Supersymmetric gauge theories and their possible applications to the weak and electromagnetic interactions

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Supersymmetric gauge field theories and their possible applications to the weak and electromagnetic interactions are discussed. The problem of spontaneous supersymmetry breaking is considered in detail. A unified model of weak and electromagnetic lepton interactions with spontaneously broken supersymmetry is constructed.

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CONTENTS

1. INTRODUCTION

Symmetry has always been one of the guiding principles in constructing new physical theories. The history of science contains many examples of how predictions made on the basis of seemingly abstract considerations of invariance were beautifully confirmed experimentally and stimulated the development of new approaches in both theory and experiment. Symmetry plays a particularly important role in the physics of elementary particles, since in this case it is very difficult to give an obvious theoretical interpretation of the experimental data. There is little hope of solving the inverse scattering problem, i.e., reconstructing the form of the potential from the scattering data, in relativistic quantum dynamics. In practice, therefore, the form of the Hamiltonian in quantum field theory is usually postulated on the basis of symmetry arguments and the natural requirement of "simplicity."

However, the intuitive ideas about symmetry that can be gained from ordinary experiments refer mainly to the invariance properties of space-time, leave too much arbitrariness in the choice of the Hamiltonian of the field system, and do not reflect the specific character of the interactions between elementary particles. A decisive role in elementary-particle physics is played by so-called internal symmetries, which are not directly related to the properties of space-time and which do not have such an intuitive interpretation as, for example, invariance with respect to displacements in space and time. In essence, the basic task of the theory is to discover such symmetries and to use them as a basis for constructing dynamical models.

The simplest and best known example of an internal symmetry is the symmetry of the wave functions of charged particles with respect to phase transformations. If a field $\psi(x)$ satisfies the Dirac equation

$$[i\gamma_\mu\partial_\mu - m + e\gamma_\mu A_\mu(x)]\,\psi(x) = 0, \tag{1.1}$$

then exactly the same equation holds for a field $\psi'(x)$ which differs from $\psi(x)$ by a phase transformation:

$$\psi'(x) = e^{i\alpha}\psi(x). \tag{1.2}$$

The fields ψ and ψ' carry the same physical information. This means that it is not the phase itself that has physical significance, but only phase differences of charged fields. In accordance with Noether's theorem, the invariance of the Hamiltonian with respect to the transformations (1.2) leads to the conservation of electric charge.

Transformations associated with the conservation of other charges—baryon charge, lepton charge, etc.—can be defined in a similar way.

The phase transformations (1.2) can be regarded as rotations in "charge" space. The symmetry means that there is no preferred direction in this space.

A natural generalization of these ideas is the concept of isotopic symmetry of the strong interactions associated with invariance with respect to rotations in a three-dimensional "isotopic" space. As in the previous case, the symmetry means that there is no preferred direction in this space. The proton and neutron emerge as different states of one and the same particle, distinguished by their projections of "isotopic" spin. Just as states with different angular-momentum projections in a spherically symmetric field are physically equivalent, the distinction between the proton and the neutron is unimportant from the point of view of the strong interactions. We can fix the "proton direction" arbitrarily, after which the "neutron direction" is uniquely determined.

Further generalizations of these ideas led to the establishment of the SU_3 and SU_4 symmetries, on which the current classification of the hadrons is based.

The symmetries discussed above manifest themselves primarily in the existence of conserved quantities—charges, isotopic and unitary spin, etc. They impose rather weak constraints on the dynamics of the inter-

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action and do not fix the specific form of the potential. In particular, from the point of view of the conservation of charge, it would be perfectly possible for the electromagnetic interaction to be mediated by massive vector or scalar particles instead of by massless vector quanta—the photons. A more restrictive symmetry is required to specify the form of the interaction uniquely.

In the case of electrodynamics, such a symmetry is well known—it is invariance with respect to local phase transformations or, as we usually say, gauge invariance. As we have already pointed out, invariance with respect to the global phase transformations (1.2) corresponds to an arbitrariness in choosing a direction in charge space. However, in fixing a direction at any single point x, we simultaneously fix it at all other points of space-time, since the phase transformations (1.2) act identically at all points x. Real experiments always refer to a limited region of space-time. It would therefore be natural to expect that there exists a possibility of choosing directions in charge space independently at different points of space-time. In other words, there exists an invariance with respect to phase transformations with a phase that depends on the coordinates:

$$\psi(x) \to e^{ie\alpha(x)}\psi(x), \quad \bar{\psi}(x) \to e^{-ie\alpha(x)}\bar{\psi}(x). \tag{1.3}$$

The Dirac equation (1.1) is invariant with respect to the transformations (1.3) if the electromagnetic field $A_\mu(x)$ is simultaneously transformed according to the law

$$A_\mu(x) \to A_\mu(x) + \partial_\mu\alpha(x). \tag{1.4}$$

Symmetry with respect to the gauge transformations (1.3) and (1.4) is much more restrictive than invariance with respect to global phase transformations. The latter holds for both Eq. (1.1) and for the free Dirac equation

$$(i\gamma_\mu\partial_\mu - m)\psi(x) = 0. \tag{1.5}$$

But the free Dirac equation is no longer invariant in the case of local gauge transformations. *Gauge invariance requires the existence of an electromagnetic field whose interaction with all charged fields is introduced by replacing the ordinary derivative by the covariant derivative:*

$$\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu. \tag{1.6}$$

This formula is a direct generalization of the well-known expression in classical electrodynamics for the momentum of a particle in an electromagnetic field:

$$p_\mu \to P_\mu = p_\mu - \frac{e}{c}A_\mu. \tag{1.7}$$

The foregoing considerations show that a local transformation of the phase of the field $\psi(x)$, which can be regarded as a coordinate in charge space, is equivalent to the appearance of an additional electromagnetic field. This situation is clearly analogous to the weak principle of equivalence in Einstein's theory of gravitation, according to which a local transformation of the coordinate system leads to the appearance of an additional gravitational field. Following H. Weyl, this analogy enables us to formulate the following *principle of relativity in charge space:*

The field configurations

$$\bar{\psi}(x),\ \psi(x),\ A_\mu(x) \tag{1.8}$$

and

$$e^{-ie\alpha(x)}\bar{\psi}(x), \quad e^{ie\alpha(x)}\psi(x), \quad A_\mu(x) + \partial_\mu\alpha(x) \tag{1.9}$$

describe the same physical situation.

The principle of relativity in charge space uniquely specifies the Hamiltonian of quantum electrodynamics—a theory whose predictions are all in excellent agreement with experiment. Invariance with respect to the gauge transformations (1.3) and (1.4) is an experimental fact that is as well established as relativistic translational invariance and other "classical" symmetries.

The electromagnetic and gravitational fields, together with the Yang-Mills fields, form the family of gauge fields. Yang-Mills fields arise in a natural way when the idea of localized phase transformations is extended to isotopic, SU_3, and other transformations. If, in analogy with electrodynamics, we require that the direction in isotopic space can be fixed arbitrarily at various points of space-time, i.e., that the theory is invariant with respect to the gauge transformations

$$\bar{\psi}(x) \to e^{-ig\tau^i\alpha^i(x)}\bar{\psi}(x), \quad \psi(x) \to e^{ig\tau^i\alpha^i(x)}\psi(x), \tag{1.10}$$

where, for example, $\psi(x) = \{\psi_p, \psi_n\}$ is an isodoublet consisting of the proton and the neutron and τ^i are the Pauli matrices, then there must necessarily exist a vector and isovector field A_μ whose interaction with the fields ψ is introduced by replacing the ordinary derivative by the covariant derivative:

$$\partial_\mu \to D_\mu = \partial_\mu - ig\tau^i A_\mu^i(x). \tag{1.11}$$

The Yang-Mills field transforms under infinitesimal gauge transformations according to the law

$$A_\mu^i(x) \to A_\mu^i(x) + \partial_\mu\alpha^i(x) - g\varepsilon^{ijk}A_\mu^j(x)\alpha^k(x). \tag{1.12}$$

A Yang-Mills field can be associated with any compact semi-simple Lie group. The form of its interaction with the other fields—the "matter fields"—is uniquely specified by the following *generalized principle of relativity in charge space*:

Suppose that the fields $\psi^a(x)$ realize some representation of a compact group G and that the vector field A_μ belongs to the adjoint representation of this group. Then the field configurations

$$\bar{\psi}^a(x), \quad \psi^b(x), \quad A_\mu^c(x) \tag{1.13}$$

and

$$[\delta^{an} - g(T^i)^{an}\alpha^i(x)]\bar{\psi}^n(x), \quad [\delta^{bn} + g(T^i)^{bn}\alpha^i(x)]\psi^n(x), \\ [\delta^{cm} - gt^{cmn}\alpha^n(x)]A_\mu^m(x) + \partial_\mu\alpha^c(x), \tag{1.14}$$

where T^i are the generators of the representation realized by the fields ψ and t^{cmn} are the structure constants of the corresponding Lie algebra, describe the same physical situation.

