didn't I think of that before?." But the reason why the ideas would pop into Mandel'shtam's head instead of someone else's was that Mandel'shtam was a physicist of broad vision. His inventions were *typical physicist's inventions*, and at the same time, those of a physicist who knew well the practical needs of his particular technical field.

During a four-year stay at Odessa (1918–1922), Mandel'shtam took an extremely active part in founding the Odessa Polytechnic Institute. He prevailed on Papaleksi and I. E. Tamm to come to the institute and designed its courses on a high level. In his introductory lecture to the general physics class he posed the question: does an engineer need physics—not only "his own" division of physics, but the entire science as a whole, and its theoretical as well as its experimental aspect? The answer was, of course, in the affirmative. Mandel'shtam argued brilliantly that broad mastery of physics is simply *indispensable to every* engineer. We might note that the founders of the Moscow Physico-technical Institute proceeded from the very same premise exactly a quarter of century later.

But Mandel'shtam continued to ponder the science everywhere and at all times. As early as 1918, it occured to him that the thermal fluctuations of a homogeneous medium should, on the scattering of monochromatic light, create a spectral doublet in the scattered light. A decisive step that he had taken earlier was the basis for this prediction-identification of Einstein's "formal" harmonic gratings with Debye acoustic waves. Much later, he would speak of this step with a touch of pride, though without a word as to his own part in it, noting that "taking it was not all that simple." L. Brillouin arrived at the same conclusion in 1922, but 12 years passed before this *Mandel'shtam-Brillouin doublet* was observed in experiment.

In 1922, Mandel'shtam moved to Moscow to work as a scientific consultant in the radio laboratory of the Light-Current Factories Trust, which was soon transferred to Petrograd and reorganized as the Central Radio Laboratory (TsRL). His work at the TsRL (with Papaleksi) continued until 1935.

Mandel'shtam's dream of *physics* research and the teaching that was inseparable from it was finally realized in 1925: he was invited to join Moscow State University (MGU) as a professor in the physics faculty and head of the theoretical department of the University's Scientific Research Institute of Physics (NIIF). I should like to note that the decisive factors attracting Mandel'shtam to MGU were not only S. I. Vavilov, but also the social organizations of the faculty and A. A. Andronov, who was then an MGU student and a very active promoter.

Mandel'shtam very quickly became a central figure in his faculty and in the institute. His Moscow school, which was eventually to produce five Academicians (not counting Mandel'shtam himself), formed just as quickly.

Having said that Mandel'shtam became a central figure, I should like to put to rest at once any incorrect but possible associations to the phrase. Mandel'shtam was exceptionally, almost painfully modest in everything—in conduct, speech, and demeanor. This does not by any means imply that he held himself in low esteem. But never in any way did he permit himself the most minute show of arrogance. Popularity, the desire to "play a role" and advance his prestige or the wish to "take charge" of any joint effort were absolutely foreign to him. Moreover, he was *not* an organizer by nature. When I speak of Mandel'shtam's school, I refer not to the organization of new institutes and the like, but to the rise of a team of talented people of diverse natures who were drawn into the orbit of his influence and his broad scientific interests solely by his attractiveness as a scientist and as a human being. Mandel'shtam's school was a special atmosphere full of mutual good will but without the least slackening of the requirements made of a scientific worker. I cannot conceive of a genuinely scientific school without such an atmosphere.

The names of Mandel'shtam's closest co-workers and his first Moscow students are well known. In radiophysics and radioengineering there was N. D. Papaleksi; in physical optics and ultraacoustics G. S. Landsberg, in theoretical physics I. E. Tamm and M. A. Leontovich, in the theory of oscillations A. A. Andronov, A. A. Vitt, and S. É. Khaĭkin. Most of these early representatives of Mandel'shtam's school went on to form their own physical schools, which were perhaps less broad in profile, but nevertheless very strong. Here I have in mind the theoretical schools of Tamm, Andronov, and Leontovich and Landsberg's experimental school. Thus, along with A. F. Ioffe, Mandel'shtam was a *founder* of "big" Soviet physics, which is now at world level.

