Resonance light pressure

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The forces acting on a resonance particle in the fields of laser radiation sources can be large. Acceleration, heating, and cooling of atoms by using these forces are discussed. Since the forces are of a resonance type, one can employ them to separate isotopes and excited atoms from unexcited atoms. Light pressure can play an important role in the spectroscopy of narrow atomic and molecular resonances. The quantum features of the motion of particles in a standing light wave are manifested in a fine structure of the absorption coefficient. This review is concerned with analyzing the resonance light-pressure forces and discussing the application of them in optics and quantum electronics.

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CONTENTS

1. INTRODUCTION

Light pressure originates from the recoil that atoms undergo when they scatter photons.

Whenever light acts on a massive dielectric or a dense gas the effects caused by light pressure prove to be small. Thus, in the first experiment of P. N. Lebedev,[1,3] he measured a force of light pressure of the order of 10^{-4} dynes/cm^2, which was considerably smaller than the radiometric forces. P. N. Lebedev also measured the pressure of light on a gas in resonance absorption. This proved to be even smaller (by two or three orders of magnitude) since the source had a low spectral density of radiation. The picture changes substantially when a monochromatic electromagnetic field acts on a rarefied gas of resonance atoms. Near resonance, the scattering cross-section for the photons rises and the force of the light pressure acting on the individual atoms can become very large. Therefore laser light sources, which have high power and a high degree of monochromaticity, open up new possibilities of observing and utilizing light pressure.

We can easily estimate the force acting on an atom in a resonance light field by quantum considerations. In the field of a strong running wave, the atom absorbs a photon from the light beam and acquires the momentum $\hbar\mathbf{k}$ of the photon. Consequently the force[8] $\hbar k\gamma/2$ acts on the atom, where γ is the spontaneous-transition frequency. It is of the order of magnitude of 10^{-3} eV/cm for strong optical transitions. The force of light pressure attains an even higher value in the field of a standing wave. Upon absorbing a quantum with the momentum $\hbar\mathbf{k}$, and undergoing stimulated emission of a quantum with the momentum $-\hbar\mathbf{k}$, the atom acquires the momentum $2\hbar\mathbf{k}$ within the stimulated-transition time $(dE/\hbar)^{-1}$. Here d is the dipole moment of the transition, and E is the field intensity. The force of the stimulated light pressure is of the order of kdE, and it coincides with the gradient force that acts on a dipole of moment d in an inhomogeneous field. It is of the order of 1 keV/cm when $E \sim 10^6$ V/cm. This force oscillates in space with a period equal to the wavelength of the resonance field. If the atom is accelerated in such a field only to a distance of half the wavelength, it acquires an energy greater than thermal. The acceleration effect can be substantially enhanced if the frequency of one of the opposing waves varies with time. Thus the mechanical effect of a resonance field on atoms can be considerable. Hence a number of problems arise in optics in which one must take into account not only the change in the states of the atom in the field (transitions from one level to another), but also account for the change in the trajectory of motion of the atom in this field.

Until recently, the effects of light pressure were studied mainly in dielectrics[13,14] and in a plasma.[15,16] The effect of stimulated Mandel'shtam-Brillouin scattering[17] also involves the action of light pressure, but it is usually studied far from resonance. Recent experiments[11,12,28] and theoretical studies[5–10,21,50] have shown a promising approach in applying effects of resonance light pressure in various fields of optics and quantum electronics.

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This review is concerned with analyzing the forces of resonance light pressure. The fundamental types of forces that act on an atom in a field are examined, and they are classified by type of photon emission. The effects of acceleration of atoms and molecules by these forces are estimated and possible applications are discussed.

The set of problems involving resonance light pressure proves to be very large, and it encompasses an energy range of the order of 10^{-10}–10^{4} eV. The lower bound of this range is defined by the recoil energy when an atom emits one photon.

Accounting for recoil in the interaction of slow particles with a field becomes necessary when one studies narrow atomic and molecular resonances.[6,7] In the field of a standing wave, one must account for the effects of quantization of the motion of the atoms.[42] Discontinuities or narrow lines can arise in the spectrum of an atom that correspond to the energies of bound states. In this case dips or narrow peaks appear in the absorption coefficient.

The quantization of the motion of the atoms in a resonance standing wave has certain features that involve the fact that the atom generally has two trajectories of motion.[24] Indeed the induced dipole moment of the atom can be directed with or against the field. In the one trajectory the atom has a positive potential, but negative in the other. The atom can go from the one trajectory to the other while moving in the inhomogeneous field. For example, this increases the width of the resonance of the bound state. Section 6 treats the effect of the quantum nature of the motion of atoms on the fine structure of the absorption coefficient.

The energy range near thermal energy is of especial interest. Here it takes a field of relatively small power to act on atoms and molecules. One can accelerate and decelerate, or heat or cool atoms by light pressure. Since the interaction with the field is of resonance type, it proves possible to affect selectively particular atoms and molecules. This situation is important in such applications as separating isotopes (Sec. 5) and excited and unexcited atoms. For example, one can accelerate metastable helium atoms and cool them to such a low temperature by using a standing wave that all the excited atoms take part in laser action in a short-wavelength transition. This does not require an inverted population of levels (Sec. 7).

As estimates show, atoms can be accelerated under special conditions to energies of the order of 10 keV.

It is interesting to compare resonance and nonresonance light pressure. In an inhomogeneous electromagnetic field a charged particle has an effective potential that is quadratic in the field.[4] The potential of an atom in a resonance field is linear in the field. In fields smaller than the critical ionization field, the resonance potential is considerably larger than the non-resonance potential. Hence the action of light (e.g., scattering in a standing wave) is much greater for resonance atoms than for electrons. And, in addition to the quantitative difference, there are the following two fundamentally different features:

1) In a resonance light field, the "atom + field" system possesses two states having different effective potentials. Hence, as we have noted, the atom generally has two trajectories of motion. Far from resonance (or when the field is slowly turned on), the atom exists in a definite state and has a single trajectory of motion. Near resonance itself (or when the field is turned on quickly), both states are occupied with about the same probability, and the atom has two trajectories of motion. This leads to double refraction of a molecular beam incident on a vacuum-electromagnetic field boundary, to certain features of the diffraction of atoms by a standing wave, and to broadening of resonances of bound states owing to a strong tunneling effect.

2) Spontaneous emission becomes important near resonance, and it destroys the coherence of the interaction of the atom with the field. Moreover, in spontaneous emission the atom goes from one quasilevel to another having a different potential. This means that it goes from one trajectory to another. The lifetime of the atom in each trajectory proves to be finite.

This effect is manifested most distinctly when atoms move in a standing monochromatic wave. If the frequency of the field is smaller than the transition frequency, then the atoms are decelerated by the field and become cooled. Conversely, if the frequency of the field exceeds the transition frequency, the energy of the atoms continuously increases.[21] This problem is treated in detail in Sec. 3.

In the general case it is a complicated problem to find the forces acting on an atom while taking account of the cited effects. The fundamental difficulty involves the fact that one generally cannot assume the motion of the atom to be fixed in finding the induced dipole moment. The rigorous description of this effect is discussed in Secs. 4 and 6. The remainder of the review will use the approximation of fixed motion of the atom in finding the forces.

2. THE FORCES ACTING ON AN ATOM IN A RESONANCE LIGHT FIELD

In a resonance electromagnetic field of the form

$$\mathbf{E}(rt)\,e^{-i\omega_0 t} + \text{complex conjugate}$$

the following force acts on the induced dipole moment $\mathbf{p}(t)\,e^{-i\omega_0 t} + \text{complex conjugate}$[18]:

$$\mathbf{F} = \sum_i p_i \nabla E_i^{*} + \text{complex conjugate}. \quad \mathbf{p}(t) = \mathrm{Sp}\,[\hat{\mathbf{d}}\rho(t)], \tag{1}$$

Here ω_0 and $\mathbf{d}$ are the frequency and the dipole-moment operator of the transition, and ρ is the density matrix for the atom. The amplitude $\mathbf{E}(rt)$ of the field varies little within the period $2\pi/\omega_0$. The force $\mathbf{F}$ is averaged over this period (the resonance approximation). In order to find the dipole moment $\mathbf{p}(t)$, we must use the equation for the density matrix taking the relaxation operator $\hat{\gamma}$ into account:

$$i\hbar\left(\frac{\partial}{\partial t} + \hat{\gamma}\right)\rho = [\mathscr{H}, \rho], \tag{2}$$

$$\mathscr{H} = -\hat{\mathbf{d}}\mathbf{E}^{*} + \text{complex conjugate}. \tag{3}$$

Henceforth we shall neglect polarization effects and treat the atom as having two levels, while the field varies only along the x coordinate. Then the formula for the force has the form

$$F = p\frac{\partial E^*}{\partial x} + \text{complex conjugate.} \tag{4}$$

Let us write Eq. (2) for the individual components of the density matrix: for the dipole moment $p = d\rho_{12}$ and for $q = \rho_{22} - \rho_{11}$ (difference in population of levels):

$$\frac{\partial p}{\partial t} + \frac{\gamma}{2} p = \frac{i}{\hbar} d^2 E(t)\, q, \qquad x = vt, \tag{5}$$

$$\frac{\partial q}{\partial t} + \gamma(1+q) = 2iE(t)\, p^* + \text{complex conjugate.} \tag{6}$$

Here v is the velocity of the atom.

The lower level of the atom is assumed to be the ground state (or a metastable state), while the lifetime of the upper level is γ^{-1}. However, Eq. (4) has a very restricted region of applicability in a resonance field, since it neglects two important circumstances. First, Eq. (1) does not account for the fluctuations of the force arising from quantum fluctuations of the dipole moment. The latter can become very great near resonance and can split the single trajectory into two. Second, Eq. (4) actually assumes that the variation of the motion of the atom in the field has little effect on the size of the dipole moment. That is, the force depends adiabatically on the velocity of the atom. We shall call this approximation the fixed-motion approximation. A rigorous formulation of the problem requires solving the quantum kinetic equations (see Sec. 4). Here we shall treat the simpler characteristic cases in which one can either disregard the fluctuations of the force in (1) or neglect spontaneous emission, or treat the motion of the atom as fixed.

The recoil that the atom suffers in interacting with the field substantially depends on the rate of emission of photons. Hence one can naturally classify the forces of resonance light pressure in terms of the type of photon emission. From this standpoint there are three fundamental types of forces, which are defined by spontaneous and stimulated emission and emission of mixed type.

A. The force due to spontaneous emission

In the field of the following plane running wave (Δ is the detuning):

$$E(xt) = Ee^{-i\Delta t + ikx}$$

the force is proportional to Im (pE^*), a quantity that determines the energy of the field that is absorbed by the atom per unit time. Upon finding the induced dipole moment from Eqs. (5) and (6), we get[8]

$$F_0 = \gamma\hbar k W, \quad W = \frac{|dE|^2}{\hbar^2[(\Delta - kv)^2 + (\gamma^2/4)] + 2|dE|^2}; \tag{7}$$

Here W is the probability of population of the upper level.

Upon absorbing a photon from the light flux and spontaneously emitting a spherical wave, the atom acquires the momentum $\hbar\mathbf{k}$ in the direction of propagation of the wave within a cycle of duration γ^{-1}.

However, the spherical wave corresponds to the classical limit. From the quantum standpoint each elementary event of photon emission bears away a momentum equal in magnitude to $\hbar k$ in an arbitrary direction. As we know, the probability of emission of a photon in a given direction is determined by the intensity of the spherical wave emitted by the dipole. The quantum nature of the emission leads to fluctuations of the light-pressure force about the mean value of (7). However, the relative contribution of these momentum fluctuations falls off approximately as $1/\sqrt{N}$ when there is a large number of scattered photons $N = \gamma t$, where t is the interaction time.[32]

We note that stimulated transitions in the field of a running wave do not contribute to the mean force of the light pressure. Therefore, in a strong field the force does not depend on the intensity of the field, and is determined only by the rate of spontaneous emission of photons $F_0 = \hbar k\gamma/2$. For example, in this case we have $F_0 = 2\times10^{-3}$ eV/cm for the resonance line of Na. The scattering of atoms by the force (7) has been studied experimentally[11,12,28] (see Sec. 5).

The force F_0 shows a resonance-type dependence on the frequency of the field. In a strong field the width of the resonance is determined by the Stark shift $dE/\hbar$, and in a weak field by the natural width $\gamma/2$ of the line.