The principle of relativity for Yang-Mills fields has proved to be extremely fruitful for elementary-particle physics. The theory of gauge fields accounts for the most important results obtained in this subject during the past decade. Remarkably, it turns out that many of the earlier successful phenomenological models can be consistently and elegantly formulated in terms of Yang-Mills theory. A good example of this is the theory of weak interactions.

Until recently, all the experimental data on weak interactions were described by means of a phenomenological four-fermion interaction of the form

$$\mathcal{L} = \frac{G}{\sqrt{2}} J^{\lambda}(x) J^{\lambda +}(x), \tag{1.15}$$

where the current J^{λ} is a sum of terms of the form

$$J^{\lambda} = \bar{e}\gamma^{\lambda}(1+\gamma^{5})\nu_{e} + \bar{\mu}\gamma^{\lambda}(1+\gamma^{5})\nu_{\mu}. \tag{1.16}$$

Here e, μ, ν_e, and ν_μ are the wave functions of the electron, the muon, and the corresponding neutrinos, and there are also analogous terms containing the wave functions of the hadrons. However, numerous attempts to develop a consistent quantum theory on the basis of the Lagrangian (1.15) have been unsuccessful. Although this Lagrangian leads to good agreement with experiment in the quasi-classical "tree" approximation, calculations of the quantum corrections give meaningless divergent expressions. As is well known, an analogous difficulty in quantum electrodynamics is resolved by means of the renormalization procedure, i.e., a redefinition of the "bare" charges and masses which characterize fictitious non-interacting particles. After renormalization, all the parameters of the actual interacting particles and the amplitudes for all physical processes become finite. However, this procedure, which provides a unique algorithm for perturbation-theory calculations in quantum electrodynamics, turns out to be inadequate in the case of the Lagrangian (1.15). The divergences cannot be eliminated by redefining a finite number of parameters—the corresponding theory is non-renormalizable.

This seems to indicate that the four-fermion interaction (1.15) is not fundamental. The form of the Lagrangian (1.15) suggests that the interaction of the vector currents J^{λ} actually occurs in the same way as in electrodynamics, via the exchange of quanta of a vector field $W^{\lambda +}$:

$$\mathcal{L}_{I} = g\,(J^{\lambda}W^{\lambda +} + J^{\lambda +}W^{\lambda -}). \tag{1.17}$$

In lowest-order perturbation theory, the Lagrangian (1.17) gives the amplitude

$$\frac{g^{2}}{2} J^{\lambda}(k) \frac{g^{\lambda\lambda'} - k^{\lambda}k^{\lambda'}m^{-2}}{k^{2}-m^{2}} J^{\lambda'}(k), \tag{1.18}$$

which at low energies $k^2 \ll m^2$ coincides with the amplitude obtained from the Lagrangian (1.15). In this scheme, the observed contact interaction of the currents is merely an approximate low-energy potential due to the exchange of a single vector meson.

For a long time, it was believed that the interaction (1.17) suffers from the same disease as the contact four-fermion interaction. Attempts to develop a perturbation-theory formalism for the Lagrangian (1.17) led to uncontrollable divergences which are characteristic of a non-renormalizable theory.

The situation becomes fundamentally different if we assume that the field W^{λ} is a Yang-Mills field. This hypothesis follows naturally from the analogy with electrodynamics. The analogy between the weak and electromagnetic interactions had already been noted by Fermi and discussed by many subsequent authors. In both cases, the interaction involves conserved vector currents. In many respects, the weak and electromagnetic currents behave like the members of a single multiplet corresponding to some algebra that unifies these interactions.

We can attempt to unify the weak and electromagnetic interactions on the basis of a common gauge group by combining the matter (lepton and quark) fields into multi-component multiplets $\psi = \{\psi_1 \ldots \psi_n\}$ which realize a representation of this group and combining the electromagnetic field and the fields of the intermediate vector mesons that carry the weak interaction into a Yang-Mills multiplet $A_\mu = \langle A_\mu^1, \ldots, A_\mu^m \rangle$. In accordance with the preceding discussion, the principle of relativity uniquely specifies the form of the interaction of the intermediate mesons with the leptons and quarks. If the gauge group is simple, this interaction is automatically universal and is characterized by a single coupling constant. In the case of the Yang-Mills theory, it is possible to formulate a renormalization procedure analogous to the corresponding procedure in quantum electrodynamics and to construct a formalism of perturbation theory.

Despite its obvious elegance, the foregoing picture is unsuitable for describing the experimental situation in this simple form. In addition to the properties which the electromagnetic and weak interactions have in common, there are important differences which this scheme does not incorporate. First of all, the electromagnetic interaction is of long range, whereas the weak interaction has a finite range. Since the effective range of an interaction is inversely proportional to the mass of the field that carries it, this means that the W mesons, unlike the photon, must have a non-zero mass. Secondly, the electromagnetic interaction conserves parity, whereas the weak interaction contains terms which are non-invariant with respect to spatial reflections. It would appear at first sight to be impossible to combine these properties within the framework of a symmetric theory—all Yang-Mills fields must have zero mass, and the currents must have the same transformation properties.

However, we recall that the symmetries discussed above referred only to the Hamiltonian and the equations of motion. Now the actual behavior of a physical system also depends on the boundary conditions or on the symmetry properties of the ground state. It is appropriate here to cite a well-known classical analogy. Consider a ball which rests at the center of the concave base of a bottle (see Fig. 1). The ball is in equilibrium. This system is symmetric with respect to reflections about the center. However, this equilibrium position is unstable. Left to itself, the ball will roll towards the wall under the action of an arbitrarily small perturba-

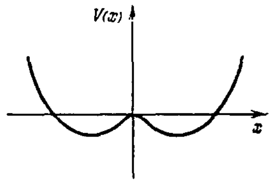

FIG. 1.

tion. This position is energetically more favorable and therefore stable. This new equilibrium position, which is the true ground state, no longer possesses the original symmetry—the symmetry is spontaneously broken. We cannot predict in which direction the ball will roll. All positions near the wall have the same energy and are therefore equivalent. This means that the ground state is degenerate.

An analogous mechanism enables us to distinguish the weak and electromagnetic interactions in a gauge-invariant theory. It leads to a non-zero mass for the Yang-Mills fields corresponding to the intermediate W mesons and to a breaking of the symmetry between the weak and electromagnetic currents.

Let us assume that, apart from the leptons and quarks, the Yang-Mills fields interact with scalar fields φ. The gauge-invariant Lagrangian for the interaction of scalar fields has the form

$$\begin{gathered}\mathscr{L}=\frac{1}{2}(\partial_\mu\varphi^a-gT^i_{ab}A^i_\mu\varphi^b)^2+\lambda\bar{\psi}\Gamma^a\varphi^a\psi-V(\varphi),\\ V(\varphi)=h^2(\varphi^2)^2\pm\frac{m^2}{2}\varphi^2.\end{gathered} \tag{1.19}$$

With the positive sign of the mass term, the potential $V(\varphi)$ has a unique, translationally invariant minimum at the point $\varphi=0$. The corresponding position of equilibrium is stable and possesses the same symmetry as the Lagrangian (1.19). On the other hand, if the mass term appears in $V(\varphi)$ with the minus sign, then the potential V has a form analogous to that considered above in the example of the bottle. Exactly as in that example, the symmetric extremum $\varphi=0$ is unstable. The system "slides" into one of the stable equilibrium positions corresponding to zero A^i_μ and constant φ having a fixed length $\varphi_0^2=m^2/2h^2$. The equilibrium position is degenerate. The minimal configurations form a sphere whose points correspond to the directions of the constant vector φ_0. All the directions of φ_0 are physically equivalent. Therefore we can arbitrarily fix the direction of the vector φ_0, assuming, for example, that it is directed along an axis n: $\varphi_0=\{0,\ldots,0,m/\sqrt{2h}\}$.

