Superconductivity and elementary particles[1)]

D. A. Kirzhnits

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR
Usp. Fiz. Nauk **125**, 169–194 (May 1978)

The article contains a brief historical review of the mutual influences on one another of elementary-particle theory and many-body theory. The main attention is devoted to the idea of spontaneous symmetry breaking and the unified theory of particles based on it. The intimate and far reaching analogy between unified theory and the theory of superconductivity is traced. Some consequences of this analogy for particle physics and cosmology are considered.

PACS numbers: 11.30.Qc, 11.80.Jy, 12.20.Hx, 74.90. + n

1. INTRODUCTION

Created about half a century ago, quantum mechanics immediately sprouted two mighty offshoots. One of them—quantum many-body theory—became the theoretical foundation of spectroscopy, quantum chemistry, solid-state physics, nuclear physics, and other sciences with a direct practical importance for humanity. The other—quantum field theory—provided the basis of the physics of elementary particles, by opening up ways of describing the fundamental regularities of the structure of matter. During their time of coexistence, these two very important physical theories have had a significant influence on one another, and this mutual influence has become particularly clear in recent years, becoming one of the sources of current progress in elementary-particle physics.

From the thirties to the fifties, the approach based on quantum field theory reigned undisputed in the theory of elementary particles. In this way, theory achieved a number of outstanding sucesses; the prediction of antiparticles and processes of production and annihilation of pairs, the prediction of mesons, the elimination of divergences in quantum electrodynamics, the creation of the theory of radiative effects, and so forth.

Despite these successes, at the end of the fifties serious doubts arose as to the very possibility of a quantum-field description of elementary particles; these doubts arose because of a number of difficulties then encountered by the theory. We may include here the impossibility of eliminating divergences for certain classes of interactions (including the weak interactions), arguments suggesting the vanishing of the interaction itself between particles ("zero charge") in all the then known field-theory models, and the difficulties of describing the strong interaction outside the framework of perturbation theory.

Simultaneously, a number of old "confounded" problems that had as a rule been forgotten during the periods of successful development of the theory resurfaced, strengthening doubts in the applicability of the ordinary quantum-field approach. These questions refer primarily to the problem of measurements in relativistic quantum physics: Is it meaningful to speak of the development of a process of interaction of particles in time or is it only permissible to consider the transition from an initial to a final state of the system; is it permissible to assume that the interaction between particles is a point interaction (local) or is this an unjustified idealization, etc.?

It therefore seemed that one must take seriously an aphorism propounded in those years: "The Hamiltonian method is dead but we must bury the corpse with all the respect it deserves." And, indeed, the successes in elementary-particle theory in the following sixties were almost entirely associated with directions far from quantum field theory—the group, dispersion, and axiomatic approaches, the phenomenological Regge theory, and so forth.

However, in recent years—to the delighted surprise of many theoreticians of the older generation—we have witnessed a genuine revival of quantum-field directions in the theory of elementary particles. At the same time, the theory appears in a new garb, less formalized and with evermore direct physical content. The gulf between elementary-particle theory and other branches of theoretical physics, still keenly felt until very recently, has now closed significantly, giving way to mutual understanding and mutual enrichment.[2)]

One of the most significant achievements to be reckoned to the revived quantum-field approach is the creation of models in which the weak, electromagnetic, and (in a preliminary form) strong interactions of the elementary particles are treated in a unified manner. In the framework of these models, the divergences of the weak interaction, the bugbear of earlier renormalization procedures, has disappeared. The theory of the weak interaction has now achieved the standing of quantum electrodynamics in the sense that one can now calculate any effect of higher order in the interaction. At a more practical level, the unified theories of the particles predicted neutral currents of the weak interaction, these leading to elastic processes already in the lowest order in the weak interaction. In addition, the

[1)] Extended text of report at the seminar of the Department of Theoretical Physics at the P. N. Lebedev Physics Institute in April 1977 dedicated to the memory of I. E. Tamm.

[2)] It is sufficient to point out that a lecture on the modern theory of elementary particles to an audience of solid-state theoreticians now stimulates a lively interested reaction, from which the lecturer himself can benefit, rather than the polite silence of earlier years.

 0038-5670/78/050470-17$01.10

unified theories have strengthened the conjecture that there exists a new property of elementary particles; charm. Both charm and neutral currents were discovered after their prediction.

We see that quantum field theory, far from dying, "slept" like the Sleeping Beauty in a state of lethargy. But of course something more than the kiss of Prince Charming was needed to reawaken her. Many factors combined, among which one of the most important was the adoption of physical ideas taken from many-body theory and, in particular, the theory of superconductivity.

It bears repeating that the phenomenon of superconductivity is not merely one among numerous effects in solid-state physics but is, without exaggeration, the most striking physical phenomenon in which quantum laws are manifested on a macroscopic scale. Accordingly, the theory of superconductivity is not merely just another model in solid-state physics but much more it is a fundamental physical theory based on deep and very general ideas, which have already found application in other branches of solid-state theory, in nuclear theory, and in theoretical astrophysics. It was not for nothing that we had to wait several decades for the microscopic theory of superconductivity.

In this paper, an attempt is made to sketch the general picture of the mutual influences of many-body theory (in particular, the theory of superconductivity) and elementary-particle theory during the last quarter of a century. The main attention is devoted to that line of mutual contact of these theories which leads directly to the modern unified theories of elementary particles. Other important lines relating, for example, to the theory of phase transition near a critical point, will hardly be touched. On the other hand, it has been the intention that the material of the paper, which straddles the junction between many-body theory and quantum field theory, should be accessible to specialists in both fields. For this reason, the exposition does not contain many significant details and is presented at a semiqualitative level, having as its main aim a general representation of the essence of the ideas and of their evolution. Further details about the questions discussed can be found in the cited literature.

2. QUANTUM-FIELD METHODS IN MANY-BODY THEORY

The successes of radio spectroscopy in the immediate post-war years led to the experimental discovery of radiative effects (effects of higher order in the interaction of electrons and photons) in quantum electrodynamics—the Lamb shift of atomic levels and the anomalous magnetic moment of the electron. In the same years, the first accelerators capable of producing elementary particles (pions, etc.) were commissioned.

All this stimulated a powerful development of the formalism of relativistic quantum field theory. The "old" formalism, which copied nonrelativistic quantum mechanics in its basis, was found to be ill suited to the calculation of effects of higher order and for carrying through the program of renormalization, i.e., the extraction of the physically meaningful part of infinite expressions.

The methodological progress achieved in elementary-particle theory by the middle of the fifties was enormous (see the collections of original papers[1] and courses of quantum-field theory[2]). Physicists—both the theoreticians and the experimentalists—were endowed with the simple, perspicuous, and capacious Feynman diagram.[3)] The calculation of effects of higher order was reduced to the almost completely automatic application of simple and unified rules. In his classical work,[3] Weisskopf had needed tens of pages to calculate the electron self-energy in the lowest order of perturbation theory (and moreover the result appeared only as the outcome of the almost complete compensation of many terms—longitudinal, transverse, magnetic, and other energies), but now the calculation of the same quantity can be given to a student in the form of a problem at the blackboard. A number of "exact" methods was also proposed; these made it possible to go beyond the framework of perturbation theory and to carry out investigations of a general nature—the methods of Green's functions, of functional integrals, of the renormalization group, etc.

These successes did not long remain restricted to the theory of elementary particles. From the middle of the fifties, the new methods of quantum field theory began to be applied to many-body theory. Not many years were needed for the creation of a flexible, economic, and effective formalism suitable for the solution of a large number of problems of macroscopic and microscopic physics (see, for example, Refs. 4 and 5).

It should be pointed out that the very possibility of using the technique of quantum field theory is based on the use in many-body theory of the method of second quantization, which was initially proposed in many-body theory but then was used for many years only in elementary-particle theory. In the framework of this method, the differences between a system consisting of a fixed number of nonrelativistic particles and a relativistic quantized field become unimportant. The method of second quantization deals directly, not with particles, but with a quantized field, which creates or annihilates particles at a given point of space; the particles themselves appear as the quanta of this field. For this reason, a system of many particles and a quantized field of elementary particles are described in the same way. This similarity goes very far: For example, the important process of excitation of a Fermi system (transition of a particle from an occupied level to a higher vacant level) takes the form of the process of creation of a pair—a particle and a "hole" in the Fermi distribution; the opposite process corresponds to annihilation of this pair.

[3)] A forerunner of Feynman diagrams (in particular, the important condition of backward motion in time of an antiparticle) had appeared already in the pre-war papers of Zisman.[6]

As in elementary-particle theory, the quantum-field methods to a considerable extent simplified and automated the calculations of effects of higher order in dynamical, statistical, and kinetic problems relating to many-particle systems. In the "old" many-body theory, many approximate methods were used (Hartree–Fock, Debye– Hückel, and many others), each of which was justified in its own way and had an insufficiently clear region of applicability. These methods now obtained a unified basis and were seen as different approximations to the exact field equations corresponding to small values of appropriate dimensionless parameters. It then became possible to improve these methods and to extend their applicability.

The "exact" methods already mentioned, in particular, the method of Green's functions, were also widely used in many-body theory. These functions contain extensive and many-sided information about a many-body system relating both to the internal properties of the system (probability distributions of physical quantities, energy spectrum) and also to the results of the influence on the system of various external factors (scattering cross sections, excitation probabilities, etc.).

It is especially important that Green's functions correspond directly to the important concept of a quasiparticle, whose introduction led to numerous achievements in many-body theory. Because of the interaction between particles, it is possible to speak not of the states of individual particles but only of the state of the system as a whole. However, if certain conditions are satisfied, it is possible to go over to the language of certain collective formations, or quasiparticles, which behave to a considerable extent in an independent manner. They have the same quantum numbers as the original particles, but their spectrum (the connection between the energy and the momentum) depends on the law of interaction, the temperature, etc.

The quasiparticles also include collective formations with different quantum numbers, these representing, so to speak, bound states of two, three, etc, "ordinary" quasiparticles; for example, an exciton in a solid can be regarded as a bound state of an electron and a "hole." If one knows what the quasiparticles are in a system and also their spectrum—and it is just this information that is contained in the Green's functions—one can obtain a fairly complete description of a many-body system (for more detail, see Refs. 4 and 5).

As a result of the penetration of quantum-field methods, not only the computational formalism but also the system of concepts and the language of many-body theory were significantly improved. This all led to important progress in many branches of the theory. The most perspicuous example of achievements in recent years is provided by the successes in the solution of the problem of phase transitions near a critical point (see Ref. 7).

What we have said also applies in full measure to superconductivity theory. Quantum-field methods played an important role in the creation of the microscopic theory of superconductivity (the methods of Bogolyubov and Gor'kov and Nambu), especially in its further development.[4,8] Today, it is hard to find a paper or a monograph in this subject in which one does not encounter Feynman diagrams, Green's functions, etc.

Quantum-field methods were a loan that many-body theory received from elementary-particle theory. Later, we shall consider how many-body theory is repaying this debt. And the debt is being repaid with, one might venture to say, a more valuable currency—new physical ideas.

