Gravitational waves in the cosmos and the laboratory

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Fundamental questions of the theory of gravitational waves are considered, and the properties of these waves are compared with those of electromagnetic waves. The efficiency of possible sources of gravitational radiation, both astronomical and in the laboratory, is analyzed. The principles on which detecting devices work are explained and their sensitivities are estimated. Particular attention is devoted to a new direction—the emission and detection of gravitational waves by means of electromagnetic systems. One of the variants of a laboratory experiment including a source and detector of electromagnetic type is described. The mechanism by which an isotropic blackbody background radiation of gravitons could have been formed is discussed, together with possibilities for detecting it.

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It is hard to overestimate the importance of the information that could be obtained from gravitational waves reaching us from the cosmos. Gravitational radiation has a tremendous penetrating capacity, and would enable one to obtain an adequate description of the details of gravitational collapse, the internal structure of supernovae, and the physical conditions during the very early stages in the evolution of the universe. (The prospects for gravitational-wave astronomy are described in[1,2].) But however grandiose may be the plans based on gravitational waves of cosmic origin, we cannot get by without persistent attempts to perform laboratory experiments. Because of its universality, the gravitational interaction cannot be put on one side by a fundamental physical theory.

1. WEAK GRAVITATIONAL WAVES

All physical fields except the gravitational can be regarded as imbedded, or given, in Minkowski spacetime. A value of a field at the points of spacetime in no way changes the interval between them:

$$ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu = c^2\, dt^2 - dx^2 - dy^2 - dz^2. \qquad (1)$$

The field may be strong or weak (for example, it can accelerate a charge to velocities that are small compared with the velocity of light c or approach c), but as long as one does not consider its ability to generate gravitation the interval between events is not changed. Producing a gravitational field simply means that one changes the distances and intervals of time between events; in other words, one changes the metric of spacetime. A change in the four-dimensional interval between events means (except for one very special case) the introduction of curvature of spacetime, and in this sense a true (i.e., one that cannot be removed by transformation) gravitational field is identical to curvature.

Curvature does not prevent one returning to the picture of flat spacetime locally, in a certain four-dimensional region. The size of the region is determined by the accuracy to which the metric does not differ from the Minkowski metric. The curvature of spacetime in the neighborhood of a given event can be associated with "radii of curvature," which have the dimensions of length and give the characteristic distances and time intervals over which the deviations from a flat world can become significant. Let us denote the characteristic value of the radii of curvature by the letter $\mathfrak{R}$. If the size of the region satisfies $\xi \ll \mathfrak{R}$, then in it one can introduce a locally inertial frame of reference. This means that the metric can be written "almost" as in (1); more precisely, the components of the metric will differ from $\eta_{\mu\nu}$ only by small quantities of order $(\xi/\mathfrak{R})^2$.

Thus, in a locally inertial frame of reference the gravitational field (curvature) is manifested only in small quantities of order $(\xi/\ \ )^2$, but, of course, it does not completely disappear. This choice of the frame of reference is convenient in that it most closely approaches to the global inertial system realized in a flat world.

A locally inertial frame of reference can be introduced not only in the neighborhood of a point but also along the worldline of a freely moving particle. For example, an observer moving freely in an arbitrary gravitational field and equipped with clocks and gyroscopes that indicate the spatial directions realizes a locally inertial frame of reference; furthermore, he does this for a very long time, though possibly in a very restricted spatial volume. This makes it possible to describe cumulative effects due to the presence of curvature. An example is the variation with time of the distance between two nearby free falling particles (the so-called geodesic deviation). The relative change of the distance $\Delta l/l$ accumulated over the time Δt is in order of magnitude $\Delta l/l \sim (c\Delta t/\Re)^2$.

There is a special case of the gravitational field when it can be, to a certain extent, treated on an equal footing with other physical fields, i.e., as a field imbedded in flat spacetime. This is the case of a weak gravitational field. It is characterized by the fact that the metric everywhere in the considered region of spacetime differs little from the Minkowski metric and can be represented in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad (2)$$

where $|h_{\mu\nu}| \ll 1$. (On the convergence of the expansions (2) with allowance for the following terms, see[3].)

We have seen that any gravitational field can be made arbitrarily weak in a sufficiently small piece of spacetime. The approximation of a weak field presupposes more—the validity of (2) in a region whose dimensions may appreciably exceed the characteristic length and time intervals over which $h_{\mu\nu}$ changes appreciably and, generally speaking, this dimension may appreciably exceed the radii of curvature determined by the metric (2).

Weak gravitational waves belong to the class of weak gravitational fields and are distinguished among these by the fact that the corrections $h_{\mu\nu}$ are oscillating functions of the Lorentz coordinates and time in which the metric $\eta_{\mu\nu}$ is expressed. In this case, one speaks of weak gravitational waves on a flat background. Weak gravitational waves have properties similar to other physical wave fields (see Chap. 2).

The approximation of weak gravitational waves on a flat background is not merely a purely mathematical idealization; it is also justified from the experimental point of view. Within the solar system, we find very weak gravitational fields produced by the Sun and the planets. To a high accuracy, we can assume that the world surrounding us is flat. It is clear that all laboratory sources of gravitational waves can create on this background only a very weak additional field of a wave nature. The waves that reach the Earth from the cosmos can also only be weak, although the weak field approximation may fail completely near the source itself.

One can also speak of gravitational waves on a flat background when the background space itself describes some gravitational field, i.e., is curved. (Moreover, it must in fact already be curved by the averaged "weight" of the gravitational waves themselves.) The only important thing is that the characteristic radii of curvature of the background world be large compared with the gravitational wavelength λ. Then in regions with dimensions $\lambda \ll L \ll \Re$ the gravitational waves behave in virtually the same way as on a flat background. Effects such as a change in the frequency of a wave and curvature of its trajectory of propagation arise only after a distance of order $\Re$ has been traversed. Of course, cardinal changes in the properties of a wave can occur under conditions when λ is not small compared with $\Re$. In particular, one can then have effects such as the amplification of classical gravitational waves and the quantum effect of graviton production in a nonstationary situation (see Chap. 7). Essentially, the very concept of a gravitational wave, which corresponds to our ideas about other wave fields, presupposes a division of the gravitational field (the curvature) into a smooth background and a weak wave "ripple."[4,5]

In situations when the waves cease to be weak, the division into the background and the wave becomes nonunique. Different divisions correspond to waves and background worlds with different properties.[6] The complete formalism of Einstein's theory of gravitation enables one to consider arbitrarily complicated metrics, although the definitions of a gravitational wave in the arbitrary case of a strong field render absolute only certain aspects of this phenomenon. In what follows, we shall consider only weak gravitational waves.

Hitherto, we have spoken of the invariant properties of the gravitational field and its description in a locally inertial frame of reference. But the choice of the frame of reference is to a certain degree a matter for the observer himself. An experimentalist who places a gravitational antenna on the surface of the Earth realizes a noninertial frame of reference. The world lines of the elements of his antenna and the complete laboratory are not geodesics. A dramatic confirmation of this can be provided by the falling onto the floor with acceleration g of a valuable instrument dropped carelessly. The acceleration of free fall $g \approx 980$ cm/sec^2 is precisely the quantitative measure of the fact that a terrestrial observer is subject to acceleration and his world line is curved. The typical radius of curvature of the world line is $\rho_{obs} \approx c^2/g \sim 10^{18}$ cm.

In order to detect a gravitational wave field by means of particles in his laboratory, the experimentalist must use particles with nearly equal four-dimensional accelerations. Neighboring particles will remain throughout time at the same distance if they have a definite but ever so slightly different acceleration. In a bound system, the difference of accelerations is compensated by the resulting stresses. From the fundamental point of view, it is precisely this situation that obtains in real experiments on the Earth. The deviation of the frame of reference from an inertial frame reduces solely to the fact that in the working body used as gravitational antenna very slight constant strains arise.

It only remains to consider the extent to which the components of the gravitational wave field are themselves changed after their recalculation in the noninertial frame of reference. Suppose that the components of a weak wave field from a source of cosmic origin are known in an inertial frame introduced far from the Earth and in which the Earth itself is fixed. Then in the frame attached to an observer at rest on the Earth they will have the same values with a relative error of order $\Re_{\oplus}/\rho_{obs} \sim 10^{-5}$. If both the source and the detector are placed in a laboratory with spatial dimension L, the relative error will be even less, of order L/ρ_{obs}. Thus, for the solution of many problems we can restrict ourselves to the approximation of weak gravitational waves on a flat background.

2. COMPARISON OF PROPERTIES OF ELECTROMAGNETIC AND GRAVITATIONAL WAVES

The analogy between electromagnetism and gravitation begins with the *laws of Coulomb and Newton*. Both laws can be expressed in the form of Poisson's equation:

$$\Delta\varphi = -4\pi\rho, \tag{3}$$

$$\Delta\psi = 4\pi G\mu, \tag{3'}$$

where φ is the electrostatic potential, ψ is the gravitational potential, ρ and μ are, respectively, the charge density and the mass density, and G is the gravitational constant. The Coulomb law of gravitation of two charges e and $-e$ is transformed into the Newton law of attraction of two equal masses m by the replacement of e by $\sqrt{G}m$.

The elliptic nature of Eqs. (3) and (3′) means that if a charge or mass changes its position in space the new field values corresponding to the changed position of the source are established instantaneously in the whole of space, at any distance from the source. It is clear that these laws can have only a restricted applicability and are true only approximately. As Einstein wrote[7]: "The conviction had to come that Newton's law of gravitation as Coulomb's laws of electrostatics and magnetostatics are of electromagnetic phenomena."

Allowance for the finiteness of the velocity of propagation of changes in the field is achieved by making the field equations of hyperbolic type, namely, wave equations. The equations of electrodynamics and the equations of gravidynamics (Einstein's equations) can be expressed in the form of *wave equations*. The Laplacian in Eqs. (3) and (3′) is replaced by the d'Alembertian, and the field variables and sources become multicomponent quantities

$$\Box A^{\alpha} = -\frac{4\pi}{c} j^{\alpha}, \tag{4}$$

$$\Box \psi^{\alpha\beta} = \frac{16\pi G}{c^4}\tau^{\alpha\beta}, \tag{4'}$$

where $\Box = \eta^{\mu\nu}\partial^2/\partial x^{\mu}\partial x^{\nu}$. On the right-hand side of Eq. (4′) we have not only the energy-momentum tensor $T^{\alpha\beta}$ of ordinary (nongravitational) matter, but also the quantities $t^{\alpha\beta}$, which combine nonlinear combinations of $\psi^{\alpha\beta}$. This expresses the universality of the gravitational interaction—all forms of matter (including the gravitational field) are subject to gravitation, and all forms of matter (including the gravitational field) produce their own gravitational field.

From the electrodynamic potentials A^{α} and the gravitational potentials $\psi^{\alpha\beta}$ one can determine *"observable"* quantities, which occur in the equations of motion of free test particles—the *electromagnetic field tensor* and the *curvature tensor*, respectively. Equations (4) and (4′) presuppose fulfillment of the gauge conditions

$$A^{\alpha}_{,\alpha} = 0, \tag{5}$$

$$\psi^{\alpha\beta}_{,\beta} = 0 \tag{5'}$$

(here and in what follows, the comma denotes simple differentiation), which really can always be satisfied by means of transformations of the potentials. In electrodynamics and the linearized theory of the gravitational field, these transformations do not change the observable quantities.

