METHODOLOGICAL NOTES

A simple demonstration of Newton's laws

L. A. Gribov and A. N. Gornostaev

K. A. Timiryazev Moscow Agricultural Academy

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There exists a large number of different demonstrations of Newton's laws used in lectures at various stages in general physics courses. Nevertheless, since we are dealing with the most fundamental propositions in the whole of physics, the development of new effective demonstrations is still important, especially since the quantitative aspect of these laws is not particularly clearly visible in any of the known experiments (see, for example, Ref. 1). For example, one of the most common demonstrations of Newton's law consists of the following: two Grimsehl carriages which can move with little friction (modern variants of this experiment use the air cushion principle) are placed on a bench and repel one another by means of a compressed spring. If the carriages are placed in the center of the bench, then when they are pushed apart by the spring they begin to move in opposite directions and reach the ends of the bench simultaneously, if the carriages have equal masses, and at different times if the masses are different, the heavier carriage moving more slowly. While the experiment with equal carriages can be interpreted as being due to the application to the carriages of forces of equal magnitude but acting in opposite directions, the situation is not obvious in the experiment with different carriages, since in this experiment one sees only the motion of the carriages with different velocities. Thus, the quantitative relationships between the masses and the velocities escape the attention of the observer.

In the present note, we describe a readily performed and simple experiment which, while completely retaining all the elements of the usual experiment, including the equipment, nevertheless makes it possible to demonstrate the quantitative aspects of Newton's laws. A photograph of the equipment is shown in Fig. 1. It consists of the bench 1, on which the Grimsehl carriages 2 are placed. At its right-hand end the bench is supported by means of a knife edge 3. The left-hand end 4 of the bench is suspended by a weak spring 5 from the support 6 and can move readily in the vertical direction. Before the start of the experiment, by moving the carriages the bench is balanced in the horizontal position, as shown by the pointer 7 on the scale 8. The bench together with the support is placed on the trolley 9, which can be set in uniform motion with a velocity of about 0.1 m/sec along the rails 11 by means of the electric motor 10. The overall size of the equipment is 1.5 m, and the rails are 3 m long.

The condition of equilibrium of the bench is, in accordance with the lever principle—well known already from school courses and readily verified (if necessary on the same instrument)—

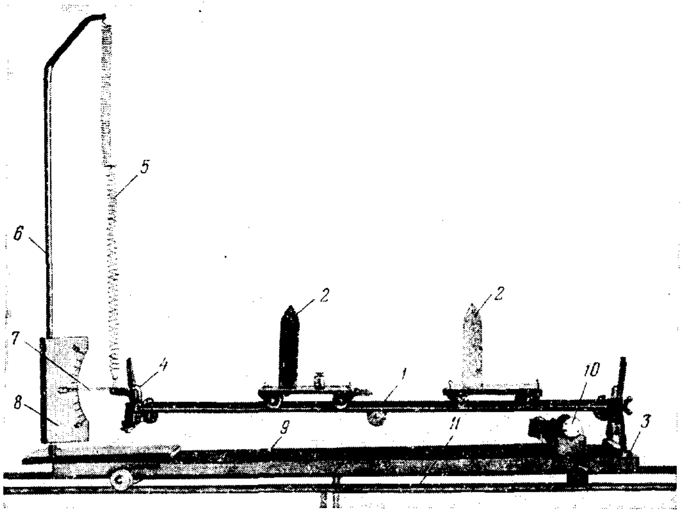

FIG. 1.

$$g m_1 x_1 + g m_2 x_2 + g M X = F l; \tag{1}$$

here m_1, m_2, and M are the masses of the carriages and the bench, l is the length of the bench, F is the tension of the spring, which is proportional to the amount by which it is extended, and x_1, x_2, and X are the coordinates of the centers of mass of the carriages and the bench measured from the right-hand edge of the bench. If the carriages are allowed to move under the influence of any internal forces (of a compressed spring, the repulsion of like poles of magnets fitted on the carriages, and so forth), the result of the experiment will be conservation of the equilibrium of the complete system (i.e., the horizontal position of the bench) until at least one of the carriages reaches an end of the bench. This is obviously only possible if the relation (1) remains in force during the whole of the time of the motion, i.e., if

$$g m_1 x_1 + g m_2 x_2 = \text{const.} \tag{2}$$

holds.

Differentiating (2), we obtain

$$m_1 \dot{x}_1 + m_2 \dot{x}_2 = 0$$

(which verifies the law of conservation of momentum), and, further,

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = 0,$$

$$f_1 = m_1 \ddot{x}_1, \; f_2 = m_2 \ddot{x}_2$$

(which verifies Newton's second law).

Moreover, it is immaterial whether the carriages move uniformly (after the spring has ceased to act) or in accordance with a more complicated law in the case of magnetic interaction. The sensitivity of the experiment depends only on the sensitivity of this somewhat unconventional balance.

It is also important to note that the same device can be readily used to demonstrate that the law of momentum conservation is no longer satisfied in the presence of external forces. For this, one can either apply to one of the carriages an additional external force (which is most readily done by attaching one of the carriages by a piece of elastic to the center of the bench) or by using a pair of carriages with different rolling friction (though this is less obvious).

Joining the carriages by a long piece of elastic and pulling them apart to the ends of the bench and then letting them move toward each other, one can also demonstrate the conservation of momentum of the system in an elastic collision using the same instrument.

Note that since the experiment combines elements of statics (equilibrium of the balance) and dynamics, the equivalence of inertial and gravitational masses is simultaneously demonstrated.

As we have said above, the bench when in equilibrium is placed on a further trolley which can be set in uniform motion along the rails. Then the complete experiment can be performed while the apparatus is in motion. This illustrates that Newton's laws hold in a coordinate system in uniform motion relative to the laboratory system. So far as we know, this has not been demonstrated in such a clear form hitherto. It is somewhat more complicated to perform the experiment in the case of a uniformly accelerated motion of the apparatus, but this is also in principle possible.

[1] V. I. Iveronova (ed.), Lektsionnye demonstratsii po fizike (Lecture Demonstrations in Physics), second edition, Nauka, Moscow (1972).

[2] E. Grimsehl, A Textbook of Physics, Vol. I, London (1930) (an English translation from the German; Russian translation published by GIZ, Moscow, 1928).

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