Clusters in hadron multiple production processes

I. M. Dremin

P. N. Lebedev Physical Institute, Academy of Sciences of the USSR

C. Quigg

Fermi National Accelerator Laboratory, Batavia, Ill., USA

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Secondary particles produced in inelastic collisions of hadrons at high energies turn out to be noticeably correlated. This fact can be explained by assuming that the process is a two-stage one involving the formation in the intermediate stage of a correlated group of particles designated by the term cluster.

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Collisions of strongly interacting particles (hadrons) at sufficiently high energies usually result in the production of many new (secondary) particles. This process has for many years provided one of the principal topics for research in high-energy physics in general and in cosmic rays in particular. The pioneering cosmic-ray studies made it possible to formulate very simple phenomenological ideas concerning the nature of the process. In recent years these ideas have been enriched and have become considerably more detailed, thanks to the accumulation of ever more precise experimental data obtained at the 70 GeV proton synchrotron at Serpukhov (USSR), at the 500-GeV proton synchrotron at Batavia (USA), and at the CERN accelerator in Geneva (Switzerland), where intersecting proton beams with energies up to 30 GeV produce proton-proton collisions equivalent to the collision of a ~2000-GeV proton with a target at rest. Since the basic survey experiments in this energy region have already been done, it is reasonable to review briefly what we have learned concerning the mechanism of multiple production of particles and what new problems face us.

Despite the steady accumulation of experimental information and the unceasing efforts of the theorists, we still do not have at our disposal a unified theory of the processes that take place in the interaction of high-energy particles. One has to deal with a large number of schemes that differ appreciably from one another, each of which emphasizes some single aspect of the experimental information and hence accounts for only a limited group of phenomena. One of the most useful general ideas is the concept of hadron clusters. According to this concept particle production takes place in two stages: first, certain unstable correlated groups of particles (the so-called clusters) are produced, and these then decay independently into the final (secondary) particles.

Experimental confirmation of the existence of clusters is usually based on indirect facts, but these, taken together, lead to the general conclusion that an average cluster "consists of" (more precisely, decays into) three or four particles and can frequently be identified with a known meson resonance. However, since the contribution from low-mass pion resonances that decay into only two or three particles is also fairly large, such an "average" cluster can be obtained only if the mass spectrum of the clusters is fairly broad, i.e., clusters that decay into a larger number of particles must also be produced. The purpose of the present article is to discuss the concept of hadron clustering—the reasons for the appearance of the concept, its experimental confirmation, and its theoretical consequences.

GENERAL INFORMATION ON HIGH-ENERGY HADRON INTERACTIONS

The total cross section for (or probability of) the interaction of hadrons is almost constant over a fairly wide range of high energies, but the ratios of the partial cross sections for different reaction channels are appreciably energy dependent. Elastic scattering dominates in the low-energy region.[1] At high energies the main contribution to the cross section comes from substantially inelastic processes. Over a considerable range of intermediate energies the inelastic processes are dominated by those having two-particle final states (each of the final particles may be either stable or un-

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[1] In the collision of a relativistic particle with a proton at rest the c.m. energy W is given by $W=\sqrt{2M(E_{\rm lab}+M)}$, where $E_{\rm lab}$ is the energy of the incident particle in the lab system and M is the proton mass ($Mc^2\approx 1$ GeV). For high energies ($E_{\rm lab}>>M$) one can obtain an approximate value for W by taking the square root of twice the beam energy in GeV.

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stable). It is just the study of such quasi-two-particle reactions that led in the early 1960s to the discovery of an enormous number of new hadron resonances. True multiple production processes, however, to which the present article is mainly devoted, are dominant at c.m. energies above 8 GeV.

The primary data on the final states resulting from the collision of two high-energy particles consists of the momenta of the produced particles (sometimes only their emission angles are known) and of usually incomplete information on the nature of these particles (their masses, charges, etc.). The three basic types of particle detectors—nuclear emulsions, bubble chambers, and electronic counters—register particles on the basis of the ionization they produce and are therefore very effective in detecting charged particles but are not sensitive to neutral particles, which can still sometimes be recognized, however, if their decay products are charged ($\rho^0 \to \pi^+\pi^-$, for example) or if they lead to the appearance of charged particles ($\pi^0 \to 2\gamma \to 2(e^+e^-)$, for example). In most experiments, therefore, each individual event is only partly recorded.