A consequence of the equations (4) and (4′)—which is actually also true before the gauge conditions (5) and (5′) are introduced—are the *differential conservation laws* (*or equations of motion*)

$$j^{\alpha}_{,\alpha} = 0, \tag{6}$$

$$\tau^{\alpha\beta}_{,\beta} = 0. \tag{6'}$$

They can be associated with integral conservation laws, though these, it is true, have a rather formal nature in the case of gravitation and acquire physical meaning for isolated systems, in an asymptotically flat region of spacetime.

In contrast to the equations of electrodynamics, which allow arbitrary motion of the sources (restricted only by the condition (6)), the equations of gravidynamics contain the equations of motion by virtue of their nonlinearity, and they must be solved simultaneously with the field equations.

If the sources are ignored, i.e., for $j^{\alpha}=0$, $\tau^{\alpha\beta}=0$, Eqs. (4) and (4′) admit the existence of *free electromagnetic* and (weak) *free gravitational waves*, which propagate with the same velocity, *the velocity of light*.

There is an additional arbitrariness in the choice of the potentials that does not affect the conditions (5) and (5′). Using this arbitrariness, one can achieve that in the absence of sources the following *additional gauge conditions* are satisfied everywhere in spacetime:

$$A_{\alpha}u^{\alpha} = 0, \tag{7}$$

$$\psi_{\alpha\beta}u^{\alpha} = 0, \tag{7'}$$

where u^{α} is a certain vector that is usually chosen in the form $u^{\alpha} = (1, 0, 0, 0)$. If the conditions (5′) and (7′) are satisfied, then either the equation

$$\eta^{\alpha\beta}\psi_{\alpha\beta} = 0 \tag{8}$$

is a direct consequence of (5′) and (7′), or not all the constraints (5′) and (7′) are independent, and then fulfillment of (8) can be achieved by using the remaining arbitrariness. (In some special cases, the gauge conditions (5′), (7′), and (8) can also be satisfied in the presence of sources.[8])

In both electrodynamics and gravidynamics one can have *plane waves*: $A_{\mu} = A^{(0)}_{\mu} e^{ik_{\alpha}x^{\alpha}}$, $\psi_{\mu\nu} = \psi^{(0)}_{\mu\nu} e^{ik_{\alpha}x^{\alpha}}$. The set of gauge conditions (5), (7) and (5′), (7′), (8) enable one to reduce all components of the potentials to zero except for two independent ones. These two components correspond to the *two independent polarization states* of electromagnetic and gravitational waves. It follows from the same gauge conditions that plane waves are *transverse* and *traceless*. Mathematically, this is expressed in the fact that the matrices of the potentials are "transverse" to the isotropic wave vector k^α and the gauge vector u^α, and Eq. (8) is satisfied. From the physical point of view, this means that the wave basically displaces test particles in the plane perpendicular to the direction of propagation (see Chap. 3), and moreover in such a way that the volume occupied by the particles remains unchanged.

The tensor of the electromagnetic field for traveling electromagnetic waves and the curvature tensor for traveling gravitational waves have *vanishing invariants* and have a similar algebraic structure. At the same time, the fields of standing electromagnetic and gravitational waves have *nonvanishing invariants* and again exhibit a correspondence from the point of view of algebraic structure.[9]

The retardation in the transmission of field changes, which is taken into account by the *retarded solutions* of Eqs. (4) and (4′), has the consequence that far from a nonstationary source, in the wave zone, the observable quantities decrease with the distance in accordance with the r^{-1} *law*, instead of the r^{-2} *law* of electrostatics and Newtonian gravitation.

The gravitational analogs of the charge, electric dipole, and magnetic dipole moments are constant in time because of the laws of conservation of the energy, momentum, and angular momentum (if one does not take into account their change due to the emission itself). A spherical body executing radial motions, or an axisymmetric body rotating about the symmetry axis, do not emit gravitational waves. Therefore, the lowest possible multipole order of gravitational radiation is the *quadrupole*, instead of the *dipole* order in the electromagnetic case. Accordingly, the graviton spin is equal to 2, while that of the photon is 1.

A system that emits electromagnetic or gravitational waves loses energy. To see this, it is sufficient to consider the solution of the field equations with appropriate initial and boundary conditions, without forming quantities of the type of the energy flux from the derivatives of the potentials. From a known solution of the field equations, one can find not only the behavior of a detector placed in the wave zone but also the variations in the source of the waves due to the presence of the emission. Since the equations of motion are solved simultaneously with the field equations in gravidynamics, one automatically takes into account the "force of radiation damping", and it need not be introduced *artificially* on the basis of additional arguments, as in the case of electrodynamics. The reaction of the gravitational radiation to a source of waves consisting of slowly moving bodies with weak field can be found in two ways. Either by extrapolating a solution of the type of outgoing waves backward, into the near zone, right down to distance of the order of a wavelength from the source, and thereby determining the "radiation" correction to the Newtonian gravitational potential.[10-12] Alternatively, and more rigorously, without any assumptions about the nature of the solution far from the source, one can solve simultaneously the field equations and the equations of motion in the near zone, but then necessarily taking into account retardation.[13,14] The two methods give the same result, whose physical significance is a reduction in the energy of the emitting system. In the equations of motion, the reaction of dipole electromagnetic radiation appears in terms of *order* $(v/c)^3$, whereas in the case of quadrupole gravitational radiation it appears in terms of *order* $(v/c)^5$, where v is the characteristic velocity of the motions in the source.

In the theory of the electromagnetic field, we have the well defined concept of the energy-momentum tensor. Using it, one can, for example, find the energy transported by electromagnetic waves independently of a calculation of the damping of the motions in the source. For a system executing stationary motion (or more precisely, a motion that would be stationary if there were no radiation), this energy is exactly equal to the work of the force of radiation damping. In the theory of the gravitational field, the concept of the energy-momentum tensor is absent. The entity $t_{\alpha\beta}$, which arises naturally when the Einstein equations are expressed in the form (4′), transforms as a so-called "pseudotensor"; that is, it transforms in accordance with the tensor law only for linear coordinate transformations. By an appropriate choice of the coordinate system, all components of $t_{\alpha\beta}$ can be made to vanish at any point of spacetime. For example, the Landau-Lifshits pseudotensor[4] can be expressed in terms of the squares of the first derivatives of the potentials $\psi_{\alpha\beta}$ in much the same way as the energy-momentum tensor of the electromagnetic field can be expressed in terms of the squares of the first derivatives of the potentials A_α. But in a locally inertial coordinate system, all the first derivatives of the metric at a given point vanish, and with them so does the energy-momentum pseudotensor. This circumstance has been reflected in the widely held opinion that it is in principle impossible to localize gravitational energy.[5] In this approach, the impossibility of localizing gravitational energy does not however prevent one from obtaining sensible physical results by using integrated and averaged quantities. For example, the reduction of the energy of a system of Newtonian type due to radiation damping is exactly equal to the energy flux calculated by means of the energy-momentum pseudotensor and integrated over a distant sphere. A different approach has also been developed, in which it is assumed that the density of the energy, momentum, and stresses of a true (i.e., one that cannot be transformed away) gravitational field must form a tensor and be expressed in terms of observable quantities—the components of the curvature tensor. This has led to the formulation of a number of interesting definitions and results,[15-17] but the justification for their derivation has more to do with the formal analogy with electrodynamics than an internal connection with the Einstein equations.

This well-known imperfection relating to the description of the energy and momentum of the gravitational field does not, of course, cast doubt on the conclusions concerning the effect of gravitational waves on a detector or the damping of motions in the radiation source. These conclusions do not in themselves require a concept such as that of the flux of gravitational energy, although they can be formulated in their simplest forms when such a concept is used.

Under certain restrictions (weak wave field or asymptotically flat metric in Cartesian coordinates) one can use the entity $t_{\alpha\beta}$, and averaging of it over spacetime regions extending over several wavelengths (for the wave situation) gives reasonable concepts for the density of energy, energy flux, and stresses.

Let us write down, for example, the *energy–momentum–stress* components in plane waves propagating along the x axis. By virtue of the gauge conditions (5), (7), and (5′), (7′), (8) the following two independent components of the potentials are nonzero: A_2, A_3, and $\psi_{22} = h_{22} = -\psi_{33} = -h_{33}$, $\psi_{23} = h_{23}$. We then have accordingly

$$T^{00} = T^{01} = T^{11} = \frac{1}{4\pi c^2}(\dot{A}_2^2 + \dot{A}_3^2), \tag{9}$$

$$t^{00} = t^{01} = t^{11} = \frac{c^2}{16\pi G}(\dot{h}_{23}^2 + \dot{h}_{33}^2). \tag{9'}$$

(Here and in what follows, the dot denotes the derivative with respect to t.)

The *power* emitted in the form of electromagnetic or gravitational waves in the case of slow motions of charges or masses can be expressed in terms of the electric dipole moment d^i or, respectively, the mass quadrupole moment Q^{ik} in accordance with

$$\frac{d\mathscr{E}_e}{dt} = \frac{2}{3c^3}\ddot{d}_i\ddot{d}^i, \tag{10}$$

$$\frac{d\mathscr{E}_g}{dt} = \frac{G}{45c^5}\dddot{Q}_{ik}\dddot{Q}^{ik}. \tag{10'}$$

Gravitational waves carry away from the system not only energy but also *angular momentum*. The conservation laws that follow from the exact theory and take into account radiation are formulated namely as follows: the decreases in the energy and the angular momentum are equal to the amounts carried away by the waves.

Thus, gravitational waves are an inescapable consequence of the relativistic theory of gravitation. Under conditions when they can be meaningfully compared with electromagnetic waves (weak gravitational waves on an unchanged or slowly varying background, and also special cases of waves with arbitrary amplitude), they exhibit a far-reaching analogy. It is true that sometimes the analogy does not hold objectively. Perhaps one of the most remarkable examples of this kind, which leads to interesting physical consequences, is the conformal invariance of the electrodynamical equations and the conformal noninvariance of the gravitational wave equations (Chap. 7).

3. TEST PARTICLES IN THE FIELD OF A GRAVITATIONAL WAVE

Just as an electromagnetic wave sets test charges in motion, a gravitational wave causes test masses to move. The motion of charges in the field of an electromagnetic wave is usually described with respect to an inertial coordinate system. The motion of test masses in the field of a gravitational wave can be described with respect to a coordinate system that most closely approaches an inertial one, i.e., a locally inertial frame.

Let us recall how charges move in the field of a plane monochromatic electromagnetic wave.[4] If the wave propagates in the direction of the x axis, the projection of the path of a charge that is at rest on the average onto the yz plane is an ellipse that degenerates into a segment of a straight line or a circle for linearly polarized and circularly polarized waves, respectively. In the general case, there is also displacement of the charge along the x axis, which is absent only in the case of a circularly polarized wave.

