The editorial board of Uspekhi Fizicheskikh Nauk announces with deep sorrow the untimely death on 8 August 1977, at the age of 51, of its member, the outstanding Soviet physicist, delegate of the Superior Soviet of the USSR, Vice President of the USSR Academy of Sciences, Rector of the Moscow State University, Lenin Prize laureate, Academician,

REM VIKTOROVICH KHOKHLOV

# To the fiftieth anniversary of the formation of quantum mechanics

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The present issue is devoted to the history of the formation of quantum mechanics—the theory of the phenomena occurring in the world of the atom—which took place essentially half a century ago.

Physicists are well aware that the creation of quantum mechanics was not a single event in time and not the result of a flash of inspiration of a single genius. Over more than a quarter-century, starting with the 90's, new experimental facts were accumulated and could not be explained by the laws of classical physics. These were the results of investigations of equilibrium radiation, of the photoeffect, of spontaneous emission, of discrete states of atoms and the regularities of atomic and molecular spectra, of the Zeeman and Stark effects, of the Compton effect, and of many other phenomena. Flourishing ideas were advanced to explain these effects, based on new quantum concepts, but it was many years before a unified theory of atomic phenomena could be developed. The successful consolidation of all the experimental results and the ideas suggested by them into a unified theory was initiated in 1925, and consumed three years of very intensive work by the outstanding physicists of that time.

To mark the semicentennial of this titanic intellectual task, so abundent in revolutionary ideas, the editors publish in this (Russian) issue a number of articles. These include translations of the trail-blazing classical papers. Among them are: de Broglie's article (end of 1923) with an exposition of its principal ideas developed in earlier articles (see Compt. rend. 177, 507, 1923), Bohr's postscript to a 1925 article, in which he concedes the need for using the corpuscular theory of light propagation and to a new group of ideas, the first papers by Heisenberg, Born and Jordan, and Dirac on matrix mechanics and the mechanics of $q$-numbers (1925), the "first report" of Schrödinger's article on wave mechanics (early 1926), Born's paper on collision theory (middle of 1926), Heisenberg's paper on the uncertainty relations (spring 1927). Most of these articles (except those of de Broglie, Bohr, and Schrödinger) appear here for the first time in Russian translation. For lack of space, we could not include other ground-breaking papers by Bohr and Schrödinger, nor papers by Einstein, Pauli, and Fermi. These, incidentally, are well known to our physicists, and were published numerous times in Russian in this journal, in individual anthologies, and in selected collected works of these authors; the reader will find references to them in the bibliography of M. A. El'yashevich's article (p. 656). All the published translations were reviewed, and the terminology unified, by M. A. El'yashevich, to whom the Editorial Board is most grateful.

The articles of that time contain epistemological statements that are sometimes formulated in inadequate terminology, and are sometimes too hasty (e.g., that the causality law is not valid), and were in their time the cause of sharp polemics. We did not deem it necessary to comment on such statements, since these questions were repeatedly discussed in the Soviet literature, and the statements themselves in no way detract from the physical significance of these articles in the formation of quantum mechanics.

The Editors.

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# From the origin of quantum concepts to the establishment of quantum mechanics

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The article deals with the origins and development, in the first quarter of the 20th century, of the fundamental ideas advanced by Planck and Einstein and by Bohr and de Broglie concerning the quantum of action, the quanta of energy and the quanta of light, the stationary states of atomic systems and jumplike transitions between them, the correspondence principle, and the particle-wave dualism. It is shown how the development of the quantum concepts have led to two approaches to the creation of quantum mechanics by Heisenberg, Born, and Dirac in algebraic form and by Schrödinger in analytic form. The main stages in the evolution of nonrelativistic quantum mechanics are described, namely the development of wave mechanics and matrix mechanics, the proofs of their equivalence, the physical interpretation of the wave function, the generalized formulation of quantum mechanics as a unified theory, and the discovery of the uncertainty relations. The most important trends in the development of quantum mechanics in the course of its evolution are considered. A complete bibliography of the main period of evolution of quantum mechanics (July 1925–March 1928), classified by topics and by time of publication, is included.

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## 1. INTRODUCTION

A special place in the history of development of natural science is taken by the remarkable period from 1925 to 1928, when the fundamental laws of quantum mechanics were discovered and it became established unusually rapidly as a new science and as an independent, full-valued branch of theoretical physics. This period was preceded by one that spanned the first quarter of the 20th Century that was marked by the origin and vigorous growth of quantum concepts, which radically differed from the stately system of concepts of classical physics that took shape in the 19th Century. Planck and Einstein, Bohr and de Broglie advanced new fundamental ideas that denoted a genuine revolution in physics—ideas of the quantum of action, quanta of energy and quanta of light, stationary states of atomic systems and jumplike transitions between them, the correspondence principle, and the particle-wave dualism for radiation and microparticles. The new ideas did not fit into the customary framework of classical views. Yet the pictorial models of atoms and molecules that combined classical and quantum concepts, according to which the electrons move around the nuclei along definite classical trajectories (the orbits of Bohr's model theory) that satisfy supplementary quantum conditions (the Bohr–Sommerfeld conditions), were internally contradictory. Besides, though these models denoted a significant step forward, yet they did not allow one to explain the entire set of experimentally established properties of atoms and molecules. The essence remained unclear of the particle-wave dualism, which sharply contradicted all the classical views of both waves and particles taken separately. In spite of all the efforts of many eminent scientists, no one had succeeded before 1925 in constructing a thorough and internally consistent theory based on quantum ideas of the microscopic phenomena that occur at the atomic-molecular level. A very important problem in constructing such a theory in the nonrelativistic approximation was solved in a time starting in mid-1925 (Heisenberg's first study on quantum mechanics) to the end of 1927, while in early 1928 Dirac gave a relativistic generalization of the theory for the case of a single particle possessing spin: the electron. We can treat this as the culmination of the period of establishment of quantum mechanics.

This was a very eventful period. A mathematical apparatus was found that adequately describes the laws of microscopic phenomena in the nonrelativistic approximation, first algebraically (summer and fall of 1925) in the form of the discrete apparatus of matrix mechanics (Heisenberg, Born, Jordan) and the algebra of $q$-numbers (Dirac), and then analytically (early 1926) in the form of the apparatus of wave mechanics (Schrödinger), which operated with continuous quantities, and which proved especially effective for solving concrete problems. The mathematical identity of matrix mechanics and wave mechanics was demonstrated in the spring of 1926 (Schrödinger, and also Pauli and Eckart), and the unified theory began to develop rapidly that we now call quantum mechanics.[1)] The fundamental problem that arose was the correct physical interpretation of the mathematical apparatus of quantum mechanics. Born took a very important step in this direction when in the summer of 1926 he gave a probabilistic interpretation of the wave function and the coefficients of its expansion for states that represent a superposition of independent states. This probabilistic interpretation was immediately used by Dirac and Jordan, who independently developed at the end of 1926 a generalized mathematical formulation of quantum mechanics as a unified theory. In early 1927 Hilbert and von Neumann (with

[1)]In the period of establishment of the unified theory, the term "quantum mechanics" often referred only to its mataix aspect, while people used the term "wave mechanics" for its wave aspect; many scientists used the latter term for the unified theory even later, e.g., Schrödinger, de Broglie, Sommerfeld, and Ya. I. Frenkel'.

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the participation of Nordheim) carried out studies of the mathematical foundations of quantum mechanics (which were then continued by von Neumann). A further, highly essential step in the physical interpretation of the formalism of quantum mechanics was the discovery by Heisenberg in the spring of 1927 of the uncertainty relationships. The most general deeply fundamental problems of quantum mechanics were treated by Bohr, who had formulated the complementarity principle. These problems became the topic of the famous discussion of Bohr with Einstein that began in the autumn of 1927.

Along with the founding of nonrelativistic quantum mechanics, which was mainly completed in 1927, attempts were made to construct a relativistic quantum mechanics. In 1926 a set of scientists independently derived the relativistically invariant second-order wave equation. Yet it proved inapplicable to the electron as possessing spin (the hypothesis of the spin of the electron, which Uhlenbeck and Goudsmit had proposed in late 1925, and which permitted one to construct a complete systematics of atomic spectra, quickly became generally accepted). Finally, Dirac in early 1928 was able to develop a relativistic quantum theory of the electron. The existence of electron spin and of spin-orbital interaction automatically stemmed from its fundamental equation, the relativistically invariant first-order wave equation, or Dirac equation.

Very importantly, in the period of establishment of quantum mechanics, not only was its mathematical apparatus developed and given its physical interpretation, but it was also applied to various problems of the physics of microscopic phenomena: to the theory of atoms and the theory of molecules, to the theory of collisions and the theory of radiation. Different methods were developed for solving quantum-mechanical problems, exact and approximate. The paths were marked out for further development of quantum mechanics and its applications.

Quantum mechanics was successfully applied in mid-1926 to many-electron systems. The studies of Heisenberg and especially those of Dirac revealed the deep connection of quantum statistics with the quantum mechanics of a system consisting of identical microparticles, which must be treated as indistinguishable in quantum mechanics. The quantum statistics proposed in 1924 by Bose and Einstein proved to be the statistics of symmetric states, while the quantum statistics proposed in early 1926 by Fermi, who started with the exclusion principle that Pauli had formulated a year earlier, proved to be the statistics of antisymmetric states of these systems. Now people generally call the statistics of the first type the Bose–Einstein statistics, and the statistics of the second type, the Fermi-Dirac statistics. A quantum theory of radiation began to be developed, first on the basis of a classical treatment of the electromagnetic field and the application of the correspondence principle. Dirac's fundamental study on the theory of radiation in early 1927 was the starting point for the development of quantum electrodynamics. In this study, Dirac treated the atom and the radiation as a unified system, and first carried out the second quantization of the electromagnetic field.

The methods of the quantum-mechanical theory of perturbations proved highly effective for solving concrete problems. People also began to apply variational methods successfully. A quasiclassical method was developed as early as 1926 that permitted one to substantiate the Bohr–Sommerfeld quantum conditions that had been applied in the model theory. The development of methods based on accounting for the symmetry properties of quantum systems was of extremely important significance for solving both general and concrete problems (in particular, the permutational symmetry for systems consisting of identical microparticles). Here Weyl and Wigner successfully applied the mathematical group theory.

In order to understand the complex process of the establishment of quantum mechanics, we must first examine the development of quantum concepts, the rise of the new ideas in the preceding period that began with the studies of Planck.

## 2. FROM PLANCK TO QUANTUM MECHANICS

The last decade of the 19th Century was marked in physics by major discoveries: x-rays (1895), radioactivity (1896), and the electron (1897). The physics of microscopic phenomena began to grow rapidly, and the laws governing them could be explained only later on the basis of the new quantum concepts. As we know, these were first introduced by Planck at the end of the same decade, at the very boundary of the 19th and 20th Centuries, in developing the theory of thermal radiation.

Planck treated thermal radiation as resulting from emission and absorption of electromagnetic waves by matter, and he applied the laws of thermodynamics and classical electrodynamics.[1,3] In order to find the spectral distribution of the radiant energy occurring in equilibrium with matter at a definite temperature, or equilibrium radiation (black-body radiation or "black radiation"), Planck sought the maximum entropy of a system that consists of sets of "resonators" that serve as a model for the matter, which are harmonic oscillators having different frequencies $\nu$ (from 0 to $\infty$), and of electromagnetic waves of the corresponding frequencies. Here the oscillators of a given frequency emit and absorb electromagnetic radiation of the same frequency. Consequently, on the basis of the concepts of classical physics, and upon making certain assumptions about the form of the entropy of the resonators, Planck derived the Wien distribution law for the spectral intensity $E_\lambda$ of the equilibrium radiation, or the flux of radiation energy per unit interval of wavelength $\lambda$ per cm$^2$ per sec per unit solid angle in the form

$$E_\lambda = \frac{2c^2b}{\lambda^5} e^{-ac/\lambda T}, \qquad (1)$$

Here $T$ is the absolute temperature, $c = 3.00\times10^{10}$ cm/sec is the speed of light, and $a$ and $b$ are constants. Planck characterized these constants as *universal constants*, and calculated their numerical values from the experimental data:

$$a = 0.4818 \cdot 10^{-10} \text{ sec}, \quad b = 6.885 \cdot 10^{-27} \text{ erg-sec}. \tag{2}$$

The constant $b$, which has the dimensions of action, amounts to the famous Planck constant $h$, while the constant $a = b/k = h/k$ is a constant that is now generally called the Boltzmann constant (its value was obtained from (2) to be $k = 1.429 \times 10^{-16}$ erg/degree). Planck reported the results (1) and (2) on May 18, 1899 at a session of the Prussian Academy of Sciences in Berlin, and this is precisely when the universal constant $h$ first appeared. Later, in 1906, [11] Planck called it the *quantum of action*. Of course, the entire fundamental significance of this quantum constant was revealed only far later, and was especially clearly manifested in the foundation of quantum mechanics (see (29), (30), and (32) below).

Interestingly, even then in 1899, Planck proposed choosing the constant $b$ as a "natural unit" of measurement, along with the speed of light $c$, the gravitational constant $f = 6.685 \times 10^{-8}$ cm$^3$/g · sec$^2$, and the constant $a$ (to be sure, only later in 1902 did Planck turn his full attention to the dependence of the constant $k = h/a$ on the choice of temperature scale[309]). We note also that even then, [1] Planck introduced the concept of an elementary length composed from the constants $b$, $c$, and $f$, which people currently apply in quantum-gravitational constructs ($\sqrt{bf/c^3} = \sqrt{hf/c^3} = 4.13 \times 10^{-33}$ cm).