We note that the formula for the force F_0 at low emission intensities was established by P. N. Lebedev, who studied the pressure of a high-frequency field on a vibrator.

B. Stimulated light pressure

Now let us study the force of light pressure in the inhomogeneous field of a standing wave:

$$E(xt) = e^{-i\Delta t} E(x), \quad E(x) = E\cos(kx + \varphi), \tag{8}$$

Here φ is the phase of the wave.

In a strong field ($dE \gg \hbar\gamma$), we can neglect damping in a first approximation. This approximation is rigorous if the atom interacts with the field for a short time less than γ^{-1}.

In this case the fluctuations of the dipole moment can be large. Since the system of equations (4)–(6) does not account for them, we shall start directly with the Schrödinger equation for an atom of mass M having two states:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2M}\frac{\partial^2\psi}{\partial x^2} - \begin{pmatrix} \hbar\Delta/2 & dE(x) \\ dE(x) & -\hbar\Delta/2 \end{pmatrix}\psi, \quad \psi = \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix}. \tag{9}$$

When $\Delta = 0$, the system of equations (4) after transforming to the wave functions $\psi_\pm = (\psi_1 \pm \psi_2)/\sqrt{2}$ breaks down into two wave equations that describe the motion of the particles in the potentials $\pm V(x)$, where

$$V(x) = dE(x), \quad |\Delta| < \Delta_0. \tag{10}$$

Although the field is a quantity that oscillates in space, the dipole moments in the states $\psi_\pm$ have the constant values $p_\pm = \pm d$. We shall call the small frequency interval near resonance where $|\Delta| < \Delta_0$ in which the dipole

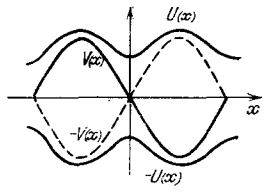

FIG. 1. The resonance $V(x)$ and the nonresonance potential $U(x)$ of an atom in a standing wave.

moment does not follow the field the nonadiabatic interval.

When the detuning is large, one can diagonalize the equations of (9) in a quasiclassical approximation. Then we have two groups of particles that move in the potentials $\pm U(x)$, where

$$U(x)=\frac{\hbar}{2}\Delta\chi(x),\quad \chi(x)=\sqrt{1+\left(\frac{2V(x)}{\hbar\Delta}\right)^2},\quad |\Delta|>\Delta_0. \tag{11}$$

The wave functions of the independent states and the dipole moments have the form

$$\psi_-=\frac{\psi_1-a(x)\psi_2}{\sqrt{1+a^2(x)}},\quad \psi_+=\frac{\psi_2+a(x)\psi_1}{\sqrt{1+a^2(x)}},\quad a(x)=\frac{2V(x)/\hbar\Delta}{1+\chi(x)},$$
$$p_\pm=\pm da(x). \tag{12}$$

In this case the dipole moments "follow" the variation of the field. Therefore we shall call the frequency interval $|\Delta|>\Delta_0$ the adiabatic interval.

Figure 1 shows the resonance and nonresonance potentials of the atom. The depth of modulation of the potential $V(x)$ exceeds that of $U(x)$ by a factor of more than two. This fact is essential in the selective scattering of atoms in the field of a standing wave[43] (see Sec. 5).

As $\Delta\to 0$, we have $U(x)=\operatorname{sign}\Delta|V(x)|$, i.e., the nonresonance potential of the atoms does not convert into the resonance potential. The lack of a limiting transition involves the breakdown of the quasiclassical approximation at small detunings at the nodes of the standing wave, where $U(x)$ has a sharp discontinuity. In the vicinity of the nodes of the standing wave, we can consider $E(x)$ in Eq. (9) to be a linear function of x. Then one can use the Landau-Zener theory[19] in the quasiclassical limit. After passing the node of the standing wave, the atom has a trajectory that corresponds to the "deep" potential $V(x)$ with the probability $\exp(-\Delta^2/\Delta_0^2)$, where $\Delta_0=\sqrt{2dEkv/\pi\hbar}$. Correspondingly, $1-\exp(-\Delta^2/\Delta_0^2)$ is the probability that it will lie in the "shallow" potential $U(x)\approx|V(x)|$.

If the atoms are trapped in the potential wells of the standing wave, then kv coincides with the characteristic oscillation frequency $\Omega=k\sqrt{dE/M}$. For trapped atoms the frequency that separates the resonance from the nonresonance region (or the nonadiabatic from the adiabatic region) has the form

$$\Delta_0=\sqrt{\frac{2dE\Omega}{\pi\hbar}}. \tag{13}$$

In a strong field the following relationship obtains between the characteristic frequencies: $\Omega\ll\Delta_0\ll dE/\hbar$. The criterion for a strong field coincides with that for quasiclassical behavior: $dE\gg(\hbar k)^2/2M$. For allowed dipole transitions the latter condition is satisfied even in weak fields $E\gtrsim 10^{-2}$ V/cm. Thus the atom can have the definite potential $V(x)$ or $U(x)$ only in the two limiting cases $|\Delta|\ll\Delta_0$ and $|\Delta|\gg\Delta_0$. When $|\Delta|\approx\Delta_0$, the atom randomly goes from the one trajectory to the other, while spending about the same time in each potential. In particular, this effect broadens the levels of bound atoms, as can be noted from the absorption coefficient (Sec. 6).

In the state described by the wave function ψ_- the atom has the potential $\overline{U}(x)$, and it has the potential $-U(x)$ in the state ψ_+. As the field is slowly turned off, the first state goes over into the ground state ψ_1 and the second into the excited state ψ_2. Henceforth for brevity we shall call the state ψ_- the "lower" state and the state ψ_+ the "upper" state.

As the field is turned off adiabatically, the atom is found all the time in the "lower" state and it has the potential $U(x)$ (apart from the small nonadiabatic frequency region). Curve 1 in Fig. 2 shows the relationship of U to Δ. The adiabatic potential of the atom is determined by the Stark frequency shift. When $|\Delta|>2dE/\hbar$, the shift is quadratic in the field, and we have $U(x)=[dE(x)]^2/\hbar\Delta$. The potential of the atom far from resonance is treated in Ref. 5. The force F_1 of stimulated light pressure is the gradient force:

$$F_1=-\frac{\partial U}{\partial x}. \tag{14}$$

From the quantum standpoint, stimulated light pressure arises from the absorption of a photon from one light flux with stimulated emission of it into the opposing flux. Since a change in the sequence of transfer of quanta reverses the sign of the force, F_1 in a standing wave depends on the phase of the wave, and it oscillates with the period $\lambda/2=\pi/k$. The frequency of quantum transfer is of the order of $dE/\hbar$, so that $F_1\sim kdE$. In a field of intensity 10^6 V/cm (alkali-metal atoms and metastable helium atoms[23]) have critical fields with respect to ionization in this range) and with $d\sim 5$ Debye, we have $F_1\sim 1$ keV/cm. The quantity $F_1/F_0\sim dE/\hbar\gamma$ is the ratio of the frequencies of stimulated and spontaneous transitions. Even in fields greater than 1 V/cm, the force of stimulated light pressure exceeds F_0.

When the field is turned on rapidly the two mixed states, the "upper" and the "lower," are excited with equal probability. Although the mean potential of the atom is zero in this case the effect of the light on the atoms does not at all vanish. Simply two groups arise containing the same number of particles that move in

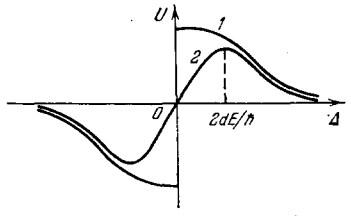

FIG. 2. Dependence of the potential (minus the constant component) on the detuning. Curve 1—adiabatic potential for $\gamma t<1$, curve 2—mean potential for $\gamma t>1$.

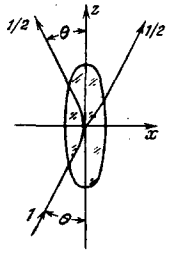

FIG. 3. The effect of double refraction of a molecular beam in a resonance electromagnetic field.

potentials that differ in sign. The trajectory of an atom is split into two trajectories. The criterion for sudden perturbation (where τ is the time for turning on the field) is:

$$\frac{2U\tau}{\hbar} < 1 \qquad (15)$$

It can be realized not only when the field is turned on rapidly, but also when the atom is excited rapidly (e.g., by electron impact) to a state on which the resonance field acts. Another example can be the passage of atoms or molecules through a vacuum-electromagnetic field boundary.

1) *Double refraction of a molecular beam.*[24] Let us examine the passage of a monoenergetic molecular beam[1)] through the boundary of a light beam. The beam amounts to a plane wave running along the y axis. We can conveniently take the cross-section of the beam (cross-hatched in Fig. 3) to be an ellipse of dimensions l_z and $l_x \ll l_z$.

At exact resonance ($\Delta = 0$), the light beam acts as a semitransparent mirror. Actually, half of the molecules has the negative potential $-dE(x)$, is attracted into the light beam, and moves on in the original direction. The other half of the atoms has a positive potential and is reflected from the light beam as from a mirror. This happens in a sufficiently strong field $dE(0) \geq Mv_x^2/2$, where $E(0)$ is the field intensity at the center of the beam, and v_x is the velocity of normal incidence. From this condition we can express the necessary radiation intensity J in terms of the kinetic energy T of the particles and the angle of incidence θ:

$$J = \frac{cl_y l_x}{16}\theta^4 \left(\frac{T}{d}\right)^2. \qquad (16)$$

Here the time of longitudinal motion must not be smaller than the time of normal motion: $l_z/v_z \gtrsim l_x/v_x$ or $l_z \gtrsim l_x/\theta$. If we assume that $l_x = 10^{-2}$ cm, $l_z = 1$ cm, $d = 0.3$ Debye, $T = 300$ °K, and $\theta = 10^{-2}$ radian, we get $J = 125$ W. Of course, a lower radiation power is required for slower molecules. The time of passage through the beam $\tau = l_x/v_x \sim 2\times 10^{-5}$ sec is shorter than the spontaneous emission time. The double-refraction effect as a function of the frequency has the form of a sharp resonance having the width $\Delta_0 = \sqrt{dE/\hbar\tau} \sim 0.5$ MHz. When $|\Delta| > \Delta_0$, the molecule has a single trajectory: it either passes through the beam when $\Delta < 0$ or it is reflected from it when $\Delta > 0$. The potential diminishes at large detunings and the reflection effect disappears. The width of the resonance with respect to reflection of molecules from the light beam is $2dE/\hbar$, and in this case it amounts to 10^3 MHz.

We note that the effect of double refraction of a molecular beam is analogous to the well-known Stern-Gerlach phenomenon.

Thus atoms and molecules have two trajectories when a resonance field is rapidly turned on. Another source of doubling of trajectories is spontaneous emission.

2) *The effect of spontaneous emission.* Thus far we have been assuming the time of interaction of the atoms with the field to be less than the spontaneous emission time. When $\gamma t > 1$, we must account for transitions from the "lower" to the "upper" state and vice versa. The lifetimes in the trajectories are estimated in Sec. 4. Outside resonance ($\hbar\Delta \gg dE$), the lifetime of the "lower" state is long, while that of the "upper" state is of the order of γ^{-1}. In a region of strong saturation ($\hbar\Delta \ll dE$), both states have about the same lifetime $4/\gamma$. In order to illustrate the influence of spontaneous emission let us study the mean potential of an atom in a standing wave. This quantity has meaning for an atom for which the kinetic energy is larger than the potential energy, and for which the fixed-motion approximation holds.

In a strong and slowly varying field ($\Delta \gg kv$), we can represent the solution of Eq. (5) as an expansion in Δ^{-1}:

$$p = \frac{d}{\hbar\Delta}\left[-V(t)q + i\left(\frac{\partial}{\partial t} + \frac{\gamma}{2}\right)\frac{V(t)q}{\Delta} + \ldots\right], \qquad V(t) = dE(t). \quad (17)$$

The second term in this expression is small, but it is the only one that contributes to $q(t)$ in Eq. (6):

$$\chi^2(t)\frac{\partial q}{\partial t} + \left[\chi(t)\frac{\partial\chi(t)}{\partial t} + \tilde{\gamma}(t)\chi^2(t)\right]q = -\gamma, \quad \tilde{\gamma}(t) = \frac{1}{2}\gamma[1 + \chi^{-2}(t)]. \qquad (18)$$

The parameters χ and $\tilde{\gamma}$ characterize the saturation and damping in the atom. They vary in time as the atom moves in the standing wave. Upon substituting the solution of Eq. (18) into the formula (4) for the force, we find the following value of the mean potential $U_{av}(x)$ for $\gamma t \gg 1$ [21]:

$$U_{av}(x) = \gamma U(x)\frac{\overline{\chi^{-1}}}{\overline{\tilde{\gamma}}}. \qquad (19)$$

The bar denotes averaging over a period of oscillation of the field. Curve 2 of Fig. 2 shows the relationship of U_{av} to Δ. Outside resonance, the atom spends practically all its time in the "lower" state, and we have $U_{av}(x) \approx U(x)$. When $\hbar\Delta \ll dE$, the mean potential declines strongly, since the lifetimes in the different states become almost equal. Here an increasing role is played by fluctuations of the gradient force, which should be described by using the kinetic equations.

