New experimental investigations of the $1/f$ noise mechanism

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Usp. Fiz. Nauk 123, 131–136 (September 1977)

PACS numbers: 72.70.+m, 73.60.Dt

Many current-carrying conductors are subject to electric fluctuations (of the voltage or current) whose spectral density $S(f)$, i.e., the mean square per unit frequency band, does not tend to a finite value with decreasing frequency f, but, on the contrary, increases. Usually $S(f)$ is proportional to $1/f$, but frequently $S(f) \sim f^{-a}$, where a is somewhat larger or smaller than unity. This type of noise is called flicker noise or $1/f$ noise, or finally excess noise.[1] It is observed in a great variety of conductors and electronic devices, and not only in solid conductors but also in liquid electrolytes.[1)] It turned out that voltage fluctuations on a membrane of a live (i.e., nonequilibrium but quiescent) nerve also satisfy the $1/f$ law.[2][2)] The $1/f$ noise is particularly large in inhomogeneous conductors, such as carbon microphones, island-type metallic films, conductors with poor contacts or with a developed surface, and in many semiconductor devices. The upper frequency limit at which the $1/f$ noise becomes noticeable, i.e., the spectral density of the noise increases noticeably with decreasing frequency, reaches many kilohertz in very noise grainy conductors. As to low frequencies, $1/f$ noise was traced in a large number of conductors and electronic devices to $\sim 10^{-5}$–10^{-6} Hz. It is important that the spectral density of the noise exhibits in this case no tendency whatever to saturation, i.e., to approach a finite value at $f=0$. Progress in the technique of measuring noise spectra at infralow frequencies merely decreases the lower frequency level, but cessation of the growth of $S(f)$ with decreasing frequency was never observed if the $1/f$ noise was observed in a large frequency band (in some electronic devices, the $1/f$ noise was traced in a band extending in frequency over ≈ 10 orders of magnitude). This is what makes the $1/f$ noise a paradox. After all, $\int_0^\infty df S(f)$ is the mean square of a fluctuating quantity and should be finite. At the same time, in the presence of $1/f$ noise this integral diverges at the lower limit. Moreover, according to the Wiener–Khinchin theorem, $S(f)$ in a stationary system is double the Fourier transform of the fluctuation correlation function $F(t_1-t_2)$ in time

$$S(f)=2\int_{-\infty}^{+\infty} d(t_1-t_2)\, e^{i\omega(t_1-t_2)}F(t_1-t_2), \quad \omega=2\pi f.$$

It follows that the derivative with respect to frequency is

$$\frac{\partial S}{\partial\omega}=2i\int_{-\infty}^{+\infty} d(t_1-t_2)\, e^{i\omega(t_1-t_2)}(t_1-t_2)F(t_1-t_2).$$

It is known that the fluctuation correlation function in time is an even function of the time difference t_1-t_2. It follows therefore that the derivative $\partial S/\partial\omega$ should tend to zero as $f\to 0$, and $S(f)$ should tend as $f\to 0$ to a finite value with a zero slope. There seems to be no way at present to reconcile the $1/f$ noise with this theorem.

Another manifestation of the same paradox is the following. According to the general premises of correlation theory of fluctuations, the frequency dependence of the spectral density $S(f)$ (and the associated dependence of the fluctuation correlation function on the time difference $F(t_1-T_2)$) are determined by the same characteristic relaxation times of the system that are observed in the response of the system to an external perturbation. However, the responses of systems with considerable $1/f$ noise do not reveal relaxation times long enough to be regarded as responsible for the $1/f$ noise.

The aforementioned paradoxes are related by some authors to the nonstationary character of systems that exhibit $1/f$ noise, i.e., to the fact that these systems have an average time variation.[4] Many *mathematical* models have been proposed, in which $1/f$ noise is obtained. However, no connection whatever was proposed between these models and any *physical* systems or models of physical systems. Moreover, there is likewise no experimental proof that conductors or devices with considerable $1/f$ noise are nonstationary to a sufficient degree.

The $1/f$ noise has been the subject of a tremendous number of studies—more than any other noise mechanism. We must bear in mind out also the applied aspect of the problem: the $1/f$ noise limits the sensitivity of many electronic devices operating at low frequencies, particularly amplifiers, and leads also to instability of the phase and frequency of high-frequency generators.

Many studies have been devoted to the elucidation of the mechanism of the $1/f$ noise in various systems. In a recent cycle of articles, Clarke and co-workers[5–9]

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1) See the brochure[14] for a survey of the investigation of $1/f$ noise in semiconductors and semiconductor devices as well as of theoretical models.