However, Wien's law did not agree at long wavelengths with the experimental data, according to which the temperature-dependence of the radiation $E_\lambda$ at high temperatures is approximately linear. In the autumn of 1900, Planck was able to find[4] by a semiempirical method (by generalizing the expression for the entropy of the resonators of a given frequency from which Wien's law was derived) the following spectral distribution law:

$$E_\lambda = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T} - 1}, \tag{3}$$

where $c_1$ and $c_2$ are constants. This law goes over at short wavelengths into Wien's law, while at long wavelengths it leads to a proportionality to the absolute temperature (to the Rayleigh–Jeans law, [2,8,9] which Rayleigh first derived in early 1900 by starting with a uniform energy distribution over the degrees of freedom; however, Rayleigh's work was not then known to Planck). Planck reported Eq. (3) (which has subsequently been termed the Planck equation) on Oct. 19, 1900 at a meeting of the German Physical Society, and it proved to agree with experiment both for short and long wavelengths. "But a very important problem in the theoretical sense remained": as Planck later recalled, [294] "to give an appropriate justification for this law, and this was an incomparably harder problem, since here the problem was to derive theoretically an expression for the entropy of an oscillator..." And Planck solved this problem[5] by going from a thermodynamic to a statistical treatment of entropy and using the Boltzmann method, which related the entropy to the probability $W$. Planck assumed the entropy $S_N$ of a system of $N$ resonators having a total vibrational energy of $U_N$ to be

$$S_N = k \ln W + \text{const}; \tag{4}$$

Here $k$ is a proportionality coefficient, which Boltzmann himself had not introduced[294] (in introducing this coefficient, Planck immediately denoted it as $k$[5]). Since, in order to find the probability $W$, "one must imagine $U_N$ not in the form of a continuous, infinitely divisible quantity, but in the form of a discrete quantity consisting of an integral number of finite equal parts,"[6] Planck made the fundamental assumption that

$$U_N = P\varepsilon, \tag{5}$$

where $\varepsilon$ is the "element of energy," while $P$ is a very large integer. The $P$ elements of energy $\varepsilon$ can be distributed in various ways over the $N$ resonators. Hence an individual resonator cannot have any given energy, but only an energy that is a multiple of $\varepsilon$. Thus, as applied to resonators, i.e., harmonic oscillators, the idea first appeared of *discreteness* of energy, or the idea of *quantizing* energy. Planck calculated the probability $W$, and according to (5), found the mean entropy $S = S_N/N$ as a function of the ratio $U/\varepsilon$ of the mean energy of a single resonator $U = U_N/N$ to the element of energy $\varepsilon$ in the form

$$S = k\left[\left(1 + \frac{U}{\varepsilon}\right) \ln\left(1 + \frac{U}{\varepsilon}\right) - \frac{U}{\varepsilon} \ln \frac{U}{\varepsilon}\right]. \tag{6}$$

On the other hand, Planck showed that the Wien displacement law can be represented in the form

$$S = f\left(\frac{U}{\nu}\right), \tag{7}$$

where $f$ is an arbitrary function, and comparison of (6) and (7) shows that

$$\varepsilon = h\nu, \tag{8}$$

where $h$ is a universal constant that coincides[7] with the constant $b$ that Planck had previously derived, and which has the dimensions of action. With $\varepsilon = h\nu$, one gets (see Ref. 6) from Eq. (6) the mean energy of a single resonator:

$$U = \frac{h\nu}{e^{h\nu/kT} - 1}. \tag{9}$$

As Planck showed, the volume energy density of the equilibrium radiation (per unit frequency interval) equals

$$u_\nu = \frac{8\pi\nu^2}{c^3} U, \tag{10}$$

and finally one gets Planck's formula

$$u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}. \tag{11}$$

Upon taking $cu_\nu/4\pi = E_\nu$, and $cE_\nu/\lambda^2 = E_\lambda$, it agrees with Eq. (3)[5] (where $c_1 = 2hc^2$, and $c_2 = hc/k$).

Planck found values of the constants $h$ and $k$ that were refined in comparison with (2) by comparison with experiment:

$$h = 6.55 \cdot 10^{-27} \text{ erg-sec},$$
$$k = 1.346 \cdot 10^{-16} \text{ erg/deg}, \qquad (12)$$

He calculated from them the value of the elementary electric charge (i.e., the charge of an electron). This was $e = 4.69 \times 10^{-10}$ cgs esu, which is close to the modern value.

People generally consider the day Dec. 14, 1900, when Planck reported his new and especially important results at the meeting of the German Physical Society[5] to be the birth date of quantum ideas. The *quantum of energy* appeared as a discrete portion of energy (we note that Planck himself used the term "element of energy" and only later did people begin to talk of "quanta of energy"). Planck's resonators could possess only energies that are multiples of $h\nu$. That is, it implied a saltatory nature of the elementary events of emission and absorption of electromagnetic radiation, in sharp contradiction to classical electrodynamics. Here the discreteness of the energy proved to be inseparably tied to the constant $h$, the quantum of action.

A characteristic feature of Planck's approach was that he attributed the discreteness of the energy to the properties of *matter*. His theory quantized the energy of the resonators, while he treated the radiation as continuous, as consisting of electromagnetic waves. Moreover, Planck had not yet posed concretely the problem of the elementary events of emission and absorption of individual portions of electromagnetic energy $h\nu$, for he was primarily interested in solving the problem of the spectral distribution of the energy of equilibrium radiation. Einstein took the extremely important next step in the development of quantum concepts in March, 1905 in the paper, "On a Heuristic Viewpoint Concerning the Production and Transformation of Light." He advanced the idea of the discreteness of the electromagnetic radiation itself—the hypothesis of *quanta of light*—and examined the elementary processes of absorption and emission of these quanta. [10] Einstein's fundamental idea is very clearly expressed in the introduction to this paper: "The wave theory of light, which operates with continuous space functions, has been excellently justified in describing purely optical phenomena, and probably will never be replaced by another theory. Yet we must remember that optical observations pertain not to instantaneous, but to time-averaged quantities. Therefore, in spite of the full confirmation by experiment of the theory of diffraction, reflection, refraction, dispersion, etc., one can imagine that a theory of light that operates with continuous space functions will lead to contradiction with experiment when people apply it to phenomena of production and transformation of light.

It actually seems to me that experiments concerning "black radiation," photoluminescence, production of cathode rays upon irradiating with ultraviolet and other groups of phenomena pertaining to the production and transformation of light seem more understandable on the basis of the assumption that the energy of the light is distributed spatially in a discrete manner. According to this hypothesis made here, when a light ray that has originated at some point propagates, the energy is not distributed continuously over ever larger volumes, but is composed of a finite number of energy quanta localized at individual points of space, which move as indivisible units and are absorbed or produced only as a whole."[2)]

The two sides of Einstein's hypothesis on light quanta proved highly essential in the development of quantum concepts. First, there was the fundamental view itself of radiation that consists of indivisible and localized quanta of energy, which corresponds to the corpuscular theory of light, and second, the discreteness of the elementary processes of production and transformation of light that stems from this view. Einstein discussed both of these aspects in his paper, and we shall briefly discuss it (for further details, see Klein's article[302]).

In the first part of the paper Einstein substantiated the corpuscular approach to radiation. He found the entropy of the radiation as a function of the volume by starting with Wien's law, i.e., for large values of the ratio $\nu/T$, and compared it with the entropy of an ideal gas or a dilute solution. Einstein writes Wien's law for the density of radiation energy in the form

$$\rho = \alpha\nu^3 e^{-\beta\nu/T}, \qquad (13)$$

which is obtained from Planck's law (11) for large values of $\nu/T$ (here $\alpha = 8\pi/c^3$, $\beta = h/k$, $\rho = u_\nu$; in this study, Einstein hadn't yet introduced the constants $h$ and $k$ as Planck had. He wrote $R/N$ in place of $k$, where $R$ is the gas constant and $N$ is Avogadro's number, and $\beta R/N$ in place of $h$).

By using thermodynamic relationships, Einstein found from Eq. (13) an expression for the entropy $S$ of the radiation occupying the volume $v$,

$$S - S_0 = \frac{E}{\beta\nu} \ln \frac{v}{v_0}, \qquad (14)$$

Here $E$ is the total energy of the radiation, and $S_0$ is the entropy of the radiation in the volume $v_0$. This expression has the same form as for the entropy of an ideal gas or a dilute solution containing $n$ particles:

$$S - S_0 = R\frac{n}{N} \ln \frac{v}{v_0} = \frac{R}{N} \ln \left(\frac{v}{v_0}\right)^n = \frac{R}{N} \ln W. \qquad (15)$$

Here $W = (v/v_0)^n$ is the probability that all $n$ of the moving particles will collect in a volume $v$ that constitutes a fraction of the total volume $v_0$. Comparison of (14) and (15) shows that $n = NE/R\beta\nu$. That is, "Monochromatic radiation of low density (within the limits of applicability of Wien's radiation law) behaves in the sense of heat theory as though it consists of mutually independent quanta of energy of size $R\beta\nu/N$"[10] (i.e., $h\nu$—M. E.). Thus Einstein arrived at a corpuscular theory of light. Further development of this theory of light quanta led him to the idea of the particle-wave dualism, which de Broglie in turn extended to microparticles, i.e., to matter (see, e.g., Ref. 303).

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[2)]In translating this and a number of the following citations, the author has made some revisions.

Einstein concludes the first part of his paper thus: "Yet if monochromatic radiation (of low enough density) behaves in the sense of the volume-dependence of the entropy like a discrete medium that consists of quanta of energy of size $R\beta\nu/N$, then the question suggests itself of whether the laws of production and transformation of light are of a type as though light consists of similar energy quanta."[10] In the second part of the paper, Einstein explains Stokes' law as resulting from emission of light quanta of frequency $\nu_2$, which is smaller than or equal to the frequency $\nu_1$ of the absorbed light quantum (in Einstein's notation, $R\beta\nu_2/N \leq R\beta\nu_1/N$, i.e., $h\nu_2 \leq h\nu_1$, and $\nu_2 \leq \nu_1$). Further, he examined the "excitation of cathode rays upon illuminating solids" and gave his famous equation for the photoelectric effect,[3)] according to which the kinetic energy of the electrons ejected by the light quantum is equal to $h\nu - P$ (in Einstein's notation $R\beta\nu/N - P$), where $P$ is the work function. Finally, Einstein discusses the "ionization of gases by ultraviolet light," while assuming that "each absorbed quantum causes the ionization of one gas molecule." This amounts to the original formulation of the law of photochemical equivalence (which Einstein also treated later from a thermodynamic standpoint[24]).

Einstein's concrete analysis of the elementary processes of absorption and emission played a great role in the development of the quantum theory of atoms and molecules. In combination with the idea of quantizing the energy of the microparticles, Bohr applied the idea of the discreteness of these processes. In his first article on the theory of the atom[30] Bohr refers to Einstein's paper and writes: "Einstein was the first to point out the universal significance of Planck's theory for discussing the behavior of atomic systems." Here Bohr meant precisely Einstein's ideas on the discreteness of elementary processes of interaction of atomic systems with radiation. We must stress that these ideas of Einstein have been developed in the studies of other scientists, in particular Stark, Nernst, and Sommerfeld, whereas the view of the corpuscular structure of radiation itself—the view of light quanta as being particles—was not widely acknowledged for a long time. Thus, in 1913 in recommending Einstein as a member of the Prussian Academy of Sciences in Berlin, Planck, Nernst, Rubens, and Warburg wrote in the pertaining presentation, after enumerating Einstein's scientific services, "The fact that in his discussions he sometimes misses the goal, as for example, in his hypothesis of light quanta, should not put him too much in the wrong. For without deciding to take a risk, one can't create anything new, even in the most exact natural science."[296] And even considerably later, in 1922, in his study on the fundamental postulates of the quantum theory,[42] Bohr wrote on the hypothesis of light quanta that it "cannot in any way be considered to be a satisfactory solution. As we know, precisely this hypothesis leads to insurmountable difficulties in explaining the phenomena of interference, which present the fundamental means of studying the properties of radiation." And Bohr recanted this viewpoint only in June, 1925 (see his epilogue to the article "On the Action of Atoms in Collisions."[55] This was after the results of Bothe and Geiger (who had proved the validity of the laws of conservation of energy and momentum in the elementary process in the Compton effect), and immediately before the foundation of quantum mechanics (for more details, see Ref. 311).

We must emphasize that ideas on the discreteness of radiation energy began to be developed also in another form besides Einstein's views on the corpuscular structure of radiation. First Ehrenfest in 1906,[12] and then Debye in 1910[17] applied these ideas to the eigenoscillations of equilibrium radiation in a cavity (the number of which per unit frequency interval is $8\pi\nu^2/c^3$. Given a uniform distribution of the energy over the vibrational degrees of freedom, with $kT$ for each of them, this leads to the Rayleigh–Jeans law $u_\nu = 8\pi\nu^2 kT/c^3$, which one gets from Planck's law (11) when $h\nu \ll kT$). According to Ehrenfest and Debye, the energy of each eigenoscillation of the electromagnetic field is quantized, and must be a multiple of $\varepsilon = h\nu$, just as for a Planck resonator. Debye started with this assumption and determined the mean energy per oscillation of frequency $\nu$ by the probabilistic method (analogous to the method that Planck had used in determining the total vibrational energy $U_N$ distributed over $N$ resonators; see (4)–(6)). This energy proved to be equal to the mean energy of one resonator; see (9). Thus he was able to derive Planck's law (11). The ideas of this approach, the ideas of *quantizing the electromagnetic field*, gave rise to quantum electrodynamics.[298] Yet the connection between these ideas and Einstein's hypothesis of light quanta was established only much later in the foundation of quantum mechanics, when the essence of the particle-wave dualism was discovered.

