Relativistic experiments in gravitational fields

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Experimental investigations in gravitation and relativity theory are reviewed. The state of experiments that test the equivalence principle, the basis of the relativistic theory of gravitation, is discussed. The latest results of measurements of the classical effects of general relativity in the solar system are given and promising programs of similar experiments in the near future are analyzed. The new possibilities for testing gravitational theories provided by the discovery of the binary pulsar are described. Finally, the problem of searching for bursts of gravitational radiation using terrestrial Weber type antennas is discussed.

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INTRODUCTION

In this review, we describe the latest achievements in the field of experimental investigation of relativistic gravitational effects and also some promising programs that may be carried out in the near future. The number of laboratories devoting attention to this problem has increased appreciably. To a considerable extent, this has come about because of the remarkable astrophysical discoveries of recent years, since these have demonstrated the increasing importance of relativity theory in large-scale phenomena. However, the principal reason for the increase is the very rapid development of radar techniques in space, the increase in the number of satellites for scientific research, the development of more versatile atomic frequency standards, and so forth. This rapid extension of the technological basis has made possible experiments that would have seemed an impossible dream 10 or 15 years ago. The technological development has been matched by a development of the theory of gravitation and relativistic astrophysics; in particular, there has been created the so-called parametrized post-Newtonian formalism (PPN), which makes it possible to analyze conveniently the results of experiments in the framework of different gravitational theories.

There are two characteristic directions for the contemporary gravitational experiments in the nonwave zone. The first, which is associated with the names of Eddington and Schiff, is directed toward the measurement of the traditional and new relativistic effects in order to calculate and make more precise the coefficients of the post-Newtonian expansion of the metric in the first, second, and successively higher orders in the weak-field parameter. This experimental determination of the geometry of space with increasing accuracy must make it possible to choose between competing variants of metric theories of gravitation. The second direction, which is sometimes known as the Dicke program, consists of devising and carrying out experiments that, *a priori*, are not associated with a definite theory but rather verify the basic postulates on which our notions about gravitation are based. Such experiments include, for example, verification of the equivalence principle, attempts to detect time variations of the gravitational constant, or local anisotropy of space, etc. The greater part of this review is devoted to a survey of the experiments in these directions. A considerable number of experiments has now been performed, but not even the smallest deviations from the predictions of general relativity in favor of the conclusions of any other theory have been found. Despite this, there still remains an

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open choice, at least until precise measurements can be made in strong gravitational fields. First steps in this direction have already been made through the observations of the binary pulsar.

Among gravitational experiments, particular importance attaches to the search for gravitational waves, the detection of which would open up a new channel of astrophysical information about the Universe. However, the restricted space available in this paper forces us to restrict attention here to just the most topical problem, namely, the construction of second-generation gravitational antennas to replace the antennas operating in the Weber series of measurements. The reader can find a discussion of other questions in, for example, the recent review of Grishchuk.[81]

1. EXPERIMENTAL VERIFICATION OF THE EQUIVALENCE PRINCIPLE

Equality of the inertial m_i and gravitational m_g mass of bodies (or rather, a constant value of the ratio m_g/m_i for different bodies) is the experimentally verified essence of the equivalence principle. In the exposition of the theoretical basis of general relativity, the preferred formulation of the equivalence principle consists of the postulation that there is local equivalence of a gravitational field to an accelerated frame of reference. This assertion contains the condition of equivalence of the masses, so that the equation $m_i = m_g$ is an empirical basis of the equivalence principle (see the discussion in Ref. 82a).

Einstein accorded the equivalence principle very great importance as the foundation of general relativity: "In my mind, my theory rests exclusively on this principle".[1] In a letter to Bergmann, he emphasized that it is more important to test once more experimentally the equivalence principle rather than the well-known consequences of general relativity—the advance of Mercury's perihelion or the deflection of a light ray in the field of the Sun.

The situation as regards the experimental verification of the equivalence principle as it stood in 1970 has been described, for example, in the review of Ref. 2. At that time, the greatest achievements were the experiments of Dicke *et al.*,[3] in which the equivalence principle was confirmed with an accuracy $\sim 10^{-11}$. In recent years, further successful steps in this direction have been made.

a) The Eötvös-Dicke experiments and weak interactions

In the first place, we must mention the important laboratory experiment made by Braginskiĭ and Panov at the Moscow State University.[4] This used Dicke's method to measure the relative acceleration of platinum and aluminum masses in the gravitational field $g_{\odot} = 0.62\ \mathrm{cm/sec}^2$ of the Sun.

The masses were fixed on the yoke of a torsion pendulum, and measurements were made of the amplitude of oscillations of the pendulum with 24-h period; this amplitude should be nonzero if the two masses have different accelerations toward the Sun. A detailed description of the experiment is contained in the book of Ref. 5. Here we only wish to draw attention to the difference between it and Dicke's experiment.

The increase in sensitivity was achieved by using a vibrational system (the torsional pendulum) with low dissipation, which is equivalent to reducing the intensity of the fluctuation force. The relaxation time was the very long time $\tau_\mu^* \gtrsim 6\times 10^7$ sec, i.e., more than two years, and appreciably exceeded the time of the measurements: $\hat{\tau} \sim 6\times 10^5$ sec ~ 7 days (30 complete periods of the pendulum). With such parameters, the gain in the sensitivity for the measured variations of the vibration amplitude is $\sqrt{\tau_\mu^*/\hat{\tau}} \sim 10$ compared with Dicke's apparatus.[3] The absolute value of the minimal detected difference in the accelerations of the test masses toward the Sun was not less than $1\times 10^{-13}\ \mathrm{cm/sec}^2$. The measured amplitude of the angular vibrations at the 24-h harmonic was $(0.55 \pm 1.65)\times 10^{-7}$ rad, whereas a violation of the equality of the ratios m_g/m_i for platinum and aluminum by one unit in the twelfth decimal place would lead to vibrations with amplitude 1.8×10^{-7} rad. This result shows that the "mass ratios" for platinum and aluminum are equal to at least 1×10^{-12}.

A judgement on the physical significance of this result entails establishing how completely the equivalence of the inertial and gravitational properties of the total rest energy of the investigated bodies has been proved. The analysis in the well-known papers of Dicke[6] and Schiff[7] shows that verification of the equivalence principle for the binding energy of the nucleons of the nuclei requires an accuracy of the experiment at the level $\sim 10^{-2} - 10^{-3}$. The contribution of the electromagnetic forces can be tested in experiments whose accuracy is $\sim 10^{-3} - 10^{-6}$. Thus, both Dicke's results[3] (and also the earlier work of Eötvös[8]) and the experiments at Moscow confirm the validity of the equivalence principle for the energy of the strong and electromagnetic interactions. As regards the weak interaction, an uncertainty in the theoretical estimates persisted until very recently. For example, it was asserted in the literature[5,9,10] that to verify the equivalence principle at the level of the weak interactions one needs an experimental accuracy $\sim 10^{-12} - 10^{-14}$; this corresponds approximately to the calculation in the second order of the corresponding dimensionless coupling constant.

Recently, greater clarity in this question was achieved. In Ref. 11, it was pointed out that the second order of the weak coupling constant does not determine the magnitude of the weak shift of the nucleon levels of the nucleus. This shift must be of order 10^{-7} of the value of the energy of the levels due to the strong interactions.[12] Hence, the estimate of Ref. 11 for the weak mass defect is 10^{-9} of the total mass of the nucleus, and this means that the accuracy of the experiments in Refs. 3 and 4 suffices to verify the equivalence principle at the level of the weak interactions.

A deeper analysis of this question with calculations made possible at the present level of development of the theory of weak interactions was made in Ref. 13. The earlier estimates[6,7,9] considered only the parity-nonconserving part of the correction to the effective

Hamiltonian due to the weak interactions $\mathscr{H}_{nc}$, which does not contribute to the energy of the nucleus in the ground state in the first order in the weak coupling constant G_W. In fact, the correction to the effective Hamiltonian due to the weak interactions can be expressed as a sum of two parts, one that conserves parity and one that does not: $\mathscr{H}_W=\mathscr{H}_c+\mathscr{H}_{nc}$. The structure of the part $\mathscr{H}_c$ is the same as that of the Hamiltonian of the strong interactions. In calculations, they are usually combined with a small renormalization of the coupling constant. An apparent individuality is preserved only for the part $\mathscr{H}_{nc}$, which was subjected to estimates in the analysis of the equivalence principle. (Note that such an approach presupposes that the part $\mathscr{H}_c$ automatically satisfies the equivalence principle, as also does the energy of the strong interactions. However, identity of methematical structure in the description of phenomena does not, in the general case, guarantee identity of the physical nature.) It is natural to test the contribution to the gravitational mass of the nucleus of the part $\mathscr{H}_c$ of the Hamiltonian as well. This radically changes the estimate of the fraction of the weak-interaction energy since the terms of the part $\mathscr{H}_c$ are proportional to G_W and are appreciably larger than the terms of $\mathscr{H}_{nc}$. Approximately, one can expect that the weak-interaction energy E_W per nucleon is proportional to the product of the number Z of protons and the number N of neutrons in the nucleus and inversely proportional to the volume V of the nucleus, i.e., $E_W \sim G_W NZ/V$. The rigorous calculation[13] leads to the following expression for the ratio of E_W to the nucleon mass m_i:

$$\Delta=\frac{E_W}{m_i}=\frac{1}{2\sqrt{2}}G_W\frac{NZ}{Vm_i}f(N,Z)[1+g(N,Z)]\approx 2{,}2\cdot 10^{-8}\frac{NZ}{A^2}f[1+g]; \quad (1.1)$$

here, $A=Z+N$, and, in addition, we have substituted the numerical values $G_W=0.896\times 10^{-43}$ MeV·cm^3, $V=5.13\times 10^{-39}\,A\cdot$cm^3, $m_i=0.931\times 10^3\,A$ MeV. The function $f(N,Z)$ depends on the chosen model and the state of the nucleus. For the free Fermi gas model, $f=1$ in the ground state. For other realistic models, this function varies only by not more than 1%. The second function $g(N,Z)$ depends on the form of the theory of neutral currents in the weak interactions. If neutral currents are ignored, then $g(N,Z)=0$. (Analysis of the variations of $1+g(N,Z)$ for different known forms of the theory of weak currents enabled Haugan and Will[13] to assert that changes of not more than a factor ~2 occur.) Then, forming the difference of the quantities Δ for aluminum ($Z=13, N=14$) and platinum ($Z=78, N=117$), we readily find from Eq. (1)

$$\left[\left(\frac{E_W}{m_i}\right)_{\rm Al}-\left(\frac{E_W}{m_i}\right)_{\rm Pt}\right]\approx 2\cdot 10^{-10}, \quad (1.2)$$

which agrees with the estimate in Ref. 11.

Comparing Eq. (2) with the experiment made at Moscow,[4] we see that the result of the experiment confirms the equivalence principle for weak-interaction energy to 0.5% (and that Dicke's experiment[3] confirms it to 5%). The recognition of this circumstance is very important, but it is still worthwhile repeating the experiments with still greater accuracy in order to be able to include the remaining part $\mathscr{H}_{nc}$ of the weak-interaction energy.

b) Nordtvedt effect

We now consider the contribution of gravitational energy to the mass of a body. It is virtually impossible to verify the equivalence of m_i and m_g at the level of the energy of gravitational interaction with laboratory bodies. The ratio of the internal gravitational energy to the total energy of a body of radius a is very small, namely,

$$\Delta\approx\frac{Gm^2a^{-1}}{mc^2}=\frac{Gm}{c^2a}\leqslant 10^{-25}\quad (1\ {\rm cm}/a)$$

for bodies of laboratory sizes. The situation changes when one considers cosmic objects. Indeed, the relative contributions of the energies of the nuclear, electromagnetic, weak, and gravitational interactions can be arranged approximately in the series $1:10^{-2}:10^{-12}:10^{-40}$ per atom of matter.[10] Because of the relatively short-range nature of the nuclear and weak forces, as well as the electrical neutrality of atoms, the ratios of the first three energies remain the same for massive bodies. But the situation is different for the internal gravitational energy. Its fraction increases with increasing mass because of the addition of the attractions between different material elements, and its contribution can move up the sequence of the above ratios.

Such a situation arises, for example, for the planets of the solar system. The idea of trajectory observations of the planets (in particular, Jupiter, for which $\Delta\sim 10^{-8}$) to test the contribution made by the internal gravitational energy to the mass m is due to Dicke.[14] However, the recent realization of this idea is associated with the Nordtvedt effect. Because of the important part played by this effect in the verification of the equivalence principle, we shall consider it in more detail.

In 1968, Nordtvedt pointed out that violation of the equivalence principle for the Earth's gravitational energy would lead to anomalous oscillations of the lunar orbit that could, in principle, be measured by laser ranging.[15] The Nordtvedt effect can be understood and its order of magnitude estimated on the basis of the following very simple analysis.

Figure 1 shows schematically the Sun–Earth–Moon system. The Earth's internal gravitational energy is the fraction $\Delta_E=0.8\pi G\rho R_\delta^2/c^2\approx 4.6\times 10^{-10}$ of the total energy; for the Moon, $\Delta_M\approx 2\times 10^{-11}$, and in an approximate calculation this contribution can be ignored. If the gravitational energy of the Earth is not subject to gravitation, the terrestrial (heliocentric) orbit for the Moon ("tied to the Earth") will no longer be the position where its attraction to the Sun is equal to the centrip-

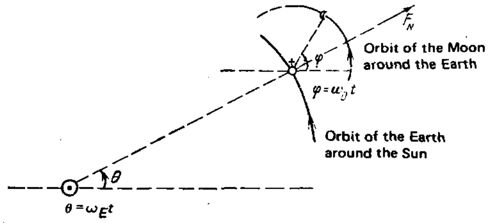

FIG. 1. Calculation of the Nordtvedt effect.

etal force. Therefore, the Moon will not be in a state of free fall toward the Sun. In a coordinate system tied to the center of the Earth, it will be subject to the force $\overline{F}_M \approx \eta\Delta_E g_{\odot} m_M$ directed along the Sun–Earth line outward ($0<\eta<1$ is the Nordtvedt parameter). The equations of motion of the Moon in the terrestrial coordinate system will have the form

$$\ddot{r} = r\dot{\varphi}^2 - \frac{GM_{\delta}}{r^2} + \eta\Delta_E g_{\odot}\cos(\varphi-\theta),$$
$$\dot{K} = -\eta\Delta_E g_{\odot}\sin(\varphi-\theta). \tag{1.3}$$

The first of Eqs. (1.3) describes the change in the radius of the Moon's orbit with respect to the Earth, and the second gives the change in the angular momentum K of the motion. In the zeroth order approximation, we consider a circular orbit, $r=r_0, K_0=r_0^2\omega_0, \omega_0^2=g_{\delta}/r_0$, and for the first-order corrections we have

$$\delta K = \frac{\eta\Delta_E g_{\odot} r_0}{\omega_0-\omega_E}\cos[(\omega_0-\omega_E)t],$$
$$\delta\ddot{r} = -\omega_0^2\delta r + 2K_0 r_0^{-3}\delta K + \eta\Delta_E g_{\odot}\cos[(\omega_0-\omega_E)t]. \tag{1.4}$$

From this, we readily obtain an oscillator equation for the variations of the radius:

$$\delta\ddot{r} + \omega_0^2 r = \left(1+\frac{2\omega_0}{\omega_0-\omega_E}\right)\eta\Delta_E g_{\odot}\cos[(\omega_0-\omega_E)t],$$

the solution being

$$\delta r \approx \frac{3\eta\Delta_E g_{\odot}}{\omega_0^2-(\omega_0-\omega_E)^2}\cos[(\omega_0-\omega_E)t]. \tag{1.5}$$

Equation (1.5) shows that the radius of the orbit oscillates with approximately the frequency of the Moon's period around the Earth (since $\omega_E \approx \omega_0/13$ and $\omega_0-\omega_E \approx \omega_0$). Effectively, this results in a transformation of the circular orbit into an elliptic orbit in a constant external force field. The situation here is analogous to the one that obtains in the classical interpretation of the Stark effect when interpreted as due to deformation of the electron orbit when an electric field is applied. In the case of the Stark effect, a classical calculation without allowance for damping would be impossible since it would lead to infinite deformations because of the coincidence between the frequency of the external force and the orbital frequency. For the Nordtvedt effect, however, our calculation is valid because of the small correction to the frequency associated with the orbital motion of the Earth. Making the replacement $\omega_0^2-(\omega_0-\omega_E)^2 \approx 2\omega_0\omega_E$, we obtain the following estimate for the amplitude of the oscillations:

$$|\delta r| \approx \frac{3}{2}\eta\frac{\Delta_E g_{\odot}}{\omega_0\omega_E} = \eta\frac{3}{2}\Delta_E R_{ES}\frac{\omega_E}{\omega_0} \sim 10^3\eta \text{ (cm)}. \tag{1.6}$$

Here, $R_{ES}=1$ a.u. is the radius of the Earth's orbit. If the equivalence principle did not extend to gravitational energy, i.e., $\eta=1$, the effect (1.6) would be ~10 m, a huge value from the point of view of the capabilities of the measuring technique. Indeed, since the accuracy of laser ranging to the Moon using the corner reflectors left on the Moon by the Soviet and American crafts is a few centimeters, one can readily understand the optimism as regards the outcome of such an experiment. In practice, however, the measurements are very laborious because of various factors leading to similar variations of the Moon's orbit.