C. The force of mixed type[21]

If we neglect spontaneous emission, then in a strong, slowly varying field the induced dipole moment of the atom follows the field adiabatically. Then the force of the stimulated light pressure has the form of the gradient

[1)] For vibrational transitions in molecules, it is easier to realize the condition $\gamma\tau < 1$.

of (14) and its average in the standing wave is zero, $\bar{F}_1 = 0$. Spontaneous emission leads to a jump in the phase of the atomic oscillator, and the theorem of the adiabatic variation of the dipole breaks down. If the dipole moment changes sign upon spontaneous emission of a photon, then the force averaged over the period of oscillation of the field becomes different from zero. Let us denote it by F_2:

$$F_2 = \overline{p\frac{\partial E^*}{\partial x}} + \text{complex conjugate.} \qquad (20)$$

The force F_2 does not depend on the coordinate, but it depends substantially on the velocity v of the atom with respect to the standing wave. The probability of spontaneous emission while the atom is passing through one period of the field is of the order of γ/kv. Hence we have the estimate $F_2 \sim F_1\gamma/kv \sim \gamma dE/v$. For an exact calculation of F_2 we must use the solution of Eq. (18). We can express the result in terms of the correlator of the parameters χ and $\tilde{\gamma}$:

$$F_2 = \frac{\gamma\hbar\Delta C}{v}, \quad C = \frac{1}{2}\left(\frac{\overline{\chi^{-1}\tilde{\gamma}\chi}}{\bar{\tilde{\gamma}}} - 1\right). \qquad (21)$$

Figure 4 shows the relationship of the force F_2 to the velocity by the solid line for $|v| > v_E$ and by the dotted line for $|v| < v_E$, where $v_E = \sqrt{dE/M}$ is the characteristic depth of modulation of the frequency. Equation (21) holds only in the region of large velocities $|v| > v_E$ where we can consider the motion of the atom to be fixed. The dimensionless coefficient C depends on the parameter $\hbar|\Delta|/dE$ (Fig. 5). Here we have $C \approx 1$ in the frequency interval $\hbar|\Delta| \leq 0.2\,dE$, while C rapidly declines outside this interval as the detuning is increased. The force F_2 reaches a maximum at the edge of this interval, and it amounts to $F_2 \approx 0.2\,\text{sign}\,\Delta\gamma dE/v$. This agrees with the estimate given above. In a weak field, the mixed-type force is proportional to the cube of the radiation intensity. We can understand this relationship by the following arguments. The emission from the atom in the external field has coherent and incoherent components (see, e.g., Ref. 25). The force F_2 is proportional to the gradient force (the first power of the intensity) and to the intensity of the incoherent photon emission. In the case of resonance fluorescence, this quantity is proportional to the square of the intensity of the external field.[26]

One can also consider the mixed-type force as resulting from Raman scattering of light. The atom absorbs a quantum of the external field of frequency $\omega_0 + \Delta$, while it spontaneously emits a quantum at the transition frequency ω_0. Depending on the sign of the detuning, the energy goes from the atom into the field or vice versa. The rate of variation of the energy of the atoms arising from the force (21) is

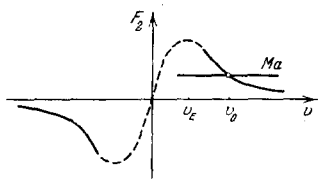

FIG. 4. Dependence of the mixed-type force on the velocity of the atom.

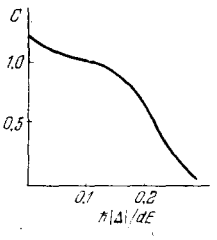

FIG. 5. Dependence of the function C that characterizes the value of the force F_2 on the detuning.

$$\frac{d}{dt}\frac{Mv^2}{2} = \gamma\hbar\Delta C. \qquad (22)$$

When $\Delta > 0$ the atoms are heated, while they are cooled when $\Delta < 0$. The entropy of the "atom + field" system does not decrease, since the entropy of the field increases in spontaneous emission.

For estimates we replace the velocity in the formula $F_2 = \text{sign}\,\gamma dE/v$ by the characteristic velocity v_E of the atom in the standing wave. Consequently we obtain

$$F_2^{\max} = \text{sign}\,\Delta \cdot 0.1\gamma\sqrt{dEM}. \qquad (23)$$

In a field of intensity 10^6 V/cm we have $F_2^{\max} \sim 1$ eV/cm. In a strong field the mixed-type force considerably exceeds F_0, but is much smaller than F_1.

3. ACCELERATION AND DECELERATION OF ATOMS BY LIGHT

Forces of light pressure can be used to accelerate or decelerate atoms and molecules. Here it is important to know not only the maximum energy that the field can transfer into the kinetic energy of the atoms, but also the efficiency of this transfer. The simplest possibility of acceleration arises when an atom moves in the field of a plane running wave under the action of the force F_0. In a saturation regime, an Na atom must travel a path of ~15 cm to be accelerated to an energy of 300 °K. Here one photon of the light beam of energy $\hbar\omega \approx 2$ eV is spent for each recoil momentum $\hbar k$ received by the atom. The number of photons scattered during the acceleration time is 2×10^4. The efficiency of this method of acceleration proves to be very low, of the order of 10^{-6}.

A. Acceleration of atoms in a field of opposing waves

In a strong field, an appreciable fraction of the atoms can lie in the potential wells of a standing wave. Thus, when $E \sim 10^6$ V/cm, the depth $dE \sim 10^{-2}$ eV of the potential well is comparable with the kinetic energy of the atoms at room temperature. The trapped atoms can be accelerated by varying the frequency of the opposing waves. This method was first proposed for accelerating charged particles by radiofrequency fields.[4] If the phase of the wave (8) has the form

$$\varphi(t) = \frac{\dot{\omega}t^2}{2}, \qquad (24)$$

then the trapped atoms move with the acceleration $a = \dot{\omega}/k$. Here the potential wells exist only under the condition $a < a_c = F_1/M$, which limits the rate of variation of the frequency. As an example, let us estimate the acceleration of metastable He ($2\,^3S$) atoms by a field that is in resonance with the $2\,^3S$–$2\,^3P$ transition ($\lambda = 1.08$ μm). In

a field $E=10^6$ V/cm we have $a_c=3.7\times10^{14}$ sec^{-1}. When $a=3\times10^{14}$ sec^{-1}, one can accelerate the atoms to an energy of 10 keV in a time 2×10^{-7} sec. The acceleration path length is 10 cm. If the diameter of the beam is 0.1 cm, then the energy of the light in the pulse is 15 J. The density of the atoms being accelerated is limited by the diffraction of the light field by the grating that arises from the modulation of the density by the field.[5,21] One can find the reciprocal scattering length q of the photons from the relationship $q=\varepsilon' k/2$, where ε' is the variable (modulated) component of the dielectric permittivity of the medium. In this case $\varepsilon'\sim10^{-20}n$, where n is the density of atoms. Hence we find for a layer of gas 1 cm thick that $n\lesssim10^{15}$ cm^{-3}.

Variation of the frequency of the field. For acceleration we must have $\dot{\omega}=2\times10^{19}$ sec^{-2}. During the acceleration time the frequency must vary by 4×10^{12} sec^{-1}. Such a frequency change can be achieved either by using the Doppler effect in a resonator with fast-moving mirrors[10] or by phase-modulating the light beam.[21] In the latter case one of the light beams must pass through a transparent dielectric in which the dielectric permittivity $\delta\varepsilon=\varepsilon-1$ varies uniformly with time throughout the volume (e.g., by varying the polarization vector in an anisotropic dielectric). The change in the phase difference after passing through a dielectric of length l is $\varphi(t)=kl\delta\varepsilon(t)/2$. The "turning on" of the dielectric should obey the law $\delta\varepsilon(t)=\delta\varepsilon_0(t/\tau)^2$, where τ is the acceleration time.[2)] Hence we get

$$\dot{\omega}=\frac{kl\delta\varepsilon_0}{\tau^2}, \tag{25}$$

We can obtain the value $\dot{\omega}=2\times10^{19}$ sec^{-1} that we are interested in for $\tau=2\times10^{-7}$ sec with $\delta\varepsilon_0=1$ and $l=15$ cm.

B. Autophasing in velocity space

Let us study the acceleration of atoms by the force F_2. For the sake of definiteness, let us take $\Delta>0$, so that the force F_2 is an accelerating one.

According to Eq. (22), the energy of the atom varies rather slowly in a stationary field: the atom acquires an energy of 1 eV in a field of 10^6 V/cm in a distance of 10^2 cm. One can considerably enhance the acceleration effect if the frequency of one of the opposing waves varies linearly with time so that the atom lies in the field of a uniformly accelerated wave. In this case we can understand the pattern of acceleration from Fig. 4. In a system with the wave at rest, the force F_2 comes into equilibrium with the inertial force Ma (a is the acceleration of the wave) at a certain velocity v_0. Here the equilibrium point v_0 is stable with respect to small perturbations of the velocity. In other words, the particles are autophased in velocity space. The acceleration must not be too great, since the condition $v_0\gtrsim v_E$ must be satisfied. When $v_0=v_E$, the trapped atoms move in the field of the force (23). Then in a limiting field the atoms acquire the energy 10^2 eV in a distance of 10^2 cm.

The acceleration effect of the force F_2 is considerably smaller than that of the gradient force. Yet the acceleration of particles in a velocity autophasing regime has two important advantages.

First, all the particles are accelerated by this method. If the depth of the potential wells is small ($dE\ll T$), then the number of atoms trapped in the wells and accelerated by the gradient force proves to be small and of the order of $\sqrt{dE/T}$. In acceleration by the force F_2, in which the phase velocity of the wave varies from $-v_T$ to $+v_T$ (v_T is the initial thermal velocity), all the atoms prove to be trapped. Consequently all the atoms in the coordinate system associated with the wave acquire the same velocity v_0 (apart from a small field modulation of the velocity). Thus the gas being accelerated becomes cooled. Second, the acceleration time can be large. In the case in which the trapped atoms are accelerated by the gradient force, the duration of acceleration is restricted by the lifetime of an atom on the trajectory (see Sec. 4). This fact is not important when the atoms are accelerated by the force F_2, provided only that v_0 does not approach its critical value v_E too closely.

C. Cooling of atoms by light

Cooling in the field of a standing wave. When $\Delta<0$, the resonance atoms are retained by the field of the standing wave. We get from Eq. (22) the characteristic cooling time τ_1 of the atoms:

$$\tau_1=\frac{(\gamma C)^{-1}T}{\hbar|\Delta|}. \tag{26}$$

Another cooling regime arises in the field of a uniformly accelerated wave. The atoms are cooled by the force $F_2^{\max}$ (they are grouped near the velocity v_0) within the time

$$\tau_2=10\gamma^{-1}\sqrt{\frac{T}{dE}}. \tag{27}$$

When $dE\ll T$ we have $\tau_2\ll\tau_1$. This method of cooling and accelerating atoms can be useful for obtaining inversion-free laser action using metastable atoms (see Chap. 7).

Cooling of resonance atoms in a standing wave can also arise from the force F_0.[33] Let us study the field of a standing wave in which the amplitude satisfies the condition $dE/\hbar\lesssim kv_T$, while the detuning is negative and approximately equal to kv_T. An atom moving with a positive velocity of the order of v_T will interact mainly with the wave propagating in the negative direction, since only the latter will be in resonance with the atom (see Eq. (7)). Therefore the atom will be decelerated by the force F_0. Within the time

$$\tau_3=\sqrt{MT}\,\frac{1}{\gamma\hbar k} \tag{28}$$

its velocity will become appreciably less than thermal. More detailed calculations of this case have been made in Ref. 55. For example, for Mg atoms we have $\tau_3\sim10^{-5}$ sec, while the required radiation power is ~1 kW/cm^2.