3. HEISENBERG'S UNIFIED THEORY OF MATTER

We have already said that at the end of the fifties doubts arose as to the possibility of a quantum-field description of elementary particles. Many people who considered at that time the fate of the theory of elementary particles came to wonder whether the difficulties were not to be sought in our attempt to construct independent theories of individual types of interactions (electromagnetic, strong, etc.) instead of a unified theory combining all particles and their interactions. In other words, they hoped for a mutual compensation of the difficulties inherent in the theories of the individual types of interaction when they are combined in the framework of an all embracing particle theory.

This is probably one of the reasons for the enthusiasm with which a program for constructing the unified theory of matter proposed at that time by Heisenberg (see Ref. 9) was greeted. He set himself the task of embodying an unequal, "aristocratic" principle in the structure of matter—the introduction of certain primary particles from which all the remaining particles are to be obtained as bound states of different numbers of the primary particles. In other words, all the elementary particles in nature should appear as quasiparticles in the system of interacting primary particles (see Sec. 2).

The primary particles must have spin $\frac{1}{2}$ (in order to obtain the complete set of spins $0, \frac{1}{2}, 1\ldots$) and they must interact with one another (in order to produce bound states of them). Therefore, the fundamental equation of Heisenberg's theory that must be satisfied by the field of the primary particles has the form of a nonlinear equation for a spinor field ψ. Starting from the ordinary Dirac equation[4)]

$$(i\gamma\partial - m)\,\psi = 0 \tag{1}$$

(γ are Dirac matrices, m is the mass of the particle, and ∂ is the four-dimensional gradient), we give Heisenberg's equation in its simplest form

$$[i\gamma\partial - \lambda\,(\bar{\psi}\psi)]\,\psi = 0, \tag{2}$$

where λ is a coupling constant.

It must be emphasized particularly that Eq. (2) contains no term with the mass of the particle [see (1)]. The result is not only a simplicity of the equation. The point is that Eq. (2), as the fundamental equation of nature, must have the highest possible symmetry (see

[4)]We use a system of units in which $\hbar = c = 1$.

also Sec. 8 below). But a term with a mass would destroy the invariance of the equation under numerous transformations (the transformation $\psi \rightarrow \gamma_5 \psi$, where γ_5 is the fifth Dirac matrix, the scale transformation $x \rightarrow \theta x, \psi \rightarrow \theta^{-1/2}\psi$, where θ is some number, and others). Anticipating, we may point out that it by no means follows from this that the primary particles whose field satisfies Eq. (2) must necessarily be massless; this question is specially discussed in Sec. 8.

The implementation of Heisenberg's program encountered great difficulties associated above all with the impossibility of eliminating the divergences inherent in Eq. (2) by means of the ordinary method of renormalization. Therefore, despite individual successes (for example, the derivation of a value near to the experimental value for the fine structure constant), Heisenberg's program was not realized (however, see Sec. 8 below). Nevertheless, it had considerable conceptual influence on the subsequent development of the theory of elementary particles and served as one of the links in the chain of events that led to the current progress in this theory. One of the most important ideas of this kind relates to the problem of symmetry.

From the very start, Heisenberg encountered the following difficulty. Everything would have been comparatively simple if all types of interaction of the elementary particles were to exhibit the same degree of symmetry. It would then be necessary for the fundamental equation of the theory to satisfy this symmetry; simultaneously, the same symmetry would be manifested in the interaction of all quasiparticles. But it is well known that the interactions of elementary particles are characterized by different degrees of symmetry; on the transition from the strong interaction to the electromagnetic one the isotopic symmetry is lost, on the subsequent transition to the weak interaction the law of conservation of parity ceases to work, and so forth. Heisenberg clearly understood that it would be inconceivable to invent some fairly simple fundamental equation that would automatically exhibit different degrees of symmetry in the interactions of quasiparticles of different types.

But not for nothing was Heisenberg the creator of the theory of ferromagnetism and made (admittedly, unsuccessful) attempts to create a theory of superconductivity. It is just these theories that suggested the way out of his difficulty, which was to use the idea of spontaneous symmetry breaking, which had already been developed long ago in the branches of many-body theory, in which one studies ordered states, phase transitions, etc.

We must now make a rather long detour into the region of many-body theory and consider with the degree of detail required for what follows the general theory of spontaneous symmetry breaking and its concrete realization in superconductivity theory. We shall return to Heisenberg's theory once more in Sec. 8.

4. SPONTANEOUS SYMMETRY BREAKING

Many-body theory considers a particular class of ordered states of many-particle systems in which there arises a certain macroscopic quantity (the order parameter) that lowers the symmetry of such states. The simplest example of an ordered state is a ferromagnet; its total magnetic moment, which plays the role of the order parameter, distinguishes a definite direction in space, and therefore destroys the rotational symmetry. Another example is the crystalline state of a solid, in which the order parameter is the deviation of the density of the ions forming the crystal lattice from a homogeneous distribution. In this case, because of the distinguished position in space of the lattice sites, the translational (and also rotational) symmetry of the system is lost. The example of a superconductor, which is more important for what follows but also more complicated, will be considered separately in Sec. 7.

It is important that the symmetry of the ordered state is lower than the symmetry of the Hamiltonian of the system. Thus, in the simplest case, the microscopic equations in the theory of ferromagnetism and the theory of crystals are completely homogeneous and isotropic. Therefore the ordered states correspond to solutions of the dynamical equations that are less symmetric than the equations themselves.

That this is possible can be seen by the following elementary example. Consider Newton's equation for a free material point: $\ddot{\mathbf{x}}=0$; it is obviously translationally and rotationally invariant. However, its solution $\mathbf{x}=\mathbf{x}_0+\mathbf{v}t$ distinguishes both a definite point in space ($\mathbf{x}_0$) and a definite direction ($\mathbf{n}=\mathbf{v}/v$). Here, to one and the same energy of the particle there corresponds a complete set of possible motions differeing by the values of $\mathbf{x}_0$ and $\mathbf{n}$. Taken as a whole, this set is symmetric with respect to translations and rotations, but the initial conditions "select" from the set a motion with distinguished values of $\mathbf{x}_0$ and $\mathbf{n}$.

In the case of an ordered state of a many-particle system, we are dealing with degeneracy of the state of the system (at zero temperature, the ground state, or vacuum), which is one of a complete set of states of the same energy. Taken as a whole, this set has the complete symmetry of the Hamiltonian, but under a given symmetry transformation the states of this set do not remain unchanged but go over into other states of the same set. It is under conditions of degeneracy that a system is unstable and anomalously sensitive to small external perturbations that lift the degeneracy.[5] Therefore, there exists such a perturbation that, despite its smallness, leads to very appreciable consequences—it distinguishes and realizes just one of the states of the complete set, and this state has lower symmetry than the Hamiltonian itself.

In the case of an isotropic ferromagnet, this set

[5] This can be seen formally by using perturbation theory to consider the response of the system to a small perturbation: to transitions within the indicated set there correspond small energy denominators whose magnitude is determined by the level splitting, i.e., by a quantity which is of the order of the perturbation itself.

combines states with all possible directions of the magnetic moment: A freely suspended ferromagnet can be given any direction without energy having to be expended on rotation, and this reflects the degeneracy of the state of the system. A small external magnetic field lifts the degeneracy and realizes a state of the ferromagnet in which the direction of the magnetic moment coincides with the field direction. In the case of a crystal, its states corresponding to all possible positions of the lattice in space are combined in a set. A weak external electric field lifts the degeneracy and fixes the position of the lattice.

We therefore arrive at a picture of spontaneous symmetry breaking that arises, not because the dynamics of the system is asymmetric, and not because the macroscopic external influences are asymmetric, but because only one of the set of states of equal energy (which as a whole is symmetric) is realized.

At first glance, the phenomenon of spontaneous symmetry breaking contradicts P. Curie's general law: "The symmetry of an effect is not lower than the symmetry of its cause." But if we regard the spontaneous symmetry breaking itself as the effect, then the cause is not only the symmetric dynamics but also the small perturbation that distinguishes one of the states of the complete set. No matter how small the perturbation in the energy sense, it plays the role of a "trigger" and has appreciable consequences.

5. PHENOMENOLOGY OF ORDERED STATES AND PHASE TRANSITIONS

Macroscopic (and strong) external influences can change the degree of ordering of a many-body system. Some of these factors directly influence the order parameter, changing its magnitude in both directions, i.e., increasing or decreasing this parameter in the ordered state, and they also lead to the appearance of the order parameter in a state of the system that would be disordered without the external influence (induced symmetry breaking). An example of this is the effect of a strong magnetic field on a ferromagnet. Other factors do not directly influence the order parameter but, by changing the characteristics of the system, ultimately affect the order parameter as well. The most important example is the effect of a sufficiently high temperature $T \geq T_c$ (T_c is the critical temperature), which leads to the disappearance (because of thermal fluctuations) of the order parameter and the restoration of symmetry. This follows directly from the condition of a minimum in the free energy $F = E - TS$: At large T, irrespective of the form of the energy E, an increase in the entropy S, i.e., disordering of the system, is advantageous.

Such a phase transition from an ordered to a disordered state, like the states themselves, can be conveniently described in the language of a simple phenomenological model. We consider the free energy $F(\Psi)$, as a function of the order parameter Ψ, which has a minimum with respect to Ψ in the equilibrium state; at $T=0$ it is necessary to speak of the energy of the system. As we are interested in spontaneous symmetry breaking with respect to some transformation, we must compute a dynamical characteristic of the system—the value of $F(\Psi)$—that does not change under such a transformation, i.e., depends only on the invariants, expressed in terms of the order parameter, of the transformation. We aim to construct the simplest expression for $F(\Psi)$ that under certain conditions leads to a disordered state with $\Psi=0$ but under others to an ordered state with breaking of the symmetry under consideration. The same expression will obviously describe the phase transition itself from one such state to another.

The simplest case corresponds to symmetry under reflection: $\Psi \to -\Psi$, where Ψ is a real scalar quantity. Assuming first that Ψ does not depend on the coordinates, there is a single invariant of the transformation; Ψ^2. If its values are sufficiently small, we can restrict ourselves to an expansion of the function $F(\Psi)$ in a series:

$$F(\Psi) = F_0 + \alpha\Psi^2 + \beta\Psi^4. \tag{3}$$

Assuming $\beta>0$, we readily see that when $\alpha>0$ the equilibrium value of Ψ is zero and we are dealing with a disordered state (curve 1 in Fig. 1). However, for $\alpha<0$ the state of the system is degenerate with respect to the sign of Ψ—there are two minima of equal depth corresponding to $\Psi = \pm(-\alpha/2\beta)^{1/2}$ (curve 2, Fig. 1). A small external perturbation that makes one of the minima only slightly deeper than the other realizes a state with spontaneous breaking of the original symmetry. For the case when Ψ is a slowly varying function of the coordinates, the invariant $(\nabla\Psi)^2$ is added, and instead of (3) we have the expansion

$$F(\Psi) = F_0 + \alpha\Psi^2 + \beta\Psi^4 + \gamma(\nabla\Psi)^2. \tag{3'}$$

More important for what follows is symmetry under a "global" gauge transformation $\Psi \to \Psi\exp(i\chi)$, where χ is a constant real phase; one can speak of such a symmetry if Ψ is a complex order parameter. This transformation, which, in contrast to reflection, is a continuous transformation, has the invariants $|\Psi|^2$ and $|\nabla\Psi|^2$. Accordingly, the expansion (3′) is replaced by

$$F = F_0 + \alpha|\Psi|^2 + \beta|\Psi|^4 + \gamma|\nabla\Psi|^2; \tag{4}$$

here, there is degeneracy with respect to the phase θ of the order parameter, $\Psi = |\Psi|\exp(i\theta)$, and the spontaneous symmetry breaking corresponds to fixing this phase. Geometrically, we must now consider the figure of revolution obtained by rotating the curve in Fig. 1 about the vertical axis. Curve 2 in Fig. 1 now corresponds to a circular through ("base of bottle") whose depth does not depend on the azimuthal angle, which is the phase θ. A small dip in the bottom of the trough at

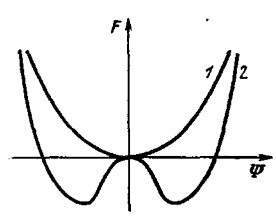

FIG. 1.

a certain value of the angle fixes a corresponding value of the phase θ.