Starting in 1905–1906, quantum views became widespread, and an ever greater number of scientists began to participate in developing them. In particular, the topic of the 1st Solvay Congress on Physics, at which almost all of the leading physicists of the time had gathered, was "The Theory of Radiation and Quanta."[301,324] The development of quantum concepts followed several different though interrelated lines that stem from Planck and Einstein. These lines were based on ideas of *quantization of the energy of matter*, on ideas of *discreteness of processes of interaction of matter and radiation*, and on ideas, on the one hand, of *discreteness of radiation* in the form of the *hypothesis of light quanta*, and on the other hand, in the form of the hypothesis of the *quantization of the energy of the eigenoscillations of the electromagnetic field.* It was precisely the development (to a considerable extent independent) of quantum concepts along different lines that later led to the two different approaches in founding quantum mechanics—to the *approach based on the ideas of the correspondence principle* (Bohr, Heisenberg), and to the *approach based on the ideas of the particle-wave dualism* (Einstein, de Broglie, Schrödinger). There is no opportunity here to describe in detail this develop-

3)This paper[10] is often referred to even now as "Einstein's paper on the photoelectric effect," which is incorrect since it doesn't reflect the main point in this paper.

ment, and we shall examine in general outline only some of its most essential stages.

A number of important papers were concerned with problems of quantizing the energy of matter:

In the first edition of his "Lectures on the Theory of Heat Radiation,"[11] Planck in 1906 treated the quantization of a resonator (harmonic oscillator) as resulting from the partitioning in phase space of elementary regions that are equal in size to the quantum of action $h$. For an oscillator on the $(pq)$ plane ($q$ and $p$ are the generalized coordinates and the momentum), the area between two ellipses corresponding to the energy values $E$ and $E+\varepsilon$ will be

$$\int dq\, dp = h \qquad (16)$$

(see Planck's report "Laws of Heat Radiation and the Hypothesis of the Quantum of Action" at the First Solvay Congress[22] 4)), and we can write the condition for quantizing the energy $E/\nu = nh$ in the form

$$\int p\, dq = nh. \qquad (17)$$

Here the *phase integral* is taken over the area of an ellipse that corresponds to the energy value $E = nh\nu$, while $n$ is an integer, which people later began to call the *quantum number*. Subsequently the condition (17) was generalized to the case of many degrees of freedom, and conditions were formulated for quantizing multiply periodic systems (they are usually called the Bohr–Sommerfeld quantum conditions) in the form[39]

$$\int p_k\, dq_k = n_k h. \qquad (18)$$

Here $q_k$ and $p_k$ are the generalized coordinates and momentum for the $k$th degree of freedom, and $n_k$ is the corresponding quantum number.

A very important step was Einstein's application in late 1906 of the law of quantization of a harmonic oscillator in the theory of heat capacity. In his study "Planck's Theory of Radiation and the Theory of Specific Heat,"[13] Einstein, following Planck, treated oscillators having the quantized energies 0, $\varepsilon$, $2\varepsilon \ldots$ (where $\varepsilon = R\beta\nu/N$, i.e., $\varepsilon = h\nu$). He very simply found the mean energy of such an oscillator by starting with the Boltzmann distribution and taking the probability $dW$ of energy values $E$ of the oscillator lying between $E$ and $E + dE$ in the form

$$dW = Ce^{-(N/RT)E}\omega(E)\, dE. \qquad (19)$$

4)We note that Planck tried to reject the discontinuity of the absorption process[20] and then that of the emission process[34] by averaging over phase space, but these attempts were not successful. The zero-point energy $h\nu/2$ of vibrations appeared for the first time in Refs. 20 and 22, but it is derived in a natural way only on the basis of quantum mechanics (see pp. xxx and xxx.)

Here $C$ is a function of $T$. Its mean equals

$$\bar{E} = \frac{\int Ee^{-(N/RT)E}\omega(E)\, dE}{\int e^{-(N/RT)E}\omega(E)\, dE}. \qquad (20)$$

Whereas the classical theory takes $\omega(E) = \text{const.}$, Einstein assumed that $\omega$ differs from zero only from 0 to $0+\alpha$, from $\varepsilon$ to $\varepsilon+\alpha$, from $2\varepsilon$ to $2\varepsilon+\alpha$, etc., where $\alpha$ is infinitesimally small in comparison with $\varepsilon$, and that

$$\int_0^{\alpha} \omega\, dE = \int_{\varepsilon}^{\varepsilon+\alpha} \omega\, dE = \int_{2\varepsilon}^{2\varepsilon+\alpha} \omega\, dE = \ldots = A. \qquad (21)$$

Equations (20) and (21) imply that

$$\bar{E} = \frac{0 + A\varepsilon e^{-(N/RT)\varepsilon} + A\cdot 2\varepsilon e^{-(N/RT)2\varepsilon} + \ldots}{A + Ae^{-(N/RT)\varepsilon} + Ae^{-(N/RT)2\varepsilon} + \ldots} = \frac{\varepsilon}{e^{(N/RT)\varepsilon} - 1}, \qquad (22)$$

i.e., we get the Planck expression (9) (with $R/N = k$, $\varepsilon = h\nu$). Thus Einstein was the first to apply the Boltzmann distribution to quantum states, as has become so customary for us. By using Eq. (10), he immediately derived Planck's formula (11). Then Einstein applied Eq. (21) to the atoms of a solid, treating each of them as an oscillator that oscillates at frequency $\nu$, and which has three degrees of freedom. For $N$ atoms, one gets the energy

$$\frac{3N\varepsilon}{e^{(N/RT)\varepsilon} - 1} = 3R\frac{\beta\nu}{e^{\beta\nu/T} - 1} \qquad (22)$$

(with account taken that $N\varepsilon = R\beta\nu$). Its derivative with respect to $T$ gives the heat capacity (per gram-atom) of the atoms of a given type:

$$3R\frac{e^{\beta\nu/T}(\beta\nu/T)^2}{(e^{\beta\nu/T} - 1)^2}. \qquad (23)$$

This heat capacity approaches zero as $T \to 0$. This study of Einstein's started the development of the quantum theory of heat capacity. An essential point is that Einstein applied the law of quantization of a harmonic oscillator, but not to an abstract model of a resonator, as Planck had done. (The latter had developed the theory of emission and absorption "without at all becoming interested in the internal structure of the resonator. For example, it is not at all essential whether the oscillations of the elementary resonators are based on conduction currents or on convection currents...".[11]) Rather Einstein applied it to a concrete, though very crude model of atoms that vibrate in the solid about equilibrium positions. Debye[27] and independently Born and von Kármán[28,29] quantized the oscillatory motion for more complex models of a solid in 1912 (Born later concerned himself greatly with the dynamics of crystal lattices, and carried out a number of fundamental studies in this field; see Ref. 304).

Along with quantizing the vibrational motion, a number of studies have also quantized the energy of rotational motion, the necessity of which Nernst had pointed out in 1911[18] (see also Ref. 309). Nernst and Lindemann[19] quantized the energy of vibrations for molecules in connection with the theory of heat capacity at low temperatures, and Bjerrum[25] did this in connection with studying molecular spectra. Ehrenfest[31] also treated the quantization of the angular momentum of a molecule in connection with the theory of heat capacity of diatomic gases according to the formula $M = \pm (h/2\pi)n$ $(n = 1, 2, 3, \ldots)$. This gives $M^2 = n^2(h^2/4\pi^2)$ and $E_n = n^2h^2/8\pi^2L$, where $L$ is the moment of inertia of the molecule.

In 1910 Haas[16,309] tried to apply quantization to the energy of an atom, and in 1912 Nicholson[26] quantized the angular momentum of an electron in an atom.

Thus ideas of quantizing the internal motion of atoms and molecules became widespread. Yet an essential feature when the topic dealt with discrete emission and absorption processes was that people assumed that the frequency of the emitted or absorbed radiation (quanta of energy $h\nu$) coincides with the frequency of motion in the atomic system. The equality of these frequencies (which holds only for a harmonic oscillator) was assumed obvious for all cases.

A very important stage on the path of foundation of the theory of microscopic phenomena was set by Bohr's study in 1913,[30] in which quantum concepts, or ideas on quantizing the energy of matter and on the discreteness of emission and absorption, were applied to atoms and molecules, primarily the hydrogen atom as being the simplest system. They were supplemented in a very substantial way by new ideas on stationary states and frequencies of quantum transitions that differ from the frequencies of motion in the system. Bohr started with the model of the atom that Rutherford had proposed in 1911, which consists of the nucleus and electrons surrounding it (which would have been unstable according to the classical laws, owing to the energy losses of the accelerated electrons). He based his theory of atomic systems on the famous two postulates that bear his name. We shall present Bohr's postulates in the formulation that Bohr later[278] gave (he refined his formulations over a span of years). According to the first postulate, "an atomic system is stable only for a certain set of states, which in the general case corresponds to a discrete sequence of values of the energy of the atom. Each change in this energy involves the complete transition of the atom from one stationary state to another." In this postulate, the idea of quantizing the energy of the atomic system is combined with the idea of *stationary quantum states*. According to the second postulate, "the ability of an atomic system to absorb and emit radiation obeys a law whereby the radiation involved in the transition must be monochromatic and have the frequency $\nu$ defined by the following relationship (the frequency condition):

$$h\nu = E_i - E_k. \tag{24}$$

Here $h$ is Planck's constant, and $E_i$ and $E_k$ are the energies of the corresponding stationary states." In this postulate, the idea of the discreteness of the emission and absorption processes is combined with the idea of a *frequency of the quantum transition that differs from the frequencies of motion in the stationary states* having the energies $E_i$ and $E_k$. Together with Bohr's model theory, which treats the motion in the stationary states by using classical mechanics, while the possible values of the energy and of other physical quantities are found by using the quantum conditions, Bohr's postulates permitted people to interpret an enormous bulk of experimental material, especially the regularities in spectra, and to give a physical explanation of the periodic law of the chemical elements (see, e.g., Ref. 313). Atomic physics began to develop rapidly. As early as 1919, Sommerfeld, who had contributed greatly to its development along with Bohr himself, published the first edition of his monograph "Structure of Atoms and Spectral Lines," which went through three more editions (each time revised) over the next five years.[39]

The importance, and withal the difficulty, of the decisive step that Bohr took is indicated by the evaluation of Bohr's theory by Rutherford and Einstein. Rutherford later (in 1931) said[293]: "I consider the original quantum theory of spectra that Bohr proposed to be one of the most revolutionary of any of those ever created in science, and I know no other theory that would have had greater success." And Einstein wrote in 1949,[295] "It has always seemed to me a wonder that there proved to be enough of this fluctuating and contradiction-filled basis (he is discussing quantum phenomena—M. E.) to permit Bohr, a man with the intuition of a genius and with fine sensitivity, to find the major laws of spectral lines and the electron shells of the atom, including their value for chemistry. This seems a wonder to me even now. This is the highest musical talent in the realm of thought."

However, apart from the very great successes of Bohr's theory and Bohr's postulates, which were confirmed on all sides by vast experimental material (in particular, they were directly confirmed by the experiments of Franck and Hertz), they had no theoretical justification, while Bohr's model theory artificially imposed quantum conditions on the motion of the electrons according to classical laws in the stationary states. The theory did not give correct results even for such two-electron systems as the helium atom and the hydrogen molecule. Bohr himself well understood the unsatisfactory nature of the model theory, when in his paper in late 1913 he said[33] while applying classical mechanics for the stationary states: "If we wish in general to create pictorial representations of the stationary states, we have no other means, *at least yet* (my italics—M. E.), besides ordinary mechanics." In closing the paper, Bohr said: "Before concluding, I wish only to voice the hope of expressing myself so clearly that all would understand how sharply the presented ideas contradict the strikingly harmonic set of concepts that are justly called classical electrodynamics. On the other hand, I have tried to have you gain the impression that it is precisely by stressing the stated contradiction that perhaps in time we can introduce some coherence into the new concepts." And during the following years, Bohr persistently sought the ways to solve the contradictions. The main line that he took, and which was one of the approaches in the founding of quantum mechanics that was realized in 1925 was that based on the ideas of the *correspondence principle*. Bohr applied the idea that the results of the classical and the quantum theory should coincide in the limiting case of low radiation frequencies (where the Planck law goes over into the Rayleigh–Jeans law) even in the first part of his work in 1913 by assuming that the frequency $\nu$ of a quantum transition between adjacent circular orbits of large radius in the hydrogen atom (corresponding to large quantum numbers $\tau$ and $\tau+1$) coincides with the frequency $\omega$ of revolution of the electron in these orbits. This is just how he was able to find in a logically most consistent way the expression for the Rydberg constant $R$ in terms of the elementary charge, the mass of the electron, and Planck's constant $h$, and to show as a consequence that one obtains the quantization condition of the angular momentum for the circular orbits in the form

$$M = \frac{h}{2\pi}\tau, \tag{25}$$

with the quantum number $\tau = 1, 2, 3, \ldots$

While developing the idea of the correspondence between the classical and quantum theories, Bohr in 1918 formulated the general principle of correspondence between classical motion in the atom and radiation in his paper "On the Quantum Theory of Line-Spectra."[38] In 1920 in his paper "On the Series Spectra of the Elements." Bohr wrote[40]: "...the process of emission involving transition from one state to another cannot be followed in detail by using ordinary electromagnetic concepts. The emission properties of the atom from the standpoint of these concepts arise directly from the motion of the system and the resolution of these motions into harmonic components. Yet it has turned out that there is a far-reaching *correspondence* between the different types of possible transitions from one stationary state to another, on the one hand, and the different harmonic components of the resolution, on the other. Thus the discussed theory of spectra can be considered to some degree to be a generalization of the ordinary theory of emission." We shall not examine here in detail the further development of the correspondence principle by Bohr and his associates (in particular, as applied to the theory of dispersion by Kramers, and then by Kramers and Heisenberg), nor the attempt by Bohr (together with Slater and Kramers) to reject the laws of conservation of energy and momentum in elementary processes.[5)] We stress only that an especially important point was that they did not correlate the classical motion in the stationary states with the frequencies of this motion. Rather, they correlated it for each pair of states with the frequency of the quantum transition according to the frequency condition (24). Here the intensity of emission is determined by the corresponding harmonic components in the expansion of the electric moment. We note also the connection of the correspondence principle with the adiabatic principle[32,35] that Ehrenfest had formulated, according to which there are quantities (adiabatic invariants) that do not change during a very slow (adiabatic) change in the parameters of the system. For example, for a harmonic oscillator in the quantum theory, the ratio $E/\nu = J = nh$ is conserved upon adiabatically varying the frequency $\nu$. Bohr repeatedly emphasized this connection (e.g., in Ref. 42), while Ehrenfest discussed it in Ref. 45.