The necessity of dealing with the loss of many details of the multiparticle final states led to the concept of inclusive cross sections. Instead of measuring all the kinematic parameters of each of the particles, it is useful to characterize the final states by a small number (e.g. one) of the final particles with specified momenta while completely ignoring the characteristics of the other (undetected) particles. Thus, for example, the single-particle inclusive cross section fixes the probability of detecting a single secondary particle with specified momentum accompanied by an arbitrary set of undetected particles. In this language the total cross section for a process could be called the general inclusive cross section without detecting any specially selected particle.

Some general qualitative features of multiple production processes have been firmly established:

1. The secondary-particle momentum components perpendicular to the collision axis of the initial hadrons are greatly limited and are small compared to the "longitudinal" components. The average transverse momentum is about 350 MeV/c and does not depend (or depends very weakly) on the energy of the colliding particles. The momentum vectors of the produced hadrons are therefore highly collimated along the collision axis, so it is useful to idealize the entire picture and treat the process as virtually one-dimensional in momentum space.[2] Here it turns out to be very helpful to use such a kinematic quantity as the (longitudinal) rapidity

$$y=\frac{1}{2}\ln\frac{E+p_l}{E-p_l},$$

where E and p_l are the energy and the longitudinal component of the momentum of an observed particle. Let us point out, in particular, a few important and widely used properties of this quantity. First, rapidity differences are invariant under Lorentz transformations along the collision axis. Thus, the concept of particles being close together or far apart in momentum space acquires a Lorentz invariant meaning. Second, the replacement of the longitudinal component of the momentum in the Lorentz invariant element d^3p/E of phase space by the rapidity, leads to its elegant separation into transverse and longitudinal parts, taking the form πdp_t^2dy. After integrating over the transverse momenta taking into account the limitations on them we obtain the one-dimensional rapidity distribution.[3] And, finally, for relativistic particles the rapidity is well approximated by the "pseudorapidity" $\eta=-\ln\tan\theta_{\text{lab}}$, which is determined by the particle emission angles alone.

These properties suggest a far reaching analogy with a one-dimensional gas, in which the kinematic limits on the rapidity correspond to the walls of the vessel confining the gas and the number of hadrons per unit rapidity corresponds to the number of gas molecules per unit volume. In the case of a collision of two protons with c.m. energy W, the length Y of the allowed rapidity interval is given by $Y=2\ln(W/M)$. At the highest accelerator energies now accessible, this length amounts to 8: $Y=8$.

2. The reaction products usually include an especially fast "leading" particle whose properties are closely connected with those of the incident particle. On the average, this leading particle carries off about half the energy of the incident particle.[4] For example, in collisions of protons with targets at rest there frequently appear fast protons or neutrons moving in the direction of the primary proton beam. The leading-particle effect is a consequence of the limited inelasticity of hadron collisions: on the average over many events, the energy expended in the production of new particles amounts to only about half the energy of the colliding hadrons.

3. The average number of secondary particles increases slowly with energy. Most of the produced particles are pions, the lightest of the hadrons. If all the secondary particles were produced at rest in the c.m. system the multiplicity would rise linearly with the c.m. energy W. Actually, however, the growth is roughly logarithmic (or follows a weak power law, increasing as $\sqrt{W}$ or slower), as is shown in Fig. 1,a. As the energy increases some processes are replaced by others, but in such a manner that the total cross section—the sum of the varying partial cross sections—remains virtually constant (Fig. 1,b). This behavior is illustrated by Fig. 1,c, which exhibits the "semi-inclusive" "topological" cross sections for the production of specified number of charged particles together with arbitrary numbers of neutral ones. These cross sec-

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[2] If the particles were emitted isotropically in the c.m. system we would have to treat the process as three-dimensional. There are some reactions (e.g. annihilation at low energies and decay of heavy particles) that do have to be treated as three-dimensional.

[3] In the first approximation one can replace the transverse momenta by their average values.

[4] The same is also valid with regard to the target particle.

