Baryon–antibaryon nuclei

I. S. Shapiro

Institute of Theoretical and Experimental Physics, Moscow
Usp. Fiz. Nauk **125**, 577–630 (August 1978)

The present state of the problem of quasinuclear systems containing antibaryons ($B\bar{B}$ and $2N2\bar{N}$ mesons and $2N\bar{N}$ baryons) is reviewed. A discussion is presented of the experimental data (the resonances which are strongly coupled with the $N\bar{N}$ channel, with masses of about 2 GeV) and of the central theoretical questions (the energy spectra and properties of the quasinuclear systems, the annihilation widths and shifts of the quasinuclear levels, the $p\bar{p}$ atom, and radiative transitions from atomic states to quasinuclear states).

PACS numbers: 14.20.Gk, 21.40. + d, 36.10. – k

CONTENTS

1. INTRODUCTION

The family of particles called "baryons" (B) includes the structural elements of nuclei, i.e., the nucleons (N) (the proton and the neutron), and the hyperons (Y), i.e., the "strange fermions" Λ, Σ, Ξ, and Ω. Corresponding to each of these particles there is an antiparticle: an antibaryon ($\bar{B}$).

In this review we will be dealing with certain unusual systems which are similar to nuclei but which contain antibaryons as well as baryons. For the most part, we will be concerned with the simplest such systems, the two-particle $B\bar{B}$ systems, and within this group we will focus on the $N\bar{N}$ systems, since we know more about nucleons than about other baryons.

The existence of nucleus-like $N\bar{N}$ systems was predicted theoretically about eight years ago, and these "quasinuclei" have recently been observed experimentally.

The $N\bar{N}$ systems are distinguished from the NN systems which are the subject of traditional nuclear physics in that annihilation can occur and the nuclear forces are different. On the basis of the data available it seems quite likely that the attraction between N and $\bar{N}$ is much stronger than that in the NN system (for roughly the same effective range of the forces). In the $N\bar{N}$ system, therefore, in contrast with the single weakly bound state of the two-nucleon system (the deuteron), there can be an entire spectrum of bound and resonant states of the nuclear type (that is, states having a mass defect much smaller than 1 GeV and an average particle separation of the order of 1 F). A factor which hinders the appearance of these quasinuclear states is annihilation: the particles may disappear before a finite-motion state is formed. An important theoretical result, however, is that annihilation, despite its large cross section, does not rule out the possible existence of quasinuclear states. The physical reason is that annihilation occurs at particle separations much smaller than the radius of the quasinuclear orbit. Theoretical estimates show that the annihilation widths of the $N\bar{N}$ quasinuclear levels can range in order of magnitude from 0.1 MeV to 100 MeV (depending on the orbital angular momentum of the relative motion of N and $\bar{N}$; relatively small widths correspond to relatively large orbital angular momenta).

The "nucleus" consisting of N and $\bar{N}$ has a zero baryon number (the number of baryons minus the number of antibaryons), so it should behave as a heavy meson (with a mass of about 2 GeV) which is comparatively "narrow" and which is strongly coupled with the $N\bar{N}$ channel (that is, this nucleus forms most readily in the interaction of $\bar{N}$ with a hydrogen or nuclear target).

The quasinuclear mesons $N\bar{N}$ (or, in general, $B\bar{B}$) are interesting in both nuclear physics and particle physics.

The reason for the interest in nuclear physics is that the N–$\bar{N}$ and N–N nuclear forces, although different, are generated by the exchange of the same light bosons. The N–$\bar{N}$ and N–N interactions should thus be related in a definite manner: by "G conjugation" (by analogy with the way in which the electromagnetic electron–positron and electron–electron forces are transformed into each other by C conjugation). If we understand the forces in the N–$\bar{N}$ channel we can thus learn about the N–N interaction. What use can be made of this additional informa-

 0038-5670/78/080645-29$01.10

tion in nuclear physics? The answer is that the conclusions which have been drawn regarding the N–N forces from data on the N–N scattering phases and the structure of the deuteron are not unambiguous: at present there are several different N–N interaction potentials which describe these data more or less satisfactorily.[1] The reason for this situation is that the scattering phases which govern the behavior of the wave function at asymptotically large distances (i.e., outside the effective range of the forces) are not sufficiently sensitive to the nature of the potential. The deuteron, on the other hand, is a weakly bound system, and a large part of the normalization integral for the wave function corresponds to the exterior region, which is essentially force-free. In other words, the N–N forces are difficult to reconstruct from the experimental data (even with the help of some physical model for the N–N interaction), because these forces are too weak in the two-particle system.[1)]

The situation is completely different in the case of the two-particle $N\bar{N}$ system, since in this system we can expect an entire spectrum of bound and resonant states which are spatially confined to the effective range of the nuclear forces. The positions of the energy levels of this system are much more sensitive to the potential than the binding energy of the deuteron or the scattering phases. Specifically, calculations show that potentials which give approximately the same NN scattering phases and $N\bar{N}$ interaction cross sections lead to very different arrangements of the particular levels (corresponding to definite quantum numbers) of the discrete spectrum (see Section 5 below). The two-particle $N\bar{N}$ quasinuclear system may thus prove to be a unique source of valuable information about nuclear forces.

Another reason for studying the $N\bar{N}$ quasinuclear mesons is to determine "who's who" in the daily-growing list of particles. The problem of quasinuclear mesons is also particularly pertinent because the same physical considerations which lead us to expect nonrelativistic $N\bar{N}$ bound and resonant states also lead to the possible existence of analogous $B\bar{B}$ states, where B is any baryon. Of particular importance here are systems of the $Y\bar{Y}$ or $Y\bar{N}(\bar{Y}N)$ type. In the latter case we are dealing with a two-particle "quasihypernucleus" in which one of the baryons is replaced by an antibaryon. Although far less is known about the interaction of nonrelativistic hyperons with each other or with nucleons than about the NN nuclear forces, it nevertheless seems likely that the discrete spectrum of quasinuclear states of the $B\bar{B}$ systems should contain many levels (including levels corresponding to doubly charged or "exotic" mesons of the $\Sigma^+\bar{\Sigma}^-$ or $\Sigma^-\bar{p}$ type).[2)] It is hardly necessary to explain how important the spectrum of these levels would be as a source of information on the hyperon–hyperon and hyperon–nucleon forces.

Heuristic variational calculations show that in addition the two-particle $B\bar{B}$ quasinuclear systems there may be more complicated "nuclei" which contain antinucleons. Among these systems are the $2N\bar{N}$ baryons and the $2N2\bar{N}$ mesons. Remarkably, even among the four-particle quasinuclear systems there are states having a comparatively small annihilation width. In particular, there is the possibility of a doubly charged $2N2\bar{N}$ bound state with a width[3)] of only some 20–30 MeV.

When three-particle and four-particle systems containing antibaryons are included, the spectrum of quasinuclear states can be extended from about 1.7 to 7 GeV. Significantly, the number of such states is expected to be large. The general implication is that in this "third spectroscopy" we are apparently entering a new field: the physics of the bound and resonant states of baryon–antibaryon, primarily nucleon–antinucleon, systems.

In the past few years we have seen the first experimental evidence of bound and resonant states in the $N\bar{N}$ system (see the review by Kalogeropoulos[2]). Particularly noteworthy is the narrow resonance $N\bar{N}$ (1940), whose existence can hardly be doubted now, since it has been observed by several experimental groups who have used very different facilities.[3]

The appearance of the first experimental data on $N\bar{N}$ quasinuclear states has of course greatly increased interest in the physics of these peculiar systems, which are now emerging as real systems, instead of the hypothetical objects which they were a few years ago.

The question of the possible existence of mesons consisting of a nucleon and an antinucleon has theoretical background going back to the well-known paper by Fermi and Yang.[4] In this paper, which actually founded the composite models of particles, the pion was interpreted as an $N\bar{N}$ bound state. Closer to the concept of the $N\bar{N}$ quasinuclear states discussed in the present review are the papers by Bethe and Hamilton[5] and Afrikyan.[6] These papers (which preceded the discovery of the antiproton) dealt with the existence of an $N\bar{N}$ bound state analogous to the deuteron.[4)] They did not, however, take up the

---

[1)]The use of more complicated systems—multinucleon nuclei—introduces uncertainties of a different type in attempts to study the N–N interaction. These other uncertainties are due to the "bad mathematics", i.e., the difficulties in solving the quantum-mechanical problem of several interacting objects in the case in which the perturbation methods cannot be used (or, at best, their applicability is questionable). Furthermore, in a multinucleon nucleus it is entirely probable that the many-body forces not found in the NN system will be significant. The possibility of these forces further complicates the extraction of information on the NN interaction directly from the nuclear data.

[2)]These expectations are based on results calculated in the OBEP model with baryon—(light boson) coupling constants approximately the same as the nucleon—boson constants [in particular, $SU(3)$ symmetry is used for these estimates]. Qualitatively, on the other hand, the situation reduces to the circumstance that for all baryons there can be an exchange of an ω meson, which always causes an attraction between particles in the $B\bar{B}$ channel (regardless of the quantum numbers of the B).

[3)]In this case the small annihilation width is attributed to the large dimension of the system (about 2F). This factor masks the increase in widths due to the increase in the number of annihilation "partners."

[4)]In fact, Afrikyan[6] asserts that the binding energy in the $N\bar{N}$ system must be precisely equal to the deuteron binding energy. He reaches this conclusion by arguing that the deuteron is bound only by one-pion exchange and that such forces are the same in the $N\bar{N}$ channel. This latter assertion was the re-effect of annihilation on the formation of $N\bar{N}$ bound and resonant states. Measurements of the cross section for $N\bar{N}$ annihilation, carried out immediately after the discovery of the antiproton, revealed a suprisingly large value: nearly three times the total pp cross section for nonrelativistic energies. This result discouraged study of possible $N\bar{N}$ quasinuclear states for a long time, although in the very first attempts to describe $\bar{N}N$ annihilation it was noted (Martin[7]) that the distances characteristic of annihilation should be far smaller than the effective range of the nuclear forces. This fact was not taken seriously, however. In the most realistic optical models (for example, the models of Nemirovskiĭ *et al.*[8] and Bryan and Phillips; see Ref. 9 and the literature cited there), although the imaginary part of the potential (which incorporates annihilation in a phenomenological manner) is assigned a mean radius of the order of 0.2 F, the amplitude of this exponentially decaying potential is chosen so high that the annihilation probability turns out to be essentially unity over the entire range of the nuclear forces in the $N\bar{N}$ system. Despite that the fact that the real part of the Bryan–Phillips potential gives a strong attraction between N and $\bar{N}$, the authors find the annihilation and scattering cross sections for nonrelativistic N and $\bar{N}$ to vary monotonically with energy, without any trace of possible resonances. The results obtained from the optical model agree satisfactorily with the available experimental information (although not much is available) on the cross sections for interaction of $\bar{N}$ and N. This circumstance supported the argument that annihilation is decisive in all $N\bar{N}$ interactions at nonrelativistic energies. This "fixation" on the high annihilation cross sections has led (and continues to lead) to several errors.[5)]

One of these errors has been the tacit belief that narrow states of a discrete spectrum in the $N\bar{N}$ system could not exist.

The question of quasinuclear (bound and resonant) $N\bar{N}$ states was taken up theoretically in Refs. 11–14, before the appearance of experimental evidence for the existence of such systems, in an effort to determine the role of annihilation in the formation of these states and to calculate upper bounds on the annihilation widths. The ideas and results of this work were reviewed in Refs. 15–18. The spectrum of bound and resonant states was calculated in Refs. 11–17 on the basis of the model of the one-boson exchange potential (OBEP) (the real part of the Bryan–Phillips optical potential). This spectrum turned out to be extremely rich (primarily because of the intense spin–orbit forces): there are about 20 quasinuclear states with nonrelativistic binding energies (i.e., with masses near two nucleon masses) and, as mentioned above, with annihiliation widths not exceeding 100 MeV. Analogous calculations were subsequently carried out by several authors. These results will be discussed in Section 5 below; at this point we simply wish to call attention to an earlier paper by Ball *et al.*,[19] who found the spectrum of heavy mesons (with masses of about 2 GeV) in a study of "bootstrap" equations (a set of self-consistent integral equations which are supposed to give as $N\bar{N}$ bound states the same mesons which are responsible for the interaction in the NN channel). This idea was not implemented in that paper (the original mesons were light, while those found were heavy). The solutions of the equations written in Ref. 19 actually reduce to the sum of several ladder diagrams for single-boson exchange and correspond to the quantum-mechanical potential approach. The heavy-meson spectrum[6)] found by Ball *et al.*[19] thus turned out to be similar to the set of $N\bar{N}$ bound states calculated in Refs. 11–17.

As can be seen from this extremely brief description of the background of the problem, one of the central points of a physical theory for the $N\bar{N}$ systems is a correct understanding of the role of annihilation. Is it possible to reconcile large annihilation cross sections with the existence of more or less narrow levels of the $N\bar{N}$ discrete spectrum? We will begin this review by answering this central question.

The organization of this review is clear from the table of contents. We would simply add that this review covers three aspects of the physics of $N\bar{N}$ quasinuclear systems: annihilation effects, the spectra of discrete states, and the observable consequences of the existence of $N\bar{N}$ quasinuclear systems. The review is devoted to the basic physics of the theory of these systems, and although the theoretical questions taken up in this paper refer to specific physical phenomena we believe that a detailed comparison of the theoretical and experimental data is premature, primarily because of important gaps in the available experimental results. The experimental activity on $N\bar{N}$ systems is now increasing rapidly, so more facts will very probably be available by the time this paper is published. With this circumstance in mind, we will simply give a brief description of the present effort to detect $B\bar{B}$ quasinuclear systems experimentally and to study their properties (Section 2).

The last of our preliminary comments deals with the references cited at the end of this paper. We do not claim that this is a complete bibliography, but we have strived to include all the pioneering papers and to give the most important key references.

2. EXPERIMENTAL DATA

Two extremely important facts emerge from the data on the cross sections for $\bar{p}$ annihilation and scattering in hydrogen.

a) The annihilation cross sections σ_a are approximately equal to the unitary limit $(2l+1)\pi\lambda^2$ in each partial wave with orbital angular momentum l (here, as usual, $\lambda=1/k$, k is the $\bar{p}$ momentum in the center-of-mass system, and $\hbar=c=1$ everywhere). This assertion holds at least over

sult of an error: in transforming from the NN system to the $N\bar{N}$ system, Afrikyan carried out C conjugation instead of the appropriate G conjugation.

5)In particular, it is asserted incorrectly in the review of Ref. 10 that the annihilation interaction is a "long-range" interaction.

6)The $N\bar{N}$ resonances were not studied by Ball *et al.*[19] Furthermore, they did not study annihilation effects. In general, Ball *et al.* were interested primarily in the structure of the bootstrap scheme instead of real physical states of the $N\bar{N}$ system.

the range $100 \leq k \leq 400$ MeV/c and is based simply on the absolute value of σ_a.

b) The scattering cross section σ_e is smaller than the annihilation cross section σ_a. For the energy range under consideration here, we can write, approximately,

$$\frac{\sigma_a}{\sigma_e} \approx 1.5-1.8$$

(the higher end of the range corresponds to lower energies). We can determine the minimum number of partial waves which contribute to the cross section. Analysis shows that all partial waves with orbital angular momenta $l \leq kR$, where R is a quantity of the order of 1.0–1.4 F, are involved (Ref. 9).

Figure 1 shows the general energy dependence of the total cross section for the $\bar{p}p$ interaction: $\sigma_t = \sigma_a + \sigma_e$ (taken from Ref. 2). Shown for comparison is the cross section for the pp interaction (the quantity plotted along the abscissa is k^{-1}, in units of c/GeV). For the nonrelativistic energies in which we are interested here, σ_t can be approximated by the empirical equation

$$\sigma_t(\mathrm{mb}) = 66 + \frac{26}{k},$$

where k is in GeV/c.

The primary consequence of facts a) and b) is that the $p\bar{p}$ interaction cannot be treated as diffraction and absorption by a homogeneous black sphere, although the annihilation cross sections are large, i.e., near the unitary limit. Indeed, in the case of a black sphere we would have $\sigma_a = \sigma_e$, in contradiction of experiment.

Nemirovskii's group (see, for example, Ref. 8) was the first to point out that in order to reconcile facts a) and b) it would be necessary to assign the "absorption" (annihilation) region a linear dimension much smaller than the effective range of the nuclear forces. A similar conclusion was reached by Phillips and Bryan.[9]

Another semiempirical method for describing the observed $N\bar{N}$ interaction cross sections is to use a model with the boundary condition of total absorption at a sphere of a definite radius within the effective range of the nuclear forces [the model with this boundary condition was first applied to $N\bar{N}$ annihilation by Ball and Chew;[20] Dal'karov and Myhrer[21] recently used this model with a realistic potential for the N–$\bar{N}$ nuclear forces to carry out cross-section calculations]. This model with the total-absorption boundary condition is essentially equivalent to an optical model in which the imaginary

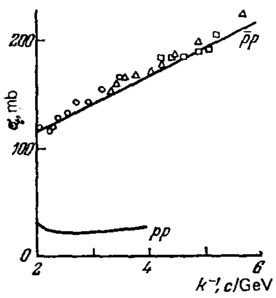


FIG. 1. Total $\bar{p}p$ cross section as function of the momentum of the colliding particles. Plotted along the abscissa is the quantity k^{-1}, where k is the center-of-mass momentum; plotted along the ordinate is the total cross section. The lower curve shows the cross section for the pp interaction.

part of the potential is confined to a certain spatial region (of the square-well type). The radius of the annihilation region in this model thus turns out to be larger than r_a in a potential with a rounded edge.

In all these semiempirical methods for describing the $N\bar{N}$ interaction cross sections, the annihilation is taken into account in such a manner that neither the imaginary part of the optical potential nor the equivalent absorption boundary condition depends on the quantum numbers of the state of the annihilating $N\bar{N}$ pair or on the energy of this pair. This assumption is obviously incorrect and represents a crude approximation of the actual annihilation process. The optical-model wave function within the effective range of the nuclear forces has little in common with the actual wave function of the $N\bar{N}$ system. It should not be surprising that a satisfactory description of the experimental data on the $N\bar{N}$ cross sections can nevertheless be obtained from the optical model (or the model with the absorption boundary condition), simply because these quantities are relatively insensitive to the nature of the interaction within the effective range of the forces (as mentioned in Section 1, even in the simpler case of NN scattering there are several potentials which give accurate descriptions of the experimental scattering phases). For the problem of the $N\bar{N}$ interaction, the optical model is no more than a semiempirical method for extrapolating (or interpolating) the experimental data on the cross sections for annihilation and elastic scattering.[7)] We emphasize that this model (as well as the model with the total absorption boundary condition, which is physically equivalent to it) cannot be used for a theory of effects which are very sensitive to the nature of the $N\bar{N}$ wave function within the effective range of the nuclear forces (in particular, it cannot be used to study the question of a discrete spectrum of $N\bar{N}$ states). The energy dependence of the cross sections σ_a, σ_e, and σ_{ce} is monotonic according to the optical model (or the model with the total absorption boundary condition), without any indication of resonance peaks. Until recently, the experimental cross sections behaved in the same way, but now, as mentioned earlier, resonances have been detected in the cross sections for the $\bar{p}p$ and $\bar{p}d$ interactions. Among these resonances are the narrow resonance $N\bar{N}$ (1940) (width $\leq$10 MeV), which is shown in Fig. 2a (Ref. 3), and at least three resonances with masses of 2000, 2150, and 2335 MeV (the widths of the two latter resonances are 95 and 110 MeV, respectively).[22] These resonances have been observed in the individual annihilation channels and in the energy dependence of the total cross section, the annihilation cross section, and the elastic-scattering cross section. We use the reservation "at least three resonances" here because the very recent "production" experiment[23] reveals narrow $\bar{p}p$ resonant states with masses of 2020 ± 12 and 2204 ± 5 MeV and respective widths of 24 ± 12 and 16^{+20}_{-18} MeV (Fig. 2b). A

7)We might add that the optical model also describes σ_{ce}, the cross section for the charge exchange ($\bar{p}p \to \bar{n}n$). This cross section is governed by the isospin-dependent Hermitian part of the Hamiltonian. The charge-exchange cross section is a weak function of the energy and amounts to no more than 25% of the elastic cross section (see Refs. 9 and 21 for more detailed information).

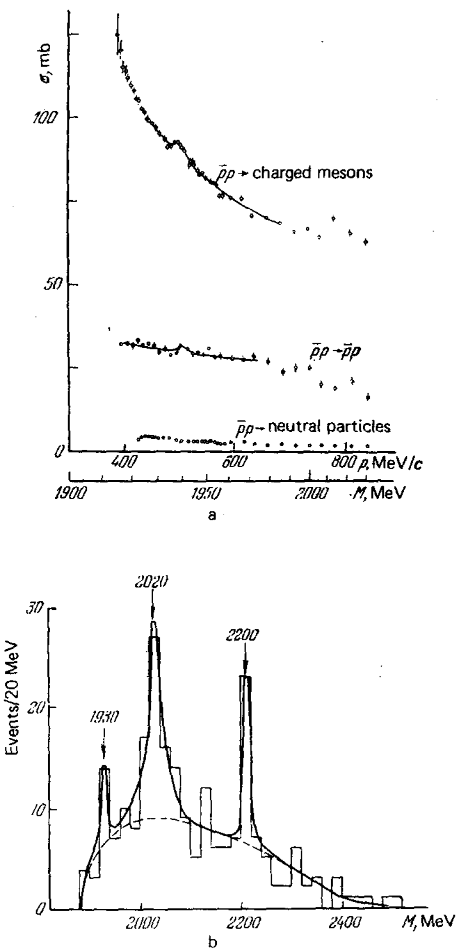


FIG. 2. a) The $N\bar{N}$ (1940) resonance; b) narrow resonances in the $(\bar{p}p)$ channel with a mass of about 2 Gev.

fine structure of the "broad" peaks near 2 GeV has been suspected for some time.[22] These resonances have also been observed in the mass spectrum of the $\bar{p}p$ pairs produced in the reaction $\pi^- p \to 2p\bar{p}\pi^-$ for initial pion momenta of 9 and 12 GeV/c.