The largest contribution to variations in the Earth–Moon distance with synodic period arises from the multipole component $P_E(\cos\theta)$ of the Sun's potential. This component induces an amplitude ~110 km. Fortunately, modern computational techniques and the accuracy with which the necessary parameters (such as the semimajor axes of the orbits, the mass ratios, etc.) are known are sufficient for calculation of the correction of this effect and other trajectory anomalies.

The raw experimental data are the time intervals corresponding to the travel time of a laser pulse sent from the Earth, reflected by a corner reflector, and returned to the Earth.

The essence of the experiment is to find the difference between the calculated trajectory of the Moon, without allowance for the Nordtvedt effect, and the true trajectory as determined by the laser ranging data. It is possible to calculate the trajectory to within a few centimeters. It is however necessary to take into account the nonsphericity of the fields of the Sun and the Earth, the influence of the planets, tidal effects, and general relativistic effects. When the true trajectory is determined from the travel times, it is necessary to take into account the corrections for atmospheric refraction, interplanetary plasma, relativistic delay of the electromagnetic pulse in the gravitational field, the same tidal deformations of the Earth and the Moon, their rotation, and so forth. Even this straightforward but by no means complete list is sufficient to demonstrate the complexity of the experiment. It was realized simultaneously by two highly qualified research groups in the United States. The first consisted of scientists from the Jet Propulsion Laboratory (JPL, Pasadena) and several universities[16]; the second group consisted of scientists from the Massachusets Institute of Technology (MIT, Boston) and the Air Force Cambridge Research Laboratories, Bedford, Mass.[17] The analyzed data were those obtained in the period 1970-1976 by laser ranging to the Moon with the corner reflector set up during the Apollo 11 mission obtained by the MacDonald Observatory (Texas) in the framework of the "planetary ephemeris program". There were altogether about 1500 ranging sessions. Frequently, duplicated computer calculations were made, so that the resulting error in the determination of the Earth-Moon distance did not exceed ~30 cm. No Nordtvedt effect was found: the estimates of the JPL group give the value $\eta=0.00\pm0.03$ for the Nordtvedt parameter; those of the MIT group, $\eta=-0.001\pm0.015$. It can therefore be assumed that the gravitational self-energy of the Earth makes the same contribution to within 1.5–3% to the inertial mass as to the gravitational mass. In other words, the ratios m_i/m_g for the Earth and the Moon are the same to ~10^{-11}. (Note that although this suffices for the extension of the equivalence principle to the energy of gravitational interaction the laboratory experiment at the Moscow State University[4] remains an order of magnitude more accurate in the absolute sense.)

The experiments we have described determine the present level of confidence in the correctness of the strong equivalence principle (for the distinction between the "strong" and "weak" equivalence principle, see, for example, Refs. 2, 7, 10). Significant steps have been made which permit us to verify very thoroughly a fundamental property of matter—the equivalence of gravitation and inertia. However, the significance of the problem is such that any new modification of the measurements with increased accuracy is justified. First, it is desirable to have a more certain verification of the contribution of the weak interactions to the gravitational mass (encompassing the parity-nonconserving terms of the Hamiltonian). Second, we need new and simpler, and therefore more reliable, tests of the equivalence principle for gravitational energy. Third, a more accurate value for the equality of the inertial and gravitational masses would make it possible to eliminate a number of scalar-tensor variants of the theory.

c) Orbital variant of the Eötvös-Dicke experiment

At the present time, there are several plans for both terrestrial and satellite experiments to test the equivalence principle with greater accuracy (some can be found in the book of Ref. 5). Below, we briefly describe a variant that was proposed by Everitt[18] and is the one closest to implementation.

This involves a satellite which preserves its orientation with respect to distant stars as it revolves in its orbit (Fig. 2). On board, there are two coaxial cylinders whose axis is directed toward the center of the Earth. One is made of a light material (aluminum) and the other of a heavy material (platinum, gold). Both are in a state of free fall in the Earth's field, but for them this field is of variable sign, the period being that of the orbital revolution, $\sim 5\times10^4$ sec. In the frame of reference associated with the satellite, the situation is analogous to the conditions of the Dicke experiment[3] on the Earth—test bodies are in a periodic gravitational field. The logic is the same as in Refs. 3 and 4. If the ratios of the masses m_i/m_g of the test bodies are not the same, there will arise periodic relative displacements which can be amplified resonantly by adding an elastic coupling element between them and arranging for the frequency of the normal vibrations to be equal to the revolution frequency of the satellite. In the project of Ref. 18, it is proposed to cover the cylinders with niobium, go over to the superconducting state, and then use the Meissner effect to establish a position of stable equilibrium, and also a quantum magnetometer to measure small vibrations. What increase in accuracy can be expected? The amplitude of the forced vibrations during the observation time $\hat{\tau}$ reaches the value $\Delta x \sim g_\delta[(m_g/m_i)_1-(m_g/m_i)_2]\hat{\tau}\omega^{-1}=g_\delta\Delta\hat{\tau}\omega^{-1}$ (the damping is assumed small, $\tau_\mu^*\gg\hat{\tau}$). On the average, the fluctuation drift has the amplitude $\sqrt{\Delta x_{\mathrm{fl}}^2}\sim(kT/m\omega^2)\sqrt{\tau/\tau_\mu^*}$. Hence, for the signal-to-noise ratio, for ideal detecting apparatus, we obtain

$$\frac{\Delta x}{\sqrt{\overline{\Delta x_{\mathrm{fl}}^2}}}\sim g_\delta\Delta\sqrt{\frac{\tau_\mu^*}{\hat{\tau}}}\sqrt{\frac{m}{kT}}. \tag{1.7}$$

Comparison of this expression with the accuracy of the Moscow experiment[4] shows that a gain in sensitivity is achieved for two reasons: 1) the main, that the Earth's field is 3.5 orders of magnitude stronger than the Sun's field (in the region of the Earth's orbit); 2) a further two orders of magnitude can be gained through the factor $\sqrt{m/kT}$. The upshot is that the accuracy of such a determination of the ratio m_g/m_i may be not worse than $\sim10^{-16}$, and this must bring into consideration the weak-interaction energy without any doubt.

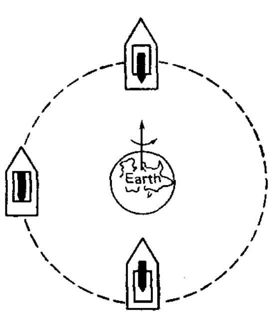

FIG. 2. Arrangement of orbital experiment to test the equivalence principle.

d) Gravitational red shift

Reflecting on the methodological basis of the Eötvös-Dicke experiment, we note that, essentially, this is a measurement of the difference in accelerations of free fall, $\Delta\bar{g}$, simultaneously for bodies with relatively simple and complicated atomic structure, respectively. For this reason, we can say that a) it is the most accurate, like every differential measurement of a quantity compared with measurement of its absolute value; b) it covers different forms of matter and forms of interaction, which makes it possible to regard it as a test of the strong equivalence principle (for more detail, see Ref. 2). However, it is also worthwhile to make a direct measurement of $\bar{g}$ for "pure" forms of matter such as elementary particles and photons. An analysis of such experiments can be found in the early review Ref. 2, but many of the described results still remain at the present level of accuracy. In particular, for neutrons the acceleration of free fall is equal to the acceleration (strength) of the Earth's gravitational field to a few percent; the analogous measurements with electrons have an accuracy of 10%. A long planned experiment for positrons has not yet been realized. As a certain "compensation" for this delay, we do have new experiments with photons, which we shall discuss in more detail. The measurement of $\bar{g}$ for photons reduces to detecting a change in the frequency of monochromatic electromagnetic radiation in the Earth's field—the so-called gravitational red shift of spectral lines. In this field, laboratory experiments have been made possible by the Mössbauer effect and were carried out with great success by Pound and his collaborators at the beginning of the seventies. The accuracy of the agreement with theory achieved was 1%.[19] Note that the effect has not always been interpreted theoretically in the same way in the literature. Some papers treat the gravitational red shift as a consequence of general relativity (we recall that Einstein himself proposed this experiment as one of the three possible tests of general relativity[20]); in other more recent studies, the effect is considered as one of the experimental bases of general relativity (for more detail, see Ref. 82a).

Since one can formally calculate the gravitational red shift of a photon even in the framework of special relativity, it does appear more natural to regard it as preceding general relativity. It is contained in general relativity as one of the forms of expression of the weak equivalence principle. This is the point of view adopted in the text books of Landau and Lifshitz[21] and Weinberg.[22] In the monograph of Misner, Thorne, and Wheeler[23] the red shift serves as experimental evidence in favor of the weak equivalence principle in a very important formulation for general relativity: "The paths of test bodies are geodesics of spacetime".

The latest successes in the measurement of this effect were achieved recently in two experiments in the neighborhood of the Earth involving the lifting of frequency standards to relatively great altitudes by means of aeroplanes and rockets. The first was carried out by a research group of the Maryland University and others.[26] The differences between the time readings of atomic clocks on an aeroplane and in a terrestrial observatory were measured. The flight time was ~15 h, the flight altitude ~10 km (this gives $\Delta\nu/\nu \sim 10^{-12}$ against the $\Delta\nu/\nu \sim 10^{-15}$ in Pound's experiments; see Ref. 27). The aeroplane flew within a given square at constant altitude and the lowest velocity possible in order to reduce the special relativistic time dilatation effect. The expected gravitational effect in accordance with general relativity was ~+50 nsec, against the special relativistic effect ~–7 nsec.

Several cesium and rubidium standards were flown; in the terrestrial laboratory, hydrogen standards were also used. The experimental clocks were compared with the reference clocks by a special system developed for programs of lunar laser ranging that had an accuracy ±0.1 nsec and this was done on board and on the ground. The flown clocks were compared with the terrestrial clocks before and after the experiment and, very importantly, during the flight.

This last made it possible to follow the dynamic development of the effect, i.e., the accumulated lagging of the terrestrial clocks behind the flown clocks. This was achieved in a relatively simple manner. A reference laser on the Earth sent short ~0.1 nsec pulses to the plane, which were there recorded and reflected and reflected back by a corner reflector. Let t_1 and t_2 be the times of transmitting and receiving of the reflected pulse according to the terrestrial clocks, and let t_2' correspond to the time of reception of the probe pulse by the aircraft. Then the required effect is determined by the difference $t_2' - (1/2)(t_1 + t_2)$ as a function of the time of flight. The results of the measurements are clearly illustrated by the graph in Fig. 3. With an error of 1.6%, the observations correspond to the theoretical predictions. This does not better the Pound–Snider experiment, but on the other hand the dynamics of the time-lag process was observed for the first time.

The accuracy was significantly increased in the second of the experiments. A group of physicists at the Smithsonian Institute led by R. Vessot had long planned a measurement of the gravitational red shift by atomic clocks on a satellite[28] and on a ballistic rocket.[29] In the summer of 1976, such measurements were made. A frequency standard with relative stability 2×10^{-10} (in $\tau\sim10^3$) sec was flown on a rocket. At the upper part of of the trajectory, at altitude ~160 km, where the velocity was low, the frequencies were compared. Receivers were operated at four terrestrial stations. This made it possible to reduce significantly the error associated with the Doppler effect, because the position of the rocket was measured to within a meter and its velocity to within 6 cm/sec. A preliminary evaluation of the results gave an error which did not exceed 0.04% of the theoretical effect.[30]

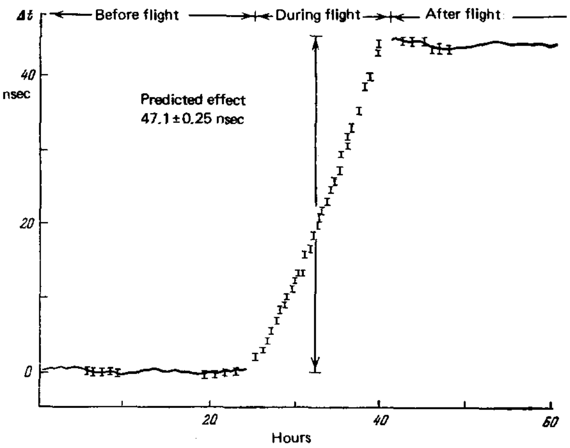

FIG. 3. Experimental measurement of the gravitational time dilatation of terrestrial atomic clocks relative to clocks on the aircraft.

Although the accuracy of the latest experiment is almost two orders of magnitude better than the results of the preceding experiments, there are grounds for a further improvement in the quality of such measurements. These relate to the problem of estimating the post-Newtonian metric coefficients and will be considered later in this review. Here we shall merely emphasize once more the importance of measurements of the gravitational red shift by quoting from Weinberg's monograph[22]: "Hence even if we suppose that the Eötvös–Dicke experiments could improve to an unlimited accuracy, and that gravitational mass were found to equal inertial mass exactly, still there would be some point in verifying the gravitational red shift of spectral lines, as an independent test of the Principle of Equivalence."

2. RELATIVISTIC EFFECTS IN WEAK FIELDS

"For the first half-century of its life, general relativity was a theorist's paradise, but an experimentalist's hell. No theory was thought more beautiful, and none was more difficult to test." (Ref. 23, Ch. 38). The reason for this is well known: at present we have access to only the space around the Sun, and this is a region of "weak" gravitational field. The measure of "weakness" of the field, the quantity φ/c^2, reaches only $GM_\odot/R_\odot c^2 \sim r_g/R_\odot \sim 10^{-6}$ even on the surface of the Sun. The specific effects that distinguish the relativistic gravitational theory from the Newtonian theory are so small that their observation required high experimental ingenuity and considerable material expenditure.

