Suppression of plasma instabilities by the feedback method

V. V. Arsenin and V. A. Chuyanov

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A new method of suppressing plasma instabilities with the aid of automatic control systems has been recently developed. The feasibility of using this method to stabilize a continuous medium is brought about by the fact that not all of the tremendous number of degrees of freedom of the medium are actually excited, but only several of the modes. The method is most suitable in practice for the suppression of large-scale oscillations. Two stabilization methods have been observed. An external control system can be used to alter the phase velocities of the waves in the plasma, thus upsetting their resonant interaction with the particles and with one another. This suppresses strong instabilities with large growth rates, particularly hydrodynamic ones (flute, helical, etc.). To eliminate instabilities of the dissipative type with small growth rates it suffices to use an external system to introduce additional damping of the plasma oscillations. The possibility of turning individual modes on or off makes feedback an outstanding tool for the study of nonlinear properties of oscillations, anoalous transport phenomena, and others. The first experiments in this direction have already been performed.

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CONTENTS

1. INTRODUCTION

As a rule, laboratory plasma is not in thermodynamic equilibrium and, therefore, exhibits various types of instability. Plasma encountered in research into controlled thermonuclear fusion is never in thermodynamic equilibrium. At the very least, it contains currents due to density and temperature gradients. The suppression of instabilities is, therefore, a central problem in this area.

There are several known ways of solving this problem. Firstly, it is desirable to ensure that the plasma produced initially is as close to thermodynamic equilibrium as possible. Secondly, stability is improved by developing special magnetic-field configurations, for example, configurations with a magnetic well. Thirdly, external high-frequency fields can be imposed on the plasma (this produces dynamic stabilization).

One more method, based on the utilization of automatic control systems, has been developed in recent years. In the literature concerned with plasma physics, it is referred to as the feedback method. The principle of this approach is that an electronic circuit interacting with the plasma through the oscillation field is set up outside the plasma, and the parameters of this circuit are chosen so that its combination with the plasma is stable as a whole.

Plasma has an enormous number of collective degrees of freedom (an infinite number in the continuum model). It would be impossible to construct a system capable of controlling all these oscillations. A typical situation, however, is that in which the instability of individual modes sets in successively as the plasma parameters are varied. Feedback stabilization is possible precisely under such conditions. In practice, the feedback method can be used to suppress only a *finite number* of modes, but whenever experiments indicate the predominance of one or a few modes, the situation must be examined in the light of both this and other nonuniversal methods.

The first published paper on feedback stabilization of plasmas is that due to Morozov and Solov'ev[1] although some isolated proposals had been put forward earlier.[1)] For example, Artsimovich and Kartashev[7] discussed the possibility of feedback stabilization of a large-radi-

[1)] We note also a number of allied problems in the sense that they involve spatial dispersion and the necessity of suppressing several modes at the same time. They include stabilization of a charged fluid jet, of a flexible conducting membrane between charged planes, and of Rayleigh-Taylor instabilities of the surface of a conducting fluid in a normal electric field.[2–6]

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us toroidal plasma ring carrying a current. Morozov and Solov'ev[1] considered the Rayleigh-Taylor instability of a sharp boundary of dense magnetized plasma in the gravitational field and the hydromagnetic instability of a column with a surface current. It was suggested that, where the plasma boundary departed from its equilibrium position, this could be corrected by currents set up in external conductors and producing an additional magnetic pressure. Similar considerations relating to the stabilization of the Rayleigh-Taylor instability were reported by Artsimovich and Kartashev.[7] However, experiments with such magnetic systems were begun only in the very recent past.

The first experiment on feedback stabilization of plasma was carried out in 1967[8,9] and was concerned with the flute instability of tenuous plasma after theoretical analysis[10] showed that stabilization near the instability threshold could be achieved by simple means, namely, with the aid of an electrostatic system. After the magnetohydrodynamic flute instability, the same method was used to suppress one of the kinetic instabilities, namely, the ion-cyclotron Harris instability.[11]

The success of these initial experiments stimulated extensive theoretical and experimental work on the suppression of MHD, drift, ionization, and other instabilities. New methods of introducing feedback were put forward in 1969–1970. The work carried out up to 1970 was reviewed at the Princeton Symposium[12] and a brief review of the results obtained up to that point was given by Thomassen.[13] In recent years, research into the application of feedback to plasma physics has been concerned not merely with demonstrating the basic possibilities of the method but also with the development of "technical" applications designed to improve the confinement of plasmas in thermonuclear installations and to provide a means of studying plasmas.

2. PRINCIPLE OF THE METHOD

A. Classification of instabilities

The basic idea of feedback stabilization of plasmas is most conveniently explained in terms of a concrete example. The question is: which is the best instability to choose for this purpose?

From the standpoint of feedback stabilization, it is important to distinguish between two classes of instability, namely, reactive and dissipative instability.[14] Feedback instability control must satisfy different conditions in these two cases.[15,16]

Reactive instabilities arise during the interaction between two oscillation branches with the same phase velocities but with energies of different sign. Mutual excitation occurs because oscillations with negative energy[17] transfer energy to waves of positive energy. Reactive oscillations include, in particular, hydromagnetic plasma instabilities in ideal single-fluid hydrodynamics (their phase velocity is zero). Collisional or collisionless dissipative processes are unimportant for reactive instabilities, at least well away from the threshold where the growth rate γ is high.

In *dissipative instabilities*, oscillations corresponding to a given branch are excited as a result of energy exchange with plasma particles or an external wall. If the wave has positive energy, it will grow if energy is transferred to it from the particles or the wall. Oscillations of negative energy will grow if the wave gives up energy. Dissipative instabilities are characterized by frequencies $\omega \neq 0$ and low growth rates $\gamma \ll \omega$.

To suppress the reactive instabilities, the dispersive properties of the plasma must be modified so that the phase velocities of the interacting waves cease to be equal and energy exchange between them is prevented. Once this problem has been solved (the growth rate then tends to zero), the behavior of the waves is determined by only the dissipative effects. Stabilization of reactive instabilities is, therefore, a more general problem which includes the stabilization of dissipative instabilities as a special case.

The well-known flute instability of plasmas in mirror traps is a characteristic example of reactive instability. We shall use this extensively-investigated example in our account.

B. Qualitative description of flute instability and its suppression

Adiabatic traps with magnetic mirrors form a class of possible systems for the confinement of thermonuclear plasma. A simple configuration of this type is illustrated in Fig. 1. The magnetic fields at the ends of the trap are stronger than at the center, and the plasma (a diamagnetic medium) is confined to the central region with reduced field between the two so-called magnetic mirrors.

Flute instability[18] in an open trap with a simple mirror-type field is also a manifestation of the diamagnetic properties of plasma. In this trap, the magnetic lines of force in the region occupied by the plasma are convex in the outward direction, so that the magnetic

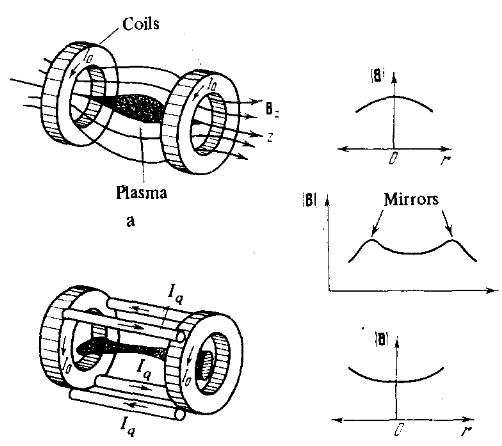

FIG. 1. Adiabatic traps with magnetic mirrors. a) Simple mirror trap, modulus of B decreases in the radial direction in the central cross section of the trap. b) Trap with a magnetic, well, modulus of magnetic field B increases in radial direction c) Axial distribution of the field modulus in mirror traps. I_0—current in the mirror coils; I_q—current in quadrupole coils producing the magnetic well.

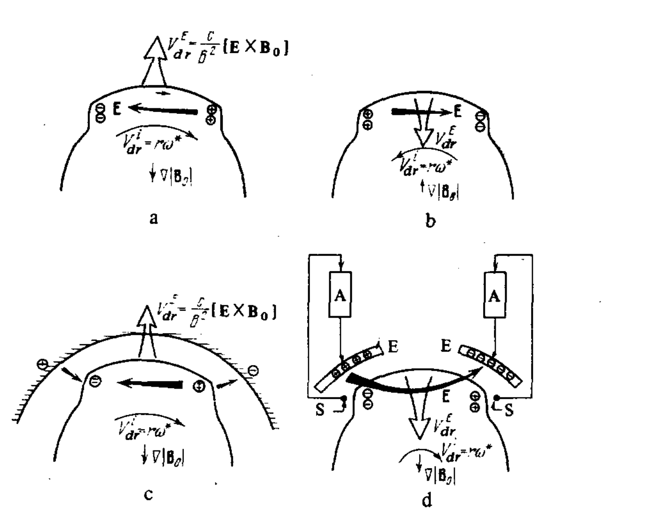

FIG. 2. Schematic distribution of charges, electric fields, and drift velocities of ions and plasma as a whole in the cross section perpendicular to the magnetic field in an open trap during the formation of a tongue: a) Simple mirror trap configuration, magnetic field decreases in radial direction, conducting wall well removed from plasma (growing tongue). b) Adiabatic trap with magnetic well, magnetic field increases in radial direction (tongue returns to plasma). c) Simple mirror configuration, metal walls close to the plasma (flute growth rate reduced). d) Simple mirror configuration with feedback system consisting of sensors (S), amplifiers (A), and electrodes (E). When the amplifier gain is high enough, the tongue returns to the plasma.

field decreases in this direction. A diamagnetic fluid will tend to occupy the region of weak field, so that a random perturbation of the plasma surface will lead to the formation of plasma "tongues" elongated along the magnetic lines of force. The result of this is that the plasma flows in the outward direction toward the chamber walls. "Flutes" are formed between the "tongues," and this is the origin of the name "flute instability."

The microscopic picture of the instability is well known and can be described as follows. Figure 2 shows the cross section of a plasma cylinder cut by a plane perpendicular to the magnetic field **B**. Suppose that, initially, the plasma is azimuthally symmetric and its axis lies along the magnetic field. The nonuniformity of **B** produces an azimuthal drift of ions and electrons in opposite directions with angular velocity $\omega \propto (1/B) \times \partial B/\partial r$. Owing to azimuthal symmetry, these currents do not lead to the separation of charges.

Let us now consider the distortion of the boundary. We shall confine our attention to the case where the gas-kinetic pressure of the plasma is $nT \ll B^2/8\pi$ (n is the plasma concentration and T is the temperature) and will consider electrostatic perturbations.

Suppose that a "tongue" has emerged from plasma with a sharp boundary (Fig. 2a). Ions will accumulate on one side of the tongue and electrons on the other. This gives rise to an azimuthal electric field **E** which produces a radial drift with velocity $c[\mathbf{E}\times\mathbf{B}]/B^2$ that is the same for both electrons and ions and gives rise to further growth of the "tongue."

The radical way of preventing this instability is to change the direction of the magnetic drift. This is achieved in a magnetic field which does not decrease with distance from the axis as in the simple mirror-type configuration but, on the contrary, increases in this direction (Fig. 2b). Stabilization by a magnetic well of this kind was first demonstrated by Gott, Ioffe, and Tel'kovskii[19] and has since been used in many experiments.

Another method of stabilization can also be proposed. Having detected a region with ion or electron excess, it is sufficient to inject charges from outside to compensate for this situation or to remove the surplus charges from the plasma. The externally introduced charges do not have to cancel accurately the separated plasma charges at each point in space. It is sufficient to achieve an average change in the field direction in the plasma.

In the case of feedback stabilization, it is important to distinguish between surface waves for which, in the absence of feedback, charges appear only on the surface of the plasma cylinder, and volume modes, for which charge perturbations are not zero practically throughout the volume of the plasma. We begin our discussion with the simplest case, namely, that of surface waves. The change in the sign of the electric field in plasma during the excitation of a surface wave need not be accomplished by introducing the compensating charges into the plasma itself. It is sufficient to place these charges near the plasma, for example, on a nearby wall, and this is very much easier to do. Moreover, from the practical point of view, it is more convenient to control not the charge density on the outer surface but the distribution of potential on it. The task of the system for the stabilization of surface waves is to control this distribution in accordance with field perturbations in the plasma. A simple analog of this system is a conducting wall surrounding a plasma column: image charges always appear in the wall (Fig. 2c). The field due to these images reduces the field due to the plasma charges, so that the presence of the conducting wall leads to a certain increase in the threshold plasma density for which instability appears. However, the image charges can never change the sign of the field in the plasma (see Fig. 2c). The effect can be enhanced with the aid of an external energy source and, if the resultant field due to all charges changes its direction (Fig. 2d), the perturbation will be suppressed.

C. Mathematical formulation of the stabilization of surface waves

Consider the stability of plasma in a mirror trap against low-frequency ($|\omega| \ll \omega_{Bi} \equiv eB/m_i c$) perturbations of the electric potential of the form $\psi = \varphi(r)\exp(im\theta - i\omega t)$, where θ is the azimuthal angle and m is the number of the azimuthal mode ($m \geqslant 1$). This wave will take the form of a flute elongated in the direction of the trap axis. The set of linearized equations of motion and of continuity for the electron and ion components, and the Poisson equation for the potential, can be reduced to a single equation for the radial part of the potential $\varphi(r)$:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right)-\frac{m^2}{r^2}\varphi(r)=-4\pi\hat{\rho}(\omega)\varphi, \qquad (2.1)$$

where $\hat{\rho}(\omega)$ is the charge density operator determined from the equations of motion and of continuity. This is a second-order equation so that, to determine the eigenfrequencies ω, we must provide two boundary conditions. One of these conditions is that the solution must be bounded at $r=0$:

$$\varphi(0) < \infty. \qquad (2.2)$$

If the plasma is located in a cylindric metal chamber of radius $r=b$, the second condition is

$$\varphi(b)=0. \qquad (2.3)$$

If there is no chamber, then $b=\infty$.

If, instead of a simple metal surface, we have an "active" surface whose potential is controlled by the feedback system, condition (2.3) must be modified so as to describe the effect of the control system. Our aim, i.e., plasma stability, can, in general, be achieved with the aid of a number of different control systems, so that the choice of the boundary (2.3) cannot be unique. However, since we are considering the linear problem, the potential on the $r=b$ surface must be a wave of the form $\varphi(b)\exp(im\theta-i\omega t)$ with amplitude proportional to one of the quantities describing the wave in the plasma, i.e., the amplitude of oscillations in density, field, potential, and so on. The experiments which we shall consider below will be such that the amplitude of the potential wave $\varphi(b)$ will be proportional to the field due to the plasma charges measured on the $r=b$ surface. The true radial electric field, $-d\varphi(b)/dr$, which can easily be determined by external probes, is the sum of the fields due to the plasma charges and the field due to the charges on the $r=b$ surface itself. The latter is determined by the "vacuum" solution of the Poisson equation $\varphi(r)=\varphi(b)(r/b)^m$ and must be subtracted from the resultant field when the field due to the plasma charges is to be determined. The result of all this is the boundary condition

$$\varphi(b)=bW(\omega, m)\left[\frac{d\varphi(b)}{dr}-\frac{m}{b}\varphi(b)\right]. \qquad (2.4)$$

This condition can be maintained by a suitably designed electronic system (Fig. 3), consisting of sensors detecting perturbations in the radial electric field $d\varphi(b)/dr$, amplifiers, and electrodes located on the $r=b$ surface. It is clear that the angular size of the electrodes must be small in comparison with the azimuthal wavelength of the stabilizing oscillations. Figure 3 shows only one circuit out of a large number of control circuits that operate in parallel and together ensure that condition (2.4) is satisfied. The feedback system shown in Fig. 3 will, in accordance with (2.4), ensure the compensation of the effect of the electrode on the sensor and the absence of self-excitation for large values of the transfer function $W(\omega, m)$ that is the only characteristic of the electronic system in our problem.[2)]

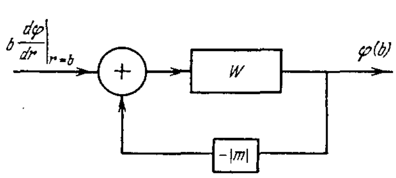

FIG. 3. Circuit ensuring that the boundary condition (2.4) is satisfied. Only one control loop is indicated.