In a coordinate system moving with respect to the original system along the x axis, the intensity of the wave and its frequency are changed but the ratio of the characteristic dimension ξ of the ellipse to the wavelength λ remains unchanged, and it always much less than or of the order of unity: $\xi/\lambda = h \lesssim 1$. It is natural to adopt this ratio as the invariant dimensionless amplitude of the wave and say that the wave is weak if $h \ll 1$. (The parameter h can be readily related to the dimensionless ratio $eE_0/mc\omega$, which can also be used to estimate the strength of the wave.[18]) For a weak wave, the displacements of the charge in the yz plane are small compared with λ, but the displacements along the x axis are considerably smaller, and they cannot exceed $x \sim \xi(\xi/\lambda)$.

Let us now consider the motion of particles in the field of a weak plane gravitational wave. Using gauge conditions analogous to those employed in electrodynamics, we can write the wave metric in the form

$$ds^2 = c^2dt^2 - dx^{1^2} - (1-a)dx^{2^2} - (1+a)dx^{3^2} + 2b\,dx^2dx^3. \tag{11}$$

There are only two nonvanishing independent components: $\psi_{23} = h_{23} = b$, $\psi_{22} = h_{22} = -\psi_{33} = -h_{33} = a$. For a monochromatic wave

$$a = h_+ \sin[q(x^0 - x^1) + \psi_+], \quad b = h_\times \sin[q(x^0 - x^1) + \psi_\times],$$

and without loss of generality we can assume $\psi_+ = \psi_\times - \pi/2 = \psi$. The two states of linear polarization correspond to the choices $h_\times = 0$ or $h_+ = 0$; the two states of circular polarization, to the choices $h_\times = \pm h_+$.

The world lines of free test particles are geodesics of the spacetime (11).[19] These include the world lines $x^i = \text{const}$, which realize the particular coordinate system in which the metric (11) is expressed.

We introduce a locally inertial frame of reference $\bar{x}^\alpha$ associated with, say, the world line $x^1 = x^2 = x^3 = 0$. Along this world line, the metric (with allowance for small terms of order h_+, $h_\times$) must take the usual form of the Minkowski metric, and all the first derivatives of the metric must vanish. A coordinate transformation satisfying this requirement is

$$\left.\begin{aligned}
\bar{x}^0 &= x^0 + \frac{1}{4}\frac{\partial a}{\partial x^0}(x^{3^2} - x^{2^2}) - \frac{1}{2}\frac{\partial b}{\partial x^0}x^2x^3,\\
\bar{x}^1 &= x^1 + \frac{1}{4}\frac{\partial a}{\partial x^0}(x^{3^2} - x^{2^2}) - \frac{1}{2}\frac{\partial b}{\partial x^0}x^2x^3,\\
\bar{x}^2 &= x^2 - \frac{1}{2}ax^2 - \frac{1}{2}bx^3 + \frac{1}{2}\frac{\partial a}{\partial x^0}x^1x^2 + \frac{1}{2}\frac{\partial b}{\partial x^0}x^1x^3,\\
\bar{x}^3 &= x^3 - \frac{1}{2}bx^2 + \frac{1}{2}ax^3 - \frac{1}{2}\frac{\partial a}{\partial x^0}x^1x^3 + \frac{1}{2}\frac{\partial b}{\partial x^0}x^1x^2,
\end{aligned}\right\} \tag{12}$$

where a, b, $\partial a/\partial x^0$, $\partial b/\partial x^0$ are taken at the points $x^1 = x^2 = x^3 = 0$.

The transformed metric is

$$\bar{g}_{\mu\nu} = \eta_{\mu\nu} + O_{\mu\nu}\left(h\frac{\bar{x}^i\bar{x}_i}{\lambda^2}\right),$$

where $h \sim \sqrt{h_+^2 + h_\times^2}$ is the characteristic value of the wave amplitude and $\lambda = 2\pi/q$ is the wavelength.

Let us now consider what is the trajectory in the locally inertial system of a nearby free particle which is at rest with respect to the system of coordinates x^α. Suppose $x^i = l^i = \text{const}$, and $|l^i| \ll \lambda$. It then follows from (12) that with respect to the system of coordinates $\bar{x}^\alpha$ the particle moves in an ellipse, which can be regarded as the result of composition of three harmonic motions with the same frequency along the directions $\bar{x}^1$, $\bar{x}^2$, $\bar{x}^3$. On the average, the particle is at rest with respect to the center of the ellipse with coordinates l^1, l^2, l^3. We restrict ourselves to considering particles that on the average are at rest in the $\bar{x}^2\bar{x}^3$ plane, i.e., we set $l^1 = 0$. The paths of these particles are described by the equations

$$\left.\begin{aligned} \bar{x}^1 &= \frac{1}{2} q h_\times l^2 l^3 \sin(qx^0 + \psi) + \frac{1}{4} h_+ (l^{3^2} - l^{2^2}) \cos(qx^0 + \psi), \\ \bar{x}^2 &= l^2 - \frac{1}{2} h_+ l^2 \sin(qx^0 + \psi) - \frac{1}{2} h_\times l^3 \cos(qx^0 + \psi), \\ \bar{x}^3 &= l^3 + \frac{1}{2} h_+ l^3 \sin(qx^0 + \psi) - \frac{1}{2} h_\times l^2 \cos(qx^0 + \psi), \end{aligned}\right\} \quad (13)$$

and x^0 is related to $\bar{x}^0$ by

$$\bar{x}^0 = x^0 + \frac{1}{2} q h_\times l^2 l^3 \sin(qx^0 + \psi) + \frac{1}{4} q h_+ (l^{3^2} - l^{2^2}) \cos(qx^0 + \psi).$$

If the wave is not linearly polarized, then each individual particle moves in an ellipse in a plane to which the unit normal vector is basically oriented along the $\bar{x}^1$ axis. The projection of the path onto the $\bar{x}^2\bar{x}^3$ plane is also an ellipse with the semi-axes

$$A = \frac{1}{2} h_\times \sqrt{l^{2^2} + l^{3^2}}, \quad B = \frac{1}{2} h_+ \sqrt{l^{2^2} + l^{3^2}}.$$

The orientation of the principal directions with respect to the $\bar{x}^2$ axis is determined by the angle θ: $\tan\theta = -l^3/l^2$.

If the gravitational wave is linearly polarized, the elliptic path of the particle lies in a plane perpendicular to the $\bar{x}^2\bar{x}^3$ plane, and the projection of the path onto that plane degenerates into the segment of a straight line. At certain points, there is no displacement of the particle along the $\bar{x}^1$ axis (Fig. 1).

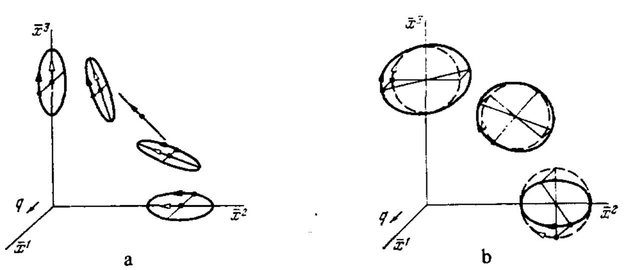

FIG. 1. Motion of a particle in the field of a gravitational wave. The black arrow indicates the spatial motion of the particle; the open arrow, the projection of the trajectory onto the $\bar{x}_2\bar{x}_3$ plane. In Fig. a) the wave is linearly polarized (+polarization, $h_\times = 0$); in Fig. b), circularly polarized (clockwise, $h_\times = h_+$).

The characteristic dimension ξ of the ellipse in the $\bar{x}^2\bar{x}^3$ plane depends on the distance $\rho = \sqrt{l^{2^2} + l^{3^2}}$ to the coordinate origin, and therefore ξ/λ is not an invariant parameter of the wave. But the ratio ξ/ρ does not depend on ρ, is proportional to h, and, as in electrodynamics, gives the invariant dimensionless amplitude of the wave. Under Lorentz transformations, the frequency of the wave and its strength (the components of the curvature tensor) do change, but ξ/ρ does not. The amplitude of the oscillations of the particle along the $\bar{x}^1$ axis is much less than ξ, and it does not exceed $\bar{x}^1 \sim h\rho^2/\lambda \sim \rho(\xi/\lambda)$, which again recalls the situation with an electromagnetic wave.

A particle which moves on the average with respect to the locally inertial frame has periodic deviations from the average direction of the motion. The energy of the particle also changes along the path. The periodic changes of the energy and the position of the particle are small (proportional to h), but they can be cumulative, causing the particle to move along a definite trajectory. To investigate these processes, it is more convenient to use the coordinate system (11), although the effect itself does not of course depend on the method of calculation.

Let us begin with the systematic variation of the energy (frequency).[19-22] Suppose a particle (or photon) moves between a pair of free perfectly reflecting mirrors, whose world lines are $x^i_{\rm I} = 0$ and $x^i_{\rm II} = \text{const}$. At the point of reflection, the energy of the particle ck^0 and the tangential components of the momentum with respect to the mirror do not change, while the normal component changes sign. A systematic change of the energy is possible if after reflection from the second mirror the particle returns to the first mirror after exactly a period of the gravitational wave, having moreover a different energy. Then, choosing the orientation of the first mirror in such a way that the particle again sets off along the same trajectory, everything can be repeated again from the start. The energy increments will then be added.

Between the reflections, the world line of the particle is described by geodesic segments. It is clear that in the principal approximation the distance L between the mirrors must be equal to $n\pi v/qc$, where n is an integer and $v = \sqrt{u^{1^2} + u^{2^2} + u^{3^2}}$ is the velocity of the particle. For $n = 1$ and $v = c$, the distance L is $\lambda/2$, while for $v < c$ we have $L < \lambda/2$. Matching segments of geodesics subject to the necessary conditions for the components of the four-momentum, we can find the increment $\Delta\omega$ after one reflection and return of the particle. The same quantity can be determined by comparing the difference between the times of departure and return of a pair of particles that follow one another. Calculating $\Delta\omega$ in one way or another, we find (for $n = 1$)[19]

$$\frac{\Delta\omega}{\omega} = 2\frac{u^1}{c}\frac{1}{c^2 - u^{1^2}}\sqrt{4h_\times^2 u^{2^2} u^{3^2} + h_+^2 (u^{2^2} - u^{3^2})}\cos\left(\frac{\pi}{2}\frac{u^1}{c}\right) \times \cos\left(-\frac{\pi}{2}\frac{u^1}{c} + \psi + \varphi\right), \quad (14)$$

where $\tan\varphi = -h(u^{2 2} - u^{3 2})/2h_\times u^2 u^3$. It can be seen from this expression that there is no effect if $u^1 = 0$ or $u^2 = u^3 = 0$, and also in certain other cases, depending on the polarization and the phase of the wave at the time of departure of the particle. The maximal value of $\Delta\omega/\omega$ is achieved when the particle moves at a certain angle to the direction of propagation of the wave. After Q reflections, the total ratio $\Delta\omega/\omega$ exceeds (14) by the factor Q and in order of magnitude is $hQ(v/c)^2$ for $v \ll c$, while for $v \approx c$

$$\frac{\Delta\omega}{\omega} \sim hQ. \qquad (15)$$

We now consider the accumulation of deviations of the particle from the mean direction of motion, which leads to a systematic drift of a particle.[19]