2) A $1/f$-type spectral density has been found also for non-electrical fluctuations. It turns out that the amount of insulin that a diabetic must have to maintain his blood sugar content fluctuates for different patients (the diet being fixed), and a correlation analysis has shown that the spectral density of the fluctuations obeys the $1/f$ law in a wide range of frequencies.[3]

 0038-5670/77/2009-0763$01.10

explain to a considerable degree, albeit not fully, the nature of the $1/f$ noise in metallic films and demonstrate the equilibrium character of $1/f$-noise sources in films of both metals and semiconductors.

Clarke and Voss have made a number of measurements[5,6] of the spectral density $S_U(f)$ of the fluctuations of the voltage at the terminals of long (250–2000 Å) homogeneous films of a number of metals and alloys at room temperature in the range from $\sim 10^{-1}$ Hz to ~ 1 kHz. The samples were in the form of narrow "necks" ($10 \times 150\ \mu$m) between broad sections of the film. Films of gold, silver, copper, bismuth, and manganin were investigated. In all cases but one, the flow of current gave rise to voltage fluctuations with a spectrum $S_U(f)$ proportional to $1/f$. The remarkable exclusion was the alloy manganin, the resistivity of which at room temperature is practically independent of temperature: $|\beta| = R^{-1}|dR/dT| < 10^{-4}\ \mathrm{deg}^{-1}$. When the average voltage U on the sample is varied, the spectral density $S_U(f)$ of the voltage fluctuation changes in proportion to U^2. These results indicate that the low-frequency voltage fluctuations on the sample are due to fluctuations of the temperature $\bar{T}$ averaged over the sample volume: these fluctuations modulate its resistance, and this leads to voltage fluctuations when current flows in the sample. Then $S_U(f) = U^2\beta^2 S_{\bar{T}}(f)$, where $S_{\bar{T}}(f)$ is the spectral density of the fluctuations of the temperature averaged over the volume of the sample.

Clarke and Voss[5,6] have also measured the correlation of the fluctuations of the voltage at the ends of two parts of the same bismuth films, separated by a certain distance L. The measured fluctuation voltages as functions of the time, $\delta U_1(t)$ and $\delta U_2(t)$ on each of two parts of the film were automatically added and subtracted, and the spectral densities $S_+(f)$ of the sum $\delta U_1 + \delta U_2$ and $S_-(f)$ of the difference $\delta U_1 - \delta U_2$ were measured. The quantity

$$C(f) = \frac{S_+ - S_-}{S_+ + S_-}$$

is a measure of the correlation of the fluctuations $\delta U_1(t)$ and $\delta U_2(t)$, since it is equal to zero if these fluctuations are not correlated and to unity if they coincide.

If the voltage fluctuations are due to temperature fluctuations, then the heat flow through the sample, described by the heat-conduction equation, should cause the fluctuations in both parts of the film to be correlated at low frequencies and uncorrelated at high frequencies. Namely, if D is the thermal diffusivity coefficient of the metal, then during one period of the oscillation (time) $1/f$ the heat traverses a distance on the order of $\lambda(f) = (D/\pi f)^{1/2}$. One should expect a decrease of $C(f)$ as a function of the frequency to take place at $\lambda(f) = L$. This was indeed the case, confirming that the $1/f$ noise is due to fluctuations of the sample temperature.

The integral $\int_0^\infty df\, S_{\bar{T}}(f) = \overline{(\delta \bar{T})^2}$ is the mean square of the temperature fluctuations. Its value in thermodynamic equilibrium is known from thermodynamics, viz., $\overline{(\delta \bar{T})^2} = kT^2/C$, where k is Boltzmann's constant and C is the sample heat capacity, equal at room temperature to $3kN_a$ (N_a is the number of atoms in the sample). It follows therefore that in metal and semimetal films, in the region of the $1/f$ noise, the ratio S_U/U^2 should be inversely proportional to the number of atoms N_a. This distinguishes the objects under consideration from homogeneous semiconducting samples, whose spectral $1/f$ noise density was found by Hooge to satisfy the empirical relation[10] (see also[11,12])

$$S_U(f) = \frac{\alpha U^2}{N_c f},$$

where α is a constant approximately equal to 2×10^{-3} and N_c is the number of carriers in the sample. Indeed, the $1/f$ noise in the films of the semimetal bismuth is approximately the same, at equal N_a, as in metals, although the free-carrier density in bismuth is lower by several orders of magnitude.

The spectral density $S_{\bar{T}}(f)$ of the temperature fluctuations can in principle be calculated by solving the heat-conduction equation with fluctuating heat sources, if the heat-transfer conditions inside the film and on the contact with the substrate are known. However, none of the theoretically investigated models of temperature fluctuation in a film has led to a spectrum in the form $1/f$ in an appreciable frequency band.