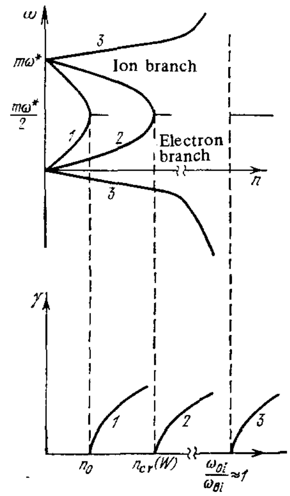

FIG. 4. Frequency and growth rate of flute oscillations for different transfer coefficients W independent of frequency. The instability threshold is determined by the point of intersection of the ion and electron oscillation branches with different signs of energy.

Figure 4 shows the eigenfrequencies of flute oscillations as functions of density in the simple mirror trap in the case of a frequency-independent transfer function W and a sharp plasma boundary:

$$\frac{dn}{dr}=-n_0\delta(r-a), \qquad 0<a<b \qquad (2.5)$$

where n is the plasma density and a the radius of the plasma column. At low densities, there are two oscillation branches, namely, the electron drift and the ion-drift branches. When $W=0$, the electron ($\omega<m\omega^*/2$) and ion ($\omega>m\omega^*/2$) oscillation branches, whose energies are of different sign, intersect at the threshold density for which $\omega_{0i}^2\sim\omega_{Bi}\omega^*$ ($\omega_{0i}=(4\pi e^2n_0/m_i)^{1/2}$ is the plasma frequency), and the result of this is the development of the reactive instability.[27] If we ensure that $W\neq 0$, this will influence the phase velocities of the waves $v_{ph}=r\,\mathrm{Re}\,\omega/m$ and will change the instability density threshold. A negative transfer coefficient will reduce the threshold density. When

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2) It is important to note that, whatever the measuring system, the boundary conditions can be reduced to the form indicated by (2.4) because all the quantities describing the plasma wave are linearly related. This is valid provided measurements are carried out on the $r=b$ surface and not inside the plasma. The additional factor which appears under these conditions depends on the frequency and mode number, and can be included in W. We note further that the presence of the term $-(m/b)\varphi(b)$ on the right-hand side of (2.4) is not essential. Systems without compensation of the electrode-sensor interaction have been considered[10] and were used in early experiments.[11,20–23] However, they are excited for $W>1/m$.[21,23] This excitation can be prevented by introducing a suitable dependence of W on ω,[24–26] but compensation is more convenient.

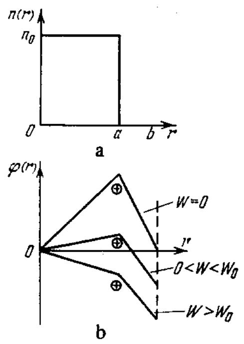

FIG. 5. Radial distribution of plasma density for a sharp boundary (a), and radial distribution of wave amplitude $\varphi(r)$ corresponding to different values of the transfer coefficient W (b). For $0<W<W_0$, the point corresponding to the effective position of the conducting wall, for which $\varphi(r)=0$, approaches plasma as W increases. For $W>W_0$, the positive charge on the plasma surface corresponds to a negative potential of the surface. The radial field in the plasma then changes sign as compared with the $W=0$ case.

$$0<W<W_0=\frac{1}{2m}\left[\left(\frac{b}{a}\right)^{2m}-1\right], \tag{2.6}$$

the threshold density corresponding to the intersection of the oscillations branches will increase. For $W>W_0$, the functional dependence of the phase velocities on density will itself be modified. This can readily be understood by considering the radial dependence of the perturbed potential $\varphi(r)$ shown in Fig. 5 for $W<W_0$ and $W>W_0$. It is clear that, in the second case, there is a relationship between the sign of the potential and the sign of the perturbed charge on the plasma boundary, which is determined by the second derivative of the potential with respect to the radial distance.

We note that the introduction of a feedback system with $0<W<W_0$ is equivalent to bringing up a metal wall closer to the plasma surface: the point where $\varphi(r)=0$ in Fig. 5 approaches the plasma surface as W increases. When $W>W_0$, the electron and ion waves propagate in opposite directions, as in the field with a magnetic well. The maximum threshold density is reached for $0<W-W_0<(b/a)^m$. When this is so, we have $\omega_{0i}^2\sim\omega_{Bi}^2/(W-W_0)$ at the threshold.

According to (2.6), the optimum value $W=W_0$ depends on m. This property was emphasized in[28] and is a general result for media with spatial dispersion. To ensure that the stabilization of a particular mode does not affect the stability of other modes, the stabilization system must also have spatial dispersion.

D. Influence of dissipative effects and phase shifts in the stabilization system

It is thus possible to ensure that the oscillations exhibit neutral stability with $\mathrm{Im}\,\omega=0$ when the transfer coefficient W is suitably chosen. However, we have ignored the weak dissipative effect and the unavoidable phase shifts in the stabilization system, which ensure that $\mathrm{Im}W\neq 0$, and this may give rise to some excitation of natural oscillations. This can be prevented through a suitable choice of $\mathrm{Im}W$ and by ensuring that all the natural oscillation frequencies are damped out. In our model, there is no dissipation in the plasma and the excitation and damping of waves must be accompanied by the exchange of energy with the active wall. If we calculate the perturbation of the magnetic field $\mathbf{B}'$ from the equation curl $\mathbf{B}=-(1/c)\,\partial E/\partial t$, and determine the average flux of energy S between the plasma and the $r=b$ surface per unit length of the plasma cylinder

$$S=\frac{c}{4\pi}\int_0^{2\pi/|\omega|}\int_0^{2\pi}E_\theta B_z'\,d\theta\,dt, \tag{2.7}$$

we find that this flux is proportional to $\omega\,\mathrm{Im}W$. When $\omega\,\mathrm{Im}W>0$, this flux is positive, and the external system takes up energy from the waves in the plasma. When $\omega\,\mathrm{Im}W<0$, the stabilization system supplies energy to the wave. It follows that oscillations corresponding to the electron branch, which have positive energy, will be damped when

$$\omega\,\mathrm{Im}\,W>0 \quad \text{for} \quad \omega<\frac{m\omega^*}{2}. \tag{2.8}$$

To suppress the ion branch, which has a negative energy, we must satisfy the opposite condition:

$$\omega\,\mathrm{Im}\,W<0 \quad \text{for} \quad \omega>\frac{m\omega^*}{2}. \tag{2.9}$$

Conditions (2.8) and (2.9) can also be obtained directly from the dispersion relation for the natural frequencies, without introducing energy considerations.

We have thus established two mechanisms for stabilization by feedback. Reactive instability is suppressed (the oscillations exhibit neutral stability) by varying (for real W) the phase velocities of the waves so that oscillation branches with energies of opposite sign cannot combine. Oscillations or waves with neutral stability, which are weakly excited for dissipative reasons, can be damped out by arranging the appropriate energy transfer between the plasma and the feedback system. The direction and magnitude of the energy flow is determined by the phase shift in the feedback system (imaginary part of W), i.e., the phase-frequency characteristic of the stabilization system. The choice of the frequency dependence of the transfer coefficient has been discussed in a general form by Sen[29] and by Uchan and Kammash.[30]

The suppression of dissipative instability with a low growth rate can be achieved even for a weak interaction between the external system and the plasma for small W. On the other hand, stabilization of reactive instabilities requires $W\geqslant 1$ (including values near the threshold) so that the simultaneous damping of two oscillation branches with close frequencies and energies of opposite sign cannot be achieved for small values of W.[16]

E. Stabilization by a surface with a complex impedance

The principle of the electronic system that maintains the boundary condition given by (2.4) is that it generates the required charge wave on the surface on which the stabilizing electrodes are arranged. It may be said that this charge wave is induced with the aid of the electronic system by the potential wave ψ, and the entire system of sensors, electrodes, and amplifiers may be regarded

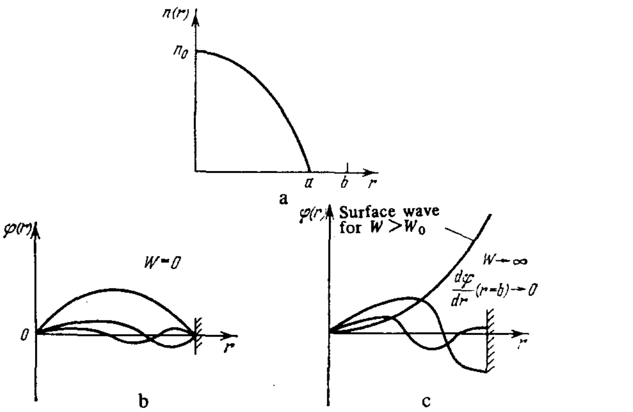

FIG. 6. Density distribution for a diffuse boundary (a), and the corresponding lowest radial modes $\varphi(r)$ for $W=0$ (b) and $W \to \infty$ (c). As $W \to W_0$, there is, in addition, a solution in the form of surface waves with imaginary radial wave number.

as a surface with a certain complex impedance $Z = -(i\omega L/c^2) - (1/i\omega C) + R$, where L, C, R depend on the structure of the sensors and the parameters of the circuit. The problem of stabilization of plasma oscillations that are sensitive to boundary conditions can be formulated right from the outset as the problem of determining the appropriate impedance of the surface (or, in general, the medium) surrounding the plasma.[31-41] In practice, the most common situation is that in which the medium is "active" (or at least one of the parameters L, C, R is negative), i.e., its realization must include amplifiers. In particular, a medium with negative capacitance is necessary for the suppression of flute oscillations.[41]

3. FACTORS COMPLICATING STABILIZATION

Before we proceed to the description of experiments, we must consider the complicating factors that distinguish the true experimental situation from the idealized model developed in Chap. 2. The most important of these are:

a) the existence of spatial modes,

b) additional damped oscillation branches, and

c) the discrete structure of the feedback system.

The problems that arise in relation to the suppression of flute instabilities in connection with these factors are also characteristic for the stabilization of other types of oscillation.

A. Spatial modes

In the model problem considered in Chap. 2, the surface feedback can be used to ensure stability for any density, i.e., to stabilize all the natural oscillations of the system. This is a consequence of the simplifying assumption that we have introduced, namely, the absence of a density gradient inside the plasma. This assumption excludes internal oscillations with an oscillating radial structure, i.e., spatial modes.

Such oscillations become possible in the presence of a radial density gradient and their radial wave number $\varkappa$, which is a function of the transfer coefficient W, remains real for all values of W (Fig. 6). The density enters the dispersion relation for flute oscillations through the ratio $n/\varkappa^2$. Truncation of the wave is therefore equivalent to an increase in the threshold density, and a restriction on the increase in $\varkappa$ as $W \to \infty$ means that high plasma densities cannot be stabilized even as $W \to \infty$. The physical interpretation of this is quite simple. As the transfer coefficient W increases, the spatial structure of the wave changes so that the electric field on the plasma surface, $d\varphi/dr(b)$, acting on the input of the stabilizing system, tends to zero (Fig. 6). The feedback system then ceases to be sensitive to perturbations in the plasma. The wave becomes unobservable from the surface, and cannot be stabilized.

Electron and ion waves with imaginary radial wave numbers $\varkappa$ appear when the transfer coefficient is large, i.e., we then have surface waves with hyperbolic radial structure. Their characteristics are identical with those obtained in Chap. 2 because surface waves are not sensitive to the detailed behavior of the density gradient. *The new feature as compared with Chap. 2 is the fact* that, for a distributed density gradient, these waves appear *not instead* of the waves with real wave numbers but *in addition* to them.

For a smooth radial density distribution on the surface, feedback can therefore only shift the instability threshold to some extent by stabilizing only the lowest spatial mode.

B. Spatial systems and observability

The fact that the higher radial modes cannot be stabilized is a basic defect of the surface stabilization system. This defect can only be completely removed with spatial feedback systems. Such systems have been put forward and have been investigated experimentally. We shall consider them later. The characteristics of such systems, if their structure corresponds to the spatial structure of the unstable oscillations, should approach those described in Chap. 2. It is important to note, however, that spatial measurements are generally much easier to perform than spatial control, and it is therefore interesting to consider a formulation of the problem in which the potential on the surface surrounding the plasma is determined by fields measured in the interior of the plasma.[42,43] It turns out that this can be used to shift the observability barrier toward shorter wavelengths, at least to some extent (Fig. 7). Numeri-

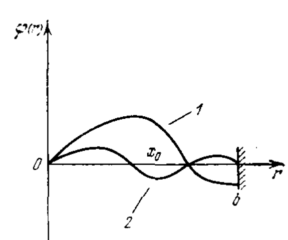

FIG. 7. Radial solutions $\varphi(r)$ for a diffuse plasma boundary, unobservable when the field sensor is at $r=x$. The wave is unobservable when $d\varphi/dr|_{r=x}=0$. 1) longest wave, unobservable from the $r=b$ surface; 2) longest wave unobservalbe on the $r=x$ surface.

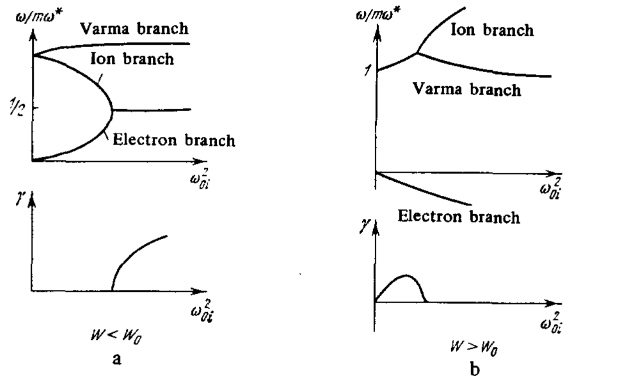

FIG. 8. Frequency and growth rate of flute oscillations as functions of density with allowance for the additional ion oscillation branch for $W < W_0$ (a) and $W > W_0$ (b). In case (b), reactive instability begins to grow at low densities because of the intersection of the ion drift and the additional (Varma) oscillation branches. The transfer coefficient W is assumed real and frequency-independent.

cal calculations of the change in the threshold density with the position of the point of observation have been reported by Lashmore-Davies.[42] However, no account is taken in this calculation of the fact that the presence of feedback makes this problem non-self-adjoint so that new types of oscillation may arise with complex radial wave numbers [radially traveling wave with amplitude $\sim \exp(\mathrm{Im} k_r \cdot r)$].[43] Such waves are unstable and their growth is accompanied by the outflow of energy from the plasma. The growth rate can, however, be made as small as desired by increasing W.