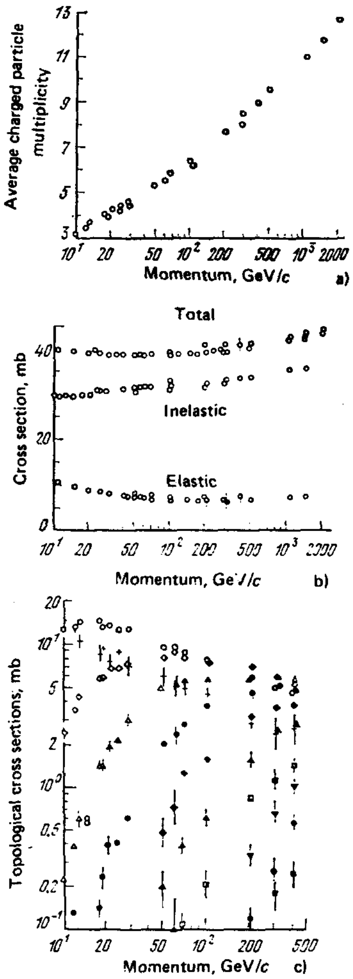

FIG. 1. Some basic features of high-energy proton-proton collisions: a) Average secondary charged particle multiplicity as a function of energy; b) the elastic cross section, the total inelastic cross section (the sum of all the topological cross sections), and the total cr cross section. c) behavior of the "topological" cross sections with increasing energy (indicated are the cross sections for events with specific numbers (2, 4,...) of prongs).

tions first rise and then fall, but their sum changes very little.

4. The short-range order of the multiparticle final states manifests itself in different ways. Thus, in the fragmentation regions, each of which occupies about two rapidity units near the edge of the rapidity distribution (the edge itself is usually appreciably shifted from the kinematic limit), the nature of the colliding particles still makes itself felt. As will be discussed below, charge correlations and other manifestations of particle clustering are also appreciable only in a limited and virtually energy independent rapidity interval.

5. Outside the fragmentation regions (in the so-called pionization region) the structure of multiparticle events is independent of the nature of the colliding particles. For example, it is in practice difficult to distinguish pion-proton reactions from proton-proton ones on the basis of the characteristics of the secondary particles if the collision energy is sufficiently high and the pion fragmentation region is not examined. This feature of the processes is frequently called the factorization property.

6. It has been found that the inclusive cross sections have a scaling property in the fragmentation regions: if the inclusive cross sections are examined as functions of the Feynman variable $x = 2p_l/W$ and the energy W, they will be found to depend only on x, i.e., they do not depend separately on the energy. In the pionization region the inclusive cross sections increase somewhat with increasing energy.

7. The structure of the events proves to be relatively stable as the energy is varied. The density of produced pions on the rapidity scale is virtually constant. In terms of the analogy with a one-dimensional gas mentioned above one could say that as the energy increases, the length of the container (the rapidity interval Y) also increases but the density of the gas within the container and the correlations between the molecules show little variation.

Of course these seven general statements are only a simplified idealization of the actual situation, and a number of improvements will be required if the situation is to be described quantitatively. Nevertheless these statements may serve as a good starting point for a discussion of all the presently known basic[5] characteristics of multiple production processes.

It is also sometimes useful to adopt a somewhat different phenomenological approach in which all multiparticle events are separated into two fundamental classes.

1. Quasielastic processes, characterized by virtually constant cross sections, in which specified final states, mainly with low mass and multiplicity, are produced. (These processes are frequently referred to as inelastic diffraction. For example, in diffraction excitation of the target particle, when the quantum numbers—charge, strangeness, etc.—do not change, all the ejected fragments have constant momenta in the laboratory system, which do not vary with the primary energy.)

2. Nondiffractive production, which is responsible for the larger part (about four-fifths) of the inelastic cross section; in this case the cross section for the reaction to proceed through some particular channel always turns out to be strongly energy dependent.

The specific models that we intend to discuss below are mainly applicable to the nondiffractive events.

HISTORY OF THE CLUSTER PROBLEM

The first indications that the generated hadrons may be incorporated into some sort of correlated system were obtained already in the early 1950s when resonances were discovered in the low-energy interactions of pions with nucleons. These findings had a definite influence on the interpretation of data on processes involving many particles.