One must note how *different* all of Mandel'shtam's students remained in spite of their common school. Mandel'shtam *developed* the individuality of his students rather than suppressing it. Andronov writes that "he had his own special kind of dialogue with each of them." As a result, there was no sterotyping of any kind—either in interests or in the style of their publications. You will not confuse a paper by Leontovich with a paper by Tamm or Andronov. They can all be distinguished at once even without their by-lines.

The work of members of Mandel'shtam's school was, of course, subject to a degree of organizing, in which the directors of the existing institutes in which Mandel'shtam had worked played a major role. They did not interfere with his school, but were able to rate it on its merits. In this respect I mention with gratitute the role of B. M. Gessen, director of the Moscow State University NIIF, and that of S. I. Vavilov, who, not only at the MGU, but also at the Physics Institute of the Academy of Sciences (FIAN) and in the Academy of Sciences, always rendered broad support to the work of Mandel'shtam and his colleagues in accordance with their steadily increasing capabilities.

In 1934, after the Academy of Sciences was moved to Moscow, Mandel'shtam took two of the FIAN's laboratories under his ideological wing: the optical laboratory, which was headed by Landsberg, and the laboratory of oscillations, which was directed by Papaleksi. Research in optics was also continued at the NIIF MGU, while in the physics faculty Mandel'shtam regularly presented his annual lecture courses and seminars, which were a totally *unique* phenomenon in the teaching of physics.

Mandel'shtam was a man of exceptional *integrity*. He strove for the truth and never compromised principle either in science or in any other life situation. This iron adherence to principle was, at the same time, combined in a surprising way with gentleness, considerateness, and genuine kindness. A. N. Krylov said of him: "Leonid Isaakovich was straightforward, honorable, totally without art and cunning...." Mandel'shtam's moral authority was extremely high and ranged far beyond the confines of his school and institute. Often, people whom he hardly knew would come to him or write him to secure his opinion or advice on some situation or other or on their projects and intentions.

On the whole, Mandel'shtam's influence on people and especially on his students and those in the classroom was indelible. He was masterful at educating without moralizing. As V.A. Fabrikant recalls: "We, the students, were taught not by rule, but by personal example, to adopt an attitude to science and to each other.

We sensed the purity of the atmosphere surrounding us, something that does not always happen. We saw how Leonid Isaakovich rejoiced in the achievements of others, how he combined great patience with people with firmness in his scientific convictions."

But it would not, of course, be possible to do justice to all of these aspects of Mandel'shtam's personality here, and I return now to his scientific career.

It will not be necessary to relate the *specific* scientific achievements of Mandel'shtam's Moscow period in detail to this audience. I believe that most of those present associate his name (I have permitted myself to omit those of his well-known coauthors) with:

discovery of the *selective* and *combination* [Raman] scattering of light;

creation of the *nonlinear theory of oscillations*, which, among other things, led to the discovery of several new nonlinear phenomena in radioengineering;

development of the theory of parametric systems, related to creation of the so-called elevated-frequency parametric generators (machines);

broad generalization of the resonance concept to parametric and nonlinear systems;

in general, *shaping of oscillation theory* into an independent scientific discipline with its own specific approaches and methods.

Also associated with Mandel'shtam's name are the development of *radiointerferometry*, with its practical offshoots in radiogeodesy and radio navigation;

discovery of the *tunnel effect* in quantum mechanics;

profound *analysis of the measurement* in quantum mechanics;

interpretation of the *energy-time indeterminacy relation*, and many other achievements that I shall probably not even have time to mention. However, this is not great calamity: there is a complete collection of Mandel'shtam's works, which, though it has become a bibliographic rarity, nevertheless still exists. The main thing is that, along with Mandel'shtam's *concrete* results, certain general "guiding points of view," to use his own expression, were no less important.