Suppose the surfaces $x^2 = 0$ and $x^2 = L$ are perfectly reflecting mirrors. When the moving particle is reflected by them, its momentum components k^0, k^1, k^3 do not change, while k^2 changes sign. Suppose that at time $x^0 = 0$ a particle leaves the point $x^i = 0$ strictly along the normal to the mirror. In other words, its four-momentum with allowance for terms of order h is $k^\alpha = \{\omega/c, 0, \omega v/c^2, 0\}$. It is necessary to choose L equal to $(n\pi/q)v/c$ to ensure that the particle returns to the first mirror when $x^0 - x^1 = n2\pi/q$, i.e., at the same phase of the gravitational wave. It is sufficient to require fulfillment of this condition in the principal approximation since $x^0 - x^1$ occurs as argument of the harmonic functions, which already have the small factor h_+ or $h_\times$. If the point of return is shifted with respect to the point of setting out, repeated reflections can increase this displacement. Fitting the segments of the geodesics, one can readily show that on its return after one reflection the particle will have the coordinates

$$x^1 = -\frac{n\pi}{q}\left(\frac{v}{c}\right)^2 h_+ \sin\psi, \quad x^2 = 0,$$
$$x^3 = 2\frac{v}{qc}[(-1)^n - 1] h_\times (\sin\psi - n\pi\cos\psi),$$
$$x^0 = n\frac{2\pi}{q}(1 + h_+ \sin\psi) - \frac{n\pi}{q}\left(\frac{v}{c}\right)^2 h_+ \sin\psi.$$

The four-momentum of the returning particle is

$$k^0 = \frac{\omega}{c}, \quad k^1 = 0, \quad k^2 = -\frac{\omega v}{c^2}, \quad k^3 = 2\frac{\omega v}{c^2}[(-1)^n - 1] h_\times \cos\psi.$$

The shift along the x^3 axis and the appearance of the component k^3 are due to the $\times$ polarization of the gravitational wave. They can be avoided by taking the number n even.

We restrict ourselves to considering the displacement along the x^1 axis. After one reflection, the displacement is $\Delta x^1 = -L(v/c)h_+ \sin\psi$, i.e., depending on ψ, the particle is displaced toward positive or negative x^1.

Thus, the system of two mirrors in the field of the gravitational wave is capable of "sorting" particles according to the time of their setting off, shifting them in opposite directions. There is no drift along the x^1 axis for $\psi = 0$ or $\psi = \pi$. These are cases when the mean trajectory of a particle that sets off along to the normal to the mirror is also normal to it. In order to obtain an effect in these cases as well, it is necessary to arrange the mirrors in such a way that the x^1 component of the normal is nonzero.

In the case of photons, the smallest admissible distance between the mirrors is $L = \lambda/2$. After Q reflections, the point of return of the photon is shifted with respect to the point of setting off by the distance Δl, which in order of magnitude is determined by

$$\frac{\Delta l}{\lambda} \sim hQ. \qquad (16)$$

Study of the motion of individual particles makes it possible to understand how a wave affects more complicated detecting systems.

4. DETECTION OF GRAVITATIONAL WAVES

Many theoretical and experimental studies have been made of the detection of gravitational waves. The main ideas and reviews of the present state of the problem can be found in the books[5,23–26] and the papers[1,27,28]. We shall give only a brief exposition of the basic ideas, and we shall consider a new and promising direction—the detection of gravitational waves by means of electromagnetic systems. The estimates made in this and the following sections may differ from the exact expressions by numerical coefficients, but they cannot contain large or small dimensionless ratios.

The mirrors considered in the previous section, which force a particle to move along a definite path, are a special case of a restoring force. Another example could be an elastic element (mechanical or electromagnetic), joining particles. If the frequency with which the restoring force acts is equal to the frequency of the monochromatic wave, then after Q periods of the wave the effect produced by the wave is multiplied by the factor Q (cf (15) and (16)).

An elementary oscillator provides the basis for studying the detection of gravitational waves. The picture must be augmented by two further important points: analysis of the inherent noise of the oscillator antenna and the study of its response to a wave field of arbitrary nature, i.e., not necessarily a strictly monochromatic wave. But these are only the first problems encountered by the experimentalist. It is also necessary to analyze subtle questions relating to the realization of a detector of small measured quantities, its noise, the back reaction on the antenna, the optimal strategy of measurements, etc.[24,29]

The interaction of a weak wave field with a gravitational antenna is described by generally covariant equations, in which one uses the metric (11) or a more complicated one, depending on the assumptions made about the structure of the gravitational field. For a solid-state antenna, one uses the generally covariant equations of elastic vibrations; for an electromagnetic antenna, the generally covariant Maxwell equations, etc. Essentially, the problem reduces to the equation of forced oscillations, in which the driving force is provided by the terms associated with the gravitational wave. Applied to an elementary mechanical oscillator considered in a locally inertial coordinate system, we obtain[23]

$$\frac{d^2\xi^i}{dt^2} + \frac{\omega_0}{Q}\frac{d\xi^i}{dt} + \omega_0^2\xi^i = -c^2 R^i_{0k0} l^k, \tag{17}$$

where R^i_{0k0} are the components of the curvature tensor of the wave field; $\pm l^k$ are the "unperturbed" coordinates of the oscillator masses; ξ^i are the small displacements of a mass from its equilibrium position; ω_0 is the natural frequency of the oscillator; and Q is its quality factor. For distributed systems, ω_0 is related to the velocity of sound v_s and the length l by the relation $\omega_0 \sim v_s/l$.

In the simplest case of a monochromatic wave, the right-hand side of Eq. (17) is proportional to $hl\Omega^2 e^{i\Omega t}$ (where Ω is the frequency of the gravitational wave) and angular factors that describe the orientation of the oscillator.

For actually existing sources, h is so small that the gravitational force on the right-hand side of (17) is comparable with the fluctuation force of Brownian motion in the antenna.

Under the influence of the inevitable thermal fluctuations, an oscillator of mass m executes randomly modulated vibrations with frequency ω_0. The mean square amplitude of the vibrations is $\overline{A_B^2} = kT/m\omega_0^2$, where T is the temperature and k is Boltzmann's constant. The amplitude and phase of the thermal vibrations change significantly over a time of the order of the relaxation time: $\tau^* \sim Q/\omega_0$. Over the time $\tau \ll \tau^*$ the probability of significant changes is very small. For $\omega_0^{-1} \ll \tau \ll \tau^*$, the probable increment of the amplitude is proportional to $\sqrt{\tau/\tau^*}$:

$$\Delta A_B \sim \sqrt{\overline{A_B^2}}\sqrt{\frac{\tau}{\tau^*}}. \tag{18}$$

(This holds for $Q\hbar\omega_0 \ll kT$, where $\hbar$ is Planck's constant. For $\tau \sim \omega_0^{-1}$ and $Q\hbar\omega_0 > kT$, right down to $Q\hbar\omega_0 \sim kT$, the quantum-mechanical discreteness of the levels of the macroscopic oscillator makes itself manifest, and Eq. (18) does not hold.[24] For $kT \sim \hbar\omega_0$ and $\tau \sim \tau^*$, it is possible to replace kT in order of magnitude by $\hbar\omega_0$: $\Delta A_B \sim \sqrt{\overline{A_B^2}} \sim \sqrt{\hbar\omega_0/m\omega_0^2}$.)

The actual motion of the oscillator is formed by superposition of its thermal vibrations and the displacement due to the gravitational wave. The response of the oscillator and its limiting sensitivity depend on the nature of the gravitational signal. In the general case, the signal changes both the amplitude and the phase of the thermal vibrations of the oscillator, and, depending on the instantaneous value of the phase, it may happen that only the amplitude changes or only the phase of the vibrations, and for this the experimentalist must be prepared. One can distinguish three typical regimes of variation of the potentials $h_{\mu\nu}$: a) monochromatic wave of frequency ω_0 with characteristic time of variation of the amplitude and frequency satisfying $\hat{\tau} \gg \tau^*$; b) short pulse with duration $\hat{\tau} \sim \omega_0^{-1}$; c) prolonged indeterminate signal (wide-band or narrow-band noise).

Under the influence of signal a), the amplitude of the oscillator may change in a time $\tau \ll \tau^*$ by the amount $\Delta\xi = hl\omega_0\tau$. During a time of order τ^*, it reaches the maximal value hlQ. The fluctuation drift of the amplitude in time τ is determined by Eq. (18). The threshold condition of detection $\Delta\xi \gtrsim \Delta A_B$ for measurement time $\tau_d = \tau$ has the form

$$h \gtrsim \frac{\sqrt{\overline{A_B^2}}}{l}\frac{1}{\sqrt{Q\omega_0\tau_d}}. \tag{19}$$

It is advantageous to use the maximal possible observation time. For $\tau_d = \tau^*$, we obtain from (19) the inequality $h \gtrsim \sqrt{\overline{A_B^2}}/lQ$, or, in other words, the increment of the oscillator energy during this time:

$$\Delta\varepsilon \sim m(\Delta l\omega_0)^2 \sim (hQ)^2 ml^2\omega_0^2 \sim (hQ)^2\left(\frac{v_s}{c}\right)^2 mc^2$$

must be greater than kT:

$$(hQ)^2\left(\frac{v_s}{c}\right)^2 mc^2 \gtrsim kT. \tag{20}$$

If one can make measurements during n intervals τ^*, the smallest detectable h is

$$h_{\min} \gtrsim \frac{\sqrt{\overline{A_B^2}}}{lQ}\frac{1}{\sqrt{n}}.$$

Let us now consider a signal of type b). A pulse of duration $\hat{\tau} \sim \omega_0^{-1}$ has a wide frequency band $\Delta\omega \sim \hat{\tau}^{-1} \sim \omega_0 \gg \omega_0/Q$. We separate the resonance harmonic $h(\omega_0)$. Substituting $\tau_d \sim \hat{\tau} \sim \omega_0^{-1}$ into (19), we find

$$h(\omega_0)_{\min} \gtrsim \frac{\sqrt{\overline{A_B^2}}}{l}\frac{1}{\sqrt{Q}}. \tag{21}$$

Thus, an oscillator with very high Q makes it possible to detect a short signal which gives rise to a change in the amplitude that is a small fraction of the equilibrium value $\sqrt{\overline{A_B^2}}$. At the same time, the amount of energy ΔE which is deposited or extracted by the pulse depends on the instantaneous value of the amplitude and phase of the oscillator[24] and may be $\Delta\mathscr{E} \sim h^2(v_s/c)^2mc^2$ or $\Delta\mathscr{E} \sim h(v_s/c)(v/c)mc^2$, where $v = \sqrt{\overline{A_B^2}}\,\omega_0$. The large difference in the possible $\Delta\mathscr{E}$ does not however change the detection condition (21). For the earliest Weber type antennas ($kT \approx 4\cdot 10^{-14}$ erg, $m \approx 10^6$ g, $\omega_0 \approx 10^4$ sec^{-1}, $l_{\rm eff} \approx 50$ cm, $Q \approx 2\cdot 10^5$) the potential sensitivity was $h(\omega_0)_{\min} \sim 10^{-18}$, which presupposes the possibility of measuring displacements at the level $\Delta l \sim hl \sim 5\cdot 10^{-17}$ cm.

