On the fiftieth anniversary of the paper by M. A. Leontovich and L. I. Mandel'shtam ("On the theory of the Schrödinger equation")[1)]

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The emergence and establishment of quantum mechanics in 1925-1927 and the creation of its mathematical apparatus gave rise to a flood of theoretical investigations in this field.[1] This led to a deeper understanding of the nature of matter and eventually made possible the construction of a fundamentally new picture of the physics of the microworld. In this connection the article by M. A. Leontovich and L. I. Mandel'shtam is of great interest.

The authors posed the question: how is the nature of the solutions of the stationary Schrödinger equation altered when the usual oscillator potential $V(x)=\alpha x^2/2$ is replaced by a potential which is arbitrarily weakly "depressed" at the edges (say, by $V(x)=(\alpha x^2/2)e^{-kx^2}$ with small k)? In the former case, as is well known, the energy spectrum is entirely discrete, while in the latter case it turns out to be entirely continuous.

The authors carried out a discussion of this somewhat paradoxical situation using the example of an oscillator with a cut-off (at large distances) potential (cf., Fig. 1 taken from their article).

It turned out that depending on the ratio of the energy of the particle and the energy of a level of a corresponding "uncut-off" oscillator two qualitatively different physical pictures are realized. In the case when the above two energies are sufficiently different the (more usual) case of free motion of the particle is realized accompanied by reflection and, it should be added, by leakage through the potential barrier (Fig. 1a). But when the two energies coincide or are close then in spite of the continuous nature of the energy spectrum, "there exist eigenfunctions which practically differ from zero only in a restricted region near the origin" (Fig. 1b). In this case the particle can be situated also outside the boundary of the potential well, and as a result of this its state within the potential well turns out to be, utilizing "temporal" language, quasistationary.

Thus, the paper of M. A. Leontovich and L. I. Mandel'shtam gave for the first time a treatment of a smooth transition between a discrete and a continuous energy spectrum and a deeper treatment of the concept of "quantization" which is not exhausted by finding the spectrum of levels but which requires an investigation of the behavior of wave functions. In a certain sense it could be said that not simply levels are quantized, but, generally speaking, states.

The paper of M. A. Leontovich and L. I. Mandel'shtam is remarkable because in it is grasped the essence simultaneously of two most important quantum concepts—the tunnel effect and quasistationary states. According to an apposite remark of Papaleksi[2] "this paper contains within it essentially all the foundations of the theory of penetration of particles through a potential barrier which is one of the most remarkable achievements of quantum mechanics. And indeed, it served as a direct impetus and a basis for subsequent papers on the theory of radioactive decay."

The depth of the analysis of the relation between the discrete and the continuous energy spectra carried out by M. A. Leontovich and L. I. Mandel'shtam is especially clear at present when the concepts of the tunnel

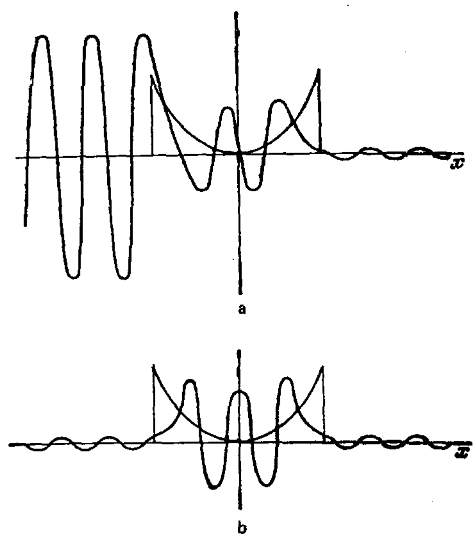

FIG. 1. The heavy curve schematically represents the wave function and the light curve schematically represents the potential. a) The energy of the particle lies midway between two neighboring levels of the "uncut-off" oscillator; b) the energy of the particle coincides with one such level.

[1)]M. A. Leontovich and L. I. Mandel'shtam, On the theory of the Schrödinger equation, Z. Phys. 47, 131 (1928).

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effect and of quasistationary states form the basis for the theory of a number of important physical phenomena. To them belong such phenomena as α-decay of radioactive nuclei, cold emission of electrons from metals, decay of atoms in an external electric field, processes of charge exchange in atomic collisions and chemical reactions with a transfer of atoms, migration of electrons in a crystalline lattice and many others.

We take this occasion to offer heartfelt congratulations to one of the authors of this paper Mikhail Aleksandrovich Leontovich on his seventy-fifth birthday, and to wish him good health and creative achievements.

[1]Usp. Fiz. Nauk 122, No. 4 (August 1977) [Sov. Phys. Usp. **20**, No. 8 (August 1977)].

[2]N. D. Papaleksi, in the "Collected Works of L. I. Mandel'shtam", S. M. Rytov, ed., v. 1, Moscow-Leningrad, Published by the Academy of Sciences of the USSR, 1948, p. 52.

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