Bohr's profound ideas on the theory of the atom, and the approach based on the correspondence principle proved to be the decisive influence on Werner Heisenberg. Heisenberg later wrote concerning his conversation with Bohr during a walk in June 1922[6)] [310]: "This walk exerted the strongest influence on my subsequent scientific development, or perhaps, to say it better, my true scientific development only began with this walk." In applying the correspondence principle in his first study on quantum mechanics in the summer of 1925, Heisenberg developed Bohr's ideas.[61]

We can trace the other approach to quantum mechanics, which was realized by Schrödinger, and which involved the idea of the particle-wave dualism, from Einstein's first study[10] on quantum theory. Einstein developed ideas on light quanta in the 1909 papers, "On the Present State of the Radiation Problem"[14] and "On the Development of Our Views on the Essence and Constitution of Radiation."[15] Upon starting with Planck's law (11) for the density of radiation (rather than Wien's law (13), as in the first paper), Einstein calculated the mean value $\overline{\varepsilon^2}$ of the square of the fluctuation of the energy of the radiation:

$$\overline{\varepsilon^2} = h\nu\eta_0 + \frac{c^3}{8\pi\nu^2\,d\nu}\,\frac{\eta_0^2}{v}, \tag{26}$$

Here $\eta_0$ is the mean energy in the volume $v$. That is, it equals the sum of two terms. The second term is inversely proportional to $\nu^2$, and it corresponds to the fluctuations according to the wave theory (owing to interference); it is the major term at low frequencies. Yet the first term corresponds to the fluctuations according to the corpuscular theory (fluctuations in the number of quanta of light), and it is the major term at high frequencies (then, if we set $\eta_0 = h\nu\cdot n$, and $\varepsilon/h\nu = \Delta n$, we have $\overline{\Delta n^2} = n$, i.e., we get the usual formula for the fluctuations $\Delta n$ in the number $n$ of particles). Thus Eq. (26) reflects the dual nature of radiation or the *particle-wave dualism*. Einstein also derived an analogous formula for the mean square $\overline{\Delta^2}$ of the fluctuation of the light pressure (in the time $\tau$) on a mirror of area $f$ that ideally reflects the radiation in the frequency interval $d\nu$ (from $\nu$ to $\nu + d\nu$) and is ideally transparent for other frequencies:

$$\frac{\overline{\Delta^2}}{\tau} = \frac{1}{c}\left(h\nu\cdot\rho + \frac{c^3}{8\pi\nu^2}\rho^2\right)d\nu\cdot f, \tag{27}$$

Here $\rho$ is the density of the radiation of frequency $\nu$ ($\rho = u_\nu$). The first term of this formula, which corresponds to the corpuscular theory, is proportional to $h\nu/c$. This corresponds to the momentum of an individual quantum of light, which is $h\nu/c$, of which Einstein began to speak explicitly only later in 1916.[37] Yet is essential that as early as 1909 Einstein was treating the momentum of

5)On this, see the article by Mehrs (CERN 76-10, 14 May, 1976, Gevena), and Refs. 36–39, 45, 47, and 48 in his article.

6)See Mehra's article, cited in Footnote 5.

radiation, along with its energy. He emphasized[15]: *"In addition to the spatial inhomogeneities in the distribution of momentum of radiation that arises from the wave theory, there are other inhomogeneities which at low energy density of the radiation greatly exceed the inhomogeneities first mentioned."*

Einstein paid especial attention to problems of the quantum theory of radiation also in his paper at the first Solvay Congress in 1911.[23] Ehrenfest carried out an important analysis of the quantum theory of radiation in the same year 1911 in his paper "What Features of the Hypothesis of Light Quanta Play an Essential Role in the Theory of Thermal Radiation?."[21]

In 1916 Einstein published some very important papers on the quantum theory of radiation.[36,37] Here he introduced the coefficients $A$ and $B$ that bear his name, which characterize the probabilities of spontaneous and stimulated quantum transitions (both in absorption and in emission). He treated the equilibrium of radiation and matter, and he derived Planck's formula (11). This "derivation of Planck's formula according to Einstein" is well known and is presented in many textbooks. Yet in addition, the article contained a very important treatment of the directionality of radiation. This is precisely where Einstein introduced explicitly the momentum $h\nu/c$ of a light quantum as a result of treating the motion of molecules in the radiation field and determining the mean square $\overline{\Delta^2}$ of the momentum imparted to the molecules. Upon pointing out that "the general quantum hypothesis for matter" implies Bohr's frequency condition (24) and Planck's formula (11), he wrote: "Yet, as I see it, the most important point is the conclusion concerning the momentum that is imparted to the molecule in spontaneous and stimulated emissions." The conclusion of the article is especially important: "If the molecule loses energy without external excitation (spontaneous emission), then this process is also *directional*. Spontaneous emission in the form of spherical waves does not exist. In the elementary process of spontaneous emission, the molecule receives a recoil momentum in the amount of $h\nu/c$, while according to the current status of the theory, the direction is determined only by "chance." These properties of the elementary process, which are required by the relationship (12) (for $\overline{\Delta^2}$—M. E.), make the creation of a genuine quantum theory of radiation almost unavoidable. The weakness of the theory consists in the fact, on the one hand, that it does not lead us to a closer union with the wave theory, and on the other hand, that the time and the direction of the elementary process are up to "chance": yet I am fully convinced of the reliability of the chosen method." Einstein's statement corresponds to the particle-wave dualism ($E=h\nu$ and $p=h\nu/c=h/\lambda$) and to the treatment of the electromagnetic field as a "phantom field" or "pilot field."[107] Einstein clearly understood the probabilistic nature of the relationship between the wave and the particle theories of radiation.

A very essential argument favoring the correctness of Einstein's views on light quanta having the energy $E=h\nu$ and the momentum $p=h\nu/c$ was the natural explanation found in the autumn of 1922 by Compton[41] of the effect (which has been named after him as the Compton effect) of scattering of x-rays by electrons with a change in wavelength of these rays. This explanation was advanced by Compton himself[43] and independently by Debye,[44] and it was based on treating the elementary process of a collision of the light quantum with a free electron that fulfills the laws of conservation of energy and momentum.

A very important stage in the development of quantum concepts was the three studies by de Broglie in 1923[46–48] and his dissertation in 1924,[52] in which the particle-wave dualism for radiation was extended to matter and to its particles (on de Broglie's studies, see Refs. 299 and 328). Even in the first of these studies, de Broglie compared a particle to a wave of definite phase velocity, and by starting with wave representations, he derived Bohr's condition (25) for quantization of circular orbits. The quantum condition had been derived in a natural way for the first time.

De Broglie generalized the results of the first three studies in the article "A Tentative Theory of Light Quanta," which was published in *Philosophical Magazine*.[49] De Broglie tried to treat the light quantum as a particle having a rest mass $m_0$ differing from zero, which was incorrect. Yet his fundamental idea of a wave that is to be correlated with a particle proved to be unusually fruitful. The famous *de Broglie wavelength* $\lambda=V/\nu$ (where $V$ is the phase velocity of the wave) appeared for the first time in explicit form in this article of his and in his dissertation; it is determined by the formula

$$\lambda=\frac{h}{p}=\frac{h\sqrt{1-\beta^2}}{m_0 v} \tag{28}$$

(where $\beta=v/c$) that relates this wavelength to the velocity $v$ of the particle.

The fundamental importance of de Broglie's studies was first of all evaluated by Einstein in Ref. 53, which was devoted to a new quantum statistics first proposed by Bose in 1924 for light quanta[50] and immediately applied by Einstein to atoms[51] (as we know, this statistics has become called the Bose–Einstein statistics). In this article, Einstein, following de Broglie (about whose dissertation he wrote that "it deserves every attention"), used the correlation of the wave field to the particle, and examined the fluctuation of this field. After this article by Einstein, Schrödinger also paid attention to de Broglie's work (see, e.g., Refs. 303 and 327); he published an important article "On Einstein's Gas Theory,"[58] in which he applied de Broglie's ideas, and then his ground-breaking first studies on wave mechanics appeared. Thus the second approach to quantum mechanics was realized.

## 3. THE SITUATION AT THE TIME OF CREATION OF QUANTUM MECHANICS

The successful study by physicists of varied microscopic phenomena, optical, magnetic, and electric, and the accumulation of extensive experimental material combined with the development of fruitful quantum concepts had prepared the conditions for the creation of a coherent theory of these phenomena—quantum mechanics. Its establishment resulted from the joint efforts of theoretical physicists from various countries, including Soviet scientists. In his reminiscences of Rutherford, Bohr later said[300]: "The years when the unrepeatable combination of efforts of a whole generation of theoretical physicists from many countries created step-by-step the logically noncontradictory generalization of classical mechanics and electrodynamics have sometimes been characterized as the "heroic" era of quantum physics. For every person who has followed this development (Bohr's inspiring and guiding role in which is hard to overrate—M. E.), an unforgettable experience remains the picture of how a new view on understanding physical experimentation had arisen from the combination of various approaches and the use of the corresponding mathematical methods. Numerous obstacles had to be overcome before this goal was reached, and time passed, and as it happens, the decisive success was attained by the youngest of us."

One of the major centers of development of quantum physics became the capital of Denmark, Copenhagen, where Bohr founded the Institute of Theoretical Physics, to which physicists came from various countries. Bohr's Copenhagen school played a very important role in the foundation of quantum mechanics. The American scientist Dennison, who had come to work at Copenhagen from the University of Michigan, recalls[323]: "Copenhagen in these years from 1924 to 1926, as in many other years, was one of the major centers for physics. Almost everyone came who was doing something at that time (of course, he is speaking of quantum physics.—M. E.). Heisenberg, Kramers, and Nishina[7)] were found there a large fraction of the time. Hund, Pauli, Fowler, Dirac, Goudsmit, Uhlenbeck, and many others came from time to time. Really it was a very exciting time."

The greatest number of leading theoretical physicists were working in Germany: Planck, Einstein, and Laue in Berlin, Sommerfeld in Munich, and Born in Göttingen (where there was also a very powerful mathematical school headed by Hilbert). Sommerfeld and Born created their own major scientific schools. In 1924 the fourth edition of Sommerfeld's monograph "Structure of the Atom and Spectral Lines"[39] was published, and in 1925 Born's "Lectures on Atomic Mechanics" was published.[56] In the first half of the twenties, two major representatives of the young generation of theoretical physicists who had been born at the boundary of the 19th and 20th Centuries studied with Sommerfeld in Munich,[293a, 294a] and then worked as assistants in Göttingen with Born: Pauli (who was 25 years old in April 1925) and Heisenberg (who was 23 in 1925). We note that Pauli in January 1925[54] formulated the exclusion principle, the famous *Pauli principle*, according to which an atom cannot contain two or more equivalent electrons having an identical quartet of quantum numbers. Pauli also wrote a fundamental review on quantum theory that reflected its status in mid-1925.[60] Born's Göttingen school acquired an especial significance in the middle twenties. Besides Heisenberg and Pauli (who moved to Hamburg in 1925), Born's associates at that time were Jordan, Hund, and Heitler. Theoretical physicists from various countries came to work with Born, including the Soviet Union: V. A. Fock, Ya. I. Frenkel', and I. E. Tamm. The eminent mathematician Wiener, the theoretical physicists Oppenheimer, Condon, Pauling, and others came from the United States.

The major theoretical physicist Fowler was working in England in the famous Cavendish Laboratory in Cambridge headed by Rutherford, and Dirac began his studies under his direction (he was 23 in August 1925). Among the English theoretical physicists, we should also recall Darwin, who worked in Edinburgh (the grandson of the famous naturalist).

In Paris in France de Broglie (he was 33 in 1925) and Brillouin were working, and in Zürich, Switzerland Schrödinger was working until 1927 (when he moved to Berlin) (he was 38 in 1925). We note that Debye (see Ref. 329) and the eminent mathematician Weyl were also working in Zürich at that time.

In Leyden in Holland, Ehrenfest founded his own theoretical school, having occupied a chair at the University from 1912. Prior to this Lorenz had held the chair, and conferred it on Ehrenfest. Ehrenfest's students were Uhlenbeck and Goudsmit, who in late 1925 advanced the hypothesis of the spin of the electron[57, 57a] (see also Refs. 305, 325, and 326).

In Rome, Italy, one of the major physicists born in the early 20th Century began his scientific activity: Fermi (he was 24 in September 1925). In 1923 he was working with Born in Göttingen, and in 1924 with Ehrenfest in Leyden. Of very great importance was Fermi's study in March 1926 "On the Quantization of an Ideal Monatomic Gas,"[59] in which he started with Pauli's principle and proposed a new type of quantum statistics that differs from the Bose–Einstein statistics, and which has been subsequently termed the Fermi–Dirac statistics.