Such considerations provide the basis of Landau's phenomenological theory (see Ref. 10), which describes a large class of phase transitions of the second kind. In this theory, α is taken equal to the simplest function $a(T - T_c)$ $(a>0)$, which goes over with increasing T from negative values to positive values at the point $T = T_c$. As this point is approached, the order parameter tends smoothly to zero, remaining equal to zero at higher temperatures. This means that the transition is of the second kind (Fig. 2a).

An alternative description of the phase transition is obtained when curve 1 in Fig. 1 at $T=0$ is an effective potential energy of the system. A maximum of this curve corresponds to an unstable disordered ground state of the system, and the minima to a stable ground state rearranged by the appearance of the order parameter; the depth of these minima determines the energy gain resulting from the rearrangement. As long as $T>T_c$, the average energy of the system, with allowance for its thermal component, lies above the central hump of the curve and a symmetric state with $\Psi = 0$ is realized (the system spends equal times in the states that differ by the sign of Ψ). But when $T<T_c$ the energy sinks below the central hump and the state of the system settles in one of the minima of the curve, and this corresponds to spontaneous symmetry breaking.

Landau's theory, which is based on a small value of Ψ, is not valid far from the critical point. It is also invalid in the immediate neighborhood of T_c. This is because the ordered and disordered phases of matter here differ so little that fluctuations of the order parameter, which are not taken into account by Landau's theory and lead from one phase to another, become decisive. It was in the description of these fluctuations that many-body theory achieved the success mentioned in Sec. 2.

Fluctuations of the order parameter can transform a phase transition of the second kind (smooth disappearance of Ψ, absence of latent heat of the transition) into a phase transition of the first kind (abrupt disappearance of Ψ, existence of at least a small latent heat). In the simplest case, this reduces to a violation of the implicit assumption that the function $F(\Psi)$ is analytic (made above when this function was expanded with respect to invariants), this being manifested in the appearance on the right-hand side of (3) of an additional term $(\Psi^2)^{3/2}$ with negative coefficient. Because of this, the function $F(\Psi)$ acquires an additional minimum (Fig. 3), which touches the abscissa at a finite value of Ψ. This then leads to an abrupt disappearance of the order

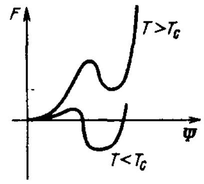

FIG. 3.

parameter at the point T_c (Fig. 2b). We shall encounter an effect of this kind later.

To conclude this section, we note that spontaneous symmetry breaking with respect to a continuous transformation is accompanied by the appearance of a quasiparticle whose energy vanishes together with its momentum. One also speaks of an acoustic (gapless) quasiparticle spectrum or, having in mind the relativistic formula $E = \sqrt{p^2 + m^2}$, of a vanishing mass of the particle. This is the well-known Goldstone's theorem (see also below). In classical language, the Goldstone particle corresponds to oscillations of the parameter with respect to which the energy of the system is degenerate. This is why the production of such a particle with vanishing momentum does not require expenditure of energy.[6] For a ferromagnet, the Goldstone particle is a spin wave (oscillations in the direction of the magnetic moment); for a crystal, sound (vibrations of the positions of the lattice ions).

It is sometimes said that the appearance of the Goldstone particle restores the spontaneously broken symmetry. This must be understood as follows. As we have already said, the essence of spontaneous symmetry breaking is the distinguishing of one of the states in a set symmetric as a whole. But the Goldstone particle corresponds to transitions within this set, and its appearance leads to mixing of the states in the set. Goldstone's theorem can be readily illustrated by the example of an ordered system with complex order parameter (see above), for which the Goldstone particle corresponds to oscillations of the phase θ of this parameter. Degeneracy with respect to θ means that the free energy does not depend on a constant phase θ and, therefore, when the free energy is expanded in terms of the small deviations of the phase $\delta\theta$, it contains the term $(\nabla\delta\theta)^2$, but not simply $(\delta\theta)^2$. Minimization of F with respect to $\delta\theta$ gives the static (corresponding to vanishing energy) equation $\nabla^2\delta\theta = 0$ for the oscillations, which leads to the conclusion that the quasiparticle momentum is zero.

6. BOSE CONDENSATION

We encounter the simplest example of an ordered state with a complex order parameter in the phenomenon of Bose condensation of an ideal nonrelativistic gas consisting of a large but fixed number of Bose particles (see Ref. 10). When we have considered this phenomen-

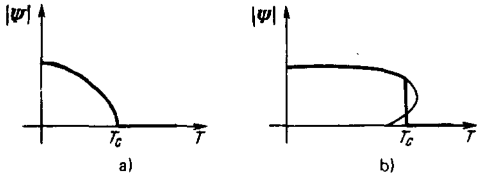

FIG. 2.

[6] Goldstone's theorem may be invalid for systems with long-range (Coulomb) forces, for which the importance of interactions between particles becomes important precisely at small momenta.

on, we shall turn to the related but more complicated phenomenon of superconductivity.

We introduce the operator of the Bose field:

$$\psi(\mathbf{x})=\sum_{p} a_p \exp(i\mathbf{px}), \tag{5}$$

where a_p is the operator of annihilation of a particle in the state with momentum $\mathbf{p}$ and energy $E_p=p^2/2m$. The population of the levels at an arbitrary temperature T is given by the occupation numbers

$$n_p=\langle a_p^+ a_p\rangle=\left[\exp\left(\frac{E_p-\bar{\mu}}{T}\right)-1\right]^{-1}, \tag{6}$$

where $\bar{\mu}$, the chemical potential, is determined by the equation $\sum_p n_p=N$, where N is the total number of particles. The symbol

$$\langle\ldots\rangle=\frac{\mathrm{Sp}[\ldots\exp(-H/T)]}{\mathrm{Sp}[\exp(-H/T)]}$$

denotes averaging over the Gibbs ensemble (H is the Hamiltonian); at $T=0$, this symbol goes over into the expectation value with respect to the ground state of the system: $\langle 0|\ldots|0\rangle$, where $|0\rangle$ is the wave function of this state.

The essence of the phenomenon of Bose condensation can be seen by letting N tend to infinity and following the behavior in this limit of the fraction of the total number of particles $\nu_p=n_p/N$ in each level. Examination of Eq. (6) reveals the following: a) at all temperatures, the lowest level $\mathbf{p}=0$ contains more particles than any other; b) at all temperatures, ν_p tends to zero for each of the upper levels $\mathbf{p}\neq 0$; c) for $T\geqslant T_c$ (T_c is the critical temperature of Bose condensation) the limit ν_0 for the lowest level is also zero; d) for $T<T_c$ a finite fraction of all the particles accumulates on the lowest level, $\nu_0\to$const, and simultaneously $\bar{\mu}\to 0$ [it is only under this condition that $n_0\propto N\to\infty$; see (6)]. This is the phenomenon of Bose condensation at $T<T_c$, which is due to the "overfilling" of the upper levels, which taken together are no longer capable of containing a number of particles that tends in the limit to N. It is for this reason that the lowest level is populated by a macroscopically large number of particles, comparable with N; it is frequently said that these particles together form a Bose condensate.

A formal manifestation of Bose condensation is the possibility of regarding the operators a_0 and a_0^+ [see (5)] as classical quantities. This is clear from the fact that their commutator $[a_0^+, a_0]=1$ vanishes compared with their product $n_0=\langle a_0^+ a\rangle_0\propto N\to\infty$. For this reason, one can set $\langle a_0\rangle=a_0$, $\langle a_p\rangle=0$ ($\mathbf{p}\neq 0$) and Gibbs averaging of the field operator (5) gives $\langle\psi\rangle=a_0$. From this, we obtain a splitting of the field operator into the condensate and "above-condensate" parts

$$\psi=\langle\psi\rangle+\psi', \tag{7}$$

where ψ' has the form (5) but with summation only over levels with $\mathbf{p}\neq 0$. The appearance in the field operator of a classical term is also a characteristic of the phenomenon of Bose condensation in the general case when there is an external field, interaction between particles, and so forth. It is the presence of such a term that describes, for example, the superfluidity of liquid ^{4}He at sufficiently low temperatures.

For the case of Bose condensation, $\Psi=\langle\psi\rangle$ then serves as a complex order parameter. In this case too, there is spontaneous breaking of the symmetry of the Hamiltonian of the Bose system with respect to the gauge transformation $\psi\to\psi\exp(i\chi)$. And here too there is degeneracy with respect to the phase of the order parameter, the symmetry breaking consisting of the fixing of this phase. Physically, the appearance of the order parameter in Bose condensation which, essentially, is a classical coherent de Broglie wave of the lowest state of the system, is associated with the matching of the phases of the particles settled in the lowest level—they form a state with a single fixed phase and not a random set of quanta.

It is sometimes said that as a result of Bose condensation there is a violation of the law of conservation of the particle number, which is directly related to the gauge transformation.[7)] This, of course, does not mean that particles are being created "out of nothing" or are being destroyed. The point is that it is only meaningful to speak of a Bose condensate in the limit $N\to\infty$, and the condensate then plays the role of an infinitely large reservoir of particles that "does not notice" the loss or addition of a finite number of particles. The vanishing of the chemical potential $\bar{\mu}=\partial F/\partial N$, where F is the free energy of the system, is saying the same thing.

We have already noted that in the case of the system under consideration with a fixed number of particles Bose condensation comes about because of the "overfilling" of the upper levels of the system. Accordingly, in a system in which the number of particles can change, Bose condensation is by no means necessary; it does not occur, for example, for a system of photons in thermal equilibrium. However, even in a system with a variable number of Bose particles the dynamics of the interactions of the particles may "induce" Bose condensation, when macroscopic population of the lowest level becomes energetically advantageous. This at least is the case if Landau's expansion (4) is valid and if there is a range of temperatures in which the coefficient α is negative. A simple example of "induced" Bose condensation (on a level with $\mathbf{p}\neq 0$) is the generation of a coherent laser wave for photons in a medium with population inversion. Below, we shall consider other examples relating to superconductivity and to scalar models of field theory.

7. SUPERCONDUCTIVITY

We now consider the phenomenon of superconductivity, which is directly related to Bose condensation.[4,8] The electrons of a metal (or the nucleons in a nucleus) satisfy Fermi statistics and cannot by themselves condense.