We note that, when the measurements are carried out in the interior of the plasma, whereas the surface held at the control potential is located outside the plasma, the fact that perturbations can be expanded into a series in terms of eigenfunctions must be separately proved. The classical Steklov theorem refers to the case where the boundary condition (which does not explicitly contain the eigenvalue) is imposed on the function or its derivative at a point at the end of the interval. In the case in which we are interested, the fact that the expansion can be carried out has been proved[44] only for the special problem $\varphi'' + q(x)\varphi = \lambda\varphi$, $\varphi(b) = \varphi(a)M(\lambda)/N(\lambda)$, where M and N are polynomials.

C. Additional damped oscillation branches

In the model considered in Chap. 2, the dispersion relation was of the second degree in frequency. When the true conditions are more accurately taken into account, the degree of this equation may become higher, i.e., new natural frequencies may appear. When these natural oscillations are stable, one is justified in ignoring them in the analysis of stability. However, in problems involving feedback stabilization, all the natural frequencies must be taken into account because they may be excited in the presence of the feedback.

In the problem of flute stabilization, the inclusion of a further ion oscillation branch, predicted by Varma,[45] is also found to lead to an increase in the degree of the dispersion relation. The Varma branch corresponds to a frequency somewhat higher than $m\omega^*$, and is not very dependent on density (Fig. 8). The energy of the Varma oscillations is positive. Under the usual conditions, these oscillations are damped and have little effect on the experimental data. However, the situation is radically changed when feedback is introduced. Thus, firstly, when the transfer coefficient is large, $W > W_0$, the introduction of feedback ensures that the Varma branch cuts the ion surface branch with negative oscillation energy. This, in turn, produces an instability that is analogous to flute instability but occurs at higher frequency. This instability does not have a density threshold but has an upper bound in density because an increase in density is accompanied by an increase in the frequency of the ion surface branch and the wave interaction is modified. This instability is absent in the absence of the surface wave, i.e., for $W < W_0$. Secondly, the Varma branch may be excited for any degree of amplification when the phase shift in the stabilization system is incorrectly chosen. Since the energies of the Varma and the ion branches have opposite signs, they require different phase shifts when damping is to be introduced. As a result, the phase-frequency characteristic of the feedback system, which ensures the damping of all the natural oscillations, becomes more complicated. The resulting conditions which the phase-frequency characteristic must satisfy are shown in Fig. 9. When $W < W_0$, the surface ion branch is absent and the phase-frequency characteristic becomes somewhat simpler.

D. Discrete structure of the feedback system

It was assumed in Chap. 2 that the boundary condition given by (2.4) was satisfied. This is so when the feedback system maintains condition (2.4) at each point on the $r = b$ surface, i.e., the system is distributed over this surface. In practice, however, the system can only consist of discrete elements, i.e., sensors and electrodes occupying certain areas on the $r = b$ surface. Since we are interested in flute oscillations that are uniform in the direction of the magnetic field, we need only consider the azimuthal structure of the feedback system.

In the expansion for the field due to an electrode of finite size, the different azimuthal harmonics are present with different weights (the weights may differ both in magnitude and sign). As the number of the azimuthal harmonic increases, the corresponding transfer coefficient decreases and then changes sign, i.e, it changes from a stabilizing into a destabilizing factor. When oscillations corresponding to the different modes are independent, the region of stability on the (n, W) plane

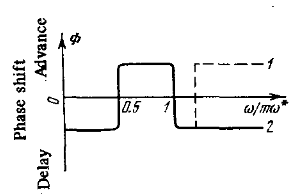

FIG. 9. Dependence of the phase shift of the transfer coefficient W on frequency, ensuring that all the oscillation branches will be damped. 1) Including all oscillation branches. 2) Without the Varma branch.

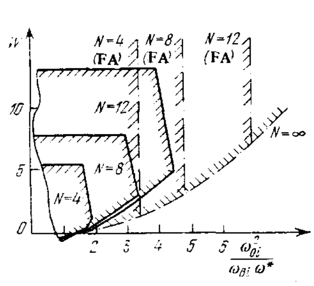

FIG. 10. Stability regions on the (density, transfer coefficient) plane corresponding to suppression of flute instabilities in plasma with a sharp boundary for different numbers N of feedback electrodes. The ratio of the plasma radius to the radius of the surface on which the electrodes are located is assumed to be $a/b = 0.8$. Solid lines—"local" stabilization system, dot-dash lines—system with spatial Fourier analysis, hatching shown on the side of stability.

is obtained by cutting off the unstable regions for all the modes. This approach was adopted by Crowley[46] but is, however, unsatisfactory. The point is that the effect of the interaction of different azimuthal harmonics through the feedback system may be a dominant factor. It was investigated numerically for a single-electrode system by Arsenin *et al.*[47] Chuyanov[48] has considered the excitation of standing waves in a single-electrode system (due to the reflection of a traveling wave from the electrode). The general solution has been given by Kostomarov *et al.*[49]

The coupling between the azimuthal modes arises for the following reason. Each sensor responds to the resultant signal due to the fields of all the azimuthal modes. Each electrode excites a superposition of azimuthal modes. The result is that pure azimuthal modes cease to be the eigenfunctions of the problem.

Kostomarov *et al.*[49] have carried out a numerical computer analysis of the equations consisting of $N = 4$, 8, and 12 electrodes. They assume that the potential of each electrode is determined only by the sensor located in the immediate neighborhood of the given electrode, i.e., the feedback system is based on the "local" principle. Figure 10, taken from the paper by Kostomarov *et al.*[49] shows the stability regions on the density-versus-transfer-coefficient plane. (Since these authors assume that the transfer coefficient is real, "stability" corresponds to real natural frequencies.) It is clear from the figure that the stability region is small for small N, and its expansion with increasing N occurs very slowly. This expansion is restricted by the excitation of oscillation branches with energies of different sign, which occurs as a result of interaction through the feedback system. These oscillations have equal frequencies but different azimuthal wave numbers. This process is accompanied by the excitation of azimuthal modes that are stable without feedback. The excitation can be suppressed by reducing the gain of the feedback system. This is, however, impeded by the excitation of lower modes whose stability at given density requires a high enough gain in the stabilization system. This means that different transfer coefficients are required for different azimuthal modes.

Such systems can be constructed by abandoning the local principle and using a nonlocal distribution. The signals from N probes can be summed with weights equal to the Fourier coefficients calculated from the azimuthal positions of the sensors, and this yields an approximate spatial Fourier analysis. This approach yields N signals proportional to the sine and cosine components of the first $N/2$ azimuthal modes with a certain admixture of higher modes. They can be independently amplified to a different degree and then, through a Fourier synthesis, they can be combined to generate the control signals for the individual electrodes. When N is large enough, the characteristics of the stabilization system with spatial Fourier analysis for the $N/2$ lowest modes approach the characteristics of the ideal local system with $N \to \infty$ (Fig. 11). For modes with $m > N/2$, feedback cannot be achieved in practice.

The utilization of spatial Fourier analysis can thus be used to weaken the degradation of the properties of the feedback system with discrete electrodes due to the mode interaction effect.

It is important to note that the mode interaction effect can be both harmful and useful. Arsenin[50] has shown how it can be used to generate a required frequency characteristic for the stabilization system. The signal from the lowest-mode unstable oscillations which it is required to stabilize is then used to produce forced oscillations corresponding to the highest stable mode. The fields of these forced oscillations are again sensed by the sensors of the stabilization system, and the resulting signal acts on the lowest unstable mode. In this scheme, the plasma itself is used as an element of the electronic circuit producing the amplitude-frequency characteristics that cannot be achieved by the usual lumped-element systems.

4. SUPPRESSION OF FLUTE INSTABILITY

A. Experiments on the stabilization of flute instability

The problem of stabilization of flute instability in adiabatic traps was solved by Ioffe and his collaborators with the aid of the "magnetic well"[19] long before the beginning of experiments on feedback stabilization, so that the development of thermonuclear studies with open traps did not in itself necessitate a search for new methods of stabilizing flute instabilities. This was so provided one could disregard the possible technologic and economic advantages of the new method under reactor conditions as and when these become attainable in systems of this type in the future.

However, because of the close connection between flute instability and other hydromagnetic instabilities characteristic for different thermonuclear confinement

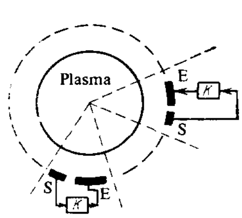

FIG. 11. Local stabilization system. Each control loop serves its own sector: S-sensor, E-electrode.

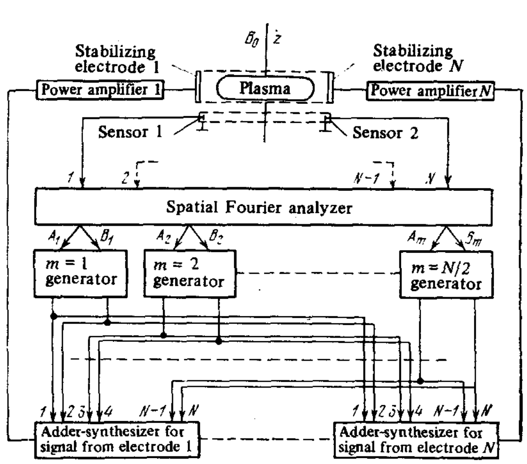

FIG. 12. Simplified block diagram for a 12-electrode electrostatic stabilization system for flute oscillations in Ogra-III. Spatial Fourier analysis is employed.

systems, and the exceptional convenience and purity of experiments with open traps, it is precisely such traps that have turned out to be the testing ground for the new method of stabilization and its detailed investigation and comparison with theory. The data obtained as a result of these experiments have led to greater confidence in the application of the new method to more complex situations in which alternative methods of stabilization are not available or, if they are, they involve an unacceptable deterioration in the parameters of the installation.

The suppression of flute instability by feedback has been investigated in detail in open mangetic traps with neutral injection such as Ogra-II,[8,9] Phoenix-II,[20–23,25,26] and Ogra-III.[51–53] In all these experiments, feedback was introduced in the simplest technical way, namely, through external electrodes surrounding the plasma, and measurements were carried out with electrostatic probes.

The first experiments[8,9,20–23,25,26,51] were carried out with systems based on the local principle (see Fig. 11) with between 1 and 6 electrodes. Chuyanov *et al.*[51] have shown that the density corresponding to stable confinement increases with increasing number of electrodes. Analysis of this result[49] led in subsequent experiments[52,53] to the development of a nonlocal feedback system with spatial Fourier analysis. Since this system is probably the best among those used, and the principles upon which it is based are of general interest, we shall take this opportunity to describe it in detail and to outline the results obtained with this system.

These experiments were carried out on a simple mirror trap by injecting 20-keV hydrogen atoms into it.

Without stabilization, the accumulation of plasma in the trap is limited to a density of up to 10^8 cm^{-3} by the excitation of the first ($m = 1$) azimuthal mode of flute instability. Stabilization was achieved with an electrostatic system consisting of 12 electrodes (Fig. 12) distributed uniformly on a cylindrical surface around the plasma. Information on the plasma electric fields was obtained with the aid of 12 electrostatic sensors, also distributed uniformly around the plasma. These sensors were metal electrodes grounded through the input resistors of the amplifiers and screened on all sides except that facing the plasma. The amplifier output is proportional to the time derivative of the radial component of the electric field on the sensor surface.

After preliminary amplification, the output of all the sensors is fed into the spatial Fourier analyzer which, as described above, generates signals proportional to the sine and cosine components of the first six azimuthal modes. These signals are then amplified and transformed independently for each mode, so that independent stabilization of the lowest azimuthal modes can be achieved and the optimum transfer coefficients can be chosen. The stabilizing signals generated for the different modes are then fed into the Fourier synthesizer where they are summed with weights calculated with allowance for the azimuthal position of each electrode, and the control signal is formed for each electrode.

An important element of the stabilization system (not shown in Fig. 12) is the additional feedback between the output and input of the system, which neutralizes the parasitic capacitive coupling between the electrodes and the sensors and prevents oscillations of the stabilizing system under high gain in the absence of plasma. This procedure is used to realize the boundary condition (2.4).

This stabilization system has been successfully used to suppress, successively, three azimuthal modes of the flute instability. Figure 13[53] shows the mean plasma density in the trap as a function of the "expected density," i.e., the density that should be produced for the given injection current and the given vacuum conditions (which determine the lifetime of fast protons prior to charge transfer on the residual gas under complete stability).

Without stabilization, the density is restricted to 10^8 cm^{-3} by the $m = 1$ flute mode. When the $m = 1$ stabilization channel is introduced, the density rises to 4×10^8 cm^{-3} and this is accompanied by the development of the second, $m = 2$, azimuthal mode. Stabilization of this

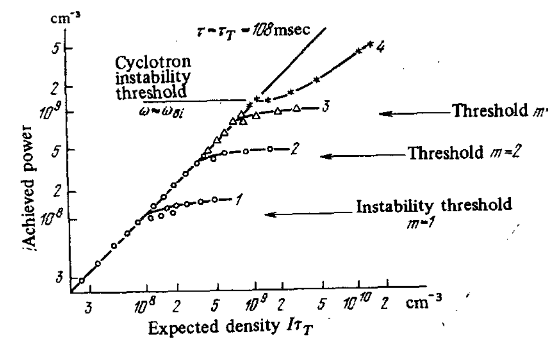

FIG. 13. Mean plasma concentration in Ogra-III as a function of the maximum possible concentration in the absence of instability.[53] The maximum possible containment time τ is equal to the time for charge transfer between fast protons and the residual gas ($\tau_T = 108$ msec). 1) Without stabilization; 2) $m = 1$ stabilization channel turned on; 3) $m = 1$ and $m = 2$ channels turned on; 4) $m = 1$, $m = 2$, $m = 3$ channels turned on.

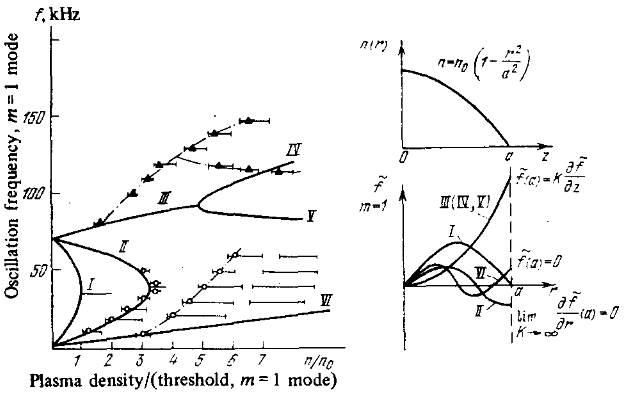

FIG. 14. Observed frequencies of flute oscillations corresponding to the $m = 1$ mode as functions of concentration in Ogra-III in the presence of feedback. [52] Bars indicate the range of concentration for which oscillations at a given frequency were observed (with amplitude greater than 10% of maximum). Open circles and triangles—concentrations for which the amplitude at the given frequency is at a maximum. Solid lines—theoretical predictions using a parabolic concentration profile. The parabolic concentration profile and the theoretical radial distribution of the oscillation amplitude are shown on the right for different branches. Curves with the same number on the left and right of the figure correspond to the same branch. The oscillations were excited through a special choice of phase characteristics of the feedback system.

mode takes the plasma up to 9×10^{8} cm^{-3}, which corresponds to the threshold for the third mode. When the $m = 3$ mode is stabilized, then, as shown by Zhil'tsov *et al.*,[53] losses at high plasma densities are connected not with flute instability but with oscillations at the ion cyclotron frequency, owing to the modified negative mass instability.[54] Losses connected with flute instability were not detected throughout the density range up to 5×10^{9} cm^{-3}.