However, the hypothesis that multiple production of particles takes place in a two stage process involving the formation of a group of correlated hadrons in the

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[5] Here we do not consider rare events with large transverse momenta or with a small admixture of particles other than pions.

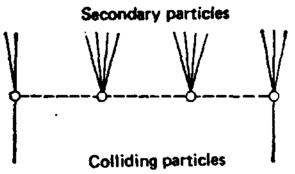

FIG. 2. Multiperipheral diagram for a process involving cluster production.

intermediate stage was not advanced until 1958. Cosmic ray data involving energies of about 1000 GeV provided the basis for this hypothesis. It was concluded from these data that there is a class of processes in which particles that move relatively slowly in the c.m system (i.e., pionization particles in current terminology) are produced from two centers that break up isotropically. The mass of such a group of hadrons (fireball) was estimated as about 3–4 GeV/c^2. The centers appeared to decay roughly isotropically into 7–8 pions

A year later events were discovered in cosmic ray experiments at energies of about 300 GeV in which a single fireball with similar properties (isotropic decay) was produced. These fireballs, however, had a broader mass distribution (from 2 to 5 GeV/c^2). At these energies the motion of the fireball in the c.m. system of the process leads to forward-backward asymmetry in the angular distribution of the secondary particles in individual events ("asymmetric showers").

It was very difficult to interpret such events in terms of the then popular statistical or hydrodynamic approach.6) However, the peripheral model of hadron interactions was developed at about that time, and it led to the sequential multiperipheral cluster scheme.7) According to this scheme, hadron collisions lead to the production of a chain of groups of correlated hadrons (cluster chains). Figure 2 shows a diagram of one such process.

A new wave of interest in particle correlations arose about five years ago when the strong short-range rapidity correlations of the produced pions were discovered and investigated at Serpukhov, Batavia, and CERN.

After that the azimuthal and charge correlations were investigated, as well as semi-inclusive correlations (for fixed multiplicity), and so on. Such experimental facts as the short-range order on the rapidity scale, local charge and transverse-momentum compensation, and the stability of the averaged characteristics of the events could be interpreted in terms of the cluster hypothesis. Very simple estimates of the sizes of the clusters were made, first using the obvious but rather primitive model in which the clusters are emitted independently. In accordance with the multiperipheral approach and with the ideas of the parton structure of hadrons, which have been successfully applied to deep inelastic electron-proton interactions,[5] it was suggested that the clusters are produced virtually independently and are uniformly distributed in rapidity. In order to be able to carry through analytic calculations the characteristics of the clusters were dealt with in a deliberately crude manner, it was assumed that all the clusters are alike (regardless of their position on the rapidity scale) and that each cluster decays isotropically into a fixed number of particles, the conservation laws were virtually ignored, etc. In this model such problems as the relation of inelastic processes to elastic scattering, the energy dependence of the total cross sections, etc., which are tractable in the multiperipheral approach, were also not touched upon. Nevertheless, this model reproduced in a fairly reasonable way the basic features of the processes derived from the multiperipheral picture. Comparison of the predictions of this crude independent cluster emission model with experiment led to the conclusion that an average cluster has a mass of ~2 GeV/c^2 and decays into 3–4 pions.

More detailed calculations within the framework of the multiperipheral scheme, in which allowance was made for the possible appearance of clusters of different types and masses and the conservation laws and interaction dynamics were taken accurately into account, showed that the above figures result from a certain averaging of the contribution from known low-mass resonances with the contribution from heavier (fireball-type) clusters. Thus, a difference in estimating the cluster parameters may arise from a difference in experimental conditions. At low energies (and also at high energies when low-multiplicity events are singled out) the main contribution comes from the production of light resonances. At higher multiplicities (corresponding to the usual selection of cosmic-ray events), on the other hand, the part played by heavy clusters will be enhanced. It is therefore not surprising that considerably higher cluster masses were obtained from cosmic-ray experiments than are obtained "on the average" from accelerator experiments.

In concluding this section we may say that the situation as regards clusters has now acquired a certain definiteness at the qualitative and semiquantitative level, although further work will of course be necessary in order to clarify the nature of the clustering phenomena and, especially, the origin, character of the decay, and other properties of heavier clusters.