Up to this point we have been talking about the total annihilation cross sections (the sum of the cross sections for all annihilation channels, averaged over the $N\bar{N}$ spin states). The dominant annihilation channel is the multipion channel. The average pion multiplicity in the annihilation of a nonrelativistic $N\bar{N}$ pair is 4–5. This number corresponds to the maximum of the Lorentz-invariant phase volume

$$\Omega = \frac{\bar{\mu}^{1-2n}}{(2\pi)^{3n-4}} \int \delta\left(p_i - \sum_f p_f\right) \prod_{j=1}^{n} \frac{d^3p_j}{2E_j},$$

where the mass used to convert to dimensionless masses is $\bar{\mu} \approx 1.43\mu$ (as before, μ is the pion mass).[8)]

Of definite interest are the isospin relations in multipion annihilation, which give the pion charge distribution. These relations are particularly informative when the initial state has a definite isospin I. In $\bar{p}p$ annihilation there is a superposition of states with isospin 0 and 1. There is an "isospin-pure" state in $\bar{p}d$ annihilation $(I=\frac{1}{2})$. If we denote by $F_{\lambda_i\lambda_\pi}(s)$ the amplitude for a transition from the initial state with isospin I_i and projection λ_i to a final state in which there is a pion with an isospin projection λ_π $(\lambda_\pi = 0, \pm 1)$ and there are other particles in states with the set of quantum numbers s, then the isospin density matrix corresponding to this process is

$$(\rho)_{\lambda_i\lambda_i',\,\lambda_\pi\lambda_\pi'} = \sum_s F^*_{\lambda_i'\lambda_\pi'}(s)\, F_{\lambda_i\lambda_\pi}(s). \tag{2.1}$$

The diagonal elements ρ give (with the appropriate normalization) the reaction probability. On the basis of isospin invariance, we conclude that the density matrix ρ should be

$$\rho = a + b\mathbf{I}_i\mathbf{I}_\pi + C\,(\mathbf{I}_i\mathbf{I}_\pi)^2, \tag{2.2}$$

where $\mathbf{I}_i$ and $\mathbf{I}_\pi$ are the isospin operators of the initial state and the pion, respectively. If $I_i \leq \frac{1}{2}$, we have[9)] $C = 0$. In particular, in the case of $\bar{p}d$ annihilation we are interested in the diagonal matrix element ρ in the case $\lambda_i = \lambda_i' = -\frac{1}{2}, \lambda_\pi = \lambda_\pi'$. Using

$$(\mathbf{I}_i)_3 = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \quad (\mathbf{I}_\pi)_3 = \begin{pmatrix}1 & & \\ & 0 & \\ & & 1\end{pmatrix},$$

and noting that the diagonal elements of the other matrices I_i and I_π are zero, we find from (2.2) the probabilities for the production of π^0, π^+, and π^- mesons:

$$\rho_{\pi^0} = a, \quad \rho_{\pi^\pm} = a \mp \frac{b}{2}.$$

It follows from these equations that in $\bar{p}d$ annihilation the number of neutral pions, N_0, and the numbers of charged pions, N_+ and N_-, should be related by

$$N_+ + N_- = 2N_0 = N_\gamma. \tag{2.3}$$

Here N_γ is the number of photons which are formed in the decay $\pi^0 \to 2\gamma$. Equation (2.3) obviously also holds for $p\bar{p}$ annihilation from a state with $I_i = 0$; in this case we have the stronger relations

$$N_+ = N_- = N_0.$$

The same is true for the channel with $I_i = 1$ (since $\lambda_i = 0$) if the coefficient C in Eq. (2.2) vanishes ($C = 0$) for some dynamical reason. We have gone into the question of the isospin relations because these relations have been checked experimentally for $\bar{p}d$ annihilation by the Kalogeropoulos group[25,26] and (less accurately) for $\bar{p}p$ annihilation.[27] The results found for the case of $\bar{p}d$ annihilation have turned out to be far from trivial. A deuterium bubble chamber was used in Ref. 25 to check a consequence of (2.3) for the average energies associated with the charged pions $\langle E_c\rangle$ and neutral bosons $\langle E_0\rangle$ per annihilation of the $\bar{p}$ particles which are stopped in the chamber. The theoretical relation should obviously be

$$\langle E_c\rangle = 2\,\langle E_0\rangle.$$

The experimental value, $\langle E_c\rangle_{\text{exp}}$, turns out to be smaller than expected[10)]:

[8)]The quantity $1/\bar{\mu}$ should not be confused with the radius of the annihilation zone, r_a. The quantity $\bar{\mu}$ is defined by the value of that S-matrix element which corresponds to annihilation. This element is strongly affected by the $N\bar{N}$ nuclear forces. The annihilation cross section is large when r_a is small because of the strong attraction between N and $\bar{N}$ (Section 4).

[9)]Equation (2.2), found here in a simple manner, is a particular case of the isospin relations in inclusive reactions, studied in Ref. 24.

[10)]This number is found after a correction is made for the electromagnetic decay of η and ω mesons (occurring with violation of isospin conservation).

$$2\langle E_0\rangle - \langle E_c\rangle_{\exp} = 58 \pm 10 \text{ MeV*}).$$

Relation (2.3) was tested experimentally by the Kalogeropoulos group.[26] The average number of charged pions per $\bar{p}d$ annihilation event was found to be

$$\langle N_+ + N_-\rangle = 3.04 \pm 0.02$$

(this number had also been found in earlier measurements[20]), while for $\langle N_\gamma\rangle$ the measurements yielded a different number[11)]:

$$\langle N_\gamma\rangle_{\exp} = 3.77 \pm 0.08.$$

The "excess" number of γ rays per $\bar{p}d$ annihilation event thus turned out to be

$$\langle \Delta N_\gamma\rangle \equiv \langle N_\gamma\rangle_{\exp} - \langle N_+ + N_-\rangle = 0.73 \pm 0.08.$$

In other words, for each "legal" photon (i.e., for each photon expected theoretically on the basis of isospin conservation) there must be

$$\frac{\langle \Delta N_\gamma\rangle}{\langle N_+ + N_-\rangle} = 0.24 \pm 0.03$$

"excess" photons with an average energy of about 180 MeV. Figure 3 (taken from Ref. 2) shows the energy spectrum of the γ rays produced in the annihilation of antiprotons stopped in a deuterium-filled bubble chamber[12)] (the solid curve in this figure corresponds to the γ spectrum from the decay $\pi^0 \to 2\gamma$). Kalogeropoulos *et al.*[26] believe that the energy spectrum of the "excess" photons indicates the possible presence of some discrete structure. A later attempt to detect discrete γ lines with a luminescent γ spectrometer[28] was unsuccessful (according to the conclusion reached by the authors, the intensity of each line in the discrete spectrum—if such lines exist—is less than 1%).[13)] In connection with these results we note that in the case in which there are bound $N\bar{N}$ quasinuclear states the theory predicts discrete γ lines which are comparatively narrow (width $\lesssim$10 MeV) in the spectrum of photons emitted in the annihilation of

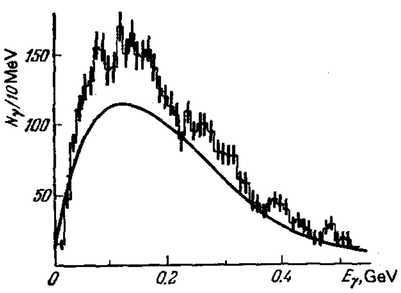


FIG. 3. Spectrum of the γ rays emitted in the annihilation of antiprotons stopped in a deuterium bubble chamber.[2,26] Solid curve—expected continuous spectrum of γ rays from the decay $\pi^0 \to 2\gamma$. The area under the curve is normalized to the average number of pions in accordance with (2.5) (see text).

11) This value was found by measuring the average photon energy, $\langle\omega_\gamma\rangle = 184 \pm 3$ MeV. The number given in the text was found from the equation $\langle N_\gamma\rangle = \langle E_0\rangle/\langle\omega_\gamma\rangle$ with $\langle E_0\rangle = 693 \pm 10$ MeV.

12) The spectrum in Fig. 3 is statistically better (by a factor of about four in terms of the number of events) than the analogous histogram in Ref. 26.

13) We are not ruling out the possibility that the negative experimental result might have been due to experimental limitations.[2,27]

$\bar{p}$ particles which are stopped in hydrogen or deuterium.[29] A theory of this problem is given in Section 7; here we simply note that these monoenergetic γ lines stem from radiative electromagnetic transitions from the s states of $\bar{p}p$ or $\bar{p}d$ atoms to $N\bar{N}$ quasinuclear bound states. The total intensity expected theoretically for the γ lines of the discrete spectrum is of the order of a few percent per annihilation event (the intensities of the individual lines should be a few tenths of a percent; the total number of lines depends on the spectrum of $N\bar{N}$ quasinuclear levels and has the value of eight for one nuclear-force version.[14)] We emphasize that the data of Refs. 25 and 26 stand alone at this point. They must be checked and refined in future experiments.[15)]

The two-particle channels ($N\bar{N} \to 2\pi, N\bar{N} \to 2K, \bar{p}p \to e^+e^-$) are of particular interest for a study of the annihilation process. The corresponding intensities are comparatively low (the probability for the annihilation $\bar{p}p \to 2\pi$, for example, is about 0.3% per antiproton stopped in hydrogen), but these channels are important because they make it possible to determine the quantum numbers of the state of the $N\bar{N}$ annihilation pair. Two-particle channels of particular interest are the $\bar{p}p$ annihilations into $2\pi^0$ and into the lepton pair e^+e^-. The process $\bar{p}p \to 2\pi^0$ can occur only for the following quantum numbers of the annihilating pair:

$$I = 0, \quad S = 1, \quad \text{with } l \text{ odd.}^{16)}$$

Here S is the total spin of the pair, and l is the orbital angular momentum of the $\bar{p}p$ relative motion. By studying the reaction $\bar{p}p \to 2\pi^0$ we can thus determine the contribution of odd-l $\bar{p}p$ states to the general two-pion annihilation channel ($2\pi^0$ or $\pi^+\pi^-$). Direct measurements of the $\bar{p}p \to 2\pi^0$ probability for the antiprotons which are "stopped" in a liquid-hydrogen target lead to an unexpectedly large value (Ref. 30):

$$\frac{(\bar{p}p \to 2\pi)_{l\text{-odd}}}{(\bar{p}p \to 2\pi)_{\text{all}}} = 0.39 \pm 0.08.$$

An analogous result has been obtained for the annihilation of a "stopped" antiproton by a proton bound in a deuteron (Ref. 31):

$$\frac{(\bar{p}p \to 2\pi)^d_{l\text{-odd}}}{(\bar{p}p \to 2\pi)^d_{\text{all}}} = 0.75 \pm 0.08.$$

It is not clear whether the difference between these two numbers is significant. What is of primary importance

14) After this paper had been written, results were published on the discrete γ spectrum accompanying the annihilation of stopped antiprotons. The Basel—Karlsruhe—Stockholm collaboration at CERN [Phys. Lett. Ser. B **72**, 415 (1978)] observed three γ lines, corresponding to energies of 183 ± 7, 216 ± 9, and 420 ± 17 MeV, with intensities ranging from a few tenths of 1% to 1% (per annihilation event). The line widths do not exceed the instrumental resolution (of the order of 10%).

15) The data published in Ref. 27 on a test of Eq. (2.3) in $\bar{p}p$ annihilation (in which case this equation may not hold, generally speaking, even in the absence of additional radiation) are insufficiently accurate: $\langle\Delta N_\gamma\rangle = 0.88 \pm 0.46$.

16) This result is easily found from the conservation of the total angular momentum, parity, isospin, G parity and the identity of the two final particles. It should be recalled here that N and $\bar{N}$ have opposite internal parities (the S state of the $N\bar{N}$ pair has a negative parity).

is what these results have in common: a high probability for $\bar{p}p$ annihilation from states with $l \neq 0$, even though we are talking about slow antiprotons.[17] In principle, this effect could be attributed to an increased probability for annihilation from the P states of the $\bar{p}p$ atom because of a quasinuclear $N\bar{N}$ state near the threshold with the corresponding quantum numbers.

Annihilation into a lepton pair, $\bar{p}p \to e^+e^-$, can also occur from the $\bar{p}p$ states with the quantum numbers of a photon, $J^{PC} = 1^-$ (for slow antiprotons, these quantum numbers correspond to the state $^3S_1$). Bassompierre *et al.*[33] have recently measured the relative probability for the annihilation $\bar{p}p \to e^+e^-$ for "stopped" antiprotons; they found

$$BR(e^+e^-) \equiv \frac{\bar{p}p \to e^+e^-}{\bar{p}p \to \text{all}} = (3.2 \pm 0.9) \cdot 10^{-7}.$$

It is easy to see that this is a very large quantity if we compare $BR(e^+e^-)$ with the analogous ratio for the annihilation $\bar{p}p \to \pi^+\pi^-$:

$$BR(\pi^+\pi^-) = (3.2 \pm 0.3) \cdot 10^{-3}.$$

We find

$$\frac{BR(e^+e^-)}{BR(\pi^+\pi^-)} = 10^{-4} \approx \alpha^2,$$

where $\alpha = \frac{1}{137}$ is the fine-structure constant. This result means that the proton form factor[18] G is approximately unity at the boundary of the physical region on the side of time-like momentum transfers q (that is, for $q^2 = 4m^2$):

$$G(q^2 = 4m^2) \approx 1.$$

Such a large value of G tells us at least that there is a strong nuclear attraction between the $\bar{p}$ and the p (which increases the particle "density" in the annihilation region). The form factor G is related to the cross section for the annihilation $\bar{p}p \to e^+e^-$ by

$$v\sigma_{e^+e^-} = \frac{\pi\alpha^2 G^2}{m^2},$$

where v is the relative velocity of the p and the $\bar{p}$. An increase in G in the limit $v \to 0$ is thus equivalent to an increase (faster than $1/v$) in the cross section for this type of annihilation. We will see below (Section 4) that a strong nuclear attraction between N and $\bar{N}$ is responsible for the large values of all annihilation cross sections and is also a factor which makes narrow quasinuclear states possible. An additional reason for the increase in the form factor in the time-like region in the limit $q^2 \to 4m^2$ can be the existence of "sub-threshold" resonances (that is, quasinuclear $N\bar{N}$ bound states with masses of about $2m$). This possibility was pointed out by Dal'karov *et al.*[33] (before experiments were carried out in the region $q^2 \approx 4m^2$). An important contribution of $N\bar{N}$ resonances to the increase in the form factor G is plausible in view of recent experimental results on the annihilation $e^+e^- \to$ hadrons. A plot of these cross sections against the energy reveals two resonant peaks, corresponding to masses of 1600 MeV (with a width of about 300 MeV)[34] and 1780 MeV (with a width of about 150 MeV).[35] There is no direct experimental proof that these resonances are quasinuclear, since the fact that they are "sub-threshold" resonances means that they cannot be detected as maxima in "formation" experiments in $\bar{p}p$ annihilation (arguments in favor of a probable quasinuclear origin for these resonances are the fact that their masses are approximately equal to $2m$ and the fact that the multipion decay channel is dominant). It should be noted here that the most promising experiments for a search for $N\bar{N}$ bound states (having masses $< 2m$) are experiments with nuclear targets, especially deuterons. Pioneering work in this field has been done by Kalogeropoulos *et al.* (see the literature cited in Refs. 2 and 15–17). In the reaction

$$\bar{p} + d \to (\bar{p}n) + p \quad \lfloor\!\to \text{pions} \tag{2.4}$$

the $\bar{p}n$ system can be formed in a state with a mass less than $2m$, since the rest of the energy can be removed by the proton which is liberated (the "spectator"). The mass M of the $(\bar{p}n)$ system is related to $\mathbf{p}$, the momentum of the incident antiproton, and $\mathbf{q}$, the momentum of the spectator proton, by

$$M = 2m + \frac{(\mathbf{p}+\mathbf{q})^2}{4m} - \frac{q^2}{2m} - \varepsilon_d, \tag{2.5}$$

where ε_d is the deuteron binding energy. It follows from the same equation that cases with $M < 2m$ are possible and also that the same mass can be found for different relative velocities of the components of the annihilating $\bar{p}$ and n pair. Indeed, if the mechanism for reaction (2.4) is simply a nuclear pickup reaction, then $(\mathbf{p}+\mathbf{q})/m = \mathbf{v}$ is the relative velocity of the $\bar{p}$ and n, and $(\mathbf{p}+\mathbf{q})/2 = \mathbf{k}$ is their relative momentum. The possibility of varying k, holding M constant, makes it possible to avoid the appearance of the factor $(kR)^{2l}$ (which suppresses the cross section) for near-threshold, large-spin resonances. Here l is the orbital angular momentum of the relative $N\bar{N}$ motion in the resonant state. By choosing a suitable range of momenta for the spectator, $\mathbf{q}$, and the initial proton, $\mathbf{p}$, it is possible to arrange a situation such that $kR \gtrsim 1$, even if $\sqrt{m|Q|}R < 1$, where $Q = M - 2m$. Figure 4, taken from a paper by Kalogeropoulos *et al.*,[3] illustrates the practical implementation of this idea. The three curves here correspond to the same range of the mass M (or of the mass defect Q) but to different values of the relative momentum $\mathbf{k}$ of the annihilation $\bar{p}$ and n. The curves in Figs. 4b and 4c show two maxima, which correspond to $(\bar{p}n)$ resonances with masses of 1897 and 1932 MeV. The 1932-MeV resonance is missing from the curve in Fig. 4a, which corresponds to smaller values of k. Kalogeropoulos *et al.* conclude from these results that the orbital angular momentum l should differ from zero for this resonance. In Ref. 3 they found the

---

[17] The experiments of Refs. 30 and 31 were carried out by very different procedures, so that the terms "stopped" and "slow" antiprotons have slightly different quantitive meanings. In both cases, however, the $\bar{p}$ momenta (in the center-of-mass system) under discussion are smaller than or of the order of 150 MeV/c. The measurements in Refs. 30 and 31 were the only such measurements until very recently; new data, which show a high probability for $\bar{p}p$ annihilation from states with $l \neq 0$, appeared while the present paper was in press.[68]

[18] We recall that in the case $q^2 = 4m^2$ the electric and magnetic form factors of the proton are equal.

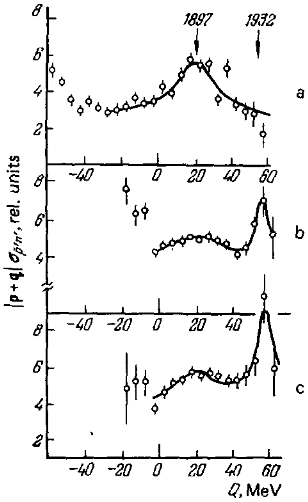


FIG. 4. The reaction $\bar{p}d \to (\bar{p}n) + p$. Distribution with respect to the mass defect Q for low values of the relative momentum of the $\bar{p}$ and n, corresponding to various values of the spectator momentum q. The curve in part a corresponds to the smallest values of the relative momentum (the spectator momentum is >200 MeV/c). The curves in parts b and c correspond to spectator momenta of 100 and 150 MeV/c, respectively (the $\bar{p}$ momentum is $\geqslant$250 MeV/c).

first indication of the resonance[19] $N\bar{N}$(1897), which lies extremely close to the threshold ($Q = 17$ MeV, width of about 20 MeV) and has an isospin $I = 1$. In addition to the near-threshold resonances, the experiments with deuterium targets reveal evidence of $\bar{p}n$ bound states with masses of 1794 MeV (Ref. 36) and 1873 MeV (Ref. 37). The respective widths of these resonances[20] are 7 and 10 MeV.

Experiments on reactions such as (2.4) involving nuclei and on the γ spectra emitted in $\bar{p}p$ and $\bar{p}d$ annihilation represent direct methods for detecting and identifying quasinuclear $N\bar{N}$ bound states.

Let us summarize the "resonance" part of our review of the experimental data on the $N\bar{N}$ interaction. The main experimental results currently available are several bound and resonant $N\bar{N}$ states with the following masses (and widths): 1794 (7), 1873 (10), 1900 (20), 1940 (<10), 2020 ($\leqslant$20), 2200 ($\leqslant$20), 2150 (95), 2335 (110). We should perhaps add the resonances 1600 ($\approx$300) and 1780 (150), which are observed in the annihilation $e^+e^- \to$ hadrons. The resonance $N\bar{N}$ (1940) is on firmer ground than the others (the data on the quantum numbers of these $N\bar{N}$ states are neither complete nor reliable). Another interesting fact is the large value of the proton form factor $G(q^2 = 4m^2)$, which demonstrates at the very least that there is a strong nuclear attraction in the $N\bar{N}$ system. Finally, there is evidence of discrete lines in the γ spectrum emitted in the annihilation of antiprotons stopped in hydrogen and deuterium.

In view of these results, it is definitely time for a theoretical discussion of the physical aspects of the problem of $N\bar{N}$ quasinuclear states.