Of course, this choice of the boundary conditions breaks the invariance of the theory with respect to global, coordinate-independent gauge transformations. The reader who is familiar with solid-state theory will undoubtedly see the analogy here with the phenomenon of spontaneous magnetization of a ferromagnet, where the formulation of the theory also requires a choice of the direction of the magnetization vector.

It might appear that by choosing the term $(m^2/2)\varphi^2$ with the minus sign we obtain a physically meaningless theory in which the scalar particles have negative mass. However, this would be a hasty conclusion. The quadratic term in φ plays the role of a mass only when the point $\varphi=0$ is a position of stable equilibrium. To determine the actual mass spectrum, we must make an expansion of the potential (1.19) in the neighborhood of the stable extremum, which is equivalent to a displacement of the fields φ by the constant vector φ_0. After such a displacement, we obtain a Lagrangian whose quadratic part actually determines the mass spectrum (apart from radiative corrections), while the terms of higher order in the fields describe the interaction. In this form, the Lagrangian can be used to construct a perturbation theory in which, as usual, the propagators are determined by the quadratic part of the Lagrangian, while the vertices which appear in the construction of the Feynman diagrams are given by the terms of third and fourth order in the fields. It can be seen from Eq. (1.19) that the transformation to the displaced fields leads to mass terms for some of the vector fields and for the fields ψ:

$$\mathscr{L}_m=\frac{g^2}{2}\frac{m^2}{2h^2}T^i_{an}T^k_{an}A^i_\mu(x)A^k_\mu(x)+\frac{\lambda m}{\sqrt{2h}}\bar{\psi}\Gamma^n\psi. \tag{1.20}$$

For example, if $T^i=i\tau^i$, the mass term takes the form

$$\frac{m^2g^2}{4h^2}[(A^1_\mu)^2+(A^2_\mu)^2], \tag{1.21}$$

i.e., two of the vector fields acquire a non-zero mass, while one remains massless. The structure of the interaction Lagrangian is also modified, and it becomes possible to obtain different transformation properties for the electromagnetic and weak currents.

This mechanism, first proposed by Higgs,[1] is the basis of the Weinberg-Salam unified model of the weak and electromagnetic interactions,[2] as well as many more recent unified models.

The reader may wonder why we insist on spontaneous symmetry breaking instead of "manually" introducing terms that break the symmetry between the weak and electromagnetic interactions. The point is that the spontaneoulsy broken symmetry is still a symmetry. After transforming to the displaced fields $\varphi'=\varphi+\varphi_0$, the Lagrangian (1.19) remains invariant with respect to local gauge transformations. Only the form of the latter changes. If a gauge transformation for the Lagrangian (1.19) had the form

$$\varphi^a\to\varphi^a+gT^i_{ab}\alpha^i\varphi^b, \tag{1.22}$$

then after transforming to the fields φ' this gauge transformation in the theory with spontaneously broken symmetry takes the form

$$\varphi'_a\to\varphi'_a+gT^i_{ab}\alpha^i\varphi'_b+gT^i_{an}\alpha^i\frac{m}{\sqrt{2h}}. \tag{1.23}$$

Unlike the original transformation (1.22), the transformation (1.23) with constant α^i is not generated by any unitary operator and therefore does not lead to the conservation of any quantity. This means that the symmetry is broken. Nevertheless, the condition of invariance with respect to the transformation (1.23) still uniquely specifies the form of the interaction of the field φ with the Yang-Mills field. Owing to the invariance of the spontaneously broken theory with respect to local gauge transformations, it is possible to extend to this theory the procedures of quantization[3] and renormalization[4] developed for symmetric theories, as was done in Ref. 5.

The unified gauge models reproduce all the successful predictions of the phenomenological four-fermion model (1.15) and, at the same time, in contrast with the latter, constitute a self-consistent theory with which it is possible to calculate uniquely quantum corrections to the amplitudes for various processes. These gauge models also yield a number of important qualitative predictions which are beautifully confirmed by experiment. The most important of these predictions is the existence of neutral weak currents and charmed hadronic states.

However, returning to the problem formulated at the beginning of this review, namely that of discovering a symmetry which completely determines the form of the interaction, we must admit that we have not yet solved this problem for gauge-invariant models with spontaneously broken symmetry. The mechanism of spontaneous symmetry breaking used in these models is a purely external one from the point of view of the gauge theories—the existence of the Higgs scalar mesons does not in any sense follow from the symmetry of the theory. The only reason for introducing them is to ensure a reasonable mass spectrum for the weakly interacting particles. In addition to its esthetic deficiency, this procedure also raises purely practical objections. The predictive power of the theory is significantly reduced. In contrast with the universal Yang-Mills interaction, the interactions of Higgs scalars with fermions and with each other are to a great extent arbitrary. The parameters which characterize these interactions are not fixed by the condition of gauge invariance. As a result, models of the Weinberg-Salam type give no predictions regarding the mass spectrum of the matter fields.

It is natural to try to take a further step towards the unification of the various interactions. Gauge invariance enabled us to combine the photon and the intermediate W mesons in a single multiplet and to specify uniquely the form of their interaction with the matter fields. Can we not postulate a larger symmetry which would unify the Yang-Mills fields and the lepton and quark fields with the Higgs scalars? It is clear that the familiar isotopic or unitary symmetries are inappropriate for this purpose. All these symmetries relate fields having the same tensorial dimensions—the irreducible multiplets consist of either scalars of spin-$\frac{1}{2}$ particles, etc. The symmetry that we require must give a nontrivial relation between fields having different tensorial dimensions, in particular fermions and bosons. Since bosons are described in quantum theory by commuting variables, while fermions are described by anticommuting variables, the algebra of the corresponding transformations must contain both commuting and anticommuting elements. In other words, the required group must include transformations of the type

$$\varphi'(x) \to \varphi(x) + \bar{\varepsilon}\psi(x), \tag{1.24}$$

where φ is a scalar field, ψ is a spinor field, and ε are anticommuting spinor parameters.

Such a group can be constructed, and the corresponding symmetry is known as supersymmetry. It turns out that most of the usual formalism of group theory, with fairly obvious modifications due to the presence of anticommuting elements, carries over to supersymmetry transformations. The irreducible representations of the supersymmetry group unify the vector, spinor, and scalar fields within a single multiplet. The existence of spinor fields (leptons and quarks) in the supersymmetric theory necessarily entails the existence of scalar fields, which are natural candidates for the role of Higgs mesons. The requirement of invariance with respect to supersymmetry transformations leads to relations between the masses and interaction constants of the gauge fields and the Higgs scalars. As a result, supersymmetric models should be of much greater predictive power than the standard unified models. However, we encounter the same problem in the supersymmetric theory as that which arose in attempting to combine the photon and the W meson into a multiplet of Yang-Mills fields. All the members of a single supermultiplet should have the same mass. Therefore the supersymmetric theory leads to a degeneracy in the masses of the scalar and spinor fields, which is not observed experimentally.

As before, we can avoid this difficulty by means of spontaneous symmetry breaking. However, the mechanism of spontaneous symmetry breaking described above cannot be directly generalized to this case. Supersymmetry imposes strong restrictions on the form of the potential $V(\varphi)$ which determines the stable ground state. In the simplest supersymmetric models, the stable extremum corresponds to a stable ground state, so that there is a degeneracy in the masses of the spinor and scalar particles. To remove this degeneracy and reproduce a realistic mass spectrum and form of interaction in the supersymmetric theory, we must resort to more refined methods. In fact, the present review will be largely concerned with this problem. We shall describe a mechanism of spontaneous supersymmetry breaking which is applicable to practically any supersymmetric theory. As in the case of the Yang-Mills theory, models with a spontaneously broken supersymmetry can again be subjected to a symmetric renormalization procedure, and the relations between the counterterms are merely replaced by finite, calculable quantities. This mechanism enables us to construct realistic supersymmetric models of the weak and electromagnetic interactions.

The plan of our review is as follows. Section 2 contains auxiliary material. Its purpose is to acquaint the reader who is unfamiliar with the theory of supersymmetry with the basic concepts of this theory. Section 3 contains a detailed discussion of spontaneous supersymmetry breaking. This section is aimed primarily at the reader with the necessary background who is interested in the further development of the theory. In Sec. 4 we discuss the application of supersymmetry to the weak and electromagnetic interactions. As an example, we consider in detail a simple supersymmetric model of the leptons. This section may be of interest to readers who are interested in unified gauge models. It can be understood without any particularly deep knowledge of the theory of supersymmetry. Finally, in the concluding section we enumerate the most characteristic features of supersymmetric gauge models of the weak and electromagnetic interactions.