[7)] Indeed, reading the left-hand side of the inequality $\langle 0|\psi|0\rangle\neq 0$ from right to left, we see that if we apply the annihilation operator to the ground state, i.e., if we reduce the number of particles in it by unity, we nevertheless return to the original state.

However, if certain conditions are satisfied,[8] they are capable of forming pairs (Cooper "pairing"), and these pairs, which have the properties of Bose particles, can condense on the lowest level. It only needs to be emphasized that Cooper pairs should not be taken too literally—they are not bound complexes such as molecules but simply strongly correlated states of a pair of particles.

A large part of what we have said above about a Bose system can be applied to the case of a superconductor. The order parameter in this case is the expectation value of the operator of a Cooper pair:

$$\Psi = \langle\psi_+\psi_-\rangle, \tag{8}$$

where ψ is the operator of the Fermi field of the electrons, and the indices (±) correspond to momenta $\pm\mathbf{p}$ and spin projections $\pm\frac{1}{2}$ (the total momentum and total spin of a pair in the absence of a current are zero). The *microscopic equations that provide the basis of* superconductivity theory and, in particular, the frequently used equation corresponding to contact interaction of electrons:

$$\left[i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m}-\lambda(\bar{\psi}\psi)\right]\psi=0, \tag{9}$$

are invariant under a gauge transformation. The appearance of the order parameter (8) with fixed phase destroys this invariance. Oscillations of this phase correspond to a Goldstone particle in the superconductor ("sub-gap" sound; see however footnote[6] in Sec. 5).

The Bose condensation of the Cooper pairs radically affects the electron-quasiparticle spectrum near the Fermi surface (energy E_F, momentum p_F), where basically the formation of pairs takes place. Transforming Eq. (9) to the momentum representation, we find the relation $E_F=(p_F^2/2m)+\lambda\langle\bar{\psi}\psi\rangle$, and by means of it the symbolic equation

$$E-E_F=\frac{p^2-p_F^2}{2m}+\lambda\,(\bar{\psi}\psi-\langle\bar{\psi}\psi\rangle).$$

Squaring both sides, averaging, and remembering that in accordance with Wick's theorem $\langle(\bar{\psi}\psi)^2\rangle-\langle\bar{\psi}\psi\rangle^2=|\Psi|^2$ [see (8)], we find, making near the Fermi surface the replacement $(p^2-p_F^2)/2m\to v(p-p_F)$, where $v=p_F/m$ is the velocity on the Fermi surface,

$$E-E_F=\pm\sqrt{v^2(p-p_F)^2+\lambda^2|\Psi|^2}. \tag{10}$$

Here, the sign in front of the radical is determined by the sign of the difference $p-p_F$, which corresponds to either an electron in an unfilled energy range or a "hole" in the Fermi sea.

Equation (10) shows that the allowed energy regions are separated by an "energy gap" $2\Delta=2\lambda|\Psi|$. Physically, this quantity corresponds to the binding energy of a Cooper pair: This is the energy which must be expended to break the pair and obtain an electron in a free state. The presence of the gap implies a definite "rigidity" of the state of the electrons in a superconductor, and their lack of response to external disturbances that are not too strong. In this way one can then understand the remarkable properties of a superconductor; the absence of Joule losses, the Meissner effect, which will be discussed below, and others.

The "rigidity" is due to the fact that in order to excite the electron component of a superconductor one must expend at least the energy 2Δ. In fact, this excitation reduces to the creation of an electron—"hole" pair whose total energy is equal in accordance with (10) to the arithmetic sum of the radicals in Eq. (10), which correspond, respectively, to an electron and a hole. We emphasize that the ratio of this excitation energy to the total momentum of the electron and the hole has a finite lower bound equal to Δ/p_F. This means that in a superconductor Landau's well-known criterion for superfludity (see Ref. 10) is satisfied, and superfluidity of electrons is none other than superconductivity.

If certain conditions are satisfied (in particular, at temperatures near T_c[9]) superconductivity can be described by the phenomenological theory based on Eq. (4) with the order parameter (8). In the presence of an external magnetic field $\mathbf{H}$ (with vector potential $\mathbf{A}$) it is necessary to make in this equation the substitution $\nabla\to\nabla-ie\mathbf{A}$, where e is the total charge of the Cooper pair, and to add the energy of the magnetic field:

$$F=F_0+\alpha|\Psi|^2+\beta|\Psi|^4+\gamma|(\nabla-ie\mathbf{A})\Psi|^2+\frac{H^2}{8\pi}. \tag{11}$$

This expression (with replacement, by analogy with the formula for the kinetic energy, of γ by $1/2m$, where m is the mass of a pair) is the basis of the semiphenomenological Ginzburg–Landau theory (see Ref. 11, and also Refs. 4 and 12). Adding to the expression (11) the energy $-\mathbf{j}\cdot\mathbf{A}$ of the external currents $\mathbf{j}$, and varying with respect to Ψ and $\mathbf{A}$, we find the Ginzburg–Landau equations

$$[(\nabla-ie\mathbf{A})^2/2m-\alpha-2\beta|\Psi|^2]\Psi=0, \tag{12a}$$

$$\nabla^2\mathbf{A}-\frac{4\pi e^2|\Psi|^2}{m}\mathbf{A}=2\pi ie(\bar{\Psi}\nabla\Psi-\nabla\bar{\Psi}\Psi)-4\pi\mathbf{j}. \tag{12b}$$

It follows from this, in particular, that a spatially homogeneous distribution of the current $\mathbf{j}$ leads to distributions of Ψ and $\mathbf{A}$ that are also independent of the coordinates, and $\mathbf{A}=m\mathbf{j}/e^2|\Psi|^2$ (the equivalent of London's equation; see Ref. 12). Substitution of this relation in (12a) shows that a positive term $mj^2/e^2|\Psi|^4$ is added to α. It is clear from this that with increasing current the value of the order parameter decreases and at a sufficient strength of the current the symmetry is restored. Thus, a current, like the temperature, destroys the superconducting order. This is due to the increase in the electrodynamic energy of the condensate [the term $e^2|\Psi|^2A^2$ in (11)], which ultimately swamps the gain in the energy due to the Bose condensation itself (see Sec. 4).

It can be seen from what we have said that for $\mathbf{j}=0$ there is no magnetic field within the superconductor (far from its boundaries). Near the boundaries, Eq. (12b) has a solution $\exp(-\varkappa x)$, which is exponentially

[8]Roughly speaking, if there is an attraction between the particles.

[9]This critical temperature, above which there is no superconductivity, is equal in order of magnitude to the energy gap Δ.

damped within the superconductor; here x is the distance from the boundary, and $\varkappa^2 = 4\pi e^2 |\Psi|^2/m$ determines the penetration depth of the field. This nonpenetration of the field into the superconductor is the Meissner effect[12] mentioned earlier. Physically, it is explained by the fact that when the field is switched on induction currents [the second term on the left-hand side of (12b)] are induced in the superconductor, and these, in accordance with Lenz's rule, screen the external sources of the field and, in contrast to a normal metal, are not damped with the time.

The Meissner effect leads to an inhomogeneous field configuration, which is energetically disadvantageous. Therefore, as in the just considered case of an external current, an external magnetic field reduces the order parameter. If the field is sufficiently strong, the superconductivity (and the Meissner effect itself disappears, and the field fills the complete volume of the superconductor.

The development of this process depends essentially on the relative magnitude of two lengths characteristic of the superconductor; the field penetration depth $\varkappa^{-1}$ (see above) and the effective size of the Cooper pair $(m\alpha)^{-1/2}$ [the characteristic distance over which Ψ varies; see (12a)]. It can be shown that the ratio of these lengths—the Ginzburg–Landau parameter—determines the sign of the surface energy on the boundary between the normal and superconducting phases of the matter. If this parameter is less than unity (type I superconductor), then the surface energy is positive and even a comparatively weak magnetic field penetrates uniformly into the superconductor, destroying the superconducting order. But if the Ginzburg–Landau parameter is greater than unity and the surface energy is negative (type II superconductor), then it is energetically advantageous to have an alternation of the normal and superconducting phases in space. The field in this case penetrates into the superconductor and is localized within special vortex filaments [Abrikosov's vortices; see Sov. Phys. JETP **5**, 1174 (1957)], forming a regular lattice within the metal. Each filament has a radius of the order of the length of a Cooper pair (and therefore "swells" as the temperature approaches T_c or the magnetic field is increased) and carries a magnetic flux equal in magnitude to $1/e$. Physically, the occurrence of the filaments is due to the Meissner effect: The expulsion of the magnetic field leads to its concentration in a minimal (for given flux) volume. In the space between the filaments superconductivity is preserved, and its final destruction occurs in fields that are so strong (up to several hundred kilogauss) that the filaments themselves come into complete contact with one another. This is the basis of the wide practical use of type II superconductors in superconducting magnets of accelerators, MHD generators, and other devices.

8. "SUPERCONDUCTING" MODELS OF ELEMENTARY PARTICLES

Having now completed our somewhat extended but necessary detour into many-body theory, we return to Heisenberg's theory (see Sec. 3). At least in principle, the idea of spontaneous symmetry breaking does enable one to solve the difficulty in this theory associated with the different degrees of symmetry of the elementary-particle interactions. For this, it is necessary to choose the fundamental equation of the unified theory of matter with maximal degree of symmetry, and the necessary violations of this symmetry for the interactions of the corresponding quasiparticles must come about spontaneously by the realization of solutions with incomplete symmetry. The resulting Goldstone particles could then be identified with the massless particles observed in nature (for example, with the photon as a result of spontaneous breaking of isotropic symmetry; in this connection see Ref. 13).

One of the most important mechanisms of spontaneous symmetry breaking in the framework of Heisenberg's program was proposed at the beginning of the sixties by Nambu and Jona-Lasinio[14] and Vaks and Larkin.[15] It was taken from the then recently discovered microscopic theory of superconductivity of Bardeen, Cooper, and Schrieffer (abbreviated BCS; see Refs. 4 and 8).

Heisenberg's equation (2) and Eq. (9), on which the theory of superconductivity is based, have a strong similarity. Accordingly, in Heisenberg's theory as well, in the case of attraction between the primary particles there will be a spontaneous symmetry breaking due to the formation of Cooper pairs of the primary particles and their Bose condensation with the appearance of an order parameter, as in (8). This conclusion is reached by application to Eq. (2) of the standard formalism of superconductivity theory, which gives relations that are a relativistic generalization of the ordinary "superconducting" formulas. It is only necessary to "cut off" divergent integrals at a certain limiting energy. It is interesting to note that a similar "cutoff" also occurs in the ordinary theory of superconductivity, in which it has a direct physical meaning corresponding to the limiting energy (Debye energy) of the phonons that transmit the interaction between electrons. This mechanism of spontaneous symmetry breaking (referred to in what follows for brevity as the BCS mechanism) solves the important problem of the mass of the primary particle. As was already noted in Sec. 3, the requirement of maximal symmetry of the fundamental equation (2) has the consequence that this equation contains no mass term, which would be noninvariant under scale and γ_5 transformations. On the other hand, the same requirement means that the interaction of the primary particles must have maximal symmetry. Therefore, the lack of mass of the primary particle would be a serious difficulty for Heisenberg's program—the only particle we know with zero mass and spin $\frac{1}{2}$ (the neutrino) does not participate in the most symmetric interaction—the strong interaction.