Cooling by the mixed-type force is more efficient than that by the force F_0 in relatively weak fields where E

[2)] The case in which $\varepsilon(t)$ is varied sharply in order to excite the opposing wave has been treated in Ref. 22.

$\gtrsim 10^2\ (\hbar k)^2/2Md \sim 1$ eV/cm. Thus, Mg atoms can be cooled in the same time of 10^{-5} sec in a field of power 0.2 W/cm^2. We stress the difference between the mechanisms of cooling by the mixed-type force and by the force of spontaneous light pressure F_0. In a strong field $dE/\hbar > kv$, an atom interacts to about the same extent with both opposing running waves. In this case stimulated transitions contribute substantially to the cooling effect. When $dE/\hbar < kv$, the atom interacts effectively only with one of the waves, and the cooling rate is determined by the rate of spontaneous transitions. Cooling by the force F_2 is most effective in a frequency-scanning regime.

Cooling and heating of atoms in a light field by collisions. In the absence of an external field collisions of atoms are usually elastic. In other words, the work done by the interaction forces between the particles is zero. In an external field (let it be the field of a plane running wave), the work done by these forces can become different from zero. This happens whenever one of the atoms spontaneously emits a photon during collision, and its dipole moment changes sign.

From the quantum standpoint, the collision becomes inelastic owing to Raman light scattering, absorption of a quantum of frequency $\omega_0 + \Delta$ and spontaneous emission of a quantum of frequency ω_0. When $\Delta < 0$, the atoms are cooled, while they are heated when $\Delta > 0$. The change δT in the kinetic energy of relative motion per collision can be estimated from the relationship (22): $\delta T \sim \hbar\Delta\gamma\tau_{\infty 1}$, where $\tau_{\infty 1}$ is the collision time. In order to find the rate of temperature variation, we must multiply this expression by the rate of collisions $na^3/\tau_{\infty 1}$:

$$\frac{dT}{dt} \sim \gamma\hbar\Delta n a^3, \tag{29}$$

Here a is the range of the forces such that the kinetic energy of the atoms changes by not less than $\hbar|\Delta|$. Of course, we also need the condition $h\Delta \sim dE$. In the case of a dipole-dipole interaction we have $d^2/a^3 \sim \hbar\Delta$. Hence we get the final estimate of the rate of variation of the temperature

$$\frac{dT}{dt} \sim \operatorname{sign}(\Delta)\gamma n d^2. \tag{30}$$

At a density $n = 10^{17}$ cm^{-3}, the temperature of Mg atoms varies by a factor of two (it increases when $\Delta > 0$ and decreases when $\Delta < 0$) within 10^{-4} sec in a field of power 1 MW/cm^2.

D. Acceleration of atoms by π-pulses [21]

We examine the acceleration of atoms by modulated light beams. Here we can deal only with small energies of acceleration in which the momentum of the atom varies by an amount of 10^2–10^3 $\hbar k$. Let a running light pulse be turned on between the instants of time t_1 and t_2 and be incident on an atom. Then after it has passed, the momentum of the atom changes by the amount

$$\int_{t_1}^{t_2} F(t)\,dt = \frac{1}{2}\hbar k\,[q(t_2) - q(t_1)]. \tag{31}$$

a) b)

FIG. 6. Acceleration by a π-pulse of an atom in the ground state (a); acceleration of an excited atom by a π-pulse running in the opposite direction (b).

This expression is derived by using Eq. (6) upon neglecting relaxation (brief light pulses).

Let a π-pulse be incident on an atom in the ground state ($q(t_1) = -1$). Then the atom goes into the excited state ($q(t_2) = +1$) and it changes its momentum by $\hbar k$ (Fig. 6a). If a π-pulse traveling in the opposite direction again acts on the atom, then the atom returns to the ground state and its momentum is increased again by $\hbar k$ (Fig. 6b). The total change in the momentum of the atom per cycle is $2\hbar k$.

It is important that the π-pulse traveling in opposite directions should act strictly alternately on the atom. These conditions can be realized approximately either inside the resonator of a laser that operates in a self-synchronized regime[45] or by using single-direction pulses and a weak constant opposing field. However, the conditions that define a π-pulse ($\Delta = 0$ and $d\int_{t_1}^{t_2} E(t)\,dt = \pi\hbar/2$) are not satisfied strictly. The extent of deviation from these conditions determines the number N of pulses for which coherent acceleration occurs. If the relative deviation from the π-pulse conditions is of the order of ε (the characteristic scale of the detuning is determined by the duration τ of the pulse), then we have either $N \sim \varepsilon^{-1}$ or $N \sim \varepsilon^{-2}$, for constant and fluctuating deviations.

The Doppler frequency shift leads to a detuning that increases during acceleration. Analysis shows that, under the condition $\varepsilon^2/\tau < kv < 1/\tau$, the change in phase arising from the Doppler effect becomes random, and we have $N \sim \varepsilon^{-2}$. Spontaneous emission also leads to breakdown of coherence of the interaction of the atom with the π-pulses. When there is a large number of scattered quanta, it is hard to attain an acceleration time shorter than γ^{-1}. Apparently the only way out is to use pulses of short duration (short duty cycle) and scattering of the atoms near a mirror, where the time interval between the oppositely running pulses is small, so that the atom is in the ground state during the relatively long interval between pulses in the same direction. Acceleration of atoms by π-pulses is of interest for resonance scattering of atoms at small angles in an atomic beam. When $N \sim 10^2$–10^3, the deviation angles are of the order of 10^{-2}–10^{-1} radians. As a function of the frequency, the mean force arising from the π-pulses has the form of a narrow peak of width $1/\tau N$ that is smaller than the characteristic field broadening $2dE/\hbar$. Acceleration of atoms by pulses of variable frequency has been treated in Ref. 27, while acceleration by a π-pulses has also been discussed in Ref. 58.

E. Acceleration of atoms by a two-frequency field [20]

A force that is constant in space (of non-gradient type) can arise not only from dissipative processes, but also

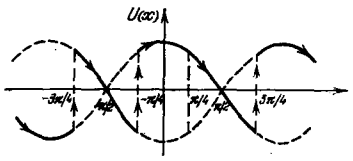

FIG. 7. Effective potential of an atom in the field of (32) (shown by the heavy line).

from a breakdown of the adiabatic approximation, which can happen in a nonmonochromatic field. Below we discuss a simple example of a special form of field in which the effective force proves to be sign-defined at a distance much greater than the wavelength, and we estimate the possible acceleration effect.

In the field of two standing waves of the form

$$E(xt) = E\cos kx + E_1\cos(kx+\pi/4)\,e^{i\delta kx - i\Delta t},$$
$$E_1 \ll E, \quad \hbar\Delta = 2dE\cos\frac{\pi}{4}, \tag{32}$$

work is done mainly by the strong field of the standing resonance wave $E\cos kx$. Here half of the atoms have the potential $+dE\cos kx$, and half have $-dE\cos kx$. We shall consider the motion of an atom to be slow: $kv \ll dE/\hbar$. The field E_1 acts everywhere as a small perturbation, except for the points $kx = -\pi/4 + m\pi$ (m is an integer) where resonance occurs: the frequency of the weak field coincides with the Stark splitting of the levels by the strong field.

If the criterion is satisfied that $dE_1 > \sqrt{\hbar k v\, dE}$, then we shall be dealing with the so-called slow passage through resonance. Then, after the point $kx = -\pi/4$ has been passed, the atom goes from one quasilevel to the other, and the potential changes sign. The effective potential energy of the atom in the presence of the weak signal is shown in Fig. 7 by the heavy line, while the transitions are indicated by the dotted lines. Resonance also occurs at the points $kx = \pi/4 + m\pi$, but the amplitude of the weak field vanishes here, and transitions from the one level to the other do not happen (fast passage through resonance). Absorption (or emission) of a quantum of the weak field occurs at the points of discontinuity. The mean variation of the energy upon passing through one period of the field (the wavelength) now differs from zero, and it equals $4dE\cos\pi/4$. That is, a force appears that is constant in space.

However, such a mechanism of acceleration cannot operate for an unlimited time. As the velocity increases, the criterion of slow passage through resonance breaks down, and acceleration ceases. Hence one can actually speak only of a small increase in velocity. For example, it takes a resonance-field power of the order of 0.3 W to scatter thermal Na atoms in an atom beam by an angle of ~0.1 radian. This value is about ten times smaller than the power of the stationary monochromatic wave (see Sec. 5) that is needed to scatter atoms by the same angle. The gain in power arises from the fact that the atom "employs" the energy of not one but many potential wells, of the order of tens.

The acceleration effect depends substantially on the frequency of the weak field. The width of the resonance region is considerably smaller than the Stark splitting $dE/\hbar$.

4. THE STIMULATED RECOIL EFFECT

As we know, the recoil effect in a weak light field is manifested in the fact that the absorption contour is shifted by $\varepsilon(\hbar k)/\hbar$ toward the violet, and the emission contour by $\varepsilon(\hbar k)/\hbar$ toward the red with respect to the transition frequency of an infinitely heavy atom. Here $\varepsilon(k\hbar) = (\hbar k)^2/2M$ is the recoil energy upon absorption or emission of one photon. The recoil effect can become appreciable under the condition

$$2\varepsilon(\hbar k) > \hbar\gamma, \tag{33}$$

This condition can be satisfied in optics only by very narrow atomic or molecular resonances. If we express λ in micrometers and A is the atomic weight, then $\varepsilon(\hbar k)/\hbar = 1.2\times10^6\, A^{-1}\lambda^{-2}$ sec^{-1}. The features of the processes of stimulated emission and absorption with account taken of recoil by perturbation theory have been studied[6,31] in connection with the theory of the Lamb dip.

In a strongly inhomogeneous field the atom reemits many quanta and its trajectory can vary substantially. The question arises of how the movement of the atom affects its response, i.e., the main dipole moment p (or conversely, how will the response affect the movement of the atom?) Let us give a simple example of strong correlation between the movement of an atom and its response. The topic is the double refraction of a molecular beam. In the case of exact resonance, the mean induced dipole moment of the molecules that cross the boundary of a light beam with the velocity v is

$$p(x) = \frac{d^2E(x)}{\sqrt{(Mv^2/2)^2 + [dE(x)]^2}}. \tag{34}$$

When $\Delta = 0$ we can assume that half of the atoms have a dipole moment directed with the field and half against it. The atoms that have the potential $-dE(x)$ are attracted into the light beam and their density becomes less than that of the atoms having the potential $+dE(x)$. Consequently a mean dipole moment arises. Thus, in a strong inhomogeneous electromagnetic field, the induced dipole moment depends not only on the magnitude and the frequency of the field, but also on the kinetic energy of the atom.

A. The kinetic equation

In order to take systematic account of the recoil effect in stimulated and spontaneous transitions, we shall start with the quantum kinetic equation for the density matrix $\rho(x_1 x_2 t)$ of the atom:

$$\frac{\partial\rho(1,2)}{\partial t} + \hat{\gamma}\rho(1,2) = \frac{1}{i\hbar}\{[\mathcal{H}_0(1) + \mathcal{H}(1)]\rho(1,2) - \rho(1,2)[\mathcal{H}_0(2) + \mathcal{H}(2)]\}; \tag{35}$$

Here $\mathcal{H}_0$ is the kinetic-energy operator, $\mathcal{H}$ is the operator for interaction of the atom with the field of (3), and $\hat{\gamma}$ is the operator for relaxation due to spontaneous emission.