Finally, let us consider a gravitational signal of type c). If in the range $\omega_0 \pm \Delta\omega$ of the noise intensity spectrum the mean square amplitude of the metric is $\overline{h^2}(t, \omega_0 \pm \Delta\omega)$, then over time τ^* the amplitude of the oscillator reaches in order of magnitude the same maximal value $\sqrt{\overline{h^2}}lQ$ as in the case of a strictly monochromatic wave with amplitude $h = \sqrt{\overline{h^2}}$. In order to distinguish the gravitational-wave noise on the background of the thermal noise of the oscillator, it is necessary to know statistical regularities that distinguish the one noise from the other. It is expedient to use various correlation schemes.[30] (It is more complicated to eliminate the influence of local gradients of the Newtonian gravitational field.[31]) During an observation time containing n intervals τ^* one can in principle detect a signal that gives rise to a change in the amplitude that is a fraction of $\sqrt{\overline{A_B^2}}$, so that (see also[23,32])

$$\sqrt{\overline{h^2(t, \omega_0 \pm \Delta\omega)}} \gtrsim \frac{\sqrt{\overline{A_B^2}}}{lQ} \frac{1}{\sqrt{n}}. \quad (22)$$

At the present time, the efforts of experimentalists are mainly directed toward the detection of signals of type b), which accompany cosmic catastrophes such as the collision of superdense objects or asymmetric collapse. In order to ensure a reasonable frequency of events (a few per month), one must be prepared to detect gravitational bursts from objects in galaxies within a radius ~ 10 Mpc. This presupposes an increase in the sensitivity of the detectors to the level $h(\omega_0)_{\min} \sim 10^{-20} - 10^{-21}$. The difficulty resides not so much in achieving this potential sensitivity by reducing T and increasing Q, l, and m as in creating sensors of small displacements capable of registering $\Delta l \sim 10^{-19}$ cm – 10^{-20} cm. It is to be hoped that this program will, nevertheless, be successful.

A locally inertial system is convenient for considering detectors that are small in size compared with the gravitational wavelength. This is the case in a solid-state antenna, since $l/\lambda \sim v_s/c \ll 1$. Gravitational waves can also be detected by electromagnetic resonators,[22] whose dimensions are, because electromagnetic and gravitational waves propagate at the same velocity, comparable with λ if the resonance phenomenon is used. In this case, it is more convenient to use a synchronous coordinate system with metric of the type (11). From the generally covariant Maxwell equations

$$F_{\alpha\beta,\,\gamma} + F_{\gamma\alpha,\,\beta} + F_{\beta\gamma,\,\alpha} = 0, \quad F^{\alpha\beta}_{;\beta} = -\frac{4\pi}{c} j^{\alpha}, \quad (23)$$

it is easy to obtain a generalization of the ordinary wave equation to the case when external gravitational fields are present:

$$F_{\mu\nu;\,\alpha}{}^{;\alpha} + R_{\mu\nu\alpha\beta}F^{\alpha\beta} + R^{\alpha}_{\mu}F_{\nu\alpha} + R^{\alpha}_{\nu}F_{\alpha\mu} = \frac{4\pi}{c} j_{[\mu;\,\nu]}. \quad (24)$$

If the gravitational field is weak, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and satisfies the gauge conditions (5′), then Eq. (24) takes the form

$$F_{\mu\nu,\,\alpha}{}^{,\alpha} + \frac{4\pi}{c} j_{[\mu,\,\nu]} = h^{\alpha\beta}F_{\mu\nu,\,\alpha,\,\beta} - (h^{\alpha\beta,\,\gamma} + h^{\beta\gamma,\,\alpha} - h^{\alpha\gamma,\,\beta})\,\eta_{\alpha[\mu}F_{\nu]\beta,\,\gamma} - h_{\alpha[\mu,\,\nu],\,\beta}F^{\beta\alpha}. \quad (25)$$

For detection, one can use either a free electromagnetic field that is not confined by reflecting walls (which is equivalent to free masses) or a resonator field (which is equivalent to a mechanical oscillator). The problem is most readily solved for an ideal resonator having perfectly conducting walls and a nonconducting dielectric. Such a resonator has infinite Q. However, every real resonator has a finite Q, which can be attributed to the effective conductivity of the dielectric σ: $\sigma = \omega/4\pi Q$. Then the currents in Eq. (25) can be expressed in terms of the electric field strength, $j_i = -\sigma F_{0i}$, and the walls of the resonator can be regarded as perfect, i.e., the boundary conditions on them can be formulated in the form $F_{0i(\text{tang})} = E_{(\text{tang})} = 0$. Since we assume that the frequency of the gravitational wave is equal to the eigenfrequency of the electromagnetic field in the resonator, the elements of the casing of the resonator behave as free particles under the influence of the wave. Indeed, the ratio of the lowest eigenfrequencies of the casing of the resonator (ω_s) and of the electromagnetic field in it (ω) is v_s/c and therefore $\omega_s \ll \omega \sim \Omega$ and the elastic force in the equations of motion of the casing of the resonator can be ignored compared with the gravitational force. The boundary conditions for the field in the resonator are specified on the world lines of the elements of its wall. In the synchronous coordinate system, they are described by the simple equations $x^i = \text{const}$.

The $F_{\mu\nu}$ are the sum of the "unperturbed" field ${}^{(0)}F_{\mu\nu}$ and the correction ${}^{(1)}F_{\mu\nu}$, which is due to the influence of the gravitational wave. Equations (25) can be conveniently solved for the ${}^{(1)}F_{0i}$, and the remaining components ${}^{(1)}F_{ik}$ determined from (23). Expanding the solution with respect to the eigenfunctions of the unperturbed boundary-value problem, ${}^{(1)}F_{0i} = \sum_n E_n(t)\psi_n(x, y, z)$, we obtain from (25) the equation of a damped oscillator with driving force consisting of the terms on the right-hand side of Eq. (25):

$$\ddot{E}_n + \frac{\omega_n}{Q}\dot{E}_n + \omega_n^2 E_n = f(t). \quad (26)$$

The actual expression for the force depends on the original field ${}^{(0)}F_{\mu\nu}$, the form of the gravitational wave, the relation between their phases and frequencies, and so forth. If the resonator contains a natural oscillation of the electromagnetic field (standing wave) with frequency ω_n, then the interaction with a gravitational wave of frequency Ω produces a driving force at the frequencies $\omega_m = \Omega \pm \omega_n$. A resonance phenomenon occurs when ω_m is a natural frequency, and the effect is the larger, the lower is the natural frequency excited.

We distinguish three characteristic cases:

a) The original field ${}^{(0)}F_{\mu\nu}$ is a constant field ($\omega_n = 0$) with strength H.

b) ${}^{(0)}F_{\mu\nu}$ is a natural oscillation of frequency $\Omega/2$ and characteristic strength E.

c) ${}^{(0)}F_{\mu\nu}$ is the sum of a constant field H and a weak natural oscillation with characteristic strength $E_1 \ll H$ and frequency Ω.

In case a), a standing wave with frequency Ω appears against the background of the constant field. The amplitude ${}^{(1)}E$ increases with time and after the time $\tau^* \sim Q/\Omega$ reaches the maximal value ${}^{(1)}E \sim hQH$. In order to calculate the change ΔE in the total energy of the electromagnetic field in the resonator, it is necessary to find all the components ${}^{(1)}F_{\mu\nu}$ and integrate the energy density over the volume $V \approx (c\pi/\Omega)^3$. The terms linear in h then disappear, and we obtain an expression of the form

$$\Delta \mathcal{E} \sim (hQ)^2\,\mathcal{E}, \quad (27)$$

where $\mathcal{E} = (H^2/8\pi)\,V \sim H^2(c/Q)^3$. We shall assume that the detection condition is satisfied if $\Delta E \gtrsim \hbar\Omega$ (cf (20)). This corresponds to the assumption that the portion of energy in one or several quanta $\hbar\Omega$ can be distinguished on the background of the electromagnetic fluctuations of frequency Ω. Then the detectable h is

$$h \gtrsim \frac{1}{Q}\sqrt{\frac{\hbar\Omega^4}{H^2c^3}}. \quad (28)$$

For $H=10^5$ G, $Q=10^{12}$, $\Omega=\pi c/l\sim10^9$ sec^{-1} we obtain the fairly low detectable $h\sim10^{-27}$ and corresponding flux $I\approx10$ erg $\cdot$ sec$^{-1}\cdot$ cm^{-2}, but, unfortunately, periodic astrophysical sources in this range are not known, and the possibilities of laboratory emitters for this level are still inadequate (see Sec. 5).

In case b) the gravitational wave leads to the appearance of an extra field at the same frequency $\Omega/2$ at which the original field exists. The oscillations will be added, and, depending on the relationship between the phases, this will change the amplitude or the phase of the field ${}^{(0)}F_{\mu\nu}$. If the amplitude is changed, then at time τ^* we shall have ${}^{(1)}E\sim hQE$. The total energy in the resonator, apart from the damping of the original field in accordance with the Q factor of the resonator, will change by the additional small quantity

$$\Delta\mathscr{E}\sim(hQ)\,\mathscr{E},\tag{29}$$

where $\bar{\mathscr{E}}=(E^2/4\pi)\,V\sim E^2(c/\Omega)^3$. Although here ΔE is linear in the small hQ (in contrast to (27)), this does not improve the detection possibilities since the resonance mode contains a large number of already existing quanta, of order $N\sim\bar{\mathscr{E}}/\hbar\Omega$. The signal can be regarded as detected if $\Delta\mathscr{E}/\hbar\Omega\gtrsim\sqrt{N}$. Then for the detectable h we obtain a condition of the same type as (28): $h\gtrsim(1/Q)\sqrt{\hbar\Omega^4/E^2c^3}$.

Finally, in case c) the gravitational wave, which interacts mainly with the strong constant field, will give rise to an oscillation at the frequency Ω, which is added to the original wave. During the relaxation time, the accumulated energy is $\Delta\mathscr{E}\sim(hQ)(H/E_1)\bar{\mathscr{E}}$, where $\bar{\mathscr{E}}\sim E_1^2(c/\Omega)^3$ is the total energy concentrated in the unperturbed oscillating field. Writing the detection condition in the form $\Delta\mathscr{E}/\hbar\Omega\geq\sqrt{\bar{\mathscr{E}}/\hbar\Omega}$, we again obtain (28). In case c), as in case b), the damping with time of the original alternating field is somewhat changed by the gravitational wave, and this can also be described as a small change in the Q of the resonator: $\Delta Q\sim(hQ)\,Q$.

Thus, the general principles for detecting gravitational waves by either electromagnetic or mechanical systems are the same. It is clear that an electromagnetic resonator is more convenient for detecting short waves, and a mechanical oscillator for long waves. Incidentally, long waves can be detected synchronously by compact electromagnetic systems realized in the form of oscillatory circuits with low natural frequencies. The efficiency of any particular type of device is of course determined by the achievable technical parameters, the simplicity of preparation, the cost, etc. A very valuable idea may be that of "quantum nondemolition (nondisturbative) measurements,"* which may make it possible to detect individual quanta in the radio range.[33]

A tremendous number of original suggestions for detecting gravitational waves have been made. They include the use of neutral and charged particles, liquids and solids, various mechanical and electromagnetic systems, and so forth (a fairly detailed bibliography can be found, for example, in[1,5,27,34]). The quantities to be measured and the method of measurement differ, but every suggestion is based on the effect of a weak gravitational field on a free mass or oscillator. In every system, one can introduce an effective Q factor and inherent noise. If one is dealing with a monochromatic wave, the relative change of the quantity affected by the wave is always of order h or hQ (excepting, of course, trivial and useless cases). Mathematically, the detecting systems are described by equations with variable (because of the gravitational field) coefficients. The situation would change if one could create a system for which $hQ\sim1$ (which at present is far beyond the range of the possible). In such a case, one could achieve parametric resonance, when the increase in the energy of the system exceeds its dissipation, and the growth in the amplitude of the oscillations or vibrations is limited only by the nonlinearity of the system.