2. SUPERSYMMETRIC GAUGE THEORIES

Since supersymmetry transformations connect Fermi and Bose fields, the corresponding algebra must contain anticommuting elements. The minimal algebra which includes anticommuting generators and contains the subalgebra of the Poincaré group has the form[6,8]

$$[P_\mu, P_\nu]_- = 0, \quad [P_\mu, S_\alpha]_- = 0, \quad [S_\alpha, S_\beta]_+ = (-\gamma_\mu C)_{\alpha\beta} P_\mu, \tag{2.1}$$

where P_μ are the generators of the four-dimensional translations, S_α are the generators of the supersymmetry transformations, which are Majorana spinors, and C is the matrix of charge conjugation. A detailed discussion of the representations of the algebra (2.1) can be found in a review by Mezinchesku and Ogievetskiĭ,[9] where references to the original papers are given. We confine ourselves here to the facts that are required for what follows.

It is convenient to realize the representations of the algebra (2.1) in the space of functions $\psi(x_\mu, \theta_\alpha)$ depending on the real parameters x (points in Minkowski space) and the anticommuting Majorana spinors θ_α satisfying the relations[7,10]

$$[\theta_\alpha, \theta_\beta]_+ = 0. \tag{2.2}$$

The supersymmetry transformations act in the space of the variables x and θ as follows:

$$x_\mu \to x_\mu + \frac{i}{2}\bar{\varepsilon}\gamma^\mu\theta, \quad \theta_\alpha \to \theta_\alpha + \varepsilon_\alpha, \tag{2.3}$$

where the parameters of the transformation, ε_a, are in turn anticommuting Majorana spinors.

A scalar superfield is defined in analogy with an ordinary scalar:

$$\Psi(x, \theta) = \Psi'(x', \theta'). \tag{2.4}$$

The superfield $\Psi(x,\theta)$ is equivalent to a multiplet of ordinary fields, since any function of a finite number of anticommuting variables is a finite polynomial. It follows from the property (2.2) that $\theta^5 = 0$, and by expanding $\Psi(x,\theta)$ as a Taylor series in θ we obtain

$$\Psi(x,\theta) = c(x) + \bar{\theta}\chi(x) + \frac{1}{4}[\bar{\theta}\theta F(x) + \bar{\theta}\gamma_5\theta G(x) + \bar{\theta}_i\gamma_\nu\gamma_5\theta A_\nu(x) + (\bar{\theta}\theta)\bar{\theta}\lambda(x)] + \frac{1}{32}(\bar{\theta}\theta)^2 D(x). \tag{2.5}$$

Thus the real superfield $\Psi(x,\theta)$ is equivalent to a multiplet of ordinary fields containing the (pseudo) scalars $c(x)$, $F(x)$, $G(x)$, and $D(x)$, the Majorana spinors $\chi(x)$ and $\lambda(x)$, and the vector field $A_\mu(x)$. These fields transform into one another under the supersymmetry transformations (2.4):

$$\delta c = \bar{\varepsilon}\chi, \ \ldots, \ \delta D = -\bar{i\varepsilon}\,\hat{\partial}\lambda. \tag{2.6}$$

The superfield $\Psi(x,\theta)$ admits an invariant expansion as a sum of superfields containing fewer components:

$$\Psi(x,\theta) = \Phi_+(x,\theta) + \Phi_-(x,\theta) + \Psi_1(x,\theta). \tag{2.7}$$

The chiral superfields $\Phi_\pm(x,\theta)$ are equivalent to multiplets of ordinary fields consisting of (pseudo) scalars $A_\pm(x)$ and $F_\pm(x)$ and two-component scalars $\psi_\pm(x)$, while the superfield $\Psi_1(x,\theta)$ includes a Majorana spinor, a four-vector, and a scalar.

To construct an action which is invariant with respect to supersymmetry transformations, it is sufficient to integrate any superscalar, such as $[\Psi(x,\theta)]^n$, with respect to the invariant measure $\int d^4\theta d^4x$, where the integral with respect to $d^4\theta$ is defined by the equations[11]

$$\int d\theta_\alpha = 0, \quad \int \theta_\alpha\, d\theta_\beta = \delta_{\alpha\beta}. \tag{2.8}$$

The simplest invariant action has the form

$$\int \Phi_+(x,\theta)\Phi_-(x,\theta)\,d^4x\,d^4\theta = \int [\partial_\mu A_+(x)\,\partial_\mu A_-(x) + i\bar{\psi}\,\hat{\partial}\psi(x) + F_+(x)F_-(x)]\,d^4x \tag{2.9}$$

and describes non-interacting spinor and scalar fields.

It follows from the definition (2.8) that

$$\int d^4\theta\Psi(x,\theta) = D(x),$$

i.e., the integral of the D-component of a scalar superfield with respect to d^4x is an invariant. This can be seen directly from the transformation equations (2.6). The D-component transforms into a total derivative, so that $\int d^4xD(x)$ is an invariant. It can be shown that the integral of the $F_\pm$-component of a chiral superfield is also an invariant. We shall often make use of this property in what follows.

Since the spinors and scalars belong to a single multiplet in the supersymmetric theory, the fermion charges of the spinor and scalar fields are interrelated. If we require that members of a single supermultiplet have different fermion charges, the transformation which determines this charge must affect the generators of the algebra (2.1). The only such transformation consistent with the commutation relations (2.1) has the form[12,13]

$$S_\pm \to e^{\pm i\alpha}S_\pm, \quad S_\pm = \frac{1 \pm i\gamma_5}{2}S. \tag{2.10}$$

The superfield $\Psi(x,\theta)$ then transforms as

$$\Psi(x,\theta) \to \Psi(x, e^{-\alpha\gamma_5}\theta) \tag{2.11}$$

or, in terms of components,

$$\begin{bmatrix} A_\nu \\ \lambda \\ D \end{bmatrix} \to \begin{bmatrix} A_\nu \\ e^{\alpha\gamma_5}\lambda \\ D \end{bmatrix}, \tag{2.12}$$

i.e., the two-component spinors λ_+ and λ_- possess fermion charges −1 and +1, while the vector field A_μ and the scalar D carry no fermion charge. If we assign fermion charges ±1 to the spinor components of the chiral superfields, their possible transformation laws have the form

$$\Phi_\pm(x,\theta) \to e^{\mp 2n\alpha}\Phi_\pm(x, e^{-\alpha\gamma_5}\theta), \quad n = 0, 1 \tag{2.13}$$

or, in terms of components,

$$n=0:\begin{bmatrix}A_\pm\\ \psi_\pm\\ F_\pm\end{bmatrix}\to\begin{bmatrix}A_\pm\\ \psi_\pm e^{\pm i\alpha}\\ F_\pm e^{\pm 2i\alpha}\end{bmatrix},\quad n=1:\begin{bmatrix}A_\pm\\ \psi_\pm\\ F_\pm\end{bmatrix}\to\begin{bmatrix}A_\pm e^{\mp 2i\alpha}\\ \psi_\pm e^{\mp i\alpha}\\ F_\pm\end{bmatrix}. \tag{2.14}$$

It can be seen that the scalar components of the chiral fields have non-zero fermion charge. The fields $F_\pm$ are not true dynamical variables, so that their charge is immaterial, but the $A_\pm$-components correspond to real particles. It can be shown that if the Lagrangian conserves fermion charge and contains a bare mass, then the supersymmetric theory necessarily contains scalar particles with non-zero fermion charge.

Supersymmetric gauge-invariant theories are of the greatest interest for applications. A supersymmetric generalization of electrodynamics was first constructed by Wess and Zumino.[14] An analogous construction was considered in Refs. 15 and 16 for the case of a non-Abelian gauge group.

It is obvious that we cannot confine ourselves to a single vector gauge field in a supersymmetric theory, since the transformations of the group connect the vector field with fields of different tensorial dimensions. The role of a gauge field in this case is played by the supermultiplet $\Psi(x,\theta)$, which contains spinors and scalars in addition to the vector field. We can describe the matter fields by means of the chiral supermultiplets $\Phi_\pm(x,\theta)$, which contain spinor and scalar fields.