Nevertheless, experiments to observe and investigate the relativistic corrections in the gravitational field of the Sun were initiated already in 1919 (immediately after the creation of general relativity) and have continued to the present day with ever increasing accuracy. For the first experiments it was sufficient to detect the effects predicted by Einstein, but the task of modern investigations has become much more complicated. It is necessary to push the accuracy of the measurements to such an extent as to make it possible to distinguish general relativity from other competing theories of gravitation proposed during the last 10 or 15 years.[22,23] Today, we can regard this problem as largely settled. It should however, be added that, despite the tremendous progress of technology, relativistic gravitational measurements have not become easier—they have merely become "performable" at a level undreamed of in the "first half-century". They still remain extremely difficult experiments that can be carried out only by advanced scientific centers with highly qualitifed personnel and impressive financing.

a) Measurements of the metric parameters

The theoretical analysis of the relativistic corrections in the field of the Sun is usually based on the idealized model of the Schwarzschild field: an isolated static sphere. The most frequently used expression for the line element in this model,

$$dS^2=\left(1-\frac{r_g}{R}\right)c^2dt^2-\left(1-\frac{r_g}{R}\right)^{-1}dR^2-R^2d\theta^2-R^2\sin^2\theta\,d\varphi^2, \quad (2.1)$$

is written in special "standard" or "Schwarzschild" coordinates, which is not very convenient for experimental estimates. A transition to "almost ordinary" spherical coordinates with origin at the center of the Sun (the so-called "isotropic" coordinates) can be made by means of the change of variables $R=r[1+(r_g/4r)]^2$, $(r_g=2GM_\odot/c^2)$, $\varphi'=\varphi$, $\theta'=\theta$, $t'=t$. Then the expression for the interval, expanded in powers of r_g/r, takes the form

$$dS^2\approx\left[1-\alpha\frac{r_g}{r}+\beta\left(\frac{r_g}{r}\right)^2\right]c^2\,dt^2 -\left(1+\gamma\frac{r_g}{r}\right)[dr^2+r^2(d\theta^2+\sin^2\theta\,d\varphi^2)]. \quad (2.2)$$

Equation (2.2) is known as the Eddington–Robertson expansion.[22] Here, we have introduced the parameters α, β, γ, which are equal to unity in general relativity. In other theories of gravitation, β and γ may have different values. The metric in formula (2.2) is universally suitable in a weak field for calculating relativistic effects that follow from different metric theories. Fortunately, the perturbations associated with the masses of the planets can be taken into account with sufficient accuracy. In a number of theories that presuppose the existence of a distinguished frame, in which the Universe as a whole is at rest, one encounters additional parameters α_i $(i=1,2,3)$ in the terms of the metric, these depending on the velocity of the Sun relative to the distinguished frame; in addition, certain parameters ζ_i $(i=1,2,3,4)$ are introduced under the assumption that the conservation laws are violated. The expression (2.2) corresponds to the approximate variant of the generalized theory—"the parametrized post-Newtonian (PPN) formalism"—whose foundation was laid by Eddington and Robertson and whose most completed form was developed by Will and Nordtvedt.[31]

The coefficient α must in all cases be equal to unity in order to obtain Newtonian gravitation in the weak field limit. The qualitative meaning of the remaining parameters is as follows: γ is a "measure of the curvature" of space and β is a "measure of the nonlinearity" resulting from the addition of fields. By measuring the effects associated with the corrections to the Newtonian metric, the experimentalists can find the actual values of the post-Newtonian parameters.

For verifying general relativity, experiments associated with observing the propagation of electromagnetic radiation in the solar system have proved to be the most convenient. The reason for this is that when an experimentalist investigates the motion of a massive body he must separate the weak relativistic corrections to the Newtonian trajectory in the field of the Sun and the planets. For electromagnetic radiation, which propagates with the maximal possible velocity, the relativistic perturbations of the trajectory are comparable with the Newtonian effects. Indeed, for a test body moving along a geodesic at velocity v at distance r from the center of the Sun, the curvature of the trajectory is determined by two terms[21-23]:

$$\rho\approx-\frac{GM_\odot}{c^2r^2}\left(\frac{c^2}{v^2}+\gamma\right). \quad (2.3)$$

The first term is the ordinary Newtonian curvature of the trajectory of the material point in the central field $\rho_N=GM_\odot/r^2v^2=r^{-1}$ (since $GM_\odot/v^2=r$). The second term is the post-Newtonian correction to the curvature. It can be seen that with increasing velocity of the test body the two terms become equal in order of magnitude. For an electromagnetic ray $v=c$, and since $\gamma=1$ in general relativity, the relativistic correction for the curving of the ray is equal to the curvature of its Newtonian trajectory. (This is the well-known fact that the "deflection of light" in general relativity is twice the classical effect which was calculated as long ago as 1801 by Soldner[33] on the basis of the corpuscular theory of light.)

The theory and details of classical experiments on the deflection and retardation of electromagnetic radiation in the field of the Sun can now be found in many reviews, monographs, and text books (for example, Refs. 2, 22, 23). Below, we shall merely briefly present the evolution of the measurement of each of the experiments, give the latest results, and discuss the prospects for increasing the accuracy.

1) *Deflection of electromagnetic ray in the Sun's field* (measurement of the parameter γ). The old "optical" form of the experiment consisted of determining the position of a star near the edge of the solar disk during an eclipse and comparing it with the position of the same star six months later when the angular distance $\hat{\alpha}$ between the Sun and the star is maximal. The relativistic effect is manifested in a variation of the angular distance by the amount[21-23]

$$\delta\hat{\alpha} \approx (1+\gamma)\frac{GM}{c^2 b}(1+\cos\hat{\alpha}). \tag{2.4}$$

For a ray passing by the edge of the solar disk, $b \sim R_\odot$, so that $\delta\hat{\alpha} \approx 0.5\,(1+\gamma)\times 1.75''$. Up to 1968, only "optical" measurements had been made. During this period, from 1919 onward, about 380 different stars were measured (one of the most extensive measurements by Campbell and Trumpler[34] was of 15 stars at once within $\hat{\alpha}\sim 2.5°$). The deviation from the general relativity prediction 1.75" was on the average 20%. The value of γ was estimated to lie in the range $0.9<\gamma<1.3$. In 1969, measurements began in the radio range of the deflection of the radio position of the quasar 3C 279, by which the Sun passes in October. The reference point was the neighboring quasar 3C 273, which is further from the Sun. As compared with the optical method, there is here no need for an eclipse of the Sun; radio interferometry is a more accurate instrument for measuring angular coordinates, and by making measurements at two frequencies one can eliminate electromagnetic refraction. In the first experiments during 1969-1972, interferometers with base 2–20 km working in the range $\lambda\sim 12$–6 cm measured the relativistic deflection with an accuracy of about 15%.[22,23] In 1974, an MIT group[35] obtained a positive result of accuracy 6% with a 800-km base interferometer. Finally, the latest measurements of Fomalont and Sramek[36] were made for three compact radio sources with angular diameters 0.1". (The sources 0116+0.8, 0119+11, and 0111+02 lie almost on a straight line subtending 10° and the first of them approaches to 1.5° of the Sun. The interferometer, which consisted altogether of seven antennas, operated at two frequencies, $\lambda\sim 10$ cm and 3.7 cm, and had a maximal baseline of 35 km.) The achieved accuracy of agreement with the general relativistic prediction (it is the accuracy with which $\gamma=1$ is satisfied) was ~2% in this experiment.

In coming years, we can expect an increase in the resolution by the increase in the baseline of the interferometers [for example, Goldstone has a ~4 000-km baseline and potential accuracy of measurement of the angular coordinate ~0.0001" (see Ref. 22)].[1)]

2) *Deceleration of the velocity of electromagnetic radiation in the field of the Sun* (measurement of the parameter γ). Whereas the deflection of light was predicted by Einstein and has been well known since the creation of general relativity, the delay of electromagnetic radiation in a gravitational field, which is similar in nature, was pointed out by Shapiro only in 1964.[37] The first experiments consisted of measuring the delay of radio pulses sent from the Earth and reflected from Venus and Mercury.[2,22,23] Calculation of the delay (for the case when the distance r_A from the Earth to the Sun and the distance r_B from the reflecting planet to the Sun are much greater than the impact parameter b of the radio ray relative to the Sun) leads to the expression[22,23]

$$\Delta\tau \approx \frac{2}{c}\left[\frac{(1+\gamma)GM_\odot}{c^2}\ln\frac{4r_A r_B}{b^2}\right]. \tag{2.5}$$

Substitution of the values $r_A\sim r_B\sim 10^{13}$ cm, $b\sim R_\odot\sim 7\times 10^{10}$ cm gives $\Delta\tau\approx 200$ µsec, while the total travel time of the signal is $\tau_0\sim 30$ min, i.e., $\Delta\tau/\tau_0\sim 10^{-7}$. Measurement of the distances r_A and r_B in (2.5) with such accuracy is impossible. Fortunately, this is not necessary. As can be seen from (2.5), the magnitude of the relativistic time delay varies logarithmically as the planet passes through superior conjunction (the parameter b passes through zero). In the experiment, it is sufficient to measure the differential variations of τ_0. However, in order to separate the logarithmic effect of the variation of τ_0 from trajectory variations of the delay, extremely accurate calculation of the trajectory of the reflector planet is required.

In the first experiments with Venus and Mercury (during the period 1966-1970) a tremendous computational program of trajectory calculations was carried out on computers (as an illustration, we can point out that these involved about 300 initial trajectory parameters and 400 radar and 6 000 optical measurements. The resolution in the timing was ~10 µsec and the relative accuracy $\sim 5\times 10^{-9}$). An effect was detected and measured, giving a 20% agreement with theory.[2] Then, the probes Mariner 6 and Mariner 7 were used as active reflectors during their flights to Mars.[38] The accuracy of the measurements was raised to 4%, of which 3% was due to the fraction of uncertainty in the estimate of the trajectory data of the satellite due to drag by the solar wind. The next step involved measurements with Mars, whose trajectory was estimated from data of Mariner 9, which was in orbit around Mars.[39a] The heavy planet is effectively free of drag by the solar wind, and Mariner was rigidly tied to its center. Thus, it was possible to combine the advantages of the planetary and satellite (active transponding) variants of the experiment. The accuracy was raised to ~2%. Finally, the latest measurements of the delay effect by means of the orbiting Viking modules gave the following preliminary results: The deviation of γ from unity, if it exists, is less than or of order 1%. If the Viking measurements are continued for a complete Martian year,[39b] it may be possible to reduce the error in the determination of γ to 0.2%.[39b 2)] The restriction on the accuracy at the present time is due to the incomplete information on the properties of the plasma in the neighborhood of the Sun, it being necessary to make a correction in the experiment for the delay in this plasma. The error can be reduced by using two synchronous transponders at different frequencies on the satellite.

Measurements at two frequencies must ensure an accuracy of 0.1% and better.

Thus, both "electromagnetic" experiments to test

[1)] *Translator's note:* It seems that the "Goldstack" (Goldstone/Haystack) baseline is meant; Weinberg quotes its potential accuracy as 0.001".

[2)] *Translator's note.* This has been achieved, as reported by C. Will in his talk at the 9th Texas Symposium (Munich, December 1978).

general relativity have ended in many independent confirmations of it. The post-Newtonian value of γ really is equal to unity with an accuracy of ~1–2%. It is customary to use γ to estimate the value of the coupling constant $\tilde{\omega}$ of the scalar–tensor theory of gravitation created by Brans and Dicke: $\gamma=(1+\tilde{\omega})/(2+\tilde{\omega})$. Taking $\gamma>0.98$, we readily find $\tilde{\omega}>48$ at the ~0.7 confidence level; at the higher confidence level 0.9, we have $\tilde{\omega}>35$ (see Ref. 36). We recall that the original Brans–Dicke estimate $\tilde{\omega}\simeq 6$ (to explain the reduced relativistic advance of Mercury's perihelion in the presence of an appreciable quadrupole moment of the Sun[22]) is increasing all the time. Since the Brans–Dicke theory goes over into general relativity in the limit $\tilde{\omega}\to\infty$, the need for this theory now begins to have only academic interest. (The absence of the Nordtvedt effect described in Sec. 1 also leads to the estimate $\tilde{\omega}>30$).

3) *Relativistic advance of orbital perihelia* (measurement of the parameter β). It is well known that this is the oldest evidence in favor of general relativity. The anomalous advance of the perihelion was itself well known before Einstein's theory explained its origin.

Calculation of the two-body problem on the basis of the metric (2.2) leads to an advance of the orbital ellipse through the angle

$$\delta\varphi_0=\frac{2-\beta+2\gamma}{3}\,\frac{6\pi GM_\odot}{a(1-e^2)c^2} \qquad (2.6)$$

for each revolution of a test body in its orbit (a and e are the semimajor axis and the eccentricity and T_0 is the period of revolution measured in years). It is obvious that planets near the Sun having small a and high eccentricity e are favorable.

Another feature of the perihelion advance compared with effects relating to the propagation of electromagnetic radiation in the gravitational field is its dependence on the post-Newtonian coefficient β. The widely quoted result for Mercury obtained by Clemence[40] by analyzing astronomical observations over many years gives the value ~42.5″+0.9″, which corresponds to the theoretical estimate of the advance in a century:

$$\delta\varphi_0\frac{100}{T_0}\approx 43''\cdot\frac{2-\beta+2\gamma}{3}.$$

In recent years, the problem of the nonsphericity of the Sun, which must also lead to an advance of the perihelion, has been raised.[23] However, a new estimate of the nonsphericity (it may be a measure of the Sun's quadrupole moment) based on data on the lunar-laser-ranging[41] gives $J_2\approx\Delta R_\odot/R_\odot<6.10^{-6}$, which is sufficient for one to be able to neglect the influence of the quadrupole moment on the advance of the perihelion.

A modern analysis has been carried out under the direction of Shapiro.[42] A large number of systematic radar measurements of the dynamics of the inner planets of the solar system has been added to the old astronomical data. The results are as follows: $(2-\beta+2\gamma)/3=1\pm 0.01$. Taking the value $\gamma\sim 2\%$ from the "electromagnetic" experiments, we find that $\beta=1$ to accuracy 7%. Hope of increasing the resolution in this important experiment has long been based on the use of artificial satellites (see, for example, Ref. 82b). Indeed, by putting a satellite into a heliocentric orbit with small a and large e, one can formally obtain from (2.6) a very appreciable value of the relativistic precession of a few or even tens of arc seconds per year.

The difficulty is however that the small satellite can readily change the parameters of its orbit under the influence of nongravitational perturbations (solar wind, radiation pressure, micrometeorites, etc.), which are particularly strong near the Sun. Nevertheless, this plan has not been abandoned. Indeed, it should be realized in the future when the technique of constructing drag-free satellites is sufficiently developed.

Concluding our brief review of the classical relativistic tests, we note that the problem of determining β—the measure of nonlinear effects in the gravitational field has turned out to be more complicated than measurement of the spatial curvature parameter γ. The single experimental estimate is not obtained directly from the perihelion displacement but by an indirect calculation using the value of γ. To determine β directly, it is necessary to be able to make measurements in the second order in the weak-field parameter $(\varphi_\odot/c^2)$. This could be done either by means of a close approach to the Sun or by means of better frequency standards.