The fact that stable confinement could be achieved for densities higher by a factor of fifty, when the surface stabilization system was introduced, indicates that the higher radial modes which are unaffected by the surface system do not, for some reason, give rise to plasma losses. It has been shown[53] that these higher radial modes are, in fact, excited but they do not give rise to losses, whilst the modification of the radial density profile is such that it approaches the "rectangular" shape, the result of which is self-stabilization of the higher

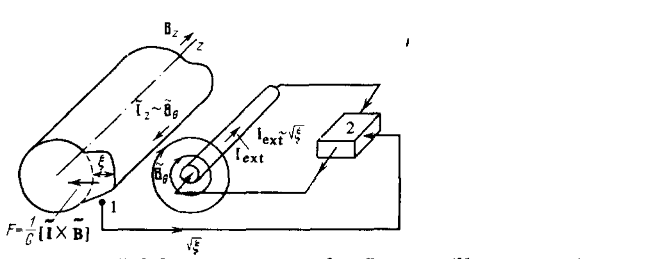

FIG. 15. Stabilization system for flute oscillations, using control by an external magnetic field at right-angles to the main trap field. [1] Only one of the external conductors is shown. The current in this conductor is varied by the control system in proportion to the square root of the displacement ξ. 1—Displacement sensor; 2—control system.

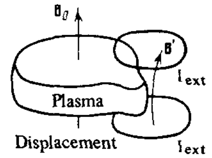

FIG. 16. Magnetic feedback system for the control of flute instability in a short trap.

radial modes. Zhil'tsov *et al.*[52] have investigated in detail the eigenfrequencies corresponding to the different oscillation branches of the $m = 1$ mode. Their results are shown in Fig. 14 which gives the theoretical and experimental eigenfrequencies as functions of plasma density, and the corresponding radial solutions. The oscillation branches were excited by introducing the corresponding phase shifts in the stabilizing system. The validity of the theory was thus verified not only for the global increase in the density under stable confinement but, what is more important, for the change in the dispersive properties of plasma in the presence of feedback.

To ensure simultaneous damping of all the oscillation branches, Zhil'tsov *et al.*[53] used a complicated phase-frequency characteristic (Fig. 9). They succeeded in using this system to reduce the oscillations corresponding to the $m = 1$ mode in plasma from 20 to 0.2 V/cm, which corresponded to the sensitivity limit of the measuring system associated with thermal noise in the stabilization system.

The optimum phase-frequency characteristic was used to investigate the influence of the dynamic range of the stabilizing system on the plasma characteristics, i.e., the influence of the maximum amplitude of the electrode potential for which the entire system was still linear. As expected, correct operation of the feedback system could be achieved with a dynamic range only slightly exceeding the thermal noise level at the output of the amplifying system.

Flute instability in mirror traps can arise not only in plasma containing hot ions, but also in plasma with hot electrons, produced by high-frequency heating. Attempts to stabilize such plasmas with an electron temperature of 30 keV and density around 10^{9} cm^{-3} with the aid of a single-electrode electrostatic surface have been reported by Haste.[55]

B. Magnetic stabilization of surface modes

Plasma can be affected not only by external electrostatic fields but also by magnetic fields. Flute instability can then be stabilized in two ways, differing by the direction of the additional stabilizing field $\mathbf{B}'$.

When the field $\mathbf{B}'$ is perpendicular to the main trap field, the additional magnetic pressure on the plasma at the points of application of $\mathbf{B}'$ is proportional to $\mathbf{B}'^2$. This pressure can be produced by inserting conductors parallel to the trap axis and analogous to the Ioffe rods used to produce the minimum-B configuration. A current is produced in these conductors whenever the measuring system detects the displacement of the plasma toward the conductors. This stabilization system was discussed in the pioneering work of Morozov and Solov'ev[1] (Fig. 15).

We emphasize that this system[1] is essentially nonlinear because the stabilizing currents are $-\sqrt{\xi}$, where ξ is the displacement of the plasma.

In a short trap in which the length l in the direction of $\mathbf{B}$ is smaller than or of the order of the radius, stabilization can be achieved with a much smaller alternating field $\mathbf{B}' \parallel \mathbf{B}$ by using the linear term in the magnetic pressure $\mathbf{B}\cdot\mathbf{B}'/4\pi$.[56–58] This type of field can be produced by azimuthal currents external to the plasma (Fig. 16). From the macroscopic point of view, stabilization is then due to the Lorentz force $F_r = (1/c)I_\theta B'$ acting on the Larmor current $I_\theta = (c/B)dp/dr$, where p is the undisturbed pressure. In "microscopic language" describing the motion of the individual particles, stabilization occurs due to radial drift in the nonuniform (in θ) field $\mathbf{B}'$. The divergence of the drift current gives the charge density

$$-\frac{c}{i\omega}\frac{1}{B}\frac{\partial p}{\partial r}\frac{\partial B'}{\partial r}.$$

Comparison of this quantity with the charge density due to the destabilizing effect of the unfavorable curvature of the undisturbed field $\mathbf{B}$ leads to the following stabilization condition: $B' > B\xi/L$, where L is the characteristic linear dimension of the change in $\mathbf{B}$.[3)]

It is readily seen that, when a field $\mathbf{B}'$ of this order is introduced, the oscillations remain almost potential and $|\operatorname{curl}\mathbf{E}| = |\omega B'/c| \ll \mathbf{E}/a$. The essential influence of weakly nonpotential oscillations on stability is explained by the compensation of (large) electron and ion currents due to electric drift. In a uniform field, this compensation is complete. A slight separation of charges occurs only in a nonuniform field $\mathbf{B}(r)$ and is proportional to $\partial B/\partial r$. It is precisely on this quantity that the field $\mathbf{B}'$, required for stabilization, is found to depend. The magnetic system will also stabilize the balloon (electromagnetic) flute instability of dense plasma.[56]

We must now note an important difference between magnetic and electrostatic stabilization systems. For a given displacement ξ_r, the charge density on the plasma surface $r=a$ is proportional to concentration. It follows that for $\omega_{0i} \gg \omega_{Bi}$ the value of $\varphi(b)$ necessary for stabilization increases as $\varphi(a)\omega_{0i}^2/\omega_{Bi}^2$. In continuum language, high potentials $\varphi(b)$ are required because the field due to the external charges is reduced in dense plasma by a factor of $\varepsilon = \omega_{0i}^2/\omega_{Bi}^2$, where ε is the permittivity. The magnetic field, on the other hand, penetrates (for $\beta \ll 1$) freely into the plasma, so that the stabilization currents are independent of density. This means that magnetic fields can also be used to influence other types of electrostatic oscillation in dense plasma.[59]

There are no published experiments on the suppression of flute instabilities by magnetic systems.

[3)]The corresponding result given by Arsenin[56] (stabilizing field $\sim\beta B\xi/a$) is incorrect.

C. Suppression of flute instability by spatial feedback

We have already noted that the basic defect of surface stabilization is that it cannot be used to suppress the higher radial modes. The fact that these oscillations were found experimentally[52,53] to self-stabilize was a piece of good fortune in this particular experiment and is probably not a general feature. The development of spatial methods of stabilization is, therefore, of undoubted interest.[4)]

To stabilize spatial modes involving radial oscillations, one must introduce compensating charges into the interior of the plasma, i.e., there must be controllable sources (sinks) of charges within the plasma.[5)] The method of controlled sources was first described in connection with drift oscillations[62,63] and was, at the same time, tested in three experiments.[62,64,65] For flute oscillations elongated in the direction of the lines of forces, volume sources may be replaced. For example, by the injection of particles through the ends of the trap.

Suppose the plasma contains sources of electrons of intensity S proportional to the perturbation of the potential:

$$S = W\varphi. \tag{4.1}$$

The continuity equation for electrons then readily yields the additional perturbation of electron concentration due to the sources, i.e., $n_{es} = -W\varphi/i\omega$, so that the additional perturbation of charge density becomes $\rho_s = -eW\varphi/i\omega$. This must be added to the right-hand side of the Poisson equation given by (2.1). Since we are interested in spatial modes oscillating in the radial direction, we can confine our attention to the quasiclassical approximation $\varphi(r) \sim \exp(ik_r r)$, in which the Poisson equation assumes the form

$$k^2\varphi = 4\pi(\rho + \rho_s), \tag{4.2}$$

where $k^2 = k_r^2 + (m^2/r^2)$ and ρ is the charge density in the absence of the source.

It is clear from (4.2) that the appearance of the source is equivalent to a change in the permittivity of the plasma by $-4\pi eW/i\omega k^2$. In general, the introduction of spatial feedback (injection of charges, transmission of currents, and so on) is equivalent to a controlled change in the permittivity tensor.

Spatial feedback as a method of suppressing flute instability was put forward by Chuyanov.[66] Since the phase of the flute wave is constant along the magnetic field, suppression can be achieved in this case by the injection of a low-intensity electron beam through the end of the trap across the entire cross section of the

[4)]In a short trap, the higher radial modes of flute instability can be suppressed (because of the two-dimensional nature of the oscillations) by a surface system consisting of electrodes placed beyond the ends of the plasma cylinder.[60]

[5)]This is analogous, at least theoretically, to the way feedback is introduced between perturbations of any physical origin.[61]

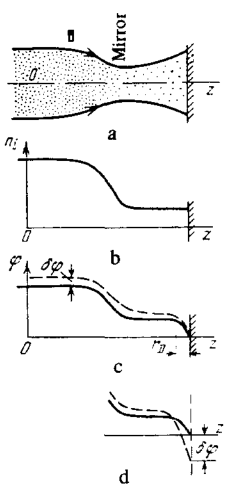

FIG. 17. Control of electron loss through the end mirrors as a method of stabilizing flute instability in thermonuclear plasma in an open trap. a) Plasma in trap; b) ion concentration distribution along a magnetic line of force; 3) distribution of potential: solid line—undisturbed potential, dashed line—potential in the flute perturbation. The increase in potential indicates that drift across the magnetic field produces an excess of ions or a deficiency of electrons on the given tube of force; d) surplus charge can be liquidated by reducing the "longitudinal" electron loss through a reduction in the potential of the sectionalized wall facing the given tube of force. r_D is the Debye radius.

plasma. By modulating the beam intensity at different points on the cross section in accordance with the fluctuations of potential in each tube of force, or fluctuations in the azimuthal electric field on a given tube of force (these quantities can be measured with the aid of the same electron beam), it is possible to control the charge fluctuations in the interior of the plasma. Although this method does provide the necessary spatial feedback structure, and the required currents are not large, it has not been tested experimentally because of technical difficulties in producing a beam of large cross section and possible effects associated with beam instability.

In cold plasma, a controllable sink can be realized by controlling the potential of a Langmuir probe introduced into the plasma. The same probe can, of course, also be used to detect the fluctuations in potential. Although this contact method does not have a "thermonuclear future," it is exceedingly simple and convenient in model experiments. It has already yielded interesting results, mainly in relation to drift waves (see Chap. 7).

Experiments on the suppression of flute instability by the controlled sink method have been carried out at Wisconsin University[16,67,68] using cold plasma with an electron temperature of 7–10 eV, ion temperature of 0.1 eV, and density of 10^{10}–10^{11} cm^{-3}. Langmuir probes were successfully used to suppress two azimuthal modes of flute instability and to investigate the diffusion of plasma under the influence of flute oscillations. The latter problem will be examined in detail in Chap. 9.

In contrast to experiments with hot collisionless plasma, which were discussed above, particle collisions and associated dissipative effects play an important role in the Wisconsin experiments. This means that the requirements imposed on the frequency properties of the stabilization system must be modified. The modified conditions have been analyzed theoretically and experimentally by Richards *et al.*[16] Their conclusions are in good agreement with the experimental data obtained for the open traps with neutral injection described above.

A different version of the controlled-sources method, namely, control of the "intrinsic" electron current due to collisional losses through the mirrors (Fig. 17), was proposed by Arsenin[69] for the suppression of flutes in an open trap with thermonuclear plasma parameters. The electrons are confined to the trap because the plasma assumes a positive potential. Its magnitude is set so that collisional losses of electrons through the potential barrier are equal to ion losses. The electron currents through the mirrors can be varied by varying the potential difference $\delta\Phi$ between the plasma and the sectionalized wall beyond the mirror. A transfer ratio $\delta\Phi/\delta\varphi \sim 10$, where $\delta\varphi$ is the oscillation in the potential in the flute, is required for the stabilization of plasma with $n \sim 10^{14}$ cm^{-3}, $T_i \sim 100$ keV, and length ~ 100 cm, in a field of $B \sim 3\times 10^4$ G.

5. STABILIZATION OF HELICAL MODES OF THE HYDROMAGNETIC INSTABILITY OF A CURRENT-CARRYING PLASMA COLUMN

A. Theoretical suggestions

Theoretical research on controlled thermonuclear fusion is, unfortunately, mainly concerned with tokamak systems.[70] The tokamak is a toroidal trap in which the plasma is kept away from the walls by an azimuthal field B_θ due to the longitudinal current I_φ (Fig. 18). MHD stability of the column is achieved by applying a longitudinal magnetic field B_φ. To stabilize perturbations localized in the interior of the column, it is evidently sufficient to ensure that

$$q \equiv \frac{rB_\varphi(r)}{RB_\theta(r)} > 1, \tag{5.1}$$

at all points in the plasma, where R is the major radius of the current ring, and r is the distance from the circular axis. Since R is much greater than the radius a of the plasma column, it is clear from (5.1) that the stabilizing field B_φ must be much greater than the field B_θ due to the current. A still more stringent condition on B_φ arises from the stability condition for large-scale helical perturbations $\psi(r)\exp(im\theta - in\varphi)$ of the surface-wave type (ψ is a maximum near the plasma boundary), where m, n are integers. This condition is

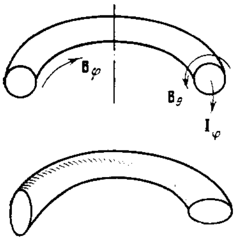

FIG. 18. Plasma ring in a tokamak. $m = 2$, $n = 1$ perturbation.

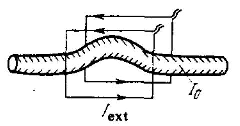

FIG. 19. Kink stabilized by an external field.

$$q > m. \tag{5.2}$$

Unless it is satisfied, an instability with a growth rate of the order of the Alfven frequency $\Delta\Omega_A = B_\theta/(4\pi\rho)^{1/2}a$ will develop (ρ is the mass density of the plasma).