The appearance of the nonvanishing order parameter (8) as a result of the BCS mechanism results in spontaneous breaking not only of the gauge invariance, as in the nonrelativistic theory of superconductivity, but also of the scale and γ_5 symmetries characteristic of Heisenberg's theory. One may therefore expect that not only the symmetry breaking but also the mass of the primary particles arises spontaneously. The same conclusion is indicated by a comparison of the spectrum (10) of electrons in a superconductor and the relativistic formula $E=\sqrt{p^2+m^2}$, which reveals a physical similarity between the concepts of quasiparticle mass and energy gap.

This is confirmed by a direct repetition of the calculation that led to (10) in the case of Eq. (2); the calculations are even simplified since now we are interested in the vacuum, and not the ground state of a metal, and therefore $E_F=0$, $p_F=0$, $\langle 0|\bar{\psi}\psi|0\rangle=0$. Equation (2) in the momentum representation gives the relation $\gamma p=\lambda(\bar{\psi}\psi)$, and squaring this with allowance for Wick's theorem (see Sec. 7), we obtain the equation $(p)^2=\lambda^2|\Psi|^2$. It can be seen from this that the spontaneously arising mass of the primary particle is $\lambda|\Psi|$. As in the case of superconductivity, it is determined by the energy needed to break a Cooper pair and obtain a particle in a free state.

In the papers of Nambu and Jona-Lasinio, the role of the primary particles was assigned to the nucleons, and the corresponding Goldstone particle was similar to the pion. This can be regarded as a definite step in the implementation of Heisenberg's program.[10] However, in subsequent years the interest in this program decreased considerably, and it is only very recently that investigations have begun to appear (see Ref. 16) in which Heisenberg's program using the BCS mechanism is formulated in connection with quantum chromodynamics ("color" model of quarks interacting by the exchange of gluons). We should also mention the use of the BCS mechanism in quantum electrodynamics,[17] which enables one to examine renormalization in this theory from a somewhat different point of view.

9. SCALAR MODELS OF SPONTANEOUS SYMMETRY BREAKING

Heisenberg's program and the "superconducting" models of elementary particles that it stimulated gave rise in the first half of the sixties to a burst of interest among field theoreticians in spontaneous symmetry breaking.[11] This interest was fanned by the understanding at that time of the fact that the world of elementary particles is characterized by a certain breaking of the majority of the symmetry types (excluding relativistic invariance, the law of conservation of electric charge, etc). It was at that time that we saw the formulation of the problem of spontaneous symmetry breaking in quantum field theory,[21] Goldstone's theorem,[22] and so forth. This, in its turn, led to a deeper understanding of spontaneous symmetry breaking in many-body theory, and, in particular, in the theory of superconductivity.

[10] Note that these papers were one of the sources of the important direction in elementary-particle theory known as PCAC (partial conservation of axial current).[18]

[11] Spontaneous symmetry breaking was one of the principal themes of international seminars on the unified theory of elementary particles at Rochester (1960, 1963) and Munich (1965)[19]; see also Ref. 20.

Goldstone introduced into quantum field theory a new mechanism of spontaneous symmetry breaking that differs from the BCS mechanism in that there is an induced Bose condensation, not of Cooper pairs, but of "ready made" Bose particles (see Sec. 6). Goldstone's model corresponds to a self-interacting scalar field φ with negative square of the mass, and is described by the Lagrangian

$$L=|\partial\varphi|^2+\mu^2|\varphi|^2-\lambda|\varphi|^4. \tag{13}$$

The structure of this expression was suggested by the so-called sigma model (see Ref. 23), which served as another source of PCAC theory. However, using (13) to find the corresponding energy of a static field:

$$E=|\nabla\varphi|^2-\mu^2|\varphi|^2+\lambda|\varphi|^4,$$

we see that Goldstone's model is simply a concrete realization of Landau's phenomenological theory [see (3′) above]. The negative sign of the square of the mass of the particle corresponds to the negative sign of α in (3′).

In Goldstone's model, there is an ordered state (Bose condensate) with complex order parameter $\Psi=\langle\varphi\rangle$. In the case of weak coupling, $\lambda\ll 1$, the calculations can be carried through to the end. Restricting ourselves for the moment to the case $T=0$, we write down the field equation that follows from (13):

$$[\partial^2-\mu^2+2\lambda|\varphi|^2]\varphi=0, \tag{14}$$

and, as in the case of a nonrelativistic Bose system [see (7)], we set

$$\varphi=\Psi+\varphi', \quad \Psi=\langle 0|\varphi|0\rangle. \tag{15}$$

Substituting (15) in (14), taking the vacuum expectation value and omitting expectation values of the type $\langle 0|(\bar{\varphi}')^n(\varphi')^m|0\rangle$ with $m\neq n$, we find the equation for the order parameter:

$$\Psi[\mu^2-4\lambda\langle 0||\varphi'|^2|0\rangle-2\lambda|\Psi|^2]=0. \tag{16}$$

The second term in the brackets, which describes fluctuations of the field about its mean value Ψ, has to be taken into account only when $T\neq 0$ (see below). Therefore, there arise the two following solutions of (16): $\Psi=0$ and $|\Psi|=\mu/\sqrt{2\lambda}$. The first corresponds to a disordered state (the maximum in Fig. 1b), and the second to a Bose condensate (minimum in Fig. 1b). As can be seen from the expression given above for the energy, the ordering of the system is associated with an energy gain of $\mu^4/4\lambda$.

In order to elucidate the stability of the solutions obtained without reference to energy arguments, it is necessary to find the spectrum of the quasiparticles (because the field is complex, there are two species of quasiparticles). The field oscillations $\delta\varphi\exp(ipx)$ correspond to them. In accordance with (14)[12]

$$[(p)^2+\mu^2]\delta\varphi=4\lambda|\varphi|^2\delta\varphi+2\lambda\varphi^2\delta\bar{\varphi}. \tag{17}$$

For the state with $\Psi=0$, the right-hand side of (17) can be omitted, from which we see that real and imaginary parts of $\delta\varphi$ oscillate in accordance with the "tachyon"

[12] Here and below, the symbol $(p)^2$ denotes the four-dimensional square E^2-p^2.

law $E=\sqrt{p^2-\mu^2}$. Therefore, long-wavelength quasiparticles ($p<\mu$) have imaginary energy and, accordingly, the field oscillations will increase with the time in accordance with the law $\exp[\sqrt{\mu^2-p^2}\,t]$. Thus, we again arrive at the conclusion that the disordered state of the system is unstable (in this connection, see Ref. 24).

In the ordered state $\Psi=|\Psi|\exp(i\theta)\neq 0$, the two possible types of quasiparticles have different spectra. In this case, the normal modes correspond to oscillations of the modulus, $\delta\varphi=\delta|\Psi|\Psi/|\Psi|$, and the phase, $\delta\varphi=i\delta\theta\Psi$, of the order parameter. Substituting these expressions in (17), finding the vacuum expectation value with neglect of the fluctuation term, and comparing the result with (16), we obtain $(p)^2=2\mu^2$ for the oscillations of the modulus and $(p)^2=0$ for the oscillations of the phase. Thus, in the stable state of the system the spectrum of quasiparticles is "corrected" (they acquire a positive or vanishing square of the mass), and this corresponds directly to the sign of the curvature at the extrema in the curve 2 in Fig. 1; in addition, we again arrive at Goldstone's theorem. We therefore see that the correct treatment of Goldstone's model obliges us to "shift" the field operator to the equilibrium point of the system [see (15) and curve 2 in Fig. 1]. Of course, this could be regarded as a purely formal operation, but it is much more fruitful to approach it on the basis of physical considerations and regard it as the manifestation of a real Bose condensation of a scalar field.

The generalization of Goldstone's model to the case of interacting scalar and vector (electromagnetic) fields was considered by Higgs.[25] Instead of (13), one must now consider the expression

$$L=-\frac{1}{16\pi}(\partial_\mu A_\nu-\partial_\nu A_\mu)+|(\partial-ieA)\varphi|^2+\mu^2|\varphi|^2-\lambda|\varphi|^4, \quad (18)$$

where the first term is the Lagrangian of a massless vector field.[13)] Going over here too to the static limit, we can readily see that the Higgs model is completely analogous to the Ginzburg–Landau theory, being its relativistic generalization [see (11) and Refs. 26 and 27]. As it happened, this conclusion had significant heuristic value, making it possible to establish direct analogies between the theory of superconductivity and theories of elementary particles, including the Higgs model.

With the first appearance of analogies of this kind, we encounter the question of the mass of the vector field. In the expression (18), this mass is taken equal to zero. However, the appearance of a nonzero order parameter leads to the spontaneous occurrence of mass of the vector field, this being equal to $\sqrt{8\pi}e|\Psi|$. Indeed the Lagrangian of a vector field with mass m is equal to the first term of (18) plus the term $m^2A^2/8\pi$. But precisely this term appears in (18) because of the presence in this expression of the term $e^2|\varphi|^2A^2$. This new mechanism (differing from the BCS mechanism) for the appearance of mass when there is spontaneous symmetry breaking is called the Higgs mechanism.

However, if we recall what we said in Sec. 7 about the Meissner effect, it becomes clear that in a superconductor we are in fact dealing with a massive photon, whose mass arises because of precisely the same Higgs mechanism.[25] Equation (12b) is the static limit of the equation $(\partial^2+\varkappa^2)A=4\pi j$ for a photon with mass $\varkappa$, and the exponential law of decrease of the field within the superconductor is Yukawa's law for a plane field source. Therefore, the Higgs mechanism could with equal right be called the Meissner mechanism.

In Sec. 7, we have already said that the Meissner effect is explained physically by the appearance of induction currents in the metal, which screen the field sources and are not damped under the conditions of the superconductor. In exactly the same way, the mass of the vector field arises in the Higgs model because of induction currents in the Bose condensate. And these currents too are not damped with time, and we can therefore say that in the Higgs model we encounter the phenomenon of superconductivity at the level of elementary particles. This conclusion is directly confirmed in the language of the Landau criterion (see Sec. 7): The ratio of the energy of the quasiparticle to its momentum, which, in accordance with what we have said above, is equal to $\sqrt{p^2+2\mu^2}/p$, has a nonvanishing lower limit.[28]

In this last argument, it is not fortuitous that we have used the expression for the spectrum of the oscillations of only the modulus of the order parameter and not its phase. The point is that when a scalar field interacts with a gauge-invariant vector field (which means that the fields A and $A+\nabla\Phi$, where Φ is some function, are physically indistinguishable) the Goldstone particle becomes unphysical and can be eliminated by a gauge transformation. Indeed, taking the function Φ equal to θ/e, we can, after the substitution (15), completely eliminate the phase θ of the order parameter from the Lagrangian (18).[14)]

This conclusion is important in connection with the following question. It is well known that a massless vector particle (for example, the photon) has two degrees of freedom—two polarizations— at a given frequency. On the other hand, the number of polarizations of a massive vector particle is three. Where does the vector field acquire its extra degree of freedom if its mass arises spontaneously? The answer is clear from what we have said above: After the spontaneous symmetry breaking, the scalar field loses one degree of freedom corresponding to oscillations of the phase of the order parameter.