Equation (35) allows an exact solution in the case of a plane running wave of arbitrary intensity.[6] Yet here the recoil is usually not large, since stimulated transitions do not contribute in this case to the recoil effect. In a standing wave of large amplitude, the change in the motion of the atoms can become appreciable, but the equations (35) in this case are complicated. However, one can substantially simplify them in the important quasiclassical limit

$$Mv \gg \hbar k. \tag{36}$$

Let us transform to a mixed Wigner representation (to coordinate space and velocity space)

$$\rho(x, v, t) = \int \frac{dx'}{2\pi\hbar} \rho\left(x + \frac{x'}{2}, x - \frac{x'}{2}, t\right) e^{iMvx'/\hbar},$$

and restrict the treatment to the first correction term in the expansion of the Hamiltonian in terms of the field gradient[24]:

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} + \hat{\gamma}\right)\rho = \frac{1}{i\hbar}[\mathcal{H}(xt)\rho - \rho\mathcal{H}(x, t)] + \frac{1}{2M}\left(\frac{\partial\mathcal{H}(x, t)}{\partial x}\frac{\partial\rho}{\partial v} + \frac{\partial\rho}{\partial v}\frac{\partial\mathcal{H}(x, t)}{\partial x}\right). \tag{37}$$

The stimulated recoil effect in the kinetic equation (37) is described by the second term on the right-hand side. When it exceeds $\hat{\gamma}\rho$, the recoil becomes substantial, and we arrive at the criterion

$$\frac{\partial}{\partial x}\frac{|dE(x, t)|}{Mv} > \gamma. \tag{38}$$

The left-hand side of this inequality determines the value of the Doppler shift arising from modulation of the velocity by the inhomogeneous field. The inequality (38) generalizes the criterion (33) to the case of a strongly inhomogeneous field and it can be satisfied for broad atomic resonances having $\gamma \gtrsim 10^7$ Hz. Let us write the equations of (37) for the individual components of the density matrix, for the distribution function $f = \rho_{11} + \rho_{22}$, for the population difference $q = \rho_{22} - \rho_{11}$, and for the induced dipole moment $p = d\rho_{12}$:

$$\frac{df}{dt} + \hat{\gamma}f = -\frac{1}{M}\frac{\partial p}{\partial v}\frac{\partial E^*(xt)}{\partial x} + \text{complex conjugate}, \tag{39}$$

$$\frac{dp}{dt} + \frac{\gamma}{2}p = -\frac{i}{\hbar}d^2E(x, t)q - \frac{\partial^2}{M}\frac{\partial E(x, t)}{\partial x}\frac{\partial f}{\partial v}, \tag{40}$$

$$\frac{dq}{dt} + \hat{\gamma}q = 2ipE^*(x, t) + \text{complex conjugate}. \tag{41}$$

Here $d/dt = \partial/\partial t + v\partial/\partial x$ is the total derivative with respect to time. The first equation describes the motion of the center of gravity of the atom when acted on by the light-pressure force of (4). Equations (40) and (41) describe the change in the atomic states when acted on by the resonance field. The distinction of these equations from Eqs. (5) and (6) involves the second term on the right-hand side of (40), which account for correlation between the motion of the atom and the induced dipole moment. We stress that Eqs. (39)–(41) account for fluctuation of the dipole moment and for the dependence of p on the state of motion of the atom.

Spontaneous transitions can alter the momentum of the atom by the amount $\hbar\mathbf{k}$. We can conveniently write the relaxation operator $\hat{\gamma}$ for the distribution functions of the atoms in the ground and excited states:

$$\hat{\gamma}\rho_{22} = -\gamma\rho_{22}, \quad \hat{\gamma}\rho_{11}(\mathbf{v}) = \gamma\int d\mathbf{n}F(\mathbf{n})\rho_{22}\left(\mathbf{v} + \frac{\hbar k\mathbf{n}}{M}\right), \tag{42}$$

Here $F(\mathbf{n})$ is the probability of spontaneous emission of a quantum in the direction $\mathbf{n}$.

The first equation of (42) describes the atom leaving the excited state, while the second describes it entering the ground state. The relaxation operator of (42) has been used[31] to find the form of the Lamb dip and for calculating[32] the fluctuations of the momentum during acceleration of atoms by a plane running wave.

In the quasiclassical limit the operator $\hat{\gamma}$ describes the slow diffusion of atoms with respect to velocity:

$$\hat{\gamma}\rho_{11} = \gamma\left[1 + \frac{1}{5}\left(\frac{\hbar k}{M}\right)^2\left(\frac{\partial^2}{\partial v_x^2} + \frac{\partial^2}{\partial v_y^2} + \frac{1}{2}\frac{\partial^2}{\partial v_z^2}\right)\right]\rho_{22}. \tag{43}$$

The diffusion coefficient is smaller by a factor of two along the direction of the polarization vector (the z axis) than in the transverse direction.

Let us study some features of the stimulated recoil effect in the monochromatic inhomogeneous field of (8) by using the quasiclassical equations (39)–(41), while neglecting slow diffusion due to spontaneous emission and assuming that $\hat{\gamma} \approx \gamma$.

1) *The nonresonance case.* Let us start with the nonresonance case $\Delta > kv_0$, where v_0 is the characteristic velocity of the atoms. Upon applying the procedure of expansion in terms of Δ^{-1} (see Chap. 2), we get a system of two kinetic equations for the functions $S_\pm = [f \pm \chi(x)q]/2$:

$$\frac{dS_\pm}{dt} \pm (\gamma_+S_+ - \gamma_-S_-) = \pm\frac{1}{M}\frac{\partial U}{\partial x}\frac{\partial S_\pm}{\partial v}, \quad \gamma_\pm(x) = \frac{\gamma}{4\chi^2(x)}[\chi(x) \pm 1]^2. \tag{44}$$

When the field is turned off we have $\chi \to 1$, $S_+ \to \rho_{22}$, and $S_- \to \rho_{11}$. Thus S_+ is a function of the distribution of the atoms in the "upper" mixed state, while S_- is a function of that in the "lower" state. In a weak field the lifetimes of these states differ strongly: when $dE \ll \hbar\Delta$, we have $\gamma_+ \approx \gamma$ and $\gamma_- = \gamma[dE(x)/\hbar\Delta]^4$. The atom spends the greater part of the time in the weak field in the "lower" state. The lifetime in this state is inversely proportional to the square of the intensity of the field. In a strong field with $\hbar\Delta \ll dE$, the lifetimes in the different states are the same: $\gamma_+ \approx \hat{\gamma}_- \approx \gamma/4$. Of course the total population of the two states is conserved since the lower level is the ground state.

Equations (44) describe particles of two types that move in potential differing in sign. The particles are unstable and are "transformed" into one another, while only the total number of particles is conserved. We can also assume that there is one group of particles having two trajectories of motion. The atom is situated on one trajectory for the time $1/\gamma_-$ and has the potential $U(x)$. Then owing to the phase jump in spontaneous emission, the dipole moment of the atom can change sign. Then the atom will have the potential $-U(x)$ during the time $1/\gamma_+$. The transitions from one trajectory to the other are statistical in nature. The coordinate and momentum of the particle are conserved in these transitions. If an atom turns out to be trapped in a potential well of a standing wave while following one trajectory, then when

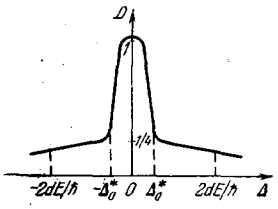

FIG. 8. Dependence on the detuning of the diffusion coefficient of atoms with respect to velocity.

it shifts to the other trajectory, the atom has only a certain probability of remaining in the bound state. Generally it becomes free. In this regard there is a fundamental distinction between charged particles and resonance atoms. Charged particles can be localized in three-dimensional potential wells.[4] The lifetime of atoms in the bound state proves to be finite. In order to increase it, one must decrease the field, i.e., decrease the depth of the potential well.[3)]

2) *The resonance case.* In a strong field with $\Delta=0$, the population difference q becomes small owing to the saturation effect and we can neglect it. From (39)–(41) we have the following equations for the quantities $R_\pm = [f \pm \mathrm{Re}\,(p/d)]/2$:

$$\frac{dR_\pm}{dt} \pm \frac{\gamma}{4}(R_+ - R_-) = \pm \frac{1}{M}\frac{\partial V(x)}{\partial x}\frac{\partial R_\pm}{\partial v}. \tag{45}$$

The atom now has two states having the resonance potentials $\pm V(x)$. The lifetime on each trajectory are the same and are equal to $4/\gamma$.

B. Heating of atoms by a nonmonochromatic field

If the resonance field of a standing wave is nonmonochromatic, then the force of stimulated light pressure is a random quantity. For simplicity, let only the phase $\varphi(t)$ of the wave fluctuate and the phase correlator have the form

$$\langle [\varphi(t) - \varphi(0)]^2 \rangle = 2\Gamma t. \tag{46}$$

Stochastic acceleration (heating) arises when the fluctuation spectrum has a large width Γ:

$$\Gamma > \Omega, \quad \Omega = k\sqrt{\frac{dE}{M}}. \tag{47}$$

When $\Gamma < \Omega$, the atoms trapped in the potential wells adiabatically "follow" the field and there is no heating. When $\Gamma > \Omega$, the adiabatic approximation breaks down. When $\Gamma \gg \Omega$, the distribution function $\langle f(v,t)\rangle$ of the atoms averaged over space and time obeys the Fokker-Planck equation:

$$\frac{\partial \langle f\rangle}{\partial t} + \frac{\partial}{\partial v}\left(\frac{\langle F\rangle \langle f\rangle}{M} - D\frac{\partial \langle f\rangle}{\partial v}\right) = 0. \tag{48}$$

If the frequency of the field is at resonance on the average, then the diffusion coefficient has the very simple form[24,29]:

$$D(v) = \frac{\Gamma (k\,dE)^2}{2M^2[\Gamma^2 + (kv)^2]}, \qquad \Delta = 0. \tag{49}$$

[3)]In the numerical example involving acceleration of $He(2\,^3S)$ to large energies, the acceleration time exceeds the lifetime of $He(2\,^3P)$ only by a factor of two. The parameter that characterizes the probability of escape from the well is $\gamma t/4 = 1/2$.

There are two characteristic regions in the frequency-dependence of $D(0)$ with respect to Δ shown in Fig. 8. In the broad frequency interval $|\Delta| \sim dE/\hbar$, the behavior of the diffusion coefficient is determined by the dependence of the gradient force F_0 on the detuning. The narrow peak in the center at $|\Delta| < \Delta_0^* = \sqrt{dE\Gamma/\hbar}$ involves the change in the correlation time of the force as we go from the nonresonance to the resonance case. Actually at resonance the question is that of correlation of amplitudes of the field, while outside resonance it is that of correlation of the intensity fluctuations. The correlation time is four times smaller in the latter case than in the former.

The energy of slow atoms increases linearly with time:

$$\langle Mv^2 \rangle = MD(0)\,t. \tag{50}$$

Fast atoms "average" the gradient force over the period, and when $kv > \Gamma$, the energy increases only as $\sqrt{t}$.

Let us estimate the heating effect on the atoms by using Eq. (49). The linear heating regime of (50) quickly breaks down for small Γ, while this process proceeds very slowly for large Γ. The most favorable case arises when $\Gamma \approx kv^*$, where v^* is the final rate of heating. By starting with this condition, we can find the radiation intensity needed for heating atoms to the effective temperature T^* in the time τ:

$$E^2 = (kv^*\tau)^{-1}\left(\frac{2T^*}{d}\right)^2. \tag{51}$$

A power of 1 MW/cm^2 is needed for heating Na atoms to 300 °K in a time of 2×10^{-5} sec.

Diffusion with respect to velocity arises in the monochromatic field of a standing wave, owing to combination of spontaneous and stimulated transitions. Just like the forces acting on the atom, one can classify the diffusion coefficients in terms of type of emission of photons.

5. SCATTERING OF ATOMS BY LIGHT AND THE ISOTOPE-SEPARATION PROBLEM

Currently the problem of separating isotopes by laser methods is being studied very intensively.[34–36] Here we shall examine the possibility of separating isotopes by using light pressure. The overall scheme of this approach is as follows. An atom beam or jet intersects a light beam at a right angle. Owing to the resonance nature of the light-pressure forces, only certain isotopes are scattered out of the beam (or jet) under certain conditions. We shall discuss below the efficiency of scattering of atoms and molecules by using the forces discussed in Sec. 2.

A. Deflection of atoms by a running-wave field

In 1973 Frisch[2] observed a deflection (very small) of Na atoms in an atomic beam when irradiated by a resonance lamp. Ashkin[8] has shown that the force of the spontaneous light pressure of a laser beam is sufficiently large to deflect atoms through large angles. References 11 and 12 describe experiments on deflection of a beam of Na atoms by using a dye laser and a resonance lamp.

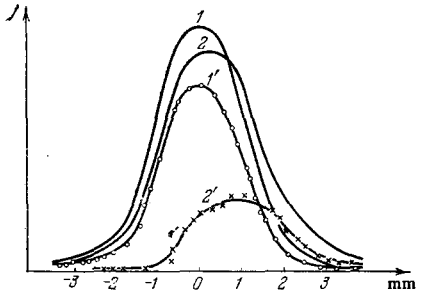

FIG. 9. Density distribution of atoms over the cross-section of the beam before irradiation by light (curve 1) and after irradiation (curve 2).