The critical number m depends on the distribution of current over the cross section.[71] Waves with $m \leqslant 3$ present a real danger for containment because, according to (5.2), we must have $q > 3$. The current breaks observed in tokamaks are commonly ascribed to the excitation of MHD helical modes.

The longitudinal field B_φ cannot be made as high as desired. Reasonable values in modern installations amount to a few tens of kilogauss. Condition (5.2) (the Kruskal-Shafranov criterion) imposes a restriction on the current and, since the field B_θ is also a plasma confining field, the criterion restricts the temperature and lifetime of the plasma. These important parameters are, therefore, determined in the final analysis by technical restrictions on the longitudinal field which does not in itself have an important influence on containment. The suppression of helical modes by a method other than the application of a strong longitudinal field would enable us to increase the current and, probably, substantially improve the characteristics of tokamaks. Since a small number of (long-wave) modes must be suppressed, it is possible to use the feedback method involving the control of the magnetic field outside the plasmas. The principle of this stabilization method is simple. Consider, for example, the plane kink $m = 1$ shown in Fig. 19. To bring the column back to the equilibrium position, we must, clearly, apply a sufficiently strong magnetic field perpendicular to the plane of the Figure.[6)]

At first sight, it would appear that, by introducing a sufficient number of displacement sensors along the torus, together with a series of correcting coils, it would be possible to control all the long-wave oscillations. In reality, the situation is much more complicated. The force acting on a given element of the column due to external currents depends not only on the displacement of the given element but also on the orientation of the entire helical perturbation relative to the line of force of the undisturbed magnetic field. In point of fact, if the perturbation has the angular dependence $\exp(im\theta - in\varphi)$, the stabilizing currents and the magnetic field $\mathbf{B}'$ due to them must have the same angular dependence. In vacuum, $\mathbf{B}' = \nabla\psi$, so that the magnetic pressure which should bring the boundary back to its equilibrium position in the approximation linear in $\mathbf{B}'$ is

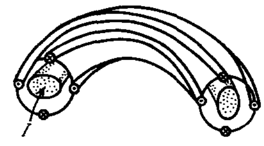

FIG. 20. Stabilizing coil for the $m = 2$, $n = 1$ mode. Only a few turns are shown.

$$\frac{BB'}{4\pi} = \frac{iB_\theta}{4\pi a}(m - nq)\,\psi. \tag{5.3}$$

We thus see that, depending on the sign of $m - nq$, external currents proportional to the displacement can have have both a stabilizing effect (which returns the column back to equilibrium) and a destabilizing effect. To obtain the stability condition in the presence of the "stabilizing" currents, we must therefore carry out a detailed analysis, taking into account the distribution of currents in plasma (this determines the values of $m - nq$ for which stabilization is required).

A practical stabilization system for the mode (m, n) should take the form of a pair of helical coils wound on a certain surface $r = b > a$ with densities $\cos(m\theta - n\varphi)$ and $\sin(m\theta - n\varphi)$ (Fig. 20). This must be complemented by the corresponding two sets of sensors measuring the "cosine" and the "sine" components of the perturbation, respectively. The sensors can measure directly the displacement of the plasma from the position of equilibrium, or can record quantities proportional to it, i.e., perturbations of temperature, magnetic field, and so on. The difficulty is to organize the measurements so that the current j in the coils, which is proportional to the measured signal, has the required dependence on $m - nq$. Feedback stabilization of helical modes was discussed by Morozov and Solov'ev[1] in connection with a column carrying a skin current. They assumed that the proportionality factor between the displacement and the additional magnetic pressure on the surface of the plasma was independent of the spatial structure of the perturbation. This means that the stabilizing current should be inversely proportional to $m - nq$, where q is taken on the plasma boundary. The result given by Wang[73] can also be reduced to this dependence. However, this dependence on q (q varies during the discharge) is difficult to realize in practice. The simplest to realize is probably $j \sim m - nq$, which is equivalent to the approach of a metal wall to the plasma.[74–76] The stabilizing effect of the wall is associated with the appearance of the antiparallel image currents whose magnetic field pushes the column back toward its position of equilibrium. Figure 21 shows the radial profile of the radial component of the perturbation in the magnetic field B_r' in the presence of a passive enclosure and in the presence of a feedback system maintaining a current proportional to B_r' at the boundary of the plasma on the

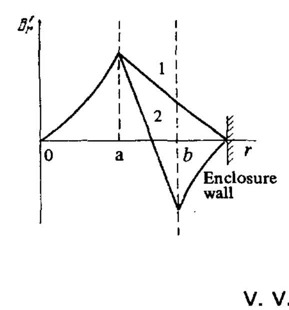

FIG. 21. Radial distribution of the perturbation magnetic field. 1—in the presence of the wall; 2—with feedback (stabilizing the current on the $r = b$ surface), a—plasma radius.

6)The development and stabilization of a helical mode are described in the language of charge kinetics by Lowder and Thomassen.[72]

$r = b$ surface. As can be seen, an increase in the ratio of the stabilizing current to $B'_r|_{r_{sd}}$ is equivalent to bringing the enclosure wall closer to the plasma. This is readily understood, again on the basis of the fact that a correctly constructed control system tends to "nullify" the signal at the input, i.e., on the sensor. When B'_r is zero on the surface containing the sensors, this is equivalent to a fictitious enclosure being present on the surface. It is clear from Fig. 21 that the presence of the enclosure wall at the boundary of the plasma prevents the appearance of surface waves. All that remains are the spatial modes for which the maximum growth rate is smaller by a factor of r/a than for the surface waves. These modes are not very sensitive to the stabilizing current.[77] 7)

An effective enclosure close to the column can be produced without special B'_r sensors by using the coils carrying the stabilizing currents.[79–81] The principle is as follows. Ohm's law for the current j induced in a coil of resistance η, connected in series with some external impedance $Z_e(\omega)$, can be written in the form

$$\frac{1}{c}\frac{d\Phi}{dt} = (\eta + Z_e)\, j; \tag{5.4}$$

here Φ is the magnetic flux linked with the coil circuit. Let us split Φ into the "primary" flux Φ_1 due to currents in the displaced column and the secondary flux due to the self-inductance L of the coil.

We have $d\Phi/dt = (d\Phi_1/dt) - (l/c)(dj/dt)$, and hence

$$j = \frac{-i\omega\Phi_1/c}{\eta - (i\omega L/c^2) + Z_e(\omega)}. \tag{5.5}$$

We can now choose the external impedance $Z_e = \eta_e - i\omega l/c^2$ so that its resistance η_e is negative and compensates for the coil resistance η. We then have

$$j = \frac{c\Phi_1}{L + L_e}. \tag{5.6}$$

The disturbed field $\mathbf{B}'_1$ on the $r = b$ surface, which produces the flux Φ_1, is proportional to the perturbation of the field on the plasma boundary. The current thus turns out to be proportional to the field perturbation on the boundary of the plasma column, as required. If we take $L_e < 0$ but $L + L_e > 0$ (since, otherwise, the system becomes self-excited), it is possible to obtain sufficiently a sufficiently large stabilizing current.

The above simple magnetic system is ineffective for "almost flute" oscillations with $m - nq \approx 0$. This is so because the magnetic sensors cannot then be sensitive to the flutes. This difficulty can be obviated by measuring not the magnetic field but directly the displacement $\xi_r = aB_r(a)/i(m - nq)B_\theta(a)$[75,76] (for example, by recording the modulation of the bremsstrahlung from the hot part of the column). Another method of ensuring that the flutes are observable is to superimpose an hf magnetic field with small but finite amplitudes.[82] 8) In the resulting composite system, the required hf field is less than that necessary for "pure" (without feedback) dynamic stabilization. Stabilization is also possible in a nonlinear system with finite current in the coils: $j = j\ \mathrm{sign}\, B'_r$.[75,84] Magnetic-field perturbations will also be produced in the flute by toroidal effects (admixture of azimuthal harmonics with $m = \pm 1$).[80] We note that the observability or otherwise of an unstable perturbation by external magnetic sensors is very dependent on the current distribution over the cross section of the column. In particular, in the case of the bell-shaped current distribution, which is of particular interest in practice, the observability problem does not appear to arise because the results obtained for a step distribution[71] indicate that all modes with $m \geqslant 2$ should be stabilized by bringing the envelope closer to the column.

When the bell-shaped current distribution is sufficiently sharp, helical modes with $m \geqslant 2$ cannot, in general, be excited in hot (perfectly conducting) plasma.[71] However, the peripheral part of the column, which does not experience Joule heating, is then found to be relatively cold and has a low conductivity. If a surface on which $q = m/n$ is introduced into the zone, the slower (with growth rate much less than Ω_A) tearing mode may develop.[85] This is a helical instability of dissipative origin. It is precisely this instability that is most likely to be responsible for the slow growth of the $m = 2$ perturbation during the initial stage of the current break (the so-called prebreak) in the T-4 installation with a relatively sharp current distribution.[86] The current break as a whole can be avoided by using feedback to suppress the tearing mode. Figure 22 shows the radial distribution of displacements and B'_r in a tearing mode. If $d^2B_r/Bdr^2 > 0$ in the neighborhood of the point r_s at which $q = m/n$, the diffusion of the field from neighboring regions will lead to a growth of the perturbation. Stabilization ($d^2B_r/Bdr^2 < 0$) is achieved if the feedback system is equivalent to an enclosure inserted into the plasma.9) This situation is realized, for example, when the stabilizing current is proportional to the measured shift of the hot part of the column. From the technical standpoint, the stabilization system for the tearing mode is simpler than that for fast instabilities of a perfectly conducting plasma. This is so because the growth rate is low, so that the response may be less rapid.

B. Experiments on the stabilization of helical modes

Stabilization of helical modes in tokamaks is, at present, the most interesting field for the application of feedback control to thermonuclear studies. This is so for many reasons, including the fact that (1) the lowest

7)We note that toroidal effects must be explicitly taken into account in the description of oscillations with a small growth rate. This is why the attempt made by Hugill[78] to find the frequency dependence of the transfer coefficient that would ensure the damping of the oscillations, carried out without taking the toroidal effects into account, was not valid.

8)Methods similar to this are used for confining fluid metals in high-frequency magnetic fields (see Gubarev and Paslavskiĭ[83] and the references cited by them).

9)Lowder and Thomassen[87] have considered the effect of an enclosure on the tearing mode.

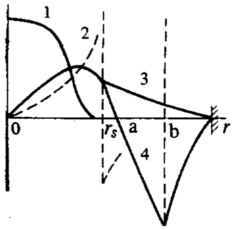

FIG. 22. Tearing-mode stabilization: a—plasma radius; r—radius of surface on which $q(r) = m/n$; 1—current profile; 2—radial displacement; 3—radial component of the perturbation magnetic field in the presence of a simple wall; 4—ditto in the presence of stabilizing current on the $r=b$ surface.

modes can readily be isolated, (2) one is dealing with a large-scale situation, and (3) there is a possibility of an improvement in the confinement and heating conditions. However, none of the experiments performed so far has been entirely successful.

An attempt to achieve the stabilization of the $m = 2$, $n = 1$ helical mode was undertaken on the installation at Princeton.[88] A magnetic feedback system was used and the perturbations of the toroidal field B'_r were measured from outside the plasma with four magnetic sensors located at 90° to each other in the median plane of the torus on its outer circle. Opposite sensors were connected in opposition, so that the signal from one was subtracted from that of the other, in order to compensate the signal due to the current field ($m = 0$) and possible oscillations with $m = 4$ and $n = 2$. Signals from the two pairs of probes (shifted in phase by 90° relative to one another since the mode number on the outer circle of the torus was $n = 1$) were summed with weighting factors which could be varied to produce the desired phase shift independently of oscillation frequency.

The resultant signal was amplified by a 0.5 MW amplifier which produced the currents in the controlling coils through an inductive coupling. Generally speaking, the required configuration of the controlling magnetic field can be produced by two helical coils with currents flowing in opposite directions, each of which closes on itself after two complete circuits around the torus. In the experiments of Bol *et al.*,[88] this coil was replaced by eight flat half-rings (Fig. 23) distributed along the generators of the torus in steps of 90° in such a way that four current frames of the form illustrated in Fig. 23b were produced. This configuration necessarily produces higher field harmonics, but this was reported[88] as having no influence on the operation of the stabilization system because the effect on the plasma did not change when some of the frames were disconnected.

The amplitude-frequency characteristic of the entire system was flat (to within 3 dB) up to 32 kHz (the characteristic frequency range of the unstable oscillations was 5–20 kHz). The phase-frequency characteristic was not at all of the desired form: the phase shift varied from 90° at 22 kHz to 180° at 55 kHz, partly due to the influence of the conducting chamber.

The total gain of the stabilization system was restricted by self-excitation due to the coupling between sensors and stabilizing coils, so that the transfer coefficient (ratio of the number of turns to the perturbation field) did not exceed 0.5 at the radius of the diaphragm and 0.25 on the resonance surface in the plasma, where $q = 2$.

Despite the unsatisfactory technical characteristics of the stabilization system, it was, nevertheless, successfully used to observe the sinusoidal dependence of the frequency and amplitude of the oscillations on the phase shift in the stabilization system. The relative change in frequency $\Delta f/f$ was found to be 40% and the change in the amplitude was $\Delta A/A = \pm 20\%$. This shows that better results can be expected at higher gain. On the other hand, the result can also be looked upon as an indication of the dissipative nature of the growth of stabilizing oscillations (tearing mode[85]).

It has been noted,[88] in our view correctly, that magnetic measurements are far from being the best method of solving the problem because, during the initial stage (this stage is the most interesting from the feedback point of view), the oscillations are localized within the plasma near the resonance surface with $q = 2$ and produce no external magnetic-field perturbations. Moreover, it is difficult to avoid or compensate the inductive coupling between sensors and stabilizing coils. A much more promising method is to observe the displacements within the plasma with the aid of collimated x-ray sensors that react to the displacement of colder or hotter layers of plasma into their fields of view. This method has already been used successfully for the diagnostics of internal modes of MHD oscillations[86,89] and can undoubtedly be successfully used in feedback systems as well.

Summarizing the experience gained in the course of the Princeton experiments, we note that, although stability was not achieved, the experiments have, nevertheless, confirmed the basic possibility of feedback stabilization of helical modes in tokomaks.

Experiments on the stabilization of the $m = 2$ helical modes have been planned for the TO-1 installation at the I. V. Kurchatov Institute of Atomic Energy. However, so far, only experiments with the excitation of the controlling helical coil by a generator producing up to 1 kA have been carried out.[90] No information is available on experiments with a closed feedback loop.

C. Stabilization of θ pinches

The θ pinch is a convenient method of producing hot dense plasma. θ pinches are systems in which the plas-

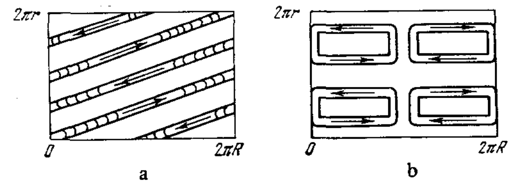

FIG. 23. Position of helical conductors on the unfolded surface of the torus corresponding to the $m = 2$, $n = 1$ mode (a) and the disposition of conductors in the Princeton experiment[88](b).

ma column is produced and compressed by a rapidly growing external magnetic field B parallel to the column axis. The vortical electric field which accompanies the alternating magnetic field produces an azimuthal current in the plasma (hence the name straight θ pinch in contrast to the Z pinch with an axial current), which interacts with the field and produces a compressive radial force.