10. MODERN UNIFIED THEORY OF ELEMENTARY PARTICLES

Although Heisenberg's "aristocratic" unified theory

13) For simplicity, we restrict ourselves here and below to an Abelian theory and do not introduce Yang–Mills fields.

14) See Ref. 30 for an analogous conclusion and its consequences in superconductivity theory.

did play an important role in the conceptual evolution of the theory of elementary particles, modern hopes of creating a unified theory of particles are associated with a different, "democratic" approach. We eschew the introduction of a distinguished species of particles, similar to Heisenberg's primary particles, and attempt to combine on an equal footing the particles which participate in different interactions and to treat them in a unified manner.

In what follows, we shall restrict ourselves to considering the most developed theory, which combines the weak and electromagnetic interactions of elementary particles (see, for example, the reviews of Ref. 29). Both these interactions exhibit a remarkable universality for a large class of processes in which very varied particles participate. The universality of the electromagnetic interaction is due to the existence of its single carrier—the photon—which interacts with charged particles, and has a single coupling constant. The carrier of the weak interaction—the intermediate vector, boson, or W meson—can fulfill an analogous function. It has much in common with the photon, differing from it in the following respects. The weak interaction, in contrast to the electromagnetic, has a finite range, and therefore the W meson must have a nonzero mass. In addition, the majority of weak processes correspond to exchange of charge between the interacting particles and therefore the W mesons (or at least some of them) must be charged.

The "democratic" principle of unifying the weak and electromagnetic interactions requires that the particles that participate in these interactions be combined into two groups (two multiplets). One of them must include the leptons (electrons, muon, neutrino and the corresponding antiparticles)—light particles with spin $\frac{1}{2}$ that do not participate in the strong interaction. The other must combine the intermediate vector particles (photon, W mesons), which transmit the interaction between the leptons.

It must be emphasized that we are speaking not of a purely mechanical but a group-theoretical unification, which would enable us to justify the structure of the multiplet, elucidate the form of the interactions of the particles in it, and so forth.[15)] For ordinary spectroscopic multiplets, the approach based on the rotation group gives such information. The use of group-theoretical methods presupposes that there is at least approximate invariance under a group transformation, i.e., at least approximate degeneracy of the multiplet (coincidence of the masses of the particles in the multiplet). But in the groups of particles listed above there is nothing of the kind—they contain both massive and massless (neutrinos, photon) particles. Accordingly, we encounter here the first reason why the existence of mass of the elementary particles is an obstacle to the creation of a unified theory of them.

A second reason is that the nonvanishing mass of the vector W meson is the source of divergences which cannot be eliminated by mass and charge renormalization. The point is that the Green's functions of massless and massive (of mass m) vector fields have the following form, respectively (in a particular gauge):

$$\frac{\delta_{\mu\nu}-[p_\mu p_\nu/(p)^2]}{(p)^2}, \quad \frac{\delta_{\mu\nu}-(p_\mu p_\nu/m^2)}{(p)^2-m^2}.$$

The worse asymptotic behavior of the Green's function in the limit $p\to\infty$ in the second case is the origin of the nonrenormalizable divergences of the weak interaction. Until we have eliminated this difficulty, we cannot expect to "raise" the weak interaction to the level of the electromagnetic interaction, in which this difficulty is not present, and we are therefore unable to obtain a usable unified theory.

Therefore, hope of success in the creation of such a theory would be justified only if the particles have no mass in the original dynamical equations. There would then be no obstacle to combining the particles into multiplets using group-theory principles and implementing the renormalization program. But in the final expressions, which are compared with the experiments, the particle masses must of course reappear. All that we have said above indicates that if this program is to be realized we must exploit the idea of spontaneous symmetry breaking by taking the original dynamical equations in massless form, and must attribute the appearance of particle masses in the final expressions to the BCS and Higgs mechanisms.

The first variants of such a unified theory of the weak and electromagnetic interactions were proposed by Weinberg and Salam.[31] Their decisive element consisted of the use of the Higgs model, in the framework of which the spontaneous symmetry breaking takes place (see Sec. 9). Referring to the reviews[29] for details, we give below a very schematic expression (which omits many important details) for the corresponding Lagrangian, which is intended to illustrate, not so much the actual unification of the particles, as rather the spontaneous appearance of their masses. Such a model, which will be used in the following section to describe the results of external influences on the elementary particles, is obtained by adding to the Higgs Lagrangian (18) the Lagrangian of the field ψ of the leptons, which interact with the scalar field (coupling constant g),[16)]

$$L=-\frac{1}{16\pi}(\partial_\mu A_\nu-\partial_\nu A_\mu)^2+|(\partial-ieA)\varphi|^2 +\mu^2|\varphi|^2-\lambda|\varphi|^4+(\bar{\psi}[i\gamma(\partial-ieA)-g|\varphi|]\psi). \quad (19)$$

From this we obtain first an equation for the scalar field:

$$[(\partial-ieA)^2-\mu^2+2\lambda|\varphi|^2]\varphi=-\frac{g(\bar{\psi}\psi)\varphi}{|\varphi|}, \quad (20a)$$

which is a generalization of (14) to the case of interaction with electromagnetic and leptonic fields. Further,

[15)] It is the group considerations, in fact, that necessitate the existence of the neutral W meson, which corresponds to the neutral currents mentioned in Sec. 1.

[16)] To avoid the introduction of "right-handed" and "left-handed" particles and other complications, we take this interaction in an extremely unrealistic form. But this does not affect the following conclusions.

from (19) we obtain the equation

$$(\partial^2 + 8\pi e^2 \mid \varphi \mid^2) A = -2\pi ie (\bar{\varphi}\partial\varphi - \partial\bar{\varphi}\varphi) - 4\pi e (\bar{\psi}\gamma\psi), \quad (20b)$$

which describes the electrodynamics of a "medium" in which there are charged scalar and leptonic fields. Equations (20a) and (20b) correspond to the Ginzburg-Landau equations (12). Finally, an equation is also obtained for the charged leptons, which interact with the scalar field:

$$[i\gamma (\partial - ieA) - g \mid \varphi \mid] \psi = 0. \quad (20c)$$

In the Lagrangian (19), both the vector field and the lepton field have zero masses. These masses become nonzero because of the spontaneous symmetry breaking—the Bose condensation of the scalar field. The mass of the vector field is equal, as in the Higgs model, to $\sqrt{8\pi}e|\Psi|$, where $\Psi = \langle\varphi\rangle$, and arises through the Higgs mechanism. Comparing (20c) and (1), we arrive at a lepton mass equal to $g|\Psi|$. Its appearance can be related to the BCS mechanism, although it is determined in the given case by the expectation value not of the same lepton field (or, more precisely, the value of $\psi\psi$; see Secs. 7 and 8), but of the scalar field, which is dynamically coupled to the lepton field.

Like the Higgs model, the unified theory of particles with spontaneous symmetry breaking has an intimate and far reaching analogy with superconductivity theory. The consequences of this analogy will be discussed in the remaining sections of the review. Note that the analogy with superconductivity would be complete if the scalar field were not introduced artificially but arose by itself as the field of Cooper pairs of leptons. Attempts in this direction have already been made, but it is by no means easy to implement this program, which would free us from the "redundant" field not observed in nature.

11. MACROSCOPIC INFLUENCES ON ELEMENTARY PARTICLES

In Sec. 4, we have already mentioned the existence of external influences on ordered many-body systems that reduce the order parameter and, when sufficiently strong, lead to a phase transition to a disordered state and to restoration of the broken symmetry. This conclusion can be completely extended to the systems of elementary particles described by a theory which includes spontaneous symmetry breaking. The corresponding influences (above all the temperature) alter fundamental characteristics of the particles such as their mass, the Fermi constant of the weak interaction, etc., ultimately transforming massive particles into massless particles, and the short-range weak interaction into a Coulomb-like long-range interaction, etc. This problem was posed in Refs. 26 and 32 and then developed by many theoreticians; we refer the reader to the review of Ref. 33, which contains a detailed bibliography.

We begin by considering the influence of temperature, assuming that a system of fields is, like thermal radiation, in a state of thermodynamic equilibrium at some temperature T. We shall show on the example of Goldstone's model (Sec. 9) that, as in many-body systems, the order parameter $\Psi = \langle\varphi\rangle$ decreases with increasing T, vanishing for $T \geq T_c$ (see Fig. 2a). To this end, we consider the expression (16), concentrating our attention on the second (fluctuation) term in the brackets. Using the relativistic analog of the expansion (5) of the field operator and omitting the contribution of the zero-point fluctuations of the field (it leads to a renormalization of the value of μ^2), we obtain

$$\langle|\varphi'|^2\rangle \propto \sum_p \frac{n_p}{E_p}, \quad (21)$$

where n_p are the occupation numbers (6), $\bar{\mu}=0$, $E_p = \sqrt{p^2+m^2}$, m is the quasiparticle mass, and summation over all species of quasiparticle is understood. Ignoring first in (21) the masses of the quasiparticles (at small λ, they are small compared with the critical temperature T_c), we find that the expression (21) is proportional to T^2. Therefore, the dependence of the order parameter on the temperature actually does correspond to the curve of a phase transition of the second kind (see Fig. 2a) with $T_c \propto |\Psi|_{T=0} \propto \mu/\sqrt{\lambda}$.

The temperature dependence of the masses of the quasiparticles (Fig. 4) is nontrivial. The mass of the Goldstone particle is zero over the complete range from 0 to T_c, and the mass of the second quasiparticle is proportional to the modulus of the order parameter and decreases monotonically from the value $\sqrt{2}\mu$ at $T=0$ to zero at $T=T_c$. At the point T_c itself, both masses disappear, and this corresponds to a growth of fluctuations (infrared singularities). When the symmetry is restored ($T>T_c$), one could expect that the square of the quasiparticle masses would, as in the original Lagrangian (13), be negative; this however would lead to instability of the system (see Sec. 9). Indeed, it can be shown that the quasiparticles in this region acquire ordinary masses, which increase with increasing T from the value zero at the point T_c. Here, we have manifestation of a purely thermal contribution to the quasiparticle mass, which exists independently of the spontaneous symmetry breaking. All of what we have said can be readily deduced from Eq. (17) after if has been averaged with allowance for the fluctuation term.

Speaking about this term earlier, we completely ignored the dependence in (21) on the mass of the quasiparticles. It can be shown that already allowance for the first term of the expansion (21) with respect to the ratio m/T [this reduces to the factor $1-(3/\pi)(m/T)$] transforms the phase transition of the second kind into one of the first kind (see Sec. 5). It is true that in the Goldstone model with small coupling constant λ the corresponding latent heat of transition is small, and the situation is changed only in the immediate proximity of

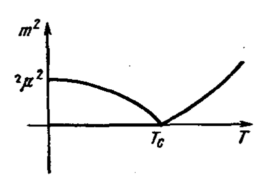

FIG. 4.