3. ANNIHILATION RADIUS

The characteristic distance for $N\bar{N}$ annihilation is of the order of the Compton wavelength of the nucleon, $1/m$, as can be shown by several methods. The most rigorous formal approach[7] is to study the nearest singularity on the annihilation-scattering diagrams (Fig. 5a) with respect to the variable $t = q^2$ (q is the 4-momentum transfer). Why is it that we must study annihilation scattering, rather than the $N\bar{N}$ annihilation into bosons itself, in order to determine the annihilation radius? The answer is that the quantity canonical conjugate to the distance between the interacting particles is the change in the momentum of any one of the particles in the course of the interaction, in other words, the momentum transferred in scattering. The amplitude for annihilation into bosons, on the other hand, does not contain quantities of this type. In this connection we emphasize that the square root of the annihilation cross section (a quantity having the dimensions of length) does not in general have the physical meaning of an annihilation radius (the cross section can reach the unitary limit, i.e., can correspond to the wavelength of the slow annihilating particles, since the distance of approach of the particles to each other for annihilation can be very small). The minimum mass which is transferred through the t channel in the diagram in Fig. 5a is $2m$. The annihilation radius corresponding to the diagram in Fig. 5a (for any number of horizontal boson lines) is thus $1/2m$ (the closest singularity along t in this case is $t_0 = 4m^2$). The result remains the same if we supplement the two fermions in the t channel with bosons; the only results are to move the singularity farther away along t and correspondingly to reduce the annihilation radius r_a. It thus follows from general theory that

$$r_a \leqslant \frac{1}{2m}. \tag{3.1}$$

This result can also be found by means of uncertainty relations, in the manner used to derive the relationship

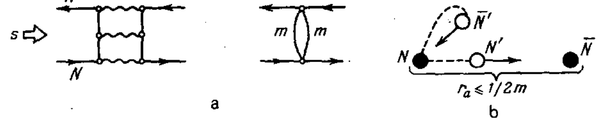


FIG. 5. a) Feynman diagram of the annihilation scattering [the diagram at the right, which is "squeezed" along the boson lines, corresponds to the nearest singularity along t ($t_0 = 4m^2$) and thus gives the minimum radius of the annihilation interaction]; b) diagram illustrating the uncertainty relation which limits the annihilation radius [an $N'\bar{N}'$ virtual pair is produced at the position of nucleon N over a time $\tau \leqslant 1/2m$ (unshaded circles); the $\bar{N}$ annihilates with the N', and the N' annihilates with the $\bar{N}$, if the $\bar{N}$ is no farther away than $\tau c = 1/2m$].

---

[19] The authors identify the $N\bar{N}$ (1932) resonance observed in that work with the $N\bar{N}$ (1940) resonance, mentioned above, in the cross sections for the $\bar{p}p$ interaction (see the other papers in Ref. 3). If this identification is correct, the isospin of this resonance should be 1.

[20] The results of Ref. 36 were the first experimental indication of the possible existence of $N\bar{N}$ bound states. The results of that work were discussed theoretically in Ref. 38. We note that the experiments of Refs. 36 and 37 are unique, and as yet there is no other information on the $N\bar{N}$ bound states detected there.

between the meson mass and the range of the forces due to meson exchange. Since the question of the annihilation radius is heuristically extremely important for the entire problem under consideration, it is useful to understand it from various points of view. We will accordingly explain the somewhat formal result in (3.1) on the basis of physical considerations. We first note that the N and $\bar{N}$ should disappear in the annihilation for otherwise the spin, the charge, and other quantum numbers would not be conserved, as they should be in all transitions. It would thus appear that annihilation could occur only if N and $\bar{N}$ were at the same point in space (since simultaneous events which occur at different points in one coordinate system are not simultaneous in another coordinate system). However, as a consequence of the energy-time uncertainty relation, at a point where one of the particles, say N, is located, another pair, $N'\bar{N}'$, can be created for a time $\tau \lesssim 1/2m$. The N and $\bar{N}'$ located at the same point can be annihilated simultaneously. If the antinucleon $\bar{N}$ (from the original pair $N\bar{N}$) is separated from this point by a distance

$$r_a \leqslant \tau c = \frac{1}{2m}$$

(here c is the speed of light), the nucleon N' may (during its lifetime τ) reach the remaining antinucleon $\bar{N}$ and be annihilated with it, thereby completing the overall annihilation process. Figure 5b illustrates the situation clearly. We see that the inequality (3.1) for the annihilation radius is a consequence of fundamental physical arguments.

4. ANNIHILATION AND QUASINUCLEAR LEVELS

In this section we will study the effect of annihilation on a quasinuclear level of the $N\bar{N}$ system. We start from the assumption that if we ignore annihilation the nuclear interaction between N and $\bar{N}$ is strong enough to form $N\bar{N}$ bound states. For the moment we are not interested in the details of this interaction, the particular nuclear potential, the configuration of the discrete spectrum, or the quantum numbers. The only property of these quasinuclear states of which we will make use here is the fact that they are nonrelativistic, that is, the fact that the average distance between particles is comparatively large, $R \gg r_a$, and the mass defect is small, $|Q| \ll 2m$.

Our problem is essentially to answer two questions. One deals with the level widths: if the annihilation cross section is large, will not the levels accordingly be so wide that the assertion of their formal existence becomes physically meaningless? The second question deals with the effect of annihilation on the level positions: will not these levels shift so much that the entire discrete spectrum due to the nuclear forces will change radically, or will these levels cease to exist at all because they are "expelled" into the continuum because of absorption? Actually, these questions dealing with the level widths and the annihilation shifts of the levels are interrelated. Both quantities are determined by the amplitude for annihilation scattering, which was mentioned earlier. Jumping ahead slightly, we can say that the annihilation shifts and widths are comparable in magnitude. We will see that, despite the widespread opinion that there is necessarily a level "expulsion" because of annihilation, the sign of the shift can also be negative; that is, the $N\bar{N}$ binding energy can in fact be increased, rather than decreased, due to annihilation into bosons. Regardless of the sign of the annihilation shift, however, it is meaningful to speak in terms of an $N\bar{N}$ bound state only if this shift is small. Otherwise we would not be dealing with the level of a system consisting of N and $\bar{N}$ but simply with a boson resonance in which the $N\bar{N}$ channel is not distinguished in any manner and whose physical nature has no direct bearing on the $N\bar{N}$ nuclear interaction. How large can the level widths and the annihilation shifts be? As a measure we can adopt the distance between levels having identical quantum numbers. If the spectrum of quasinuclear levels is not to be greatly distorted by annihilation, the widths and shifts must be smaller than these distances. In the absence of degeneracy, the distances between levels with identical and different quantum numbers are equal in order of magnitude. As a natural scale we can thus adopt the distance between adjacent levels on the same Regge trajectory. Using the familiar equation for the derivative dl/dE (see, for example, Ref. 39), we find

$$\delta E = \frac{2l+1}{m} \left\langle \frac{1}{r^2} \right\rangle.$$

With $r \approx 1$ F we have $l \approx 1$ and thus

$$\delta E \approx 100 \text{ MeV}.$$

The annihilation widths and shifts must thus be smaller than (or, at most, of the order of) 100 MeV. We immediately note that in the $N\bar{N}$ quasinuclear system there are only one or two pairs of levels which have identical quantum numbers, so the permissible values of the widths are generally set by the experimental capabilities for observing broad resonances, rather than by the theory. We will see, however, that the annihilation widths and shifts can actually be much smaller than the permissible upper limits stated above.

Let us now formulate the problem of the annihilation widths and shifts.

The $N\bar{N}$ interaction includes processes involving the exchange of light bosons and the annihilation diagrams. Boson exchange in the t channel corresponds to the OBEP model. In particular, the sum of ladder diagrams of this type (Fig. 6a) gives the scattering amplitude f of nonrelativistic $N\bar{N}$ in this model. Figures 5 and 6b show examples of diagrams of the annihilation type.

In general, the energy poles E_λ in the "potential" amplitude f lead to poles W_λ in the total scattering amplitude (in which annihilation effects are incorporated).

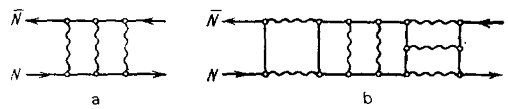


FIG. 6. a) Ladder diagram of the boson exchange [the sum of an infinite series of such diagrams gives the scattering amplitude in the model of the one-boson exchange potential (OBEP model)]; b) example of the annihilation diagrams [the sum of an infinite series of such diagrams (including the simple one in Fig. 4) gives the amplitude for annihilation scattering f_a].

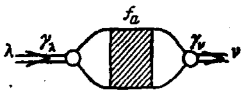


FIG. 7. Diagram corresponding to the mass operator $\hat{M}(E)$. The unshaded circle is the vertex part of the $N\bar{N}$ quasinuclear state; the hatched rectangle is the amplitude for annihilation scattering, f_a.

Our goal is to find the W_λ.

We consider the mass operator $\hat{M}$, defined by

$$M_{\lambda\nu}(E) = -\frac{1}{2^4\pi^5 m}\int \varphi_\nu(\mathbf{k})\, f_a(k, k', E)\, \varphi_\lambda(\mathbf{k})\, d\mathbf{k}\, d\mathbf{k}', \tag{4.1}$$

where E is the kinetic energy of the N and $\bar{N}$ in the center-of-mass system of these particles, m is the nucleon mass, φ_λ and φ_ν are the wave functions of the $N\bar{N}$ system in the bound states due to the potential interaction, and f_a is the amplitude for annihilation scattering off the energy shell. This quantity does not incorporate boson exchange in the initial and final states, and is the sum of several annihilation diagrams, shown in Figs. 5 and 6b. The wave functions in (4.1) are normalized so that

$$\varphi(\mathbf{k}) = \int \psi(\mathbf{r})\, e^{-i\mathbf{kr}} d\mathbf{r}, \qquad \int \psi^+(\mathbf{r})\, \psi(\mathbf{r})\, d\mathbf{r} = 1.$$

Within a factor $i/(2\pi)^4$, the mass operator in (4.1) corresponds to the diagram in Fig. 7, where the hatched triangle is the amplitude $(4\pi/m)f_a$. The vertex parts $\gamma_\lambda(\mathbf{k})$ of this diagram are related to $\varphi(\mathbf{k})$ by

$$\gamma_\lambda(\mathbf{k}) = \frac{k^2 - mE_\lambda}{m}\varphi_\lambda(\mathbf{k}), \quad E_\lambda < 0.$$

We introduce the Green's matrix $\hat{D}(E)$, which satisfies the equation

$$\hat{D} = \hat{d} + \hat{d}\hat{M}\hat{D}, \tag{4.2}$$

where

$$d_{\lambda\nu}(E) = d_\lambda(E)\,\delta_{\lambda\nu}, \quad d_\lambda(E) = \frac{1}{E - E_\lambda}. \tag{4.3}$$

Equations (4.2) and (4.3) are written for the case in which the amplitude for annihilation scattering, f_a, has no poles at all or its poles are far from E_λ. To find W_λ, we should solve Eq. (4.2) and diagonalize the matrix $\hat{D}$. The poles of its eigenvalues will be the quantities W_λ which we are seeking.

We also note that, by virtue of the invariance of f_a under rotations, reflections, and isorotations, the only nonvanishing elements of $\hat{M}$ are those off-diagonal elements which correspond to states λ, μ having identical quantum numbers (spin, total angular momentum, parity, and isospin). In the $N\bar{N}$ system, such states arise as a result of tensor forces (two pairs of levels, $^3S_1 + {}^3D_1$ and $^3P_2 + {}^3F_2$). In principle, the mass operator could mix states with different numbers of radial nodes, but in the $N\bar{N}$ system nearly all bound states are nodeless (the abundance of levels—more than ten—is due to spin-orbit coupling rather than to radial excitations). In the case in which we are interested, the matrices $\hat{M}$ and $\hat{D}$ thus contain only a few 2×2 off-diagonal "boxes," which are arranged along the main diagonal; all other off-diagonal elements of $\hat{D}$ vanish. For the diagonal elements outside these boxes, that is, for states having unique quantum numbers, Eq. (4.2) simplifies, and we find

$$D_\lambda = d_\lambda + d_\lambda M_{\lambda\lambda} D_\lambda. \tag{4.4}$$

The solution of this equation is the function

$$D_\lambda(E) = \frac{d_\lambda(E)}{1 - M_{\lambda\lambda}(E)\, d_\lambda(E)}. \tag{4.5}$$

The zeros of the denominator in (4.5) are the poles of $D_\lambda(E)$. Substituting $D_\lambda(E)$ from (4.3) into (4.5), we find

$$W_\lambda - E_\lambda = M_{\lambda\lambda}(W_\lambda). \tag{4.6}$$

The levels W_λ are found immediately by solving Eq. (4.6). The level shift due to annihilation effects, $\mathrm{Re}W_\lambda - E_\lambda$, is simply equal to the real part of the diagonal element of the mass operator at the pole. In order to estimate this shift, we examine (4.1) in the case $\lambda = \mu$, and assuming for simplicity that the state λ is an S state we initially assume that f_a has no poles of its own along E. Then the only dimensionless parameter governing the rate of change of f_a as a function of $\mathbf{k}$ and $\mathbf{k}'$ is the nearest singularity of this amplitude along the transferred momentum. For annihilation diagrams (Fig. 5) the singularity is at the point $t_0 = 4m^2$. The interval in which there is a significant change in f_a as a function of the momenta must be of order m in size. The wave function $\varphi_\lambda(\mathbf{k})$, on the other hand, which describes the quasinuclear bound state of the $N\bar{N}$ system, changes significantly over an interval of size R^{-1}, where R is the average distance between particles. According to our condition (the state λ is nonrelativistic),

$$R^{-1} \ll m.$$

Actually, for quasinuclear states in the OBEP model[1] the values of R are in the range 1–1.5 F, so that

$$R^{-1} = 130\text{–}200 \ \mathrm{MeV}/c.$$

The small parameter $1/2mR$ is thus of the order of 0.1 for our problem. Using this circumstance, we can extract f_a from the integrand in (4.1), since it varies slowly in comparison with $\varphi_\lambda(\mathbf{k})$. Then noting that

$$\int \varphi_\lambda(\mathbf{k})\, d\mathbf{k} = (2\pi)^3\, \Psi_\lambda(0),$$

we find

$$M_{\lambda\lambda}(E) = -\frac{4\pi}{m} f_a(k_0, k_0', E)\, |\Psi_\lambda(0)|^2, \tag{4.7}$$

where k_0 and k_0' are certain effective values of k and k', which have values of the order of R^{-1}. Equation (4.7) is a familiar equation, used widely to calculate the level shifts in hadronic atoms (see, for example, Ref. 40a; in this case the role of f_a is played by the amplitude for hadron–nucleus scattering). We emphasize that Eq. (4.7) is not a result of the use of perturbation theory in the amplitude for annihilation scattering, f_a, or, especially in the constants of the fermion–boson interaction. Furthermore, this equation is independent of any potential model for f_a. The condition under which this equation is applicable is that the variant of f_a as a function of k and k' be comparatively slow. This is the only distinction between (4.7) and the exact equation in (4.1). To estimate the level shift we consider the first approximation of Eq. (4.6); that is, we replace E by E_λ in (4.7). In order of magnitude, we should have

$$|\psi(0)|^2 \approx \frac{3}{4\pi R^3}, \tag{4.8}$$

$$|\mathrm{Re}\, f_a| \lesssim \frac{1}{m}. \tag{4.9}$$

Substituting these estimates into (4.7) we find

$$|\mathrm{Re}\, M_{\lambda\lambda}(E_\lambda)| \leqslant \frac{3}{2}\frac{1}{m^2R^3} = 8-25 \text{ MeV}.$$

According to this estimate, the probable value of the annihilation shift of the quasinuclear S level is about 10 MeV. Such a shift would have little effect on the overall spectrum, since the binding energies of the quasinuclear S levels in the $N\bar{N}$ system are on the order of 100 MeV. The annihilation shifts of levels with nonvanishing l should be even smaller. For this case we find from (4.1)

$$|\mathrm{Re}\, M_{\lambda\lambda}(E_\lambda)| \approx \frac{R^{-1}}{(mR)^{2l+2}},$$

so that, for $l=1$, for example,

$$|\mathrm{Re}\, M_{\lambda\lambda}(E_\lambda)| \approx 0.2-0.6 \text{ MeV}.$$

In other words, the annihilation shift of the p level is less than or of the order of 1 MeV, while for most P levels we have, in order of magnitude, $|E_\lambda| \gtrsim 10-100$ MeV. The annihilation level shift is due to the real part of the amplitude f_a, so it is nonvanishing only if (and only to the extent that) the annihilation scattering differs from scattering by an absolutely black sphere (for which the scattering amplitude is purely imaginary, so that the level shift vanishes).

Let us estimate the possible value of the annihilation width. Here we have

$$-\mathrm{Im}\, W_\lambda = \frac{\Gamma_{\lambda a}}{2} = \frac{4\pi}{m}\,\mathrm{Im}\, f_a(E_\lambda)\,|\psi(0)|^2. \qquad (4.10)$$

If $E_\lambda \leqslant 0$, then $\mathrm{Im} f_a$ can be expressed in terms of the annihilation cross section (rather than the total cross section). We assume $\mathrm{Im} f_a(E_\lambda) = \mathrm{Im} f_a(0)$ and find

$$\frac{4\pi}{m}\,\mathrm{Im}\, f_a(E_\lambda) = \frac{1}{2}(v\bar{\sigma}_a)_{v\to 0}, \qquad (4.11)$$

where v is the relative velocity of the particles, and $\bar{\sigma}_a$ is the annihilation cross section. Here the superior bar is used to distinguish this cross section from the observed cross section (which is strongly affected by t-channel boson exchange; see Section 4). Substitution of (4.11) into (4.10) yields

$$\Gamma_{\lambda a} \subset |\psi(0)|^2 (v\bar{\sigma}_a)_v \to 0. \qquad (4.12)$$

Guided by the same considerations regarding the radius of the annihilation interaction as in the estimate of the level shift, we naturally assume

$$\bar{\sigma}_a = \frac{\pi}{m^2 v}. \qquad (4.13)$$

A factor of $1/v$ should be introduced because the nonrelativistic N and $\bar{N}$ should be treated as slow particles in this case (the wavelength is much larger than the radius of the interaction region). Substituting (4.8) and (4.13) into (4.12) we find

$$\Gamma_{\lambda a} \approx \frac{3}{4}\frac{R^{-1}}{m^2R^2} \approx 2-7 \text{ MeV}, \qquad (4.14)$$

that is, a quantity of the same order of magnitude as for the annihilation level shift. In Refs. 11–17, $\bar{\sigma}_a$ was replaced by the observed annihilation cross section (of the order of 45 mb/v), with the result that the values of Γ_a for the S states turned out to be 20–30 times higher than the estimate in (4.14). As mentioned in those papers, the motivation for this approach was to find an upper confidence limit on Γ_a, clearly spanning the probable uncertainties in the estimates of $\bar{\sigma}_a$ and in the values of $|\psi(0)|^2$ (for the same purpose, the centrifugal potential was cut off at small distances in the calculation of the annihilation widths of the $l \neq 0$ levels; this approach increased the widths of these states by one or two orders of magnitude). All these estimates, which are clearly on the high side, lead to values of Γ_a which are no greater in order of magnitude than the widths of the well-known boson resonances. This result is evidence in favor of the existence of $N\bar{N}$ quasinuclear states and an argument for an experimental search for these states.