It follows from simple physical considerations that in the film temperature-fluctuation spectrum should have two characteristic frequencies that are connected with the film length and width l and w: $f_1 = D/\pi l^2$ and $f_2 = D/\pi w^2$. They are the reciprocals of the times of propagation of the heat along and across the film, respectively. Calculation yields at $f \gg f_2$ a spectral density $S_{\bar{T}}(f) \sim f^{-3/2}$ and $S_{\bar{T}}(f) = \mathrm{const}$ at $f \ll f_1$. Clarke and Voss have constructed a model-dependent function $S_{\bar{T}}(f)$ with a constant value at $f \leqslant f_1$, proportional to $f^{-3/2}$ at $f \geqslant f_2$, and was set proportional to $1/f$ in the interval between f_1 and f_2 in accord with the experimental data. The requirement that this function be piecewise continuous and the equality $\int_0^\infty df\, S_{\bar{T}}(f) = kT^2/C$ define completely the model-dependent function $S_{\bar{T}}(f)$ and consequently also $S_U(f)$. In the region between f_1 and f_2 we have

$$S_U(f) = U^2 \frac{\beta^2 kT^2}{C\,[3 + 2\ln(l/w)]} \frac{1}{f}. \qquad (1)$$

This function agrees very well quantitatively with the measured values of $S_U(f)$, a fact regarded as one more argument in favor of the assumption that the $1/f$ noise observed in metallic films is due to the equilibrium fluctuations of the sample temperature. In experiment, however, the $1/f$ noise was observed by the authors also at frequencies much lower than f_1, although in the derivation of (1) it was assumed that at these frequencies $S_{\bar{T}}(f)$ does not depend on the frequency. It must apparently be recognized that the right-hand side of (1) depends very weakly (logarithmically) on the length l and on the frequency f_1.

We emphasize once more that Clarke and Voss have postulated that $S_{\bar{T}}(f)$ is proportional to $1/f$. What remains unclear is which of the possible physical reasons (say singularities of heat transfer on the film–substrate interface) gives rise to a fluctuation spectrum of this form. The extent to which the conditions of heat transfer on the film–substrate interface are important was demonstrated by Clarke and Hsiang.[7,8] They measured the spectral density of the voltage fluctuations at the ends of thin tin and lead films in the region of the transition from the superconducting to the normal state. Some of the films were deposited directly on glass (type A samples) and the other part was deposited not directly on a glass or sapphire substrate, but with a thin layer of aluminum 50 Å thick between the film and the substrate (type B samples). It was established that the aluminum interlining improves greatly the thermal contact between the film and the substrate, and decreases its thermal resistance.

In current-carrying samples of type A, the spectral density of the voltage fluctuations was proportional to $1/f$ and $U^2\beta^2/V$, where V is the volume of the sample. In type A tin films $S_U(f)$ followed strictly the semi-empirical formula (1), while in lead films it was smaller by a factor of 5. On the other hand in type B tin films there was no $1/f$ noise: $S_U(f)$ did not vary changing frequency below approximately 30 Hz. The presence of aluminum interlinings altered the form of the spectrum in the Pb films: to proportionality to $f^{-1.1}$ (type A) gave way to a proportionality to $f^{-0.8}$ (type B), i.e., the noise increased more slowly with decreasing frequency. In addition, in samples of type B there was a much smaller correlation of the fluctuations in two parts of the film separated by a definite distance L (see above) The understanding gained from these experiments of the factors that determine the $1/f$ noise in films in the region of the transition from the superconducting to the normal state has made it possible to develop a superconducting bolometer with an appreciably lower noise.[13]

A very important observation, which can help understand the mechanism whereby the $1/f$ noise is produced in metallic films, was made by Clarke and Voss.[5,6] They measured the time variation (relaxation) of the temperature of a metallic film that had been heated by current (the temperature was determined by measuring the resistance). In one case the heating was pulsed, and the authors monitored the return of the temperature to the preceding value. In another case Joule heating was turned on instantaneously and subsequently kept constant, and the approach of the temperature to the new higher value was measured. It is interesting that although the relaxation of $\overline{T}_p(t)$ after the application of the pulse power lasts only several hundredths of a second, the relaxation of $\overline{T}_s(t)$ after a steplike application of the power is much slower and is noticeable after several dozen seconds. The Fourier cosine transform of the function $\overline{T}_s(t)$, i.e., $\int_0^\infty dt\,\overline{T}_s(t)\cos\omega t$, behaves like $1/f$ while the frequency $f=\omega/2\pi$ changes by several orders of magnitude. At the same time, the Fourier cosine transform of the function $\overline{T}_p(t)$ is independent of f already at $f \lesssim 10$ Hz. Thus, in the case of the $1/f$ noise, the noise spectrum coincides with the spectrum of the real part of the response of the temperature to the power input to the sample, but just to a stepwise application of the power. There is no theoretically incontrovertible explanation of this regularity.