Future relativistic experiments in space are connected with the hope of using drag-free satellites with a truly geodesic trajectory in the gravitational field. Such a satellite is based on the idea of screening the proof mass by a spherical shell from external nongravitational perturbations[43,44] (Fig. 4). By tracking the central position of the proof mass within the shell by sensors and using jets for correction, one can in principle ensure geodesic motion (another way of reconstructing a geodesic was proposed in Ref. 45). An analysis of the requirements imposed by gravitational measurements on such a satellite, in particular its construction, can be found in Refs. 44, 46, 47 (there is a detailed review in Ref. 5).

The first drag-free satellite Triad I was lauched in the United States in 1972 and operated for a year.[48] The proof mass was made of an alloy of platinum and gold, i.e., a diamagnet and a paramagnet. This reduced the total magnetic permeability by two orders of magnitude compared with ordinary nonferromagnetic materials. The tests showed that the level of compensation of nongravitational accelerations was ~10^{-8} cm/sec^2. The height of the orbit was ~800 km. The main forces deflecting the satellite from a geodesic were compensated on the average at the level of ~10^{-3}. The position of

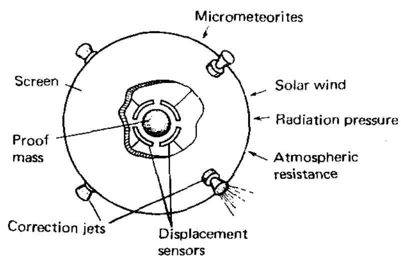

FIG. 4. Schematic drawing of drag-free satellite.

Triad I was forecast with an accuracy of 100 m two weeks in advance, whereas for ordinary satellites the daily correction is hundreds of meters. In the SOREL space probe project,[49] the attainable level of compensation would be theoretically even greater, $\sim 1\times 10^{-10}$ cm/sec^2. The accuracy of the two-week forecast is then improved to centimeters.

The actual commissioning of drag free probes will give the gravitational experimentalists a qualitatively new instrument, and, if its quality is sufficiently good, a series of important investigations will become possible. Below, we discuss three experimental programs whose fulfillment depends on the practical possibility of eliminating drag from the geodesic.

b) Solar probe

This program of the European Space Agency (ESA), which combines the efforts of the scientific centers of several European countries, embodies the idea, which has long been the dream of astrophysicists, of approaching the Sun as close as possible. It undoubtedly has very great scientific interest but entails considerable technical difficulties. In recent years, a group of specialists in different fields of physics and technology (Haskell, Bertotti, Balogh, *et al.*) have worked out a complete project for a flight to the Sun, which, for reasons of space navigation, could be realized in 1982-1985. (Part of this plan was presented by Professor Bertottie at the Fourth Soviet Gravitational Conference at Minsk (July, 1976). Below, use is also made of the material of the technical report[50] of the ESA.)[3)]

What gravitational experiments are possible during the flight of a solar probe?

1) First of all, measurements can be made of the structure of the solar gravitational field, and these will make it possible to draw definite conclusions about the inner structure of the Sun. In particular, a fairly accurate (better than 10^{-6} of the absolute value) measurement of the solar quadrupole moment is possible only under the condition that a probe approaches the Sun sufficiently close. In no other way can this be done. It is well known that the existing model of the Sun is not completely satisfactory. One of the main parameters of the relativistic [Transl. note: perhaps misprint for "a realistic"] model is the quadrupole moment J_2, which is directly related to the angular momentum of the inner core. If it is assumed that the Sun rotates rigidly, i.e., the angular velocity of a point does not depend on the distance to the rotation axis, the quadrupole moment can be readily estimated from the observed rotation. One then obtains $J_2\sim 10^{-7}$.

Note that the variable J_2 traditionally used for estimates is not in reality the quadrupole moment in the literal sense. It is a dimensionless parameter of the form

$$J_2=\frac{I_{\uparrow}-I_{\rightarrow}}{M_{\odot}R_{\odot}^2}, \tag{2.7}$$

where $I_{\uparrow}$ and $I_{\rightarrow}$ are the moments of inertia about the rotation axis and an equatorial axis. The expression for the gravitational potential of the Sun with allowance for its quadrupole moment in spherical coordinates (ρ,θ) takes the form

$$\varphi=\frac{M_{\odot}}{\rho}\left[1-J_2\frac{R_{\odot}^2}{\rho^2}\left(\frac{3\cos^2\theta-1}{2}\right)\right]. \tag{2.8}$$

There are a number of reasons for thinking J_2 is greater than 10^{-7}. After the experiments in which Dicke and Goldenberg[51] measured the oblateness of the Sun, it seemed that J_2 was determined at the level $\sim 10^{-5}$, which did not lead to serious astrophysical objections. But repeat measurements by Hill *et al.*[52] showed that optical observations of this kind are subject to errors due to surface thermal effects, so that J_2 may deviate from 10^{-7} in either direction. We have already mentioned the estimate of Ref. 41 based on the data of lunar-laser-ranging: $J_2\lesssim 6\times 10^{-6}$. The only reliable way of determining J_2 is to measure the departure of the Sun's gravitational potential from spherical symmetry [see (2.8)]. Knowledge of J_2 is very important for finding the geodesic trajectories of planets and satellites. (For example, the relativistic advance of the planetary perihelia must be recalculated if J_2 has an appreciable value.[23])

In addition, knowledge of J_2 is important for studying the behavior of the solar corona. The solar wind, which carries away hydrogen and helium, must facilitate loss of angular momentum by the outer layers, and must therefore change the surface rotation. The depth from the surface to which this process penetrates can be determined from the value of J_2.

In the solar probe program, it is intended that J_2 should be determined from the data on the variations of the probe's acceleration obtained from the ranging parameters: the range and the range rate. The nongravitational forces must be compensated if the probe is drag-free or (in the second variant) they can be taken into account on the basis of accelerometer data taken on the satellite with the necessary accuracy. The variations in the acceleration due to the influence of the quadrupole moment can be calculated in accordance with the simple formula

$$\Delta a=J_2\frac{r_g}{R_{\odot}}\frac{c^2}{R_{\odot}}\frac{1}{r^4}=J_2\cdot\frac{3\cdot 10^4}{r^4}, \tag{2.9}$$

where $R_{\odot}$ and r_g are the ordinary and the gravitational radius of the Sun, and r is the distance to the center of the Sun in units of $R_{\odot}$. The accuracy foreseen by the program[50] is $\Delta J_2\sim 10^{-8}$; hence, for distances $r=4$, Eq. (2.9) gives $\Delta a\approx 1\times 10^{-6}$ cm/sec^2, which is quite capable of measurement by, for example, a probe of the Triad I class.

Equation (2.9) demonstrates the strong dependence of the effect on the distance r to the center of the Sun, and this explains the need for a close encounter.

2) Such a probe will permit measurement of the basic

[3)] *Translator's note.* This project has now been taken up actively by NASA [see JPL Publication 78-70 ("A Close-up of the Sun"), produced by JPL and NASA, September, 1978]. It will probably not be undertaken by ESA.

post-Newtonian parameters γ and β with much greater accuracy than has hitherto been achieved. The anomalous acceleration of the probe due to the "curvature" and "nonlinearity" of the Sun's field must be in order of magnitude

$$\Delta a = (\gamma, \beta)\cdot\frac{6\cdot 10^{-2}}{r^3} \approx 10^{-3}\ \text{cm/sec}^2 \tag{2.10}$$

(as above, $r\approx 4$ in units of $R_\odot$). But it follows from (2.10) that the acceleration in this mission must be measured with an accuracy not worse than $\Delta a \sim 10^{-6}$ cm/sec^2. Then the accuracy in the measurement of γ and β will be 0.1%. The gain is again achieved through the closeness of the flyby.

3) In its turn, the improved accuracy in the measurement of γ and β makes it possible to verify some nonstandard effects (outside the framework of general relativity). For example, the program of Ref. 50 includes the task of measuring a putative deviation of the Sun's field from spherical symmetry resulting from motion relative to distant masses. (In fact, this conjecture is in line with Mach's suggestion that the motion of distant matter influences the local dynamics of particles.) According to the estimates of Will and Nordtvedt,[32] such a deviation would result in corrections to γ and β of order v/c, where v is the velocity of the Sun's motion in our Galaxy. Since $v\approx 300$ km/sec, the required corrections would be of order $\sim 10^{-3}$, which happens to be at the accuracy level of the measurements in this experiment.

4) Finally, it is possible to observe the gravitational frequency red shift with an accuracy sufficient to measure the effect in the second-order in $\varphi_\odot/c^2$, this being due to the close approach and, accordingly, increase in $\varphi_\odot/c^2$. We have mentioned the importance of such a measurement for general relativity—it is a pure determination of the parameter β. From a more general point of view, this is a test for theories that violate (partly) a postulate of general relativity such as the equivalence principle.

That "gravitation does not affect the rate of clocks" in the sense that in a freely falling frame of reference the period of the clocks does not depend on $\bar{g}$ but is determined solely by the values of the atomic constants is a consequence of the strong equivalence principle (the laws of nature are universal in any locally Lorentzian frame of reference). Theoretically, one can consider if it is possible to construct theories which violate the strong equivalence principle but not the weak equivalence principle (identical law of free fall of test bodies). Lightman and Lee,[53] and also Will[54] have shown that such theories are logically possible. In them, frequency standards based on the hyperfine structure of atomic levels can change their frequency in a gravitational field in accordance with the law

$$\nu = \nu_0\left[1 - \alpha_0\frac{r_g}{r} + \alpha_1\left(\frac{r_g}{r}\right)^2 + \dots\right], \tag{2.11}$$

where the second coefficient α_1 differs from β.

The accuracy of measurement of the red shift in the experiments of Vessot[30] was $\sim 4\times 10^{-3}$. With the solar probe, one could in principle achieve the accuracy

$$\Delta\alpha_0 = \frac{\Delta\nu}{\nu_0}\cdot 5\cdot 10^5 r. \tag{2.12}$$

For $\Delta\nu/\nu_0 = 10^{-13}$, $r=4$, we already have $\Delta\alpha_0 \approx 2\times 10^{-7}$. The resolution in the measurement of the second-order effect is related to the formula $\Delta\alpha_1 = \Delta\alpha_0 \times 10^6$, so that a departure of α_1 from 0 can be established at the 20% level. Under terrestrial conditions, a frequency standard with stability $\Delta\nu/\nu \sim 5\times 10^{-18}$, which does not yet exist, would be needed for this purpose.

The method of the experiment must consist in a comparison of the frequencies of the standards on the probe and on the Earth. It follows from this that data must be transmitted by a system in which one can separate the Doppler frequency shift from the gravitational shift.

These experiments are unique and possible only because of the very close encounter with the Sun, right down to $\sim 4R_\odot$. How could such an encounter be engineered?

The authors of the plan of Ref. 50 found an original solution using the so-called gravitationally-assisted trajectory, or swingby. The probe is not sent directly toward the Sun, but toward Jupiter. The gravitational field of Jupiter changes the momentum of the probe and makes it go over into an orbit with a very high eccentricity, and the probe now flies toward the Sun. Three classes of orbits are possible: an "unaesthetic", pointed orbit with vanishing perihelion ("Kamikaze orbit") and orbits with perihelia of a few solar radii in the plane of the ecliptic and out of the ecliptic. Optimization of the parameters indicates that an orbit in the plane of the ecliptic with perihelion $4R_\odot$ is to be preferred. The Kamikaze orbit, although it goes right into the Sun, is unsatisfactory in that its final section is ineffective due to technological restrictions. All that happens is that one loses half the data on the outward part of the flyby; in addition, the relativistic parameters of this orbit are strongly correlated.

The principal technical result of the project is the finding of an orbit for which it is possible to observe the perihelion from the Earth when the probe passes through perihelion. For greater clarity, the dates and mutual configurations are shown in Fig. 5. The flyby of the probe will of course make it possible to obtain rich information about the plasma in the neighborhood of the Sun, which we shall not consider here since it goes beyond the scope of the present paper. But even without this it is clear that the solar probe mission is of exceptional importance; under the most modest assumptions about the accuracy of the instruments, the scientific results must be unique and must provide fundamental new

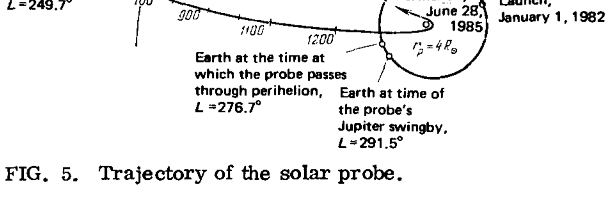

FIG. 5. Trajectory of the solar probe.

information on the gravitational field that cannot be acquired by other methods.

c) Lense-Thirring effect

The Lense–Thirring effect is the name given to the "systematic ("secular") displacement of the orbit of a particle moving in the field of a central body due to the rotation of that body" [Teoriya polya (The Classical Theory of Fields), §104[21]]. The corresponding problem was solved by the authors after whom the effect is named as long ago as 1918.[55] The possibility of observing this effect in practice has since then been investigated frequently.[56] However, for the planets and their satellites it is swamped by the much larger perturbations of Newtonian origin.

Under laboratory conditions, when the Lense–Thirring effect is frequently understood to be the change in the attraction of two bodies when one of them rotates,[57] the magnitude of the effective perturbation in all the discussed variants is usually much lower than the fluctuation noise. Recently, an interesting project has been proposed by Van Patten and Everitt,[59] and this really could be embodied in a space program using drag-free satellites.

The nature of the perturbation of the orbit of an artificial satellite by the Earth's rotation can be described as precession of the vector of the orbital angular momentum of the satellite about the vector of the intrinsic angular momentum of the Earth. In this precession, the line of the nodes (the line joining the nodes, the points of intersection of the orbit with the equatorial plane) rotates with angular velocity $\Omega = d\varphi/dt$, so that the right ascension of the ascending node increases all the time (the right ascension is the angular distance from the point of the vernal equinox to the ascending node, i.e., the node at which the satellite rises above the equatorial plane). The value of Ω can be calculated in accordance with the formula[22]

$$\bar{\Omega} = \dot{\bar{\varphi}} = \frac{2GJ\omega}{c^2 a^3 (1-e^2)^{3/2}} \bar{n}_E, \qquad (2.13)$$

in which J and ω are the moment of inertia and the angular velocity of the Earth's rotation, a and e are the semimajor axis and the eccentricity of the satellite's orbit, and $\bar{n}_E$ is the unit vector in the direction of the Earth's angular momentum.

A numerical calculation in accordance with (2.13) for a satellite at a distance of a few hundred kilometers above the surface of the Earth gives the value $\Omega \approx 0.1$–0.2 arcsec/year. At this rate of precession, the displacement of the node of the orbit along the arc of the equator is 10–20 m in two or three years. For comparison with the relativistic precession of a gyroscope due to the rotation of the Earth, we note that in identical polar orbits the Lense–Thirring precession rate is approximately four times greater than the rate of spin-orbit precession of a Schiff gyroscope with spin normal to the plane of the orbit (See Sec. 2d below). Could this effect be observed with a satellite of the Triad I class? We have already mentioned the experimental drag of Triad I, which was approximately 10^2 m in two weeks, or several tens of kilometers in a year. However, this drag rate is characteristic for the direction along the orbital trajectory. But even this can be appreciably reduced if the satellite is forced to rotate about an axis of its own—then the vector of the residual acceleration will be averaged by the rotation.[5] In the transverse direction, the drag is appreciably less. The satellite, as a giant gyroscope, manifests stability against perturbation of the orbital plane.