The supersymmetry transformations do not commute with the ordinary gauge transformations, so that the supersymmetric action must necessarily be invariant with respect to the larger gauge group

$$\Phi_\pm(x,\theta)\to\Omega_\pm(x,\theta)\,\Phi_\pm(x,\theta),\quad e^{g\Psi}\to\Omega_- e^{g\Psi}\Omega_+^{-1}, \tag{2.15}$$

where the matrices $\Omega_\pm(x,\theta)$ satisfy the condition $\Omega_+^+=(\Omega_-)^{-1}$. The role of the usual gauge function is now played by the chiral matrix superfield $\Omega_\pm(x,\theta)$, which depends on eight arbitrary functions $U_\pm(x)$, $V_\pm(x)$, and $W_\pm(x)$, where $U_\pm$ and $W_\pm$ are (pseudo) scalars and $V_\pm$ is a two-component spinor. In the case of a non-Abelian group, each of these functions in turn has several components.

The gauge-invariant kinetic term has the form

$$\int d^4x d^4\theta\,[\Phi_+^+ e^{g\Psi}\Phi_+ + \Phi_-^+ e^{-g\Psi}\Phi_-]. \tag{2.16}$$

In contrast with ordinary gauge theories, this expression is essentially non-linear: the series (2.16) contains arbitrary powers of the fields $c(x)$, the fourth power of $\chi(x)$, etc. At first sight, the Lagrangian (2.16) therefore corresponds to a non-renormalizable theory and is meaningless in the framework of perturbation theory. However, the gauge invariance (2.15) allows us to specify eight components of the field $\Psi(x,\theta)$ arbitrarily. In particular, following Wess and Zumino, we can put

$$c(x)=\chi(x)=F(x)=G(x)=\partial_\mu A_\mu(x)=0. \tag{2.17}$$

In this gauge, the infinite series (2.16) terminates and the action takes the form (we write here the supersymmetric generalization of the Lagrangian for a Yang-Mills field interacting with the matter fields)

$$\begin{aligned}S=\int d^4x\,\Big\{&\Big|\partial_\mu A_+^+ + \frac{ig}{2}A_+^+A_\mu\Big|^2 + F_+^+F_+\\ &+i\bar\psi\gamma_\mu\Big(\partial_\mu-\frac{ig}{2}A_\mu\Big)\psi+M(A_-^+F_+-\bar\psi\psi)+\frac{g}{2}[A_+^+DA_+-A_-^+DA_-]\\ &+\frac{ig}{\sqrt{2}}A_+^+\bar\lambda\frac{1+\gamma_5}{2}\psi+\frac{1}{8}\mathrm{Tr}\,(\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu])^2\\ &+\frac{i}{4}\mathrm{Tr}\,\{\bar\lambda\gamma_\mu(\partial_\mu\lambda+g[A_\mu,\lambda])\}+\frac{1}{4}\mathrm{Tr}\,(D^2)+(+\leftrightarrow-)+\mathrm{c.c.}\Big\},\\ &A_\mu=A_\mu^k\tau^k,\quad \lambda=\lambda^k\tau^k,\quad D=D^k\tau^k.\end{aligned} \tag{2.18}$$

(The Lagrangian for an Abelian theory is obtained from (2.18) by making the substitution $\tau^k\to I$.) The fields D and $F_\pm$ are not true dynamical variables, since the Lagrangian (2.18) contains no derivatives of D and $F_\pm$. Eliminating D and $F_\pm$, we obtain the mass term and the contact interaction for the scalar fields:

$$-M^2(A_+^+A_+ + A_-^+A_-)-\frac{g^2}{4}(A_+^+\tau^kA_+ - A_-^+\tau^kA_-)^2. \tag{2.19}$$

The Lagrangian (2.18) contains all the necessary ingredients of unified gauge models: the vector field A_μ interacts in a gauge-invariant manner with the spinor and scalar fields, and the scalar fields in turn interact with each other and with the spinor fields. In other words, the scalar components of the chiral superfields can in principle play the role of Higgs mesons. In contrast with the Weinberg-Salam model, the Lagrangian (2.18) now depends on only a single dimensionless coupling constant g.

It can be shown that this property is preserved in the renormalized theory.[17-20] All the ultraviolet divergences are eliminated by renormalizing the charge and the wave functions of the superfields. This implies in particular that the supersymmetric Yang-Mills theory is asymptotically free, despite the presence of scalar particles. Owing to the supersymmetry, the invariant constant of the fourfold interaction of scalar fields, which usually violates asymptotic freedom, coincides with the Yang-Mills constant, which, as is well known, tends to zero for large values of the argument (if the number of multiplets of the matter fields is not too large).

A remarkable property of supersymmetric theories is the absence of an independent mass renormalization. If the bare mass is equal to zero and there are no conservation laws which forbid the appearance of a mass, then the physical mass can be uniquely calculated.

However, the Lagrangian (2.18) is "too symmetric." In nature there is no degeneracy in the masses of the scalar and spinor particles; consequently, the supersymmetry must be broken and, if we wish to preserve the symmetry relations between the renormalized amplitudes and coupling constants, broken spontaneously. The problem of spontaneous supersymmetry breaking has proved to be non-trivial, since the condition of invariance imposes strong constraints on the form of the effective potential. In particular, the potential (2.19) corresponds to a stable symmetric extremum and does not produce spontaneous supersymmetry breaking.

3. SPONTANEOUS SUPERSYMMETRY BREAKING

The first successful attempt to solve this problem was made by Fayet and Iliopoulos,[21] who pointed out that is is possible to produce spontaneous supersymmetry breaking in the case of an Abelian gauge group by adding a term linear in the field D to the action (2.18), namely

$$\xi \int d^4x D(x). \tag{3.1}$$

The action remains supersymmetric after the addition of this term, and there is no violation of gauge invariance in the Abelian case. However, the presence of the linear term leads to a non-zero vacuum average $\langle D\rangle_0 \neq 0$. The transition to a stable ground state, $D \to D - \xi$, produces an additional mass term for the scalar fields, namely

$$-\xi g [A_+^+ A_+ - A_-^+ A_-]. \tag{3.2}$$

The degeneracy of the masses within a supermultiplet is removed, and the supersymmetry is spontaneously broken. If $\xi g > M^2$, the fields $A_\pm$ in turn acquire non-zero vacuum expectation values, and we have the usual Higgs effect, leading to spontaneous breaking of the internal symmetry and the appearance of a mass for the vector particles.

The Fayet-Iliopoulos mechanism is applicable only in the case of an Abelian gauge group, since the addition of the term $\int D^a(x) d^4x$ obviously violates gauge invariance in the non-Abelian case. (Still another possible mechanism of spontaneous supersymmetry breaking for Lagrangians of a special form was discussed in Refs. 22 and 23.) Both of these methods are applicable only to a very limited class of theories, and attempts to use them to construct realistic models have not met with success.[24,25]

Still another problem arises in this connection. Spontaneous supersymmetry breaking involves the presence of a spinor particle of zero mass—a Goldstone fermion (Goldstone's theorem carries over to this case practically unchanged, except that we have a massless fermion instead of a Goldstone boson). It was originally assumed that the Goldstone fermion can be identified with the electron neutrino. It is easy to show, however, that this identification leads to a contradiction with experiment.[26,27]

Invariance with respect to the supersymmetry transformations leads to a conserved current, which in the case of a spontaneously broken symmetry contains a term proportional to the field of the Goldstone fermion:

$$j^\mu(x) = -ic\gamma^\mu \nu(x) + \ldots \tag{3.3}$$

Making use of the conservation of the current j^μ, we obtain

$$0 = \int d^4x e^{iqx} \partial_\mu \langle B | j_\mu(x) | A\rangle = (2\pi)^4 \delta(q + p_B - p_A) [cM_\nu(q) + q^\mu R_\mu], \tag{3.4}$$

where $M_\nu(q)$ is the neutrino pole term and R_μ denotes the contribution of the remaining (non-pole) terms in (3.3). It follows from (3.4) that $M_\nu(q) \to 0$ as $q \to 0$. In particular, the amplitude for β decay must tend to zero as the neutrino momentum tends to zero or, equivalently, at high energy of the charged lepton. The experimental data are incompatible with such a behavior. In principle there is still a possibility of identifying the Goldstone fermion with the muon neutrino,[28] although the low-energy theorems impose strong constraints on the behavior of the corresponding amplitudes.