5. EMISSION OF GRAVITATIONAL WAVES

Hitherto the amplitude h of the gravitational-wave perturbation has remained undetermined. It is clear that under terrestrial conditions we can find only extremely small h of either laboratory or cosmic origin, but the actual value of h depends on the source. To get an idea of the "typical" quantities, let us suppose that in the center of the galaxy, i.e., at distance $R\approx3\cdot10^{22}$ cm from the Earth, a mass m of the order of the solar mass, with characteristic radius $l\sim r_g\sim Gm/c^2\sim3\cdot10^5$ cm is entirely transformed into gravitational radiation in a time $\tau\sim r_g/c\sim10^{-5}$ sec. Making a very crude estimate in accordance with (10), we find that the emitted power does not depend on m and is equal to $d\mathscr{E}_g/dt\approx c^5/G$, and the value of h at the characteristic frequency $\nu\sim c/r_g\sim10^5$ Hz is $h\sim c_g/R\sim10^{-17}$. A rigorous calculation will, most probably, only significantly reduce this quantity.

The theory of emission has been well developed for the case of a weak field (for a review of the different methods of calculation, see[35]). The retarded solution of Eq. (4′) is

$$\psi_{\alpha\beta}=-\frac{4G}{c^4}\int\frac{(\tau_{\alpha\beta})_{t'}}{R}\,dV,\tag{30}$$

where

$$\psi_{\alpha\beta}\equiv h_{\alpha\beta}-\frac{1}{2}\eta_{\alpha\beta}h_{\mu\nu}\eta^{\mu\nu},$$

and $\tau_{\alpha\beta}$ contains in addition to the energy-momentum tensor $T_{\alpha\beta}$ of the matter terms quadratic in $\psi_{\alpha\beta}$ as well if the emitting system is gravitationally bound.

The solution (30), found for an isolated source, automatically satisfies the conditions (5′). If the source has several components, i.e., consists of several parts with different $T_{\alpha\beta}$, it is sufficient to find ψ_{ik} from that one of the parts that makes the greatest contribution to ψ_{ik}, and calculate $\psi_{0\alpha}$ directly from (5′).[36] From known $\psi_{\alpha\beta}$ in the wave zone one can find the energy flux and the total emitted power; for this, it is necessary to know only the "physical" components of $\psi_{\alpha\beta}$ satisfying the conditions (5′), (7′), and (8). If the dimensions of

*Translator's footnote. These are the English expressions coined by Prof. Braginskii.

the source are small compared with the wavelength, the calculation of ψ_{ik} reduces to finding the second derivatives with respect to the time of the quadrupole moment, and the total power is expressed by Eq. (10′).

Let us compare the efficiency of mechanical and electromagnetic emitting systems, doing this in a form suitable for estimating the possibilities of a laboratory experiment.[37] We do not write out the indices of the field components, nor numerical coefficients. We consider elementary mechanical (m) and electromagnetic (e) emitters that produce gravitational radiation at the same wavelength λ. The expression "elementary" means that in the first case the dimensions of the emitters are of order λ_s, and in the second λ_e, where λ_s is the wavelength of an acoustic wave, and λ_e is that of an electromagnetic wave. As examples of elementary emitters we can take a vibrating solid beam and a standing electromagnetic wave in a resonator. We shall compare the amplitude of the gravitational-wave field at the boundary of the wave zone. We take the factor $1/R$ outside the integral in (30) and replace it by $1/\lambda$. For $R > \lambda$, the wave amplitude decreases as $1/R$.

Let A be the amplitude of elastic vibrations in an elementary m emitter. The components T_{ik} are the elastic stress tensor σ_{ik}, which is proportional to the spatial derivatives of the displacement vector. The proportionality factor, the modulus of elasticity, can be expressed in terms of the material density ρ_m and v_s^2. Therefore, the characteristic amplitude of the tensor σ_{ik} is $\sigma_m \sim \rho_m v_s^2 A/\lambda_s$. Since $\omega \sim c/\lambda \sim v_s/\lambda_s$, we have $\lambda_s \sim (v_s/c)\lambda \ll \lambda$. This means that the elementary m emitter is deep in the induction zone and the retardation in (30) is negligibly small. For the amplitude of the gravitational wave at the boundary of the wave zone, we obtain

$$h_m \sim \frac{G}{c^4}\frac{1}{\lambda}\sigma_m\lambda_s^3 \sim \frac{r_{gm}}{\lambda}\frac{v_s}{c}\frac{v}{c} \sim \frac{\tilde{r}_{gm}}{\lambda}\left(\frac{v_s}{c}\right)^2, \tag{31}$$

where $r_{gm} \sim G\rho_m\lambda_s^3/c^2$ is the gravitational radius of the m emitter, $\tilde{r}_{gm} \sim (G\rho_m\lambda_s^3/c^2)A/\lambda_s$ is the gravitational radius of the variable part of the emitter mass, $v \sim (A/\lambda_s)v_s$ is the velocity of motions in the source.

Applied to an astronomical, gravitationally bound system with mass M, radius R, and characteristic velocity $(v/c)^2 \sim GM/c^2R$ of the motions, Eq. (31) gives

$$h_M \sim \frac{r_{gM}}{\lambda}\left(\frac{v}{c}\right)^2, \tag{32}$$

where $r_{gm} \sim GM/c^2$. In the favorable case of a pair of stars of solar mass rotating with a period of 0.5 day and at a distance $3\cdot 10^{19}$ cm from the Earth, we obtain $h \sim 5\cdot 10^{-17}$ at the boundary of the wave zone and $h \sim 5\cdot 10^{-21}$ on the Earth and wave frequency $\nu \approx 10^{-5}$ Hz.

The emission of an isolated rotating star depends on the asymmetry with respect to the axis of rotation. In Eq. (32), r_g must refer to the "asymmetric" fraction of the mass. The most optimistic assumptions for the pulsar in the Crab lead to $h \sim 10^{-25}$ at the Earth with frequency $\nu \approx 60$ Hz.[1] However, the gradual drift in the frequency of the radiation makes it impossible to use a prolonged resonance separation of the signal from the noise unless the frequency of the antenna itself can be adjusted.

Finally, application of Eq. (32) to short gravitational bursts from strongly asymmetric explosions, collapses, or vibrations in neutron stars that have just formed, etc., leads in the case of objects with a solar mass in the center of the Galaxy to $h \sim 10^{-20} - 10^{-17}$ at characteristic frequencies $\nu \sim (1 - 10^5)$ Hz. (For a discussion of pulsed gravitational radiation, see also[38–42].) Let us now consider an elementary e emitter. Since $\omega \sim c/\lambda \sim c/\lambda_e$ in this case, the volume of the emitter is of order λ^3, and its size is at the limit of applicability of Eq. (30) without allowance for retardation. The amplitude σ_e of the electromagnetic stress tensor is equal in order of magnitude to the amplitude of the energy density of the alternating electromagnetic field ε_e. (If the field is the sum of an appreciable constant field with energy density ε^c and an additional alternating field with energy density $\tilde{\varepsilon}$, then the amplitude in which we are interested is $\sigma_e \sim \sqrt{\varepsilon^c\tilde{\varepsilon}}$.) At the boundary of the wave zone, we obtain

$$h_e \sim \frac{G}{c^4}\sigma_e\lambda^2 \sim \frac{r_{ge}}{\lambda}, \tag{33}$$

where $r_{ge} \sim G\rho_e\lambda^3/c^2$ is the gravitational radius of the e emitter and $\rho_e c^2 = \varepsilon_e$. The ratio of (31) to (33) is $h_m/h_e \sim (\rho_m/\rho_e)(v_s/c)^5 A/\lambda_s$. It is much less than unity for moderate values of the parameters here, so that an elementary e emitter is much more effective than an elementary m emitter. For example, if $\rho_m \sim 1$ g/cm^3, $\rho_e \sim 10^{-18}$ g/cm^3, $(v_s/c) \sim 10^{-5}$, $(A/\lambda_s) \sim 10^{-3}$ then $h_m/h_e \sim 10^{-10}$. However, the volume of the e emitter ($\sim\lambda^3$) is much greater than an m emitter's. Let us consider their comparative efficiency at equal volumes. In a volume λ^3 one can place $N \sim (\lambda/\lambda_s)^3 \sim (c/v_s)^3 \gg 1$ elementary m emitters. If their gravitational-wave fields are to be added in the region of the detector and not cancel each other, the emitters must be specially arranged in phase. Of course, in practice an m emitter with volume λ^3 need not consist of N individual bodies. It may be realized as a single body working coherently. Coherence can be achieved by means of an external influence, such as, for example, electrostriction, as Weber proposed.[23] However, the technical realization of such coherence is probably not a simple matter. We note that the coherence of an e emitter with volume $\sim\lambda^3$ is achieved automatically, since $\lambda_c \sim \lambda$.

Suppose that the coherence of the m emitter is realized to the same extent as is achieved automatically in an e emitter of the same volume ($\sim\lambda^3$). Then the coherent m emitter produces at the boundary of the wave zone

$$h_{mc} \sim Nh_m \sim \frac{G}{c^4}\sigma_m\lambda^2$$

and therefore

$$\frac{h_{mc}}{h_e} \sim \frac{\rho_m}{\rho_e}\left(\frac{v_s}{c}\right)^2\frac{A}{\lambda_s} \sim \frac{\sigma_m}{\sigma_e}.$$

Thus, the comparative efficiency of emitters of the same volume is determined by the ratio of the achievable stresses, but under the condition that the m emitter operates coherently.

The greatest stresses (at the limit of the static rupture point) for known materials lie in the region $\sigma_m \sim 10^9$ dyn/cm^2. Suppose they are realized in the dynamical regime in a coherent m emitter. Then it produces the same gravitational wave amplitude as an electromagnetic emitter of the same volume with characteristic fields $E \sim H \sim 10^5$ G. If one is speaking of the possible laboratory systems, the appreciably greater coherence volume and comparatively simple principles of preparation probably decide in favor of the electromagnetic variant.

Coherence of the emitter can also be realized in volumes appreciably exceeding λ^3. In this case, one could achieve interference "focusing" of gravitational radiation. (A detailed calculation of the emission produced by electromagnetic resonators can be found in[43,44].) Here we merely note that a review of the suggestions made for the use of macroscopic quantum objects as generators and detectors of gravitational waves is contained in[48a].