According to the estimates of Ref. 58, the transverse drag for a satellite of the type of Triad I must be less than 10 cm in a year. Against this background, the Lense–Thirring precession is a very large and readily measurable effect.

In reality, drag is not the main difficulty with the observation; there is an analogous precession of the satellite's orbit due to geophysical factors, namely, a nonvanishing quadrupole moment $J_{2\oplus}$ of the Earth. The angular velocity of this geophysical precession is described by the expression

$$\Omega_{\mathrm{gph}} = -\frac{3}{2}\omega_0 \left[\frac{R_\oplus}{a(1-e)^2}\right]^2 J_{2\oplus} \cos\theta \bar{n}_3 = \mathrm{const}\cdot J_{2\oplus} \cos\theta \bar{n}_3, \qquad (2.14)$$

where ω_0 is the mean orbital angular velocity of the satellite, $R_\oplus$ is the radius of the Earth, and θ is the inclination of the plane of the orbit to the equator. It is easy to see that this precession is much greater than the relativistic precession (2.13), since it does not contain the small factor G/c^2.

It would seem that the situation here is largely the same as in many already performed relativistic experiments such as observations of the advance of Mercury's perihelion, time delay of the radar echo from planets and satellites, and, finally, measurements of the Nordtvedt effect. In all cases, a small relativistic shift could be distinguished as the uncompensated remainder from a large but exactly calculable background. Unfortunately, in the present case such a method does not apply. It is impossible to make an exact calculation of Ω_{gph} and its evolution because of the large error in the determination of the angle θ, which, at the present technological level of measurements of satellite trajectories, leads to an uncertainty that is 6–7 times greater than the Lense-Thirring effect. An ingenious way out of this dilemma is that one can find a method of continuous experimental checking of the geophysical precession. This exploits the difference between (2.13) and (2.14) associated with the dependence of the geophysical precession on the inclination of the orbit. Considering two satellites in symmetric orbits that are close to an exact polar orbit but one on each side of such an orbit, one can readily see that the sum of the absolute magnitudes of the geophysical precessions is equal to

$$(\Omega_{1\mathrm{gph}} + \Omega_{2\mathrm{gph}}) = \mathrm{const}\cdot J_{2\oplus}(\cos\theta_1 + \cos\theta_2) \approx \mathrm{const}\cdot J_{2\oplus}(\theta_1' + \theta_2') = \mathrm{const}\cdot J_{2\oplus}\cdot 2\alpha; \qquad (2.15)$$

where $\theta' = (\pi/2) - \theta$, and $2\alpha = \theta_1' + \theta_2'$ is the angle between the planes of the orbits of the satellites. For the total displacement of the orbits due to the geophysical precession in time t, we obtain

$$(\varphi_1 + \varphi_2)_{\mathrm{gph}} = \mathrm{const}\cdot 2J_{2\oplus}\int_0^t \alpha\, dt. \qquad (2.16)$$

Equation (2.15) shows that one can exactly calculate the geophysical relative rotation of the orbits if the function $2\alpha(t)$ is available (the errors in our values of $J_{2\delta}, \omega_0, R_\delta$ are fairly small). This can be done by measuring the distance between the two satellites by radar or laser ranging in each orbit. The experiment can be arranged as follows. The satellites revolve in slightly different orbits in opposite directions, so that they meet each time above the poles; this can be achieved by correcting the initial flight parameters. The point of closest approach is determined by the time at which the Doppler frequency shift passes through zero in the satellite—satellite communications link. At this instant, laser ranging is used to measure the mutual separation $\sim 2\alpha a$ to an accuracy of a few centimeters. The result is transmitted to the Earth. Stations tracking the satellites on the equator measure the right ascension (longitude) of the orbits. On the basis of their data, the relative angle of rotation of the ascending nodes during the time of observation, $(\varphi_1+\varphi_2)_{\rm meas}$, is determined. The geophysical rotation (2.15) is calculated from the data on the time evolution of the angle $2\alpha(t)$. Finally, the Lense–Thirring relativistic displacement must be the difference between the measured and calculated shifts:

$$\Delta\varphi_{LT} = (\varphi_1+\varphi_2)_{\rm meas} - (\varphi_1+\varphi_2)_{\rm gph}.$$

The satellites must orbit in opposite directions so that one can eliminate displacement of the nodes of the orbit due to gradients of the gravitational field produced by the influence of the planets, the Moon, tidal effects, and so forth. These Newtonian perturbations give rise to displacements of the two orbits that are equal in magmagnitude but opposite in sign, and therefore the total angular shift $\varphi_1+\varphi_2$ is not changed.[56]

Following the evolution of the angle $2\alpha(t)$ is a convenient and practical method for obtaining information about about the figure of the Earth and its mass distribution. Therefore, the overall value of such an experiment is not restricted to merely measurement of relativistic corrections.

Analysis of the sources of error (under the condition that the nongravitational accelerations are compensated at the same level as for Triad I) indicates that an accuracy at the 1–2% level could be achieved in the measurement of the Lense–Thirring effect.[59] In the case of success, this would be a new verification of general relativity.

d) Relativistic gyroscope

In Newtonian mechanics, an ideal gyroscope (a rotating sphere) does not precess whatever motion it makes in a gravitational field. Its spin is conserved in time. Einstein's theory predicts a relativistic precession of the spin. Almost immediately after the creation of general relativity, the possibility was considered of observing this precession with "natural gyroscopes": De Sitter[60] considered the gyroscope formed by the Earth–Moon system, and Eddington[61] considered the precession of the Earth in the field of the Sun. In these cases, the relativistic perturbation was found to be minute (for the Earth, it is 10^{-2} arcsec/year, which is much less than the accuracy with which the total precession, which is ~50 arcsec/year, can be measured). Shortly after the launching of the first artificial satellites of the Earth, Schiff[62a] in 1960 proposed an experiment with an artificial gyroscope in a terrestrial orbit and calculated the rate of precession.

The effect consists of two parts, which can be well distinguished in a polar orbit. A gyroscope with spin in the plane of the orbit undergoes a relatively large geodetic (or Thomas[4)]) precession, which is 7 arcsec/year for an orbit at altitude ~500 km. (The spin of the gyroscope rotates around the vector of the normal to the orbit.) A gyroscope whose spin is normal to the plane of the orbit precesses around the Earth's axis; this is "spin-spin precession"[5)] at the rate ~0.05 arcsec/year. The first effect does not depend on the rotation of the Earth, while the second is due to it. A detailed theory of the relativistic gyroscope is given in the monographs of Refs. 21–23; The experimental situation has been described, for example, in Ref. 2.

Although the idea of the experiment is very old, it has not yet been implemented. For many years, a group at Stanford (in the United States) has been working on such a project.[2,63] The group is made up of members of the W. W. Hansen Laboratories of Physics and the Faculty of Aeronautics and Astronavigation, and its members include Fairbank, Everitt, De Bra, Van Patten, Anderson, Lipa, and others. Deadlines for the completion of the work keep on being postponed. The reason for this is the extreme complexity of the experiment, which is a combination of many difficult technological and constructional problems that must all be solved simultaneously if the mission is to be a success. Below, we briefly describe the main details of the project and its present state of preparation.

The general scheme of the satellite, which is drag free and carries gyroscopes, is shown in Fig. 6.[18,63] The orientation telescope, four gyroscopes, and the proof mass of the drag-free tracking system are placed in a single superconducting dewar. The dewar is designed to take 800 liters of liquid helium and function for two years. An individual gyroscope is a ball (4-cm diameter) of pure quartz (uniformity $\sim 10^{-6}$, sphericity accurate to $\Delta r/r \sim 10^{-7}$). There is an outer covering of niobium. The gyroscope is kept in position by electrostatic suspension with three mutually perpendicular sets of condenser plates (accuracy of centering 10^{-5} cm, voltage required for suspension under space conditions ~0.5 V). The gyroscope is spun up by a helium vapor jet to 200 Hz in 30 min with vacuum $\sim 10^{-9}$ torr.

Each gyroscope is surrounded by a superconducting magnetic screen. The proof mass of the tracking system, which is also made of quartz, is placed together with the gyroscopes in a quartz jacket, to which the

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4) An analogous effect, calculated on the basis of the special theory of relativity, exists for the spin of an electron in orbit around a nucleus (see Ref. 62b).

5) The analogy here is the interaction between the orbital angular momentum of an electron with the spin of the nucleus, which is responsible for the hyperfine structure of the spectral lines.

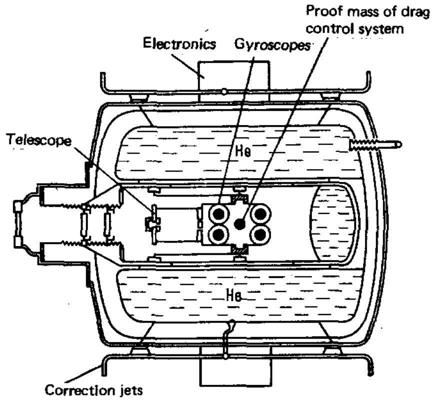

FIG. 6. Design of the satellite for the relativistic gyroscope experiment.

quartz telescope is rigidly fixed. All joints are made without the use of glue with optical contact. Further magnetic screening is achieved by a superconducting dewar filled with a nonmagnetic material; the residual field is at a level less than $\sim 10^{-7}$ G.

All these measures have but one aim—to make a gyroscope with residual drift not greater than $\sim 10^{-16}$ rad/sec (6×10^{-4} arcsec/year). Then the accuracy with which the geodetic precession can be measured is 0.01%, while the spin-spin precession can be measured to 30%. (Transl. note: 1% according to Ref. 18). The positions of the spins of the gyroscopes are measured by observing the London magnetic moment which the rotating superconducting sphere must have. Three mutually perpendicular pickup loops surround each gyroscope and send a signal to quantum magnetometers of the Josephson junction type. The accuracy with which the angular position of the spin can be measured by this method reaches $\sim 10^{-3}$ arcsec in a time of 10^4 sec.

The design requirements on the extent to which the satellite must be drag free do not exceed the level achieved by Triad I. The motors for correcting the drag use the main store of liquid helium. The overall control of the craft derives from the error signal obtained from the telescope.

The telescope locks onto the guide star, and the two reference gyroscopes align their axes along this direction. The two remaining gyroscopes are used for the measurements; one has its spin oriented approximately along the direction of the Earth's axis, while the other is perpendicular to it. Both lie in the plane of the normal optic axis of the telescope; the first measures mainly the geodetic precision, the second the spin–spin precession. The reference gyroscopes also measure a precession, but a mixed one, and their signal will be periodic as the satellite rolls about the axis of the telescope. The rolling is helpful since it also makes it possible to eliminate drift effects; a convenient period is ~20–30 min. Naturally, the telescope cannot be continuously corrected because of the orbital motion. The correction is done on each orbit during a time interval when the star is in the telescope. (A convenient reference star could be, for example, Procyon or Rigel.)

At the time of writing of the present review, all the separate details are ready and bench tests have been made of the gyroscopes, the suspension system, the gyroscope spinup system, and the signal reception system.[18] A trial flight to test the complete system as a whole is foreseen for the program on the Shuttle 10 in 1980.

Let us consider briefly the importance of the last two experiments described in Secs. 2c and 2d. Both projects are rather expensive and labor consuming, and it is therefore worth considering once more what new knowledge they can give us about the nature of gravitation as compared with other tests. From the point of view of the determination of the metric coefficients the answer is not particularly much. The geodesic precession of the gyroscope is related to the same "curvature" coefficient γ, though it is true that this could be measured with accuracy 0.01% if the planned resolution is achieved. Observation of the Lense–Thirring effect and spin–spin precession (the nature of which is approximately the same) admits in principle an estimate of one of the parameters associated with the hypothesis of a privileged frame of reference, α_1, but the estimate is poor, with an accuracy of not more than 50%. It is however important to emphasize a new feature of these experiments. Namely, they will be the first measurements of relativistic features of gravitational systems with rotation of the distortion of the Newtonian field of a rotating mass, and of the behavior of the angular momentum vector in a gravitational field.

In the evolution of ideas about space and matter, experiments involving rotation played an important role. We recall that (thought) experiments with a rotating frame of reference served for Newton as a proof for the existence of absolute space. Mach saw in these same experiments a confirmation of his principle that the inertia of bodies is due to the attraction of distant masses of the Universe.[6)] In general relativity, these ideas are replaced by the equivalence principle, which states that in an inertial frame of reference (which is locally Lorentzian) all fields, both those of the distant and the nearby matter, are compensated. A freely falling gyroscope does not precess in an inertial frame of reference (i.e., in its own).

The precession is observed in a system tied to the stars. The inertial system (the axis of the gyroscope) is displaced with respect to the star because of the influence of the Earth's field, which changes because of the rotation. For this reason, the experiment with the relativistic gyroscope is sometimes called a measurement of the dragging of the inertial frame of reference.[23] Note that the use of the gyroscope spin is merely a practicable method of specifying an inertial frame; any other free vector must be subject to precession, the rate of which does not depend on the absolute magnitude of the spin.

Laboratory projects and estimates for measurements of the effects described in the two last sections can be found in Refs. 57, 67, 68.

[6)]Experiments that measure the anisotropy of inertial mass[65,66] are a direct refutation of Mach's principle.

3. OBSERVATION OF RELATIVISTIC EFFECTS IN STRONG FIELDS (BINARY PULSAR)

Objects that must have strong gravitational fields have been discovered during the last ten years. They are neutron stars (pulsars) and x-ray binaries with a compact invisible component, which may be a black hole. It is natural that attempts were immediately made to establish relativistic gravitational features on the basis of observational parameters of these objects. Unfortunately, this could not be done because of the strong background of hydrodynamic and plasma processes in the surrounding medium, which distort the picture and rule out an unambiguous specification of the causes of any particular effect. However, recently, the situation has changed.

At the end of 1974, a pulsar in a binary system was discovered. The pulsar is a neutron star with a mass of the order of the solar mass and radius ~10 km, so that is has $\varphi/c^2 \sim 0.1$, which is five orders of magnitude greater than the corresponding quantity for the Sun. Whatever the nature of the second component, it is in the relatively strong field of the pulsar, so that the dynamics of the system must have an appreciably relativistic nature. The most remarkable thing, and it distinguishes this object among all other compact binaries, is the presence of an extremely stable radio source—the pulsar—in orbit in the binary system. Nature has presented to us a "relativistic laboratory" with a ready-made tool for its investigation.

Having discovered this pulsar, PSR 1913+16, Taylor and Hulse estimated the parameters of the system by the usual methods employed for double stars. They found that the period of the orbital motion is $T = 27907 \pm 30$ sec ≈ 7.5 h; the maximal repetition period of the pulsar pulses is $P_{max} = 59.045$ sec and the minimal is $P_{min} = 58.967$ sec (the change in P is a consequence of the Doppler effect and other more subtle factors); the distance from the Earth to the pulsar is of order 5 kpc. It has not proved possible to observe the second component either optically or by radio methods.[7)]

An analysis of tests which this unique object makes possible can be made from the following points of view. How well does this system confirm the classical effects of general relativity? To what extent can the general relativistic effects be used to determine the characteristics of the binary itself and its physics? What new tests of gravitation can be made with such a system?

a) Traditional relativistic tests

Naturally, one first considers the relativistic precession of the orbit, or the advance of the periastron (for the Sun, perihelion). If $M_1 = M_2 \sim M_\odot$, then an estimate in accordance with the well-known formula for the angular advance of the periastron in one orbit gives

$$\Delta\varphi_P = 6\pi \frac{M_1+M_2}{c^2 a(1-e^2)} = \frac{6\pi^{5/3}\cdot 2^{2/3}[G(M_1+M_2)]^{2/3}}{c^2(1-e^2)T^{2/3}} \approx 5\cdot 10^{-5} \text{ rad/revolution}, \tag{3.1}$$

which means a shift of order ~3 arc degrees in a year. This is a huge effect compared with the advance ~0.43 arc second per year for Mercury. Experimentally, Taylor and Hulse have found ~$3.6 \pm 1.6°$ for the advance of the periastron, which agrees well with theory.