Another recently proposed mechanism of spontaneous supersymmetry breaking[29,30] gives a significant extension of the class of admissible theories and is free from the difficulty connected with the Goldstone fermion. This mechanism makes it possible to obtain arbitrary mass terms for the scalar components of the chiral superfields in a theory with spontaneously broken supersymmetry.

We shall first show how to obtain mass terms of the form

$$\varkappa (A_-^+ A_+ + A_+^- A_-). \tag{3.5}$$

Let S_s denote an arbitrary supersymmetric action. We introduce auxiliary chiral fields $R_\pm$ and $\tilde{R}_\pm$ which interact with the "physical" fields as follows:

$$S = S_s + \int [\{\Phi_+^+ \Phi_- R_- + \Phi_-^+ \Phi_+ R_+\}_F + \{R_+ \tilde{R}_- + \tilde{R}_+ R_-\}_D + \varkappa (\tilde{R}_+ + \tilde{R}_-)_F] \, dx. \tag{3.6}$$

The action (3.6) is manifestly supersymmetric. Owing to the presence of the linear term $(\tilde{R}_+ + \tilde{R}_-)_F$, the vacuum averages $\langle R_+\rangle_F \langle R_-\rangle_F$ are non-zero, and the supersymmetry is spontaneously broken. The transition to the stable vacuum produces a term

$$\int [\{\Phi_+^+ \Phi_- + \Phi_-^+ \Phi_+\}_A] \, d^4x \cdot \varkappa = \int \varkappa (A_-^+ A_+ + A_+^- A_-) \, d^4x, \tag{3.7}$$

which removes the degeneracy in the masses of the spinor and scalar fields. The variation of the action (3.6) with respect to $\tilde{R}_+$ and $\tilde{R}_-$ leads to free equations for R_+ and R_-. This means that the auxiliary fields $R_\pm$ and $\tilde{R}_\pm$ are decoupled from the physical fields. The diagrams describing physical processes contain no internal lines for $R_\pm$ and $\tilde{R}_\pm$, and the S-matrix is unitary in the physical sector. The only observable effect of the fields $R_\pm$ and $\tilde{R}_\pm$ is the mass term (3.7). However, the explicit supersymmetry of the original expression (3.6) enables us to subject it to the invariant renormalization procedure developed in Refs. 17–20. It is possible to write generalized Ward identities and to show that the relations between the renormalized coupling constants and masses which are valid in the symmetric theory are modified only by finite, calculable terms. The only possible new counter-term which may be required to eliminate ultraviolet divergences is

$$\tilde{Z} \{\Phi_+^+ \Phi_- R_- + \Phi_-^+ \Phi_+ R_+\} \tag{3.8}$$

and merely leads to a redefinition of the arbitrary constant $\varkappa$:

$$\tilde{Z}_\varkappa (A_+^+ A_- + A_-^+ A_+) \tag{3.9}$$

(in fact, even this renormalization is absent in most models).

An analogous procedure can be used to obtain mass terms of the form

$$\xi_+ (A_+^+ A_+) + \xi_- (A_-^+ A_-). \tag{3.10}$$

To do this, we must introduce general auxiliary superfields P and $\tilde{P}$, which interact as follows:

$$S_P = \int \left\{ [(1+\xi_i P)\mathcal{L}_i]_D + \frac{1}{2}[P_1 \Box \tilde{P}_1 + P_+ \Box \tilde{P}_- + \tilde{P}_+ \Box P_-]_D - \varkappa [\tilde{P}]_D \right\} d^4x, \quad (3.11)$$

Here P_1, P_+, P_- and $\tilde{P}_1$, $\tilde{P}_+$, $\tilde{P}_-$ denote the irreducible components in the expansions of the fields P and $\tilde{P}$ (see Eq. (2.7)), and $\mathcal{L}_1$ and $\mathcal{L}_2$ are general superscalars which depend quadratically on the chiral fields Φ_+, Φ_+^* and Φ_-, Φ_-^*, respectively. For example, $\mathcal{L}_1$ and $\mathcal{L}_2$ take the following form in the supersymmetric Yang-Mills theory:

$$\mathcal{L}_1 = \frac{1}{4}\Phi_+^+ e^{g\tau^a\Psi^a}\Phi_+, \quad \mathcal{L}_2 = \frac{1}{4}\Phi_-^+ e^{-g\tau^a\Psi^a}\Phi_-. \quad (3.12)$$

(In order to preserve the gauge invariance of the theory, the fields P and $\tilde{P}$, as well as the fields $R_\pm$ and $\tilde{R}_\pm$ considered above, must be taken to be singlets of the gauge group.)

Owing to the presence of the linear term, the supersymmetry is spontaneously broken, as in the previous case. The displacement of the fields P given by $[P]_D \to [P]_D + \varkappa$ produces the mass term (3.10). The fields P and $\tilde{P}$ are again decoupled from the physical fields, and the S-matrix is unitary in the physical sector.

Another method of introducing auxiliary fields is discussed in Ref. 30, where a detailed analysis of the renormalization procedure is also given.

An interesting special case of the Lagrangian (3.12) is the model

$$S = \frac{1}{4}\int \left\{ [(1+\xi P)\Phi_+^+ e^{g\tau^a\Psi^a}\Phi_+]_D + \frac{1}{2}[P_1 \Box \tilde{P}_1 + P_+ \Box \tilde{P}_- + \tilde{P}_+ \Box P_-]_D - \varkappa [\hat{P}]_D \right\} d^4x, \quad (3.13)$$

where Φ_+ is a chiral isodoublet, Ψ^a is a supersymmetric Yang-Mills field, and $\mathcal{L}_{YM}$ denotes the Yang-Mills Lagrangian. The transition to the stable vacuum produces the mass term

$$\frac{\varkappa\xi}{2}\int (A_+^+ A_+)\, d^4x. \quad (3.14)$$

If $\xi > 0$, the Lagrangian (3.13) also leads to spontaneous breaking of isotopic invariance. The fields A_+ acquire non-zero vacuum averages. The usual Higgs effect leads to the following mass spectrum: all three components of the vector field, two complex fermions, and one Hermitian scalar acquire non-zero masses proportional to ξ. Three Goldstone scalars, which can be eliminated by a gauge transformation, and one two-component fermion remain massless. The model is infrared-finite. Being a Yang-Mills theory with one dimensionless coupling constant, this model is asymptotically free. Thus the Lagrangian (3.6) is an example of an asymptotically free theory with no infrared divergences.

In conclusion, we note that by dropping the auxiliary fields and considering from the very beginning Lagrangians containing the mass terms (3.5) and (3.10), we obtain a theory with an explicit but "mild" supersymmetry breaking. A special case of such a "mild" breaking in a Φ^3 model was considered by Iliopoulos and Zumino.[31] The foregoing arguments provide a simple explanation of their result, which is a particular case of the mechanism that we have described.

This approach is free from the difficulty connected with the Goldstone neutrino. The Goldstone fermion is a component of the auxiliary multiplet and does not interact with the physical fields.[29,30,32] The low-energy theorems do not impose any restrictions on the form of the interaction. As we shall show below, the existence of massless neutrinos may be related to the conservation of lepton charges. In models with nonconservation of lepton charge, the neutrinos acquire a uniquely calculable non-zero mass. Supersymmetric models with non-conserved lepton charge can be used to account for the neutrino oscillations discussed by Pontecorvo and others.[33,34]

4. UNIFIED GAUGE MODELS WITH SPONTANEOUSLY BROKEN SUPERSYMMETRY

In this section we shall illustrate the possible applications of supersymmetry to models of the weak and electromagnetic interactions by means of the simplest examples. In principle, we could also apply the formalism developed above to unified models which include the strong interactions. However, it seems to us that the existing experimental information is not good enough to enable us to make any reliable choice of a particular model; moreover, the mechanism of producing the masses of the strongly interacting particles is not clear at the present time. We shall therefore confine ourselves here to the weak and electromagnetic interactions, bearing in mind that the "symmetry group of the world" G is already broken down to $G_{w+e} \otimes G_s$, and we take the group G_{w+e} to be $SU_2 \times U_1$.