EXPERIMENTAL EVIDENCE FOR CLUSTERING

In order to elucidate how the effects due to secondary-particle clustering have actually been recognized we shall have to discuss some methods for distinguising clusters. It is not possible to describe here all the methods that have been proposed, since their number is already approaching twenty. However, only a few basic methods are actually used.

As was noted above, clusters do not ordinarily manifest themselves directly in an experiment, but are recognized through various indirect effects such as correlations.

To separate out an individual cluster is a rather dif-

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6)The hydrodynamic approach was proposed by Landau[1]. For recent reviews see Ref. 2.

7)The multiperipheral model was developed in Ref. 3; see Ref. 4 for the part played by clustering in this model.

ficult task for two reasons: first, the decay products of two or more clusters produced in a single specific event may overlap in rapidity, and then it is impossible to separate the clusters; and second, isolated groups of particles may arise as a result of ordinary fluctuations which, in particular, may appear if, for example, neutral particles are not detected. In order to eliminate the effect of such fluctuations one would have to collect a great deal of data on many completely identified and analyzed events of the same type, and this can be a very laborious task. Hence it is very important to find what characteristics of the secondary particles are most sensitive to the production mechanism—to the nature of the cluster that gave rise to them.

Cosmic-ray events have been found in which two groups of correlated pions are rather far apart in pseudorapidity. Each group covered a pseudorapidity interval ($\Delta\eta\approx 2$) that should be obtained in the case of isotropic separation of the particles in the c.m. system of the group (cluster). These findings led to the hypothesis that the particles were emitted from two centers. Of course the statistics and precision of these first experiments were relatively modest, so the results are to be regarded as an indication rather than as proof.

The manifestation of the clustering effect in the full set of inelastic events was recognized only when two-particle pion correlations began to be studied. Experimentally, one measured the probability for the simultaneous appearance in an event of a pion with a specified rapidity y_1 and a second pion with rapidity y_2. If the particles were produced independently this probability would be the product of the probabilities for detecting pions independently at the points y_1 and y_2 on the rapidity scale. Hence in the absence of dynamic and kinematic correlations, the correlation function (defined as the difference between the two-particle distribution and the product of the one-particle distributions)[8] would vanish.

Experiment showed that this correlation function differs appreciably from zero for coinciding pion rapidities and falls off exponentially with increasing rapidity difference, so that the correlations are concentrated in a limited rapidity interval about two units long (Fig. 3). This was an important argument in favor of particle clustering.

It can be shown on the basis of the independent cluster emission model that the peak of the correlation function at $y_1=y_2=0$ is the higher, the greater the number of particles in the cluster, while the width of the peak is determined by the angular distribution of the particles resulting from the breakup of the cluster. Comparison of the experimental data with theoretical formulas showed that the hypothesis that the particles are produced independently is in conflict with known facts, while agreement can be achieved if the particles are produced from clusters that decay isotropically on the average into 3—4 particles.

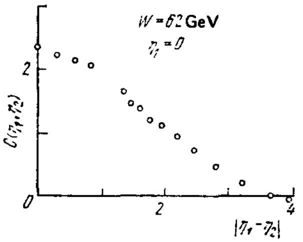

FIG. 3. Two-pion correlations in 62-GeV pp collisions, showing the peak in the correlation function at coincident pseudorapidities ($\eta_1=\eta_2$). The figure was taken from Ref. 6.

More detailed information on clusters can be derived from a study of analogous functions for events having a specified number of charged particles (semi-inclusive correlations). Analysis based on the independent cluster emission model shows that semi-inclusive correlations can provide information not only on the average number of particles in a cluster, but also on the width of the distribution with respect to the number of particles produced when the cluster decays. The average characteristics thus found for the clusters are the same as before, but it turns out that the clusters produced in events of relatively low multiplicity (multiplicities of the order of and lower than the mean multiplicity) are mainly ones that decay into a fixed number (three, on the average) of particles, whereas more massive clusters whose decay is described by a nonresonance Poisson distribution with respect to particle number (Fig. 4) are produced in higher-multiplicity processes. These data were obtained at CERN energies, i.e., in the 1000-GeV region where fireballs were first discovered.