In principle, gravitational red shift of the pulsar frequency could be observed by using the variations in the distance between the components during the orbital motion, but this is difficult to separate from the second-order relativistic Doppler effect. The two can be observed together and, as will be shown below, such an observation plays an important role in the investigation of this binary system.

In principle, the delay of the pulsar pulses in the field of the second component can also be measured. However, since the inclination of the orbit is not equal to the angle $i = \pi/2$ most favorable for this experiment, the effect of the delay will show up in the repetition frequency of the pulses only in terms of order $(v/c)^3$, and an improvement of the instrumental technique will be needed for its measurement.[70,71]

In Refs. 72 and 73, the possibility of observing the Schiff effect, i.e., the precession of a gyroscope in a Keplerian orbit, was discussed. The gryoscope is the pulsar itself. If its rotation axis is not perpendicular to the plane of the orbit, it will undergo geodetic precision, i.e., it will rotate around the vector of the orbital angular momentum of the pulsar. (The spin–spin, or hyperfine precession can hardly be observed because of its smallness.) The integrated precession of the pulsar axis over a revolution, averaged over the period of revolution, is

$$\Delta\varphi_g = \frac{6\pi}{2}\frac{GM_2(3M_2+4M_1)}{a(1-e^2)c^2\cdot 3(M_2+M_1)}. \tag{3.2}$$

In the limit when the mass of the second component greatly exceeds the mass of the pulsar, $M_2 \gg M_1$, the value of $\Delta\varphi_g$ is half the relativistic advance $\Delta\varphi_p$ of the periastron. If $M_1 \sim M_2$, we obtain $\Delta\varphi_g \approx (7/24)\Delta\varphi_p$. Since the experimental result for the advance of the periastron is $\Delta\varphi_p \sim 4°$ year^{-1}, one must expect a geodetic precession $\Delta\varphi_g \sim 2°$ year^{-1} of the pulsar axis. (We recall that a Schiff gyroscope would precess by only ~7″ year^{-1}.)

How could one observe precession of the pulsar axis from the Earth? The answer depends on the model of the radiation mechanism. In the pulsar model of a rotating lighthouse, radiation leaves the star within a cone of opening angle 2α whose axis makes an angle ξ with the axis of rotation. An observer on the Earth detects radiation if the line of sight falls within the angle 2α. The rotation of the pulsar modulates the radiation, which then comes in the form of pulses of given duration. The polarization of the radiation is scanned in a definite interval. Precession of the axis changes the angle at which the line of sight pierces the radiation cone. As a result, the pulses received at the Earth change their duration and interval of polarization scan-

[7)]Translator's note. Optical identification of an object close to the position of the binary pulsar was reported at the Ninth Texas Symposium (Munich, December, 1978). A spectrum was not yet available, and the object could be merely a projected star.

ning. In principle, these changes could enable one to observe and measure the relativistic geodetic precession of the pulsar gyroscope.[72,73] For a definite geometry, the radiation may disappear from the field of view and return after a time equal to the precession period. However, this is the too long interval of 180 years for PSR 1913-16, so that actually all that is sensible is to look for small variations in the width and polarization of the pulses. At the precession period ~180 years, the changes in the polarization are a few percent in a year. Of course, the width of the pulses can be measured with greater accuracy than the polarization. However, in practice both these parameters vary randomly from pulse to pulse because of perturbations by various interstellar processes. All that one can hope for is a procedure of averaging over a large number of pulses, which could separate the secular variations in the duration and polarization.

We see that the measurements here are very complicated and labor consuming and require many years of observation and detailed analysis of the signal-to-noise ratio. However, such an experiment is far cheaper than the terrestrial project of a relativistic gyroscope (see Sec. 2d). It is also very important that, besides observations of the relativistic effect, this experiment can give valuable information on pulsar parameters such as the width of the radiation cone, the inclination of the cone to the rotation axis, and the inclination of the rotation axis itself to the orbital angular momentum, all of which will make it possible to estimate the validity of the theoretical models.

b) Determination of the parameters of astrophysical objects by means of general relativity

In the preceding subsection, we have considered a second aspect of the relationship between general relativity and this unique binary. Namely, general relativity appears here, not as a hypothesis requiring confirmation, but as a theory which makes it possible to find critical tests of astrophysical models of the pulsar. Indeed, the general relativistic effects are sufficiently well assured for one to be able to use them to find unknown parameters of relativistic objects and therefore sharpen hypotheses about their structure.

This new feature of gravitational experimental investigations is already well demonstrated by the results of observation of the periastron precession. The agreement between the measured value of the precession and the relativistic estimate makes it possible to draw a conclusion about the compactness of the pulsar's companion; for if the companion were a main sequence star, tidal effects alone would cause the periastron precession to be 50 times greater than the observed precession.[70]

If a successful measurement can be made of the geodetic precession of the pulsar, a possibility is provided for determining the masses of both components from (3.1) and (3.2). A less sophisticated experiment, involving measurement of the relativistic frequency shift of the pulsar's radio emission, also makes it possible to find the masses of the components and the inclination of the orbit.[70,74]

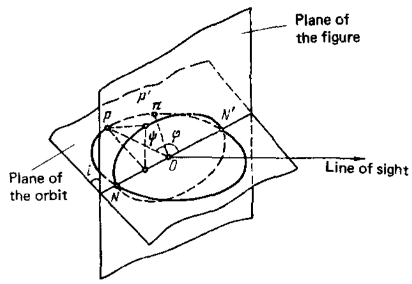

FIG. 7. Geometry of the orbit of the binary pulsar for a terrestrial observer.

In view of the importance of the question, we shall consider it in more detail, following Ref. 74.

Figure 7 shows schematically some very simple geometrical constructions that provide a picture of the binary system for a terrestrial observer. The line of sight from the observer to the binary system is perpendicular to the plane of the figure, in which the apparent projection of the orbit lies. The orbital plane is inclined to the plane of the figure at angle i. The point π is the periastron of the orbit, p is the instantaneous position of the pulsar, the point O is the center of mass of the binary system, and NN' is the line of the nodes. The angle $\sphericalangle N'O\pi$ is the longitude of the periastron, πO is the line of the apsides, and the angle $\sphericalangle \pi OP$ is the true anomaly of the pulsar.

As the pulsar moves in its orbit, the repetition period of the radio pulses changes in accordance with the law

$$P \approx P_0\left(1-\frac{v}{c}\cos\theta+\frac{v^2}{2c^2}+\frac{GM}{c^2 r}\right) \tag{3.3}$$

(it is assumed that the center of mass is at rest relative to the observer), M is the mass of the companion, r is the distance between it and the pulsar, and $v\cos\theta$ is the projection of the pulsar velocity onto the line of sight.

Equation (3.3) takes into account the ordinary and the relativistic Doppler effect, and also the gravitational frequency red shift. Using the laws of a Keplerian orbit and introducing the mean value $\overline{P}$ over an orbital period T [when $v\cos\theta$ is to be regarded as the radial rate of change of the segment $PP' = OP\sin i\times\sin(\varphi+\psi)$], we can readily obtain from (3.3) an equation for the variation of the period:

$$\Delta p = p-\overline{p} = p\left[\left(\frac{K}{c}\cos\varphi+\frac{eA}{c^2}\right)(\cos\varphi+e)-\frac{K}{c}\sin\varphi\sin\psi\right]. \tag{3.4}$$

The constants K and A are determined by the expressions

$$K=\frac{(2\pi G)^{1/3} M\sin i}{T^{1/3}(M+m)^{2/3}\sqrt{1-e^2}}, \quad A=\frac{(2\pi G)^{2/3} M^2\left(2+\frac{m}{M}\right)}{T^{2/3}(M+m)^{4/3}(1-e^2)}. \tag{3.5}$$

The position of the periastron, the angle φ, can be assumed fixed during one or a few revolutions. Making a Fourier analysis of the variations of the period at the revolution frequency $2\pi/T=\dot{\psi}$, an observer is in a position to determine the quantities

$$\widetilde{\alpha}=\frac{K}{c}\cos\varphi+\frac{eA}{c^2}, \quad \widetilde{\beta}=\frac{K}{c}\sin\varphi. \tag{3.6}$$

The constant A includes both relativistic effects, i.e., the second-order Doppler effect and the gravitational red shift. It can be seen from (3.6) that if the longitude of the periastron remains constant, it is impossible to separate the relativistic term. But, as we know, φ varies slowly because of the general relativistic gravitational precession, the angular velocity of which has already been measured. It then becomes possible to determine A. For example, using (3.4) to form the combination

$$\Delta P = P_{\max} - P_{\min} = 2\bar{P}\left(\frac{K}{c} + \frac{eA}{c^2}\cos\varphi\right), \qquad (3.7)$$

we can readily see that by measuring the difference between the $\Delta P/\bar{P}$ values at $\varphi = 0$ and $\varphi = 180°$ we can separate cleanly the relativistic term $2eA/c^2$. In practice, this is inconvenient because a half-period of the precession lasts ~90 years. A realistic method could consist of measuring the rate of change of the quantities $\tilde{\alpha}$ and $\tilde{\beta}$. From (3.6), we then obtain

$$2eA/c^2 = \frac{d}{dt}(\tilde{\alpha}^2 + \tilde{\beta}^2)\Big/\frac{d\tilde{\alpha}}{dt}.$$

For estimating A, the most favorable position of the orbit is the one at which $d\tilde{\alpha}/dt$ is maximal, i.e., the position when the longitude of the periastron is $\varphi = \pi/2$ or $\varphi = 3\pi/2$. At the present time, the observations of Taylor and Hulse[69] give $\varphi = 179°$. In this situation, the linear Doppler effect is maximal, and it is hard to separate the relativistic correction A against the background of the linear effect (by the indicated method or in other ways). We shall have to wait for a more favorable position. However, we want to emphasize the fundamental importance of such a measurement. Determination of A gives the value of the independent combination $B = M(2M+m)/(M+m)^{4/3}$ of the masses, and this, in conjunction with the known precession $\Delta\varphi_p$ (3.1) of the periastron and the classical mass function $f(M) = (M\sin i)^3/(M+m)^2$, makes possible separate estimates of the masses of the components and the inclination of the orbit. Thus, general relativistic effects appear here in the role of an *experimentum crucis* in the investigation of a new object in the Universe.

c) Gravitational radiation and non-Einstein effects

The idea of observing the decrease in the period of a binary to obtain an indirect proof of the existence of gravitational radiation has long been known.[75] Its realization in practice has always been prevented by the low accuracy with which the orbital period is measured. The binary pulsar makes possible an appreciable increase in the accuracy of measurements of relative variations of the period by means of the Doppler shift of the frequency.

Energy losses through gravitational radiation must lead to a reduction of the period and of the eccentricity of the orbit at the rate[64]

$$\frac{\dot{T}}{T} = -\frac{96}{5}G^3\frac{M_1M_2(M_1+M_2)}{a^4}F(e) \approx 6.3\cdot 10^{-11}\frac{\chi(1+\chi)^3}{\sin^5 i}\ \text{year}^{-1},$$
$$\frac{T}{e}\frac{de}{dT} = 0.342, \quad \chi = \frac{M_1}{M_2}. \qquad (3.8)$$

The second of these equations does not require knowledge of the masses of the components or the angle of inclination of the orbit, but, unfortunately, we are not able to measure the eccentricity with sufficient accuracy. The first equation in (3.8) for $\chi = 1$ and $i \lesssim 60°$ gives[69] the estimate $(\dot{T}/T) \gtrsim 10^{-9}$ year^{-1} (Refs. 74 and 76), and this could be readily measured with existing frequency standards. In reality, measurement of such a slowing down of the period would not result in an unambiguous argument in favor of gravitational radiation since there exist other causes of a reduction in the period with time. The most important of these is a possible acceleration of the center of mass of the binary, which could, for example, result from its motion relative to the center of the Galaxy. Such acceleration could also be due to a third invisible component. If this has mass $\sim M_\odot$, it is sufficient to place this component at distance ~100 astronomical units from the center of the binary.[71] We should emphasize that precisely the monotonic nature of the variation of the period due to gravitational radiation prevents one as yet from separating it with confidence from the background of other perturbations.

In practice, the experimentally measured slowing down of the orbital period of the pulsar has been found to be $\dot{T}/T \gtrsim 5\times 10^{-8}$ year^{-1} (Ref. 77), which is somewhat greater than the estimate in (3.8) and, therefore, favors the hypothesis that the center of mass is accelerated.[8)]

The majority of theories of gravitation which are competitors of general relativity, beginning with the Brans–Dicke theory, predict the existence of dipole gravitational radiation. In several variants, the post-Newtonian effects associated with the parameters γ and β are almost identical with the effects predicted by general relativity and they cannot be eliminated by ordinary tests. Dipole gravitational waves provide one of the few ways in which an experimental distinction can be made.

The intensity of dipole radiation in the various theories—Brans–Dicke, Lightman–Lee, Rosen, Eardley—can be characterized in a uniform manner by a certain dimensionless parameter ξ with a particular velue in each theory.[78] It is a general rule that dipole radiation is important if $\Delta > (v/c)^2$, where Δ is the ratio (introduced in Sec. 1) of the gravitational energy of the system to its total energy. For a neutron star $\Delta \sim 0.1 \gg (v/c)^2 \sim 10^{-6}$, so that the binary pulsar is of interest in this respect. An estimate of the rate of change of the orbital period due to dipole radiation was made by Will.[78] The effect must be of order

$$\frac{\dot{T}}{T} \approx -\xi\mu\Delta^2\left(\frac{2\pi}{T}\right)^2\left(1+\frac{e^2}{2}\right)(1-e^2)^{-5/2} \approx \xi(3.09\cdot 10^{-7})\left(\frac{\Delta}{0,1}\right)^2\frac{\mu}{M_0}\quad \text{year}^{-1}, \qquad (3.9)$$

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8) *Translator's note.* At the Ninth Texas Symposium (Munich, December 1978), J. H. Taylor reported that the most natural interpretation of the latest measurements of the slowing down of the period is that the binary pulsar *is* losing energy through gravitational radiation as predicted by general relativity. The rate of slowing down is currently measured to be somewhat larger than is expected in accordance with Einstein's quadrupole formula. (It was also pointed out at the Symposium, especially by Ehlers, that there is as yet no really sound theoretical derivation of Einstein's quadrupole formula.)

where $\mu = Mm/(m+M)$ is the reduced mass. If ξ is large, dipole radiation must give an effect appreciably larger than other reasonable mechanisms for variation of the period. In general relativity, $\xi = 0$; in the Brans–Dicke theory, $\xi = 2/(2+\omega) \lesssim 2/30 \sim 0.07$; in the remaining theories, $\xi \gg 1$.

Experimental measurement of the slowing down gives an upper limit for ξ. Arguments in favor of a particular theory can be made if the masses of the components are known.

