V. G. Sidorovich. *The mode theory of the three-dimensional hologram.* At this time, the theory of the three-dimensional hologram has been developed satisfactorily only in the kinematic approximation, which assumes that the wave incident on the hologram remains unchanged in the latter's volume.[1] This paper discusses a method for analysis of the highly efficient conversion of light waves by a three-dimensional hologram based on representation of the wave to be converted as a superposition of waves matched to this hologram (modes of the hologram); see Refs. 2–4. The modes are characterized by the property that polarization waves induced by them in the hologram reproduce them exactly by radiating electromagnetic waves. Therefore the complex-amplitude ratios of the plane mode components are constant throughout the volume of the hologram. The existence of electromagnetic waves that are consistent in the above sense with a medium having a permittivity that is periodic in space was first used by P. P. Ewald in his work on the dynamic theory of x-ray diffraction.[5,6]

We introduce the following nomenclature: $E(r)=\sum_{n=0}^{N} a_n e^{i\mathbf{k}_n r}$ is the electric-field amplitude of a light wave whose energy density distribution has been registered in the volume of the hologram in the form of local deviations of the complex dielectric permittivity from the average value; a_n and $\mathbf{k}_n$ are the amplitudes and wave vectors of the plane components of the registered wave; $\mathbf{r}=(x, y, z)$, and $E'(x, y, 0)=\sum_{n=0}^{N} b_n e^{i\mathbf{k}_n^{\perp}\mathbf{r}^{\perp}}$ is the electric field amplitude distribution of the transformed light wave on the surface of the hologram (the sign $\perp$ indicates the projection onto the $X0Y$ plane, in which the hologram's surface lies). Matching of the transverse wave-vector components at the interface between the two media enables us to represent the field of the transformed wave in the volume of the hologram in the form

$$E(x, y, z>0)=\sum_{n=0}^{N} c_n(z) e^{i\mathbf{k}_n\mathbf{r}}. \qquad (1)$$

In the approximation of slowly varying amplitudes, substitution of (1) into the Helmholtz equation gives for paraxial beams

$$\frac{dc}{dz}=D(\hat{A}+\hat{B})c, \qquad (2)$$

where z is the longitudinal coordinate; $D=k_0\sqrt{\varepsilon_0}\varkappa/2$; k_0 is the wave number of the wave incident on the hologram; ε_0 is the dielectric permittivity of the unexposed recording medium, $\varkappa$ is the complex proportionality coefficient between the energy density of the recorded wave and the local change in the dielectric permittivity of the light-sensitive medium; $c=[c_0(z), c_1(z), \ldots, c_N(z)]$ is the amplitude vector of the plane components of the light wave propagating in the hologram; $\hat{A}$ is the matrix of the hologram, with elements $A_{mn}=a_m a_n^*(1-\delta_{mn})+\delta_{mn}L$; δ_{mn} is the Kronecker delta; $L=\sum_{n=0}^{N}|a_n|^2$; $\hat{B}$ is the matrix for scattering of plane waves propagating in the hologram by volume gratings in the recording of which they had not participated; $B_{mn}=\sum_{pq} a_p a_q^* e^{iA_{pqn}^{(m)}z}(1-\delta_{mn}$ the summation extends over p and q for which $[\mathbf{k}_n+(\mathbf{k}_p-\mathbf{k}_q)]^{\perp}=[\mathbf{k}_m]^{\perp}$, $A_{pqn}^{(m)}=[(\mathbf{k}_n-\mathbf{k}_m)-(\mathbf{k}_q-\mathbf{k}_p)]_z$, and $A_{pqn}^{(m)}$ different from zero indicates that the Bragg condition is not satisfied for scattering of a wave with wave vector $\mathbf{k}_n$ by the grating formed by waves with wave vectors $\mathbf{k}_p$ and $\mathbf{k}_q$. If the recorded radiation can be characterized by the angular divergence parameter θ, then $\langle A_{pqn}^{(m)}\rangle \sim k_0\theta^2$, where $\langle\ \rangle$ indicates averaging over p, q, m, and n. We shall denote by l the thickness of the hologram layer with significant diffraction efficiency. For $k_0\theta^2 l \gg 1$, most of the exponentials $e^{iA_{pqn}^{(m)}z}$ in $\hat{B}$ execute many oscillations while the wave incident on the hologram undergoes significant transformation. In this case, the contribution to the transformation from scattering by "foreign" gratings, which is described by the matrix $\hat{B}$, can be neglected. Since $l \sim 1/k_0\Delta\varepsilon$, where $\Delta\varepsilon=\varkappa\varepsilon_0 L$, the condition under which waves that do not carry useful information are of negligibly low intensity, which corresponds to scattering by "foreign" gratings, has the form $\theta^2 \gg \delta\varepsilon$. If this condition is satisfied, the second term in the right-hand side of (2) can be dropped. Since the matrix $\hat{A}$ is self-adjoint, its eigenvector system is complete in $N+1$-dimensional space. Therefore the solution of Eq. (2)

can be represented in the form

$$E'(\mathbf{r})|_{z>0} = \sum_{n=0}^{N} \left(\sum_{m=0}^{N} d_m c_n^{(m)} e^{D\alpha_m z} \right) e^{i\mathbf{k}_n\mathbf{r}}, \tag{3}$$

where the d_m are the coefficients in the expansion of the vector $c(0)=(b_0, b_1, \ldots, b_N)$ in the system of eigenvectors $\mathbf{c}^{(m)}$ of the matrix A and the α_m are the corresponding eigenvalues. Formula (3) indicates that the hologram transformation of any light wave with an angular spectrum localized within the limits of the angular spectrum of the recorded wave reduces to a change in the amplitude-phase relationships between its component light waves with amplitude vectors $\mathbf{c}^{(m)}$. The latter propagate in the hologram as in free space, without any change in structure, and are therefore naturally called modes of the hologram. The difference between the complex phase velocities of the modes, which are characterized by the values of α_m, results from the fact that the energy densities of different modes are distributed differently in space with respect to the distributed differently in space with respect to the distribution of the hologram's complex permittivity.

The effectiveness of this mode approach in study of the transformation of light waves by three-dimensional holograms can be attributed to the simplicity with which the eigenvalue problem is solved for the hologram matrix $\hat{A}$ in a number of practically important cases. The mode theory has now made it possible to calculate the diffraction efficiency of a three-dimensional phase hologram in the very general case in which the object and reference waves have extended discrete angular spectra and arbitrary intensity ratios.[2,3] These calculations have been used to formulate recommendations for choosing the parameters of the radiation used to record the hologram and of the recording medium that are optimal from the standpoint of diffraction efficiency and the level of reconstructed-wave distortion. Another important application of the mode theory has been quantitative interpretation of the basic properties of the so-called "Brillouin mirror," which have been studied experimentally.[7] Since in stationary-regime induced Mandel'shtam-Brillouin scattering the local coefficient of stimulated amplification of the Stokes wave is proportional to the pump energy density (see, for example, Ref. 7), a nonlinear medium situated in the field of a spatially inhomogeneous pump is an amplitude-amplifying hologram with respect to Stokes waves with frequencies within the stimulated-amplification range.[8] This is why development of the methods in Refs. 2, 3 made it possible in Refs. 4, 9 to offer an exhaustive explanation of the experimental facts established in Ref. 7 and to derive formulas for estimating experimental parameters at which wave front inversion of the radiation reflected from a "Brillouin mirror" should be observed.

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