Two-particle correlations of a somewhat different form were studied by the rapidity gap method. This method is based on the idea that the distance between neighboring particles on the rapidity scale (the rapidity gap), which under ordinary circumstances would characterize the average particle density (on the rapidity scale), would actually determine the cluster density in the case of large rapidity intervals. Then, knowing the particle density and the cluster density (both on the rapidity scale), one can easily find the average number of pions per cluster. Again the same value was found: 3—4 pions per cluster. Similar estimates were also

FIG. 4. The quantity $\langle K(K-1)\rangle/\langle K\rangle$ (K is the number of charged products of cluster decay) as a function of the number n of prongs in pp collisions at 62 GeV. The points represent experimental data, and the dashed line shows a possible dependence for three-particle resonances (e.g. ω resonances, etc., in place of heavy clusters). A linear growth would correspond to a Poisson distribution of the number of particles in a cluster. (The figure was taken from Ref. 6.)

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[8] It is clear that if effects associated with energy and momentum conservation are neglected, this function can depend only on the difference $|y_1-y_2|$.

made by a number of different methods. Thus, the average characteristics of the clusters are now fairly well known.

However, despite its heuristic value in describing multiple production processes, this reduction to average clusters that are all alike is too great a simplification. In principle, particles can be produced singly, and two-particle resonances like the ρ meson can be produced in great abundance. But then, in order to obtain the average values cited above it would be necessary for fairly heavy clusters also to be produced. This means that the mass spectrum of the clusters may turn out to be very broad, so that considerably more than two or three particles may sometimes be produced in the decay of a single cluster.

To investigate these possibilities one must examine not only the two-particle correlations, but also many-particle ones. A simple generalization of the correlation functions to the case of three or more particles leads to multidimensional distributions that are difficult to analyze. It is considerably more convenient to generalize the rapidity-gap method by examining rapidity intervals that include various different numbers (1, 2, ...) of particles. According to the independent cluster emission model the peaks of the distributions of such intervals shift toward lower values as the clusters become heavier, while their standard deviations are related to the decay characteristics of the clusters. And again, comparison with experiment confirmed the average values cited above, and in addition, it showed, in accordance with the semi-inclusive data, that heavy clusters play a part in high-multiplicity processes.

Although the clustering phenomenon is extremely important for understanding multiple production processes, it does not by itself lead to an exhaustive understanding of them. What is required is a broad understanding of the entire phenomenon as a whole. Analytic calculations within the limitations of the crude independent cluster emission model played an important part in establishing the average characteristics of the clusters, but they are quite incapable of elucidating the more detailed interaction picture because of the simplifying assumptions on which they are based (such as renunciation of the conservation laws, neglect of the leading particle effect and of the transverse momentum of the clusters, complete neglect of the energy dependence of the cross section, of the distribution with respect to multiplicity, etc.). To take all these factors into account one would have to resort to computer calculations. It is preferable to use a more realistic scheme—the quantized-field multiperipheral cluster model—in which the production of a chain of clusters, possibly having different properties, is determined by specific hadron interaction dynamics together with the kinematic possibilities for the process. But in this case more is required of the scheme: one would hope that it would not only account for the clustering phenomenon but that it would also to some extent account for all the basic features of multiple production processes that were listed at the beginning of this article and, as far as possible, for some of the features of elastic diffraction. The requirements that the total cross section be only weakly energy dependent and that the average multiplicities be correctly reproduced turn out to be very important for such a scheme. It follows precisely from these requirements (in accordance with what was said above) that within the framework of the model one must admit the production not only of ordinary resonances, but also of heavy clusters with masses of 3–4 GeV/c^2. The limitation on the transverse momentum arises from the peripheral character of the interaction. (Allowance for cluster decay does not alter this assertion since the momenta of the pions from cluster decay are limited and small). Besides, it is easy to explain the leading-particle effect as a result of the peripheral character of the interaction of the incident particle which, passing by, leaves behind only a finite fraction of its energy to form new hadrons, and, since its fragmentation is energy independent and does not affect the pionization particles, both the scaling property in this region, and also the factorization property, become understandable. The short range of the correlations is assured here much as it is in the simplest independent cluster emission model.