The straight θ pinch is absolutely stable. Plasma containment is restricted by losses through the ends, and these losses are so large that thermonuclear utilization of the θ pinch is impossible for reasonable linear dimensions of the system. Attempts to modify this configuration with a view to reducing these losses always lead to instabilities. For example, the introduction of magnetic mirrors produces the flute instability corresponding to the $m = 1$ mode (the upper modes are probably suppressed by effects associated with the finite Larmor radius). Since only one large-scale mode is unstable under these conditions, feedback stabilization is an attractive possibility.

The Culham Laboratory has therefore carried out a detailed study of the motion of the θ-pinch plasma under the influence of external alternating fields,[91,92] but experiments with a closed feedback system have not as yet been carried out.

A more radical method, as compared with mirrors, for preventing end losses is the closure of the θ pinch into a torus. This configuration is not in itself an equilibrium one because the plasma ring is stretched in the direction of the major radius. One way of achieving equilibrium is to pass a sufficiently large longitudinal current (screw-pinch configuration) whose interaction with the image current in the enclosure, or an external magnetic field perpendicular to the plane of the torus, produces a force preventing the torus from expanding. Another way is to impose an additional magnetic field (high-β stellarator), and this has been adopted, for example, in the Scyllac installation. Equilibrium can be obtained by adding to the main toroidal field a combination of a corrugated field (proportional to $\sin k\varphi$) and a helical field [proportional to $\sin(\theta - k\varphi)$]. The angle θ is measured over the minor circle of the torus and the angle φ over the major circle.

However, in both cases, the $m = 1$ helical instability sets in [the displacement is proportional to $\sin(\theta - n\varphi)$]. This mode is well defined, so that it may well be possible to stabilize it by feedback. The stabilization principle which is important for the practical realization of such systems was, of course, previously checked on simpler direct systems.

Experiments on the stabilization of the $m = 1$ helical mode[93] in the linear screw-pinch have been carried out on a long ($L = 142$ cm), thin ($r = 0.8$ cm), and relatively cold ($T = 11$ eV) plasma column with a high density (in excess of 10^{16} cm^{-3}) and pressure ($\beta = 0.2$), in a strong longitudinal field (16 kG) with longitudinal currents exceeding the Kruskal-Shafranov limit by a factor of 1.8 for $m = 1$. With these parameters, the instability developed with a growth rate $\gamma = 0.55 \times 10^6$ sec^{-1}.

The field perturbations were measured with magnetic sensors. The effect on the plasma was produced by two mutually perpendicular current frames producing a field perpendicular to the axis of the plasma column. A feedback system of up to 2.8 MW per coordinate (i.e., each frame) ensured frame currents of up to 2000 ampere turns (for a plasma current of 3800 A). Stabilization was observed for transfer coefficients greater than unity but less than the 1.4 at which the system was self-excited. Special measures were undertaken to compensate inductive coupling between the frames and the sensors. The frame currents were proportional to the displacement and the velocity of the plasma column. The time delay in the feedback channel was $\tau = 0.6$ μsec. By varying the initial pressure of the gas in the discharge chamber, it was possible to vary the instability growth rate. It was found that stabilization was possible only for sufficiently small growth rates, for which $\gamma\tau < 0.42$.

It is important to consider separately the role of the parameter $\gamma\tau$ in experiments with rapidly growing aperiodic instabilities because it appears that it is precisely this parameter that imposes particularly difficult conditions on the technical characteristics of systems for the suppression of such instabilities.

The parameter $\gamma\tau$ appears for two reasons. Firstly, it is readily shown considering, for example, the model of an inverted pendulum feedback-stabilized in position and in velocity[10)] with a delay τ in the feedback circuit, that even for optimum feedback parameters no stabilization is possible at $\gamma\tau \gtrsim 1$ as τ increases, where γ is the instability growth rate in the absence of feedback. Another restriction on this parameter is due to the finite dynamic range of the stabilization system. For example, in the screw-pinch experiments mentioned above, the stabilization system could return the column to its original position only for displacements ξ_{max} less than 3 mm. Large initial perturbations ξ_0 are possible during the fast transient processes occurring while the plasma is being prepared. During the period corresponding to the delay in the stabilization system, these perturbations increase to $\xi_0 e^{\gamma\tau}$. It is clear that stabilization is not possible at $\gamma\tau > \ln(\xi_{max}/\xi_0)$. Numerical calculations[94] have shown that, for practical systems, the restriction on this parameter can be even more stringent: $\gamma\tau < 0.5$. The problem of ensuring the required rapid response turned out to be absolutely basic in the work on the stabilization of the Scylla toroidal θ pinch at Los Alamos. Although this work has been in progress for many years, the desired end has not been achieved. Nevertheless, the stabilization of the toroidal θ pinch in Scyllac deserves detailed consideration because of the scale of this work and the demonstration of the technical difficulties that arise when attempts are

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10)On the short time scale corresponding to the θ pinch, the slow buildup of natural frequencies due to incorrectly chosen phase shift is of no interest. It is therefore sufficient at present to use the simplified inverted pendulum model and introduce feedback with respect to position and velocity.

made to stabilize fast MHD oscillations in a thermonuclear experiment.

It has been shown theroetically[95,96] that feedback can be produced by utilizing the volume force due to the corrugated and helical fields that is used to achieve equilibrium. The direction of the force vector can be varied by varying the relative spatial "phase shift" between the corrugated and helical fields. The programmed single-turn coil in Scylla, which ensures compression and equilibrium is therefore augmented by single-turn coils that vary the corrugation of the magnetic field and are controlled by the feedback system. These coils are located under the main coil producing the longitudinal compressive field. There are four pairs of such coils per period of the helical field. The period of the helical field in these experiments was 43 cm, which was much less than half the wavelength along the major circle of the torus for the shortest toroidal modes. (The radius of Scyllac is 4 m, and both theory and experiment suggest that the maximum toroidal wave number for the unstable $m = 1$ oscillations is $n = 6$ and the wavelength is 420 cm.) Consequently, the direction of the stabilizing force should remain practically constant over several periods of the helical field. It was therefore intended in the initial project to use one power amplifier (one module) to supply two pairs of coils. A total of 60 such modules was to be used (in accordance with the number of helical field periods). Each module produced 24.5 MW and could be used to generate a field of up to 230 G (with allowance for the shielding effect of the closely-lying coil producing the main compressive field for the θ pinch) with a rise time of 2.1 μsec (consisting of the true time delay in the feedback channel of 0.9 μsec and a rise time of 1.2 μsec of the current in the coil, which was determined by the inductance and resistance). The force acting on the plasma column in the presence of the combined helical and corrugated fields was demonstrated experimentally[97] and the motion of the plasma column under the action of this force was analyzed theoretically.[98]

The displacement of the plasma relative to the equilibrium position in each coordinate was determined with optical sensors (a pair of silicon diodes connected dif-

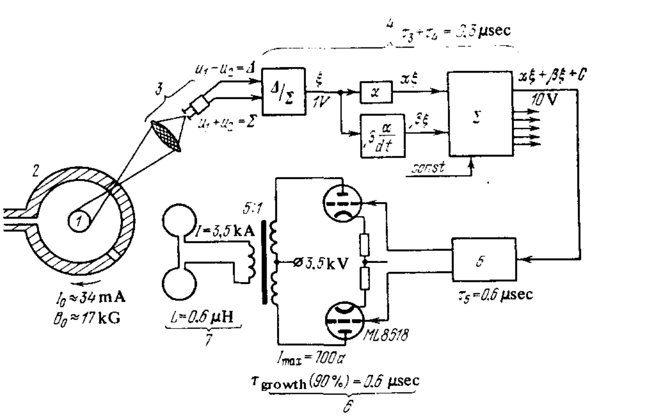

FIG. 24. Block diagram of the feedback channel in the Scylla installation.[94] 1—Plasma; 2—toroidal field coil; 3—optical sensor; 4—amplifier-shaper; 5—power amplifier; 6—terminal amplifier; 7—control coils with stepdown transformer.

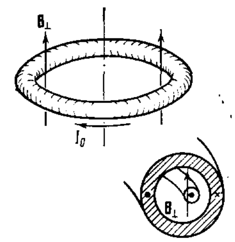

FIG. 25. Equilibrium in the tokamak (see the second half of the legend to Fig. 24). The required field $B_\perp$ appears when the plasma is surfounded by a metal envelope.

ferentially). Each diode in a pair received radiation from the opposite edge of the plasma column. The displacement ξ of the plasma column was measured by the ratio of the difference Δ between the light signals and their sum Σ. For a Gaussian luminosity profile, this signal is linear up to displacements of 7 mm and yields useful information on displacements up to 11 mm (for a plasma radius of 1 cm). The resulting signal is amplified, added to its own derivative (so that velocity feedback can be introduced), a constant term representing the equilibrium position of the column is added, and the result of all this is used to control the power amplifiers. Figure 24 shows the feedback system of Scylla.

Each such measuring "station" can service a number of power amplifier modules. The number of stations must, of course, exceed $2n_{max}$. For Scyllac, $n_{max} = 6$. Computer calculations have shown that it is sufficient to have 15 such stations.[94]

A similar system was tested on the linear θ pinch of Scyllac IV-3.[99] 10 modules and one detector station were employed. The instability was specially produced by a helical coil, and control was carried out with respect to one coordinate only. These experiments have demonstrated that the system was capable of operating satisfactorily.[99–102]

Feedback experiments have been planned for a toroidal sector and a complete torus with the following parameters: compressing field 40 kG, plasma density 2.7×10^{16} cm^{-3}, electron temperature 500 eV, ion temperature 1.3 keV, and $\beta = 0.8$. The instability growth rate corresponding to these parameters was found to be $\gamma = 0.7 \pm 0.3\times10^6$ sec^{-1} and the parameter $\gamma\tau$ was found to be greater than the permissible value. Feedback experiments on the complete torus were therefore not performed, and it was decided to reduce the plasma parameters so that they became consistent with the technical possibilities of the stabilization system. An experiment for an 8-m, 120° sector was therefore proposed.[103] The compressing field was reduced to 17 kG, but the density and size of the plasma column remained almost the same. The value of β was reduced to 0.6–0.7 and the temperature $T_e \approx T_i$ down to 130–150 eV. This resulted in a reduction in the instability growth rate to 0.2×10^6–0.3×10^6 sec^{-1}. The properties of the feedback system were improved at the same time. Each module was used only for one pair of coils, and this reduced the total signal rise time to 1.5 μsec. The parameter $\gamma\tau$ was thus reduced to the required value of < 0.5.

Further improvement of the system can be achieved by replacing the corrugated field coils with helical coils producing a $\sin(2\theta - k\varphi)$ field. Other things being equal, this results in an increase in the stabilizing force by a factor of two.[103]

A recent report[104] states that this improved system has been successfully used to stabilize helical modes in the toroidal Scyllac sector.

6. CONTROL OF THE EQUILIBRIUM POSITION OF THE PLASMA COLUMN IN THE TOKAMAK

Control of the equilibrium position of the plasma column in the tokamak is a special problem, distinct from instability suppression, because it is concerned with the stable position. However, the philosophy of this work and the techniques upon which it relies are not very different from those in the case of stabilization. Since the process is slow, the equilibrium control systems are relatively simple and are already in use in thermonuclear installations. Future large-scale experiments and, even more so, the tokamak as a thermonuclear reactor, are unthinkable without equilibrium control systems. Because of the practical importance of this problem, we must consider it separately.

A toroidal plasma ring carrying a longitudinal current I_p tends to expand and increase its major radius R. To ensure equilibrium, a magnetic field must be introduced at right-angles to the plane of the torus: $B_{\perp 0} \sim I_p/R$ (this is the so-called vertical field, Fig. 25).

This magnetic field is produced by external coils and the currents induced in the metal walls around the plasma during the motion of the plasma ring. The field $B_{\perp 0}$ appears automatically when the plasma is surrounded by a conducting metal enclosure. Since the conductivity is finite, the image currents in the metal wall which produce the field $B_{\perp 0}$ decay with time constant $\tau_k = 2\pi\sigma db/c^2$, where d is the thickness, b the radius, and σ the conductivity of the wall.[105] The plasma-ring time constant is of the order of τ_k. The time constant can be increased by increasing the thickness of the conducting wall and by cooling it. This was, in fact, done in the previous tokamak experiments but is not, however, a promising approach for modern installations where a confinement of about 1 sec is required. It is even less satisfactory for future reactors with pulse lengths of some hundreds of seconds. Such reactors will require nonuniform wall thicknesses, and this will complicate the utilization of the neutron flux generated by the plasma.

The requirements on τ_k can be reduced by ensuring that the external field $B_\perp$ is kept close to $B_{\perp 0}$. This can be achieved with the aid of an automatic control system, i.e., by specifying a suitable relationship between $B_\perp$ and the true position of the plasma ring.

Although the idea of using a tracking system for maintaining equilibrium was proposed a long time ago,[2] actual experiments have begun only in recent years. In the first experiment with the small TO-1 tokamak,[106–109] the control system and a thin wall produced the vertical field necessary for equilibrium. Inductive coupling between the control coil and the plasma was enhanced (to produce large W) by connecting this coil to a two-terminal circuit with a negative resistance (which compensated the resistance of the control coil) and a negative inductance[79] (see Chap. 5 above). With shorted coil, the constant τ_k was 16–18 msec. When the two-terminal circuit was connected, this increased the time constant to 150–200 msec (more accurate compensation of the resistances could be used to increase this still further). For a given displacement of the plasma ring, the current induced in the coil was greater than in an ideal wall by a factor of 2.5–3.

In contrast to stabilization systems for MHD modes, the equilibrium control systems do not have to be fast because the metal enclosure or liner takes on the role of the "fast" controller. Moreover, when $\omega > \tau_k^{-1}$, external fields do not penetrate under the liner. The controller should augment the liner only for $\omega < \tau_k^{-1}$. This is why equilibrium control systems have rapidly become very popular. All large tokamaks under construction, or planned, include such systems.[110] It appears that it is better to base them not on the impedance scheme,[106–109] as in the TO-1, but on the classical scheme with individual displacement sensors. High-power impedance regulators are difficult to design and to correct. The main point is that, in large systems, it is not reasonable to pass the entire current ensuring equilibrium through the amplifiers. It is better to control most of the current by a programming device, and use the tracking system only for error correction. The programming plus feedback combination is used in the French tokamak TFR.[111,112] A possible feedback system with independent sensors is described by Fijwara *et al.*[113] (the process is slow so that a computer can be used in the system).

A feedback system can also be used to control the motion of the plasma ring in the direction perpendicular to the plane of the torus.[112] In this direction, there are no gas-kinetic forces, and only the leakage fields have to be compensated. A low-power control system can therefore be employed.