Using (4.7), we can express the level shift ΔE_1 in terms of the width $\Gamma_{\lambda a}$ without resorting to any additional assumptions:

$$\Delta E_\lambda = \mathrm{Re}(W_\lambda - E_\lambda) = \alpha\frac{\Gamma_{\lambda a}}{2}, \quad \alpha = \frac{\mathrm{Re}\, f_a}{\mathrm{Im}\, f_a}. \qquad (4.15)$$

Since in hadron–hadron scattering with many inelastic channels the inequality

$$\alpha < 1 \qquad (4.16)$$

generally holds, we should expect on the basis of (4.15) that

$$\Delta E_\lambda \leqslant \Gamma.$$

Let us consider an example in which ΔE_λ and $\Gamma_{\lambda a}$ are calculated explicitly. Specifically, we assume that f_a has, along E, the pole $E_a - i(\Gamma_a/2)$, which lies far from E_λ ($\sqrt{m|E_a|} \gg \sqrt{m|E_\lambda|}, R^{-1}$). Then we have

$$f_a(k, k', E) = -\frac{1}{2\sqrt{kk'}}\,\frac{\sqrt{\Gamma_{ae}(k)\,\Gamma_{ae}(k')}}{E - E_a + i(\Gamma_a/2)}. \qquad (4.17)$$

The variation of $\Gamma_{ae}(k)$ with k is governed by the radius r_a for the state corresponding to the pole. According to these arguments, we should assume $r_a \gg R$, since either $r_a \approx 1/m$ or $r_a \approx (m|E_a|)^{-1/2}$. The effective values of k and k' in the integral in (4.1) are R^{-1}, so that

$$kr_a \approx \frac{r_a}{R} \ll 1.$$

We can thus write

$$\Gamma_{ae}(k) = 2\gamma_{ae} k r_a \qquad (4.18)$$

(we recall that we are considering the S state). Substituting (4.18) into (4.17), and then substituting (4.17) into (4.7), we find

$$M_{\lambda\lambda}(E) = \frac{4\pi}{m}\,\frac{r_a\gamma_{ae}}{E - E_a + i(\Gamma_a/2)}\,|\psi(0)|^2. \qquad (4.19)$$

Substituting (4.19) into (4.6), we find a quadratic equation for E. For simplicity, we solve this equation in the case

$$\frac{16\pi}{m}\, r_a\gamma_{ae}\,|\psi(0)|^2 \ll \sqrt{(E_\lambda - E_a)^2 + \frac{\Gamma_a^2}{4}}.$$

Then

$$\Delta E_\lambda = \frac{8\pi r_a\gamma_{ae}|\psi(0)|^2}{m}\,\frac{E_\lambda - E_a}{(E_\lambda - E_a)^2 + (\Gamma_a^2/4)}, \qquad (4.20)$$

$$\Gamma_{\lambda a} = \frac{8\pi r_a\gamma_{ae}|\psi(0)|^2}{m}\,\frac{\Gamma_a}{(E_\lambda - E_a)^2 + (\Gamma_a^2/4)}. \qquad (4.21)$$

The shift ΔE_λ given by Eq. (4.20) is positive (since $E_\lambda > E_a$); that is, the level E_λ is "expelled." The annihilation width Γ_a, however, reduces rather than increases the shift. If we had used the optical potential with a nonvanishing imaginary part to take the inelastic processes into account, we would have found the opposite effect.[41] It is seen from (4.20) that the level shift due to the short-range forces (in this case, annihilation forces) depends not so much on the annihilation intensity (which completely governs the width $\Gamma_{\lambda a}$) as on the presence of poles in the amplitude f_a and the positions of these poles. Making use of this circumstance (setting $\Gamma_a=0$), we can use the potential model for f_a to study the shift ΔE_λ in a case not covered by Eq. (4.6): that in which f_a has poles E_a near E_λ. An analogous problem was studied in Ref. 42 in connection with the problem of the shift of the Coulomb levels of a $\bar{p}p$ atom due to the $N\bar{N}$ interaction.[21] The potential model for f_a consists of identifying this quantity with the scattering amplitude in the separable short-range potential with effective radius r_a:

$$V(\mathbf{r}_1, \mathbf{r}_2) = G\xi(r_1)\,\xi(r_2),$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the interacting particles, and G is the interaction constant. We also assume that the levels E_λ are produced by some potential well and that the linear dimensions R of the states λ are much larger than r_a. The shifted levels can be found from the equation

$$\sum_\lambda d_\lambda(E)\,|\langle\xi|\psi_\lambda\rangle|^2 + \int \frac{|\langle\xi|\psi_k\rangle|^2\,dk}{E-(k^2/m)} = \frac{1}{G}, \tag{4.22}$$

where $\psi_k(r)$ are the wave functions of the continuum and

$$\langle\xi|\psi\rangle = \int dr\,\xi(r)\,\psi(r), \quad \int \xi^2(r)\,dr = 1. \tag{4.23}$$

In order of magnitude,

$$|\langle\xi|\psi_\lambda\rangle|^2 \approx \left(\frac{r_a}{R}\right)^3,$$

and the second term in (4.22) can be replaced by $-mr_a^2$. As a result, Eq. (4.22) can be rewritten approximately as

$$\sum_\lambda d_\lambda(E) = \left(\frac{R}{r_a}\right)^3\left(\frac{1}{G}+\frac{1}{G_c}\right), \quad G_c \approx \frac{1}{mr_a^2}. \tag{4.24}$$

If there is only a single S level E_λ, we find from (4.24)

$$\Delta E_\lambda = \frac{r_a}{R}\,\frac{G}{G+G_c}\,\frac{1}{mR^2}. \tag{4.25}$$

We thus see that the shift ΔE_λ can be large near the critical value $G=-G_c$. Because of the small coefficient r_a/R, however, the "dangerous" region of G is quite narrow: even at $G+G_c=(r_a/R)G$, the shift is of the order of $1/mR^2 \approx |E_\lambda|$, and this shift becomes small again as G is increased further. Figure 8 shows the motion of the level in a square well as a function of the depth of the short-range potential (here we have a ratio $r_a/R=0.1$, and the depth of the "wide" well is chosen such that the 1S level exists in the well, while the 2S level is "at the threshold of appearance"; that is, $E_{2S}\approx 0$).[22] This behavior of the shift ΔE_λ is qualitatively independent of the particular nature of the potentials (long-range or short-range). The entire picture[23] is governed by the small parameter r_a/R. The existence of this small parameter also determines the stability of the spectrum of quasinuclear levels of the $N\bar{N}$ system, where "stability" means the independence of this spectrum from the details of the annihilation interaction. In particular, the quasinuclear states are also stable with respect to mixing with very relativistic mesons which are strongly coupled with the NN channel. If mesons of this type could exist (this seems improbable), their linear dimensions would have to be of the order of $1/m$ or smaller. Under this condition, as was just explained, there can be a radical reorganization of the quasinuclear spectrum if the interaction constant (which is responsible for the appearance of the relativistic state) falls in a narrow interval of "dangerous" values. The latter situation is almost as difficult to imagine as the complete destruction of the Coulomb spectrum of the hadronic atom because the depth of the effective hadron–nucleus potential happens to be equal to some critical value.

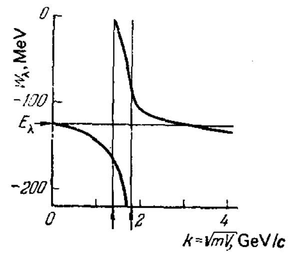


FIG. 8. Motion of the levels in a square well V of radius R as a function of the depth of the short-range potential U of radius r_a. $V=200$ MeV, $R=2$ F, $r_a/R=0.1$. Plotted along the ordinate is the level position W_λ (E_λ is the unperturbed level, corresponding to $U=0$). The vertical lines define the region in which there is a complete change in the structure of the spectrum ($\delta U/U\approx r_a/R$). This change in structure occurs upon the appearance of an eigenlevel in the short-range potential. Outside the vertical corridor the spectrum is again spproximately the same as the unperturbed spectrum (the position of the original level, which moves far off downward, is assumed by a level with the same quantum numbers from the continuum).

In studying the question of the annihilation shifts and widths, we have not used any specific dynamical model for the amplitude for annihilation scattering, f_a, or for the Hamiltonian of the $N\bar{N}$ interaction. In this connection we emphasize that the optical model should not be used (as it is in Ref. 41) to calculate the shifts and widths of levels of the discrete spectrum, because in the optical model the annihilating particles disappear "irreversibly," thus dropping out of the interaction which is responsible for the appearance of states of the discrete spectrum. This occurs because the Hamiltonian of the optical model is not Hermitian and is not T-invariant (time is not reversible). In a correct theory, a "stronger" annihilation would be accompanied by a larger amplitude for reannihilation—the inverse process—and the question of the level shift would be decided by the coupling between the nucleon–antinucleon and boson channels. This coupling is unitary in the sense that the matrix which diagonalizes the Hermitian Hamiltonian is unitary. A calculation of the annihilation shifts thus leads us to the coupled-channel problem. Since there are many annihilation channels in $N\bar{N}$ annihilation, and the particles in the boson channels are relativistic, the problem is extremely complicated. It would be completely unrealistic to try to solve this problem at present. It is nevertheless useful with the aid of simple heuristic examples to compare the results found through a solution of the coupled-channel problem with the levels calculated for the discrete spectrum on the basis of the

[21] See Section 7 below for more details on the shift of the $p\bar{p}$ atomic levels. The same problem was studied in Ref. 40 for other hadronic atoms.

[22] The curve in Fig. 8 was calculated by V. E. Markushin.

[23] An analogous behavior of the shift was found for the nS Coulomb levels near the critical value of the nuclear charge (corresponding to the coincidence of the 1S level with the boundary of the lower continuum).[43]

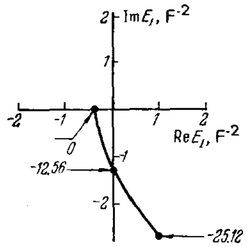


FIG. 9. Trajectory of the $N\bar{N}$ quasinuclear level for the separable complex potential of the optical model (Myhrer and Thomas[41]). The arrows show the values of the constant (in reciprocal fermis) which determines the magnitude of the imaginary part of the potential. In the absence of annihilation, the level energy is 9.1 MeV.

optical-model Hamiltonian. As mentioned above, such calculations were carried out in Ref. 41. A common result of those papers is that the annihilation necessarily "expels" the level. Figure 9, taken from the paper by Myhrer and Thomas,[41] illustrates the situation. It shows the level position in a complex separable potential as a function of the magnitude of the imaginary part of the potential. We see that as the imaginary part of the potential increases (that is, as the annihilation intensity increases) the level width increases, while the level itself (the real part of the energy) is "expelled": it moves monotonically in the positive direction along the abscissa. This is a completely natural result within the framework of the optical model, and it has an obvious physical meaning: the ejection of particles from the $N\bar{N}$ channel with strong attraction weakens the effective $N\bar{N}$ interaction and thereby reduces the binding energy. A variation of the shape of the optical potential (in particular, a variation of the radii of its real and imaginary parts) causes no qualitative change in the curve in Fig. 9, although there can be a quantitative change: the increase in the width and the postive shift of the level can be slowed or accelerated.[24)]

Now, following Ref. 44, we consider the problem of the annihilation shift and the level width in the nonrelativistic model of two coupled channels. We assume that each of the channels is a two-particle channel. Channel l corresponds to two noninteracting "light" particles which have the same mass μ (an analog of the boson annihilation channel). Channel h contains a "heavy" particle and a "heavy" antiparticle, of mass $m>\mu$ (the analog of the $N\bar{N}$ channel). We assume that the particles in both channels are spin-zero, nonrelativistic particles (this latter condition holds in real $N\bar{N}$ annihilation, for example, in the processes $N\bar{N}\to\rho^+\rho^-$ and $N\bar{N}\to 2\omega$). The Hamiltonian H in the equation

$$\hat{H}\Psi = E\Psi \tag{4.26}$$

is a 2×2 Hermitian matrix,

$$\hat{H}=\begin{pmatrix}\hat{H}_l & \hat{H}_{lh}\\ \hat{H}_{hl} & \hat{H}_h\end{pmatrix}. \tag{4.27}$$

The diagonal elements in (4.27), H_l and H_h, are the Hamiltonians of the individual channels:

$$\hat{H}_l=\frac{\mathbf{k}^2}{\mu}-(2m-2\mu),\quad \hat{H}_h=\frac{\mathbf{p}^2}{m}+V(\mathbf{r}). \tag{4.28}$$

The constant in $\hat{H}_l$ means that the origin for the energy scale is the point $2m$. The off-diagonal elements,

$$\hat{H}_{lh}=\hat{H}_{hl} \tag{4.29}$$

couple the channels with each other. The symmetry in (4.29) is a consequence of the Hermitian nature of H and the T-invariance of Eq. (4.26); this T-invariance requires that the Hamiltonian of a system of spin-zero particles be real. The wave function Ψ is a two-component wave function:

$$\Psi=\begin{pmatrix}\Psi_l\\ \Psi_h\end{pmatrix},$$

where $\Psi_l(\mathbf{r})$ and $\Psi_h(\mathbf{r})$ are the wave functions for channels l and h. Equation (4.26) can be analyzed comparatively easily if $\hat{H}_{lh}$ is assumed to be a nonlocal separable potential, by analogy with the discussion above:

$$\hat{H}_{lh}\Psi(r)=\int H_{hl}(\mathbf{r},\mathbf{r}')\Psi(\mathbf{r}')\,d\mathbf{r}',\quad H_{hl}(r,r')=g_a\sqrt{\mu m}\,\xi(r)\xi(r'), \tag{4.30}$$

$$\xi(r)=\sqrt{\frac{2}{r_a}}\frac{e^{-r/r_a}}{r}. \tag{4.31}$$

We now denote the wave functions of the discrete spectrum of Hamiltonian $\hat{H}_h$ by Ψ_λ, and we write

$$\int\xi(r)\Psi_\lambda(\mathbf{r})\,d\mathbf{r}=\left(\frac{r_a}{R}\right)^3\alpha_\lambda, \tag{4.32}$$

where R is the radius of the state λ. The quantities α_λ should be of the order of unity. Proceeding as above in the calculation of the shift in a single-channel problem with an additional short-range potential, we easily find

$$\sum_\lambda\frac{\alpha_\lambda}{E-E_\lambda}=\left(\frac{R}{r_a}\right)^3\left\{-\frac{[1-ik(E)r_a]^2}{g_a^2\mu^2mr_a^2}+\frac{mr_a^2}{(1-i\varkappa(E)r_a)^2}\right\}; \tag{4.33}$$

here

$$k(E)=\sqrt{\mu(E+2m-2\mu)},\quad \varkappa(E)=\sqrt{mE}. \tag{4.34}$$

The square roots in (4.34) are defined in such a manner that their imaginary parts are positive. The second term on the right side of Eq. (4.33) is found from the integral over the states of the continuum, which is analogous to the sum over λ on the left side of the equation. The functions of the continuum are replaced by plane waves; this is a legitimate procedure if r_a is small ($|V(r_a)|\ll 1/mr_a^2$) and if the behavior of $V(r)$ in the limit $r\to 0$ is not extremely singular.

In the case of weak coupling between channels, with

$$g_a^2\ll\frac{1}{\mu^2m^2r_a^4}, \tag{4.35}$$

the problem can be solved by perturbation theory. From Eq. (4.33) we find the following results[25)] for the shift ΔE and the width Γ:

$$\Delta E\equiv\operatorname{Re}W_\lambda-E_\lambda=-\left(\frac{r_a}{R}\right)^3g_a^2\mu^2mr_a^2\alpha_\lambda^2\{1-[k(E_\lambda)r_a]^2\}\{1+[k(E_\lambda)r_a]^2\}^{-2}, \tag{4.36}$$

[24)]In Ref. 41 the level "was expelled" quite rapidly as the imaginary part increased because the authors chose the ratio $r_a/R\approx 0.5$, that is, a rather large value (in comparison with the value ≈ 0.1 which follows from the general theoretical considerations in Section 3). As explained in Section 2, a large radius is required for the imaginary part of the potential if the optical model is to be used to describe the experimental data on the annihilation cross section σ_a. We will see in section 4 that large values of the cross section σ_a are in fact compatible with small values of $r_a=1/m$

[25)]In order to derive (4.36) and (4.37) it is necessary to retain a single term in the sum over λ in Eq. (4.33), discard the second term on the right side of the equation, and then replace E by E_λ.

$$\Gamma = 4\left(\frac{r_0}{R}\right)^3 g_a^2 \mu^2 m r_a^2 \alpha_\lambda^2 \{1-[k(E_\lambda) r_a]^2\}^{-2}. \tag{4.37}$$

It is easy to see that within the range of applicability of Eq. (4.35) we have

$$\Delta E < 0,$$

if

$$r_a \leqslant \frac{\sqrt{2}}{m}. \tag{4.38}$$

The annihilation thus does not "expel" the level; on the contrary, the level drops (that is, the binding energy in channel h increases). The physical reason for this effect is quite simple: the coupling between the channels generates an additional diagonal interaction in channel h between particles which correspond to the "potential"

$$V_h = \hat{H}_{hl} d_l \hat{H}_{hl}, \tag{4.39}$$

where d_l is the Green's function of the system of two free light particles. An analogous "potential" arises in the light-particle channel

$$V_l = \hat{H}_{lh} D_h \hat{H}_{hl}; \tag{4.40}$$

where D_h is the Green's function of the system of interacting heavy particles (interacting with a potential V). The sign of the "potential" in (4.39) determines the sign of the annihilation shift ΔE. Equation (4.39) corresponds to the diagram in Fig. 10a. In the realistic quantum-field model we would have the diagram in Fig. 10b. Denoting by M_h the Feynman amplitude for the diagram in Fig. 10b, we can write

$$V_h(\mathbf{r}) = -\int M_h e^{-i\mathbf{q}\mathbf{r}} \frac{d\mathbf{q}}{(2\pi)^3}, \tag{4.41}$$

while M_h is given by the integral

$$M_h = 4m\mu g_a^2 \int d\mathbf{p}_1\, dE_1 (2mE_1 - p_1^2 + i0)^{-1} (2mE_2 - p_2^2 + i0)^{-1} \times (2mE_3 - p_3^2 + i0)^{-1} (2mE_4 - p_4^2 + i0)^{-1}. \tag{4.42}$$

The energies and momenta E_i and $\mathbf{p}_i$ are related, as usual, by the conservation laws at each of the vertices and for the overall process. Expressing all the momenta and energy in terms of $\mathbf{p}_1$ and E_1 with the help of these relationships, we easily see (without evaluating the integral explicitly) that the real part of the integral is postive definite, so that the potential in (4.41) is attractive. The diagram in Fig. 10b corresponds to $r_a \approx 1/2m$, so that condition (4.38) is satisfied for such a model. The amplitude M_h is also positive if $m < \mu$. Here we are seeing the meaning of the familiar theorem for the coupled-channel model (a conversion to closed channels leads to an attraction[45]).

Another important result of the solution of the two-channel problem is the demonstration that the shifts and widths are small if $r_a/R \ll 1$, in accordance with the general considerations outlined above. This assertion can be seen clearly from Eqs. (4.36) and (4.37), where this ratio appears raised to the third power. These equations have a broad range of applicability. For small values of r_a, inequality (4.39) holds if $g_a^2 \gtrsim 1$, i.e., in the case of a strong annihilation (we will see in Section 5 below that the annihilation cross section for such coupling constants and for a strong attraction in the h channel can reach the unitary limit). At $r_a \approx 1/2m$, condition (4.35) thus converts to the inequality

$$g_0^2 \ll 16\frac{m^2}{\mu^2}.$$

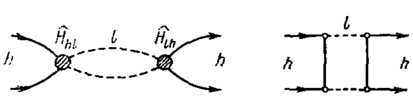
If the h particles are "nucleons" while the l particles are "ρ mesons," we find $g_a^2 \ll 24$. In practice, the perturbation-theory equations give results which are correct in order of magnitude up to those values of g_a for which a coupled state arises in channel l due to the potential V_l [Eq. (4.40)]. This value, $g_a^{(0)}$, of the constant g_a is given by the following equation when the interaction in channel h is "turned off" ($V=0$):

$$g_a^{(0)} = \frac{(1+r_a\varkappa_0)^2}{\mu^2 m^2 r_a^4}, \qquad \varkappa_0 = \sqrt{m(2m-2\mu)}. \tag{4.43}$$

For the example discussed above, of the coupling of "nucleon" and "ρ-meson" channels with $r_a = 1/2m$, we find $g_a^{(0)} = 44$. If perturbation theory cannot be used, Eq. (4.33) can be solved numerically. Figure 11 shows the motion of a level in a potential well in the h channel as the annihilation constant g_a^2 is increased ($r_a/R = 0.1$). It can be seen from this figure that the shifts and widths remain small for any value of g_a^2. For the case $N\bar{N} \to 2\rho$, the binding energy for $g_a = 0$ is 85 MeV. The maximum shift ΔE (for $g_a^2 \gtrsim 100$) is about 15 MeV, and the width is $\Gamma = 5$ MeV. A simple comparison of Figs. 9 and 11 shows that the level trajectories in the optical model and in this example of the two-channel problem have nothing in common. This example is crude and overly simplified, but with the same crudeness and oversimplification in the optical model we always find a trajectory like that in Fig. 9 rather than like that in Fig. 11.

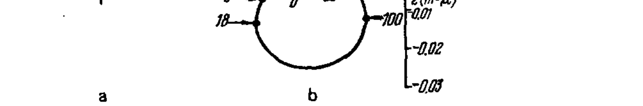
From this analysis of the annihilation shifts and widths we draw the following conclusion: although it is hardly possible at present to calculate these quantities accurately, we can assert with some confidence that these quantities are small if the ratio r_a/R is small, where r_a is the annihilation radius and R is the radius of the $N\bar{N}$ quasinuclear state.

How do we reconcile a small annihilation radius (and thus the spectrum of narrow quasinuclear levels) with



a b

FIG. 10. Diagram of the annihilation "potential." a) General case; b) a model close to quantum field theory ($\hat{H}_{hl}$ corresponds to the pole diagram, while v_h corresponds to the four-vertex Feynman diagram in the figure).



a b

FIG. 11. Motion of a level in the nonrelativistic model of coupled channels.[44] a) Cuts in the complex energy plane: the solid cut runs from the threshold for the creation of heavy particles, which is adopted as the origin, while the dashed cut runs from the threshold for the creation of light particles; b) level trajectory (the arrows give the value of the dimensionless channel coupling constant g_a^2; see text).

the large observed cross section for $N\bar{N}$ annihilation?

To answer this question, we begin by examining a simpler and well-known example: positronium and e^+e^- annihilation in states of the continuum ("in-flight" annihilation). In this case, because of the small electron charge, we can use perturbation theory to calculate the annihilation widths and cross sections. The results found in this manner incorporate those qualitative aspects of the annihilation process in which we are interested.

For definiteness we consider two-photon annihilation. Then we have the following equations for the annihilation width of the 1^1S_0 state of parapositronium and the cross section for annihilation from the S states of the continuum (see, for example, Ref. 46):

$$\Gamma_a = \frac{m_e}{2}\alpha^5, \qquad v\sigma_a = \frac{\pi\alpha^2}{m_e^2}\frac{2\pi\alpha/v}{1-e^{-2\pi\alpha/v}}, \tag{4.44}$$

where m_e is the electron mass, $\alpha = 1/137$, and v is again the relative velocity of the annihilating particles (which are assumed to be nonrelativistic). Both these equations are found from the following expressions:

$$\Gamma_a = |\Psi(0)|^2 v\bar{\sigma}_a, \qquad v\sigma_a = |\Psi_v(0)|^2 \frac{v\bar{\sigma}_a}{4}. \tag{4.45}$$

Here Ψ and Ψ_v are the wave functions of the discrete and continuous spectra, and $\bar{\sigma}_a$ is the annihilation cross section when the interaction in the initial state is ignored. In this case,

$$v\bar{\sigma}_a = \frac{4\pi\alpha^2}{m_e^2}. \tag{4.46}$$

For the values of the wave functions at $r = 0$ we have

$$|\Psi(0)|^2 = \frac{m^3\alpha^3}{8\pi}, \quad |\Psi_v(0)|^2 = \frac{2\pi\alpha/v}{1-e^{-2\pi\alpha/v}}. \tag{4.47}$$

A comparison of Eqs. (4.4)–(4.7) shows clearly why the probability for the same process, two-photon annihilation, is proportional to α^5 in one case (parapositronium) and proportional to α^2 in the other case (annihilation from the continuum) (if $v \gtrsim \alpha$; in the e^+e^- bound state, $v \approx \alpha$). The reason is that the annihilation width Γ_a contains, in addition to the small parameter which is governed by the number of photons (α^2), the ratio of the cube of the annihilation radius, $(1/m_e)^3$, and the cube of the Bohr radius of positronium, $(2/\alpha m_e)^3$. The fact that the annihilation region is small in comparison with the dimension of the e^+e^- bound state thus has a most important effect on the width of this state, reducing it by six orders of magnitude. In this example we can also clearly see that the size of the annihilation region is actually of the order of the Compton length of the annihilating particles. We further note that Eqs. (4.45) and (4.46) explicitly demonstrate the fact (mentioned in Section 4) that the observed cross section σ_a is not equal to the quantity $\bar{\sigma}_a$.