Since "nonexotic" pion clusters have electric charges of 0 or ±1, there is little charge transfer between clusters and the charges in a group of particles within a rapidity interval corresponding to a single cluster should almost cancel one another. This explains the experimentally discovered property of local charge compensation (compensation within a given small y interval). The growth (with increasing energy) of the inclusive spectra in the pionization region can be understood as a consequence of the growth of the particle density (on the rapidity scale) in pionization clusters, i.e., of a small increase in the mass of the clusters[9]. This, together with an increase in the number of clusters with increasing energy, may also be the physical reason for the small growth in the total cross sections that has been noted in recent years in accelerator experiments, and also for the appreciable growth in the yield of heavier particles (kaons, antiprotons, etc.). Thus, the multiperipheral cluster model makes it possible to understand the basic qualitative characteristics of multiple production processes, and specific forms of this model reproduce fairly well the quantitiative experimental results obtained at Serpukhov, Batavia, and CERN.

In summarizing, we can say that while the data on short-range correlations of course provide the strongest argument in favor of particle clustering in inelastic processes, a large number of other characteristics indirectly support that hypothesis and are reasonably explained within its framwork.

THEORETICAL APPROACHES TO AN UNDERSTANDING OF THE NATURE OF CLUSTERING

But what is the nature of those hadron clusters in

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[9] Regardless of into how many particles a cluster decays, they will all be contained in a rapidity interval of length $\Delta y \approx 2$ provided the cluster decays isotropically.

favor of which all the experimental results speak so strongly? Are they actually dynamical objects or are they a consequence of the "play" of certain kinematic limitations, fluctutations, and the like? In other words, do they give a precise unique description of the processes taking place at high energies or do they merely provide us with simple mnemonic rules? Although the clustering phenomenon seems to be firmly established, its interpretation from the standpoint of actual existence of dynamical clusters cannot be regarded as rigorously proved. We feel, however, that the observed stability of the properties of clusters speaks in favor of such an interpretation. Most of the theoretical approaches treat clusters from just this point of view.

In the multiperipheral model the multiple production process at high energies is reduced to a set of lower-energy interactions between virtual particles. It is therefore natural that the low-mass clusters reproduce the meson-meson scattering spectrum, including the low-energy resonances. The nature of the massive clusters is not so clear. One might regard a massive cluster as a set of heavy resonances that overlap in energy (mass). Heavy resonances, however, are usually produced with low probability, so it is difficult to explain the frequent appearance of massive clusters without assuming that there are many more heavy resonances than light ones.

At the same time, if we consider the space-time evolution of the system produced by the collision of primary hadrons, we see that if a large number of light resonances are produced at the initial instant many of them will not succeed in escaping from the region in which the forces are acting, will suffer secondary interactions, and in the end may form a massive thermalized system reminiscent of a heavy cluster. In its subsequent decay, such a system should conform to the laws of statistics and thermodynamics, and indeed the isotropic decay of the cluster and the observed Planck momentum distribution of the secondary particles can be interpreted in this manner. Thus, it is natural to introduce massive clusters and elements of the statistical treatment of the subsystems into the multiperipheral model.

Clusters can perhaps be most economically described as collective excitations of hadronic systems having many degrees of freedom. In this case, theoretical approaches developed to solve nonrelativistic many-particle problems may prove to be very successful in clarifying the nature of the clustering phenomenon. At the same time, the connection with the description in terms of the quark-parton model of the hadron jets observed in deep inelastic electron-proton interactions may also prove to be useful. All this may lead to a unified description of the entire cluster spectrum.

Such an approach would make clustering a common characteristic of hadronic matter. Hence one should expect clustering to manifest itself not only in hadron-hadron collisions, but also in lepton-initiated reactions in which hadronic final states are produced, and in general, whenever a large amount of energy emerges in the form of two or more hadrons capable of subsequently interacting with one another and generating new hadrons among which the energy may be redistributed. The first indications of clustering effects were obtained in proton-antiproton annihilation reactions and in the conversion of electron-positron pairs into hadrons at high energies. The new possibilities of experimenting with colliding electron-positron beams with total c.m. energies up to 40 GeV that will soon appear will doubtless enable us to look more deeply into the problem of scaling in multiparticle reactions initiated by different primary particles.

In concluding we want to emphasize that the cluster concept has proved to be very useful in studying multiple production processes and that further study of the particle-clustering phenomenon may provide one of the keys to an understanding of the nature of the strong interactions.

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