We note in conclusion that, since perturbations propagate rapidly (with the Alfven velocity) along the torus, any slowly-varying applied force will be averaged over the length of the torus. It is therefore sufficient to apply $B_\perp$ not to the entire circuit of the torus but over short segments, for example, under slots in the enclosure.[114]

Control of the major radius of the tokamak can also be used as a means of affecting plasma instability. The possible effect of escaping electrons on the instability is discussed by Furth and Rutherford.[115]

7. STABILIZATION OF OSCILLATIONS WITH A LOW GROWTH RATE

A. Suppression of dissipative instabilities with a low growth rate

This section is concerned with the suppression of oscillations with a low growth rate, which are due to weak energy exchange between the wave and the plasma particles. They result, formally, when small anti-Hermitian terms in the plasma permittivity are taken into account.

The stabilization mechanisms discussed in Chap. 2, namely, variation of phase velocities of the waves and direction of the energy flux to (or from) an external circuit, apply fully in the present case as well.

A typical example is the "universal" drift instability of magnetized inhomogeneous plasma against drift waves propagating at right angles to the magnetic field and to the direction of the inhomogeneity.[116] Its source is the inversion of the distribution of the corresponding electron-velocity component at low velocities (Fig. 26). The phase velocity V_{ph} of drift waves lies in the range where $\partial f_e/\partial v > 0$, so that the electrons transfer energy to the wave with which they interact. The natural frequency can be increased by means of feedback with large real W, so that the velocity falls into the $\partial f_e/\partial v < 0$ range and the waves becomes damped.[117] It is simpler to achieve stability in a system with small but complex W (phase shift), in which energy flows to an external circuit, and this energy is greater than that transferred by resonance electrons to the wave.

Stabilization of this class of oscillations is facilitated by the fact that the growth rate is low:

1) It is possible to suppress spatial modes by a surface system.[118] In point of fact, since the role of the feedback reduces to ensuring a small outflow of energy from the oscillations (inflow in the case of negative-energy waves), it is immaterial whether the energy is removed directly from the interior of the plasma or there is an energy flow to the walls. Consider a quasiclassical perturbation $\psi \exp(ik_r r)$. Let us suppose that the solution of the Poisson equation $\Delta\psi = -4\pi\rho_\omega(\psi)$ for k_r is $k_r = k_r(\omega)$. The equation for the frequencies is obtained by equating $k_r(\omega)$ to the values of k_n allowed by the conditions which match the solution to the vacuum solutions (which depend on W): $k_r(\omega) = k_n(W)$. The stabilization condition then becomes

$$\frac{[dk_n(W)/dW]\,\mathrm{Im}\,W}{dk_r/d\omega} + \gamma < 0. \tag{7.1}$$

A restriction on the wavelength that can be stabilized arises when $|dk_r/d\omega|$ increases with k_r. Under these conditions, the growth rate γ decreases with increasing number of the radial mode that can be suppressed for given W.

2) Stabilization is also possible with discrete, including local, feedback. In fact, a small additional energy flow, i.e., small W is sufficient for the stabilization of waves with low growth rates. When W is small, perturbation theory can be used to determine the corrections to the natural frequencies due to the interaction with the external circuit. To be specific, let us suppose that the application of feedback is equivalent to the introduction of charges (into the plasma, or into some external region, as in the case of surface stabilization) with density $W\iint G(\mathbf{r}-\mathbf{r}', t-t')\psi(\mathbf{r}', t')d\mathbf{r}'dt'$.[119] The frequency correction due to feedback is given by

$$\delta\omega = W\frac{\iint g(\mathbf{r}-\mathbf{r}', \omega)\,\psi^*(\mathbf{r}')\,d\mathbf{r}'\,d\mathbf{r}}{\int \psi^*(\mathbf{r})\,\hat{\rho}\psi(\mathbf{r}')\,d\mathbf{r}'\,d\mathbf{r}}, \tag{7.2}$$

where g is the time Fourier transform of G, and $\hat{\rho}$ and ψ are, respectively, the charge density operator and the eigenfunction of the problem in the absence of feedback. The magnitude of W necessary for stabilization is determined by the form of the kernel G, i.e., the distribution of sensors and "suppression points." The case of a sensor and a suppressor separated by a distance $\mathbf{r}_0$, for which $G \sim \delta(\mathbf{r}-\mathbf{r}'-\mathbf{r}_0)$, is not special in any way. The fact that, to determine the required W it is sufficient to know only the spatial structure of the instability in the absence of feedback, determines in the final analysis the success of the numerous relatively crude (1 sensor + 1 suppressor) experiments on the suppression of drift-type oscillations.

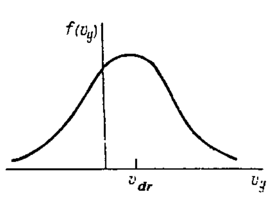

FIG. 26. Transverse-velocity distribution function in inhomogeneous plasma. y is the direction at right-angles to the magnetic field and the density gradient.

We note, however, that not all the drift-type oscillations belong to the above class. In particular, the well-known temperature-drift instability[120] is not, in general, dissipative because its growth rate is, in some cases, $\gamma \gtrsim \mathrm{Re}\,\omega$. Feedback suppression of this instability, analyzed formally by Lakhine and Sen,[121] will, in practice, encounter all the difficulties characteristic for reactive instabilities.

B. Experiments on the stabilization of drift and cyclotron oscillations

Successful stabilization of drift oscillations by feedback was reported practically simultaneously by three groups.[65,64,62] Although these experiments were carried out under very different conditions (arc discharge plasma in argon at $n \sim 10^{13}$ cm^{-3}, $T_e \sim 5$ eV, $B \sim 1$ kG,[64] weakly ionized plasma of a reflex discharge with $n \sim 10^{10}$ cm^{-3}, $T_e \sim 5$ eV, $B \sim 100$ G,[65] and potassium plasma in the Q-machine with $n \approx 7\times10^{10}$ cm^{-3}, $T_e \sim 0.3$ eV, $B \sim 2$ kG[62]), the techniques employed, the results obtained, and the interpretation used are completely analogous. In each case, one Langmuir probe was used as the fluctuation center and suppression was achieved with a Langmuir probe[62] or plane electrode[64,65] touching the plasma. Since the sensor and the electrodes were used under the conditions of a "controlled sink of electrons," their shape was of no particular importance. In all cases, the same method of connection of the sensor and suppressor through a broad-band amplifier and phase shifter was employed (Fig. 27). It was first used by Arsenin *et al.*[11] and appears to be the universal method for the suppression of all dissipative instabilities.[122] In all three cases, correct choice of the amplitude and phase ensured the suppression of unstable-mode fluctuations and a small (by 20–30%) increase in density. Parker and Thomassen[65] recorded not only the suppression of the $m = 2$ unstable mode, but also the exci-

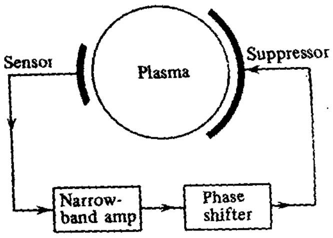

FIG. 27. Stabilization of dissipative instabilities.

tation of the $m = 1$ mode under high gain. Mode decoupling was achieved by the correct choice of the phase shifts between the four suppressing electrodes (such as the $m = 1$ harmonic was absent from the electrode field).

Stabilization of collisionless drift waves was achieved by Lindgren and Birdsall[123] in the Q-machine with rarefied potassium plasma ($n \sim 10^8$ cm^{-3}, $T_e \sim 0.2$ eV, $B \sim 1$ kG). They used one sensor and three suppressors connected in parallel in the usual way (Fig. 27) with the addition of frequency filters. This resulted in three independent frequency channels that could be used to suppress the three spatial modes corresponding to $m = 2$, 3, and 4. It is clear that this simplified frequency decoupling between the modes is possible only when the natural frequencies are very different.

We must now consider one further possible approach to the stabilization of a number of modes. So far, the simultaneous suppression of a number of spatial modes has been achieved by increasing the number N of discrete elements, i.e., sensors and suppressors (electrodes, probes, and so on). The frequency dependence $W(\omega)$ is relatively simple under these conditions. However, if the oscillations corresponding to different spatial modes have different frequency spectra, the feedback system can detect all the modes with the aid of only one sensor[124] (this was, in fact, done by Lindgren and Birdsall[123]) provided there are no standing waves with zeros on the sensor. Since the response corresponding to each plasma mode to the suppressor is also a function of frequency, suitable choice of the form of $W(\omega)$ will ensure that one discrete suppressor (electrode, coil, probe, and so on) will, in principle, suppress all the modes. This applies to oscillations with both $\gamma \ll \omega$ and instabilities with $\gamma \gtrsim \omega$. This principle has not as yet been realized in practice. We note that, in the case of stabilization of plasma in thermonuclear installations, engineering considerations demand that the number N should be as small as possible. It is probable that a combination of spatial and frequency mode separation will be the optimum solution.

Soon after, the probe method was used to suppress balloon-type collisional drift waves in the stellarator[125] (xenon plasma, $n \sim 10^{11}$ cm^{-3}, $T_e \sim 1$ eV, $B \sim 1$ kG).

Simonen *et al.*[62] and Furth and Rutherford[63] have given a theoretical interpretation of the stabilization of drift waves by the "controlled sink method" and obtained criteria for the phase shift, which were found to agree with experimental results.[62]

All the experiments considered above demonstrate a similarity between the dependence of the oscillation amplitude on $|W|$ and on the phase shift (Fig. 28). This was first established in connection with the suppression of the Harris cyclotron instability[126] in an open trap[11] using an electrostatic system consisting of sensors and electrodes distributed around the plasma (similarly to that described in Chap. 4).[11] Strictly speaking, this instability cannot be classified as dissipative. It occurs as a result of the interaction (resonance) between magnetized Langmuir oscillations and the Larmor rotation of ions. However, the ions provide a small ($\sim m_e/m_i$) contribution to the perturbed charge density, so that it is relatively easy to prevent resonance and thus suppress the instability by introducing some weak damping into the Langmuir branch with the aid of feedback. The condition for this damping, in fact, determines the shaр of the curves in Fig. 29. The direction and magnitude of the energy flow are determined by the sign of the ph shift (see Chap. 2), so that the dependence of the amp tude on the phase shift is also sinusoidal. For the o mum phase shift, the damping introduced in this wa creases with $|W|$, but only up to $|W| \sim 1$, because t spatial structure of the wave changes for large $|W$ its coupling to the external system deteriorates. best "matching" of the wave ("generator") to the occurs when small damping is introduced with $|W$ For $|W| > 1$, there is, in addition, the danger of excitation of the stabilization system.

A dependence of the form shown in Fig. 29 is

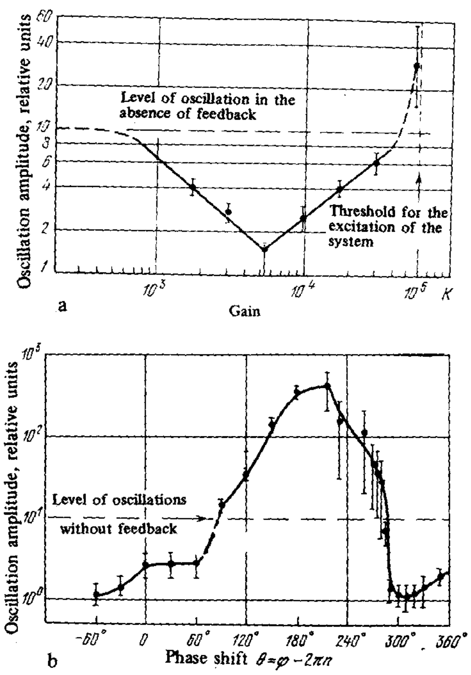

FIG. 28. Amplitude of ion-cyclotron oscillations as a function of gain (a) and phase shift (b) in the feedback system.[11]

[11]Arsenin *et al.*[11] succeeded in suppressing the mo $= \varphi(Z)$, which is symmetric with respect to the equa plane of the trap (Z is measured from the center of ma in the direction of the magnetic field). A syste electrodes cut into two was used to stabilize the an metric mode $\varphi(-Z) = -\varphi(Z)$ as well.[127,128]

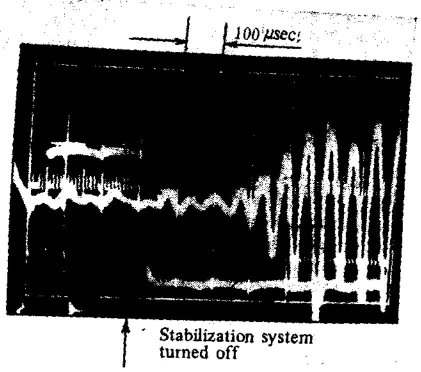

FIG. 29. Development of flute instability when feedback is turned off.[183] Cutoff pulse in the stabilization system is shown.

teristic for all dissipative instabilities; see, for example, Thomassen.[13]

C. Contactless methods of introducing feedback

Drift instabilities are convenient for experimental investigation and have therefore served as a testing ground for the development of new contactless feedback methods. Microwave oscillations have been used for this purpose.

The heating of electrons during resonance absorption of microwave power at the frequency of the upper hybrid resonance $\omega_h = (\omega_{pe}^2 + \omega_{Be}^2)^{1/2}$ have been used[129] for the suppression of the drift-dissipative instability in the cesium plasma of a Q-machine at density $n = 5\times10^{10}$ cm^{-3}. The microwave power from the source (11 GHz) was modulated with the signal from a Langmuir probe measuring the saturation current (i.e., density fluctuations). For a steady-state fluctuation amplitude of $\Delta n/n \sim 5\%$, the total suppression of fluctuations was achieved at microwave power ~100 mW. To ensure that the microwave oscillations at the upper hybrid frequency reached the absorption region through the opacity zone, the experiments were performed in a weakly inhomogeneous magnetic field. The localization of the source of heat could then be varied by varying the frequency of the microwave oscillations.

Local microwave heating at the frequency of the electron-cyclotron resonance has been used to suppress the ion-acoustic instability with $m = 0$ in the positive column of an arc discharge in neon[130,131] ($n \approx 3\times10^{11}$ cm^{-3}, $T_e \approx 5.4$ eV, $B \approx 180$ G). Density perturbations were recorded with photodiodes whose output was fed through a phase shifter into the modulator of the oscillator. Under the conditions of optimum stabilization, the change in T_e due to the hf field did not exceed $\Delta T = 0.6 \pm 0.1$ eV. Modulation of the radiation at $\omega \approx \omega_{Be}$ has also been used[132] to suppress collisional drift-type instabilities.

Another method of introducing feedback through microwave oscillations was investigated by Wong *et al.*[133] At first sight, this experiment is similar to that reported by Hendel *et al.*[129] The latter experiment was also performed on a Q-machine but at a lower density ($n \cdot 10^9$ cm^{-3}) when collisions were unimportant. In both cases, drift waves were stabilized by modulating the microwave amplitude at the upper hybrid frequency. However, under collisionless conditions,[133] feedback consists not in local heating but in the nonlinear excitation of low-frequency oscillations at the beat frequency of the modulated signal. Wong *et al.*[133] succeeded in developing a completely contactless system by using the reversibility of the nonlinear mechanism. They measured the perturbations by sounding the plasma with another microwave beam of low constant power (<1 mW). The high-frequency oscillations in the plasma modulated the amplitude of the transmitted microwave signal, the signal was detected, and its hf component was used to modulate the suppressing microwave signal.