Without them, my incomplete citation of his scientific achievements might appear spotty and disjointed. In light of these guiding Mandel'shtam viewpoints, however, it is seen to be held together by an internal unity. These apparently unrelated results are found to be the outgrowth of a few profound ideas whose emergence, development, and remarkable utilization can be traced clearly from the Strasbourg period throughout Mandel'shtam's entire lifetime. It was these ideas that shaped his views and led to the internal relationships that combine all of physics into a single entity in his perception. Mandel'shtam was unquestionably a *universal man* in physics, and possibly one of the last of them. Andronov put it nicely: "There were no rooms closed to him in the enormous edifice of physics."

It is this physical "ideology" of Mandel'shtam's (may the philosophers forgive me this use of the word, which they may feel to be somewhat irreverent) that I should like to make the main subject of my further remarks.

I do this not only because this "ideology" has already become a part of the flesh and blood of modern physics (which is undoubtedly a good thing), but also because it has become almost a folklore, which, by definition, has no author (and this is not good at all and there is no justification for it). There are various reasons for this, and is not worthwhile or even particularly pleasant to go into all them, but one thing that has worked for forgetfulness has been the disappearance of the two Mandel'shtam *memorial* prizes (in physics and in radio) that were approved by the Presidium of the Academy of Sciences after his death. One of them was awarded only twice, the other three times.

Before turning to the announced last part of my presentation, I stress once again the organic merging of theoretical and experimental, of mathematical and technical physics in Mandel'shtam's genius—the *breadth* that ranks him among the classics of the science. And this is not all. Presenting a paper on the scientific works of A. N. Krylov (in observance of his 80th birthday) to the Academy of Sciences in 1943, Mandel'sshtam himself pointed out certain typical attributes of the classics. The first was their attitude toward the question: *pure or applied science*. The classics did not know the difference, and the two aspects of knowledge complimented and fertilized one another in their works... Secondly, their *choice of problems*. Mandel'shtam explained that we may, in a schematic way, distinguish problems that bear the imprint of cleverness from problems that flow naturally from the organic development of any field of knowledge. The former may be useful at times, but they contribute little to scientific *progress*. The latter, on the other hand, are *vital* and therefore productive. To this Mandel'shtam adds a characteristic reservation: "I do not wish to be misunderstood. When I say 'vital,' I do not mean 'utilitarian.' As the most striking examples, he cites Einstein with his theory of relatively and Planck and Bohr and others with quantum mechanics, which responded to profoundly vital problems of physics. "Science is advanced by the solution of vital problems." To these two attributes of the classics, Andronov added Ehrenfest's statement that the classics are typified by profound understanding of the *need* for the new by virtue of their exceptionally complete knowledge of the old.

Mandel'shtam spoke of Krylov, Ehrenfest, Einstein, and Bohr, but he himself was also gifted with these attributes. The problems that he solved were vital, and it was for this reason that their solution opened up new pathways in both physics and engineering.

One other thing. When the wellsprings of physical theories were being discussed, Mandel'shtam never used the deprecation "obvious." He was always interested in problems of gnosiological nature, and he tried, more and more as the years passed, to clarify to himself (and hence also to others) the deepest roots of physical postulates and theories, roots that are both historical and logical. He was not only a universal physicist, but also a deep thinker.

Andronov drew Mandel'shtam's scientific profile perhaps better than others. In the paper that he wrote on Mandel'shtam after the latter's death, he succeeded in drawing together a number of his subject's characteristics very precisely. I shall cite only that which I have not yet mentioned.

Andronov stresses the special "Mandel'shtam" clarity or, to use Mandel'shtam's own word "transparency." It was, of course, on the one hand a result of thorough meditation on the problem and, on the other, Mandel'shtam's unusual skill in *idealization* of a problem. This enabled him to revert to extremely simple examples that still retained all of the essential premises. Mandel'shtam gave a great deal of attention to the fundamentally inevitable, but correct, idealization. He believed that even a grade-school pupil should understand clearly that in theory we are always dealing with idealized models of real things and processes.

