D. M. Chernikova and L. P. Gor'kov. *Instability of the charged surface of liquid helium.* The paper discusses a purely classical phenomenon—the instability of a plane liquid helium surface in the presence of electrons localized near it. It is known[1,2] that the possibility of applying a rather substantial charge to the surface of helium results from the fact that electrons have a negative affinity for helium. In other words, an electron must overcome a potential barrier of about 1 eV to enter the condensed phase of helium. Figure 1 shows a typical formulation of the problem. The helium surface in the bath lies between two capacitor plates. Region 1 in the figure is that of the vapor, while region 2 corresponds to the liquid phase of the helium. By applying a potential difference to the plates of the capacitor, it is possible to "charge" the surface from an external source and manipulate both the values of the fields E_1 and E_2 in the gaps and the actual sizes h_1 and h_2 of the gaps. Comparatively small field strengths, $\sim 10^3$ V/cm, can be used to produce surface charge densities n_s as high as 10^8–10^9 cm^{-2}.

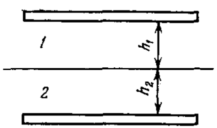

FIG. 1.

When charges are present near the surface, competition arises between gravitational and capillary forces on the one hand, which tend to preserve the flat shape of the boundary, and the stretching action of electrostatic forces on the other, with the ultimate result that the charged surface becomes unstable when appropriate critical parameters are reached. The phenomenon is very similar to the instability of liquid dielectrics placed in a strong electric field,[3] but, in addition to more favorable values of the critical parameters, it is distinguished physically by the fact that the electrons have high mobility along the surface, forming a conductive film along it.

Three problems are of importance in the theory of the phenomenon: determination of the instability criteria, the question as to the manner in which instability develops near the critical values of the parameters and, finally, the question as to the final state of the system in the transcritical region.

1. The stability of the system is determined by its linear vibration spectrum. The constancy of electron

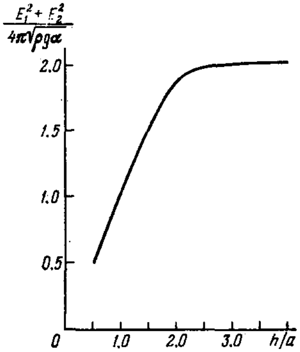

FIG. 2. Plot of $(E_1^2+E_2^2)/4\pi\sqrt{\rho g\alpha}$ against h/a.

potential along the surface, which results from their high mobility, it something new as compared to the hydrodynamic problem of the gravity-capillary wave spectrum. For arbitrary parameters, the spectrum of the vibrations (see Fig. 1) has the form[4,5]

$$\omega^2 = k\,\mathrm{th}\,(kh_2)\left\{\rho g+\alpha k^2-\frac{k}{4\pi}[E_1^2\,\mathrm{Cth}\,(kh_1)+E_2^2\,\mathrm{Cth}\,(kh_2)]\right\}. \qquad (1)$$

The variation of $\omega^2(k)$ due to the electrostatic contribution in the right-hand side of (1) is nonmonotonic in nature. If the thicknesses h are large compared to the capillary constant $a=\sqrt{\alpha/\rho g}\sim 0.5$ mm, the third term in (1) results in the appearance of a deep minimum at $k_0\sim a^{-1}$, which reaches $\omega=0$ as the field increases. This condition will determine the threshold of appearance of shortwave instability, when the turbulent motion that arises as a result of the instability is characterized by the scale $k_0^{-1}\sim a$. Another possibility is shown by (1) to appear at small gaps ($kh\ll 1$). In this case, the coefficient in the expansion of $\omega^2\propto k^2$ in (1) vanishes. The dependence of the critical value of $(E_1^2+E_2^2)$ on h (if $h_1=h_2=h$ in Fig. 1) is shown in Fig. 2, where the region $h>\sqrt{3}a$ corresponds to the development of shortwave instability and $k\leq\sqrt{3}a$ to that of longwave instability.

2. We investigated the manner of instability development in Refs. 5, 6. To formulate the problem, it is necessary to ascertain whether the turbulent motions that arise at small excesses over the critical parameter values are also of small amplitude ("soft" regime) or whether the abrupt onset of the instability leads at once to strongly developed irregular oscillations of the surface with amplitudes of the order of the capillary constant ("hard" regime). Investigation shows that in the case of shortwave instabilities the abrupt onset should as a rule take place according to the "hard" regime whereas for $h\ll a$ the longwave oscillations that develop when the stability threshold is reached are of small amplitude.

3. The question as to the final state of the system assumes that the energy stored by the system is ultimately dissipated by viscosity. The viscosity of helium is very low at low temperatures. Estimates[6] indicate that turbulence can exist for a long time (several tens of seconds). It is difficult to investigate the nature of the state of the surface at rest in the general case, but a study of certain limiting cases reported in Ref. 6 indicates that for shortwave instability the surface at rest may be expected to assume the form of a periodic structure with hexagonal symmetry of the unit cell. The amplitude and the period of the structure are both of the order of the capillary constant, $a\approx 0.5$ mm.

It was observed in the experiments of Ref. 7 that as a result of gradual damping of the oscillations the surface of the helium is discharged rapidly instead of forming a periodic structure. Although this problem requires special study, it can be stated even now that the experiments of Ref. 7 were conducted at temperatures below the λ point of helium, where nondissipative motion of the superfluid component of the liquid is of special importance.

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