T_c. However, in the Higgs model, to the consideration of which we now turn, the picture of a phase transition of the first kind is the more clearly expressed, the larger is the ratio e^2/λ of the two dimensionless constants of this model.[28]

Indeed, in the Higgs model the fluctuations of the vector field, described by the term $e^2A^2\varphi$ in (20a), are added to the fluctuations of the scalar field. Their contribution to the brackets in (16) is given by $e^2\langle A^2\rangle$, which is expressed by the same formula (21) with $m \propto e|\Psi|$. Taking into account in (16) the correction associated with the mass of the vector field, we do indeed arrive at the picture shown in Fig. 2b. In connection with Sec. 5, we add that this correction makes a contribution to the Landau expansion (4) proportional to $|\Psi|^3$ with a negative coefficient; the nonanalyticity of the free energy as a function of $|\Psi|^2$ is due to the appearance in (21) of a term proportional to the mass $m \propto |\Psi|$.

The effect of transformation of a phase transition of the second kind into one of the first kind because of the influence of the fluctuations of the electromagnetic field must also occur in a superconductor.[17)] We did not mention it in Sec. 7 since, being small, it is not seen in the experiments, and its theoretical prediction was made quite recently[34] (after the similar effect was discovered by A. D. Linde in the Higgs model).

Concluding here our consideration of the effects of temperature, we now consider the effect of an external magnetic field. In the Higgs model, as in a superconductor, such a field reduces the order parameter, leading ultimately to the complete restoration of symmetry. Discussing this question in Sec. 7, we employed arguments associated with an inhomogeneous field configuration, but in the presently considered case of an infinite vacuum these do not apply directly. Therefore, we shall give a direct proof that in sufficiently strong fields H the order parameter must vanish. To this end, we consider the vacuum expectation of Eq. (20a) in the static limit and for $g=0$. Our aim is to demonstrate the fact that Ψ vanishes already at a finite value of the field. To this formulation of the problem there corresponds the equation $[(\nabla - ie\mathbf{A})^2 + \mu^2]\Psi = 0$, which is analogous to the Schrödinger equation for an oscillator and does not have nontrivial solutions above the field value $H=\mu^2/e$ (see Ref. 11).[18)]

As in a superconductor, the vacuum in the Higgs model behaves in two different ways in the presence of a strong magnetic field—either it remains homogeneous or it forms a system of vortex filaments, in which the field is concentrated. Whether the vacuum belongs to a type I or type II system (see Sec. 7) is determined by whether the ratio e^2/λ is greater than or less than unity, i.e., whether the ratio of the masses of the vector and scalar fields is greater than or less than unity.[28]

It should be emphasized that what we have said applies directly only to the simplest Higgs model with the Lagrangian (18). In realistic models of the unified theory of particles, the corresponding Lagrangian contains an entire multiplet of vector fields, and only the massive vector fields, and not the true electromagnetic field, interact with the scalar particles directly; it is for this reason that the photon mass remains, as is required, equal to zero. Therefore, all that we have said above applies to the effect on the vacuum not of the the true magnetic field, but rather of the "quasimagnetic" field corresponding to the massive vector particles (which, however, cease to be massive after the symmetry has been restored).

Interest in the effect of a magnetic field took on a practical nature in connection with the following proposal made by Salam and Strathdee.[35] It is known that the rate of weak decay of strange particles differs, under otherwise equal conditions, from the same quantity for particles with zero strangeness, this difference being determined by the so-called Cabibbo angle. The actual appearance of this angle breaks the symmetry of the Lagrangian of the weak interaction. If it is assumed that this breaking is spontaneous, then an external magnetic field of sufficient strength will annihilate the Cabibbo angle and "stabilize" a strange particle with respect to weak decay. In this connection, one can consider, in particular, a possible influence of the magnetic field within Λ nuclei on the rate of decay of the Λ particle.[36] Referring to the review of Ref. 33 for details, we mention that the possibility of observing an effect of this kind seems to be very remote. In the framework of realistic models, the influence of a true magnetic field on spontaneously broken symmetries is much weaker than that of a quasimagnetic field, and the value of the fields required to achieve an observable effect is exceptionally high.

The last type of influence that we consider arises from the existence in the system of a lepton current $j=e(\bar{\psi}\gamma\psi)$. Essentially, we repeat the arguments given in Sec. 7 for a superconductor, but the result of the arguments is now more interesting. We proceed from Eqs. (20a) and (20b), setting $g=0$; the direct influence of the right-hand side of (20a), which is small compared with the influence of the current j itself, would correspond to induced breaking or restoration of the symmetry (see Sec. 5). Calculations analogous to those made in Sec. 7 have the consequence that one must add to $-\mu^2$ in (20a), not a positive term, as in the case of a superconductor, but a term of variable sign: $(j)^2/4e^2|\Psi|^4$, where $(j)^2 = j_0^2 - \mathbf{j}^2$ is the four-dimensional square of the current. As in a superconductor, the vector part of the current reduces the order parameter but the zeroth component of the current, i.e., the lepton density, increases the symmetry breaking. It is clear that this effect is related to the relativistic invariance of the theory, which provides a rare example of a factor that does not directly influence the order parameter [as an influence determined by the right-hand side of (20a)] and, at the same time, leads, not to restoration of

[17)] This effect is due physically to the long known fact that an external magnetic field leads to just such a change in a superconductor.

[18)] Without going into details, we note that in this way one can calculate only the "upper" critical field, which does not exhaust the problem.

symmetry, but rather increases the extent to which it is broken.[37]

This leads one to ask whether there is not an analogous effect in superconductivity. This is of interest since we know of no direct influence on the superconducting order parameter [it would have to have the form $V\psi\psi$ in the Hamiltonian (see (9)) and make is possible to increase the symmetry breaking]. Therefore, a positive answer to this question would open up new possibilities in the important and difficult problem of a radical increase in the critical temperature of the superconducting transition (see Ref. 38).

Completing our discussion of external influences on spontaneously broken symmetries in elementary-particle theory, we emphasize that an appreciable variation of the order parameter requires very extremal values of the external factors. For example, the critical value T_c of the temperature is of the order of the energy scale of the weak interaction: $\mu/\sqrt{\lambda}\sim 1$ TeV $\sim 10^{16}$ deg. Therefore, the effects could be manifested only under exceptional conditions; in a collision of particles of superhigh energies, in the early stages of the evolution of a "hot" Universe, and so forth. The first of these possibilities has only just started to be developed[47]; with regard to the cosmological consequences of the effect, quite a number of results, which make important corrections to the standard cosmological scheme, have already been achieved. We refer the reader to the review of Ref. 33 and the investigations of Ref. 48, where a detailed bibliography of cosmological applications can also be found.

12. VORTEX FILAMENTS, MONOPOLES, AND MAGNETIC CONFINEMENT OF QUARKS

The analogy between the unified theory of particles and the theory of superconductivity finds still other applications in elementary-particle theory. We are here referring to the vortex filaments already mentioned more than once above (Secs. 7 and 11), which, as Nielsen and Olesen[27] have shown, really do arise as classical solutions of the equations of the Higgs model and more complicated models of the same type, being clearly expressed for $e^2\ll\lambda$ (type II theory; see Sec. 11). As we have already said, each filament carries a fixed magnetic flux and has an energy proportional to the length of the filament. The magnetic field is localized within the filament, but the order parameter in this region is close to zero.[19)]

Nielsen and Olesen associated the vortex filaments with "strings"—the linear relativistic objects introduced some time ago in the theory of the strong interaction. Referring the reader to the review of Ref. 39 for details, we restrict ourselves to the following remarks. One of the promising directions in the theory of the strong interaction—the dual resonance model—makes it possible to describe in a unified manner the asymptotic behavior of the scattering of strongly interacting particles at high energies and the characteristics of resonances at low energies. It was found that the equations of the dual resonance model can be given a dynamical meaning if one formally associated this model with the Lagrangian of a "string." However, this concept was introduced into the theory to a large extent at the formally mathematical level and it is only the picture of vortex filaments that makes it possible to give it a direct physical content. In addition, the quantization of purely linear "strings" of zero thickness encountered serious difficulties, which make it necessary to introduce spaces with a large number of dimensions.[40] The identification of the "strings" with vortex filaments, which have a transverse structure, removes these difficulties.

A picture of vortex filaments was also used by Nambu,[41] who proposed a special "magnetic" mechanism of confinement of quarks to prevent their appearance in the free state. Consider, for example, a meson consisting of a quark and an antiquark coupled by a vortex filament. As we have already said, the energy of the filament, and, therefore, the interaction energy of the quark and the antiquark, is proportional to the length of the filament, i.e., the distance between these particles. But this means that it is energetically impossible for quarks to appear separately (potential well with linearly increasing walls). The mechanism of confinement of three quarks within a baryon is more complicated, but this too is possible (see Ref. 42).

It is important to note that a vortex filament carries a magnetic flux and is similar in this sense to a magnetic line of force. Therefore, this mechanism of quark confinement requires the quark to have a magnetic charge (to be a Dirac monopole, see Ref. 43). Moreover, a vortex filament terminating at one end in a monopole can be regarded as a physical realization of a filamentary "tail" of a monopole, which arose as a certain linear singularity of the Dirac solution and caused much trouble to the theoreticians.[20)]

To conclude this section, we pose the question concerning the fate of a strongly interacting particle composed of quarks confined by the magnetic mechanism if the temperature is raised sufficiently high. It is intuitively clear that there must then occur a dissociation and the break up of the particle into its constituent quarks, as occurs, for example, when a molecule dissociates in a hot gas. Indeed, as in a superconductor (see Sec. 7), the vortex filament "swells" when the temperature is raised and at the specific point T_c the filament as such completely ceases to exist. At the same time, the linear law of attraction, which does not allow dissociation, is replaced by the ordinary Coulomb law of attraction characteristic of the interaction of two monopoles of opposite sign of the magnetic charge. And

19) Here and in what follows, we shall not, for simplicity, distinguish between magnetic and quasimagnetic fields.

20) Note that the monopole itself also arises as a classical solution of the equations of the unified theory of particles.[44]

for such a law, nothing can then hinder the breakup into quarks. It is important to emphasize that this is apparently specific for the magnetic mechanism, which is based ultimately on spontaneous symmetry breaking. It is therefore possible that the question of the quark confinement mechanism will be elucidated by data obtained from collisions at high energies, from cosmological data, and so forth.

13. CONCLUSIONS

Twenty years ago, the theory of elementary particles, which was based on quantum field theory, was in a critical state. The way out of the dilemma was seen by some in an increase in the mathematical "armament" of the theory, by others in a transition to a description of interaction processes less detailed than in ordinary quantum mechanics, by a third group in the need for a radical break with our ideas about spacetime, the causal connection of events, etc.

However, we are today justified in saying that quantum field theory has emerged from the crisis and has overcome the difficulties with, so to speak, its own resources. Today, we are no longer dismayed by the nonrenormalizability of the weak interaction—the unified theory of particles has overcome this difficulty. We have now overcome our fear of the "zero charge" problem—in the unified theory we encounter the opposte situation (asymptotic freedom, see Ref. 45). And, finally, we are no longer particularly afraid of the difficulties in describing the strong interaction on account of that same asymptotic freedom (weakening of the interaction with increasing energy) and other factors.

The quantum-field approach which has emerged has led to impressive successes in many directions of elementary-particle theory (above, we have been able to mention only some of them). This approach promises even more in the future. It therefore appears that in the theory of elementary particles a more or less prolonged stage of supremacy of the ideas and methods of quantum field theory is once more beginning.