Andronov then points out the collective nature of scientific creative work and the absence of a sharp boundary between scientific work and teaching. Finally, he speaks of:

the idea of developing nonlinear thought;

the Mandel'shtam understanding of the role of oscillation theory in physics, and

the idea of "oscillatory mutual aid" between various areas of physics and technology.

It is on these three "guiding viewpoints" of Mandel'shtam that I should like to dwell, since Andronov's paper appeared 35 years ago.

Nonlinearity, of course, is nothing new for physics. Celestial mechanics, fluid dynamics, electrical engineering with iron, the relativistic equations of motion, Einstein's theory of gravitation-all of these things are nonlinear "by nature." But in every one of these areas, nonlinearity has always been perceived as a kind of specific *inconvenience*, as "the" nonlinearity of the particular field. It was Mandel'shtam who was first to recognize the need for development of "nonlinear physical thinking." He stated the problem clearly in his remarkable paper at the 1931 conference on oscillations, stated it in application to the theory of oscillations and to radio. Nonlinearity had found its way into radio back at the time of the spark-over and Branly's coherer, but it became a *vital* problem with the appearance of vacuum tubes and self-excited nondamping oscillators. At that time, no one understood more clearly than Mandel'shtam that the broad powers of the classical linear theory of oscillations, which began with Lagrange and was largely perfected by Rayleigh, were still limited, and that an enormous range of physical and engineering problems required creation of a new approach and a new, *nonlinear* theory, Mandel'shtam's role in solution of this problem cannot be overstated.

The nonlinear mathematical formalism of oscillation theory was found and developed, its nonlinear language was expanded, and nonlinear concepts and nonlinear institution were developed by Mandel'shtam himself or with his direct involvement. Summarizing the results of this process in 1944, Andronov proposed to compare the state of affairs in 1927, when there were only the pioneering works of van der Pol and a few other authors, with the situation in 1944, when the arsenal contained Poincaré's qualitative theory of differential equations and Lyapunov's stability theory, a theory of discontinuous self-oscillations had been developed, advances had been made toward a theory of automatic control, and the first steps had been taken in the field of *distributed* nonlinear systems.

It is appropriate to continue this comparison today, i.e., to compare the states of affairs as of 1944 and 1979.

Nonlinear thinking has broken into *field-theory* and *wave* problems, including the propagation of electromagnetic waves and, specifically, optics (by virtue of the lasers). Both nonlinear acoustics and fluid dynamics in general underwent further development. The qualitative theory burst the bonds of two-dimensional phase space. Even at 1.5 degrees of freedom, a new type of limit set that is stable in the large was discovered-the so-called "strange attractor," which has all of the necessary attributes of *stochasticity*, i.e., what I would call *noise self-generating* systems (Lorentz, 1963). This was the first observation of the *emergencies of stochasticity in a deterministic* system with a *small* number of degrees of freedom.

Finite-dimensional analogs of systems of the hydrodynamic type were developed (Obukhov, 1969), the significance of the number of phase-space dimensions was pointed out, and, specifically, the possible loss of stability of the strange attractor in augumented phase space was observed (Dolzhanskiĭ and Pleshanova, 1979). These methods have come into extensive use in modeling various fluid-dynamic phenomena in liquids-thermal convection and self-oscillations-as well as turbulence and other cases in which "disorder" (stochasticity) arises out of "order," in a deterministic system (see the reviews by Monin and Rabinovich in this journal, 1978).

Nor may we omit mention of processes that are in a certain sense the "reverse"—the appearance of regularity, of order from disorder, as when stable spatially periodic structures from in convection (for example, the well-known Benard cells).

The theory of nonlinear oscillations was further developed and applied to plasmas, chemistry, biochemical and membrane systems, astro-physics, and ecology. Speaking of chemistry, we might note concentration fluctuations and observation of self-wave chemical reactions (Zaĭkin, Zhabotinskiĭ, 1970). In general, it has been found that practically all types of oscillations known in nonlinear mechanical and electrical systems can occur in a homogeneous chemical system (Zhabotinskiĭ, 1974).