The minimal model corresponds to an interaction of gauge fields B_μ and A_μ with chiral isodoublets $\Phi_\pm$. After spontaneous breaking of isotopic invariance, such a model would contain two charged and two neutral leptons, which we might try to identify with the electron, the muon, and their corresponding neutrinos. However, it can be seen from Eq. (2.18) that, owing to the presence of the vector Yang-Mills term

$$\frac{i}{2} g e^{ijk} \bar{\lambda}^i \gamma^\mu \lambda^j A_\mu^k,$$

this identification would lead to the existence of right-handed neutrinos interacting with the same strength as the left-handed ones. Therefore a model containing a total of four Dirac leptons is certainly incompatible with experiment. Supersymmetric models of the weak and electromagnetic interactions must necessarily include heavy leptons. A model that leads to a reasonable form of interaction can be constructed on the basis of two chiral isodoublets $\Phi_{\pm 1,2}$ which interact with gauge fields A_μ and B_μ.[30] In addition to the electron, the muon, and the e- and μ-type neutrinos, this model also predicts heavy charged and neutral leptons. It turns out that the mass of the neutral lepton is of the order of the muon mass, so that this lepton would have to be seen, for example, in the decay of the kaon. Thus this model is unsatisfactory from the experimental point of view, and we must further increase the number of heavy leptons.

The simplest model[35] which gives a lepton spectrum that is acceptable from the experimental point of view can be constructed on the basis of three chiral complex isodoublets

$$\Phi_{i\pm}=\{A^k_{i\pm},\ \psi^k_{i\pm},\ F^k_{i\pm}\},\qquad i=1,\ 2,\ 3,$$

where k is an isotopic index, and two Hermitian singlets

$$S^{1,2}_{+}=S^{1,2+}_{-},\quad S^{1,2}_{\pm}=\{A^{1,2}_{S\pm},\ \psi^{1,2}_{S\pm},\ F^{1,2}_{S\pm}\}.$$

The most general supersymmetric and gauge-invariant Lagrangian describing the interaction of these multiplets with gauge superfields

$$\Psi^a=\{c^a,\ \chi^a,\ M^a,\ N^a,\ A^a_\mu,\ \lambda^a,\ D^a\}\qquad (a=1,\ 2,\ 3),$$
$$\Psi=\{c,\ \chi,\ M,\ N,\ \lambda,\ A_\mu,\ D\},$$

has the form

$$\mathscr{L}=\frac{1}{2}\{\Phi^+_{+i}e^{g\Psi+g_1\Psi^a\tau^a}\Phi_{+i}+\Phi^+_{-i}e^{-g\Psi-g_1\Psi^a\tau^a}\Phi_{-i}\}_D+\frac{1}{2}\{S^l_+S^l_-\}_D$$
$$-\{M_{ij}\Phi^+_{-i}\Phi_{j+}\}_F-\{a^l_{ij}S^l_-\Phi^+_{+i}\Phi_{-j}\}_F+\mathscr{L}_0(\Psi,\ \Psi^a)+\text{c.c.}\tag{4.1}$$

where $\mathscr{L}_0(\Psi,\Psi^a)$ is the supersymmetric Lagrangian for the Yang-Mills field and the Abelian gauge field Ψ.

The mechanism of spontaneous supersymmetry breaking described in the preceding section can be used to produce arbitrary mass terms for the scalar components of $\Phi_\pm$ and $S_\pm$. We shall not repeat the appropriate arguments here, and merely write the result (dropping the auxiliary fields):

$$\Delta\mathscr{L}=\xi_{i\pm}A^+_{i\pm}A_{i\pm}+\eta_{ij}(A^+_{i+}A_{j-}+A^+_{j-}A_{i+})+\xi^l_SA^l_{S+}A^l_{S-}.\tag{4.2}$$

The Lagrangian (4.1) and (4.2) depends on the parameters ξ_i, η_{ij}, M_{ij}, and a^k_{ij}, whose possible values are constrained by the postulated conservation laws. We require conservation of lepton charge, which is associated with invariance with respect to the transformations

$$\begin{pmatrix}A_{1,2+}\\ \psi_{1,2+}\\ F_{1,2+}\end{pmatrix}\to\begin{pmatrix}A_{1,2+}\\ \psi_{1,2+}e^{i\varphi}\\ F_{1,2+}e^{2i\varphi}\end{pmatrix},\quad \begin{pmatrix}A_{1,2+}\\ \psi_{1,2-}\\ F_{1,2-}\end{pmatrix}\to\begin{pmatrix}A_{1,2-}e^{2i\varphi}\\ \psi_{1,2-}e^{i\varphi}\\ F_{1,2-}\end{pmatrix};$$
$$\begin{pmatrix}A_{3\pm}\\ \psi_{3\pm}\\ F_{3\pm}\end{pmatrix}\to\begin{pmatrix}A_{3\pm}\\ \psi_{3\pm}e^{-i\varphi}\\ F_{3\pm}e^{-2i\varphi}\end{pmatrix},\quad \begin{pmatrix}A^l_{S-}\\ \psi^l_{S-}\\ F^l_{S-}\end{pmatrix}\to\begin{pmatrix}A^l_{S-}\\ \psi^l_{S-}e^{-i\varphi}\\ F^l_{S-}e^{-2i\varphi}\end{pmatrix};\tag{4.3}$$
$$\lambda_+\to e^{-i\varphi}\lambda_+,\quad \lambda_-\to e^{i\varphi}\lambda_-,$$

as well as separate conservation of the electron and muon charges, corresponding to global phase transformations

$$\Phi_{1\pm}\to e^{i\beta}\Phi_{1\pm},\quad \Phi^1_{S-}\to e^{-i\beta}\Phi^1_{S-},$$
$$\Phi_{2\pm}\to e^{i\gamma}\Phi_{2\pm},\quad \Phi^2_{S-}\to e^{-i\gamma}\Phi^2_{S-}.\tag{4.4}$$

Invariance with respect to the transformations (4.3) and (4.4) requires the vanishing of all the parameters M_{ij}, a^l_{ij}, and η_{ij} except the following:

$$M_{11}\equiv m_e\neq 0,\quad \eta_{33}\equiv\eta\neq 0,\quad a^1_{31}\equiv a_1\neq 0,\quad a^2_{32}\equiv a_2\neq 0,$$
$$M_{22}\equiv m_\mu\neq 0.\tag{4.5}$$

It is easy to show that the effective potential corresponding to (4.1) and (4.2) has a stable extremum when

$$\langle A_{1,2\pm}\rangle_0=\langle A^l_{S\pm}\rangle_0=0,\quad \langle A_{3\pm}\rangle_0=\alpha_\pm(\eta,\ \xi,\ g,\ g_1)\neq 0.\tag{4.6}$$

The transition to fields with zero vacuum averages leads to the following mass spectrum: we have charged vector mesons

$$W_\pm=\frac{A^1_\mu\pm iA^2_\mu}{\sqrt{2}},\quad M^2_W=\frac{g_1^2}{2}(\alpha^2_++\alpha^2_-);\tag{4.7}$$

and neutral vector mesons

$$Z_\mu=(g^2+g_1^2)^{-1/2}(gA_\mu-g_1A^3_\mu),\quad M^2_Z=2^{-1}(g^2+g_1^2)(\alpha^2_++\alpha^2_-),$$
$$a_\mu=(g^2+g_1^2)^{-1/2}(g_1A_\mu+gA^3_\mu),\quad M_a=0.\tag{4.8}$$

The charged fermions acquire masses

$$-\frac{g_1}{\sqrt{2}}\{\alpha_+(\bar\lambda_2-i\bar\lambda_1)\psi^1_{3+}+\alpha_-(\bar\lambda_2-i\bar\lambda_1)\psi^1_{3-}\}+\text{c.c.}\tag{4.9}$$

Bearing in mind the lepton charges determined by the transformations (4.3) and (4.4), we can make the identifications

$$\left.\begin{array}{ll}\psi_{1+}=e_+ & \psi_{1-}=e_-\\ \psi_{2+}=\mu_+ & \psi_{2-}=\mu_-\\ \psi_{3+}=E^2_+ & \psi_{3-}=E^1_-\\ 2^{-1/2}(\lambda_2+i\lambda_1)_+=E^1_+ & 2^{-1/2}(\lambda_2+i\lambda_1)_-=E^2_{1-}\\ m_e=M_{11},\quad m_\mu=M_{22}, & M_{E_1}=g_1\alpha_-,\quad M_{E_2}=g_1\alpha_+.\end{array}\right\}\tag{4.10}$$