6. POSSIBILITIES OF A LABORATORY EXPERIMENT

It is clear that the first attempts to detect gravitational waves were directed toward natural and not artificial sources and detectors. Unfortunately, the Earth as a detector of waves of cosmic origin leads to a too large upper limit of the intensity, which exceeds even the estimates based on cosmological data.[23,45] The present stage is characterized by the creation of sensitive artificial gravitational antennas, but, as before, it relies on cosmic sources. These investigations were strongly stimulated by Weber's well-known observations.[46] However, with regard to Weber's experiments, the most remarkable thing would be if they had been confirmed, because they indicated quite fantastic processes for whose existence no serious astrophysical justifications could be found.[1,41] The main hopes of experimentalists are now concentrated on more probable but still rather exotic sources. There is no doubt that the detection capabilities will be improved and the "cosmic" program itself continued until signals from sources of one type or another have been reliably detected. It is however curious that the requirements that must be imposed on a detector of radiation from definitely existing astronomical sources do not greatly exceed those that we encounter when we consider essentially feasible laboratory variants (here, of course, we are talking about "orders of magnitude" and not "units").

The power of a laboratory emitter is negligible compared with that of a cosmic source, but it has the advantage that one can place it next to the detector, optimize the shape, use prolonged resonance accumulation of a signal, and so forth. If necessary, one can create an emitter with dimensions greatly exceeding λ, and "focus" the gravitational radiation. An important advantage is the possibility of controlling the emission, which makes the interpretation of the observations less ambiguous.

Various sources could produce either a monochromatic wave or short bursts of gravitational radiation (by, for example, the asymmetric explosion of a bomb[49b]). Although the amplitude of the gravitational field may be somewhat higher in the second case than the first, the impossibility of synchronous separation of the signal greatly reduces the use of this method, to say nothing of the fact that ultimately we are interested not so much in the fact of detection of gravitational radiation as the possibility of using it in physical experiments.

Let us describe one of the laboratory variants that includes an emitter and detector of electromagnetic type (for the details, see[44]). Their principles of operation and advantages were discussed in Chaps. 4 and 5. The experimental scheme is shown in Fig. 2.

In an emitting resonator of torus shape an alternating electromagnetic field that does not depend on the coordinate φ is produced at the natural frequency $\omega = ck$. The electromagnetic stresses emit a gravitational wave at the frequency $\Omega = 2\omega = cK$. Converging on the symmetry axis, the wave is then transformed into an outgoing wave, and, as a result of interference in the focal region of the emitter, a standing cylindrically symmetric gravitational wave is formed. The components of the gravitational field in the focal region have the form

$$\left.\begin{aligned} h_{xx} &= -\tfrac{1}{2} A \cos(\Omega t + \psi)\,[J_0(Kr) + \cos 2\varphi\, J_2(Kr)],\\ h_{yy} &= -\tfrac{1}{2} A \cos(\Omega t + \psi)\,[J_0(Kr) - \cos 2\varphi\, J_2(Kr)],\\ h_{zz} &= A \cos(\Omega t + \psi)\, J_0(Kr),\\ h_{xy} &= -\tfrac{1}{2} A \cos(\Omega t + \psi) \sin 2\varphi\, J_2(Kr), \quad h_{xz} = h_{yz} = 0, \end{aligned}\right\} \tag{34}$$

where J_0 and J_2 are Bessel functions. The components (34) satisfy the gauge conditions (5′), (7′), and (8). In cylindrical coordinates, the wave terms are expressed as follows:

$$h_{rr} = -A\cos(\Omega t + \psi)\,\frac{J_1(Kr)}{Kr}, \quad h_{\varphi\varphi} = -A\cos(\Omega t + \psi)\,\frac{dJ_1(Kr)}{d(Kr)},$$
$$h_{zz} = A\cos(\Omega t + \psi)\, J_0(Kr).$$

For a suitable choice of the size of the emitting resonator and the configuration of the electromagnetic field in it, the wave amplitude is equal to $A \approx (G/c^4)\,\mathcal{E}/R_1 \approx (1/2)\, r_g/R_1$, where $\mathcal{E} = (E^2/4\pi)\, V$ is the total energy of the electromagnetic field in the resonator, $r_g = 2G\mathcal{E}/c^4$ is the corresponding gravitational radius, E is the characteristic field strength in the resonator, and V is its volume. Thus, the complete volume of the emitter operates coherently. If other resonators are placed

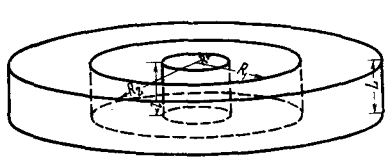

FIG. 2. Scheme of laboratory experiment with emission and detection of gravitational waves.

coaxially outside it, they give rise to the same amplitude A in the focal region, since the reduction of A by the greater distance of the resonator ($\sim 1/R_1$) is completely compensated by the increase in its radius and volume ($\sim R_1$). If the phase of the electromagnetic oscillations is correctly chosen, the contributions of all the coaxially arranged resonators are summed.

In the focal region of the emitter, a detector resonator of cylindrical form, radius R, and height l is placed. In it, for example, a constant magnetic field H oriented along the z axis is produced. The gravitational field (34) excited an alternating electromagnetic field at the resonance frequency Ω. Of Eqs. (25), the only nontrivial one is the equation for ${}^{(1)}F_{0\varphi}/r \equiv E_\varphi$:

$$\Box E_\varphi + \frac{1}{r^2} E_\varphi + \frac{\Omega}{Q} E_\varphi = AK^2HJ_1(Kr)\sin(\Omega t + \psi),$$

which, after expansion with respect to eigenfunctions, reduces to the form (26). The calculation of the change in the energy of the field in the detector resonator gives $\Delta W = 0.1\,(AQ)^2 W$, where $W = (H^2/8)R^2 l$.

Since the torus-shaped emitter is in the field of the gravitational radiation produced by itself, it can in principle also be used as a detector. Indeed, the emitted ingoing cylindrical wave is transformed, after it has passed through the symmetry axis, into an outgoing wave, which passes through the emitter. If an oscillation of frequency ω is excited in it, the frequency of the created gravitational wave is 2ω, and its interaction with the field of the emitter leads to the appearance of additional fields at the frequencies $2\omega - \omega = \omega$ and $2\omega + \omega = 3\omega$. If the size of the emitter is specially chosen, one can arrange that 3ω also be a natural frequency of the resonator. In this case, one would have resonant excitation of oscillations at this frequency. At the frequency ω, the original and the additional oscillations are added if the phase relationships are appropriate, which can always be achieved by the choice of R_1.

Let us now substitute actual values of the parameters of the system that, on the one hand, ensure matching of the natural frequencies of the emitter and the detector ($2\omega = \Omega$) and, on the other, leave the expressions we have derived correct in order of magnitude. Suppose $R_1 = 2\lambda$, $R_2 = 7\lambda/2$, $R \approx 2\lambda/3$, $L \approx l \approx \lambda$, where $\lambda = 2\pi c/\Omega$. Thus, the dimensions of the emitter and the detector and also the distance between them are comparable with the wavelength λ. Under our assumptions, $A \approx (G/c)E^2\lambda^2$. We take the detection condition to be $\Delta W \approx \hbar\Omega$. It has the form $AQ\sqrt{H^2c^3/\hbar\Omega^4} \approx 1$ (cf. (28)). Or, finally,

$$\lambda^4 E^2 HQ \approx 30\,\frac{c^4}{G}\sqrt{\hbar c}. \tag{35}$$

This equation relates the field strengths in the resonators, the Q factor of the detector, and the wavelength, which ultimately determines the size of the complete system. It is assumed that the emitter operates for at least the time $\tau^* \sim Q/\Omega$ needed to detect the signal.

Of course, the relation (35) imposes exceptionally high requirements on the quantities in it. For example, for $E \sim H \sim 3\cdot 10^5$ G, $Q \sim 7\cdot 10^{13}$, $\lambda \sim 10^2$ cm, the left-hand side of (35) is smaller than the right by 4 orders of magnitude. (This discrepancy is approximately of the same order as that which exists in the case of the detection of radiation from double stars by means of devices capable of realization at the present time.) In order to satisfy (35), it is necessary to increase λ to 10^3 cm, or, at $\lambda \approx 10^2$ cm, to raise the product E^2HQ by the same 4 orders of magnitude.

As one more example, let us consider an accelerator of elementary particles of annular form with total volume $2\cdot 10^{10}$ cm^3 and mean radius 10^5 cm of the ring. Suppose that in the complete volume a constant field $5\cdot 10^4$ G is produced and an alternating field $3\cdot 10^2$ G. Then in the center of the ring, in the focal plane, an amplitude $h \sim 10^{-38}$ of the metric is achieved. (A compact system with the same fields and total volume increases h by a further two orders of magnitude.) A coherently operating detector placed in the focal volume with total volume 10^9 cm^3, field $3\cdot 10^5$ G, and Q factor $3\cdot 10^9$, is capable of detecting $h_{\min} \sim 10^{-30}$ in a time $3\cdot 10^5$ sec (of course, the emitter must work for as long). The elimination of the remaining difference requires a significant improvement in the relevant parameters.

It is clear that the realization of an experiment in which gravitational waves are emitted and detected requires one to overcome tremendous difficulties, but it is undoubted that such an experiment will lead to a fundamental extension of our knowledge of nature and in the future possibly to the use of gravitational waves for practical ends.

7. BLACKBODY GRAVITONS AND THEIR DETECTION

It is well known that the Earth moves through electromagnetic radiation emitted by localized astronomical sources during comparatively recent (on a cosmological scale) times, and also the isotropic microwave background, which is a relic of the primordial plasma in the distant past of our universe. The background electromagnetic radiation has an equilibrium Planck spectrum with temperature $T = 2.7\,^\circ$K. This spectrum, even if it did not exist "from the very start," could perfectly well have been formed during the prolonged period of intense interaction between the primordial photons and matter.

This situation as regards gravitational radiation is similar to the extent that the Earth probably passes through gravitational waves produced by individual astronomical objects as well as an isotropic background gravitational radiation of primordial origin. It would be natural to assume that it resembles the microwave electromagnetic radiation and has a Planck spectrum; then in the framework of ordinary ideas, we can estimate its temperature to be (1–2) °K.[26] However, it is extremely important that, if there existed mechanisms for forming primordial gravitons with nonequilibrium spectrum, then because of the very weak interaction of gravitons with matter[26a,49] this spectrum must have persisted unchanged to the present epoch, except, perhaps, for only the short-wave region of the spectrum (waves with wavelength shorter than fractions of a centimeter). It is remarkable that the production of gravitons with nonequilibrium spectrum must occur by virtue of the mechanism of superadiabatic enhancement of gravitational waves and spontaneous creation of gravitons in the gravitational field of the Metagalaxy during the earliest stages of its evolution.[50] This mechanism already operates under the simplest assumptions contained in the standard cosmological model, i.e., nonstationarity and isotropy of the smoothed (background) gravitational field of the Metagalaxy. A gravitational field of more complicted nature leads to a stronger manifestation of this effect, but does not change it. (The only possible exceptional case in which the mechanism does not work will be pointed out below.)