A number of young theoretical physicists started their work in the United States, but the scientific schools were founded there far later, while at that time most of the young scientists went to study in Europe, mainly to Bohr in Copenhagen and to Born in Göttingen.

We should especially spend time on the participation of Soviet theoretical physicists in founding quantum mechanics. Before the revolution in Russia, only a few young theoretical physicists were working, in particular Yu. A. Krutkov (see Ref. 312 concerning him). He had been a student of Ehrenfest, who worked in Russia (in St. Petersburg) from 1907 to 1912. Only after the revolution did physics begin to develop rapidly in our country, the first scientific research institutes were founded, and major scientific schools began to build, primarily in Leningrad; they were headed by A. F. Ioffe and D. S.

[7)]As Dennison points out, Nishina, the coauthor of the Klein-Nishina formula, was occupied at that time with experimentation.

Rozhdestvenskiĭ. Major theoretical physicists were actively conducting research in the early twenties: in Moscow L. I. Mandel'shtam, and in Leningrad: A. A. Fridman, Yu. A. Krutkov, V. R. Bursian, and V. K. Frederiks. Noted for their studies were Ya. I. Frenkel' and V. A. Fock in Leningrad, I. E. Tamm in Moscow, and in the middle twenties also the younger theoretical physicists: G. A. Gamow, D. D. Ivanenko, L. D. Landau in Leningrad, and M. A. Leontovich in Moscow. The Soviet theoretical physicists were attentively following the studies of the leading foreign scientists, and in the period of establishment of quantum mechanics they became its active participants. This participation facilitated the development of theoretical physics in the Soviet Union and the creation of major scientific schools by L. D. Landau, I. E. Tamm, V. A. Fock, and Ya. I. Frenkel'.

A very important role in the foundation of quantum mechanics was played by the rapid publication of papers and the numerous discussions of the most exciting problems of the new branch of theoretical physics being born, both by personal contact and by correspondence.

Most of the studies on quantum mechanics were published in the German physics journals, which in the period 1925–1928 were to a considerable extent international, primarily the *Zeitschrift für Physik* and *Annalen der Physik*. Beginning in September 1925, the former journal published the largest number of studies on quantum mechanics: Heisenberg and Born, Pauli and Jordan, Wentzel and Klein, Fock and Landau, London and Hund, and many other theoretical physicists. The latter published the fundamental articles of Schrödinger. The British scientists published in the journal *Proceedings of the Royal Society*, and all of Dirac's most important papers appeared there. Articles on quantum mechanics also began to be published in the journals of other countries. The publication time of articles in this period, when the total number of papers in physics was incomparably smaller than now, usually did not exceed two months, and therefore the specialists became acquainted with new articles very rapidly.

The work on creation of quantum mechanics became an example of the union of scientists of different countries for solving fundamental problems of science.

## 4. THE FUNDAMENTAL STAGES IN THE ESTABLISHMENT OF NONRELATIVISTIC QUANTUM MECHANICS

The first studies on quantum mechanics that start with the correspondence principle and are based on applying algebraic mathematical apparatus were published beginning in September 1925, when Heisenberg's groundbreaking article of July 29[8)] "On the Quantum-Theoretical Interpretation of Kinematic and Mechanical Relationships"[61] was published in *Zeitschrift für Physik*. A characteristic feature of Heisenberg's approach was his rejection of the classical description of motion that used "quantities thus far unobserved (such as the position, and the time of revolution of an electron)" and his striving to "construct a quantum-theoretical mechanics analogous to classical mechanics into which only relationships among observable quantities would enter." This approach agreed with Bohr's general ideas on the inapplicability of classical concepts to describe microscopic phenomena,[9)] and it proved highly effective. In solving the posed problem, Heisenberg, following Bohr, relied on the correspondence principle, and in particular, on the results of studies on the quantum theory of dispersion, in which this principle had been successfully applied and refined.[10)] Heisenberg correlated the classical oscillation frequencies of an electron in an atom with the frequency of the transitions, in line with the Ritz combination principle, while he correlated the classical amplitudes of the oscillations with the amplitudes that determine the intensity of emission in these transitions. An essential point was the change in kinematics while the form of the equations of motion was conserved. Here Heisenberg directly faced the noncommutativity of the expressions that he had introduced for the dynamical variables. He specially emphasized this as a difficulty, and hence he treated the case of an anharmonic oscillator, in which one doesn't face this difficulty. Heisenberg showed that energy is conserved (both for a harmonic oscillator and for the simplest form of anharmonic oscillator), and found the energy levels. For the harmonic oscillator, the energy is $W=(n+\frac{1}{2})h\omega_0/2\pi$ (see[61], Eq. (23)). That is, not only did he derive the quantization law in a natural way, but also the "zero-point energy" $h\omega_0/2\pi$ appeared. Heisenberg also found the quantization law of the energy of a rotator, and he derived formulas for the intensity in the Zeeman effect and for multiplets. All of this indicated the effectiveness of the new approach (see the end of the just-cited Heisenberg's article). Bohr immediately evaluated the importance of Heisenberg's work in the concluding section of his paper "Atomic Theory and Mechanics," which was published in late 1925.[269] He wrote that Heisenberg "had apparently taken a very significant step forward on the path to a new formulation of the problems of the quantum theory."

A further, very important step forward was taken by Born. When he had become acquainted with Heisenberg's article, he saw that the "symbolic product" of discrete quantities composed of classical quantities that had been introduced in this article constitutes a product of matrices.[11)] In association with Jordan in the article "On Quantum Mechanics,"[62] Born applied the method

[8)]The dates cited are those of submission of the articles to the editors of the corresponding journals.

[9)]Just at that time in July 1925, in the epilogue to his article "On the Action of Atoms in Collisions,"[55] Bohr wrote of the inapplicability of "space-time concepts that rely on classical mechanics."

[10)]See the article by Mehra (cited in footnote 5) for more details on the studies on the quantum theory of dispersion and Heisenberg's participation in them. It also discusses how Heisenberg arrived at the results (during spring and summer of 1925) that he presented in his first article on quantum mechanics.

[11)]See Mehre's article, cited in Footnote 5.

of matrix calculation, and was able to carry out "a deeper mathematical study" (about which Heisenberg wrote at the end of his article) and to get a number of fundamental results. His article contained the *commutation relationship* ("sharpened quantum condition") for the matrices of the coordinate $q$ and the momentum $p$ in the form (see[62], Eq. (38))

$$pq - qp = \frac{h}{2\pi i} \mathbf{1} \tag{29}$$

(here **1** is the unit matrix). This expression contained Planck's constant $h$, which is characteristic of microscopic phenomena. He introduced and studied the Hamiltonian $H$ as a function of $p$ and $q$, and showed that it obeys the law of conservation of energy ($\dot{H}=0$), and derived Bohr's frequency condition. Thus he created the foundations for solving concrete problems by starting with the explicit form of the Hamiltonian and applying the mathematical apparatus of matrix mechanics. He treated the cases of the harmonic and the anharmonic oscillator by using this apparatus. Here he found the form of the matrix for the coordinate $q$ for the harmonic oscillator (see[62], Eq. (53)) not by using the correspondence principle (as Heisenberg did), but from the fundamental relationships of the theory. In the article he also derived, with certain assumptions, an expression for the probabilities of dipole spontaneous transitions by starting with the Maxwell equations (see p. [62], Eq. (109)).

Both Heisenberg's study and that of Born and Jordan treated systems having a single degree of freedom. The generalization of matrix mechanics to a system having any number of degrees of freedom and the development of its methods were carried out in a joint fundamental article by Born, Heisenberg, and Jordan "On Quantum Mechanics. II" of Nov. 16, 1925.[64] It examined the canonical transformations for a system having a single degree of freedom (Chap. 1), and it developed the methods of perturbation theory, both time-independent and time-dependent (with an explicit dependence of the Hamiltonian on the time). Moreover, it examined the foundations of the theory of systems having an arbitrary number of degrees of freedom (Chap. 2). Here they assumed that the commutation relations for the coordinates $q_k$ and the momenta $p_k$ have the form

$$\left.\begin{array}{l} p_k q_l - q_l p_k = \frac{h}{2\pi i}\delta_{kl}, \\ p_k p_l - p_l p_k = 0, \\ q_k q_l - q q_{lk} = 0. \end{array}\right\} \tag{30}$$

They also applied perturbation theory to degenerate systems. Chapter 3 studied in detail the relation of the theory to the eigenvalues of Hermitian forms, with application to perturbation theory and to the continuous energy spectrum. Chapter 4, which was concerned with physical applications of the theory, started with the commutation relations to treat primarily the highly important problem of quantizing the square of the angular momentum $M^2$ and its projection $M_z$. They showed that

$$M^2 = j(j+1)\left(\frac{h}{2\pi}\right)^2. \tag{31}$$

Here $j$ can be *"only "half-integral" or "integral"."* They also found formulas for the intensities in the Zeeman effect. An essential point is that they treated the statistics of wave fields as being systems of quantized oscillators (with references to Debye[17] and also to Ehrenfest[12]); here they derived Einstein's general formula (26) for the energy fluctuations of radiation. They especially emphasized the relation of the first term in the given formula (not given by the classical theory) with the zero-point energy $C = (\frac{1}{2})h\sum_k \nu_k$. Here they stressed that "the essential difference between the theory presented here and the earlier ones consists not in the difference in the mechanical laws, but in the kinematics that is characteristic of this theory. One might even treat Eq. (55) of Chap. 4 (Eq. (26)—M. E.), which uses no mechanical principles, as one of the most graphic examples of the difference of quantum kinematics from the previous forms."

Independently of the study of Born and Jordan and that of Born, Heisenberg, and Jordan, Dirac developed the mathematical apparatus of the "algebra of $q$-numbers" by starting only with Heisenberg's work.[12)] Dirac's first article "The Fundamental Equations of Quantum Mechanics" of Nov. 7, 1925[63] was published in the *Proceedings of the Royal Society*, and it was the start of a remarkable series of his articles, which played a very large role in the establishment of quantum mechanics (see the collected volumes,[314,316] which were devoted to Dirac's 70th birthday, and especially Mehra's article in the first of these collected volumes[315]). In his first article, Dirac treated the properties of quantum quantities by close analogy with those of classical quantities. Together with the sum and product of two quantum quantities $x$ and $y$, he defined the concept of "quantum differentiation," and what is most important, he formulated the general quantum conditions in the form (cf. Eq. (11) on p. 617)):

$$xy - yx = \frac{ih}{2\pi}[x, y], \tag{32}$$

Here the

$$[x, y] = \sum_r \left\{\frac{\partial x}{\partial q_r}\frac{\partial y}{\partial p_r} - \frac{\partial y}{\partial q_r}\frac{\partial x}{\partial p_r}\right\} \tag{33}$$

are classical Poisson brackets. Since $[q_r, p_s] = \delta_{rs}$, while $[p_r, p_s] = [q_r, q_s] = 0$, one gets from (32) Born, Heisenberg, and Jordan's commutation relationship (30) as a special case. Dirac was immediately able to derive the canonical equations of motion for quantum systems. Even this study manifested Dirac's skill at representing all results in a highly general and very elegant mathematical form.

Dirac's article was published in December 1925, while in the study "On Quantum-Theoretical Kinematics and Mechanics" of Dec. 21, which was published in *Mathematische Annalen*,[66] Heisenberg supplied the first results of the development of quantum mechanics in its algebraic form, while referring both to Refs. 61, 62,

[12)]See Mehra's article cited in footnote 5 for more details.

and 64 and to Dirac's article as well as that of Kramers[65] (who had noted the agreement of results of Heisenberg's study with that of Born and Jordan in the theory of dispersion).

A very important point was the successful application of matrix mechanics to the hydrogen atom in Pauli's study "On the Spectrum of Hydrogen from the Standpoint of the New Quantum Mechanics."[69] Pauli was able (though indeed by a rather unwieldy method) to derive the fundamental Balmer formula for the energy levels of an atom having a single electron, and also the correct formulas for the Stark effect. As Pauli emphasized, "in the new quantum mechanics, in which one does not introduce a pictorial representation of the stationary states by using definite electronic orbits, the special additional exclusion rules become superfluous, and the described difficulties (of the earlier theory—M. E.) disappear of themselves."

Dirac also derived independently a partial solution of the problem of the hydrogen atom in the paper "Quantum Mechanics and a Preliminary Investigation of the Hydrogen Atom."[70] Here he further developed his algebraic method, and began to call quantum (in the general case, noncommutative) variables $q$-numbers, as constrasted with $c$-numbers (classical variables that obey the commutative law). Upon treating multiply periodic systems, Dirac discussed as a concrete example the case of the orbital motion in the hydrogen atom, but he never presented the solution of the problem thoroughly, as Pauli had done (to whom he refers in a remark).

Heisenberg and Jordan[75] applied matrix mechanics to calculate the splitting in the anomalous Zeeman effect, and Brillouin[72] and Lucy Mensing[81] did this to calculate molecular spectra (rotational and rotational-vibrational). Dennison[84] studied the rotation of molecules. Independently, I. E. Tamm applied matrix mechanics to a rotator.[82] This was the first study by a Soviet scientist in quantum mechanics. At the beginning of his paper, Tamm wrote that the "new quantum mechanics of Heisenberg, Born, and Jordan appears to be laying out new pathways and it offers hope that it will become possible with its aid to solve the problem of the atom, at least formally."

Studies also appeared that treated more general problems of the theory. Among them, one must note first of all the study by Born and Wiener (the eminent American mathematician, who was later one of the founders of cybernetics)[68] "A New Formulation of the Quantum Laws for Periodic and Nonperiodic Processes." This paper applied for the first time in quantum mechanics the theory of linear operators, and each physical quantity was correlated with a Hermitian operator. As examples they treated the harmonic oscillator and uniform free motion (a case that presented difficulties in the matrix treatment).