Figure 9 shows the density distribution of the atoms over the cross-section of the beam before irradiation by light (curve 1) and after (curve 2).[12] The width of the spectrum of the laser radiation allowed them to excite only a certain transition between components of the hyperfine structure. Therefore nonresonance, unscattered atoms (curve 1′) and resonance, scattered atoms (curve 2′) contribute to the distribution function shown by curve 2. The number of excitations of resonance atoms by light amounted to about 60, while corresponds to a scattering angle of $(3-2)\times10^{-3}$ radians. Bernhardt *et al.*[28] have separated Ba isotopes by using spontaneous light-pressure forces.

Deflection by the field of a running wave required a long exposure of the atoms to the light beam. This situation can substantially impair the parameters of an atomic beam or jet, since when resonance excitation takes place the dipole-dipole interaction is "switched on" and the collisional scattering cross-section of the atoms increases sharply. Another reason why prolonged irradiation is not desirable is the transition of atoms during irradiation to metastable levels, e.g., as in the case of Ba atoms.

As the radiation intensity increases, the force F_0 becomes saturated, while the selectivity of action is impaired by field broadening. Evidently the conditions $dE \ll \Delta_{iso}$ must be satisfied, where Δ_{iso} is the isotopic frequency shift. The impossibility of using a strong field restricts the density of atoms in the beam or jet, and thus it limits the throughput of the method. Owing to the small radiation width, it is difficult to apply the force F_0 for scattering molecules in an infrared field.[4)] Also the energy balance is extremely unfavorable in spontaneous scattering of photons. All these circumstances substantially restrict the possibilities of applying the force F_0 for separating isotopes. However, one can eliminate them by using the force of stimulated light pressure for scattering atoms.

B. Scattering of atoms in a standing light wave

A standing wave gives rise to a diffraction grating with the half-wave period of $\lambda/2$. Kapitza and Dirac[37] have estimated the efficiency of scattering of electrons by such a grating. With almost normal incidence on the light wave, the small diffraction angles of scattering θ_n are determined by the Vul'f-Bragg condition

$$\theta_n = \frac{2\hbar k n}{p}, \tag{52}$$

Here p is the momentum of the incident particles.

The theory of the Kapitza-Dirac effect has been treated in Refs. 38 and 39. Scattering of electrons in powerful light fields has been observed in the first diffraction maximum.[40,41] For an electron energy of 10 eV that was used in Ref. 40, we have $\theta_1^e \sim 10^{-3}$ radians.

Let us examine the features of the scattering of atoms and molecules in the resonance field of a standing wave.[30] The angles between the diffraction maxima for atoms are somewhat smaller than for electrons. For example, for thermal Na atoms we have $\theta_1^a \sim 10^{-4}$ radians. However, the number of quanta scattered by the atoms can be large. Therefore, in a strong resonance field the maximum deflection angle for atoms is far larger than for electrons. In view of the large number of scattered quanta, one can treat the problem of the diffraction of atoms by a standing resonance wave from the classical standpoint. Let the light wave of (8) be monochromatic, $\varphi = \text{const}$. The atoms of the beam or jet move along the y axis. The atoms in the impinging beam have no transverse velocity, $v_x = 0$, and the longitudinal velocities v_y are all the same. For the sake of simplicity we shall assume the boundary of the light beam to be sharp.

In the region of the light beam, the gradient force F_1 directed along the x axis acts on an atom. One can find the transverse energy of the atoms from the law of conservation of energy

$$\frac{Mv_x^2}{2} + U(x) = U(x_0), \tag{53}$$

Here x_0 is the entrance point of the atom into the light beam.

On leaving the light beam the atom has a certain transverse velocity that is determined by the initial coordinate x_0 (x_0 is a random quantity) and by the time of interaction l_y/v_y with the field (l_y is the thickness of the beam). The atom can acquire the maximum transverse energy in the stationary field $U(x)$ only in a light beam of sufficiently great thickness $l_y \gtrsim l_c$. Here the critical thickness is determined by the condition of matching the time of transit with the period (more exactly, one-fourth the period) of oscillation in the potential well

$$\frac{l_c}{v_y} \sim \frac{\lambda}{2v_E}; \tag{54}$$

For estimates we can assume that $l_c = \lambda/\theta_m$, where θ_m is the maximum scattering angle of the atoms for the given entrance velocity:

$$\theta_m = 2\sqrt{\frac{dE}{Mv_y^2}}. \tag{55}$$

Here we have used the formula for the resonance potential and have taken account of the fact that the depth of modulation of the potential is $2dE$.

[4)] A report has been published[59] of observing a fall in pressure of the order of 10^{-5} Torr in a bulb containing SF_6 gas irradiated by the resonance field of a CO_2 laser.

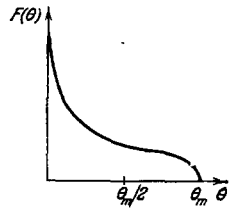

FIG. 10. Distribution function with respect to scattering angles.

By averaging the scattering result with respect to x_0 we obtain the angular distribution function $F(\theta)$ for a monoenergetic beam (or jet), as shown in Fig. 10. About 30% of the particles is scattered into the angular range from $\theta_m/2$ to θ_m.

Let us find the intensity J of the standing wave required for steady-state scattering of atoms and molecules at the characteristic angle θ_m. If we assume that the light beam is focused (compressed) along the y axis to the thickness $l_y = l_c$, the height of the light beam is $l_z = 0.1$ cm, and $Mv_y^2 = 300$ °K, we find that

$$J = 2.5 \cdot 10^5 \frac{\lambda \theta_m^3}{d^2}\ \text{W}, \tag{56}$$

Here λ is expressed in micrometers, θ_m in radians, and d in Debyes.

Table I gives J values characteristic of atoms and molecules. J increases several thousandfold in going from atoms to molecules owing to an increase in the wavelength and a decrease in the dipole moment.

If there were no spontaneous emission, then the scattering of the atoms would be elastic. For example, in deflection of Na atoms through the angle $\theta_m = 5 \times 10^{-2}$ radians, we find that the inelasticity parameter is $\gamma t \lesssim 1$. That is, each atom scatters no more than one photon from the light beam.

1) *The pulsed scattering regime.* Let atoms cross an unfocused light beam having the transverse cross section of 1 cm^2 ($l_y \sim l_z \sim 1$ cm). The duration of the light pulse is $\tau_{\text{pulse}} = 10^{-8}$ sec, while the amplitude of the field at the maximum is 7.5×10^3 V/cm. Under these conditions, the resonance atoms are scattered in a regime in which the times τ_{pulse} match the period of oscillation $\lambda/2v_x$ in the potential well. Therefore the atoms can acquire the transverse energy that they could get in a static potential. The total energy of the light pulse needed to scatter Na atoms by the angle $\theta_m = 0.1$ radian is 2 mJ.

The corresponding parameters for molecules scattered by infrared radiation are: scattering angle $\theta_m = 3 \times 10^{-2}$ radians, $\tau_{\text{pulse}} = 10^{-6}$ sec, total pulse energy = 0.2 J.

2) *Selectivity of scattering.* When $\Delta_{\text{iso}} \gg dE/\hbar$, selectivity of scattering can be ensured by a sharp decrease in the depth of the potential $U(x)$ for large detunings.

Let us examine whether one can ensure selectivity of scattering when $\Delta_{\text{iso}} \ll dE/\hbar$. We shall assume that the frequency of the field of the standing wave is tuned exactly to the transition frequency of the given isotope: $\Delta = 0$. Then the resonance isotopes are scattered approximately uniformly into the angular range $|\theta| < \theta_m$, as is shown in Fig. 11.

The nonresonance isotopes, for which the frequency shift satisfies the condition $\Delta_{\text{iso}} > \Delta_0$, will be scattered in the potential $U(x) \approx |dE(x)|$. The depth of modulation of this potential is only half that of the resonance potential. Therefore the nonresonance isotopes will sweep out the angular range $|\theta| < \theta_m/\sqrt{2}$, while only the resonance isotopes will be scattered at the angles $\theta_m/\sqrt{2} < |\theta| < \theta_m$.

Thus the isotopes that satisfy the condition

$$\Delta_0 < \Delta_{\text{iso}} \ll \frac{dE}{\hbar}, \tag{57}$$

can be separated with a high degree of enrichment.

For Na in a field of intensity 7.5×10^3 V/cm, we have $\Delta_0 = 500$ MHz. In this case the effective width of the resonance scattering Δ_0 exceeds by a factor of 50 the width of the upper working level, and is 60 times smaller than the field width $2dE/\hbar = 3 \times 10^{10}$ Hz. In other words, we lose somewhat in selectivity of scattering in the field of a powerful standing wave as compared with a weak running wave. On the other hand, the time of interaction of the atoms with the field is substantially shortened (by a factor of more than 2000), and the free flight path of the photons increases considerably. This circumstance allows one to rely on getting a high productivity of isotope separation in the standing wave.

"Collisionless" isotope separation methods have an upper bound of productivity that is determined by the flow rate and the allowable (with regard to collisions) density of atoms. From this standpoint, powerful gasdynamic jets are of great interest, in which the density of atoms attains values of 10^{15} cm^{-3}.[46] The flow rate is supersonic, while the temperature of the particles is low. That is, the flow is monoenergetic.

3) *Scattering of atoms by a nonmonochromatic field.* When the condition (47) is satisfied, then the atoms are randomly accelerated in the transverse direction as they pass through the beam. The angular distribution function is Gaussian with the variance $\overline{\theta^2}$:

TABLE I.

	λ, μm	θ_m, radius	J, W	$2dE/\hbar$, Hz
Na	0.6	$5 \cdot 10^{-2}$	2	10^{10}
Na	0.6	$2 \cdot 10^{-2}$	0.1	10^9
SF_6	10.6	$2 \cdot 10^{-2}$	200	10^9

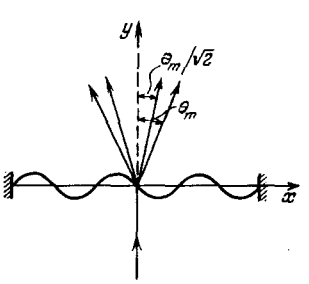

FIG. 11. Resonance isotopes are scattered into the angular interval $|\theta| < \theta_m$, while the nonresonance isotopes are scattered into the angular interval $|\theta| < \theta_m/\sqrt{2}$.

$$\bar{\theta} = \frac{1}{2_i}\theta_m \Omega \sqrt{\frac{\tau_{\text{pulse}}}{\Gamma}}. \quad (58)$$

Thus, in a 7.5×10^3-V/cm field with $\tau_{\text{pulse}} = 10^{-8}$ sec and $\Gamma = 4\times10^6$ Hz, we have $\bar{\theta} = 0.02$ radian. This scattering regime can also be used for isotope separation. Since the mean angle is expressed in terms of the diffusion coefficient of the atoms $\bar{\theta} = \text{const}\cdot\sqrt{D}$, the dependence of the scattering angle on the detuning has two characteristic frequencies, $2dE/\hbar$ and Δ_0^*. When $|\Delta| > \Delta_0^*$, the scattering angle $\bar{\theta}(\Delta)$ decreases by a factor of two as compared with $\bar{\theta}(0)$. Therefore one can separate isotopes in a monoenergetic flux of atoms under the conditions of the inequality (57) (with Δ_0 replaced by Δ_0^*).

4) *Other possibilities.* If one deflects trapped atoms by the field of a uniformly-accelerated wave, then one can get a narrower angular distribution of the scattered particles as compared with the function $F(\theta)$. This type of deflection of atoms requires a field of lower power. For example, for deflecting thermal Na atoms through an angle of 0.1 radian in a time $\tau_{\text{pulse}} = 3\times10^{-8}$ sec, the energy of the pulse should be 0.5 mJ. The variation of the frequency of the opposing waves within the acceleration time amounts to 100 MHz. This can be carried out in a phase-modulation scheme with $l\delta\varepsilon_0 \sim 10^{-4}$ cm (see Sec. 3).