Equations (4.45) for Γ_a and σ_a are found through calculations for the diagrams in Figs. 12a and 12b [Eq. (5.2) for Γ_a is equivalent to the diagram in Fig. 12a]. The oval in Fig. 12b represents the amplitude for e^+e^- Coulomb scattering. This amplitude is the sum of several ladder boson-exchange diagrams of the type in Fig. 6a (in this case the bosons are photons). The wave func-

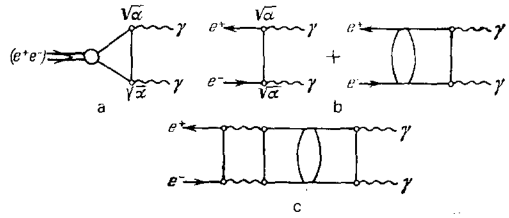


FIG. 12. The annihilation $e^+e^- \to 2\gamma$. a) Annihilation of positronium from the $^1S_0$ state; b) amplitude for annihilation from the continuum (the unshaded oval is the amplitude for e^+e^- Coulomb scattering); c) annihilation unit in a higher order in α.

tions $\Psi(0)$ and $\Psi_v(0)$ appear in (4.45) because the amplitude for two-photon annihilation, which varies significantly over an interval of size m_e, is a weaker function of the momenta of the virtual fermions than are the Fourier transforms of their wave functions (for which the scale interval is governed by the reciprocal of the Bohr radius, $m\alpha/2$). Accordingly, this amplitude can be extracted from the integral, in precisely the same manner as in the calculation of the mass operator. The wave function Ψ_v incorporates the scattering amplitude, and the factor $|\Psi_v(0)|^2$ in the equation for the observed cross section σ_a results from the interaction of the annihilating particles in the initial state. Why is the annihilation taken into account in the calculation of σ_a in the lowest order in α (for example, if the diagram in Fig. 12c and the similar diagrams are discarded), while the "potential" diagrams for the t-channel exchange are summed over all orders of the same parameter α? This question has been answered elsewhere; the reason is that the parameter which actually governs the convergence of the ladder series for boson exchange is

$$\eta = \frac{\alpha}{v}, \tag{4.48}$$

while for the annihilation diagrams this parameter is α itself. In the nonrelativistic case we have $\eta \gg \alpha$, so the ladder diagrams of the "potential type" are "senior" diagrams: they must be summed, and the resulting scattering amplitude must be inserted into any diagram which can be cut vertically with only fermion lines being intersected. The presence of the "senior" parameter η thus determines the order in which several diagrams are summed. If we increased the fermion–boson coupling constant in the annihilation diagrams, we would need to augment the annihilation diagrams in Figs. 12a and 12b with a chain of units as in Fig. 12c and then sum the resulting series. It is important to note that otherwise we would not be able to meet the requirement of unitarity, since the cross section σ_a in Eq. (4.44) would exceed the unitary limit as the annihilation coupling constants increased. Obviously, it is this need to sum the "augmented" diagrams in Figs. 12a and 12b which distinguishes e^+e^- annihilation from $N\bar{N}$ annihilation. The problem of finding Γ_a and the annihilation cross section σ_a for the $N\bar{N}$ system thus reduces to a calculation of the diagrams in Fig. 13. The hatched rectangle with the external meson and fermion lines is the sum of several annihilation diagrams (Fig. 13c). Figure 13d shows the structure of the amplitude for annihilation scattering, f_a (which appears, in particular, in the mass operator in Fig. 7). The imaginary part of this amplitude (the verti-

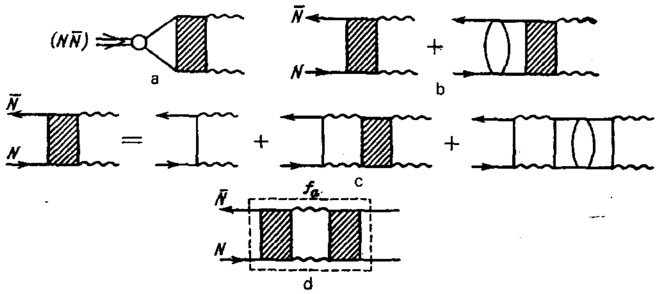


FIG. 13. Annihilation of an $N\bar{N}$ pair. a) Annihilation from the bound state; b) amplitude for annihilation from the continuum (the unshaded oval is the amplitude for potential scattering); c) structure of the hatched unit in the diagrams in parts a and b (this amplitude does not contain the potential interaction in the final state); d) relationship between the amplitude for annihilation scattering, f_a, and the annihilation unit in part c (the imaginary part of f_a contains the squared unit in part c).

cal cut along boson lines) in the case $E < 0$ is expressed in terms of the square of the diagram in Fig. 13c, that is, in terms of the cross section $\bar{\sigma}_a$. The singularities of the diagram in Fig. 13d along t, which are governed by the masses of the hatched rectangles, lie at values $t \geqslant 4m^2$, as mentioned in Section 3. It follows that the characteristic length corresponding to the diagrams in Fig. 13c is smaller than or comparable to $1/m$ and that the scale interval for an important change in this amplitude as a function of the momenta of the virtual fermions in the diagram in Fig. 13b is correspondingly equal to the nucleon mass m. The amplitude for potential scattering (the unshaded circle), on the other hand, in this diagram undergoes a change over an interval R^{-1}, where R is the effective radius of the potential well, which is essentially equal to the orbital radius of the finite motion for states with good binding. This means that in the diagrams in Figs. 13a and 13b the unit in Fig. 13c can be extracted from the integral over the virtual momenta of the fermions, so that we again find Eqs. (4.45) for Γ_a and the observed cross section for $N\bar{N}$ annihilation. A distinction from e^+e^- annihilation is that now the quantity $\bar{\sigma}_a$ should be understood as the cross section corresponding to the amplitude in Fig. 13c rather than Fig. 12b; the wave function Ψ_v describes the continuum S state for the OBEP nuclear potential.

For $\bar{\sigma}_a$ we have the following equation, which is analogous to (4.46):

$$v\bar{\sigma}_a = \frac{4\pi\bar{g}_a^2}{m^2}, \tag{4.49}$$

where $\bar{g}_a$ is some effective constant. This constant depends on the nucleon–boson interaction constants, the structure of the unit in Fig. 13c, and the effective momenta of the virtual fermions at which the amplitude in Fig. 13c is extracted from the integral in Fig. 13a. If the $N\bar{N}$ nuclear interaction is assumed to be attractive, and if it can be approximated well by a Yukawa potential (or by a sum of Yukawa potentials), then we find an equation like (4.47) for $\Psi_v(0)$ [because of the short range of the forces, the actual equation will thus differ from Eq. (4.50) in that the wave function will be finite in the limit $v \to 0$]:

$$|\Psi_v(0)|^2 \approx \frac{2\pi g^2/v}{1 - e^{-2\pi g^2/v}}, \tag{4.50}$$

where g is the effective constant of the potential interaction (this constant may be different for the singlet and triplet states).

We can now answer the question asked at the beginning of this section. We equate the unitary limit for the annihilation cross section in the S wave to be calculated cross section σ_a:

$$\pi\lambda\!\!\!^-{}^2 = \frac{4\pi}{m^2v^2} \approx \frac{2\pi^2 g^2 \bar{g}_a^2}{m^2v^2(1 - e^{-2\pi g^2/v})}. \tag{4.51}$$

Discarding the term with the exponential function from the denominator in (4.51), we find

$$g^2\bar{g}_a^2 \approx \frac{2}{\pi}. \tag{4.52}$$

For the observed cross section σ_a to be "large" (that is, comparable to the unitary limit) it is thus sufficient that the constants $\bar{g}_a$ and g be of the order of unity. As shown above, however, in the case $\bar{g}_a \lesssim 1$ we find narrow quasinuclear states and small annihilation shifts. Up to this point in this section we have been talking about S waves. For waves with higher orbital angular momenta we could find corresponding estimates. If we express $v\bar{\sigma}_a$ or $\bar{g}_a^2$ in terms of the annihilation widths of the quasinuclear levels, we can relate the observed annihilation cross sections and widths to the effective constant of the potential interaction, g^2. It turns out that for $g^2 \approx 1$ and $R^{-1} \approx 200$ MeV/c the unitary limit of the observed annihilation cross section, $\sigma_a^{(l)}$, is reached at the following values of the annihilation widths of the quasinuclear levels of the $N\bar{N}$ system:

$$\Gamma_{a\lambda}(\text{MeV}) = \begin{cases} 8.6, & l=1, \\ 2.9, & l=2, \\ 0.7, & l=3. \end{cases} \tag{4.53}$$

We thus see that the narrow quasinuclear levels are compatible with observed annihilation cross sections σ_a which are large (that is, which reach the unitary limit in each partial wave). The physical reason for the appearance of large annihilation cross sections in the $N\bar{N}$ system is the attractive interaction between the particles in the initial state, which in a sense "focuses" the particles into the region of small separations, at which annihilation is possible. The reason that the annihilation widths of the states of the discrete spectrum are small, on the other hand, is that this same attraction creates finite-motion orbits which are well outside the annihilation region. As mentioned above, this situation is quite familiar in the case of e^+e^- annihilation: the annihilation width for positronium contains the attenuation factor $(m_e a)^{-3} = \alpha^3$, while the cross section for annihilation from the continuum contains an amplifying factor, $|\Psi_v(0)|^2 > 1$. The only distinction between $N\bar{N}$ annihilation and e^+e^- annihilation is that the small "annihilation" constant α^2 is replaced by a quantity on the order of unity, and instead of $(m_e a)^{-3} \approx 10^{-6}$ the widths of the $N\bar{N}$ quasinuclear levels are proportional to $(mR)^{-3} \approx 10^{-3}$, where R is the radius of the quasinuclear orbit.

Using graphic examples, we can show that the unshaded block in the second diagram in Fig. 13b acts as a focusing lens, while the three-particle vertex in Fig. 13a "holds" the particles far from the annihilation zone.

Figures 14–17 show how the optical model differs

FIG. 14. Ladder diagram of the optical model. The sum of an infinite series of such diagrams gives the scattering amplitude in the optical model. The half-hatched rectangle is the Born amplitude (the optical potential).

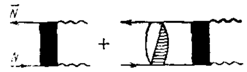


FIG. 16. Amplitude corresponding to the annihilation cross section in the optical model, σ_a^{opt}. The half-hatched oval is the amplitude for $N\bar{N}$ scattering in the optical model.

from the systematic approach described above. These figures and their captions show that in the optical model each boson-exchange event is accompanied by absorption. The attraction between particles thus acts as a weakly focusing semitransparent lens. When the absorption region has a small radius, it is thus necessary to use complex potentials with anomalously large amplitudes for the imaginary part (in order to obtain the experimentally observed annihilation cross sections). This is why the effective (average) annihilation radius in the optical model is always larger in comparison with that expected on the basis of the general physical considerations discussed above.

5. SPECTRA OF $B\bar{B}$ QUASINUCLEAR LEVELS

The results of this analysis of the annihilation-shift problem clearly show that annihilation effects can be ignored altogether in finding a general picture of the spectra of quasinuclear levels of the $B\bar{B}$ systems.

In this section we will examine those characteristic features of the spectra of $B\bar{B}$ quasinuclear states which are essentially independent of the dynamics of the annihilation processes and which are governed entirely by the nuclear interaction between B and $\bar{B}$. We will proceed on the basis of the OBEP model, since this version of the nuclear potential is based on definite physical arguments regarding the mechanism for the nuclear interaction of nonrelativistic baryons, and as a result of this provides a possibility of relating the effects observed in the BB and $B\bar{B}$ channels. On the other hand, we know that the NN scattering phases are described by the OBEP model at least as well as by the phenomenological potentials of the Hamada–Johnston, Read, etc., types.[1] In summary, the OBEP model can be used as a basis for a realistic model of nuclear forces.

The OBEP model for the NN interaction is widely known and has been discussed many times in reviews and monographs.[1,9,15,47-50] Furthermore, as mentioned earlier, we do not have as yet enough experimental information on the spectrum of quasinuclear $N\bar{N}$ mesons for a quantitative comparison with theory. Accordingly, we will write neither the equations nor the constants governing the OBEP in its various modifications, and we will simply recall the physical content of the model. The nuclear interaction of nonrelativistic baryons according to this model results from the t-channel exchange of the "light" bosons π, η, ρ, and ω. Recent versions of the model also incorporate two-pion exhanges, and for this purpose two "effective" scalar mesons are sometimes introduced: an isoscalar meson σ_0 and an isovector meson σ_1. A method for incorporating two-pion exchange which is formally more systematic was recently discussed by Richard *et al.*[50] The potentials $V_{N\bar{N}}$ and V_{NN} generated by the exchange of a boson X are related by

$$V_X(B\bar{B}) = G_X V_X(BB), \tag{5.1}$$

where G_X is the G parity of boson X ($G_\pi = G_\omega = G_{\sigma_1} = -1$, $G_\eta = G_\rho = G_{\sigma_0} = +1$). The sign of V_X generally depends on the isospin and spin states of the interacting particles. Significantly, the σ_0 and ω exchanges contribute an attraction in the $N\bar{N}$ channel, and the attraction due to the ω meson is particularly strong. Another important feature of the OBEP model is that spin-orbit forces are important (they result from the exchange of scalar and vector mesons). The spin-orbit coupling partially cancels the centrifugal barrier and generates a large number of states in the discrete spectrum. The potential in this model of $N\bar{N}$ nuclear forces is deep enough for the appearance of bound states, but it is not so "broad" that the wave functions have radial nodes. The spectrum of $N\bar{N}$ quasinuclear levels would thus be quite sparse in the

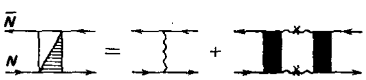


FIG. 15. Potential of the optical model; the second term is the imaginary part of the optical potential. The crosses on the wavy lines denote bosons on the mass shell; the black rectangles are the effective annihilation units governing the magnitude of the imaginary part of the potential.

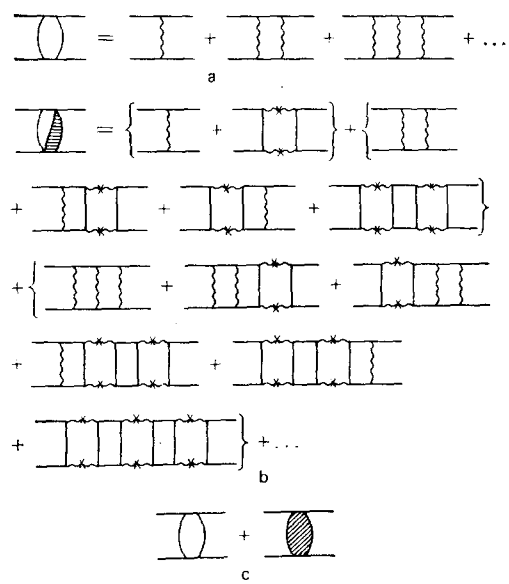


FIG. 17. Summary of the comparison of the scattering amplitudes in the OBEP model (a) and the optical model (b). The "microstructure" of the series reproduced here in part b corresponds to the ladder diagrams in Fig. 14. The sum of diagrams in part c could have been found by regrouping the terms of the series in part b (the hatched oval is what is left after the subtraction of the series in part a). The sum in part c, however, is not equal to the scattering amplitude in the optical model, in part b.

absence of spin-orbit forces. In this case we are seeing how much valuable information on nuclear forces can be obtained from a study of the $N\bar{N}$ quasinuclear levels: a qualitative picture of the spectrum is sufficient for arguing for the occurrence of a strong spin-orbit coupling, which is difficult to analyze in the nonrelativistic NN system. In addition to the spin-orbit interaction, the OBEP model contains spin–spin and tensor forces (they derive from the exchange of pseudoscalar and vector mesons). The tensor and spin-orbit forces are excessively singular in the limit $r\to 0$ ($\propto r^{-2}$ and r^{-3}). The nonrelativistic theory thus contains at least one parameter in addition to the boson–nucleon coupling constants: the cutoff radius of the singular forces.

To end this brief description of the Hamiltonian of the $N\bar{N}$ interaction, we note that this Hamiltonian has the same quantum numbers as for the NN interaction. In particular, the total spin S of the nucleon and antinucleon is a good quantum number, and all the $N\bar{N}$ states can be classified as singlet ($S=0$) or triplet ($S=1$) states.

A calculation of the levels of the $N\bar{N}$ system in the potential approximation can be justified only for nonrelativistic states. This assertion means that, first,

$$|Q| \ll 2m, \quad Q \equiv M-2m \tag{5.2}$$

(here M is the mass of the $N\bar{N}$ state) and, second,

$$R \gg \frac{1}{m}, \tag{5.3}$$

where R is again the radius of the state. Condition (5.3) is of further importance in this problem because it makes the annihilation widths and shifts comparatively small (Section 4). In calculating the spectrum of $N\bar{N}$ quasinuclear levels we must thus not only find the eigenvalues but also check to see that condition (5.3) is satisfied. In the nonrelativistic potential approach, any states which satisfied condition (5.2) but not condition (5.3) would not be of any serious interest.

There are two ways to estimate the state radius R. The first, which is useful for bound states ($M<2m$), reduces to a simple calculation of the wave function and the use of this wave function to calculate the mean square radius. The second way, which is useful for both bound states and resonances ($M>2m$), is to calculate the Regge trajectories $l(E)$ for the given potential and the derivative $d(\mathrm{Re}\,l)/dE$. As mentioned above, this quantity contains $\langle r^{-2}\rangle$ in the case $E<0$ (Ref. 39):

$$\frac{d(\mathrm{Re}\,l)}{dE} = \frac{m\langle r^{-2}\rangle^{-1}}{2l+1}. \tag{5.4}$$

The fact that Eq. (5.4) can be used to estimate $\langle r^{-2}\rangle = R^{-2}$ is particularly important for resonances, since in this case the wave function is not quadratically integrable, and the mean radius cannot be calculated directly. If, on the other hand, we extrapolate (5.4) to small but positive values of Q (under the condition $\mathrm{Im}\,l \ll 1$), we can estimate R. Knowing the derivative of the Regge trajectory, we can also find the partial width $\Gamma_{N\bar{N}}$ for decay through the "elastic channel": $(N\bar{N})_{\mathrm{res}} \to N+\bar{N}$. This quantity is given by

$$\Gamma_{N\bar{N}} = \frac{2\,\mathrm{Im}\,J}{d(\mathrm{Re}\,J)/dM}. \tag{5.5}$$

In order of magnitude we have

$$\Gamma_{N\bar{N}} \leqslant \frac{2(2J+1)}{mR^2}, \tag{5.6}$$

so that for $R=1-1.5$ F we should have

$$\frac{\Gamma_{N\bar{N}}}{2J+1} \leqslant 80-40\ \mathrm{MeV}. \tag{5.7}$$

It should be noted that Eq. (5.6) holds for sufficiently small values of $\mathrm{Im}\,J$ near the threshold for decay to $N\bar{N}$. The value of $\mathrm{Im}\,J$ is extremely sensitive to the analytic properties of the potential in the complex r plane. The cutoff of the potential in the OBEP model which disrupts the analyticity (this cutoff is required because of the r^{-3} singularity) can strongly affect the values of $\Gamma_{N\bar{N}}$, especially if this cutoff is not made at a very short distance (as in the static version of the OBEP model, used in Refs. 11–17). Accordingly, it is not possible to calculate the widths $\Gamma_{N\bar{N}}$ accurately with the existing uncertainties regarding the potential, and the same is true of the level positions.

In contrast, the experimental data on the resonance masses M and widths $\Gamma_{N\bar{N}}$ can fill in the gaps in our knowledgeof the nuclear forces (we emphasize that annihilation processes have no greater effect on the values of $\Gamma_{N\bar{N}}$ than on the level positions). In other words, the resonance masses M and widths $\Gamma_{N\bar{N}}$ are quantities of the same class, which are governed essentially entirely by nuclear forces. Condition (5.6) is a necessary condition for a calculation of the widths $\Gamma_{N\bar{N}}$. Those resonances for which the values of $\Gamma_{N\bar{N}}$ do not satisfy this inequality are just as irregular in the nonrelativistic approach as bound states for which R is too small to satisfy condition (5.3).

The annihilation widths Γ_a can only be estimated from Eq. (4.12), since an accurate calculation would require a knowledge of the "unobservable" annihilation cross section $\bar{\sigma}_a$ (Section 4). As mentioned above, $\bar{\sigma}_a$ was replaced by the observable annihilation cross section σ_a in Eq. (4.12) in Refs. 11–17 in order to estimate upper limits[26] on Γ_a. We also note that in specific calculations Eq. (4.12) is used in a modified form: the quantity $|\Psi(0)|^2$ in this equation is replaced by $|\bar{\Psi}|^2$, averaged over the volume of the annihilation region [for states with $l\neq 0$, this averaging is essentially equivalent to extracting derivatives of the annihilation amplitude f_a from the integral in the mass operator in Eq. (4.1)]. The use of (4.12) to estimate the annihilation widths of resonances also requires some explanation. The wave function which appears in this equation has the dimensionality of $\mathrm{cm}^{-3/2}$, as a result of the normalization. For a resonant state the wave function is normalized to the volume of the region whose radius is calculated from data on the width $\Gamma_{N\bar{N}}$.

To illustrate these arguments, we briefly examine some calculated results. We first examine the spectrum of bound states and resonances of the $N\bar{N}$ system. Table I, taken from Ref. 51, shows the masses, annihilation widths Γ_a, and partial elastic widths $\Gamma_{N\bar{N}}$ of the $N\bar{N}$ quasinuclear states calculated in the static version of the OBEP model by the procedure described above. This

[26]We again emphasize that the values of Γ_a found in this manner may be too high, even in order of magnitude.

