On discussions concerning the problem of ponderomotive forces

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A discussion is presented in this note from the point of view of modern concepts of the reasons for some controversies which occurred in the past concerning some problems involving ponderomotive forces. In particular it is pointed out that the reason for the different expressions for the force consists of the nonuniqueness of the separation into components of the total force acting on a body situated in an electromagnetic field.

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Recently considerable attention has been attracted to problems associated with ponderomotive effects. In particular this is attested to by the appearance of a considerable number of reviews and methodological articles (Refs. 1–7 and others), and also by the appearance of a number of important experimental papers (Refs. 8–11 and others).

The problems of ponderomotive forces and ponderomotive effects attracted the interest of many scientists already in the last century. And in spite of the large number of papers devoted to this subject and carried out over a long period of time the questions under consideration continue to retain their problematic character and are of current interest (cf., for example Ref. 12), since they are not only of theoretical, but also of applied significance.

In the present communication we would like to discuss from the point of view of modern concepts the reasons for certain disputes which occurred in the past.

1. As *the first example* we cite the discussion known already from the last century concerning the expressions for the density of ponderomotive forces with which a static electrical field acts on electrically neutral liquid dielectrics.

Two solutions are known. One going back to the work of Kelvin[13] 1) has the form

$$\mathbf{f}^{\mathrm{Kelv}}=(\mathbf{P}\nabla)\mathbf{E}, \tag{1}$$

where $\mathbf{f}$ is the density of the ponderomotive force, $\mathbf{P}$ is the polarization, and $\mathbf{E}$ is the intensity of the field.

The other solution based on energy concepts belongs to Helmholtz[14] (cf., also the papers by Lorberg, Korteweg, Kirchhoff[15–18]):

$$\mathbf{f}^{\mathrm{Helm}}=-\frac{E^2}{8\pi}\nabla\varepsilon+\nabla\left(\rho\frac{\partial\varepsilon}{\partial\rho}\frac{E^2}{8\pi}\right), \tag{2}$$

where ε is the dielectric permittivity of the dielectric medium, and ρ is the density of the dielectric.

After the work of Helmholtz a difference became apparent between the results obtained and a discussion arose concerning the correctness of the one or the other approach, i.e., concerning the correctness of the expression for the force (that due to Kelvin or due to Helmholtz).

Larmor[19] made critical statements concerning Helmholtz's theory. He was supported by Livens.[20–21] Since within the framework of an energy-based investigation essential expressions were those for the energy of the field in dielectric and magnetic substances, and also the formulation of the first law of thermodynamics for polarizable and deformable media, stormy discussions concerning these questions took place (we cite only the papers by Stoner, Livens and Guggenheim[22–26]), which ceased only after the death of Livens.

Stratton[27] notes that criticism due to Larmor and Livens is hardly likely to have been well founded. At the end of the forties and the beginning of the fifties Smitt-White published a series of articles (for brevity we shall cite only the most substantial one[28]) in which he criticized the Helmholtz method as being inadequate. For the history of this problem the articles of Slepjan are of interest (his paper[29] and a number of other papers on ponderomotive forces).

In the papers of Prigogine, Mazur, de Groot[30–31] and other Dutch scientists that appeared subsequently a solution was obtained anew within the framework of thermodynamics and statistical physics and, apparently, due to the work of these scientists a solution was finally found of the controversial question of the correctness of expressions for the force due to Kelvin (1) and to Helmholtz (2).

The essence of this solution consists of the fact that in electromechanical systems the physically determined parameter is not simply the force (1) or (2) but the combination

$$\mathbf{f}^{\mathrm{pond}}-\mathrm{grad}\ p, \tag{3}$$

where p is the pressure in the fluid.

1)As far as we know, reference to this paper is generally either not made at all, or it is cited incorrectly.

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Therefore the alternative approach to expressions for the forces (1) and (2) turns out to be incorrect. In actual fact both the one and the other expression for the force are correct if they are supplemented by appropriate values of the pressure p. In particular, it turned out that if for p one uses the value p_0 which existed in the fluid in the absence of a field, then into expression (3) in place of $\mathbf{f}^{\text{pond}}$ it is necessary to substitute the Helmholtz force (2). But if for $\mathbf{f}^{\text{pond}}$ we take the Kelvin expression (1) then for p we should utilize the expression

$$p = p_0 + \frac{\mathrm{EP}}{2} + \left(\frac{\partial \varepsilon}{\partial \rho}\right) \frac{E^2}{2} \rho. \qquad (4)$$

It can be easily shown that in both cases the sum (3) remains the same.

In Ref. 32 a detailed investigation is given of the question of ponderomotive forces acting on a liquid (or gaseous) and solid dielectric in an electric field. In particular, for liquid dielectrics the volume density of the forces has the form

$$\mathbf{f} = -\operatorname{grad} p_0(\rho, T) + \frac{1}{8\pi} \operatorname{grad} \left[E^2 \rho \left(\frac{\partial \varepsilon}{\partial \rho}\right)_T \right] - \frac{E^2}{8\pi} \operatorname{grad} \varepsilon, \qquad (5)$$

i.e., this is in fact the ponderomotive force due to Helmholtz combined with the pressure p_0.[2)]

Nevertheless, in spite of the substantial papers mentioned above, new papers on the problems in question kept on appearing, particularly in foreign journals. In 1962, Hakim in a theoretical paper[33] devoted to nonpolar dielectrics showed that the expression for the Helmholtz force (2) can be obtained not only on the basis of energy considerations but also starting with model concepts. And in the experimental paper of Hakim and Higham[34] it is shown that at least in the case of nonpolar dielectrics expression (2) for the force is valid and not expression (1). An extensive discussion concerning this and allied problems occurred in 1966 on the pages of the journal "Proc. IEE" (cf., for example Ref. 35).