There is no doubt that the experience now being acquired in the theory of macroscopic nonlinear waves—with all of their self-actions, interactions, mutual transformations, formation of discontinuities, various types of structures, solitons, etc.—will play its own part in the construction of a unified theory of so-called elementary particles and their various types of interactions. As a matter of fact, this extension of nonlinear thinking to elementary-particle theory is already taking place.

Thus, Mandel'shtam's idea of broadening nonlinear thinking covers all of physics, and Andronov was clearly correct when he concluded his address with the words: "the ideas of L. I. Mandel'shtam in the theory of nonlinear oscillations... are of surpassing importance. They have before them vigorous development and a life rich in events."

Mandel'shtam himself perceived the grounds for extension of oscillation-theory ideas to physics as a whole. This was essentially the broad understanding of oscillation theory at which he arrived not at once, but toward the end of his life, after the evacuation to Borovoye, and of course, he wished to convey this understanding immediately to others as he prepared a new course in oscillation theory. Though quite ill, he began to teach this course in 1944, but he succeeded in completing only four lectures. His thrust was the premise that all of the basic discoveries of physics, beginning with Kepler's laws were essentially *oscillatory.*

The original division of physics into its traditional (scholastic) fields was, of course, anthropomorphic in nature. It was a division in accordance with our perception of phenomena. A rather complete knowledge of the laws operating within each of these regions developed with the framework of this division, and this prepared the soil for establishment of the *commonality of the laws*, for that which Andronov later called the "isomorphism of relationships." This was a later step, and, accordingly, a higher type of classification of phenomena-based not on their perception, but on commonality of relationships. This classification has enormous heuristic power.

Rayleigh himself, in his "Theory of Sound," wrote a supplementary chapter on electrical oscillations, stressing that the two types of *small* oscillations—acoustic and electrical—are one and the same. This is a triviality today. As Poincaré noted: "Every truth enjoys one instant of triumph between the infinity of time during which it is regarded as untrue (or is simply not known–S. R.) and the infinity in which it is considered trivial." Nevertheless, Mandel'shtam said regarding even this rudimentary *linear* isomorphism that: "But what is not trivial is the fact that it is trivial." And in fact an even more interesting subject metamorphosed during the further development of the *nonlinear* theory of oscillations. The isomorphism principle was elevated from identity of equations to a higher level: not simple identity of equations, but the inclusion of *dissimilar* equations in the same type or class, in the sense of the topological structure of the phase space. Andronov presented striking examples as far back as 1944. For example, two problems–those of the rotation of the synchronous motor and the symmetric flight of an airplane at a constant angle of attack, which are described by dissimilar equations that cannot be reduced to each other, are one and the same problem from the standpoint of oscillation theory, a problem with three identical stationary regimes of identical stability or instability.

Following Mandel'shtam's example, we now speak not only of *general-oscillatory*, but also of *general-wave* laws, although a general qualitative theory of some functional phase space for nonlinear fields does not yet exist.

Since equations are involved even in this more developed form of the isomorphism principle, Mandel'shtam noted all of this is in a sense mathematics. But he still stressed that this principle cannot be treated as "pure" mathematics. The whole thing comes down to equations for *physical* quantities, which we must be able to *measure* in one way or another. Our equations, including even the most general axioms (for example, variational principles) from which they can be derived, are not the result of agreement in the abstract, but are subject to control by experiment and reflect the way in which nature is structed. Therefore not even the isomorphism of relationships is a conclusion drawn from logical exercises or something that can be assumed *a priori.* This is why Mandel'shtam said that "it is physics that teaches us how to cross-examine differential equations."