TABLE I. Spectrum of bound and resonant $N\bar{N}$ quasinuclear states found by Bogdanova *et al.*[51] **for the static version of** the OBEP model. The asterisks show states which are mixed by tensor forces (not taken into account in this calculation).

Spectroscopic designation	$I^G(J^P)$	Mass, MeV	Γ_a, MeV	$\Gamma_{N\bar{N}}$, MeV
$^1S_0$	$1^-(0^-)$	1714	151	
	$0^+(0^-)$	1678	149	
$^3S_1-{}^3D_1^*$	$1^+(1^-)$	1710; 1970	163; 1.3	—; 57
	$0^-(1^-)$	1395; 1515	88; 1.1	
$^1P_1$	$1^+(1^+)$	1858	7.4	
	$0^-(1^+)$	1824	8.3	
$^3P_0$	$1^-(0^+)$	1768	13.3	
	$0^+(0^+)$	1330	8.2	
$^3P_1$	$1^-(1^+)$	1825	9.2	
	$0^+(1^+)$	1438	11.5	
$^3P_2-{}^3F_2^*$	$1^-(2^+)$	1878: —	6.8; —	
	$0^+(2^+)$	1684; 1860	9.8; 0.1	
$^3D_2$	$1^+(2^-)$	2030	1.2	157
	$0^-(2^-)$	1775	0.9	
$^3D_3$	$1^+(3^-)$	—	—	—
	$0^-(3^-)$	1978	1	78
$^3F_3$	$1^-(3^+)$	—	—	—
	$0^+(3^+)$	2085	0.12	73
$^3G_3$	$1^+(3^-)$	—	—	—
	$0^-(3^-)$	2230	0.01	76

table lists a total of 17 states in the discrete spectrum. We see that the rough, order-of-magnitude estimates made earlier on the basis of qualitative physical considerations are confirmed by the detailed calculations. The annihilation widths, even when clearly overestimated by the substitution of $v\sigma_a = 45$ mb (in place of $v\sigma_a \approx 2$ mb) in Eq. (4.12), are small for the overwhelming majority of the levels. They decrease by about an order of magnitude when the orbital angular momentum l is increased by unity, so that for the P states they are already down to some 12 MeV or less. Figure 18 shows the squared modulus of the radial wave function for the $N\bar{N}$ bound state and resonance. The annihilation region is much smaller than the radius of the quasinuclear state, which is of the order of 1 F. Figure 19 shows one of the quasinuclear Regge trajectories, on which there are two bound states and one resonance. There are eight such trajectories in all: two singlet trajectories ($S=0$, $I=0, 1$) and six triplet trajectories ($S=1$, $I=0, 1$). Along a nonrelativistic trajectory, P parity and G parity are not conserved, in contrast with the situation in the relativistic theory. The reason is the absence of cross symmetry in the nonrelativistic approximation. If quantum-field relativistic corrections were made, the single trajectory in Fig. 19 would be replaced by two closely spaced trajectories, for even and odd values of l. The G parity of the $N\bar{N}$ system is thus given by

$$G = (-1)^{S+I+l}. \qquad (5.8)$$

Each of the two relativistic trajectories would correspond to a definite G parity. It can also be seen from Fig.

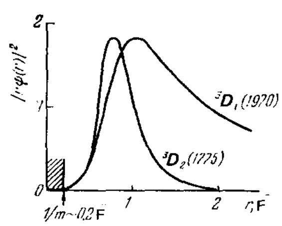


FIG. 18. Wave functions of the $^3D_2$ (1775) bound state and the $^3D_1$ (1970) resonant state of the $N\bar{N}$ system. The hatching shows the annihilation region.

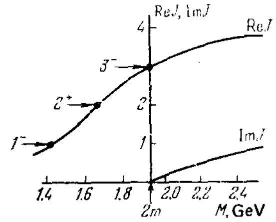


FIG. 19. One of the eight Regge trajectories for the $N\bar{N}$ quasinuclear system. The quantum numbers corresponding to this trajectory are $I=0$, $S=1$, $S'=J-l=1$.

19 that ImJ increases rapidly with distance from the threshold ($M=2m$), while the increase in ReJ slows down. Both factors tend to increase the elastic widths $\Gamma_{N\bar{N}}$, so that most of the resonances in Table II have a slight inelasticity. We have already mentioned that the widths $\Gamma_{N\bar{N}}$ are very sensitive to the details of the nuclear potential. This sensitivity is illustrated in Table II (taken from Ref. 16). Shown here are data on the resonances calculated, again by the static version of the OBEP model, except that at $r \leqslant 0.6$ F the centrifugal potential as well as the nuclear potential is cut off (this approach is obviously equivalent to the introduction of certain l-dependent attractive forces).[27)] As can be seen by comparing Table I and II, this change in the potential affects the spectrum of resonant levels but does not change the basic picture. The widths $\Gamma_{N\bar{N}}$ vary rapidly; Table II shows several narrow resonances[28)] ($\Gamma_{N\bar{N}} \lesssim 30$ MeV). In both versions of the calculation, the widths $\Gamma_{N\bar{N}}$ satisfy the inequality (5.7).

As mentioned above, a variation of the potential shifts the quasinuclear levels. This situation is illustrated in Fig. 20 (Ref. 49), which compares the level schemes found on the basis of the various OBEP versions which satisfactorily describe the experimental data in the NN channel (the scheme at the left corresponds to the Nagels–Rijken–De Swart (NRD) potential,[52] while the scheme at the right corresponds to the Bryan-Phillips (BP) potential[9]). It is seen from this figure that the two interactions, which are actually indistinguishable in the NN channel, lead to very different results for the positions of specific levels of the discrete spectrum in the $N\bar{N}$ system.

TABLE II. Spectrum of resonant $N\bar{N}$ quasinuclear states.[16] Static version of the OBEP model with a cutoff of the centrifugal barrier at short distances.

Spectroscopic designation	$I^G(J^P)$	Mass, MeV	$\Gamma_{N\bar{N}}$, MeV
$^1D_2$	$1^-(2^-)$	1955	28
	$0^+(2^-)$	1930	15
$^3D_2$	$1^+(2^-)$	1925	10
$^3D_3$	$1^+(3^-)$	2025	122
	$0^-(3^-)$	1880	0.0
$^3F_3$	$1^-(3^+)$	2165	76
	$0^+(3^+)$	1880	0.0

[27)]The centrifugal potential was cut off in Refs. 11-17 in order to estimate reliable upper limits on the annihilation widths of levels with $l \neq 0$.

[28)]In this connection we note that if we switch to OBEP versions with a smaller cutoff radius we also reduce the widths $\Gamma_{N\bar{N}}$.

Completely analogous calculations of the discrete-spectrum levels have been carried out for certain other $B\bar{B}$ systems. Figure 21 shows the quasinuclear levels of the $Y\bar{Y}$ system.[49] The $Y\bar{Y}$ interaction potential is found from the $N\bar{N}$ potential on the basis of the $SU(3)$ symmetry. This is of course an extremely rough procedure, since $SU(3)$ symmetry is violated in different ways in different phenomena, so that its overall accuracy is difficult to determine. It should also be noted that in calculations of the spectra of the $\Lambda\bar{\Lambda}$ and $\Sigma\bar{\Sigma}$ states it is important to take into account the coupling of the Λ and Σ channels, but this was not done in the present case. Nevertheless, the heuristic data in Fig. 2 are extremely interesting. They show that a rich spectrum of quasinuclear $Y\bar{Y}$ states is quite probable according to the theory.

Figure 22 shows the spectrum of "exotic" states ($I=\frac{3}{2}$) for the $\Sigma\bar{N}$ system.[53] The two diagrams here correspond to the Nagels–Rijken–De Swart potential (a) and the Bryan–Phillips potential (b). The potential in the $Y\bar{N}$ channel is found from the $N\bar{N}$ interaction on the basis of $SU(3)$ symmetry. It is seen from Fig. 22 that the existence of several exotic strange resonances of a quasinuclear nature is completely probable. Dal'karov and Mandel'tseig[54] were the first to carry out calculations of the quasinuclear spectrum for systems of the $Y\bar{N}$ type, specifically, for $\Lambda\bar{N}$.

In completing this section, we wish to draw attention to several facts.

First, theoretical calculations of the spectra of quasinuclear states indicate that the positions of the specific levels and the elastic widths $\Gamma_{N\bar{N}}$ of the resonances are very sensitive to the nature of the $N\bar{N}$ interaction. Then the $B\bar{B}$ quasinuclear spectra can yield exceedingly valuable information about the nuclear forces between nonrelativistic baryons.

Second, despite the differences in the specific level schemes corresponding to the different versions of the $B\bar{B}$ interaction, all the results agree with regard to the general structure of the spectrum of quasinuclear states: there should be many such states, and the overwhelming majority of them should have nonvanishing orbital angular momenta (there are no S states with radial excitations). The expected abundance of quasinuclear states is one of the main predictions of the theory. It follows from this prediction that many heavy mesons with masses of about 2 GeV and higher may actually be peculiar $B\bar{B}$ nuclei.

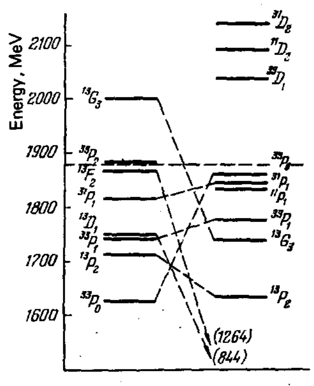


FIG. 20. Spectra of $N\bar{N}$ quasinuclear levels for two versions of the OBEP model[49]: the Nagels–Rijken–De Swart (NRD) potential (scheme at the left) and the Bryan–Phillips (BP) potential (at the right). The levels in the column at the right differ from the data listed in Table I in that the diagonal matrix elements of the tensor forces are taken into account.

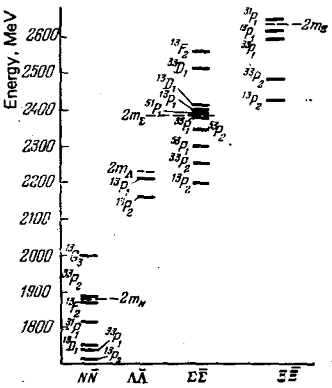


FIG. 21. Spectra of the $N\bar{N}$ and $Y\bar{Y}$ quasinuclear states.[49] The nuclear potential in the $Y\bar{Y}$ channel is found from the $N\bar{N}$ potential (the NRD version) on the basis of $SU(3)$ symmetry.

Third, it should be emphasized that experiments with deuteron targets in $\bar{p}$ beams would be extremely useful for determining the quantum numbers of the $N\bar{N}$ bound states. For slow antiprotons, the pickup mechanism is the most effective (Fig. 23a), while for fast antiprotons the replacement mechanism can be used (Fig. 23b). Figures 24 and 25 demonstrate the possibility of determining the orbital angular momentum of the relative motion of N and $\bar{N}$ from the momentum spectra of the particles formed in the reaction.

6. THE $2N\bar{N}$ AND $2N2\bar{N}$ QUASINUCLEAR SYSTEMS

The possible existence of quasinuclear $2N\bar{N}$ baryons and $2N\bar{N}$ bosons was studied theoretically in Refs. 55 and 56. Such systems should have masses of about 2.8 and 3.7 GeV, respectively. There can be no doubt that forces which are capable of binding two particles together would also be capable of binding three or four particles in a common system. The primary question thus concerns the annihilation width; specifically we are con-

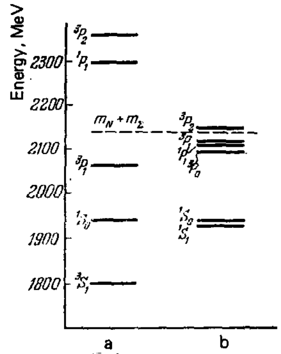


FIG. 22. Spectra of exotic ($I=3/2$) quasinuclear $\Sigma\bar{N}$ states.[53] The $\Sigma\bar{N}$ potential is found from the $N\bar{N}$ potential on the basis of $SU(3)$ symmetry. The scheme in part a corresponds to the NRD potential; scheme b corresponds to the BP potential.

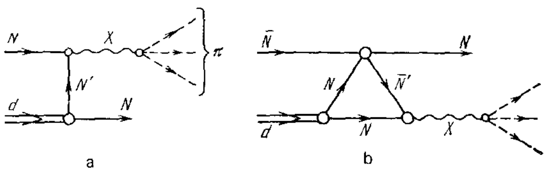


FIG. 23. Various mechanisms for the reaction $\bar{p}+d \to N+X$. a) Pickup; b) replacement.

cerned that this width would be too large because of the increase in the number of annihilation partners. This question is especially important because the orbital angular momentum of the relative motion of the annihilating pair may turn out to be zero, regardless of the total angular momentum of the system and regardless of the parity of the state. It would thus seem that the annihilation widths of three-particle and four-particle systems would have to be larger than the widths of the S states of the $N\bar{N}$ system. In certain near-threshold states, however, the linear dimensions of many-particle systems may be larger than those of two-particle systems, so there may also be an increase in the average separation of the annihilating particles, R. Since the annihilation probability is proportional to R^{-3}, a comparatively slight change in R can cancel the increase in the widths due to the increase in the number of annihilating pairs. For an estimate of the annihilation widths of three-particle and four-particle systems containing antinucleons we can use a generalization of Eq. (4.12), which was written for two-particle systems:

$$\Gamma_a = v\bar{\sigma}_a \int d\xi_1 \ldots d\xi_{n-1} \sum_{i>j=1}^{n} |\hat{Q}(ij)\Psi(\xi_1 \ldots \xi_{n-1})|^2 \delta(\mathbf{r}_{ij}). \qquad (6.1)$$

Here n is the number of particles in the system, $\mathbf{r}_{ij}$ is the distance between particles i and j, $\hat{Q}(i,j)=\overline{Q}(j,i)$ is the projection operator which acts on the baryon-number variables of the fermions and the $N\bar{N}$ pair under consideration, and the ξ_i are the Jacobi coordinates. The simple equation in (6.1) is not rigorous and is suitable only for estimates. To find these estimates, we need the wave function of the system, $\Psi(\xi_1, \ldots, \xi_{n-1})$. Here we run into the nuclear many-body problem, and we can use the approximate methods which have been developed for this field. One of these methods is the method of multidimensional spherical harmonics.[57] With this method it is possible to construct wave functions which meet the requirements of permutation symmetry and which correspond to given quantum numbers (angular momentum,

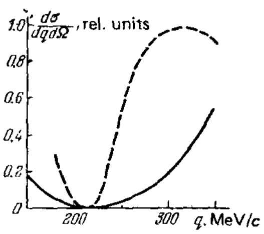


FIG. 24. Differential cross section for the pickup reaction as a function of the spectator momentum q (in the laboratory coordinate system). The momentum of the incident antinucleon is $p=200$ MeV/c; $\theta_{p_q}=180°$. Solid curve—the $^3D_1$ state; dashed—$^1P_1$.

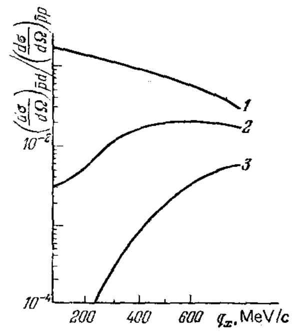


FIG. 25. The ratio $(d\sigma/d\Omega)_{\bar{p}d}/(d\sigma/d\Omega)_{\bar{p}p}$ as a function of q_x, the momentum of the quasinuclear meson formed in the replacement reaction (in the laboratory system). The momentum of the incident antinucleon is $p=1$ GeV/c. Curves 1–3 correspond to the states $^3S_1$, $^1P_1$, and $^3D_1$ (Ref. 59).

parity, and isospin). In this method, the wave function $\Psi(\xi_1, \ldots, \xi_{n-1})$ is expanded in spherical harmonics of a space $(\xi_1, \ldots, \xi_{n-1})$ with $3n-3$ dimensions. When this series is substituted into the Schrödinger equation, we find a system of coupled ordinary differential equations in the multidimensional radial variable

$$\rho^2 = \sum_{i=1}^{n-1} \xi_i^2.$$

This chain of equations, which is generally infinite, can be truncated since a series in spherical harmonics converges rapidly if the system is "well-bound" (if the average distances between the particles are comparable to the effective range of the forces).[29)] For the theory of three-nucleon and four-nucleon nuclei, this method yields satisfactory results (satisfactory values of the binding energies, electromagnetic form factors, etc.[57]). There is thus reason to believe that the wave functions found by the method of multidimensional spherical harmonics will be useful at least as trial wave functions which are approximately the same as the actual wave functions.

This method has been used to calculate the energies and to estimate the widths of several states of the three-particle quasinuclear system $2N\bar{N}$ and the states of the four-particle system $2N2\bar{N}$ with quantum numbers $2^+(4^+)$. In contrast with the two-particle $N\bar{N}$ system, the state spectra of the $2N\bar{N}$ and $2N2\bar{N}$ system have radial excitations (in terms of the variable ρ). As the number of radial nodes increases there are increases in the average distances between the particles, with the results that there is a decrease in the annihilation width Γ_a (this quantity falls off roughly in proportion in n_ρ^{-3}, where n_ρ is the number of nodes along ρ). Table III shows the masses and annihilation widths of the quasinuclear states of the $2N2\bar{N}$ system mentioned above. As in the case of the two-particle $N\bar{N}$ systems, we use the observed annihilation cross section σ_a instead of $\bar{\sigma}_a$ for the estimates (the widths found in this manner may be an order of magnitude too high). It can be seen from Table III that even with this overestimate of the annihilation widths there is

[29)]For clusterized systems, the harmonic series converges slowly, and the method is not useful. In this case it is necessary to solve integral Faddeev equations.

TABLE III. Spectrum of bound states of the $2N2\bar{N}$ system with quantum numbers $I^G(J^P = 2^+(4^+)$ (Ref. 56).

Number of radial nodes, n_ρ	Mass, MeV	Binding energy, MeV	Annihilation width, MeV
0	3375	381	1472
1	3611	145	871
2	3722	34	339
3	3756	0.22	34

at least one near-threshold state which is narrow enough to be observed experimentally. The situation for the three-particle $2N\bar{N}$ system is analogous. Table IV lists data on several of the narrowest near-threshold states of this system. Actually, because of the considerations discussed above, it seems quite likely that there is a greater number of narrow states.

We turn now to certain features of the decay of these three-particle and four-particle quasinuclear systems.

The primary decay channel for the quasinuclear baryon $2N\bar{N}$ should be $(2N\bar{N}) \to N + (4-5)\pi$. It is thus unlikely that a quasinuclear baryon with a mass of about 2.8 GeV would be observed as a resonance in πN scattering.

Analogously, the primary decay channel for the quasinuclear boson $2N2\bar{N}$ with a mass of about 3.7 GeV should be the multipion mode (8–10 pions). Despite its quasinuclear nature, this boson resonance should be manifested extremely weakly in the $N\bar{N}$ interaction cross sections, since the partial width for this channel, $\Gamma_{N\bar{N}}$, is expected to be small. On the basis of structural considerations the decay

$$(2N2\bar{N}) \to N + \bar{N} + (4\text{–}5)\,\pi$$

should not be suppressed, although its relative probability should be low because of the small phase volume. Since boson resonances with the quantum numbers of a photon and with masses of about 4 GeV are presently the subject of intense study in experiments with colliding e^+e^- beams, it is interesting to estimate the width $\Gamma_{e^+e^-}$ for the decay of the quasinuclear boson $2N2\bar{N}$ by the e^+e^- channel. We can write

$$\Gamma_{e^+e^-} \approx \alpha^2 M \left(\frac{1}{2MR}\right)^3,$$

where R is the size of the $2N2\bar{N}$ system, and M is its mass. Setting $R^{-1} \approx 140$ MeV/c and $M \approx 3.7$ GeV, we find

$$\Gamma_{e^+e^-} \approx 75\text{eV}.$$

This result corresponds to an increase of about 0.5 in the ratio $(e^+e^- \to hadr)/(e^+e^- \to \mu^+\mu^-)$ at resonance. At present we do not have an unambiguous interpretation of the experimental data on the $2N\bar{N}$ and $2N2\bar{N}$ quasinuclear systems (although there is evidence for the existence of

TABLE IV. Some bound states of the $2N\bar{N}$ system.[55]

$I(J^P)$	Mass, MeV	Binding energy, MeV	Annihilation width, MeV
1/2 (1/2⁻)	2814	6	20
1/2 (1/2⁺)	2806	14	66
1/2 (3/2⁻)	2810	10	33
1/2 (3/2⁺)	2814	6	37
3/2 (3/2⁺)	2795	25	105
3/2 (5/2⁺)	2798	22	17

resonances in the multipion channels with a mass of about 4 GeV; Ref. 58).

In concluding this section we would like to point out that large numbers of three-particle and four-particle quasinuclear states are probable according to the theory. The large number results not only from the spin-orbit forces (as in the case of the $N\bar{N}$ system) but also from radial excitations. Accordingly, for the $2N\bar{N}$ and $2N2\bar{N}$ systems there are typically several states with the same quantum numbers $I^G(J^P)$ which are generally overlapping ($D \leq \Gamma$, where D is the level spacing, and Γ is the width of one of the levels). This result means that a narrow resonance of this nature should in general interfere with the "background" (that is, with broader levels). The analysis of the resonance spectra (the extraction of the masses and widths from the experimental data), on the other hand, should be carried out on the basis of the multilevel resonance equations. The quasinuclear states of the $2N\bar{N}$ and $2N2\bar{N}$ systems may turn out to be important for deciphering the mass spectra of the baryon and boson resonances at 3–4 GeV.

7. THE $\bar{p}p$ ATOM AND THE NUCLEAR INTERACTION

In the section we will consider two questions: first, the shift and width of the atomic levels due to the strong $N\bar{N}$ interaction; second, radiative transitions from atomic states to $N\bar{N}$ quasinuclear levels.