2. As *a second example*, we quote the expression for the ponderomotive force obtained by Einstein and Lamb[36]:

$$\mathbf{f}^{\text{E.-L.}} = \rho \mathbf{E} + \frac{1}{c} [\mathbf{jH}] + (\mathbf{P}\nabla)\mathbf{E} + (\mathbf{M}\nabla)\mathbf{H} + \frac{1}{c}\left[\frac{\partial \mathbf{P}}{\partial t} \mathbf{H}\right] - \frac{1}{c}\left[\frac{\partial \mathbf{M}}{\partial t} \mathbf{E}\right]. \qquad (6)$$

This expression determines the density of the force with which an electromagnetic field acts on a polarizable and magnetizable body.

The reason for the controversy connected with equation (6) is (cf., Refs. 37–38) the expression for the term representing the action of the magnetic field on a current in the form

$$\frac{1}{c}[\mathbf{jH}], \qquad (7)$$

and not in the form

$$\frac{1}{c}[\mathbf{jB}], \qquad (8)$$

as occurs in the Lorentz theory.

Without going into great detail it can be noted that the reason for the disputes noted above consists of the nonuniqueness of breaking up the total force into components corresponding to forces acting on charges and currents and on polarization and magnetization. Utilizing the Maxwell equations it can be shown that the Einstein-Lamb equation (6) can be put in the form

$$\mathbf{f}^{\text{E.-L.}} = \rho \mathbf{E} + [\mathbf{jB}] + (\nabla \mathbf{E})\mathbf{P} + (\nabla \mathbf{H})\mathbf{M} + \frac{1}{c}\frac{\partial}{\partial t}([\mathbf{PB}] - [\mathbf{ME}]). \qquad (9)$$

It can be seen that in this expression the force acting on the current is determined now not in terms of the intensity of the magnetic field H, but in terms of the induction B, and the reason for the different expressions thus consists, as in the preceding case, of the nonuniqueness of the decomposition of the total force.

3. The discussion concerning the correctness of the expressions for the energy-momentum tensors and for the ponderomotive forces corresponding to them in the forms due to Abraham[39] and to Minkowski[40] can serve as yet another example. We shall not examine here the relativistic expressions for the quantities indicated above, since differences remain also in the simple case of stationary substances placed in an electromagnetic field.

From Abraham's theory it follows that in addition to the expression for the force which follows from the Minkowski theory there must also exist an additional force which is equal to

$$\mathbf{f}^{A} = \frac{1}{4\pi c} \frac{\partial}{\partial t} ([\mathbf{DB}] - [\mathbf{EH}]). \qquad (10)$$

Although this force is exceedingly small nevertheless its existence would significantly affect the fundamental principles of the theory of electromagnetism. This circumstance is what gave rise to prolonged discussions lasting over half a century.

As was noted in Ref. 9, this force was finally measured in 1975. However, papers soon appeared (for example, Ref. 41), which cast doubt on the validity of the interpretation of the experiments[7] on the measurement of the Abraham force.

The reason both for past discussions and for the discussions continuing at present is the lack of a possibility of uniquely defining the ponderomotive force and the energy-momentum tensor for the electromagnetic field associated with it by the relation

$$f_\alpha^{\text{pond}} = -\frac{\partial T_{\alpha\beta}}{\partial x_\beta}. \qquad (11)$$

An additional consideration is the fact that both

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2) We note that the authors, in quoting in their paper an expression for the total force density, do not pick out separately the pressure and the ponderomotive force, in the same manner as the elastic force and the electromagnetic force. Such a separation is made by certain specialists, including also those who took part in the controversy of 1966. For mention of it, cf., below.

Minkowski and Abraham considered only the field tensor and did not introduce in their papers the material or the real part of the tensor. But, as has been already noted by Moller,[42] 3) the most general equation which enables one to find different parameters in the expression

$$\frac{\partial(T_{\alpha\beta}^{\text{field}} + T_{\alpha\beta}^{\text{matter}})}{\partial x_\beta} = 0 , \qquad (12)$$

i.e., the vanishing of the four-dimensional divergence of the total (matter plus field) energy-momentum tensor for a closed system. Consequently, the choice of only the field portion of the total energy-momentum tensor (and consequently, also of the ponderomotive force) is an arbitrary operation (with the exception of the case in which not taking the material part into account will not lead to an error).

Summarizing the solutions investigated above and the special features of the problem, it can be concluded that the methodological and theoretical development of the possibilities of an experimental determination both of the total force, and also of its parts remains as before a currently pressing scientific problem.

3) As far as could be ascertained, the equation for describing the system "field plus matter," i.e., $\partial T_{ik}^{\text{tot}}/\partial x_k = 0$, was for the first time introduced in 1927 by Thirring. Prior to that this expression appeared in the 1916 work of Einstein as applied to the gravitational field, and in the 1906 paper of Poincaré devoted to the theory of the electron.

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