When $\omega_h - \omega_{Be}$ is much greater than the frequency of the oscillations that are to be suppressed, the nonlinear effect of ω_h on low-frequency waves occurs through the Miller force.[134]

The different nonlinear mechanisms offer a very fruitful field for searches for new contactless methods of introducing feedback.[135] One possibility[136] is to use the nonlinear interaction between two powerful CO_2 laser beams with frequency difference close to ω_h. The role of the laser beams is to transport energy through the opacity zone. Nonlinear interaction of the beams at the point of crossing leads to the excitation of oscillations at the difference frequency, which produce electron drift in the direction opposite that of the Poynting vector associated with the incident wave[137] and this can be used for the suppression of instabilities. Unfortunately, estimates of the laser power necessary for stabilization[136] are subject to considerable uncertainty. Optimistic and pessimistic estimates differ by nine orders of magnitude.

Experiments on hf and microwave heating of plasmas are being carried out on tokamaks. It would be desirable to examine in greater detail the possibility of using part of the hf power for stabilization by feedback.

Chen and Furth[138] propose a further method for introducing feedback into the interior of the plasma, namely, injection of a beam of neutral particles. Ionization of the neutrals results in the appearance of equal numbers of ions and electrons in each volume element. It is then clear from the continuity equation $(\partial n_{e,i}/\partial t) + \mathrm{div}(n_{e,i} v_{e,i}) = S$ that, because of the difference between the electron and ion velocities, perturbations of the electron and ion concentrations $n_{e,i}$ vary in different ways in the wave. The ionization of neutrals is, therefore, in effect a source of charges. Chen and Furth[138] considered this feedback method in relation to the drift-dissipative instabilities (they took into account the fact that fast neutrals introduced momentum into the plasma so that there was a change not only in the continuity equation but also in the equation of motion of the ions). The method is also applicable to other low-frequency long-wavelength instabilities.

Provision for powerful neutral injection is made in a number of thermonuclear installations being planned at present. A fraction of the neutral flux could be controlled and used in a feedback circuit. This method of suppressing trapped-particle instabilities has also been discussed in the literature.[139,140]

8. OTHER INSTABILITIES

Some of the work on stabilization was undertaken because a particular instability was a convenient object for the experimental investigation of nonlinear phenomena or because of applied interest.

Anker-Johnson *et al.*[141] used a system consisting of one sensor and one suppressor for the sausage mode ($m = 0$) of the electron-hole plasma in the semiconductor InSb.

The effect of feedback on the Kelvin-Helmholtz instability in the Q-machine, in which there is relative rotation of plasma layers due to a radial electric field, has also been investigated experimentally.[142] A sensor + suppressor pair was used to suppress the $m = 2$ mode in plasma with $n \sim 5\times10^{10}$ cm^{-3} with angular velocity $\omega_E \sim 10^5$ sec^{-1} in a field $B \sim 1.5$ kG. It has been found experimentally[39] (at lower plasma density, i.e., $n \leq 8\times10^9$ cm^{-3}) that a surface with a complex impedance, placed outside the plasma (Chap. 2), has a stabilizing effect.

The suppression of the Kelvin-Helmholtz instability has also been analyzed in another case,[143,144] namely, in the presence of a tangential discontinuity in plane-parallel flow with and without a longitudinal magnetic field. Without feedback, the discontinuity is stable provided the velocity jump is $u < v_A = B/\sqrt{4\pi\rho}$, where ρ is the density.[145] The problem is to achieve stability for $u > v_A$. When the longitudinal field is present, coils with negative impedance (Chap. 5) have a stabilizing effect and control the perturbation of the magnetic field on the channel walls. Ladikov-Roev and Mashkovskiĭ[146] have suggested a controller capable of maintaining the required boundary conditions for the pressure perturbation on the walls in the absence of the field. Stabilization of flow across a magnetic field has also been discussed.[147]

Electrostatic[148,149] and magnetic[59] feedback suppression of the current-convective instability of weakly ionized plasma in the positive column of a gas discharge[150] has been analyzed (this is a dissipative instability but, in general, the growth rate is low). Successful control of this instability would extend the possibility of experimental verification (with a convenient object) of the various hypotheses on nonlinear effects. A contactless method was used by Chen *et al.*[151] to suppress an instability similar to the current-convective instability in the plasma of a high current arc ($n \sim 10^{16}$ cm^{-3}, $T_e \sim 4$ eV, $I = 200$ A, $\beta = 8\%$). The displacement of the column was measured with the aid of an HCN laser, and the position of the column was controlled by a magnetic field perpendicular to the discharge axis ($B_\perp \leq 8$ G). The electrostatic oscillations of plasma in the positive column were suppressed by feedback in an experiment by Mase and Tsukishima,[152] but the nature of the oscillations was not indicated by the authors.

Arsenin and Chuyanov[153] have shown that feedback can be used to stabilize drift-beam oscillations (collisionless current-convective instability).[154–156] These oscillations are regarded as responsible for the limitation of the current (below the Pierce Limit)[12] in small-radius compensated electron beams.[160] On the other hand, the excitation of these oscillations has been used for ion heating in an adiabatic trap.[161] Excitation by feedback has also been used to produce the heating of denser plasma in which "natural" excitation is not possible.[162][13]

Garscadden and Bletzinger[165] and Sato[166] have reported experiments on external feedback control of ionization waves in the plasma of the positive column (see also the review by Pekarek[167]). Arsenin[168] has considered the suppression of ionization instability, i.e., magnetic losses (see the review by Nedospasov[169]) in nonisothermal magnetized plasma in which electrons are heated by a current flowing across the magnetic field (this is one of the methods of producing high conductivity at low gas temperature in the MHD generator). Since the overheating character of this instability, which develops even in homogeneous plasma, is relatively simple, there is considerable applied and general interest in both its properties and the properties of the turbulence associated with it.

We know that, since the introduction of feedback is equivalent (see Chap. 3) to a change in the plasma permittivity ε, any oscillations can be formally stabilized by suitably choosing the permittivity increment $\delta\varepsilon$. The whole problem is, however, how to achieve this increment. There have been analyses[61,170] of, for example, the stabilization of drift-cyclotron oscillations[161] and the conical instability of Dory, Guest, and Harris.[172] However, it is not clear how feedback can be realized for such short-wave oscillations. Nor has it been indicated how feedback could be realized for the theoretically possible[173,174] suppression of the spatial modes of beam instability[175,176] (the longest wave perturbation along the beam is stabilized, at least for a small ratio of beam density to background plasma density, by controlling the cathode and anode potentials[174]).

Baswell and Christiansen[177] suppressed ion-acoustic oscillations[178] corresponding to $m = 0$ in plasma produced by a high-frequency discharge ($n \sim 5\times10^{11}$ cm^{-3}). Stabilization was achieved by modulating the hf power in accordance with fluctuations in the plasma emission. A similar method was used to suppress oscillations of

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[12]The Pierce instability[157] is due to feedback through the external circuit (maintaining constant potential difference between the cathode and the anode). The influence of the characteristics of the external circuit on the stability limit has been investigated by a number of authors.[158,159]

[13]We note that Kammash and Uchan[163] have investigated (for conical modes[164]) the destabilizing influence of feedback during the quasilinear stage. They report (without giving details) that, although the excitation of these oscillations is faster in the presence of feedback, the deformation of the distribution function produced in this way ensures that instability sets in before the entire plasma is thrown into the loss cone. However, it is not indicated how feedback can be realized for such high-frequency and short-wave instability.

unknown (!) origin, reported by Brown *et al.*[179]

Finally, experiments have been carried out on the excitation and suppression of plasma-potential oscillations in a plasma accelerator with closed electron drift and extended acceleration zone.[180]

9. APPLICATIONS OF FEEDBACK IN PLASMA PHYSICS

Feedback can be used not only as a means of improving containment systems but also as a convenient instrument for investigating the physics of plasma (in the first instance, for investigating instabilities). Even the simple facility of being able to "turn on" and "turn off" instabilities at will provides the experimenter with possibilities that are difficult to achieve by other means. Let us consider various ways of using feedback in physical studies.

a) Growth rates have been measured by turning off feedback and observing the development of oscillations in practically all the successful experiments on stabilization by feedback. This method was used to determine the growth rate for flute,[9] drift,[181] and other instabilities.

b) Nonlinearity coefficients have been measured by examining the establishment of nonlinear oscillations after perturbation through the introduction of feedback. It is known[182] that the oscillation amplitude can be written as a function of time in the form

$$\frac{dA^2}{dt}=\Big(2\gamma-\alpha A^2-\sum_{k>1}\beta_k A^{2k+2}\Big)A^2. \tag{9.1}$$

Wong and Hai[181] have discussed a method of determining the coefficients α and β with the aid of feedback. The establishment of nonlinear oscillations, and of flute[183] (Fig. 29) and drift[181] instabilities, in semiconductor plasmas[141] has been investigated experimentally.

In many cases, the coefficients β_k in (9.1) are found to be zero, and the nonlinear behavior of unstable oscillations may be describable by the Van der Pol equation.[184] The application of this equation to nonlinear oscillations in the presence of feedback has been investigated in detail for the drift-dissipative and ion-acoustic instabilities.[185–187] It was found that a correct prediction could be obtained for the amplitude and oscillation frequency as functions of gain and phase shift in the feedback system.

c) Studies of stable natural oscillations can also be carried out with the aid of feedback systems. The application of feedback with the necessary phase shift can be used to excite normal damped natural oscillations[52] and to determine their damping. Such experiments yield not only the frequency and growth rate but also the sign of the oscillation energy, since the latter determines the sign of the phase shift for which the growth of the oscillations takes place.

d) Studies of nonlinear wave processes are facilitated by the ability to control the damping and frequency of the various oscillation branches. This control can be used to investigate the dependence of decay processes on the characteristics of natural oscillations,[187] for example, to measure the threshold amplitudes of decay instability.[188] The interaction and competition of modes[68] can also be investigated in this way.

e) Transport processes associated with instabilities can be seen against the background of other transport processes by simply turning on or turning off the feedback system.[9,53] The ability to vary the phase velocities of the waves, the linear growth rates, the amplitudes of established oscillations, and the wavelengths of unstable waves enable us to determine the diffusion coefficient as a function of the parameters of the unstable oscillations.[68]

In our view, this is the most valuable of all the possibilities offered by the feedback method because it may provide experimental data necessary for constructing the nonlinear theory of anomalous plasma diffusion. Even the few experiments performed so far in this area[53] have stimulated the development of a new theory describing diffusion in open traps in the presence of monochromatic waves and rare collisions, which is analogous in some degree to the "banana" diffusion in toroidal systems.[189]

f) Diffusion in velocity space can be investigated by the methods described in the previous section. A variation in the energy and angular distribution in the direction of the magnetic field under the action of cyclotron oscillations controlled by feedback was observed by Chuyanov *et al.*[127]

This brings us to the end of our list of attempts made so far to apply feedback to fundamental plasma physics measurements. We note that, as a rule, stabilization by feedback cannot be achieved without knowing the "plasma transfer function." Measurements and calculations of this function in the course of stabilization research[91,92,98,190] reveal the possibility of a very detailed precise comparison between theory and experiment. Thus feedback stabilization of any particular instability means that a deep level of understanding has been achieved of the corresponding plasma processes.

In our opinion, fundamental studies by the feedback method, which can be carried out with the aid of relatively simple technical means (in contrast to the thermonuclear applications of this method), will undergo a very much greater development than has taken place so far.

10. CONCLUSIONS

Investigations of feedback control of plasma instabilities have now led to a reasonably clear understanding of the possibilities and limitations of this method and the position which it occupies or may occupy in the arsenal of plasma physics and thermonuclear studies.

Comparison of the feedback method with other known methods of stabilization (such as the magnetic-well and dynamic stabilization) shows that the feedback method is energetically the most convenient. Although feedback systems should, as a rule, be broad-band and low-Q systems (in contrast to dynamic stabilization systems operating at a given frequency so that they are, in principle, high-Q systems), their very principle guarantees that they will act on oscillations even at the thermal noise level.[14)] This is why the feedback method does not suffer from nonlinear (decay-type) processes that are characteristic for dynamic stabilization and, since they lead to the absorption of the hf field by the plasma, reduce to zero the high Q of the external devices in the presence of plasma.

Having enumerated the successes of the feedback method and its potential advantages, we must not, nevertheless, forget the basic difficulties and restrictions from which it suffers. The feedback method is, in principle, universal, i.e., suitable for the suppression of different instabilities, so that it is, in practice, selective in the sense that a given device cannot influence many oscillation modes (in contrast to the magnetic-well or dynamic stabilization). The suppression of each mode requires a separate device, and this leads to considerable technical complications whenever several modes have to be suppressed simultaneously. Stabilization is basically impossible when the number of unstable modes is not restricted by some additional factors (such as viscosity, and so on). Stabilization is readily achieved only in experiments with well-defined sufficiently long-wave or weak (with low growth rate) instabilities for which there are not restrictions connected with the spatial structure of the oscillations. In such cases, the feedback method is a valuable addition to the very limited range of methods at the disposal of plasma physics for the suppression of instabilities.

The other limitation which is intrinsic for the feedback method, but does not affect other methods of stabilization, is the finite time taken by the signal to pass through the feedback circuit. This imposes very stringent restrictions on the control of rapidly growing instabilities. (It is interesting that this difficulty was noted right at the outset of thermonuclear studies.[192]) At present, these limits are determined by the electronics used in these experiments, i.e., the feedback characteristics can be improved but at a high cost.

Evidently, feedback systems are, in general, technically the most complicated among the methods used to stabilize plasmas. Nevertheless, there are a number of areas that are potentially capable of benefiting from the application of feedback methods. They include:

a) stabilization of helical modes in tokamaks;

b) stabilization of individual modes in high-β toroidal systems;

c) suppression of flute instabilities in open traps with Coulomb losses through the mirrors;

d) suppression of microinstabilities by using a fraction of the microwave and neutral-injection power intended for plasma heating;

e) studies of anomalous transport processes, and

f) studies of nonlinear wave processes.

Automatic control of the position of the plasma ring in tokamaks is, of course, by now established in thermonuclear studies.

There are also considerable possibilities insofar as improvement of the feedback method itself is concerned. Many of the ideas developed in automatic control theory and suggested for plasma experiments have not as yet been realized. They involve nonlinear feedback,[84] pulse modulation,[193] and self-adjusting and search devices.[194] Suppression by neutral beams has not as yet been tested. There is a need for new ways of using nonlinear phenomena in feedback control with the aid of microwave and light fields. It would be interesting to re-examine the various possible control systems, bearing in mind possible extension and improvement through the application of feedback. The feedback method has not been extensively investigated in relation to MHD generators and gas lasers.

There is no doubt that further applications of ideas taken from automatic control will benefit the development of plasma physics. The first steps toward the investigation of feedback control in thermonuclear experiments have already been made.

[14)]The question of noise in the presence of a feedback system has been considered by Selidovkin[191] who has shown that the noise itself will depend on the properties of the stabilization system and, when these properties are suitably chosen, the noise level can be reduced very substantially even in non-equilibrium plasma. Estimates obtained for particular systems have shown that the necessary feedback power is very low.

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