Acceleration of the center of mass of the binary hinders observation of energy losses through gravitational radiation. However, it can be used to test nonconservative metric theories, in which the conservation laws, in particular the momentum conservation law, are violated. Calculations show[79] that in this case the center of mass is accelerated, but in a distinguished direction, namely, the direction of the periastron of the orbit.

As was pointed out in Sec. 2a, nonconservative theories contain in the post-Newtonian expansion of the interval the additional constants α_i and ζ_i, and it is these that determine the required acceleration toward the periastron:

$$\bar{a} = \frac{\pi M m (M-m) e}{T (M+m)^{3/2} P^{3/2}} (\alpha_3 + \zeta_2 - \zeta_W) \bar{n}_P. \tag{3.10}$$

Here, $\bar{n}_p$ is a unit vector in the direction of the periastron (we have used the notation of the monograph of Ref. 23 for the post-Newtonian coefficients). Without going into detail, we mention that in general relativity and the Brans–Dicke theory $\alpha_3 = \zeta_2 = \zeta_W = 0$; according to Lightman and Lee, and also from experiments in the solar system, limits on α_3 and ζ_W are known[23]: $|\alpha_3| < 2\times 10^5$, $|\zeta_W| < 10^{-2}$. Hence, an estimate of the acceleration of the center of mass of the binary toward the periastron gives a possibility for setting a limit on ζ_2, which, roughly speaking, is a measure of the contribution of the gravitational energy of the body to the active gravitational mass (see the review of Ref. 2 for a discussion of the ratio of the active and passive gravitational masses).

The acceleration (3.10) can again be determined from the change in the frequency of the radio emission of the pulsar. A feature is that the relativistic motion of the periastron must code the effect harmonically. This will make it possible to distinguish the effect, though only over an appreciable length of time. On the basis of (3.10), Will[79] gives an estimate for the variation of the orbital period due to the acceleration toward the periastron:

$$\frac{\dot{T}}{T} = 2.5\cdot 10^{-8} \frac{\chi(1-\chi)}{1+\chi} \left(\frac{m+M_\odot}{M_\odot}\right)^{2/3} \sin\varphi\,(\alpha_3 + \zeta_2 - \zeta_W)\ \text{year}^{-1},$$

where φ is the angle of the periastron (see Fig. 7). Since we have noted above that at the present time $\varphi \sim 180°$, this measurement is currently impossible; but after a few years, because of the displacement ~4° of the periastron per year, such an experiment will become meaningful. It was pointed out in Ref. 80 that anisotropy of the gravitational mass can be tested by means of this binary system.

4. SEARCHES FOR GRAVITATIONAL WAVES

Setting aside the problems of subtle distinction between relativistic gravitational effects, we must say that general relativity has been well established by many measurements in weak fields and by the first observations in a strong field. There is no single case in which the predictions of general relativity have been violated. With the advent of drag-free satellites, allowance for relativistic trajectory corrections will become an ordinary navigational operation in space. This will mean that general relativity will have passed from the status of "pure" science to that of an applied discipline.

Unfortunately, one of the most important predictions of Einstein's theory—gravitational waves—still remains beyond the bounds of experiment. If general relativity is confirmed, there is no doubt in the existence of gravitational radiation or its physical reality. For the majority of theoreticians, its discovery is only a matter of time.[82] Seen in this light, the detection of gravitational waves has much greater significance than a further ordinary confirmation of general relativity. It is necessary to discover gravitational waves and learn to "work" with them in order to have access to a new channel of astrophysical information that cannot be obtained in any other way.

Even in the case of failure, we can be consoled by a remark of Weinberg: "...gravitational radiation would be interesting even if there were no chance of ever detecting any, for the theory of gravitational radiation provides a crucial link between general relativity and the microscopic frontier of physics." (Ref. 22, p. 251) For the experimentalist, this same motive is one further stimulus to action—the value of the effect depends strongly on whether we have mastered it or not.

After the promising results obtained by Weber with the first gravitational antennas, it seemed that "gravitational-wave astronomy" had begun.[83,85] Theoretical analysis and subsequent checking of the data dispersed the initial optimism.

Today, the experimentalists are involved in the arduous pursuit of quality of gravitational detectors, their aim being to achieve a theoretically reasonable sensitivity level.

The nature of gravitational waves, calculations of various sources of radiation, and the first experimental attempts at detection have been frequently discussed in the literature.[2,21-23,81,83] In the present review, we shall consider only one aspect—the fundamentals and potentialities of Weber-type gravitational antennas.

a) Bursts of radiation from the cosmos

The pioneer in the search for extraterrestrial pulsed gravitational waves was J. Weber, Professor at the Maryland University (USA), who in 1968-1971 made a series of measurements with his gravitational antennas[84] and observed coincident bursts recorded by independent instruments separated by a thousand kilometers (details are given in the reviews of Refs. 2 and 85).

Theoretical analysis of Weber's results immediately revealed serious difficulties with the gravitational-wave interpretation of the coincidences.[86,87,101] At the antenna sensitivity $I\sim(10^6-10^7)\text{erg.sec}^{-1}.\text{cm}^{-2}$, the energy flux and the frequency of events (more than five per month) imply an energy loss by the source in the center of the Galaxy exceeding $10^3 M_\odot c^2$ per year, which is anomalously high considering the age of the Galaxy.

In 1972, experiments were made at the Moscow State University in collaboration with the Institute of Cosmic Research, USSR Academy of Sciences.[88,89] Antennas with parameters close to the Weber values had an improved detection system and were separated by ~20 km. During 20 days of pure observation time, no coincident bursts exceeding the noise level were detected.

In 1973-1974, similar experiments were repeated at Rochester University by the Bell Telephone[90] and IBM[91] Laboratories (USA), by a collaboration between centers at Frascati, Munich, and Meudon[92] (Italy, German Federal Republic, France), by the University of Glosgow[93] (Scotland), and by other groups. In no case was the sensitivity of the instrument worse than Weber's, but the Weber effect was not found.

On the basis of theoretical arguments that powerful bursts of gravitational waves should be accompanied by radiation of other kinds, attempts were made to observe radio[94] and neutrino[95] bursts correlated with Weber's events. The outcome was negative. At the same time, a correlation between the Weber events and solar and geomagnetic activity and cosmic-ray bursts was noted.[96,97]

Obviously, new measurements are necessary. However, the second-generation antennas can no longer be designed for a fortuitous case. They must correspond to sound theoretical predictions.

The prognostication of extraterrestrial pulsed sources proved a difficult task. Some clarity has been achieved recently. A complete analysis can be made for a model cluster of black holes,[98-100] with all the characteristics of the bursts of gravitational waves, including their profile, predicted.[100,102] However, the experimentalist will prefer to rely on data which may be less exhaustive but in compensation are free of the conjectural model. The estimates given below satisfy this requirement to a considerable extent.

Powerful bursts of gravitational-wave radiation can arise only if superdense stars with $r\sim r_g$ participate. Normal stars are capable of producing only an extremely weak flux of radiation at the Earth with very low frequency $\omega<10^{-4}$. The existence of black holes is still to some extent conjectural, but relativistic objects such as neutron stars have been observed and their existence is in no doubt.

We note that solid-state antennas of the Weber type can cover the frequency range $\nu\sim10^2-10^4$ Hz. Such frequencies must be emitted during a supernova explosion from its dense core. This is a fairly reliable event, and, moreover, one can very reasonably expect an asymmetric collapse and collision of dense stars.

The statistics of star masses in our Galaxy permits us to consider objects with masses $3M_\odot-30M_\odot$. More massive stars are encountered very rarely. Without making a large error, one can assume that the fraction ε of gravitational radiation is 10^{-1} to 10^{-3} of the rest energy of the object. On the basis of the observed supernova explosions in our Galaxy, the probability of an event is approximately $p\sim10^{-2}-10^{-3}$ event/year per galaxy. However, one must also take into account the possibility of unobserved events (explosion hidden by dust clouds, star clusters, etc.). Then the predicted frequency is $p\sim5\times10^{-2}$ event/year per galaxy.

The size of the part of the Universe that must be considered is dictated in practice by a reasonable integrated frequency $\gtrsim 10$ bursts per year. At mean density $n\sim3$ galaxies/Mpc^3, the distance to the most distant source will then be $R\sim\sqrt[3]{3N_0/4\pi np}\sim3-10$ Mpc. From this, the universal estimate of the expected energy density of gravitational-wave radiation per pulse on the Earth, $W=\varepsilon(Mc^2/4\pi R^2)$, must lie in the range from the maximal ~10^4 erg/cm^2 to the minimal ~1 erg/cm^2. The burst duration is $\hat{\tau}\sim(2-3)r_g/c\sim(10^{-3}-10^{-4})$ sec.

These consideration are fairly general and almost certainly reflect reality correctly. More concretely, one usually refers to the Virgo cluster of galaxies, which is ~10 Mpc from us[103] and contains between 2.5 and 3 thousand members. In this cluster, between three and four supernovae are observed on the average in a year, so that the total number of supernovae, bearing in mind invisible explosions, is probably not less than 10 per year (we recall that the number of galaxies in a sphere of radius 10 Mpc is of order 10^4, which increases the integrated number of events by a further factor 3). Making estimates like those above, we find that gravitational-wave bursts with $W\sim10^3-1$ erg/cm^2 can be expected from the Virgo cluster.

b) Sensitivity of the second-generation antennas

In the first series of gravitational-wave experiments, the sensitivity of the antennas was characterized on the average by $I_{\min}\sim10^6$ erg/sec^{-1}.cm^{-2} and band $\Delta f\sim0.5$ Hz. The tasks for the second-generation antennas dictated by estimates of the previous subsection are much more complicated and are at the limit of technical feasibility. Nevertheless, we can certainly reckon with an appreciable increase in the sensitivity as compared with Weber antennas.

The interaction of radiation with a cylindrical gravitational detector can be calculated rigorously. However, all practically important consequences can already be obtained by analyzing the simplest model of a gravitational detector in the form of a quadrupole oscillator with equivalent point masses m at distance l from each other. For the forced vibrations of the gravitational detector, one obtains an equation of Weber type:

$$\ddot{x}^\alpha+2\delta_\mu\dot{x}^\alpha+\omega_\mu^2x^\alpha=-c^2R^\alpha_{0\beta0}(t)\,l^\beta+f_{\text{n}}(t). \tag{4.1}$$

The first term on the right-hand side is the perturbing or "signal" acceleration, equivalent to the action of gravitational waves with the components of the curvature tensor $R^{\alpha}_{0\beta0}(t)$. The second term is the fluctuation acceleration produced by various noises; in the case of ideal insulation, $f_{fl}=F_{fl}/m$ is the Nyquist fluctuation force with spectral intensity $\overline{F}^2_{\nu}=4kTH$, where $H=2m/\delta_{\mu}$ is the coefficient of friction of the material of the gravitational detector. For an experimentalist, it is more convenient to go over from the "magnitude of the curvature" $R^{\alpha}_{0\beta0}$ to energy-density units W, for which the substitution must be made in accordance with

$$\left(\frac{F}{m}\right)_c=|c^2R^{\alpha}_{0\beta0}l^{\beta}|\approx\omega_{\mu}l\sqrt{\frac{8\pi G}{c^3}\frac{W}{\hat{\tau}}}. \tag{4.2}$$

In (4.2), we substitute the following parameters: frequency averaged over the range $\omega_{\mu}=3\times10^4$ rad/sec and "laboratory length" $l=50$ cm. Then for the two extreme predictions a) $W\approx10^4$ erg/sec, $\hat{\tau}\sim10^{-3}$ sec and b) $W\approx1$ erg/sec, $\hat{\tau}\approx10^{-4}$ sec, we find estimates for the amplitudes of the "signal" accelerations which excite vibrations in the gravitational detector:

$$\left(\frac{F}{m}\right)_{a)}\approx10^{-9}\ \text{cm/sec}^2,\quad\left(\frac{F}{m}\right)_{b)}\approx3\cdot10^{-11}\ \text{cm/sec}^2. \tag{4.3}$$

If the frequency of mechanical vibrations of the gravitational detector is equal to the fundamental harmonic of the gravitational burst, the change in the amplitude of the vibrations due to the burst still remains negligibly small (we assume $\hat{\tau}\ll\delta_{\mu}^{-1}=\tau^*_{\mu}$, so that $\Delta x\approx F_0\tau/2m\omega_{\mu}$):

$$\Delta x_{a)}\approx2\cdot10^{-17}\ \text{cm},\quad\Delta x_{b)}\approx1\cdot10^{-19}\ \text{cm}. \tag{4.4}$$

These estimates show how sensitive the new gravitational antennas have to be. The intrinsic thermal noise of the gravitational detector, the fluctuations due to external perturbations, and the (very complicated) noise of the detection system of the vibrations must be reduced to such an extent that the small perturbations (4.3) and (4.4) can be measured.

Programs to develop a second generation of gravitational antennas of the Weber type are currently under way in a number of foreign scientific centers, namely, at the Universities of Maryland, Stanford, Louisiana, and Rochester in the United States, and at the University of Rome and the Marconi Institute in Italy. A Soviet group is based on the Institute of Physics of the Earth (USSR Academy of Sciences), the Moscow State University, and other Institutes. A number of new methods and new technical solutions are to be used in order to achieve the required sensitivity. Let us consider the main aspects of these programs.

Suppose that we can eliminate seismic and acoustic noise by using good mechanical insulation. There remains the thermal noise of the gravitational detector. Under the condition of optimal separation of the signal from the noise of the gravitational detector, the signal can be detected if the condition

$$\frac{F_s}{m}\geqslant\frac{1}{m}\sqrt{F^2_{\nu}\frac{1}{\hat{\tau}}}=\frac{2}{\hat{\tau}}\sqrt{\frac{kT_{\mu}\omega_{\mu}\hat{\tau}}{mQ_{\mu}}} \tag{4.5}$$

is satisfied. In practice, the condition (4.5) can be satisfied by the choice of the parameters m, Q_{μ}, T_{μ}. Two variants are known. The first reflects the ideas of the program of Ref. 88 and consists of using moderate m and T_{μ} and large values of Q_{μ} achieved by making the body of the detector out of single-crystal blocks of sapphire or ruby. If in Eq. (4.5) we set $\omega_{\mu}=3\times10^4$ rad/sec, $\hat{\tau}=2\times10^{-4}$ sec, $m=2\times10^4 g$, $T_{\mu}=2^{\circ}$K, and $Q_{\mu}=10^{10}$, then we can satisfy (4.3) even in the most difficult case b); indeed, the substitution gives $F/m\approx3\times10^{-11}$ cm/sec^2.

In the second variant, the same aim is achieved by supercooling of very large masses: $m=2\times10^6 g$, $T_{\mu}\sim2\times10^{-20}$K with moderate $Q_{\mu}\approx10^6$. This program has been favored by Stanford University.

The practical realization of the technical parameters of the gravitational detectors that we have just mentioned is perfectly possible at the present level of experiments. However, this will mean that only the first step has been made toward realization of the antennas; namely, it is possible to reduce the thermal noise of the detector to the required level (4.5). It has been found, however, that the main difficulties are associated with constructing the detection system, i.e., the electromechanical transducer, or simply, sensor, by means of which small variations in the vibrations of the gravitational detector are measured. In Weber's antennas (see Ref. 85) and the majority of other antennas[90-93] the sensors were piezoelectric transducers covering the central part of the cylindrical body of the detector or clamped between the ends of two identical cylinders.[93] This type of sensor (which belongs to the class of passive transducers that do not require an external source of energy) is not suitable for antennas of the second generation.[104,105] This can be seen by simple considerations.