Let us estimate the efficiency of scattering of atoms by the mixed-type force in a frequency-scanning regime. Here all the atoms are deflected at the same angle, which can be represented in the form

$$\theta = 0.05\gamma\tau_{\text{pulse}}\theta_m . \quad (59)$$

Hence we see that deflection by the force $F_2^{\max}$ becomes more efficient as compared with elastic scattering by a standing wave when $\gamma\tau_{\text{pulse}} > 20$. Thus, for Na with $\gamma\tau_{\text{pulse}} = 60$, the atoms are deflected through an angle of 0.1 radian at a pulse energy of 7.5 mJ. The width of the resonance-scattering region is $\Delta_{\text{res}} \approx 0.2\ dE/\hbar \sim 300$ MHz. The advantage of this scattering method relates to the high selectivity when $\Delta_{\text{iso}} > \Delta_{\text{res}}$.

One can also employ acceleration of atoms by π-pulses and a two-frequency field for resonance scattering. As has been noted in Sec. 3, the width of the resonance scattering in this case is appreciably smaller than the field width $2dE/\hbar$. The variation in isotopic composition of an atom beam resulting from "longitudinal" illumination by a resonance field has been studied in Ref. 44.

Thus atoms can be scattered rather selectivity by all the types of forces. From the standpoint of the problem of isotope separation, the stimulated light-pressure force is of greatest interest. The large size of this force allows one to use short light pulses. If the width of the emission line is small, then a high degree of selective scattering persists even in a strong field. Here the atoms scatter a small fraction of the photons from the light beam, so that we can consider the scattering to be elastic. Apparently, one can attain the "collisionless" light of the rate of isotope separation in this method.

6. QUANTIZATION OF THE MOTION OF ATOMS AND MOLECULES IN AN ELECTROMAGNETIC FIELD

A strong inhomogeneous electromagnetic field acts on atoms and molecules in a twofold manner: both the energy levels and the velocities of the particles are altered by the Stark shift.

The level-modulation effect (the dynamic Stark effect) has been studied in detail, mainly in connection with the theory of the gas laser.[47–49] This theory assumes that the atoms move with constant, unperturbed velocities. Yet a situation can happen in which both effects of modulation of levels and of velocities are essential. In other words, the mechanical and optical phenomena become entangled in a strong resonance inhomogeneous field. It is of interest to study this problem for the spectroscopy of narrow atomic and molecular resonances. We shall call a resonance narrow if it satisfies the condition (31) and broad if it does not. The resonance is usually very broad for strong atomic transitions. For example, we find that $\varepsilon(\hbar k)/\hbar\gamma \sim 0.01$ for sodium atoms. For weakly allowed transitions resonance can be narrow. For this to happen, the upper level, which is weakly coupled to the ground state, must not have strong transitions to other states. There are apparently few such transitions, but the intercombination transitions in Mg, Ca, and Zn satisfy this condition.[50] Thus, for Ca in the $4\,^1S_0$–$4\,^3P_1$ transition ($\lambda = 6572$ Å), the width of the line is practically determined by the reciprocal of the transit time through the beam $\sim 3\times10^4$ sec^{-1}, and we have $\varepsilon(\hbar k)/\hbar\gamma \approx 2$. For slow atoms and molecules (e.g., methane) or in beams of large diameter, the transit width is small and the resonance becomes narrow.[51] The shape of the Lamb dip is altered for narrow resonances. Since the absorption and emission contours are shifted in different directions in weak fields by the recoil effect, two dips arise of somewhat smaller depth, which are shifted from the center of the line by the recoil energy[6] $\varepsilon(\hbar k)/\hbar$. An indirect observation has been reported[52] of the recoil effect in an He–Ne laser with a methane cell.

In the case of broad resonances under the condition $U/\hbar < \gamma$, a small peak of width γ can arise in the center of the line on the background of the Doppler absorption contour, owing to atoms trapped by the standing wave.[7] The depth of the Stark modulation $U/\hbar$ (where U is the depth of the potential well) must be smaller than γ so as to avoid field broadening.

When $U/\hbar > \gamma$, the width of the absorption line of the trapped atoms becomes of the order of $U/\hbar$ as the field increases. This continues until the field intensity reaches the critical value I_0. When $I > I_0$, we must take account of the quantization of the motion of the atoms and molecules in the standing wave.[42,50] Here the absorption line of the trapped atoms consists of narrow peaks of width γ arising from the bound states, while the envelope of these peaks has the field width $U/\hbar$. We can easily find the critical intensity I_0 by using the criterion of the stimulated recoil effect (38). For trapped atoms of energy $\varepsilon(p) \sim dE$, the latter has the form

$$\Omega > \gamma. \quad (60)$$

Here the spacing between the energy levels becomes

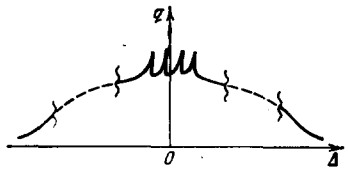

FIG. 12. Absorption coefficient under the conditions of the inequality (60).

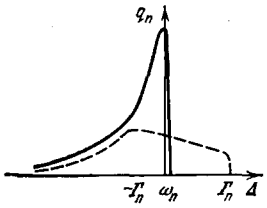

FIG. 14. Shape of an individual peak for $\Delta = 0$ (solid line) and for $\Delta \neq 0$ (dotted line) in the case of broad resonances.

larger than $\hbar\gamma$. We find from the condition $\Omega = \gamma$ that $I_0 = (c/4\pi)(M/d)^2(\gamma/k)^4$. For Na atoms we find that $I_0 \sim 30$ W/cm^2. For Ca atoms, if we use the known value of the oscillator strength of the intercombination transition,[55] we have $I_0 = 10^{-4}$ W/cm^2 when $\gamma = 3\times10^4$ sec^{-1}. For methane molecules with the same transit width and with $k = 2\times10^4$ cm^{-1}, we have $I_0 = 10^{-5}$ W/cm^2. As we see from this, the critical intensity can vary over very broad ranges, mainly depending on the line width. The number of bound states at the threshold for Na is of the order of 10^2, and about three for molecules.

If the field broadening exceeds the Doppler width ($dE/\hbar > kv_T$; we shall denote the corresponding critical intensity by I_1), then the absorption contour substantially differs from the Doppler contour. For Na atoms we find $I_1 = 500$ W/cm^2, and for molecules $I_1 > 2.5$ kW/cm^2.

Following Refs. 42 and 50, let us study the shape of the absorption line of the weak field $E_0 \exp[-i(\omega_{20} + \Delta_0)t + ik_0 x]$ produced by a three-level system with states 0, 1, and 2. The frequency of the weak signal is close to the $0-2\omega_{20}$ transition frequency; the adjacent transition is acted on by the strong field of the standing wave of (8) with a frequency close to the $1-2\omega_{21}$ transition frequency. In order to take into account the quantization of the motion of the atoms occupying states 1 and 2, we must start with the system of wave equations (9) with relaxation taken into account. The fact that an atom generally has two trajectories (two states in the field) leads to certain quantization features that become appreciable near resonance.

The quantization effects are manifested most clearly in two cases: $dE \gg \varepsilon(\hbar k)$, $\hbar\gamma$, and $\hbar\gamma \ll dE \ll \varepsilon(\hbar k)$. The first case corresponds to the quasiclassical limit in which there are many bound states and the second to the quantum limit in which bound states are lacking, but discontinuities appear in the spectrum of the particles.

A. The quasiclassical limit

Let us represent the absorption coefficient of the weak signal in the form

$$q(\Delta_0) = \sum_n q_n(\Delta_0) + q^{(c)}(\Delta_0),$$

where $q_n(\Delta_0)$ is the contribution from the bound state having the energy $\hbar\omega_n$, and $q^{(c)}(\Delta_0)$ is the contribution from the continuous states of the atoms.

We shall assume that the condition (60) is satisfied with some room to spare, so that we can speak of the structure of an individual agsorption peak. When $\Delta = 0$, the system of equations (9) breaks down into two wave equations that describe the particles in the potentials $\pm V(x)$. The energies of the bound states lie between $-dE$ and $+dE$ and they are doubly degenerate. The peak has a characteristic radical form $q_n \sim (\omega_n - \Delta_0)^{-1/2}$, with $\Delta_0 < \omega_n$, since the weak signal mixes the continuous states of the unexcited atoms and the discrete states of the excited atoms.[5)] Figure 12 shows the overall shape of the curve of the absorption coefficient. When $\Delta_0 = \omega_n$, we have $q_n \sim \gamma^{-1/2}$. The degree of contrast of the peaks becomes greatest in the center of the absorption line, where we can neglect the curvature of $q^{(c)}(\Delta_0)$. We can characterize the relative peak height near the threshold by the relationship

$$\frac{q_n(\omega_n)}{q^{(c)}(\omega_n)} \sim \begin{cases} (\varepsilon(\hbar k)/\hbar\gamma)^{3/2}, & \varepsilon < \hbar\gamma, \\ (\varepsilon(\hbar k)/\hbar\gamma)^{1/2}, & \varepsilon > \hbar\gamma. \end{cases} \qquad (61)$$

This quantity is small for broad atomic resonances. For example it is of the order of 10^{-3} for Na atoms. The peak height can be large for narrow resonances.

At finite detunings of a strong field, the degeneracy with respect to energy is removed and the resonances are shifted (split) and broadened. The broadening of the energy of the bound states in the quasiclassical situation is somewhat unexpected. We can explain this fact by the effect of the strong tunneling (which is not at all exponentially small) at finite Δ. We see from Fig. 13 that the wave functions of the bound states that arise in the different potentials ($+V(x)$ or $-V(x)$) can overlap spatially. One potential is depicted in the diagram by the solid line and the other by the dotted line. When acted on by a small perturbation proportional to Δ, a particle of the one type can virtually go from the level n to a state of the particles of the "other" type in the level $\bar{n}$ and fall into a level having the energy $\hbar\omega_n$ in an adjacent well. In line with the Franck-Condon principle, the turning points of the states n and $\bar{n}$ should lie close together. States having the same negative energy are mixed in the second order with respect to Δ, while states of positive energy (e.g., l and $\bar{l}$) are mixed in the first order with respect to Δ.

Owing to these transitions, the width of the resonance is broadened by the frequency of jumping into the adjacent well Γ_n. For negative energies, the latter is of the order of $\Gamma_n \sim \Delta^2(\omega_n - \omega_{\bar{n}})^{-1}$, while $\Gamma_n \sim \Delta$ for zero energy. This broadening is manifested in different ways for broad and narrow resonances. Figure 14 shows the

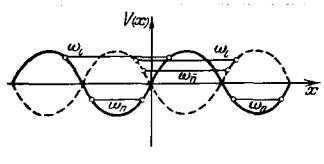

FIG. 13. Diagram of the resonance levels in a periodic potential.

[5)] If the weak signal were to act on the 1–2 transition, then transitions would also arise between the discrete-discrete states, and the peak shape would become Lorentzian.

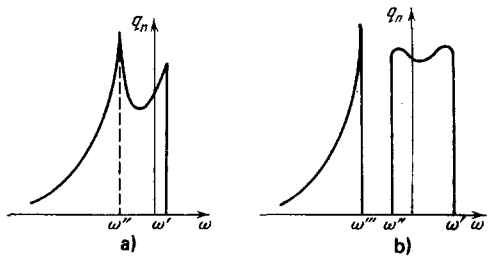

FIG. 15. Change in the peak shape with increase in the transition frequency Γ_n in the case of narrow resonances.

change in the peak shape for $\varepsilon(\hbar k) < \hbar\gamma$ in the case of finite detunings. The peak spreads out while maintaining its area.

The situation is more complicated for narrow resonances. With increasing Γ_n, an additional symmetric peak first arises, as shown in Fig. 15a. With further increase in Γ_n, when $\hbar\Gamma_n \gtrsim \varepsilon(\hbar k)$, the symmetric peak splits into two asymmetric peaks (of radical shape), and a band of width $\varepsilon(\hbar k)/\hbar$ (Fig. 15b) splits off the original peak. This process of splitting of narrow bands continues as Γ_n increases, while their height diminishes.

When $\Delta \sim \Delta_0$, the potential in which the particles move readjusts from the resonance potential $V(x)$ to the nonresonance potential $U(x)$. When $\Delta \gg \Delta_0$ in the quasiclassical approximation, we have two groups of particles that move in the potentials $\pm U(x)$. In this case transitions also arise from one potential well to the other one. However, the frequency of these transitions rapidly decreases with increasing detuning: $\Gamma_n \sim \Delta^{-3}$.