Dirac[79] treated the problem of dynamical variables in quantum theory for an atom having several electrons, and Wentzel[80] (on the theory of multiply periodic systems, in developing Dirac's work[70]) and London[77] (who gave a proof of the law of conservation of energy that was independent of the combination principle) published first works in quantum mechanics.

Thus, in early 1926 quantum mechanics in its algebraic form continued to develop. At the same time, the solution of a number of concrete problems in matrix form involved considerable difficulties.

We should also take up the interesting study by Lanczos of Dec. 22, 1925 "On the Field Representation of the New Quantum Mechanics"[67] (then Lanczos published two more studies[74,86] of lesser interest). This study tried to transform from the discrete matrix form of quantum mechanics to its analytic ("continuous") form by using concepts of matrix elements in integral form. Lanczos expressed the matrix elements in terms of eigenfunctions of the nondegenerate symmetric kernel $K$ that formed a complete orthogonal system. The eigenvalues of this kernel were equated to the reciprocals of the energy $W_i$ of the system divided by $h$. However, Lanczos did not find the explicit form of the kernel $K$, and he was treating only the integral, rather than the differential equation for the eigenfunctions (for Schrödinger's evaluation of Lanczos' work, see below, p. 670).

The next stage in the establishment of quantum mechanics involves the studies of Schrödinger, who arrived at quantum mechanics in its analytic form by taking a completely different path. His first article on wave mechanics, "Quantization as an Eigenvalue Problem (First Communication)"[71] was published in *Annalen der Physik* in March 1926 (submitted for publication Jan. 27). He introduced the function $\psi$, which involves the action function $S$ by the relationship $S=K \ln \psi$, and he derived the following differential equation for the hydrogen atom in the nonrelativistic approximation (cf. Eq. (5) of[71]):

$$\Delta\psi+\frac{2m}{K^2}\left(E+\frac{e^2}{r}\right)\psi=0. \tag{34}$$

(Here Schrödinger set the constant $K$, which has the dimensions of action, equal to $h/2\pi$.) This is *Schrödinger's stationary equation*, which very rapidly became widely known (see, e.g., Ref. 327). Upon solving it by separation of variables (see also Brillouin's study[110]), Schrödinger obtained the Balmer terms for the hydrogen atom in a far simpler and more natural way than Pauli had by the methods of matrix mechanics.

This article lacks any references to the studies on matrix mechanics, and there is only a reference to de Broglie's dissertation,[52] "which contains many profound ideas," and to Schrödinger's own earlier paper.[58] We should note that at the end of the article Schrödinger speaks of the graphic picture of beat formation for a radiating atom.

It was followed in less than a month by the "Second Communication"[73] of the same series of articles (on Feb. 23), and then appeared the "Third Communication"[90] (on May 10) and the "Fourth Communication"[97] (on June 21). These communications contained a very complete presentation of wave mechanics. And in the meantime he published a very important article[78] (on the relation between matrix and wave mechanics, see below) and an article[89] on the continuous transition from micro- to macromechanics for a harmonic oscillator (in early 1927 all of the first six articles by Schrödinger on wave mechanics were published as a separate collected volume[272] in which Schrödinger put as an introduction a very interesting "Systematized (Sachlich geordnete) presentation of contents").

We note that Ref. 89 treated wave packets which do not blur out for a harmonic oscillator. We also note that Markov treated the problem of the minimal properties of these packets in Ref. 175.

The fundamental second communication[73] treated in detail the optical-mechanical analogy of Hamilton, and made a comparison of "geometric" and "wave" mechanics that permits one to substantiate the wave equation (34). To supplement the first communication in the second one, he discussed a number of important examples: an oscillator, a rotator having a fixed axis, a solid rotator with a free axis, and an elastic rotator (a diatomic molecule). All of these problems are solved in a natural and very beautiful way, and we can understand the great impression that Schrödinger's results made (which for us have long since become classical). In the third communication,[90] Schrödinger developed perturbation theory (time-independent) and applied it very successfully to the Stark effect. The paper also contains a mathematical appendix devoted to generalized Laguerre polynomials and orthogonal functions, definite integrals of products of two Laguerre polynomials, and integrals containing spherical functions. While the third communication is more mathematical in nature, the fourth communication[97] gave for the first time the time-dependent wave equation (*Schrödinger's nonstationary equation*), treated time-dependent perturbation theory, and treated dispersion theory on that basis, took an account of the continuous energy spectrum, discussed the case of resonance, carried out a relativistic generalization of the fundamental equations, and finally, treated the problem "On the physical meaning of the field scalar," i.e., the wave function $\psi$. Here he found an expression for the current density (see also Fermi's article[139]). Thus the fourth communication contains an entire series of important results.

The effectiveness of the methods of wave mechanics was very quickly revealed, and they began to be widely applied. Moreover, two fundamental problems immediately arose: first, the relationship between matrix mechanics and wave mechanics, and the second, the physical meaning of the wave function.

The results of concrete calculations by wave mechanics and by matrix mechanics proved to agree in all cases. As early as the second communication, Schrödinger gave references to the studies[61–64] of Heisenberg, Born, Jordan, and Dirac, and he wrote (see p. 39 of the Russian translation of Ref. 73): "... In terms of the methods applied, the proposed attempt to solve the problem differs so much from Heisenberg's approach that I haven't yet been able to find the link that connects these two methods. I am fully convinced that these two attempts not only will not contradict one another, but even, vice versa, owing to the complete contrast in the initial assumptions and methods, they will prove mutually supplementary." And in the case of the harmonic oscillator, upon deriving the formula for its energy levels $E_n=(2n+1)h\nu_0/2$, which contains the zero-point energy, he notes: "Remarkably our quantum levels are exactly equal to the levels obtained by Heisenberg's theory!"

In the paper "On the Relation of the Heisenberg–Born–Jordan Quantum Mechanics to Mine" of Mar. 18, 1926,[78] Schrödinger was able to solve the problem of the relation of the two approaches to quantum mechanics—algebraic and analytical. In beginning it, Schrödinger writes: "It is very strange that these two new quantum theories coincide *with one another* as regards the concrete results obtained up to now, and also in the region where they differ from the old quantum theory." And further on he says: "I shall reveal below the very close *internal interrelation* of the Heisenberg quantum mechanics and my wave mechanics. From the formal mathematical standpoint, it can perhaps be treated as an *identity* (of the two theories)." Schrödinger correlates the operators with the physical quantities (independently of Born and Wiener, whose study[68] he didn't yet know), in particular, the operator $(h/2\pi i)\partial/\partial q_l$ with the momentum $p_l$, and examines in operator form the equation

$$-[H,\psi]-E\psi=0 \qquad (35)$$

for the function $\psi$ of the coordinates, of which he writes that it is "*identical with the wave equation on which my wave mechanics is based.*" He finds the matrix elements by using the wave functions. At the end of his article, Schrödinger compares the two theories and examines the "prospectives of the classical understanding of the intensity and polarization of emitted radiation" as resulting from beat formation by the electric moment of the atom (see Ref. 78, Eq. (38), p. 74 of the Russian translation).

It became clear after this study by Schrödinger that there is a unified theory, quantum mechanics, and that matrix mechanics and wave mechanics represent only different forms of this unified theory.

Pauli and the young American theoretical physicist Eckart arrived at the same result independently of Schrödinger. In an article of May 31, 1926,[92] Eckart treated the connection between wave mechanics and matrix mechanics for a harmonic oscillator, and in an article[93] of June 7, 1926 in more general form (here Eckart refers in a remark also to Schrödinger's article[78]). Pauli presented the results of his analysis of the connection between wave mechanics and matrix mechanics in a very interesting letter to Jordan of Apr. 12, 1926, which was published only in 1973 in an article by Van der Waerden[320] in the collected volume, Ref. 316. Pauli never published these results, apparently in line with the fact that they mainly agreed with Schrödinger's results in the article[78], which was published in early May 1926.

We note that Van der Waerden, a mathematician who is widely known for his works on higher algebra (and who is the author of a monograph on application of group theory to quantum mechanics[288] and studies on spinor analysis) has also occupied himself greatly with problems of the history of quantum physics (see the article "Exculsion Principle and Spin" in the collected volume[297] and the introductory historical article in the collection of original papers[308]). Van der Waerden's article[320] represents a report that he gave at the symposium in Trieste in September 1972, "On the Development of the Concepts of Physics on Nature in the Twentieth Century" (devoted to Dirac's 70th birthday; at this symposium Dirac himself, [317] Heisenberg, [318] Jordan, [319] Wigner, [321] and a number of other major theoretical physicists spoke). Van der Waerden examined in detail how the transition occurred from matrix mechanics to the unified quantum mechanics, he gave the text of Pauli's letter of Apr. 12, 1926 addressed to Jordan, and in particular, he stressed the role of Lanczos' study[67] by showing that Schrödinger's wave equation gives rise to an integral equation having a kernel whose eigenvalues amount to the reciprocals of the energy $W_i$ (divided by $h$), as Lanczos had found (see above, p. 668). Van der Waerden pointed out that Schrödinger had erred when, in commenting to the article[78] (see p. 73 of the Russian translation of this paper), he wrote of the difference of the Green's function of his own wave equation from Lanczos' symmetric kernel. This Green's function has the eigenvalues $h/W_i$, rather than $W_i/h$, as Schrödinger had supposed. Lanczos (who was already 79 years old in 1972) was also present at Van der Waerden's report, which Van der Waerden found out only at the time of the report. The audience warmly acclaimed Lanczos, whose work[67] had played a definite role in the foundation of quantum mechanics, although of course, Schrödinger's studies had the decisive significance in developing the analytical apparatus of quantum mechanics, in which he found the explicit form of the Hamiltonian.

In mid-1926, when the unified theory had already begun to develop, Born carried out a fundamental study on the theory of collisions, in which he solved also the very important problem of the physical meaning of the wave function and gave it a probabilistic interpretation (for which Born received the Nobel Prize in 1954).

We must emphasize that the problem of interpreting the wave function and the mathematical apparatus of quantum mechanics in general was not yet clear (see, e.g., Ref. 108). Whereas Heisenberg had taken the path of rejecting pictorial representations (and only later determined the limits of applicability of the concepts of the coordinate and the momentum in Ref. 177 on the uncertainty relationships; see below, p. 671), Schrödinger tried, as we have pointed out above, to give a pictorial interpretation of the function $\psi$, by treating $e\psi\psi^*$ as being the charge density and explaining emission by the change in the dipole moment owing to beat formation by the eigenoscillations corresponding to the two stationary states between which the transition occurs. In the second communication he wrote (see p. 39 of the Russian translation of the article[73]): "Personally I especially like the interpretation of the emitted frequencies given at the end of[71] as "beats," and here I think that one will get also a pictorial interpretation of the formulas for the intensity." In a footnote to the article, [78] while pointing out that "my theory was stimulated by the work of L. de Broglie[52] and by the short but highly perspicacious remarks of Einstein, [53]" he said the following on Heisenberg's theory (see p. 56 of the Russian translation): "Of course, I knew of his theory, but I was frightened off, if not to say, repelled, by the methods of transcendental algebra that seemed very difficult to me and by the lack of pictorial quality." Indeed Schrödinger understood that the "$\psi$-function in the general case cannot and should not be directly interpreted in three-dimensional space, as is done in the case of the one-electron problem, since in general it is a function in configuration space, rather than in real space" (see p. 135 of the Russian translation of Ref. 97).

We note that unsuccessful attempts were subsequently made at mechanical interpretation of Schrödinger's equation (see the recent monograph by Jammer[322]), in particular by Madelung. [130]

We note also that de Broglie, who valued Schrödinger's work very highly, [115] tried to construct a "double-solution theory"[179] by treating the particles as singular points in a wave process.

The physical interpretation of the wave function that Born proposed in a preliminary communication of June 25, 1926, which was published in the *Zeitschrift für Physik*, [99] 13) and which he developed in an article of July 27, 1926, which was published in that journal, [107] was based on Einstein's idea of the relation of the wave field to light quanta and on the idea of the "phantom field" (see[107] and also above, p. 664). Born treats the wave function, which depends on the coordinates of all the particles of the system and on the time, as being a "probability wave," which propagates in accord with the Schrödinger equation. Born gives the following very important formulation (see[107]): "One could generalize this concept in the following, though somewhat paradoxical way: the motion of the particles follows probabilistic laws, but the probability itself propagates in accord with the law of causality." And in a footnote on the law of causality, Born says: "This means that a knowledge of the state at all points at some instant of time determines the distribution of states for all subsequent times." The outlook that Born formulated became habitual for physicists, and it correctly reflects the specifics of the laws of microscopic phenomena. It agrees with the dialectical-materialistic understanding of causality and the relationship between dynamic and statistical laws.

13)The Zeitschrift für Physik did not publish preliminary communications as a rule. In a footnote to Ref. 99, Born wrote: "This communication was originally intended for Naturwissenschaften, but couldn't be accepted there for *lack of space* (my italics—M.E.). I suppose that its publication here will not seem superfluous." The editors of Zeitschrift für Physik show showed greater understanding and published Born's preliminary communication in the July 10 issue, i.e., a half a month after receipt.

Born's probabilistic interpretation of the wave function played a very important role in the subsequent development of quantum mechanics, and was the basis for the correct physical interpretation in developing the generalized mathematical apparatus of quantum mechanics as a unified and consistent theory of microscopic phenomena.

We note that de Broglie agreed in Ref. 166 with Born's probabilistic interpretation of the wave function, and adhered to it for a number of years. Yet later, as late as the fifties, he rejected it and began to support an ill-substantiated viewpoint that had been developed in particular by Vigier (see de Broglie's statements in the book[299]).