Let us set forth the main principles of this mechanism. The gravitational field of a nonstationary isotropic universe (with, for simplicity, flat three-dimensional space) is described by the metric

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu = a^2(\eta)\,(d\eta^2 - dx^2 - dy^2 - dz^2).$$

Weak gravitational fields on this background, i.e., the corrections $h_{\mu\nu}$ to the metric $g_{\mu\nu}$, can be made to satisfy the complete set of gauge conditions (5′), (7′), and (8). After this, Einstein's equations in the linear approximation reduce to the wave equation

$$h_i^{k''} + 2\frac{a'}{a} h_i^{k'} - a^2 g^{lm} h^k_{i,l,m} = 0, \tag{36}$$

where the prime denotes the derivative with respect to η and the comma the derivatives with respect to the spatial coordinates. Following[51], we can represent the wave corrections to the metric in the form of the sum of terms $h_i^k = (\mu/a) G_i^k$, where G_i^k is tensor eigenfunction of number n of the Laplace operator formed from the metric $dl^2 = dx^2 + dy^2 + dz^2$. Then from (36) we obtain

$$\mu'' + \mu\left(n^2 - \frac{a''}{a}\right) = 0.$$

The effective potential $U(\eta) = a''/a$ distinguishes this equation from the ordinary wave equation in a Minkowski world. Note that $U(\eta) = 0$ not only for $a = \text{const}$, which corresponds to a flat background universe, but also in the unique exceptional case $a = a_0\eta$. The fact that $U(\eta) \neq 0$ is a manifestation of the so-called conformal noninvariance of gravitational wave equations.

To solve (36), we use a modification of Lagrange's method, similar to what is done in[52]. Consider some component h (we omit the indices) of a monochromatic wave field that depends on η and x. We seek a solution of (36) in the form

$$h = \frac{A(\eta)}{a} e^{-in(\eta - x)} + \frac{B(\eta)}{a} e^{in(\eta + x)}$$

with the additional condition $A' e^{-in\eta} + B' e^{in\eta} = 0$. We obtain the two first-order equations

$$A' = \frac{1}{2n} i \frac{a''}{a}(A + Be^{2in\eta}), \quad B' = -\frac{1}{2n} i \frac{a''}{a}(B + Ae^{-2in\eta}) \tag{37}$$

and their consequence $|A|^2 - |B|^2 = \text{const}$. Suppose that as $\eta \to -\infty$ and $\eta \to +\infty$ the value of a tends to the constants a_1 and a_2, respectively, and that a varies fairly smoothly in between. As $\eta \to -\infty$ and $\eta \to +\infty$, A and B tend to constant values A_1, B_2 and A_2, B_2 and $|A_1|^2 - |B_1|^2 = |A_2|^2 - |B_2|^2$. The characteristic time of variation of the background metric $\theta = a/a'$ as $\eta \to \pm\infty$ is much greater than the wave period $T = 2\pi/n$, and we have here short high-frequency waves with adiabatically varying amplitude $h \sim \text{const}/a$.[51]

As can be seen from (37), A and B are strictly constant only for $a'' = 0$. But if $a'' \neq 0$ and as $\eta \to -\infty$ there is specified a traveling wave of only one direction (for example, $A_1 \neq 0$, $B_1 = 0$), then in what follows its A amplitude increases and, in addition, there appears a wave of the opposite direction—that is, a B amplitude. As a result, as $\eta \to +\infty$ we obtain the original wave amplified in comparison with the adiabatic law: $|A_2|^2 - |A_1|^2 = |B_2|^2 > 0$ and a generated wave in the opposite direction with amplitude $|B_2|^2$. If a standing wave exists as $\eta \to -\infty$, $|A_1|^2 = |B_1|^2$, then as $\eta \to +\infty$ it remains a standing wave, $|A_2|^2 = |B_2|^2$, and its amplification or attenuation depends on the initial phase. After averaging over the phase, amplification of the wave is always obtained. The quantum process of spontaneous creation of particles is precisely the result of amplification of the initial vacuum fluctuations. The initial amplitude of the corresponding classical wave with frequency Ω ($\Omega = nc/a_1$) can be found from the condition that the contribution of this wave to the field energy in the volume $\lambda^3 = (2\pi c/\Omega)^3$ be equal to $\hbar\Omega/2$. Subtracting the energy of the zero-point oscillations at the end of the process, i.e., as $\eta \to +\infty$, we obtain the spectrum and intensity of the created waves (particles).

In principle, superadiabatic amplification of the wave occurs for any law of variation $a(\eta)$ (except the case $a'' = 0$), but significant superadiabatic amplifications (by several times or more) of the wave occurs only when there are such rapid variations of the background metric that $\theta \lesssim T$. This condition is always satisfied near the singularity in Friedmann cosmological models. A universe filled with matter with the equation of state $p = q\varepsilon$ ($0 \leqslant q \leqslant 1$) has the scale factor $a(\eta) = a_0\eta^{2/(3q+1)}$, and for any n there exists η such that $n\eta \lesssim 1$. To a certain η_1 there corresponds the so-called Planck time $t_{Pl} = \sqrt{G\hbar/c^5} \approx 5 \cdot 10^{-44}$ sec, which establishes the lower limit of applicability of modern gravitational theory. If at the time η_1 the initial spectrum $h_1(n)$ is specified by appropriately defined amplitudes of the gravitational waves, then by the contemporary epoch η_2 it is transformed into the spectrum $h_2(n)$. After averaging over the initial phase, the connection is expressed by $h_2(n) = V(n\eta_1, q)\, h_1(n)$.

The function V has the following properties. For $q = 1/3$, which corresponds to $a = a_0\eta$, $V \equiv 1$ for all n, and the waves change strictly in accordance with the adiabatic law. For n such that $n\eta_1 \to \infty$, $V \to 1$ for all q. For $n\eta_1 \ll 1$ and $|(1-3q)/(1+3q)| \ll n\eta_1$, $V - 1$ in accordance with the law $V - 1 = (1/4)[(1-3q)/(1+3q)]^2/(n\eta_1)^2$. Finally, for $n\eta_1 \ll 1$ and q not too near $1/3$, $V \sim (n\eta_1)^{-2/(1+3q)}$. Thus, the initial vacuum spectrum $h_1(n) \sim n$ in the range $n\eta_1 \ll 1$ is transformed into a power spectrum of some form, which depends on q (for more details on the spectrum and energy density of primordial gravitons, see[59]). In the region $n\eta_1 \gg 1$, the spectrum falls rapidly. In the region of small n, it is meaningful in any case to consider only $n \gg 1/\eta_2$ since $n\eta_2 \sim 1$ corresponds to perturbations with characteristic length of the order of the distance to the contemporary horizon ($l \sim 3 \cdot 10^{28}$ cm). The critical value $n_c \sim 1/\eta_1$ corresponds in the contemporary epoch to waves with a length of tenths or hundredths of a centimeter, $\lambda_c = 2\pi a(\eta_2)/n_c$. In other words, the Planck length $l_{Pl} = ct_{Pl}$ increases to these values during the time of expansion from t_{Pl} to the present time. It is curious that the maximum of the electromagnetic background radiation is in precisely this range.

If the scale factor $a(\eta)$ passed through a minimum at Planck densities or lower (for example, because of a modification of Einstein's equations[53,54] or the existence of a fundamental length[55,56]), the spectrum of the created gravitons will have a turnover at correspondingly longer wavelengths. In addition, in its low-frequency range the spectrum will contain amplified gravitational waves of the contraction epoch. The low frequency part of the spectrum of gravitons created on the transition from contraction to expansion in accordance with a power law is calculated in[57].

It should not be thought that in an isotropic nonstationary universe wave fields of any kind are amplified. Quite the opposite: of the wave fields corresponding to the known massless particles only gravitational waves have this property. Free electromagnetic waves always change in accordance with the adiabatic law in an isotropic universe. In particular, if the evolution of the scale factor $a(\eta)$ begins with constant value a_0 and ends with it, then however complicated the behavior of a in the intermediate region, the amplitude of an electromagnetic wave and the energy density at the beginning and end of the evolution are equal. The mathematical reason for this difference is to be found in the conformal invariance of Maxwell's equations and the conformal noninvariance of the gravitational-wave equations deduced from Einstein's equations.

Let us now turn to the possibilities for detecting isotropic background gravitational radiation.[30b] As we have seen, it must now exist in the form of gravitational-wave noise in a very broad spectrum. Let us represent the spectral density of the perturbations of the metric for $\nu < \nu_c$ by the power law $(h^2)_\nu = H\nu^{-\gamma}$, where $\nu_c = c/\lambda_c \approx 10^{11}$ Hz. The various values of the exponent γ correspond to the predictions for the spectrum under various models of the singular state and the passage through the singularity. The spectral density of the flux F_ν is related to $(h^2)_\nu$ by the equation $F_\nu \approx (c^3/G)\nu^2(h^2)_\nu$. The energy density ε_g concentrated in a definite frequency range is obtained by integrating F_ν/c over the frequencies.

At the present time, several indirect bounds on ε_g are known.[26a] They use additional arguments[4] that do not have rigor but are nevertheless plausible. The strongest restriction is obtained by considering nucleosynthesis in the early universe[58] and is to the effect that ε_g for waves with $\lambda < \lambda_m = 3 \cdot 10^{17}$ cm ($\nu_m \approx 10^{-7}$ Hz) cannot appreciably exceed the energy density of the electromagnetic microwave background radiation: $\varepsilon \approx 4 \cdot 10^{-13}$ erg/cm³. We shall take the rather strong bound

$$\varepsilon_g = \frac{1}{c}\int_{\nu_m}^{\nu_c} F_\nu \, d\nu \approx 10^{-12} \text{ erg/cm}^3,$$

although the available direct cosmological observations would not contradict an ε_g that is 3–4 orders of magnitude greater.

The feasibility of an experiment to detect primordial gravitons depends strongly on γ. For $\gamma > 0$, the effective temperature $T_{eff} = c^2 F_\nu / 2k\nu^2$ at low frequencies appreciably exceeds the equilibrium temperature, which facilitates detection of the radiation. Since the natural frequencies of the existing solid-state antennas lie in the range $\nu_0 \sim 10^3$ Hz, i.e., at approximately the center of the range between ν_m and ν_c, the most favorable spectrum for detection has γ near 3. This spectrum corresponds to reasonable models of the initial state, though it is by no means necessary. Let us take $(h^2)_{\nu_0} \approx (G/c^2)\varepsilon_g/\nu_0^3$; then $\bar{h}^2(t, \nu_0 \pm \Delta\nu) \approx (h^2)_{\nu_0}\Delta\nu \approx (G/c^2)(\varepsilon_g/\nu_0^2)/Q$, and the detection condition (22) takes the form

$$\frac{\sqrt{n}\, G\varepsilon_g m l^2 Q}{c^2 kT} \gg \mu, \tag{38}$$

where μ is a combination of ignored numerical coefficients, which may reach values $\mu \sim 10$–10^2. The relation (38) imposes very high requirements on the experimental level, but they could probably be achieved by solid-state antennas of the following generation.[24]

Another promising possibility would be to use drag-free objects in space.[24] The point is that two space probes at a distance l from each other can acquire a relative velocity $v \sim l\sqrt{(G/c^2)\varepsilon_g}$ over a time $\Delta t \sim l/c$. If $l = 3 \cdot 10^{13}$ cm, we have $v \approx 2 \cdot 10^{-7}$ cm/sec, which could probably be measured by the technology of the near future.

There is no doubt that experiments to detect primordial gravitational radiation are very difficult, but the detection of this radiation or a direct restriction on the possible profile of its spectrum would give fundamental information about extremely early stages in the evolution of the Metagalaxy. Such information is important not only for astronomy, but also physics quite generally.

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