A large number of workers searching for the mechanism of the $1/f$ noise have attributed this noise to the onset of instabilities in the conductor under the influence of the current flowing through it. These theories seem to contradict the fact that in many conductors the spectral density $S_U(f)$ is proportional to the square of the average current (or to U^2) in the region of the $1/f$ noise. This means that the current makes it possible to observe only those resistance fluctuations which are present also in the absence of the current. The following experiment by Clarke and Voss[6,9] confirms this once more in a rather illustrative manner.

It is known that the spectral density of the fluctuation of the voltage on the terminals of a resistor R shunted by a capacitor C is given by the Nyquist formula $S_U(f) = 4kTR/(1+\omega^2\tau^2)$, where $\tau = RC$. Voss and Clarke passed a fluctuating voltage picked off an InSb semiconducting film through a spectrum analyzer in a frequency band $\Delta\nu$ from ν_- to ν_+. The lowest frequency of the transmission band was $\nu_- \gg 1/2\pi\tau = 500$ Hz. The obtained signal $\delta U_{\Delta\nu}(t)$ was squared. The quantity $P(t) = \delta U^2_{\Delta\nu}$ fluctuated about the mean value given by the Nyquist formula:

$$\overline{P} = \overline{\delta U^2_{\Delta\nu}(t)} = \int_{\nu_-}^{\nu_+} d\nu\, 4kTR\,(1+4\pi^2\nu^2\tau^2)^{-1} \approx \frac{kT}{\pi^2 RC^2}(\nu_-^{-1}-\nu_+^{-1}).$$

The right-hand side of this formula contains the mean values of the fluctuating T and R. The low-frequency ($f \ll \nu_-$) fluctuations of $P(t)$ about $\overline{P}$ are caused by two factors: the fluctuations of T and R, contained in the Nyquist formula, and the random character of the Nyquist sources themselves of the voltage fluctuations, i.e., the randomness of the scattering of the carriers in R. The first cause would produce fluctuations of $P(t)$ (we shall designate them $\delta P_{\text{ext}}(t)$ even if T and R were not to fluctuate). Therefore

$$\delta P(t) = P(t) - \overline{P} = \delta P_{\text{ext}}(t) + \frac{\partial\overline{P}}{\partial T}\delta T(t) + \left(\frac{\partial\overline{P}}{\partial R}\right)_T \delta R_T(t).$$

The last term corresponds to fluctuations of R which are not connected with δT.

The spectral density of the fluctuations of $P(t)$ at low frequencies $f \ll \nu_-$ is given by a formula which follows from the fact that there is no correlation at all between the three sources of the fluctuations of $P(t)$:

$$\frac{S_P(f)}{\overline{P}^2} = \frac{S_{P_{\text{ext}}}(f)}{\overline{P}^2} + (1-\beta T)^2 \frac{S_T}{T^2} + \frac{S_{R_T}}{R^2}.$$

At $f \ll \nu_-$ the function $S_{P_{\text{ext}}}$ does not depend on f and it can be shown that it is equal to $2\int_{\nu_-}^{\nu_+} d\nu\, S_U^2(\nu)$. Therefore if S_T or S_R contains a part proportional to $1/f$, then at sufficiently low frequencies it is precisely this part which becomes decisive. The measurements of Clarke and Voss[6,9] have shown that below 1 Hz the function $S_p(f)$ varies like $1/f$.

It must be emphasized that the investigated samples were in a state of thermodynamic equilibrium: no current was made to flow from an external source. It can therefore be concluded that the sources of the $1/f$ noise are in equilibrium and that this noise is not due exclusively to violation of the equilibrium by the flowing current or to the appearance of various current instabilities.

As already mentioned, investigation of the $1/f$ noise has been the subject of a tremendous number of studies. Most experiments were made on systems that produce an intense $1/f$ noise or are important from the point of view of practical applications, but are physically so complicated, that it is extremely difficult to identify the processes that may possibly be responsible for the noise. A tendency to study simpler systems has been observed recently. Although these systems have a much lower $1/f$ noise, they lend themselves to physical experimentation. One can hope that this will lead to a solution of the problem that is half a century old.

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Translated by J. G. Adashko