Beginning in mid-1926, a considerable number of studies was devoted to developing the general theory.

These include Dirac's paper "On Quantum Algebra"[112] and his paper "On the Theory of Quantum Mechanics,"[117] Heisenberg's papers "The Many-Body Problem and Resonance in Quantum Mechanics"[94,157] and "Fluctuation Phenomena and Quantum Mechanics,"[135] Born's paper "The Adiabatic Principle in Quantum Mechanics,"[128] Jordan's papers "On Canonical Transformations in Quantum Mechanics"[83,104] and "On the Quantum-Mechanical Representation of Quantum Jumps,"[144] London's papers "On Jacobian Transformation in Quantum Mechanics"[91] and "Wave Variables and Canonical Transformations in Wave Mechanics,"[124] the paper of Fermi and Persico "The Adiabatic Principle and the Notion of Vis Viva in the New Wave Mechanics,"[136] and Klein's paper "Electrodynamics and Wave Mechanics from the Standpoint of the Correspondence Principle."[149]

The theory was developed in very general form in Jordan's papers "On a New Substantiation of Quantum Mechanics,"[155,202] and especially in Dirac's paper of Dec. 2, 1926, "The Physical Interpretation of the Quantum Dynamics."[148] These studies treated the theory of transformations, and Dirac's paper first introduced his famous δ-function. This permitted him to present the results in a very elegant form, and in particular, to derive in a natural way Schrödinger's equation as the equation for the transformation functions (Dirac had treated Schrödinger's equation even earlier in Ref. 117). Both Dirac and Jordan started in the physical interpretation of the mathematical apparatus from Born's probabilistic views (we note that Jordan speaks in Ref. 155, in referring to Pauli's study,[152] of the transformation functions as being *probability amplitudes*). Dirac's fundamental study was the basis of his widely known book *The Principles of Quantum Mechanics*,[282] which was published in its first edition in 1930.

Studies by Jordan,[170,226a] Beck,[188a] and Van Vleck[249] were concerned with various problems involving the probabilistic interpretation of quantum mechanics.

The mathematical methods of quantum mechanics were further developed in a paper by Hilbert, von Neumann, and Nordheim, "On the Fundamentals of Quantum Mechanics" of Apr. 6, 1927,[183] which was published in *Mathematische Annalen*. This paper, which arose as a result of Hilbert's reading in the winter of 1926–1927 of lectures on the newest developments of quantum mechanics, gave a systematic construction of the theory with the introduction of the probability amplitudes, while using the theory of linear operators (in Hilbert space) together with conditions of possessing real values. This line of studies was further developed in the papers of von Neumann of May 25[196] and Nov. 11, 1927,[234,235] which were published in *Göttingen Nachrichten*. The latter of these studies was devoted to "the thermodynamics of quantum-mechanical sets," and it treated mixed ensembles. Later von Neumann generalized his results in the well-known monograph *Mathematical Foundations of Quantum Mechanics*,[287] which was published in 1932.

A very important final stage in the establishment of nonrelativistic quantum mechanics was the establishment by Heisenberg of the *uncertainty relationships* and Bohr's formulation of the *complementarity principle*. An article by Heisenberg of Mar. 23, 1927, "On the Graphic Content of the Quantum-Theoretical Kinematics and Mechanics," which was published in *Zeitschrift für Physik*,[177][14)] was concerned with the uncertainty relationships. This article is very rich in content. It formulates the uncertainty relationships for the coordinate and the momentum, and also for the energy and the time (see[177], Eqs. (1) and (2)), it carries out a mathematical treatment of the uncertainty relationships based on the Dirac–Jordan theory and analyzes in detail a thought experiment with an atom beam that passes successively through two inhomogeneous magnetic fields, it discusses the problem of the transition from micro- to macromechanics also on the basis of the Dirac–Jordan theory, it discusses other thought experiments, and finally, it examines problems involving the uncertainty relationships for the energy and the time. Heisenberg discussed the results of this study with Bohr, and took account of Bohr's remarks.

Physicists quickly appreciated the importance of Heisenberg's study, and the uncertainty relationships became one of the foundations of quantum mechanics that reflect the physical essence of microscopic phenomena for which the classical concepts have only limited applicability.

Among the studies that discussed problems involving the uncertainty relationships, we can note the study that Kennard carried out in Copenhagen on July 17, 1927,[211] "On the Quantum Mechanics of Simple Types of Motion," in which he treated the wave packets as "probability packets" (Bohr referred to this paper later in Ref. 224) and the study of Landé.[213] The later papers in which the uncertainty relationships were applied include Kennard's paper[244] "Note on Heisenberg's Indetermination Principle" (see also his paper[256]) and Ruark's papers.[261,262]

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14)See[177]. For more details on the development of Heisenberg's ideas, see Mehra's article (cited in Footnote 5) and also Heisenberg's book.[310]

Bohr gave a very profound treatment of the fundamental problems of quantum mechanics in a generalizing report that he gave in September 1927 in Como, Italy at an international congress of physicists held in memory of Volta, and then in Brussels in October of the same year at the fifth Solvay Congress. The report was published in the spring of 1928 in the journals *Nature* (in English) and *Naturwissenschaften* (in German).[224] In this report ("The Quantum Postulate and the Recent Development of Atomic Theory"), Bohr expounded the concept of complementarity. At the Solvay Congress, the famous discussion between Bohr and Einstein began, which continued almost thirty years until Einstein's death (on this discussion, see in particular Jammer's book[322]).

## 5. THE MOST IMPORTANT LINES OF DEVELOPMENT OF QUANTUM MECHANICS THAT AROSE IN ITS PERIOD OF ESTABLISHMENT

In the period of establishment of quantum mechanics, there arose different pathways of generalizing it and of developing its methods and applications that were based on the mathematical apparatus that was created in this period and on its physical interpretation.

The *problem of the relativistic generalization of quantum mechanics* arose from the very outset. As we have discussed above, people were able to establish its laws in the nonrelativistic approximation. We note that Schrödinger originally tried to solve the relativistic problem of the hydrogen atom (see, e.g., Ref. 327, p. 374) on the basis of a four-dimensional formulation of de Broglie's particle-wave dualism, though he did not succeed; only then did he arrive at the treatment of the nonrelativistic case and derive his wave equation in the form of (34) (see p. 668). It was natural to seek the relativistic wave equation in the form of a second-order differential equation in the coordinates and the time. The first to publish such an equation (starting with a five-dimensional formulation of the theory of relativity) was Klein in a paper of Apr. 28, 1926[87] (he cited Schrödinger's papers[71,73]). In addition to the second derivatives with respect to the coordinates, this equation contains the second derivative $\partial^2/\partial t^2$ with respect to the time. It was then derived independently in a series of papers—by Fock,[95,114] Schrödinger (in his fourth communication[97]), by De Donder and van den Dungen,[103] by de Broglie,[111] by Kudar,[120] and by Gordon[125] (in a study on the theory of the Compton effect). This equation is often called the Klein–Gordon equation; it would be more correct to call it the Klein–Fock equation. It was also derived as early as April 1926 by Pauli, who cited it in a letter to Jordan (see above, p. 695; for more details, see Ref. 320), but he didn't publish it, along with his results on the relationship between wave and matrix mechanics. A simple method of deriving the Klein–Fock equation was proposed by Ivanenko and Landau in a paper of Oct. 8[127] (this was the first paper by the eighteen-year-old Landau on quantum mechanics); it has been discussed in studies by Ehrenfest and Uhlenbeck,[122a] Gamow and Ivanenko,[123] Schrödinger,[150] de Broglie,[158] Bateman[178] (see also the papers of London[171] and of Wiener and Struik[180]). However, this second-order relativistic equation for the scalar wave function $\psi$ proved to be inapplicable to the electron; we know now that it is applicable to particles lacking spin.

The *electron spin*, whose magnetic interaction with the orbital angular momentum of the electron (spin-orbital interaction) is of relativistic origin, was taken into account phenomenologically as early as the paper of Pauli and Jordan on the anomalous Zeeman effect[75] and in other studies (e.g., those of Brillouin,[154] Richter,[191] and Rosenfeld[207]). In the spring of 1927, Pauli was able to include the spin in nonrelativistic quantum mechanics in the important paper of May 3 "On the Quantum Mechanics of the Magnetic Electron"[188] by introducing two-component wave functions and the corresponding two-dimensional matrices (the *Pauli matrices*, which are widely applied even now). In late 1927 von Neumann and Wigner[243] started with Pauli's work to discuss this problem by applying the methods of group theory (see below, p. 675). Darwin[214] proposed a somewhat different approach in the summer of 1927: to replace the scalar function $\psi$ with a vector function, while Frenkel[255] proposed replacing the scalar function with a tensor function.

Dirac made a decisive advance in treating the electron as a particle possessing a spin of $\frac{1}{2}$ in early 1928 in the famous papers (of Jan. 2 and Feb. 2) "The Quantum Theory of the Electron."[247,254] Dirac started by requiring that the relativistic wave equation of the electron should be linear in the energy, i.e., linear with respect to $\partial/\partial t$, just like the nonrelativistic Schrödinger equation. It proved possible to satisfy this requirement while maintaining relativistic invariance by replacing the second-order equation with an equation that was first-order, both with respect to the operator $p_0 = ih\,\partial/c\partial t$ and with respect to the momentum operators $p_r = ih\,\partial/\partial q_r$ $(r = 1, 2, 3)$:

$$(p_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \beta)\,\psi = 0. \tag{36}$$

Upon subjecting the time- and coordinate-independent operators $\alpha_1$, $\alpha_2$, $\alpha_3$, and $\beta$ $(\beta^2 = m^2c^2)$ in this *relativistic wave equation* to certain permutational conditions and introducing four-dimensional matrices instead of Pauli's two-dimensional matrices (this corresponds to four-component, rather than two-component wave functions), Dirac showed that his equation implies the existence of the electron spin and the spin-orbit interaction, and one gets the correct fine structure of the energy levels of the hydrogen atom. This was a very important achievement by Dirac that was immediately acknowledged, and articles began to appear on the study and application of Dirac's equation. Gordon's article[257] is dated in February of 1928, and articles by Darwin,[258] Ivanenko and Landau,[260] von Neumann,[264] and Landé[267] in March of 1928. Without taking up the later papers based on Dirac's equation (cf. the collected volumes[314,316] and also the collected volume[297]), we note that the fundamental significance of this equation became especially manifest later on, when it was shown (in particular, by Dirac himself in his "hole theory") that Dirac's equation implies the possible existence of antiparticles, pair creation and annihilation, and then such antiparticles were discovered (beginning with the discovery of the positron in 1932).

A very essential point was the establishment of the *relation of quantum statistics to the quantum mechanics of systems that consist of identical particles*. In the summer of 1926, Heisenberg in his paper "The Many-Body Problem and Resonance in Quantum Mechanics"[94] and Dirac in the paper "On the Theory of Quantum Mechanics"[117] (already mentioned above, see p. 671) independently showed that the states of a system that consists of identical, indistinguishable particles are divided into those that are symmetric and antisymmetric with respect to permutations of two such particles. Around this time, the identity of wave and quantum mechanics had been shown, and both Heisenberg and Dirac introduced the wave function of a system of electrons in the form of the product of the wave functions of the individual particles (while neglecting interactions), and they studied the sums of such products. For two electrons in the states $m$ and $n$, Dirac writes the complete wave equation in the form

$$\psi_{mn} = a_{mn}\psi_m(1)\,\psi_n(2) + b_{mn}\psi_m(2)\,\psi_n(1)\,. \tag{37}$$

He points out that either the case of symmetric states holds, $a_{mn} = b_{mn}$, or that of antisymmetric states, $a_{mn} = -b_{mn}$ (Heisenberg also writes analogous expressions). The case of symmetric states corresponds to the Bose-Einstein statistics, and that of antisymmetric states to the statistics that satisfies the Pauli principle, i.e., the statistics that Fermi had proposed not long before, and at which Heisenberg and Dirac arrived independently (the papers[94, 117] contain no reference to Fermi's work[59]). Dirac discussed in detail the problem of the statistics of antisymmetric states, soon to be named Fermi–Dirac statistics. Subsequently the problem of quantum statistics from the standpoint of quantum mechanics with introduction of wave functions that are symmetric and antisymmetric with respect to permutations of identical particles was discussed in papers by Fowler,[133] Ehrenfest and Uhlenbeck,[151, 164] and Ivanenko and Landau.[169] We note that the Fermi–Dirac statistics was later applied very successfully to electrons in a metal by Sommerfeld[241] and by Frenkel'.[255a, 268]

Dirac stressed in his study[117] that a "solution having symmetric eigenfunctions should hold for light quanta, since, as we know, the Bose–Einstein statistical mechanics leads to Planck's law for black-body radiation." Dirac was greatly interested in problems of the theory of radiation, and he was not satisfied with the semiclassical theory of radiation (which was discussed, in particular, in the studies of Schrödinger,[97] Slater,[146, 165] Beck,[172] Landé,[174] and Fermi[187]). In a fundamental paper of Feb. 2, 1927, "The Quantum Theory of the Emission and Absorption of Radiation,"[167] which was carried out in Copenhagen and presented by Bohr in the *Proceedings of the Royal Society*, Dirac developed the foundations of the *quantum theory of radiation*. He was able to find the Hamiltonian for a unified system consisting of an atom and a radiation field. He was the first to apply the method of *second quantization* to the radiation by starting with the permutation relationships for the dynamical variables that describe the free radiation field. Here he described spontaneous and stimulated emission from a unified standpoint. In the next study (of Apr. 4, 1927), Dirac also discussed the problems of the theory of dispersion.[182]

Dirac's studies on dispersion theory organically combined particle and wave representations, and these studies began the development of quantum electrodynamics and generally of the theory of quantized fields (see Wentzel's review[298]). We note that the application of the second-quantization method for a system of microparticles obeying the Bose–Einstein statistics was studied in a paper by Jordan and Klein,[226] while that for a system of particles obeying the Fermi–Dirac statistics was studied in papers by Jordan[210] and by Jordan and Wigner.[251] We note also that only later did the studies of Pauli (see Ref. 297) elucidate and substantiate the fact that microparticles obey the Bose–Einstein statistics that have an integral spin (including zero), while particles of half-integral spin obey the Fermi–Dirac statistics.