The condition of detection requires at the least that the power of the signal deposited in the detector during the time $\hat{\tau}$ be equal to the power of the electrical fluctuations of the sensor in the frequency band $\Delta f\sim2/\hat{\tau}$ occupied by the spectrum of the signal (under the assumption that the Brownian thermal noise of the detector is reduced by one of the methods discussed above). We have

$$\frac{(F_0\hat{\tau})^2}{2m\hat{\tau}}\beta\approx kT_e\frac{2}{\hat{\tau}}; \tag{4.6}$$

here, T_e is the temperature of the sensor and β is a very important parameter of the sensor, the coefficient of electromechanical conversion of energy, and it is equal to the ratio of the electrical energy of the signal at the output of the sensor to the mechanical energy of the antenna vibrations. The maximal value of β obtained in practice[93] for piezoelectric transducers is close to 0.1 (theoretically, β cannot exceed 0.5, which corresponds to the condition that the electrical and mechanical degrees of freedom of the antenna be matched). Thus, the minimal detectable value of F_0/m is

$$\frac{F_0}{m}\geqslant\frac{2}{\hat{\tau}}\sqrt{\frac{kT_e}{m\beta}}. \tag{4.7}$$

Substituting parameters characteristic of the variant of the Stanford program, $m=1\times10^6 g$, $T_e=0.02^{\circ}$K, $\beta\approx0.1$, and $\hat{\tau}=10^{-3}$, we find $F/m\gtrsim10^{-8}$ cm/sec, which is obviously inadequate in the face of the estimates (4.3) (the smallest detectable displacement is then $\Delta x\sim3\times10^{-16}$ cm).

Of much greater promise are transducers with an additional energy source; these are free of the restriction $\beta \lesssim 0.5$. The best instrument is a parametric transducer (fed from a pump generator) in which the mechanical vibrations of the gravitational detector modulate the capacitance or inductance of the electrical resonator. The natural frequency ω_e of the resonator is much greater than the frequency ω_μ of the gravitational detector. Under the influence of the "signal", there arise new electromagnetic harmonics at $\omega_e \pm \omega_\mu$, and these must be detected by means of a sensitive amplifier. A sensor of this type with variable capacitance and pumping frequency $\omega_e \sim 10^7$ was used successfully on the antenna of Ref. 88.

The maximal value of β for a parametric sensor under the condition of matching is equal to the ratio $\beta = \omega_e/\omega_\mu$ of the electrical and the mechanical frequencies. Hence, the formula for the sensitivity that follows from (4.6) (it is valid to within a small numerical factor) takes the form

$$\frac{F_0}{m} \geqslant \frac{2}{\hat{\tau}} \sqrt{\frac{kT_e\omega_\mu}{m\omega_e}}. \tag{4.8}$$

The matching condition under which (4.8) is satisfied consists of the requirement that the pumping voltage have a definite value U_0. Note that the condition of matching (maximum of the signal at the transducer output) and the condition of high sensitivity (minimum of the detectable value of F_0/m) require, in general, different values of the pumping voltage U. It was shown in Ref. 106 that in the case of optimal filtration of the sensor signal an excess $U > U_0$ leads to reduction of the right-hand side of (4.8) by U_0/U times. However, this process is accompanied by a decrease of β by $(U_0/U)^2$ times, and therefore higher requirements are imposed on the noise of the subsequent amplifier. The physical reason for the decrease in the signal response of the antenna is simple—the increase in the coupling between the gravitational detector and the sensor brought about by increasing U lowers the frequency ω_μ, moving it away from the frequency ω_0 of the gravitational wave. For $U > U_0$, the shift $\Delta\omega$ exceeds the width $\sim 2/\hat{\tau}$ of the spectrum of the signal, and then the reaction of the gravitational detector to nonresonance excitation begins to fall appreciably. If one artificially tunes the frequency of the gravitational detector by some third method keeping ω_μ within the spectral band of the signal, the sensitivity still remains frozen at the level (4.8) because of the effective amplification of the noise in the strongly coupled parametric system (the so-called feed-back effect).[107] Let us substitute in (4.8) parameter values characteristic for the variant of the program with high Q gravitational detector, i.e., $m = 2 \times 10^4$ g, $T_e = 2°$K, $\omega_\mu = 3 \times 10^4$. It is advantageous to take the pumping frequency as high as possible, though technical restrictions associated with the absence of high-frequency generators of a harmonic signal with sufficient frequency stability permit the choice of pumping with only $\omega_e \lesssim 2 \times 10^{10}$ rad/sec (microwave range).[107] Then for $\hat{\tau} \sim 10^{-3}$ sec, we find that $F_0/m \gtrsim 3 \times 10^{-10}$ cm/sec^2, i.e., we achieve the sensitivity $\sim 10^2$ erg/cm^2, which is below the upper limit of the optimistic prediction for the energy density of a burst.

How can one approach the lower limit ~ 1 erg/cm^2 with $\hat{\tau} \sim 2 \times 10^{-4}$ sec? In the framework of a classical parametric sensor, the only possibility that remains is to use the reserves for increasing the sensitivity associated with sacrificing the magnitude of the useful signal. Suppose that the experimentalist has at his disposal a low-noise amplifier which is such that one can lower the amplitude of the signal by a factor 50 compared with the maximal "matched" value. Then, multiplying (4.8) by the factor $U/U_0 = 30$ and substituting $\hat{\tau} \sim 2 \times 10^{-4}$ sec, we obtain $F_0/m \gtrsim 3 \times 10^{-11}$ cm/sec^2. Thus, a parametric sensor does in principle make it possible to reach the lower limit of the predicted burst intensity. We now formulate more concretely the requirements on the noise temperature T_n of the amplifier which receives the signal from the sensor. If the sensor operates in the matched regime, the noise temperatures of the amplifier and the sensor must be equal (we assume that the pump generator has no noise). In the regime $U > U_0$, the noise temperature of the amplifier must be appreciably lower. We find the condition on T_n from the condition that the power of the signal be equal to the power of the noise in the band $\Delta f \approx 2/\hat{\tau}$ (Refs. 105 and 106):

$$\frac{(F_0\hat{\tau})^2}{2m\hat{\tau}} \frac{\omega_e}{\omega_\mu} \left(\frac{U}{U_0}\right)^2 \geqslant kT_n \frac{2}{\hat{\tau}}. \tag{4.9}$$

Remembering that the signal pulse contains approximately one period of the frequency ω_μ, i.e., $\hat{\tau} \sim 2\pi/\omega_\mu$, and using (4.2), we obtain from (4.9) ($U \gtrsim U_0$)

$$T_n \leqslant \frac{8\pi^2 m\omega_e l^2}{c^3 k} \left(\frac{U_0}{U}\right)^2 W. \tag{4.10}$$

If we substitute here $m = 2 \times 10^4$ g, $\omega_e = 2 \times 10^{10}$, and $l = 50$, for $U = U_0$, then we find that $T_n \lesssim 10°$K for $W = 10^4$ erg/cm^2 and $T_n \lesssim 10^{-3}$°K for $W = 1$ erg/cm^2. Modern maser amplifiers have a noise temperature in this frequency range of order 1–10°K. This means that in the case of a low-mass detector one can only hope for a sensitivity level $W \sim 10^3$ erg/cm^2. For more massive detectors, the requirements on the noise temperature are reduced and the detectable values of F/m and W are lowered. However, there is a simultaneous increase $U_0 \sim m^{1/2}$ and it is difficult to adjust the antenna even to the matched regime, so that an excessive increase of m does not save the situation.

These estimates demonstrate the complexity of exploiting the sensitivity reserve in the regime $U > U_0$. The situation is such that further progress in measurement technique is necessary. This will involve either the development of amplifiers with very low noise temperature or the finding of new low-noise sensors with high conversion coefficient.

c) Quantum magnetometers and gravitational antennas

Attempts to overcome the difficulties noted above have been associated in a number of studies[108-111] with the idea of using quantum magnetometers based on the Josephson effect, the so-called squids. Squids have made it possible to achieve record sensitivity in measurements of magnetic fields and magnetic flux with a relatively simple construction. It seemed that the advantageous combination of a gravitational detector with a sensitive magnetometer would give good results. However, the theoretical analysis in Ref. 112 showed that single-junction squids with high-frequency external pumping are parametric transducers and that in the best case the sensitivity of the antenna would be subject to Eq. (4.7). In particular, for the most widespread squid—the Zimmerman–Silver magnetometer in the hysteresis regime—we have the following formula for the minimal detectable acceleration[112]:

$$\frac{F_0}{m} \geqslant \frac{4\pi}{\hat{\tau}} \sqrt{\frac{kT_s\omega_\mu}{m\omega_e} \frac{1}{\gamma^{1/3}}}, \tag{4.11}$$

where the parameter $\gamma = 2\pi k T_s / \Phi_0 I_0 \ll 1$ is the relative measure of the thermal energy and the energy of the quantum transition in the sensor, T_s is the temperature of the squid, and I_0 and Φ_0 are the critical current of the Josephson junction and the magnetic flux quantum. Comparison of (4.8) and (4.11) illustrates what we have said above.

The situation is changed on the transition to a quantum interferometer—a magnetrometer with a constant feed current containing two Josephson junctions. A gravitational detector with such a sensor is shown schematically in Fig. 8. It was pointed out in Ref. 113 that such an antenna can in principle achieve the sensitivity level determined by formulas (4.3) and (4.4).

The quantum interferometer can be regarded as a sensor of parametric type with self-pumping and self-detection. Once the constant feed current exceeds the critical value I_0 at which the superconductivity is destroyed, in at least one of the junctions, a high-frequency current arises in the loop because of a Josephson effect of the second kind (i.e., an intrinsic rf "pumping" generator is switched on). The external magnetic flux, which varies slowly in step with the vibrations of the gravitational detector, penetrates the loop and modulates the variable Josephson current in amplitude and phase. Because of the nonlinear properties of the junctions, a detection operation is realized, and a "slow" voltage appears on the terminals of the loop. The resonance circuit, tuned to the frequency ω_μ, filters this slow voltage. It is remarkable that since all three elements of the parametric sensor—the pumping generator, the rf resonance circuit, and the detector—are combined, the only source of noise is the normal resistance R of the junction (the flicker noise of the displacement current lies in the range of frequencies below 1 kHz). The frequency of the Josephson generation depends on this resistance and the critical current: $\omega_0 = 2\pi R I_0 / \Phi_0$; by a choice of parameters it can in principle be raised to $\omega_0 \sim 10^{12}$. It is precisely the increase in the effective pumping frequency by two orders of magnitude compared with the $\omega_e \sim 10^{10}$ in (4.7)–(4.10) that makes it possible to reckon with attainment of the lower limit $F/m \sim 3 \times 10^{-11}$ cm/sec^2 (for $m \sim 10^6$, $T_e \sim 2°$K) of the theoretical prediction.

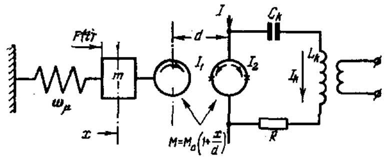

FIG. 8. Gravitational wave detector with quantum interferometer.

Here, it is true, one must draw conclusions carefully and bear in mind quantum restrictions on the sensitivity, which begin to play a role as $\omega_e \to \infty$ (or $T_e \to 0$). In this limit, the detectable acceleration no longer depends on the pumping parameters, but satisfies the formula

$$\frac{F_0}{m} \geqslant \frac{1}{\tau} \sqrt{\frac{\hbar\omega_\mu}{m}}. \tag{4.12}$$

This last limit is due to the fact that quantum fluctuations become decisive as compared with thermal fluctuations. It is important to estimate the critical pumping frequency at which the relation (4.8) still holds. This critical frequency depends on the efficiency of conversion of the high-frequency quantum fluctuations into the region of low frequencies (to the frequency ω_μ of the measurements). An analysis for the quantum interferometer made in Ref. 113 shows that $(\omega_0)_{cr} > 10kT/\hbar \sim 10^{12}$, $T_e \sim 2$–$4°$K, i.e., one can reckon with a pumping frequency $\omega_e = \omega_0 \sim 10^{12}$ even when quantum restrictions on the sensitivity are taken into account.

CONCLUSIONS

To summarize this review, we should like to emphasize that gravitational experiments are now being concentrated more and more on the wave zone rather than the nonwave zone. This is because there are now virtually no doubts about the validity of the conclusions of general relativity for the nonwave zone—they have been frequently verified with an accuracy of about 1%. A deeper verification in the second order in the small parameter $(\varphi/c^2)^2$ in weak fields will require long preparation. In addition, the practical value of high-precision measurements of the post-Newtonian coefficients is obviously less than that of steps which could lead to the discovery of gravitational waves and their subsequent exploitation. There is no doubt that searches for bursts of gravitational radiation from space using second-generation antennas will be made, and this will require not less than five years. However, the situation will become much more complicated if waves are not detected at this new level. Then further searches with terrestrial antennas will have to be postponed because of the restrictions of the existing measuring techniques. It is possible that new progress here will be associated with the development of the idea of quantum nondemolition measurements.[114]

If negative results are obtained with the terrestrial second-generation antennas, the center of gravity will evidently be displaced to experiments with space antennas with a very long baseline of several astronomical units (the feasibility of space gravitational antennas is discussed in Ref. 115).

Radio interferometers with very long baseline in space promise to provide the basis for the development of one further (nonwave) direction in relativistic gravitational experiments. We are here referring to the possibility discussed in Ref. 116 of a direct measurement of the mean density of matter in the Universe and its variations on the basis of the relativistic parallaxes of distant objects at cosmic distances.

However, it is at present difficult to say how soon the experimentalists will attack the development of this new direction.

In conclusion, let us point out a further circumstance. Today, the activity of the gravitational experimentalists is usually regarded as "pure science" or, "fundamental research". One can hardly speak of the applied importance of relativistic gravitational measurements, although, with the advent of drag-free satellites, relativistic effects will become "navigational corrections to the trajectories of space probes" in flights in space. Nevertheless, the efforts expended even now on experiments in relativistic gravitational physics have significant practical fruits. Indeed, the pursuit of sensitivity in the investigation of minute effects on the Earth and in space has stimulated an appreciable development of technology and experimental techniques; new instruments suitable for investigations in various scientific and technical problems have been developed. As an example, we can mention the drag-free satellites that were born out of the idea of geodesic motion and have become sensitive accelerometers which are irreplaceable in studies of the density of the atmosphere and the figure of the Earth.

It would seem that the as yet fruitless work on gravitational antennas has already led to the creation of a new large-scale cryogenic technique, which operates with masses of several tons at temperatures of thousandths of a degree. Here we should also include the development of secondary microwave frequency standards with stability unavailable to modern masers. And not only experimental techniques but numerous fundamental questions of applied and basic physics have been partly developed during gravitational experiments; for example, the experimental investigation of the dissipative properties of various alloys and single-crystal materials and the development of the technology for obtaining mechanical and electromagnetic resonators with record Q values. Finally, there is the measurements of extremely small forces and deformations. Here, the experimental exigencies have given rise to many new problems relating to the determination of the classical and quantum limits of the accuracy of measurement. We have here a newly developing field—the theory and methods of ultimate measurements in physics—and this is of undoubted fundamental and practical importance.

Thus, even now the work of the gravitational experimentalists must be recognized to be very furitful, although, of course, the discovery of gravitational waves would place this evaluation beyond doubt and provide the stimulus for new efforts in this difficult sphere of activity.

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