Since the isomorphism of relationships is not known *a priori*, it may not be universal either. But in the theory of oscillations with its general physical significance it has been established with very high certainty and has long been acquitting itself well and broadly in both study of known phenomena and discovery of new ones. I have in mind here Mandel'shtam's idea of "oscillatory mutual aid" between the traditional divisions of physics, the "international" language of oscillation theory in which these divisions, which speak in their own "national" languages, can communicate with one another.

It is hardly necessary to quote the statements from Mandel'shtam's lectures and papers that apply here. I should like only to stress the astonishing skill with which Mandel'shtam himself made use of this oscillatory mutual assistance both in his own research and in his lectures. This skill made it easy for him to build bridges between traditionally "unlike" phenomena: from radio to optics while he was at Strasbourg, from optics to radio while he was at Moscow—in the creation of radio-interferometry and again from radio to optics in Rayleigh and Raman scattering. In problems of resonance, modulation, in parametrics, wherever you pleased, he drew without the slightest effort on either mechanics or electrodynamics or optics or acoustics or molecular physics or quantum mechanics. It can be said that the traditional division of physics was immaterial to him, but... only where the isomorphism of relationships actually worked. His view of the aspects specific to various fields of physics was no less lucid.

In general, he managed a striking combination of his love for the broad generalization and sharp-slighted perception of the concrete fact—in Tamm's words, a combination of the abstract and the plastic mind.

Perhaps the most brilliant and significant example in which the specifics of a given area of physics were brought out was Mandel'shtam's analysis of the meaning of *measurement* in quantum as contrasted to prequantum physics. His theory of *indirect* measurements in quantum mechanics was an original and weighty contribution to our understanding of quantum theory itself according to the testimony of those in the field (in particular, Tamm).

The last thing that I must not fail to mention is Mandel'shtam's teaching activity—his lectures, seminars, and public appearances in general. Much has been written, by many, on the altogether singular emotional atmosphere that Mandel'shtam's appearances generated. His lectures and papers captivated audiences, filling them with joyful excitement and blocking out everything in the world but his words. Even persons only remotely concerned with physics were moved. Mandel'shtam's "weapon" was not the orator's craft, but his remarkable ability to stimulate his listeners, to ignite their interest and force them to perceive problems where, it would appear, all was clear by virtue of what had been said before. He induced listeners to hang on every word and think intensely. One particular device that served this purpose was the paradox, to which he loved to resort. But this was only a "weapon." What was the real root or "secret" of this effect?

In the reminiscences that I wrote for the Anniversary Collection, I mentioned Mandel'shtam's acute sense of style and form, his ability to set forth a problem not only convincingly and "transparently," but also elegantly. P. L. Kapitza had good reason to note on one occasion that, in his opinion, Mandel'shtam was surely an esthete. But I soon understood that this was not the most important thing.

Mandel'shtam's teaching had nothing in common with traditional university instruction. It was not repetitious reading of textbooks from a study plan, even one with annual supplements and revisions. Each of his eight seminars and six lecture courses at Moscow had its own unique subject matter or content. Even the course in oscillation theory, which was started in 1944, was in no sense a revival of the course given under the same name in 1930-1932, but was conceptually totally new. I now realize that the main thing was not that Mandel'shtam liked teaching and knew how to teach, but the fact that *he himself needed* his lectures, seminars, and papers. He needed to share his ideas, the fruits of his deliberations—on both narrower problems and those of general physics. His teaching was therefore not conventional instruction in physics, but first and foremost a whole school of physical thought.

This is why his presentations were of absorbing interest to persons other than physicists.

This is why his lectures have not and cannot age in spite of the years that have passed.

In the words of S. I. Vavilov at Mandel'shtam's graveside: "The great and dear name of Leonid Isaakovich will not die in the Soviet nation or in the world as long as there is still one living physicist who understands the truly great and excellent in science." I heard these words. Vavilov spoke them not as a phrase becoming to a burial ceremony, but with the firm conviction that this is the way it would be. And all of his listeners were also convinced. Even today, I continue to hope that what Vavilov said will ultimately come to pass.

Translated by R. W. Bowers