B. The continuous absorption spectrum in a strong field

The absorption coefficient $q^{(c)}(\Delta_0)$ is determined by the contribution from the fast atoms for which the modulation of the velocity in the standing wave is small. Absorption of the weak signal occurs only at the space points x_0 at which the law of conservation of energy is obeyed:

$$\hbar\Delta_0 = \pm U(x_0) + \frac{\hbar\Delta}{2}. \tag{62}$$

We can neglect the Doppler frequency shift when $I \gg I_1$. The absorption intensity is proportional to the time of transit through the neighborhood of the point x_0 or to the quantity $[dU(x_0)/dx_0]^{-1}$.

We can derive from this condition a simple formula for the absorption contour

$$q^{(c)}(\Delta_0) = \frac{Q}{\pi}\sqrt{\Delta_0 - \Delta}\,\sqrt{\Delta_0\left[\left(\frac{dE}{\hbar}\right)^2 + \Delta_0\Delta - \Delta_0^2\right]}, \quad \frac{\Delta}{2} - \frac{U}{\hbar} < \Delta_0 < 0, \quad \Delta < \Delta_0 < \frac{\Delta}{2} + \frac{U}{\hbar}, \tag{63}$$

Here $Q = 2\pi^2 k_0 d_{20}^2/\hbar$ is the integral absorption coefficient. In the absence of an external field, the integral absorption coefficient is also Q (sum rule). In line with the fact that there are two groups of particles that move in the potentials $\pm U(x)$, two absorption bands arise (Fig. 16b). They merge into one when $\Delta = 0$ (Fig. 16a).

Thus the absorption line is smeared out in the field of a strong standing wave into a band whose width is $2dE/\hbar$ when $\Delta = 0$. Formula (63) breaks down near the edges of the band. The resonance levels do not overlap, but touch at the very edge. In this case the time of transit of the particles through the resonance region is effectively increased and the absorption reaches a maximum. The law of conservation of energy (62) is not satisfied outside the absorption band, and $q^{(c)}(\Delta_0)$ sharply declines.

C. The quantum limit

Discontinuities arise in the quantum limit where $\hbar\gamma \ll dE \ll \varepsilon(\hbar k)$. When $\Delta = 0$, discontinuities of width $2dE$ arise at momenta of the particles of $\pm\hbar k/2$. When the detuning of the weak signal satisfies the condition

$$\hbar\Delta_0 = \varepsilon\left(\frac{\hbar k}{2}\right) - \varepsilon\left(\frac{\hbar k}{2} - \hbar k_0\right), \tag{64}$$

the parabola that corresponds to the spectrum of the atoms in the ground state falls in the forbidden band, and the absorption declines to the small relative value of $\hbar\gamma/dE$ or to $dE/\varepsilon(\hbar k)$. A dip arises in the absorption coefficient. Its width is determined by the value of the field $dE/\hbar$, while its position does not depend on the field, the depth of the dip can be large. The width increases with increasing field, while the depth diminishes. The dip disappears when $dE \gtrsim \varepsilon(\hbar k)$.

Thus the effect of light pressure on the spectral characteristics of atoms and molecules can be very substantial. Even in very weak fields (of intensity of the order of 0.1 mW/cm^2), atoms having narrow lines begin to manifest effects of quantized motion: a fine structure arises in the absorption spectrum in the form of peaks or dips. The relative height of the peaks or depth of the dips can be of the order of unity for narrow resonances.

7. THE SHORT-WAVE LASER BASED ON THE STIMULATED-RECOIL EFFECT

We shall show on two simple examples that light pressure can be employed for selection of excited and unexcited atoms. One can perform this selection most simply in terms of velocities. Here one can have two characteristic regimes: slow (during a transit time) using metastable atoms, and fast for strong resonance transitions.

A. The laser based on metastable helium atoms[21]

Let us study a gas of helium atoms with an appreciable admixture of He (2^3S) which is still small with respect to the concentration of unexcited atoms. The lifetime of orthohelium is long at low pressures. Hence one can

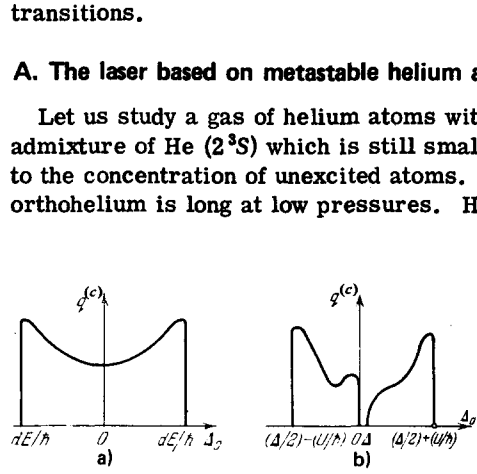

FIG. 16. Absorption contour in the strong field of a standing wave for $\Delta = 0$ (a) and for $\Delta \neq 0$ (b).

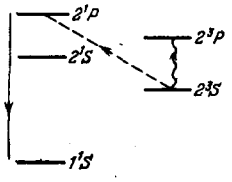

FIG. 17. Diagram of the levels of He and transitions in resonance fields.

try to alter the velocity of the metastable atoms with respect to the atoms in the ground state. For this purpose it suffices to use the resonance field for the 2^3S–2^3P transition, $\lambda = 1.08$ μm. Figure 17 shows a level diagram. Owing to the F_2 forces, one can accelerate the metastable atoms with a light pulse of energy 2 J and duration 2×10^{-5} sec to a velocity $v_0 > v_T$. A quite essential point is that the atoms become cooled during such an acceleration. This means that subsequently all the excited atoms can participate in laser action.

As seen as v_0 exceeds the thermal velocity of the atoms in the ground state, the emission and absorption contours begin to differ substantially. Yet the decay of the 2^3S and 2^3P states into 1^1S occurs too slowly. One can overcome this difficulty by using an additional light pulse that is in resonance with the 2^3S–2^1P transition ($\lambda = 0.87$ μm).[6)] The intercombination transition should occur in a time of the order of or shorter than the lifetime τ of the 2^1P state: $2dE \gtrsim \hbar/\tau$ ($\tau = 0.5$ nsec). We find from this condition with $d = 2\times10^{-3}$ Debye[56] that the needed power of the pulse is 100 MW/cm².

In this way we can get a negative absorption coefficient in the short-wave 2^1P–1^1S transition, $\lambda = 584$ Å. Since the excited atoms have no Doppler broadening, the amplification coefficient is $\sigma_{res} n = 10^{-11} n$, where $\sigma_{res} = 3\pi\lambda^2$ is the resonance absorption cross section of the photon. In order to get appreciable amplification ~1 cm⁻¹, one needs a small initial concentration of metastables ~10^{11} cm⁻³. The gain in density as compared with the inhomogeneously broadened contour amounts to two orders of magnitude. Perhaps it is more expedient to use helium molecules to get laser action in such a scheme.

B. The laser based on the recoil effect[54]

We shall now discuss a one-step laser scheme in a gas of atoms having the working levels 0, 1, and 2. The transition 1–2 lies in the microwave region, while the transition 0–2 lies in the short-wave region of the spectrum.

The initial concentrations n_1 and n_2 of excited atoms are smaller than n_0, the concentration of unexcited atoms. Let us denote by $1/\gamma_{20}$ the lifetime of state 2 with respect to spontaneous transition to the ground state. In the strong field of two opposing nonmonochromatic waves that are in resonance with the transition 1–2, the excited atoms are heated. The power needed to heat the atoms to the temperature T^* is determined by the relationship (51). In order to avoid broadening of the emission contour by the strong field, the duration of the pulse should be shorter than $1/\gamma_{20}$. If the acceleration of the excited atoms yields

$$T^* > T, \tag{65}$$

where T is the temperature of the unexcited atoms, then the absorption coefficient in the short-wave transition 2–0 (Δ_0) in the frequency region $k_{20}v_T < \Delta_0 < k_{20}v_T^*$ can become negative (Fig. 18). This happens under the condition

$$\frac{n_2}{n_0} > \frac{\gamma_{20}}{k_{20}v_T}. \tag{66}$$

The criterion (66) replaces the condition of an inverted population. In the examples to be given below we have $\gamma_{20}/k_{20}v_T \sim 10^{-2}$–$10^{-3}$. The condition (65) and (66) determine the power of the pumping light pulse and the concentration of excited atoms. As an example, Table II gives the values of these parameters that are needed for getting laser action in hydrogen, helium, and lithium. In order to estimate the parameters, the inequalities are replaced by equalities, the temperature of the neutral atoms is taken as 300 °K, and that of the ions as 1000 °K.

TABLE II.

λ_{12}, Å	I, W·cm²	λ_{20}, Å	q, cm⁻¹
H. $2s-3p$, 6560	$3\cdot10^8$	1025	$4\cdot10^{-14}\,n_2$
He. $2s-3p$, 5015	$4\cdot10^9$	537	$10^{-14}\,n_2$
He II. $4s-6p$, 6500	$5\cdot10^9$	234	$3\cdot10^{-16}\,n_2$
Li II. $4s-6p$, 6100	$5\cdot10^{10}$	157	$6\cdot10^{-16}\,n_2$

We see that it takes a density of excited atoms of 3×10^{13}–3×10^{15} cm⁻³ for an appreciable amplification. One can select similar transitions for any neutral atoms. Here the requirements imposed on the light beam are less rigid than in the case of helium atoms, since the widths of the resonance levels are smaller for the other atoms. Evidently the method of velocity-selection of atoms discussed above is impeded by collisions of the excited and unexcited atoms, which equalize the velocities of the particles. The rate of relaxation Γ_{20} due to collisions involving excitation exchange is related to γ_{20} by the relationship $\Gamma_{20} \approx 5.3\,\gamma_{20} n_0 \lambda_{20}^3$. The condition $\Gamma_{20} < \gamma_{20}$, which is satisfied up to densities $n_0 \lesssim 10^{17}$–10^{18} cm⁻³, is required for laser action. In the case of He II and Li II, owing to Coulomb collisions, we have the value $\Gamma_{20} \sim 10^{19}$ sec⁻¹ for $T = 10^3$ °K and density $n_0 = 10^{16}$ cm⁻³. This value is of the order of the rate of spontaneous relaxation γ_{20}.

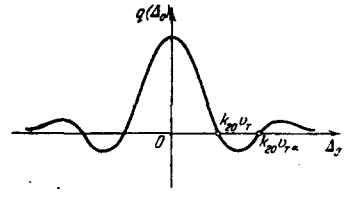

FIG. 18. Absorption line shape in the adjacent transition under conditions of the inequalities (65) and (66).

[6)] A scheme for getting laser action in a recombining plasma by using an intercombination transition has also been treated in Ref. 57.

Thus, one can "control" the emission and absorption contours by using light pressure and obtain laser action without inversion.

8. CONCLUSION

At the beginning of our century people assigned only a certain modest role to light pressure in astrophysical phenomena. Now the situation is becoming different. In the high-power fields of coherent laser sources of radiation, the pressure forces on a resonance particle can be very large. A set of phenomena arises in optics and in quantum electronics in which resonance light pressure becomes essential or can be utilized in some way.

Usually the mechanical action on individual atoms and molecules is realized by scattering of particles having dipole or magnetic moments in an inhomogeneous electric or magnetic field. Scattering of atoms and molecules in a resonance light field also can be effective. In the field of a standing wave, in which the field inhomogeneity is determined by the wavelength, the gradient force of light pressure can be so great that it suffices to use short light pulses for scattering the particles. In a pulse of duration 10^{-8} sec and energy 1 mJ, the characteristic time of transit by an atom of a distance of half a wavelength is also of the order of 10^{-8} sec. The energy obtainable in a pulse of tunable-frequency lasers suffices for scattering atoms through an angle of the order of 5–10°.

Since the mean free path of the photons is large in a strong field, one can use in such a scattering scheme high-density particle fluxes obtainable in high-power gas-dynamic jets. The scattering of the atoms in this case differs little from elastic scattering. The selectivity of scattering is sufficiently great even in a strong field. This circumstance can be essential for the problem of isotope separation. One can use light pressure not only to heat but also to cool atoms. Metastable atom accelerated and cooled by the mixed-type force can all contribute to coherent emission in a short-wavelength transition. In order to get laser action in this case a small concentration of metastables is required. Light pressure can play a very substantial role in the spectroscopy of narrow atomic and molecular resonances.

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Note added in proof. The splitting of the trajectories of motion of an atom in the field of a plane running wave has been studied recently in Ref. 60. The motion of atoms in the weak field of a standing wave $dE \lesssim \hbar\gamma$ (without taking account of the two trajectories of motion) has been studied in Ref. 61.

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