A number of papers were concerned with radiation theory during 1927 and early 1928, including those of Jordan,[205] Gorovits,[225] and Kronig and Kramers.[263]

We especially note the paper by Landau of July 21, 1927, "The Problem of Damping in Wave Mechanics,"[212] in which the very important concept of the *density matrix* was first introduced, and the paper by Jordan and Pauli of Dec. 7, 1927, "On the Quantum Electrodynamics of Charge-Free Fields,"[237] which was a development of the ground-breaking study of Dirac[167] on the topic of accounting for relativistic invariance. We recall also the studies of Fues[194] and Oppenheimer.[231]

A very important feature was the application of quantum mechanics for solving varied problems of the *theory of atoms* and the *theory of molecules*.

Many studies were concerned with the theory of the hydrogen atom. They include Eckart's paper "The Hydrogen Spectrum in the New Quantum Theory,"[106] studies on the intensities in the hydrogen spectrum (including the theory of the intensities in the continuous spectrum) by London,[116] Oppenheimer,[132, 160] Sugiura,[162] Slack,[242a] and studies on the Stark effect and the Zeeman–Waller effect[98] by Epstein,[113, 126] Slack,[159] and Ruark.[236]

A very important paper was that of Heisenberg of July 24, 1926, "On the Spectra of Atomic Systems with Two Electrons,"[109] in which he started with the symmetry of the wave function to treat the energy levels and the spectra of para- and orthohelium. The helium atom (and $Li^+$) were also treated in papers by Slater,[186] Sugiura,[197] Kellner,[201, 206] Hylleraas[266] (who calculated the ionization energy of this atom to high accuracy by the variation method), Jane Dewey[119, 215] (on the intensities in the Stark effect, performed in Copenhagen). Studies by Pauling[131] and Unsöld[156] were concerned with individual problems of the theory of complex atoms. Slater in the United States and Hartree in England then began to take up the development of general methods of calculating complicated atoms. Slater's paper "Central Fields and Rydberg Formulas in Wave Mechanics"[245] dates to late 1927. As we know, Hartree developed his method of the self-consistent field, which Fock then generalized (in 1930), and it became known as the *Hartree–Fock method*.

Applications of quantum mechanics to molecules also developed successfully. Studies by Fues,[85,105] Reiche[118] and Rademacher and Reiche,[146a] Kronig and Rabi,[134] Landau ("On the Theory of the Spectra of Diatomic Molecules"),[138] Witmer,[163] and Condon[217] were concerned with the theory of the rotational and vibrational energy levels and spectra of molecules. A paper of fundamental importance was that of Born and Oppenheimer of Aug. 28, 1927 "On the Quantum Theory of Molecules,"[219] in which they justified (by expanding in a power series in the parameter $(m/M)^{1/4}$, where $m$ is the mass of the electron, and $M$ is of the order of the mass of the nuclei) the separation of the energy of the molecule into electronic, vibrational, and rotational energy. The problems of interpreting molecular spectra were discussed in detail in a series of papers by Hund.[143,168,200] The symmetry properties of the energy levels of molecules were examined by Heisenberg in the second part of the paper "The Many-Body Problem and Resonance in Quantum Mechanics"[157] and in a paper by Hund.[199] Problems of molecular spectra were the topic also of the later papers by Hulthén[233] and Kronig.[238] Of importance was a paper by Condon,[176] who examined the connection between electronic and vibrational motions in diatomic molecules, and gave a quantum-mechanical interpretation of the principle of conservation of the distance between nuclei in electronic transitions, which had been formulated earlier by Franck, and which is now called the *Franck–Condon principle*.

Papers by Burrau,[143a] Unsöld,[192] Wilson,[242] and Gorovits and Finkel'shtein[252a] were concerned with quantum-mechanical calculation of the ground state of a very simple molecule, the molecular ion of hydrogen $H_2^+$. The ground state of the hydrogen molecule $H_2$ was calculated by Condon[184] and by Sugiura.[220] The properties of two modifications of the hydrogen molecule, para- and ortho-, were studied by Dennison[203] (see also Ref. 323).

A very important paper was that of Heitler and London of June 30, 1927 "Interaction of Neutral Atoms and Homopolar Binding According to Quantum Mechanics,"[209] which started the development of *quantum chemistry*. Following it, other papers have appeared on the quantum-mechanical theory of the homopolar chemical bond, by Heitler,[227,252] London,[239] and Pauling.[259] Heitler also treated the motion of an electron in a lattice, while taking account of the periodicity of the latter.[195]

The quantum-mechanical *theory of the dielectric constant and the magnetic susceptibility* was the topic of studies by Mensing and Pauli,[100] Kronig,[101,122] Van Vleck,[140,171a,290,253] Epstein,[185] and Wang.[232]

Born's paper on the theory of collisions[107] (in which he gave the probabilistic interpretation of the wave function) was the starting point of the development of the *quantum-mechanical theory of collisions*; the method of studying collisions applied in this study became known as the *Born method*.

The theory of collisions was the subject also of papers by Fermi,[129] Dirac,[208] Faxén and Holtsmark,[216] Elsasser,[218] and Lucy Mensing.[223] The quantum theory of electron capture was studied by Oppenheimer,[246] and the theory of scattering of slow electrons by Holtsmark.[257a]

We should especially note the paper by Mandelstam and Leontovich "On the Theory of the Schrödinger Equation,"[240] which studied the behavior of the wave function as depending on the trend of the potential, and which essentially contained a theory of passage through a barrier (see Ref. 304a). The idea of passage through a barrier was also expressed in a paper by Nordheim.[239a]

Papers by Oppenheimer[221] and Darwin[230] were concerned with problems of the theory of aperiodic motions.

An entire set of papers was concerned with the *quantum-mechanical theory of the Compton effect* (which allows one to calculate the scattering probabilities at different angles). The first were the papers of Beck[76] and Dirac,[86] and they were followed by the papers of Gordon,[125] Schrödinger[147] (which was included in the second edition of the collection of his works,[272] where also Ref. 204 is to be found), Klein,[149] and Wentzel.[181,198] Papers by Wentzel[142] and Beck[161] were concerned with the *theory of the photoelectric effect*.

Various approximate methods were developed and applied for solving varied quantum-mechanical problems. The methods of time-independent and time-dependent perturbation theory proved very effective (see, e.g., Refs. 173, 193, 248, 267a). People began to apply variation methods successfully. The *quasiclassical method* was developed as early as 1926, and it was independently proposed by Wentzel,[96] Brillouin,[102] and Kramers.[121] This method, which is based on representing the wave function in the form $\psi = \exp(2\pi i S/h)$ and expanding the function $S$ in powers of $h$, became known as the *Wentzel–Kramers–Brillouin method* (WKB method for short). In a first approximation, one gets from it the Bohr–Sommerfeld conditions for the phase integrals, and here the connection between quantum and classical mechanics is manifested. One can represent this connection especially perspicuously by treating the time-dependence of the means of the physical quantities, as Ehrenfest showed in his well-known paper of Sep. 5, 1927[222]: one gets equations of motion for the mean values that coincide in form with the classical equations (*Ehrenfest's theorem*). As concerns the quasiclassical approximation and the agreement between quantum and classical mechanics, see also the studies of Debye,[153] Niesson,[250] Eckart,[141,265] and Gamow, Ivanenko, and Landau.[229] We note that Feynman later developed the "path integral" method, which permits one to correlate very graphically quantum with classical mechanics (see the book by R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, 1975; Russ. Transl., "Mir," M., 1968).

Some very important papers were those on the *application of group theory to quantum mechanics* for treating the symmetry properties of quantum systems; the importance of accounting for them was stressed already at the beginning of this article (see p. 000). Some ground-breaking studies were those of Wigner of Nov. 12 and 26, 1926 "On the Non-Combining Terms in the New Quantum Theory" (Parts I and II), [137, 145] and Weyl's paper of Oct. 13, 1927 "Quantum Mechanics and Group Theory,"[228] which was the basis of his monograph of the same title.[277] We note also the paper by Wigner.[189]

## 6. CONCLUSION

The results of the process of establishment of quantum mechanics as a new science became reflected in numerous generalizing works, at first in reviews and collected volumes, and then in monographs and textbooks. They presented the mathematical apparatus of quantum theory, gave its physical interpretation, and described the methods and results of solving varied problems of the physics of microscopic phenomena.

Heisenberg's review[270] on quantum mechanics was published as early as the second half of 1926 in *Naturwissenschaften*, and at the end of the year, the generalizing article of Schrödinger[271] in *Physical Review*. Born's review[273] "Quantum Mechanics and Statistics" was printed in early 1927 in *Naturwissenschaften*, and a collection of Schrödinger's articles was published as a book[272] (see above, p. 668). We note that the translations of the reviews[270,271] were quickly published in *Uspekhi Fizicheskikh Nauk*, and the collected volume *Foundations of the New Quantum Mechanics*[274] was published in Leningrad in early 1927. It contained the articles of P. S. Tartakovskiĭ: "Difficulties of the Theory of Quanta Prior to the 'New Quantum Mechanics'," of G. A. Grinberg: "Fundamentals of the New Heisenberg–Born Quantum Theory," of N. N. Andreev: "The Analog Between Mechanics and Optics," of V. R. Bursian: "Schrödinger's Wave Mechanics," of V. K. Frederiks: "Schrödinger's Theory and the General Theory of Relativity," of B. N. Finkel'shteĭn: "Quantum Mechanics and the Compton Phenomenon," and of V. A. Fock: "The Mathematical Apparatus of Schrödinger's Theory." In the preface to the collection, its editor A. F. Ioffe wrote of the "significance of these new views as a new epoch opening in physics."

In 1928 were published the collection of selected studies by de Broglie and Brillouin[276] and the second supplemented edition of the collected papers of Schrödinger's papers.[272] The first monographs on quantum mechanics appeared, including *Wave Mechanics* by de Broglie[275] (which was quickly translated into other languages) and the fundamental book by Weyl, *Group Theory and Quantum Mechanics*.[277]

In 1929 were published Sommerfeld's book *Atomic Structure and Spectral Lines. Wave-Mechanical Supplementary Volume*[279] (which was then translated into Russian under the title *Volnovaya Mekhanika* (Wave Mechanics)) and the *Introduction to Wave Mechanics* of Ya. I. Frenkel'[280] (in German; this was the first book by a Soviet scientist on quantum mechanics). Very important monographs appeared in 1930: Heisenberg's *The Physical Principles of Quantum Theory*[281] and Dirac's *The Principles of Quantum Mechanics*,[282] a book that has not been excelled in grace and beauty of presentation, which has enjoyed a number of editions, and which remains to this day an essential reference for every theoretical physicist.

Also published in 1930 were Born and Jordan's *Elementary Quantum Mechanics*[283] (whose "elementary" quality consisted in applying exclusively the algebraic apparatus; the authors proposed to write a volume presenting the Schrödinger analytic apparatus, but did not do so), and de Broglie's *Introduction to the Study of Wave Mechanics*.[284] Published in 1931 was Wigner's monograph on application of group theory in quantum mechanics[285]; Van der Waerden's monograph, which appeared in 1932, was concerned with the same topic.[288] V. A. Fock[286] (*Principles of Quantum Mechanics*) and von Neumann[287] (*Mathematical Foundations of Quantum Mechanics*, which was mentioned on p. 000) published their books in 1932. The fundamental view by Pauli[289] in *Handbuch der Physik* ("The General Principles of Wave Mechanics") and the first part[290] of Ya. I. Frenkel's *Wave Mechanics* appeared in 1933. Its second part[291] was published in 1934. Also in 1934 was published de Broglie's book[292] *The Magnetic Electron*. The listed monographs and a number of others published in the early thirties presented the new science, quantum mechanics, with great completeness, and they reflected the ground-breaking results obtained in the period of its establishment.

As early as 1927, lecture series on quantum mechanics were delivered, the first courses being in universities and other institutions of higher education; many of the monographs of those listed above were written on the basis of lectures given. In the Soviet Union the first courses on quantum mechanics were given in Leningrad in the Polytechnic Institute by V. A. Fock and Ya. I. Frenkel' starting in the autumn of 1928. Quantum mechanics as an important branch of physical science became an organic part of the education of physicists, not only theoreticians but also experimentalists, and subsequently of the education of representatives of other specialties, primarily the related sciences.

The significance of quantum mechanics is great as one of the branches of modern natural science, for the ideas based on it, and for the formation of the dialectical materialist world outlook.

The modern quantum theory of elementary particles and of high-energy processes is based in its development on the quantum mechanics born 50 years ago as an integral part of the physics of microscopic phenomena.

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The more complete (yet not exhaustive) section is that containing the references to the original articles of the period of establishment of quantum mechanics. References to original articles on quantum mechanics of the later period (from April 1928) are not given; one can find them in the historical studies, in particular, in Refs. 297, 306, 314, 316, and 322. The references to the materials of archives on the history of quantum physics (in particular, to the letters of the participants in the foundation of quantum mechanics and to the notes of conversations with them) can be found in Ref. 307.

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