The first Bohr radius of the $\bar{p}p$ atom in 57.5 F, the binding energy of the 1S state is 12.5 keV, and the energy of the $K\alpha$ ($2P-1S$) transition is 9.375 keV. These numbers have not been corrected for relativistic and radiation effects (the Lamb shift and the vacuum polarization) or the finite dimensions of the particles (that is, their electromagnetic form factors). All these corrections are small, however, in comparison with the nuclear shift of the level and its annihilation width. Since the Bohr radius a satisfies the inequality $a \gg R$, where R is the radius of the nuclear forces, the equations derived in Section 4 can be used for the shifts and widths of the atomic levels. For the shift and width of the atomic S level we can write

$$\Delta E_n(S) \equiv E_n(S) - E_n^{(c)} = -\frac{4\pi}{m}\,\mathrm{Re}\,f\,|\Psi_{nS}(0)|^2, \tag{7.1}$$

$$\Gamma_{na}(S) = (v\sigma_a)_{v\to 0}\,|\Psi_{nS}(0)|^2. \tag{7.2}$$

Here $E_n^{(c)}$ and $E_n(S)$ are the energies of the unperturbed and shifted levels, Ψ_{nS} is the atomic wave function of the nS state, f is the $\bar{p}p$ scattering length due to the strong interaction, and σ_a is the annihilation cross section. We emphasize that Eq. (7.1) incorporates the "true" $\bar{p}p$ scattering length, which is governed both by the nuclear forces and the annihilation interaction (in contrast with the amplitude f_a in Section 4, which correspond to annihilation scattering alone). In precisely the same manner, Eq. (7.2) contains the observed annihilation cross section σ_a [instead of the cross section $\bar{\sigma}_a$ in Eq. (4.12), which does not incorporate the nuclear interaction of the annihilation pair]. Using (7.1) and (7.2), we can estimate the corrections to $\Delta E_n(S)$ and $\Gamma_{na}(S)$. Setting

$$\mathrm{Re}\,f \equiv -\frac{2}{3}\,mU_0R^3, \tag{7.3}$$

where U_0 is the depth of some effective potential well,[30] we find from (7.1)

$$|\Delta E_n(S)| \approx 2|U_0|\left(\frac{R}{a}\right)^3 \approx \frac{1}{n^3}\,(\mathrm{keV}) \qquad (7.4)$$

with $R \approx 1$ F and[31] $U_0 = 100$ MeV. This shift of the 1S level of the $\bar{p}p$ atom should thus be of the order of 1 keV. Extrapolating the experimental data (Section 2) to $v \to 0$, we adopt $(v\sigma_a)_0 \approx 45$ mb. From (7.2) we find

$$\Gamma_{na}(S) \approx \frac{1}{n^3}\,(\mathrm{keV}). \qquad (7.5)$$

We see that the nuclear shifts and annihilation widths of the levels of the $\bar{p}p$ atom are equal in order of magnitude. If the annihilation probability is a weak function of the quantum numbers, we can use the following estimates for a level with $l \neq 0$:

$$|\Delta E_n(l)| \approx \left(\frac{R}{a}\right)^{2l} |\Delta E_n(S)|, \quad \Gamma_{na}(l) \approx \left(\frac{R}{a}\right)^{2l} \Gamma_{na}(S). \qquad (7.6)$$

The factor $(R/a)^{2l}$ is very small, so the shifts and widths of the atomic levels with $l \neq 0$ should generally be much smaller than the corresponding values for the S levels. There may be an exception to this rule in the case of a $N\bar{N}$ quasinuclear resonance or bound state which lies very near the threshold, in which case Eqs. (7.1) and (7.2) are not applicable (see Section 4). The numerical estimates in (7.4) and (7.5) are of course extremely crude, and this refers not only to the shift $\Delta E_n(S)$, but also to the width $\Gamma_{na}(S)$, since the annihilation cross section is not known for very small momenta ($\ll 200$ MeV/c). We can say that it will be measurements of $\Gamma_{na}(S)$ which will make it possible to find this cross section, through the use of Eq. (7.2).

For an accurate calculation of the shift $\Delta E_n(S)$ we need to know Ref. If the annihilation radius satisfies $r_a \ll R$, then the annihilation shift is a small quantity of the order of $(r_a/R)^3$, according to the earlier discussion (Section 4). With regard to the atom this assertion means that the annihilation shift can be neglected in comparison with the nuclear shift; in other words, we can use simply the potential of the nuclear interaction in calculating Ref, ignoring the annihilation effects. Calculations of this type have been carried out by Dal'karov and Samoĭlov[60] with the BP potential.[32] The following result was found for the shift of the 1S level:

$$\Delta E_1(S) = \begin{cases} +0.53 & ({}^1S_0), \\ +0.60 & ({}^3S_1). \end{cases} \qquad (7.7)$$

What is important in this result is not the number itself, which will change with the particular version of the OBEP model used, but the *sign of the shift*, which is positive. This means that the 1S level is "expelled" by nuclear forces, although these forces are attractive forces in the $N\bar{N}$ channel. We emphasize that the shift in (7.7) was calculated for the real BP potential, so the "expulsion" of the 1S level found in Ref. 60 is not a consequence of annihilation effects. What is the physical reason for this seemingly paradoxical result? According to Eq. (7.1), a positive shift is found for a negative scattering length. The scattering length, on the other hand, can be negative in the case of attractive forces if the potential well is so deep that there is a bound state in it. In the case of a "shallow" potential well the sign of the scattering length is opposite the sign of the potential, according to the Born-approximation equation; that is, the sign of the scattering length is positive for attractive forces. A classic case of a linkage between the sign of the scattering length and the presence or absence of a bound state is the NN scattering length. With a zero spin, there is no bound state, and the scattering length is positive. In the triplet channel there is a bound state (the deuteron), and the scattering length is negative. Accordingly, when there is a change in the depth of the potential, the scattering amplitude and thus the level shift change a sign at the point corresponding to the appearance of an eigenlevel in the nuclear potential. As mentioned in Section 4, in this case Eq. (4.7) and Eq. (7.1), which corresponds to it, are not applicable, and in order to study the effect we need an exact solution of the eigenvalue equation. This approach was used to study the motion of atomic S levels in Ref. 42. In particular, calculations were carried out there for the motion of Coulomb levels as a function of the depth of a square potential well corresponding to the nuclear $p\bar{p}$ attraction. Figure 26 shows the calculated results. They are in complete correspondence with the motion of quasinuclear levels as a function of the intensity of the annihilation interaction, studied in Section 4. As the depth of the nuclear potential well, V, increases, the 1S level moves downward $[\Delta E_1(S) < 0]$. When the variable V reaches a certain critical value V_{1c}, which corresponds to the appearance of an eigenlevel in the nuclear potential well, the 1S level moves far downward, and "its

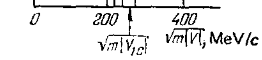


FIG. 26. Shift of the S levels of the $p\bar{p}$ atom as a function of the depth V of the square well corresponding to the nuclear interaction.[42] The vertical dashed lines show the narrow region in which there is a change in structure: $|\delta V/V| \approx R/a$, where $\delta V \equiv V - V_{nc}$, is the critical value of V at which a level with $n-1$ radial nodes appears in the nuclear potential well.

---

[30]To avoid any confusion, we emphasize that Eq. (7.3) is the definition of the quantity U_0. The true potential of the $N\bar{N}$ nuclear interaction would be approximately equal to U_0 if the Born approximation were valid.

[31]We can estimate $|U_0|$ for a given R by assuming that the effective well is not "shallow" ($|U_0| > 1/mR^2$).

[32]The actual calculations in Ref. 60 were direct calculations of the position of the 1S level in the potential $V_c + V_{BP}$ (V_c is the Coulomb interaction, and V_{BP} is the BP nuclear potential). This calculation is equivalent to a calculation of Ref for the BP potential and the use of Eq. (7.1) if there is no quasinuclear S level near an atomic level. Other approaches to the calculation of the shifts of $\bar{p}$ atoms are discussed in Refs. 61-63.

position" is nearly assumed by the $2S$ level, which is, however, shifted slightly upward with respect to the Coulomb $1S$ level. This "former" $2S$ level now appears as a slightly shifted $1S$ level with $\Delta E_1(S)>0$. The fact that the "former $2S$ state" actually has a radial node is now inconsequential for atomic effects, since this node shifts to very small nuclear distances. The situation is analogous for the other Coulomb levels: the position of the nS level is assumed by the $(n+1)S$ level. In other words, at $V=V_{1c}$ there is a complete restructuring of the atomic spectrum, which, however, is completed very rapidly, as soon as the quantity $\delta V=|V-V_c|$ becomes larger than $(R/a)V_{1c}$ (we recall that $R/a\lesssim 0.03$). As V increases further the shift remains small until the next critical value, V_{2c}, is reached; in a small neighborhood of this critical value there is again a complete change in the structure of the spectrum. This second critical point corresponds to the appearance of a second level, with the same quantum numbers and with a single radial node, in the nuclear potential well. The pattern repeats itself at the succeeding critical points $V_{3c},\ldots, V_{nc}$, at which bound states with $2,\ldots n-1$ radial nodes appear in the nuclear potential. We can say that the positive shift of the atomic S levels occurs because, after a change in the structure of the spectrum, the former $(n+1)S$ level "does not manage" to reach the nS level. We emphasize again that since the ratio R/a is small the range of well depths V within which the shifts are large and the atomic spectrum is changed beyond recognition is extremely narrow. At positions between the critical values V_{nc} the shifts of the atomic levels are small and are described by Eq. (7.1). It is seen from Fig. 26 that a positive shift of the atomic $1S$ levels in the attractive nuclear potential is possible only if the potential well is so deep that there is a nuclear eigenlevel in it. Experimental observation of the "expulsion" of the $1S$ level of the $p\bar{p}$ atom would thus be evidence of the existence of an $N\bar{N}$ quasinuclear state. A positive shift of the $1S$ level of the $p\bar{p}$ atom was also predicted theoretically by Caser and Omnes.[61] They used Eq. (7.1), but instead of the actual value of $\mathrm{Re} f$ they used $\mathrm{Re} f^{\mathrm{opt}}$, where f^{opt} is the scattering length calculated from the optical model. The reason that $\Delta E_1(S)$ turns out to be positive in this calculation is completely different: the positive value simply reflects the fact that in the optical model generally all the levels are expelled because of absorption (Fig. 9). The applicability of the optical model (especially in the case of strong absorption) for calculating the spectrum of discrete states was covered in Section 4. Nevertheless, the different physical nature of the level expulsion in the $\bar{p}p$ atom due to the strong interaction found in Refs. 60 and 61 could be reflected experimentally in the following manner: in the optical model,[61] positive shifts should be found for all levels of the $\bar{p}p$ atom, while according to the approach used in Ref. 60 (a real potential for the $\bar{p}p$ nuclear interaction) the only atomic levels which are expelled are those whose quantum numbers are the same as those of the $N\bar{N}$ quasinuclear states. Specifically, taking into account the theoretical spectrum of quasinuclear levels (Section 5), we could expect that the shifts of the atomic terms would be negative beginning with $l\gtrsim 4$. In other words, the theory predicts a definite correlation between the shift of an atomic level and the existence of an $N\bar{N}$ quasinuclear state with the same quantum numbers.

Up to this point we have been talking primarily about the shift of atomic S levels. We have estimated the annihilation width $\Gamma_{na}(S)$ in order of magnitude; a more accurate estimate yields[60]

$$\Gamma_{na}(S)=\frac{0.3}{n^3}\,(\mathrm{keV}). \tag{7.8}$$

A similar result was found by Caser and Omnes[61] [in contrast with the calculation of $\Delta E_n(S)$ with the help of the optical model, an estimate of $\Gamma_{na}(S)$ through a replacement of σ_a by σ_a^{opt} in Eq. (7.2) is completely legitimate, provided, of course, that there is no quasinuclear resonance in the S wave which lies very close to the threshold].

The magnitude of the annihilation widths of the levels of the $\bar{p}p$ and $\bar{p}d$ atoms has an important effect on the intensities of the x-ray lines in the emission spectrum of these atoms.[33)] An experimental study of the yield of monoenergetic x radiation in the annihilation of antiprotons stopped in liquid and gaseous hydrogen (or deuterium) could yield extremely valuable information on the probability for $N\bar{N}$ annihilation from states with a definite orbital angular momentum.[34)] We cannot examine these interesting effects in more detail here; we will simply refer the reader to the detailed theoretical papers.[64]

Quasinuclear $N\bar{N}$ bound states should be manifested directly in electromagnetic transitions from states of the $\bar{p}p$ atom to quasinuclear levels (Fig. 27). This statement means that the annihilation of the antiprotons stopped in hydrogen and deuterium should be accompanied by the emission of monoenergetic γ rays with an energy ~100 MeV. The predominant transitions should evidently be $E1$ transitions from S states of the $\bar{p}p$ atom. The radiation width $\Gamma_{n\gamma}(a)$ for such a transition, with energy ω, is easily estimated. Specifically, it can be found from the equation

$$\Gamma_{n\gamma}(\omega)\approx\frac{\alpha}{4}\left(\frac{R}{a}\right)^3(R\omega)^2\omega. \tag{7.9}$$

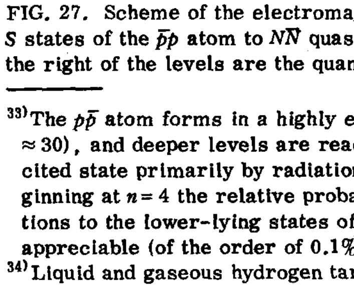


FIG. 27. Scheme of the electromagnetic transitions from the S states of the $\bar{p}p$ atom to $N\bar{N}$ quasinuclear states. Shown at the right of the levels are the quantum numbers J^P.

[33)] The $p\bar{p}$ atom forms in a highly excited state ($n=\sqrt{m/2m_e}\approx 30$), and deeper levels are reached from this highly excited state primarily by radiationless transitions. Beginning at $n=4$ the relative probability for radiative transitions to the lower-lying states of the $\bar{p}p$ atom becomes appreciable (of the order of 0.1%).

[34)] Liquid and gaseous hydrogen targets differ in the role played by the Stark effect due to the field of the hydrogen (or deuterium) atoms which the newly formed $\bar{p}p$ atom "enters."[64]

TABLE V. Energies and relative intensities of the γ rays (per annihilation event) emitted in electromagnetic transitions from the S states of the $\bar{p}p$ atom to a $N\bar{N}$ quasinuclear state.[29] Annihilation width of the S states, $\Gamma_{na} \approx 0.3$ keV$/n^3$ (the asterisk shows an $M1$ transition).

Transition energy, MeV	Relative probability, $(\Gamma_{n\gamma}/\Gamma_{na})\cdot 10^3$	Transition energy, MeV	Relative probability, $(\Gamma_{n\gamma}/\Gamma_{na})\cdot 10^3$
30	0.4	260	4.9
66	2.4	470	3.8
103	3.5	498*	0.9*
109	1.3	591	1.3
156	0.6		

Setting $R \approx 1$ F and $\omega \approx 100$ MeV, we find

$$\Gamma_{n\gamma}(\omega) \approx \frac{0.25}{n^3}\,(\mathrm{eV}). \tag{7.10}$$

Now using (7.8) we find the relative intensity of the γ line per annihilation event:

$$\frac{\Gamma_{n\gamma}}{\Gamma_{na}} \approx 10^{-3}. \tag{7.11}$$

More accurate calculations, based on a version of the OBEP model, were carried out by Dal'karov *et al.*[29] Table V shows the relative intensities of the γ lines per annihilation event [for the annihilation widths corresponding to Eq. (7.8)]. These intensities correspond in order of magnitude to the estimate in (7.11). Figure 28 shows the theoretically expected γ spectrum (the widths of the quasinuclear levels are taken to be 10 MeV; this value also determines the width of the γ lines). It is seen from this figure that the eight lines fall in three groups: near 100 MeV, in the interval 200–300 MeV, and in the interval 400–800 MeV (the 591-MeV lines is the most doubtful line according to the nonrelativistic approximation). According to Table V, the overall intensity of hard γ radiation in the discrete spectrum is about 2% per annihilation of a $\bar{p}p$ atom and thus in general per annihilation of a slow antiproton, since about 95% of all the antiprotons with momenta below or of the order of 200 MeV/c are annihilated from states of the $\bar{p}p$ atom.[35)]

In this connection, let us consider a possible interpretation of the experimental data on $\bar{p}d$ annihilation obtained in Refs. 25 and 26 and reproduced in Section 2 of the present review. According to Refs. 25 and 26, about 70% of the $\bar{p}d$ annihilation events are accompanied by the emission of hard γ rays, with an average energy of about 180 MeV. This energy is approximately the average energy (264 MeV) of the theoretical γ spectrum corresponding to Table V and Fig. 28. The intensity of the hard γ radiation is seen to be 35 times that predicted theoretically for $\bar{p}p$ annihilation. In principle, there are several possible explanations for this discrepancy. The first argument is that the annihilation widths of the atomic S levels are much smaller than assumed in the calculations (we recall that the annihilation cross section σ_a, which governs the atomic widths, has not been measured for very small momenta; instead, it has been found by some extrapolation procedure or other). For example, if we assume that the annihilation widths of the ${}^3S_1$ triplet states are two orders of magnitude smaller than the widths predicted by (7.8) [because of the presumed smallness of $\sigma_a({}^3S_1)$], then we can obtain agreement with the experimental data of Refs. 25 and 26 (the cross section for $\bar{p}p$ annihilation from the S state, averaged over the spin, decreases by a factor of only four if the cross section for annihilation from the ${}^1S_0$ singlet state is left unchanged).[36)] This hypothesis also leads to the conclusion that intense lines of hard γ radiation should be observed not only in $\bar{p}d$ annihilation but also in $\bar{p}p$ annihilation. Furthermore, in the annihilation of antiprotons stopped in hydrogen or deuterium we should observe an intense K_α $(2P \to 1S)$ line of the atomic x-ray spectrum. Its intensity per annihilation event should be about 10%, instead of the 0.1% corresponding to the usual annihilation width of the S state [Eq. (7.8)].[37)]

Another possible explanation for the large excess of hard γ rays in $\bar{p}d$ annihilation is that the excess emission is due to transitions between quasinuclear levels instead of to radiative transitions from atomic states to quasinuclear states (this was the interpretation offered along with the experimental results in Ref. 26). In order to explain the intense emission it would then be necessary to assume that the $\bar{p}d$ annihilation involves an intermediate formation of a "narrow" quasinuclear state, with an annihilation width comparable to the radiation width. In particular, such a state could be formed in a process like (7.1), although the quantitative aspects of the question (the dominance of this channel) are not clear. Finally, another possible source of a discrete spectrum of hard γ rays is represented by radiative transitions to quasinuclear levels of the $2N\bar{N}$ three-particle system. The magnitude of the corresponding transition probabilities, however, is not clear. Further experiments involving a search for and a study of hard discrete γ radiation accompanying $\bar{p}$ annihilation in hydrogen and deuterium would undoubtedly be extremely interesting.

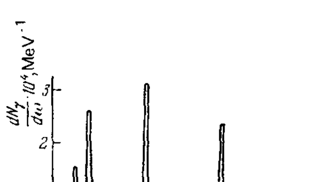


FIG. 28. Spectrum of γ rays emitted in electromagnetic transitions from the S states of the $\bar{p}p$ atom to $N\bar{N}$ quasinuclear states. The annihilation widths of the quasinuclear P levels are taken to be 10 MeV (the width of the γ lines is thus determined).

[35)]Strictly speaking, the number given in the text refers to annihilation events from S states alone. Annihilation from states with $l \neq 0$, however, makes only a small contribution to the total probability for the annihilation of $\bar{p}p$ atoms.

[36)]The physical reason for the presumed smallness of $\sigma_a({}^3S_1)$ may be the short-range repulsion which arises from the exchange of heavy mesons, not taken into account in the OBEP model, or from relativistic effects.

[37)]This result was found by Markushin.[64]

As mentioned already in Section 2, there is evidence of an increased probability for annihilation from P states of $\bar{p}p$ and $\bar{p}d$ atoms (the process $\bar{p}p \to 2\pi^0$). In principle, this increase could be due to a near-threshold nuclear P state of $N\bar{N}$ with a low binding energy. Then the average distance R between the p and $\bar{p}$ in this state could be comparatively large, 3–4 F, so that there would be a substantial increase in the annihilation width of the atomic P state, because of the relation

$$\Gamma_{na}(P) \propto (R/a)^5.$$

A consequence should be a substantial decrease in the yield of x rays in $nP \to 1S$ transitions (in particular, the intensity of the K_α line should fall by about an order of magnitude[64]).

In concluding this section, we wish to emphasize that experimental data on $\bar{p}p$ and $\bar{p}d$ atoms can yield very useful information about the nuclear interaction and the dynamics of the $N\bar{N}$ annihilation process.