The invariance with respect to the transformations (4.3) and (4.4) leads to the existence of four massless fermions

$$\nu_{e+}=\frac{(M_1a_1g_1^{-1})\psi^2_{1+}-m_e\psi^1_{S+}}{(m_e^2+M^2_{E_1}a_1^2g_1^{-2})^{1/2}},\quad \nu_{\mu+}=\frac{(M_2a_2g_2^{-1})\psi^2_{2+}-m_\mu\psi^2_{S+}}{(m_\mu^2+M^2_{E_2}a_2^2g_1^{-2})^{1/2}},\tag{4.11}$$

$$\frac{i(g_1\lambda+g\lambda_3)}{(g^2+g_1^2)^{1/2}}=\nu_1,\quad \frac{M_{E_2}\psi^2_{3+}-M_{E_1}\psi^{2c}_{3-}}{\sqrt{M^2_{E_1}+M^2_{E_2}}}=\nu_2.\tag{4.12}$$

There also exist three heavy neutral leptons with masses

$$M_{N_1}=(M^2_{E_1}a_1^2g_1^{-2}+m_e^2)^{1/2},\quad M_{N_2}=(M^2_{E_2}a_2^2g_1^{-2}+m_\mu^2)^{1/2},$$
$$M_{N_3}=(g^2+g_1^2)^{1/2}g_1^{-1}(M^2_{E_1}+M^2_{E_2})^{1/2}.\tag{4.13}$$

The masses of the heavy charged leptons are related to the mass of the intermediate meson by the sum rule

$$M^2_W=\frac{1}{2}(M^2_{E_1}+M^2_{E_2}).\tag{4.14}$$

If we identify one of these leptons with the recently discovered particle ($M\sim 1.9$ GeV), then the mass of the second lepton is very large.

As to the neutral leptons, their masses depend on the ratio $a_ig_1^{-1}$. Since M_{N_i} must exceed the mass of the kaon, for $M_{N_1}\sim 1.9$ GeV we have $a_1g_1^{-1}\gtrsim 0.25$.

There is a large arbitrariness in the spectrum of scalar mesons. By choosing the parameters ξ and η appropriately, it is possible to ensure that all the scalar mesons are much heavier than the W meson, so that all processes in which they take part are strongly suppressed.

The explicit form of the interaction can be easily obtained from Eq. (4.1). To do this, it is sufficient to transform to the Wess-Zumino gauge

$$c=\chi=M=N=0\tag{4.15}$$

and eliminate the auxiliary fields D, $F_\pm$, and F_S. Here we shall write explicitly only the part of the Lagrangian which is responsible for the interaction of the light leptons:

$$\mathscr{L}_I=gg_1(g^2+g_1^2)^{-1/2}a_\mu(\bar e\gamma^\mu e+\bar\mu\gamma^\mu\mu)$$
$$+2^{-1}(g^2+g_1^2)^{-1/2}Z_\mu\{(g^2-g_1^2)(\bar e\gamma^\mu e+\bar\mu\gamma^\mu\mu)$$
$$+(g^2+g_1^2)(\bar\nu_{eL}\gamma^\mu\nu_{eL}+\bar\nu_{\mu L}\gamma^\mu\nu_{\mu L})\}$$
$$+2^{-1/2}g_1W^\rho_-\{M_{E_1}a_1g_1^{-1}(M^2_{E_1}a_1^2g_1^{-2}+m_e^2)^{-1/2}\bar e_L\gamma^\rho\nu_{eL}$$
$$+M_{E_2}a_2g_1^{-1}(M^2_{E_2}a_2^2g_1^{-2}+m_\mu^2)^{-1/2}\bar\mu_L\gamma^\rho\nu_{\mu L}\}+\text{c.c.}+\dots,\tag{4.16}$$

where ... denotes the terms containing heavy leptons and scalar mesons.

The first term describes the standard electromagnetic interaction, with $e = gg_1(g^2 + g_1^2)^{-1/2}$.

The second term contains neutral weak currents. We note that although this model is vector-like (apart from spontaneous symmetry breaking, the interaction Lagrangian contains no axial-vector currents), the neutral current is not purely vector in character. The neutral current, which contains the heavy leptons E_1 and E_2, has both vector and axial-vector parts. Unlike the vector-like models considered by a number of authors,[36-40] supersymmetric vector-like models automatically lead to the existence of an axial-vector part of the neutral current. This is so because the fermions in these models belong to different representations of the internal symmetry group: some of the fermions appear in the gauge superfield and hence transform according to the adjoint representation, while the others are components of the chiral superfields. (The neutral current is necessarily vector in character in a vector-like model only if all the fermions transform according to the same representation.)

The interaction constants of the charged left-handed currents $(\overline{\mu}_L \nu_{\mu L})$ and $(\overline{e}_L \nu_{eL})$ differ by a factor

$$\frac{M_{E_1} a_1 g_1^{-1}}{(M_{E_1}^2 a_1^2 g_1^{-2} + m_e^2)^{1/2}} \frac{(M_{E_2}^2 a_2^2 g_1^{-2} + m_\mu^2)^{1/2}}{M_{E_2} a_2 g_1^{-2}} \approx 1. \qquad (4.17)$$

The universality is exact only in the limit $m_e = m_\mu = 0$. However, since $m_e, m_\mu \ll M_{E_1}, M_{E_2}$, the deviation from universality is negligibly small.

It is also easy to include a weak quark interaction in this scheme. If the quarks are not mixed with the leptons, they must be described by chiral supermultiplets, and the vacuum averages of the scalar components of these multiplets must be equal to zero. If this were not so, there would be a mass term of the form (4.9), which would lead to quark-lepton mixing. (Of course, such models are also possible in principle.) We can introduce supersymmetric mass terms for the quarks at will. All the quarks will then have non-zero masses, which are uncorrelated with the masses of the leptons and intermediate vector mesons. It seems to us, however, that the origin of the quark masses cannot be understood without taking into account the strong interactions. Therefore we shall not write any specific quark Lagrangian here. We simply note that the quark sector is completely independent of the lepton sector, and the standard quark models can easily be written in a supersymmetric form. Of course, the quarks are then accompanied by scalar particles.

5. CONCLUSIONS

The model described in the preceding section shows that gauge theories of the weak and electromagnetic interactions admit a supersymmetric generalization which allows us to include Higgs scalars in the gauge theories in a natural way. The model which we have considered is the minimal possibility (in the sense of the number of leptons) of obtaining a reasonable mass spectrum and form of interaction. A more detailed comparison with experiment will probably require more complex models. The specific form of the weak neutral current may prove to be crucial. The mechanism of spontaneous supersymmetry breaking described in Sec. 3 is applicable to a wide class of gauge theories, and we might hope that it would allow us to construct a model which satisfies all the requirements of experiment. From an esthetic point of view, there is a preference for models which contain only the minimal gauge interaction and which do not include any direct interaction of the chiral multiplets. Such models also have greater predictive power, since they contain fewer arbitrary parameters.

In conclusion, we briefly enumerate the most characteristic features of unified gauge models with spontaneously broken supersymmetry.

Supersymmetric models necessarily include scalar fields, which can give rise to spontaneous symmetry breaking via the Higgs mechanism.

The parameters characterizing the interactions of the scalar fields are related to the constants of the gauge interaction, thus greatly enhancing the predictive power of the theory.

In general, the scalar fields have non-zero fermion charge.

Supersymmetric models necessarily include heavy leptons.

Supersymmetry imposes strong constraints on the masses of the leptons and determines a relation between the masses of the leptons and vector mesons.

Supersymmetric theories involve no independent mass renormalization, thus making it possible to calculate uniquely finite mass differences and, in the case of zero bare masses, the masses themselves.

The neutral current in vector-like supersymmetric models is not purely vector in character.

Supersymmetric theories based on semi-simple gauge groups may be asymptotically free, despite the presence of scalar particles.

All these features make supersymmetric theories extremely attractive from the point of view of a possible description of the weak and electromagnetic interactions.

Looking to the future, supersymmetry may also unify other interactions, including the strong and gravitational interactions, within the framework of a single theory. The first attempts to construct a supersymmetric generalization of gravitation have given encouraging results.[41-45] In particular, it turns out that the non-renormalizable ultraviolet divergences which occur in theories describing the gravitational interaction with matter are no longer present in the single-loop diagrams in the supersymmetric theories of gravitation. This offers hope that the divergences may also cancel in more complex diagrams. At the present time, however, these investigations are still at an initial stage which is very remote from the actual experimental situation, and we shall not discuss them in further detail.

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