Of course, the history sketched above of the rebirth of the almost buried quantum field theory demonstrates clearly how dangerous it is to make predictions by extrapolating to the future the modern tendencies in the development of science. With this important reservation, the immediate future of the theory of elementary particles would seem to be as follows.

The general line of development of the theory will tend to the creation of a unified theory of all particles and their interactions, including gravitation. Most probably, essential use will here be made of the idea of the theory of supersymmetry (unification of Fermi and Bose particles in a single multiplet; see Ref. 46), which represents a further step toward a unified theory and has already given clear examples of mutual compensation of the difficulties inherent in the theories of individual types of particles.

The theory will be based on relatively simple and perspicuous field-theory models (of the type of quantum chromodynamics; see Sec. 8), the transition from these models to observable quantities being made by the methods generally adopted in theoretical physics. In this way, no need will be seen for a radical change in the fundamental ideas or methods of description; indeed, rather, we can expect that even more use will be made of old and well tested ideas taken from macroscopic physics. In brief, the tendencies for increasingly close approach of elementary-particle theory to other branches of theoretical physics will prevail.

If these expectations are fulfilled, we shall be convinced once more of the unity of the physical picture of the world in the sense that it is constructed in general in accordance with a "standard model" principle (if not from standard parts, at least according to standard design). On the other hand, this will amount to a certain downgrading of the theory of elementary particles, which, though it will remain at the frontier of theoretical physics, will occupy for a time an honored but not the exceptional position of "first among equals" among the other branches of theoretical physics.

Whether these expectations will be confirmed or elementary-particle physics will before then encounter new fundamental laws is something that only the future can show.

I thank V. L. Ginzburg, L. V. Keldysh, A. D. Linde, and E. L. Feinberg for discussions and valuable comments.

[1]Sdvig urovneĭ Atomnykh Élektronov (Level Shifts of Atomic Electrons; Russ. Transl., IL, 1950). Noveĭshee Razvitie Kvantovoĭ Élektrodinamiki (Latest Developments in Quantum Electrodynamics; Russ. Transl., IL, M. 1954).

[2]A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya Élektrodinamika, Nauka, M. 1969 (Transl. of earlier edition: Quantum Electrodynamics, Interscience, New York, 1965); S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Row, Peterson, and Co., Evanston, Ill. 1961 (Russ. Transl. IL, M. 1963; N. N. Bogolyubov and D. V. Shirkov, Vvedenie v teoriyu kvantovannykh poleĭ, Nauka, M. 1976 (Transl. of earlier edition: Introduction to the Theory of Quantized Fields, Interscience, New York, 1959); V. B. Berestetskiĭ, E. M. Lifshits, and L. P. Pitaevskiĭ, Relyativistskaya Kvantovaya Teoriya, Vol. 1, Nauka, M. 1968 (Trans: Relativistic Quantum Theory, Oxford, 1971); E. M. Lifshits and L. P. Pitaevskiĭ, Relyativistskaya kvantovaya teoriya (Relativistic Quantum Theory, Vol. 2), Nauka, M. 1971.

[3]V. Weisskopf, Phys. Rev. 56, 72 (1939).

[4]V. L. Bonch-Bruevich and S. V. Tyablikov, Metod Funktsiĭ Grina v statisticheskoĭ mekhanike, Fizmatgiz, M. 1961 (English Transl.: The Green's Function Method in Statistical Mechanics, North-Holland Publ. Co., Amsterdam, 1962); A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike, Fizmatgiz, M. 1962 (English Transl.: Methods of Quantum Field Theory in Statistical Physics, Englewood Cliffs (1963)); A. B. Migdal, Teoriya konechnykh Fermi-sistem i svoĭstva atomnykh yader, Nauka, M. 1965 (English Transl.: Theory of Finite Fermi Systems, Interscience, New York 1967).

[5]D. A. Kirzhnits, Polevye metody teorii mnogikh chastits (Field Methods in Many-Particle Theory), Atomizdat, 1963;

E. S. Fradkin, Tr. Fiz. Inst. Akad. Nauk SSSR 29, 7 (1965); L. V. Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964) [Sov. Phys. JETP 20, 1018 (1965)].
[6]G. A. Zisman, Zh. Eksp. Teor. Fiz. 10, 1163 (1940); 11, 631 (1941).
[7]A. Z. Patashinskiĭ and V. L. Pokrovskiĭ, Fluktuatsionnaya teoriya fazovykh perekhodov (Fluctuation Theory of Phase Transitions), Nauka, M. 1975; K. Wilson and J. Kogut, Renormalization Group and ϵ Expansion (Russ. Transl., Mir, M. 1975); Kvantovaya teoiya polya i fizika fazovykh perekhodov (Quantum Field Theory and the Physics of Phase Transitions; Russ. Transl., Mir, M. 1975.
[8]N. N. Bogolyubov, V. V. Tolmachev, and D. V. Shirkov, Novyĭ metod v teorii sverkhprovodimosti, Izd-vo Akad. Nauk SSSR, M. 1958 (English Transl.: A New Method in the Theory of Superconductivity, New York 1959); J. R. Schreiffer, Theory of Superconductivity, Benjamin (1964) (Russ. Transl., Nauka, M. 1968).
[9]Nelineĭnaya kvantovaya teoriya polya (Nonlinear Quantum Field Theory; Russ. Transl., IL, M. 1959); W. Heisenberg, Introduction to the United Field Theory of Elementary Particles, Wiley (1967) (Russ. Transl., Mir, M. 1968).
[10]L. D. Landau and E. M. Lifshitz, Statisticheskaya fizika, Nauka, M. 1964, 1976 (English Transl.: Statistical Physics, Pergamon Press, 1969).
[11]V.L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).
[12]P. G. De Gennes, Superconductivity of Metals and Alloys, Benjamin, New York, 1966 (Russ. Transl., Mir, M. 1968).
[13]J. D. Bjorken, Ann. Phys. (New York) 24, 174 (1963); J. des Cloizeaux, J.Phys. A6, 597 (1973).
[14]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
[15]V. G. Vaks and A. I. Larkin, Zh. Eksp. Teor. Fiz. 40, 282 (1961) [Sov. Phys. JETP 13, 192 (1961)].
[16]H. Pagels, Phys. Rev. D 14, 2747 (1976); D. Caldi, Phys. Rev. Lett. 39, 121 (1977).
[17]P. I. Fomin, Fiz. Elem. Chastits At. Yadra 7, 687 (1976) [Sov. J. Part. Nucl. 7, 269 (1976)].
[18]A. I. Vainshtein and V. I. Zakharov, Usp. Fiz. Nauk 100, 225 (1970) [Sov. Phys. Usp. 13, 73 (1970)].
[19]Proc. of Seminars on Unified Theory of Elementary Particles (Rochester, 1963, München, 1965).
[20]Proc. of Symposium on Basic Question in Elementary Particles Physics, München (1971).
[21]M. Baker and G. Glashow, Phys. Rev. 128, 2462 (1962).
[22]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962); S. Bladman and A. Klein, Phys. Rev. 131, 2362 (1963).
[23]M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).
[24]D. A. Kirzhnits and V. N. Sazonov, in: Éinshtenovskiĭ sbornik, 1973 (Einstein Collection, 1973), Nauka, 1974.
[25]P. W. Higgs, Phys. Lett. 12, 132 (1964).
[26]D. A. Kirzhnits, Pis'ma Zh. Eksp. Teor. Fiz. 15, 745 (1972) [JETP Lett. 15, 529 (1972)].
[27]H. Neilsen and P. Olesen, Nucl. Phys. B61, 45 (1973).
[28]D. A. Kirzhnits and A. D. Linde, Preprint Lebedev Inst. No. 101, M 1974).
[29]S. Weinberg, Usp. Fiz. Nauk 118, 505 (1976); V. B. Berestetskiĭ, in: Élementarnye chastitsy (Elementary Particles), No. 1, Atomizdat, 1973; L. B. Okun', Élementarnye chastitsky (Elemenatry Particles), No. 1, Atomizdat, M. (1973); A. I. Vaĭnshteĭn and I. B. Khriplovich, Usp. Fiz. Nauk 112, 685 (1974) [Sov. Phys. Usp. 17, 263 (1974)].
[30]V. L. Ginzburg and G. F. Zharkov, Usp. Fiz. Nauk 125, 19 (1978) (in the present volume of this journal) [Sov. Phys. Usp.].
[31]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in: Elementary Particle Theory, Stockholm (1968).
[32]D. A. Kirzhnits and A. D. Linde, Phys. Lett. B42, 471 (1972).
[33]D. A. Kirzhnits and A. D. Linde, Ann. Phys. (New York) 101, 195 (1976).
[34]B. Halperin, T. C. Lubensky, and S. Ma, Phys. Rev. Lett. 32, 292 (1974).
[35]A. Salam and J. Strathdee, Nucl. Phys. B90, 203 (1975).
[36]A. Salam and J. Strathdee, Preprint IC-76-62, Trieste (1976); S. M. Polikanov, Neobychnye yadra i atomy (Unusual Nuclei and Atoms), Nauka, M. 1977.
[37]A. D. Linde, Preprint Lebedev Inst. No. 25, Moscow (1975).
[38]L. N. Bulaevskiĭ, V.L. Ginzburg, G. F. Zharkov, D. A. Kirzhnits, Yu. V. Kopaev, E. G. Maksimov, and D. I. Khomskiĭ, Problema vysokotemperaturnoĭ sverkhprovodimosti (The Problem of High-Temperature Superconductivity), Nauka, M 1977.
[39]M. S. Marinov, Usp. Fiz. Nauk 121, 377 (1977) [Sov. Phys. Usp. 20, 179 (1977)].
[40]J. Scherk, Rev. Mod. Phys. 47, 123 (1975).
[41]Y. Namub, Phys. Rev. D10, 4262 (1974).
[42]S. Mandelstam, Phys. Lett. B53, 476 (1975); A. D. Linde, Preprint IE-76-33, Trieste (1976).
[43]P. A. M. Dirac, Proc. R. Soc. A133, 60 (1931); Phys. Rev. 74, 817 (1948).
[44]C. t'Hooft, Nucl. Phys. B79, 276 (1974); A. M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. 20, 430 (1974) [JETP Lett. 20, 430 (1974) [JETP Lett. 20, 194 (1974)].
[45]V. B. Berestetskiĭ, Usp. Fiz. Nauk 120, 439 (1976) [Sov. Phys. Usp. 19, 934 (1976)].
[46]V. I. Ogievetskiĭ and L. Mezincescu, Usp. Fiz. Nauk 117, 637 (1975) [Sov. Phys. Usp. 18, 960 (1975)].
[47]S. Eliezer and R. Weiner, Phys. Rev. D13, 87 (1976); I. V. Krive, P. I. Fomin, and E. M. Chudnovskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. 25, 215 (1977) [JETP Lett. 25, 196 (1977)].
[48]Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun', Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys. JETP 40, 1 (1974)]; I. V. Krive, A. D. Linde, and E. M. Chudnovskiĭ, Zh. Eksp. Teor. Fiz. 71, 825 (1976) [Sov. Phys. JETP 44, 435 (1976)].

Translated by Julian B. Barbour