8. CONCLUSION

It has been shown in this review that the existence of narrow quasinuclear states of the $N\bar{N}$ system is compatible with large annihilation cross sections. In a certain sense, the large cross sections and the appearance of the discrete quasinuclear spectrum result from the same factors, of which there are two: the rather strong attraction between the particles and the small size of the annihilation region. A decrease in the depth of the effective potential well erases the discrete spectrum and reduces the annihilation cross section, because of a decrease in the modulus of the continuum wave function at small interparticle distances, corresponding to the size of the annihilation region. An increase in the radius of this region, on the other hand, at a fixed depth of the potential, "expels" the levels and leads to a decrease in the annihilation cross section, since the wave function of unbounded motion in the case of attractive forces has its largest value at small distances (in other words, if the annihilation radius is too large, the wave function does not manage to increase to the limiting values). We have also learned that the small size of the annihilation region in comparison with the radius of the quasinuclear orbit makes the annihilation shifts and widths of the energy levels of the discrete spectrum small. This same smallness parameter makes it improbable that there would be any radical change in the expected level spectrum if the energy of the quasinuclear state should by chance be approximately equal to the mass of a meson of a different nature. All this means that the general picture of the spectrum of quasinuclear states is stable with respect to uncertainties in the $N\bar{N}$ interaction at small distances (of the order of $1/m$) and, especially, with respect to annihilation processes. It follows that annihilation need be taken into account only for estimating the widths of quasinuclear states, at least in calculations whose only purpose is to determine the qualitative effects due to the existence of these states.[38)] We thus assume that the observable consequences of the theory should, on the whole, be confirmed by experiment if the current ideas regarding the NN and $N\bar{N}$ interactions at nonrelativistic energies are correct. Let us list the most important facts which are predicted by the theory.

a) There are several (10–20) nuclear-like bound and resonant $N\bar{N}$ states with comparatively small total widths (from 1 to 100 MeV in order of magnitude). We emphasize that there could hardly be a situation in potential theory in which there are narrow above threshold resonances [for example, $N\bar{N}$ (1940)] but no bound states. As a rule, there should be at least one bound state for each above threshold resonance.[39)]

b) A distinctive feature of the $N\bar{N}$ bosons is of course their "strong coupling" with the $N\bar{N}$ channel. These words have a quantitative meaning in the theory of quasinuclear bosons: the reduced widths in the $N\bar{N}$ channel are of the order of $1/mR^2$, where R is the radius of the state. It follows that the theory of quasinuclear resonances, while predicting a strong coupling of the quasinuclear boson with the $N\bar{N}$ channel, still limits the values of $\Gamma_{N\bar{N}}$ for the above threshold resonances which can be interpreted physically: excessively large widths $\Gamma_{N\bar{N}}$ (>200 MeV) indicate either that the radius of the state is too small (and the "two-particle" nonrelativistic approximation could hardly be applicable, even for drawing qualitative conclusions) or that the properties of the resonance are unacceptably sensitive to the details of the potential.

c) Several conclusions follow from b).

1. It is preferable to seek quasinuclear bosons ($N\bar{N}$) and to study their properties in "formation" experiments with beams of slow antiprotons. Especially interesting for detecting $N\bar{N}$ bound states would be experiments with nuclear targets (in particular, a deuteron target). It is also important to study the energy dependence of the nonresonant annihilation of slow antiprotons: as demonstrated in Section 4, the strong attraction between N and $\bar{N}$ leads to a deviation from the $1/v$ law. It is important to know the magnitude and behavior of this deviation in order to find information about $N\bar{N}$ forces.

2. Bound $N\bar{N}$ states should be seen in electromagnetic transitions from a state of a Coulombic $\bar{p}p$ atom to quasi-

---

[38)] We might add that at present we do not see any reliable method for calculating (rather than estimating) small annihilation shifts, since the amplitude for the annihilation scattering, f_a is not known. Any models for this quantity should correctly incorporate the analytic properties of the amplitudes of the multichannel problem. In order to find more or less reliable quantitative results, we thus need, at a minimum, calculations by the coupled-channel method. For the case of interest here, however, this method is not easy to parametrize on the basis of observable quantities. We note that a multichannel approach to $N\bar{N}$ quasinuclear states was announced (in words alone) in Refs. 65.

[39)] This follows from the behavior of the Regge potential trajectories if the potential is independent of the energy (by changing the Hamiltonian from state to state it is possible, of course, to obtain any prespecified result).

nuclear states. These transitions should be accompanied by the emission of a discrete spectrum of γ rays with an energy of ~100 MeV. The theory predicts several γ lines, corresponding to $E1$ and $M1$ transitions from the S states of the $\bar{p}p$ atom to quasinuclear P and D states. The intensities of these lines depend on the annihilation widths of the atomic S levels. For an annihilation width of the order of 300 eV for the 1S level, the total intensity of the lines in the discrete γ spectrum is about 1% per annihilation event, while the intensity of an individual line is correspondingly of the order of 0.1%. The widths of these γ lines should be of the order of 10 MeV or less.

3. The presence of bound quasinuclear $N\bar{N}$ states has important effects on the shift and widths of the levels of the $\bar{p}p$ atom. The theory links these quantities with the properties of the quasinuclear states, so a study of the level spectrum of the $\bar{p}p$ atom and of the energies and intensities of the corresponding x-ray lines in liquid and gaseous hydrogen targets would yield useful information on the spectrum of quasinuclear bosons and on the $N\bar{N}$ nuclear interaction. In particular, the theory predicts an upward shift (of about 1 keV) for the atomic 1S level, although the $N\bar{N}$ interaction is attractive. A shift of this type resulting from a short-range attractive potential is possible only if this potential is so deep that there is at least one eigenlevel in it (in this case an $N\bar{N}$ quasinuclear level). Accordingly, if a positive shift of the 1S level of the $p\bar{p}$ atom is observed experimentally (that is, if there is an observed decrease in the measured energy of the K_α line of the x-ray spectrum in comparison with the "purely Coulombic" value), this fact alone would constitute evidence for the existence of an $N\bar{N}$ quasinuclear state and thus for a strong attractive interaction between N and $\bar{N}$ in the S state. These remarks seem to demonstrate clearly that significant information about the forces and the quasinuclear levels can be extracted from sufficiently accurate measurements of the nuclear shifts and the widths of atomic levels.

d) Since the nuclear attraction in the $N\bar{N}$ system is due primarily to ω exchange, which can occur between baryons with any isospins, it seems extremely likely that quasinuclear systems of the $Y\bar{N}$ and $Y\bar{Y}$ types exist.

e) Variational calculations show that $2N\bar{N}$ and $2N2\bar{N}$ quasinuclear systems, i.e., heavy fermion and boson quasinuclear resonances, with masses near 3 and 4 GeV, respectively, are completely realistic. Remarkably, even in the $2N2\bar{N}$ system there can be comparatively narrow states (with a width on the order of 20–30 MeV, including "exotic" states with isospin 2.

It can be seen from the discussion above that the theory of quasinuclear states of the $N\bar{N}$ systems and similar systems gives quite definite predictions regarding several effects which can be detected and studied experimentally. In this sense the model is physically meaningful and amenable to testing.

The idea that there are mesons which are strongly coupled with the $N\bar{N}$ channel also arises within the framework of quark dual models (see Ref. 66 and the literature cited there). A question which naturally arises is how these two approaches are related, but it is not a simple matter to answer this question, for two reasons.

First, the quark dual model has not yielded observable predictions or consequences like those listed above (aside from an indication of the very existence of mesons which are strongly coupled with the $N\bar{N}$ channel, which is actually postulated). What is the general nature of the spectrum of "$N\bar{N}$ dual mesons?" Why do their masses group near the $(2m)$ threshold and only there? Should subthreshold mesons (with a mass $<2m$) exist or not? What are the expected pion-channel widths in order of magnitude? What causes these widths? What does "strong coupling with the $N\bar{N}$ channel" mean quantitatively? (What is the order of magnitude of the partial widths for this channel, and how are these widths related to other measurable characteristics?) How does the existence of such mesons affect the variety of processes by which slow antinucleons interact with nucleons and nuclei? (For example, how does the existence of such mesons affect the behavior of the nonresonant annihilation cross section at low energies, the properties of $\bar{p}p$ atoms, etc.?) Since the dual model does not answer these questions, it is hardly possible to distinguish between dual and quasinuclear $N\bar{N}$ mesons from the purely phenomenological standpoint, that is, by comparing the physical consequences of these objects (consequences which have been detected or which are detectable in principle).

Second, the quark dual model is rather hazy on this point even in the purely theoretical aspect. According to this model, mesons which are strongly coupled with the $N\bar{N}$ channel consist of four quarks (two quarks and two antiquarks). The quasinuclear mesons $N\bar{N}$, on the other hand, are six-quark systems (like the deuteron), according to the quark theory. It would seem that there is an obvious difference, but the fact of the matter is that in the dual model "strong coupling with the $N\bar{N}$ channel" is postulated. As mentioned above, it is not completely clear what the physical meaning of this postulate is, since this "strong coupling" of a four-quark system with a six-quark system occurs only at the time of a "mass breakout of quarks from prison."

When certain channels $|a\rangle$ and $|b\rangle$ are strongly coupled, the usual meaning in quantum theory is that the system is simultaneously in both these states, $|a\rangle$ and $|b\rangle$. For the case at hand, the meaning is that a meson which is "strongly coupled with the $N\bar{N}$ channel" is a "six-quark" system to the same extent that it is a "four-quark" system. If this is the case, the answer to the question of whether the dual and quasinuclear models are similar or different depends on which properties are assigned to the four-quark state. If it is assumed, for example, that the four-quark system is of the order of $1/m$ in size, while the radius of the six-quark $N\bar{N}$ state is $R \gg 1/m$, and if it is assumed that the mass spectrum of these mesons near $2m$ is governed primarily by the long-range interaction between N and $\bar{N}$, then the two models are obviously compatible, and the quark model actually converts into the quasinuclear model. If, on the other hand, in contradiction of the requirements of unitarity, and despite the "strong coupling" between the

$|a\rangle$ and $|b\rangle$ channels, the system is in state $|a\rangle$ because of the mystical rules of quark theory, then the dual and quasinuclear models are fundamentally different. A further discussion of the relationship between the quark and quasinuclear models would seem to go beyond the scope of the present paper; this topic is also discussed by Chew and Rosenzweig.[67]

In summary, it could be said that a study of $B\bar{B}$ systems, especially $N\bar{N}$ systems, is becoming an extremely interesting field of modern physics—a field which lies at the boundary between nuclear physics and particle physics and which is erasing this boundary to some extent. It is quite probable that we will learn much about "ordinary" NN nuclear forces by studying the properties of $N\bar{N}$ systems. On the other hand, when the nuclear-like $B\bar{B}$ systems are singled out of the spectrum of heavy resonances, particle physics can gain from a study of these systems, because otherwise unobtainable information can be acquired on the interaction of unstable baryons.

The author thanks L. N. Bogdanova, O. D. Dal'karov, B. S. Kerbikov, A. E. Kudryavtsev, V. E. Markushin, and N. A. Aksenova for many discussions and for assistance in preparing this paper for publication.

[1]G. E. Brown and A. D. Jackson, in: The Nucleon–Antinucleon Interaction, North-Holland, Amsterdam; American Elsevier, New York, 1976.

[2]T. E. Kalogeropoulos, in: Proc. of the Sixth Intern. Conf. on Nuclear Structure and High Energy Physics, Santa Fe, June 1975.

[3]A. S. Carroll, I-H. Chiang, T. F. Kycia, K. K. Li, P. O. Mazur, D. N. Michael, P. Mocket, D. C. Rahm, and R. Rubinstein, Phys. Rev. Lett. **32**, 247 (1974); T. E. Kalogeropoulos and G. S. Tzanakos, Phys. Rev. Lett. **34**, 1047 (1975); V. Chaloupka, H. Dreverman, F. Marzano, L. Montanet, P. Schmid, J. R. Fry, H. Johnstad, J. M. Lesceuk, J. S. Skura, A. Bettini, M. Cresti, L. Peruzzo, R. Rossi, R. Bizzari, M. Iori, E. Castelli, C. Omero, and P. Poropat, Phys. Lett. Ser. B **61**, 487 (1976); W. Bruckner, B. Granz, D. Ingham, K. Killian, V. Lynen, J. Niewisch, B. Dietrzyk, B. Povh, H. D. Ritter, and S. Schroder, Preprint, CERN, 1976.

[4]E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

[5]H. Bethe and J. Hamilton, Nuovo Cimento **4**, 1 (1956).

[6]L. M. Afrikyan, Zh. Eksp. Teor. Fiz. **30**, 734 (1956) [Sov. Phys. JETP **3**, 503 (1956)].

[7]A. Martin, Phys. Rev. **124**, 614 (1961).

[8]P. É. Nemirovskiĭ and Yu. F. Strokov, Zh. Eksp. Teor. Fiz. **46**, 1379 (1964) [Sov. Phys. JETP **19**, 932 (1964)].

[9]R. J. N. Phillips, Rev. Mod. Phys. **39**, 681 (1967); R. A. Bryan and R. J. N. Phillips, Nucl. Phys. Ser. B **5**, 201 (1968).

[10]R. Omnes, Phys. Rept. Ser. C **3**, No. 1 (1972).

[11]O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 402 (1969) [JETP Lett. **10**, 257 (1969)]; Yad. Fiz. **11**, 889 (1970) [Sov. J. Nucl. Phys. **11**, 496 (1970)].

[12]O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. **59**, 1363 (1970) [Sov. Phys. JETP **32**, 744 (1971)].

[13]O. D. Dalkarov, V. B. Mandelzweig, and I. S. Shapiro, Nucl. Phys. Ser. B **21**, 88 (1970).

[14]L. N. Bogdanova, O. D. Dal'karov, V. B. Mandel'tsveĭg, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. **61**, 2242 (1971) [Sov. Phys. JETP **34**, 1200 (1972)].

[15]I. S. Shapiro, Usp. Fiz. Nauk. **109**, 431 (1973) [Sov. Phys. Usp. **16**, 173 (1973)].

[16]L. N. Bogdanova, O. D. Dalkarov, and I. S. Shapiro, Ann. Phys. (N.Y.) **84**, 261 (1974).

[17]L. N. Bogdanova, O. D. Dalkarov, B. O. Kerbikov, and I. S. Shapiro, in: Proc. of the Fourth Intern. Symp. on Nucleon–Antinucleon Interactions, ed. T. E. Kalogeropoulos and K. C. Wali, Syracuse, N.Y., Vol. 2, Part VIII, 1975, p. 1.

[18]C. Dover, in: Proc. of the Fourth Intern. Symp. on Nucleon–Antinucleon Interactions, ed. T. E. Kalogeropoulos and K. C. Wali, Syracuse, N.Y., Vol. 2, Part VIII, 1975, p. 37.

[19]J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

[20]J. S. Ball and G. F. Chew, Phys. Rev. **109**, 1385 (1958).

[21]*NN* Compilation, Particle Data Group, LBL-58, May 1972.

[22]a) B. French, CERN/D.PH. II/Phys. 75–38.-August 1975; b) E. Eisenhandler, in: Third European Symp. on Antinucleon–Nucleon Interactions, Stockholm, July 1976.

[23]P. Benkheiri, G. Bocurot, *et al.*, Preprint LAL P 1977/3-21/, Orsay, April 1977.

[24]V. G. Grishin, V. A. Nikitin, and M. I. Podgoretskiĭ, Preprint R-480, Joint Institute for Nuclear Research, Dubna, 1960; H. J. Lipkin and M. Peshkin, Phys. Rev. Lett. **28**, 862 (1972).

[25]T. E. Kalogeropoulos, T. A. Fillipas, *et al.*, Phys. Rev. Lett. **33**, 1631 (1974).

[26]T. E. Kalogeropoulos, A. Vayaki, G. Grammatikakis, *et al.*, Phys. Rev. Lett. **33**, 1635 (1974).

[27]G. Ghesquire, CERN Report No. 74-18.-1974, p. 436.

[28]T. E. Kalogeropoulos, D. N. Michael, D. I. Lowenstein, A. M. Nathan, N. P. Samios, and A. Z. Schwarzschild, Phys. Rev. Lett. **35**, 824 (1975).

[29]O. D. Dal'karov, V. M. Samoĭlov, and I. S. Shapiro, Yad. Fiz. **17**, 1084 (1973) [Sov. J. Nucl. Phys. **17**, 566 (1973)].

[30]S. Devons, T. Kozlowski, *et al.*, Phys. Rev. Lett. **27**, 1614 (1971).

[31]L. Gray, T. Papadopoulou, *et al.*, Phys. Rev. Lett. **30**, 1091 (1973).

[32]G. Bassompierre, G. Binder, P. Dalpiaz, P. F. Dalpiaz, G. Gissinger, S. Jacquey, C. Peroni, A. Ruzza, M. A. Schneegans, and L. Techio, Cited in Ref. 22b.

[33]O. D. Dal'karov, V. B. Mandel'tsveĭg, and V. A. Khoze, Pis'ma Zh. Eksp. Teor. Fiz. **14**, 131 (1971) [JETP Lett. **14**, 87 (1971)]; O. D. Dal'karov, V. A. Khoze, and I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 534 (1974) [JETP Lett. **19**, 282 (1974)].

[34]G. Bassompierre, G. Binder, *et al.*, Phys. Lett. Ser. B **65**, 397 (1976); see also the literature cited there.

[35]CERN Courier **17**, No. 1/2 (January–February 1977).

[36]L. Gray, P. Hagerty, and T. E. Kalogeropoulos, Phys. Rev. Lett. **26**, 1491 (1971).

[37]L. Gray, T. Papadopoulou, *et al.*, Phys. Rev. Lett. **30**, 1091 (1973).

[38]L. N. Bogdonova, O. D. Dalkarov, and I. S. Shapiro, Phys. Rev. Lett. **28**, 1418 (1972).

[39]V. De Alfaro and T. Regge, Potential Scattering, American Elsevier, New York, 1965.

[40]a) T. E. O. Ericson, Review Talk at the Fourth Intern. Conf. on High Energy Physics and Nuclear Structure, Dubna, September 1971; TH-1410-CERN; b) J. H. Koch, M. M. Sternheim, and J. F. Walker, Phys. Rev. Lett. **26**, 1456 (1971).

[41]F. Myhrer and A. Thomas, Phys. Lett. Ser. B **64**, 59 (1976); F. Myhrer and A. Gersten, Nuovo Cimento. Ser. A **37**, 21 (1977); I. Ulehla, Preprint, Saclay, 1976.

[42]A. E. Kudryavtsev, V. E. Markushin, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. **74**, 432 (1978) [Sov. Phys. JETP **47**, 225 (1978)].

[43]V. S. Popov, Zh. Eksp. Teor. Fiz. **60**, 1229 (1970) [Sov. Phys. JETP **33**, 665 (1971)].

[44]B. O. Kerbikov, A. E. Kudryavtsev, V. E. Markushin, and I. S. Shapiro, Pis'ma Zh. Eksp. Teor. Fiz. **26**, 505 (1977) [JETP Lett. **26**, 368 (1977)].

[45]R. G. Newton, Scattering Theory of Waves and Particles, McGraw—Hill, New York, 1966 (Russ. Transl., Mir, M., 1969).
[46]V. B. Berestetskiĭ, E. M. Lifshits, and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory), Nauka, M., 1968 [English transl. Addison–Wesley, Reading, Mass., 1971].
[47]M. Nagels and T. Rijken, Theses, Nijmegen University, 1975.
[48]G. Schierholz and S. Wagner, Nucl. Phys. Ser. B **32**, 306 (1971).
[49]C. B. Dover and M. Goldhaber, BNL, Preprint, June 1976.
[50]J. M. Richard, M. Lacombe and R. Vihn Mau, Phys. Lett. Ser. B **64**, 121 (1976).
[51]L. N. Bogdanova, O. D. Dal'karov, and I. S. Shapiro, Zh. Eksp. Teor. Fiz. **70**, 805 (1976) [Sov. Phys. JETP **44**, 417 (1976)].
[52]N. Nagels, T. Rijken, and J. J. De Swart, Preprint, Nijmegen University, 1976.
[53]L. M. Bogdanova, O. D. Dalkarov, and I. S. Shapiro, Preprint ITEP-5, Institute of Theoretical and Experimental Physics, Moscow, 1977.
[54]O. D. Dal'karov and V. B. Mandel'tsveĭg, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 429 (1969) [JETP Lett. **10**, 275 (1969)].
[55]O. D. Dal'karov, B. O. Kerbikov, I. A. Rumyantsev, and I. S. Shapiro, Yad. Fiz. **17**, 1321 (1973) [Sov. J. Nucl. Phys. **17**, 688 (1973)].
[56]E. S. Birger, B. O. Kerbikov, N. B. Konyukhova, and I. S. Shapiro, Yad. Fiz. **17**, 178 (1973) [Sov. J. Nucl. Phys. **17**, 92 (1973)].
[57]A. M. Badalyan and Yu. A. Simonov, Yad. Fiz. **3**, 1032 (1966) [Sov. J. Nucl. Phys. **3**, 755 (1966)]; **5**, 88 (1967) [**5**, 60 (1967)].
[58]G. Alexander *et al.*, Nucl. Phys. Ser. B **45**, 29 (1972).
[59]O. D. Dal'karov and V. E. Markushin, Yad. Fiz. **21**, 878 (1975) [Sov. J. Nucl. Phys. **21**, 452 (1975)].
[60]O. D. Dal'karov and V. M. Samoĭlov, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 353 (1972) [JETP Lett. **16**, 249 (1972)].
[61]S. Caser and R. Omnes, Phys. Lett. Ser. B **39**, 369 (1972).
[62]T. E. O. Ericson, Cited in Ref. 22b.
[63]L. Tauscher, Cited in Ref. 2; AIP Proc. No. 26, 541 (1975).
[64]O. D. Dalkarov, V. O. Kerbikov, and V. E. Markushin, Preprint ITEP-25, Institute of Theoretical and Experimental Physics, Moscow, 1976; V. E. Markushin, Preprint ITEP-164, Institute of Theoretical and Experimental Physics, Moscow, 1976.
[65]J. Koplik, Nucl. Phys. Ser. B **82**, 93 (1974); G. F. Chew and J. Koplik, Nucl. Phys. Ser. B **79**, 365 (1974).
[66]G. F. Chew and C. Rosenzweig, Phys. Lett. Ser. B **58**, 93 (1975); Phys. Rev. Ser. D **12**, 3907 (1975); Nucl. Phys. Ser. B **104**, 290 (1976).
[67]G. F. Chew, Preprint LBL-5391, 1976; C. Rosenzweig, Phys. Rev. Lett. **36**, 697 (1976).
[68]G. Bassompierre, G. Biuder, P. Dalpiaz, P. F. Dalpiaz, G. Gissinger, B. Jacquey, C. Peroni, M. A. Schneegans, and L. Tecchio, Contributed paper to European Conference on Particle Physics, Budapest, Hungary, July 4-9, 1977.

Translated by Dave Parsons